QCD and jet physics

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1st Lecture

European School of High Energy Physics – November 2022

Today's high energy colliders

Today's high energy physics program relies mainly on results from

Collider	Process	status
LEP/LEP2	e^+e^-	1989-2000
Hera	$e^\pm p$	1992-2007
Tevatron	pp	1983-2011
LHC- Run I	pp	2010-2012
LHC- Run II	pp	2015-2018

- LEP high precision measurements of masses, couplings, EW parameters ...
- Hera: mainly measurements of proton structure / parton densities
- Tevatron: mainly discovery of top and many QCD measurements
- LHC designed to
  - discover the Higgs [done in Run I]
  - unravel possible BSM physics [elusive up to now]

Today's status of particle physics

- The Higgs boson: the last missing ingredient of the Standard Model of particle physics
- The SM is a consistent theory up to very high energy, but it can not be a complete theory because of many theory and experimental puzzles (Dark matter and dark energy, neutrino masses, flavour physics, hierarchy problem, no theory of quantum gravity…)

The days of guaranteed discoveries or no-lose theorems are over. Progress will be driven by LHC data.

The LHC program in the coming decades will focus on

- Direct searches: production of new (heavy status) …
- Indirect searches: Higgs couplings and branchings, rare decay modes …
- Consistency tests: precision electroweak data

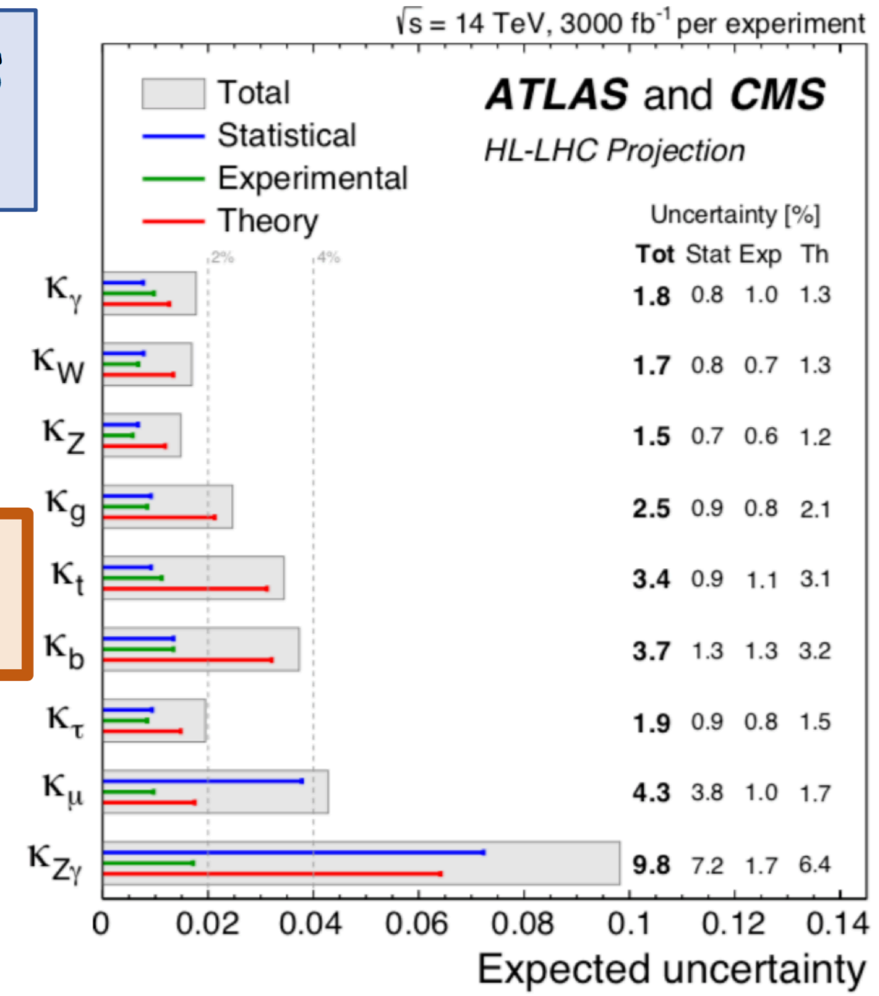
Today’s status of particle physics

Improving our theoretical description of data crucial to enhance sensitivity when looking for new physics.

<u>Example</u>: extraction of Higgs couplings limited by theory uncertainties

Higgs couplings after HL-LHC

Largest contribution to the uncertainty from theory

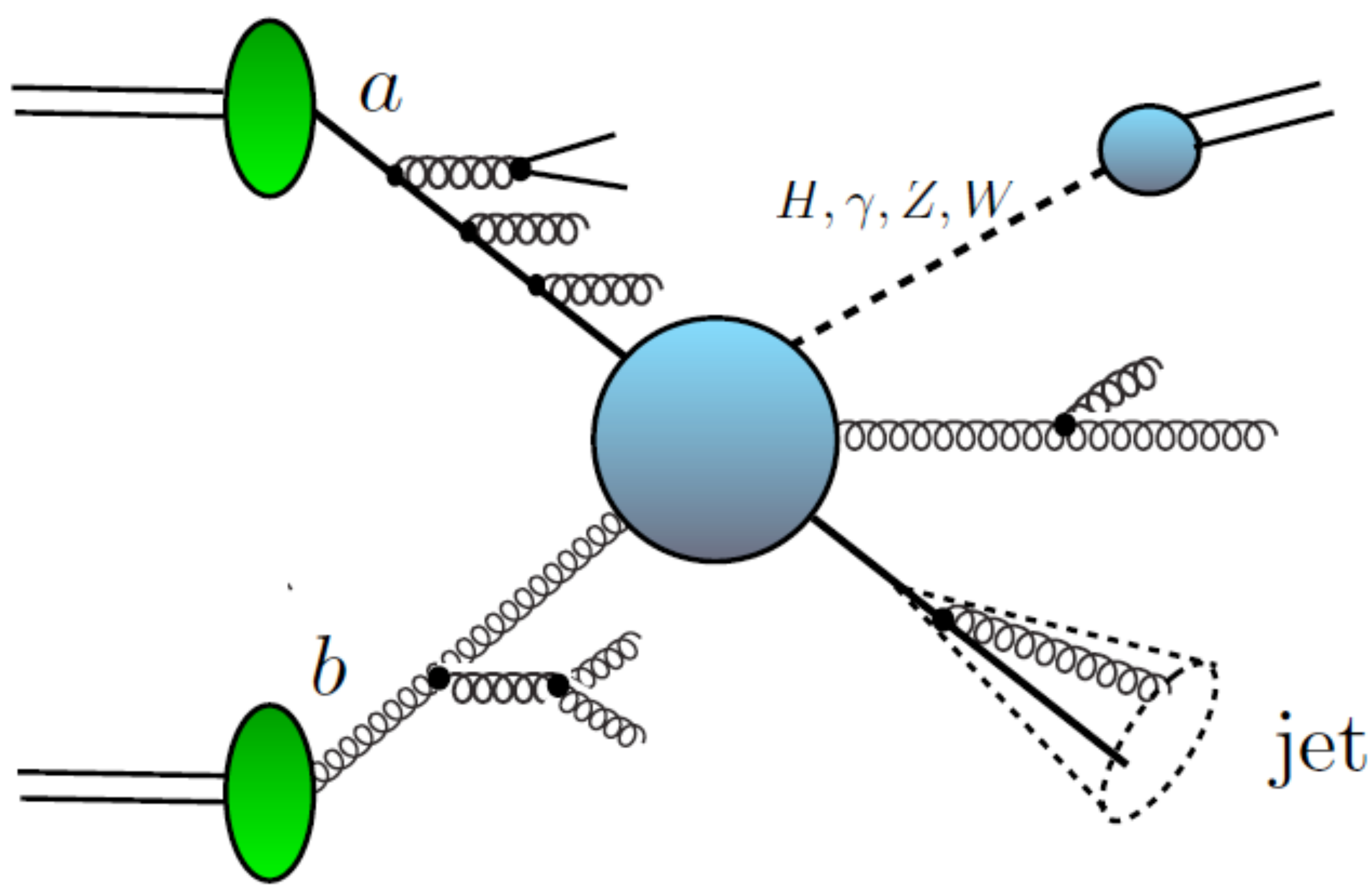


Sharpen the tools ⇒ get more accurate predictions ⇒ get improved indirect sensitivity to New Physics (which would modify SM couplings)

Propose of these lectures

In these lectures: perturbative QCD as a tool for precision QCD at colliders

- Introduction to QCD (or refresh your knowledge of QCD)
- Basic concepts which appear over and over in different contexts
- Understand the terminology
- Recent developments in the field

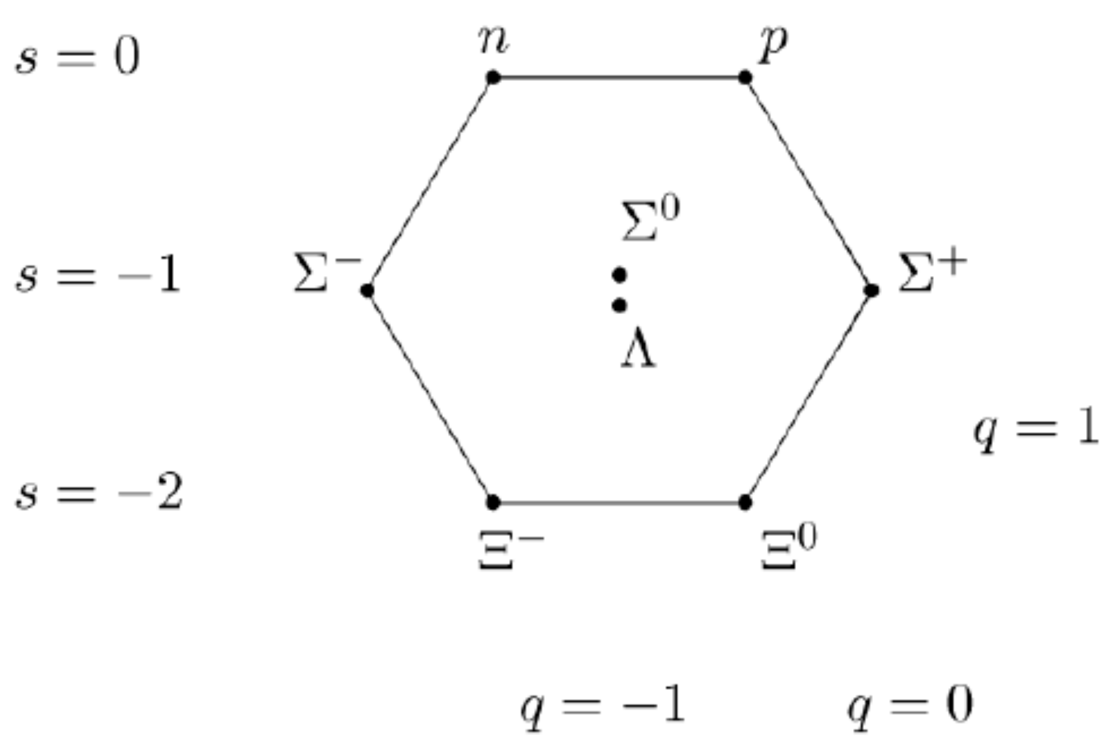


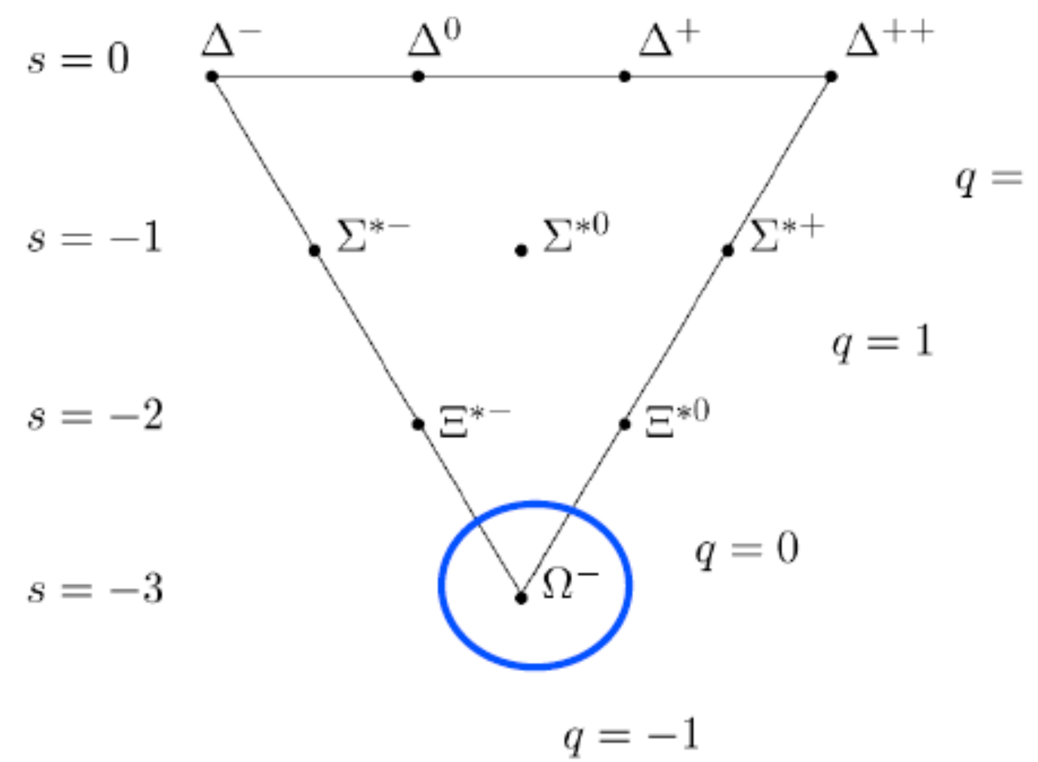
Some bibliography

- A QCD primer
  G. Altarelli (TASI lectures 2002) hep-ph/0204179
- QCD and Collider Physics (a.k.a. The Pink Book)
  R.K. Ellis, W. J. Stirling, B. R. Webber, Cambridge University Press (1999)
- Foundations of Quantum Chromodynamics
  T. Muta, World Scientific (1998)
- Quantum Chromodynamics: High Energy Experiments and Theory
  G. Dissertori, I. Knowles, M. Schmelling, International Series of Monograph on Physics (2009)
- The theory of quark and gluon interactions
  F. J. Yndurain, Springer-Verlag (1999)
- Gauge Theory and Elementary Particle Physics
  T. Cheng and L. Li, Oxford Science Publications (1984)
- The Black Book of Quantum Chromodynamics: A Primer for the LHC Era
  J. Campbell, J, Huston, F. Krauss (2018)
- many write-ups of QCD lectures given at previous other schools ...

The beginning: the eightfold way

Organise hadron spectrum to manifest some symmetry-pattern



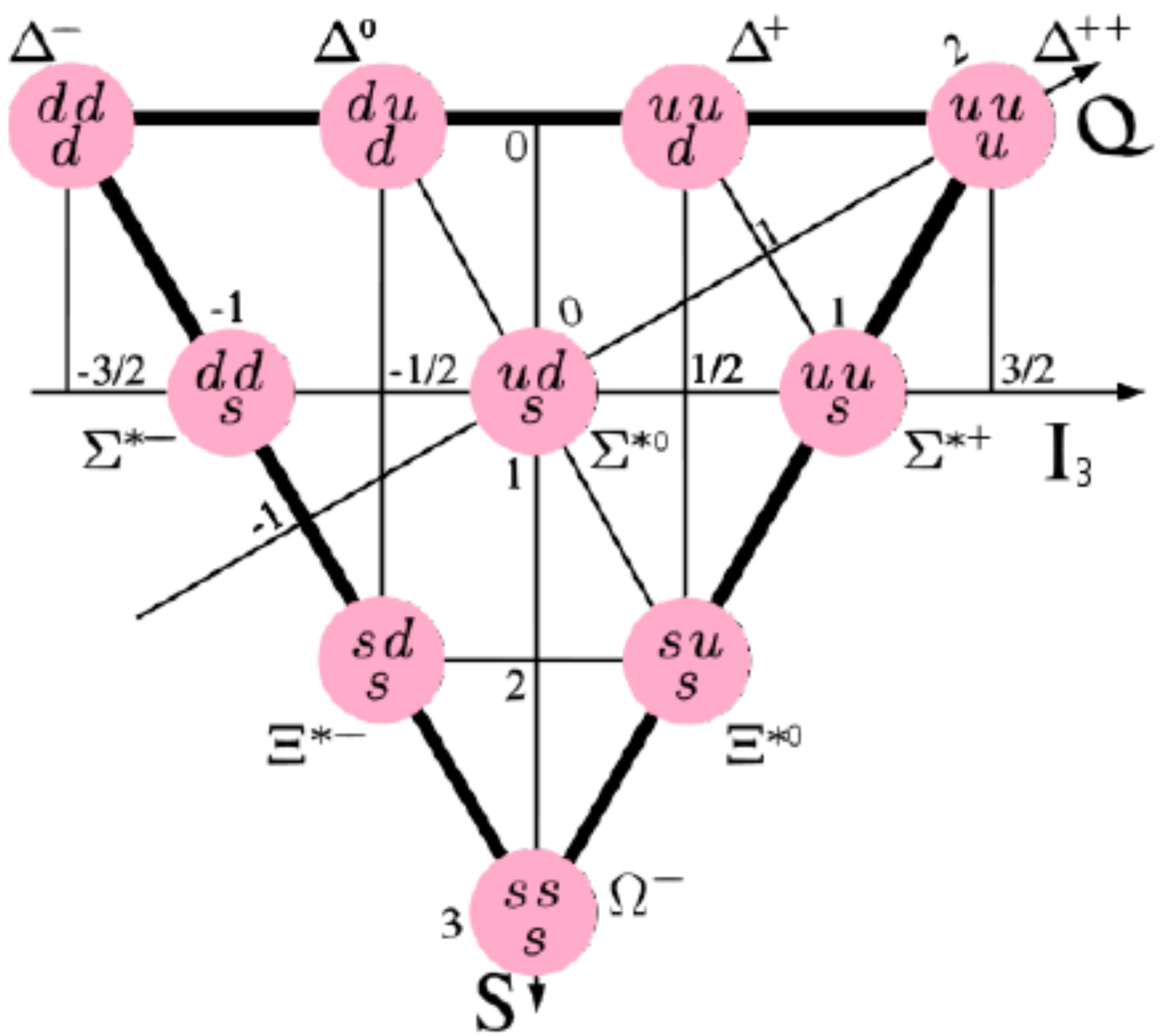


One hadron missing, but predicted following the pattern

But what is the reason for this pattern?

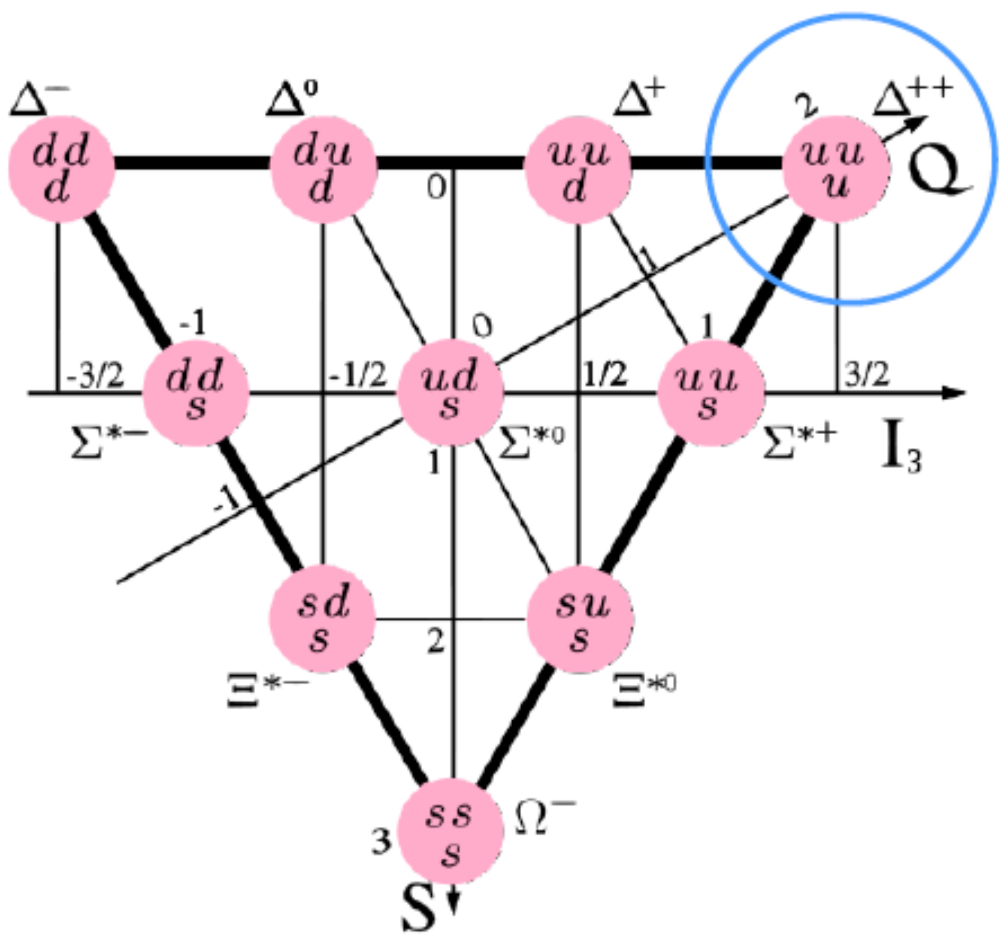
Quarks (1964)

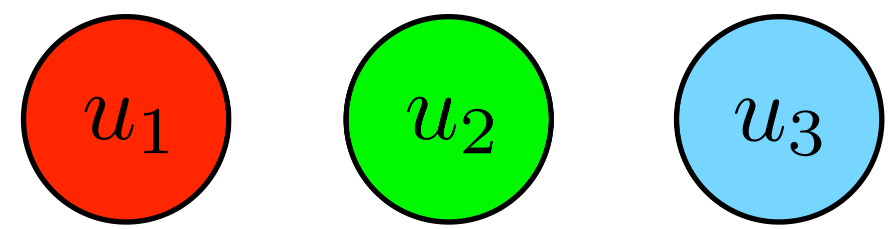
Gell-Mann and Zweig propose the existence of elementary spin 1/2 particles (the quarks). Three types of quarks, up, down, strange (plus their antiparticles) can explain the composition of all observed hadrons



First experimental evidence for colour

I. Existence of Δ^{++} particle: particle with three up quarks of the same spin and with symmetric spacial wave function. Without an additional quantum number Pauli's principle would be violated
⇒ color quantum number





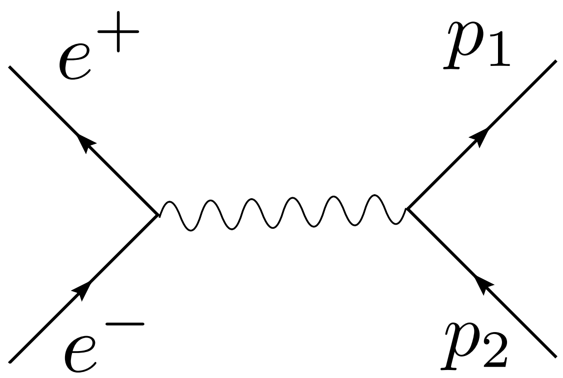
$$\Delta^{++} = \epsilon_{ijk} u_i u_j u_k$$

New quantum number solves spin-statistics problem (wave function becomes asymmetric)

First experimental evidence for colour

II. R-ratio: ratio of ($e^+e^- \rightarrow$ hadrons)/($e^+e^- \rightarrow \mu^+\mu^-$)

$$R \equiv \frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} \propto N_c \sum_f Q_f^2$$



Experimental data confirms $N_c = 3$. (Will come back to R later.)

Color becomes the charge of strong interaction. Interaction is so strong, that the only observed hadrons in nature are those where quarks are combined in colour singlet states

Colour SU(3) is an exact symmetry of nature

QCD

Model for strong interactions: non-abelian gauge theory SU(3)

$$U^\dagger U = U U^\dagger = 1 \quad \det(U) = 1$$

Hadron spectrum fully classified with the following assumptions

- hadrons (baryons, mesons): made of spin 1/2 quarks
- each quark of a given flavour comes in N_c=3 colors
- color SU(3) is an exact symmetry
- hadrons are colour neutral, i.e. colour singlet under SU(3)
- observed hadrons are colour neutral ⇒ hadrons have integer charge

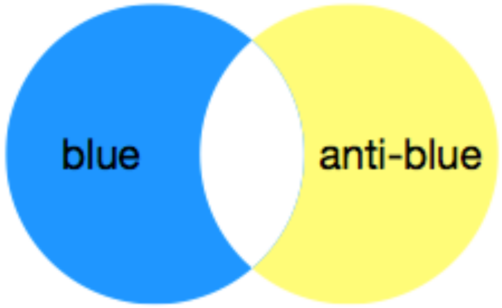
$$\begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$

Color singlet hadrons

Quarks can be combined in 2 elementary ways into color singlets of the $SU_c(3)$ group

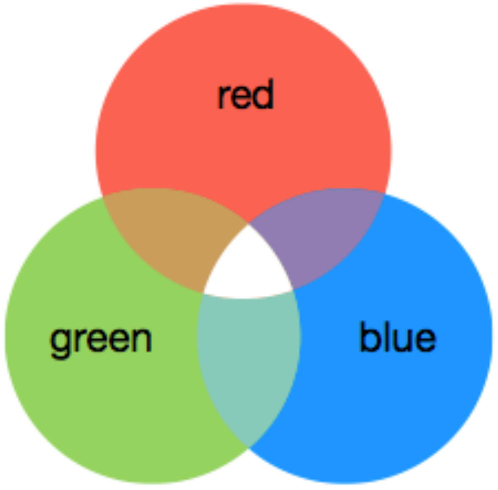
<u>Mesons</u> (bosons, e.g. pion ...)

$$\sum_i \psi_i^* \psi_i \rightarrow \sum_{ijk} U_{ij}^* U_{ik} \psi_j \psi_k = \sum_k \psi_k^* \psi_k$$

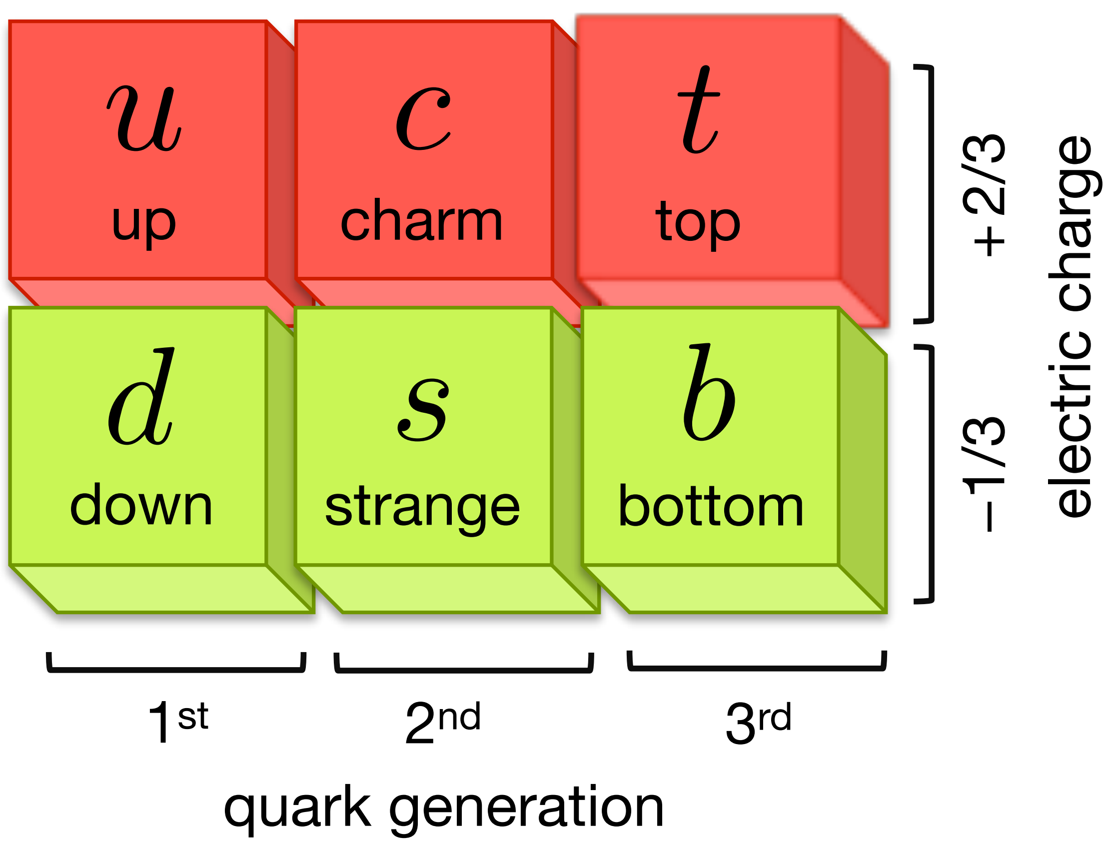


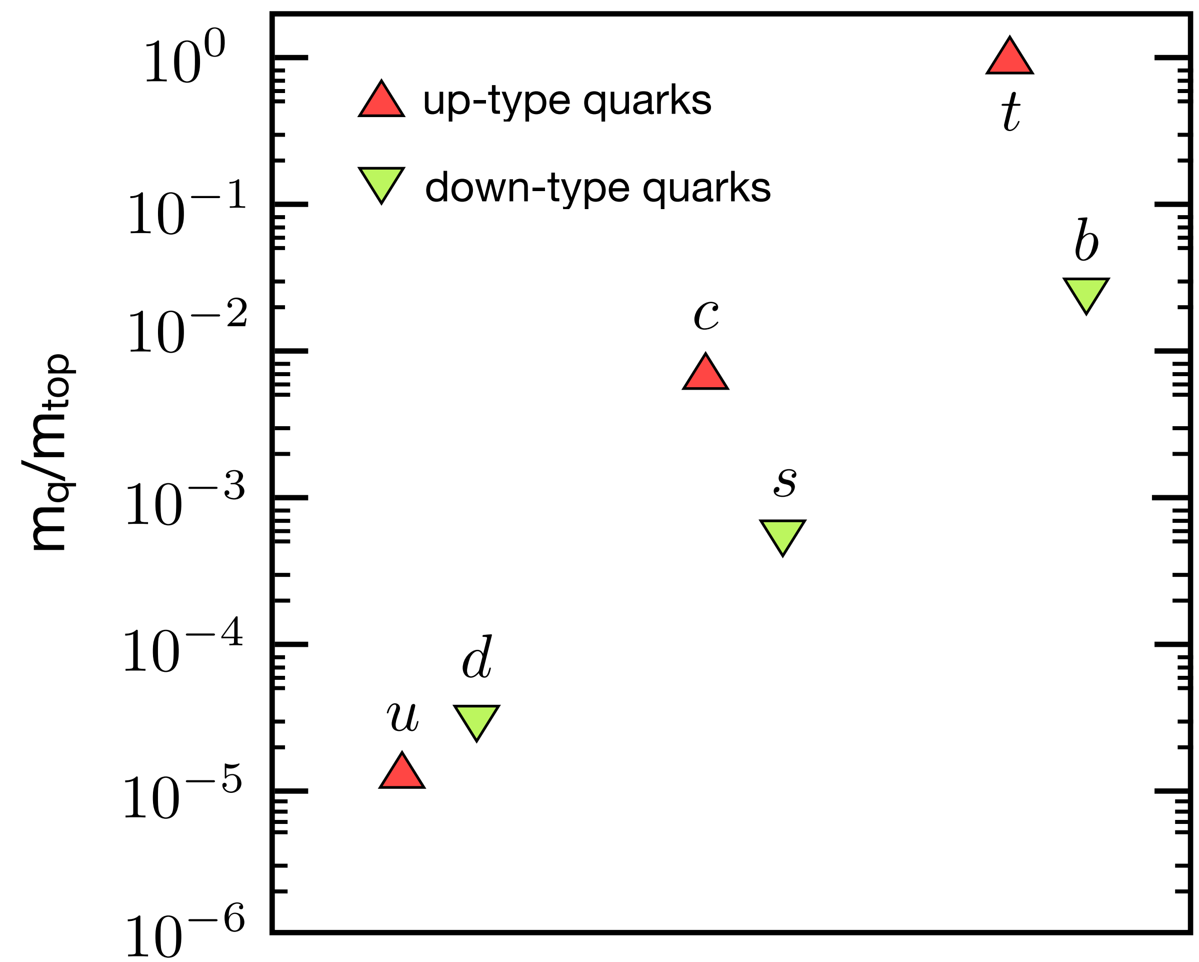
<u>Baryons</u> (fermions, e.g. proton, neutrons ...)

$$\sum_{ijk} \epsilon_{ijk} \psi_i \psi_j \psi_k \rightarrow \sum_{ii'jj'kk'} \epsilon_{ijk} U_{ii'} U_{jj'} U_{kk'} \psi_{i'} \psi_{j'} \psi_{k'} = \sum_{i'j'k'} \epsilon_{i'j'k'} \det(U) \psi_{i'} \psi_{j'} \psi_{k'}$$



Quark mass spectrum

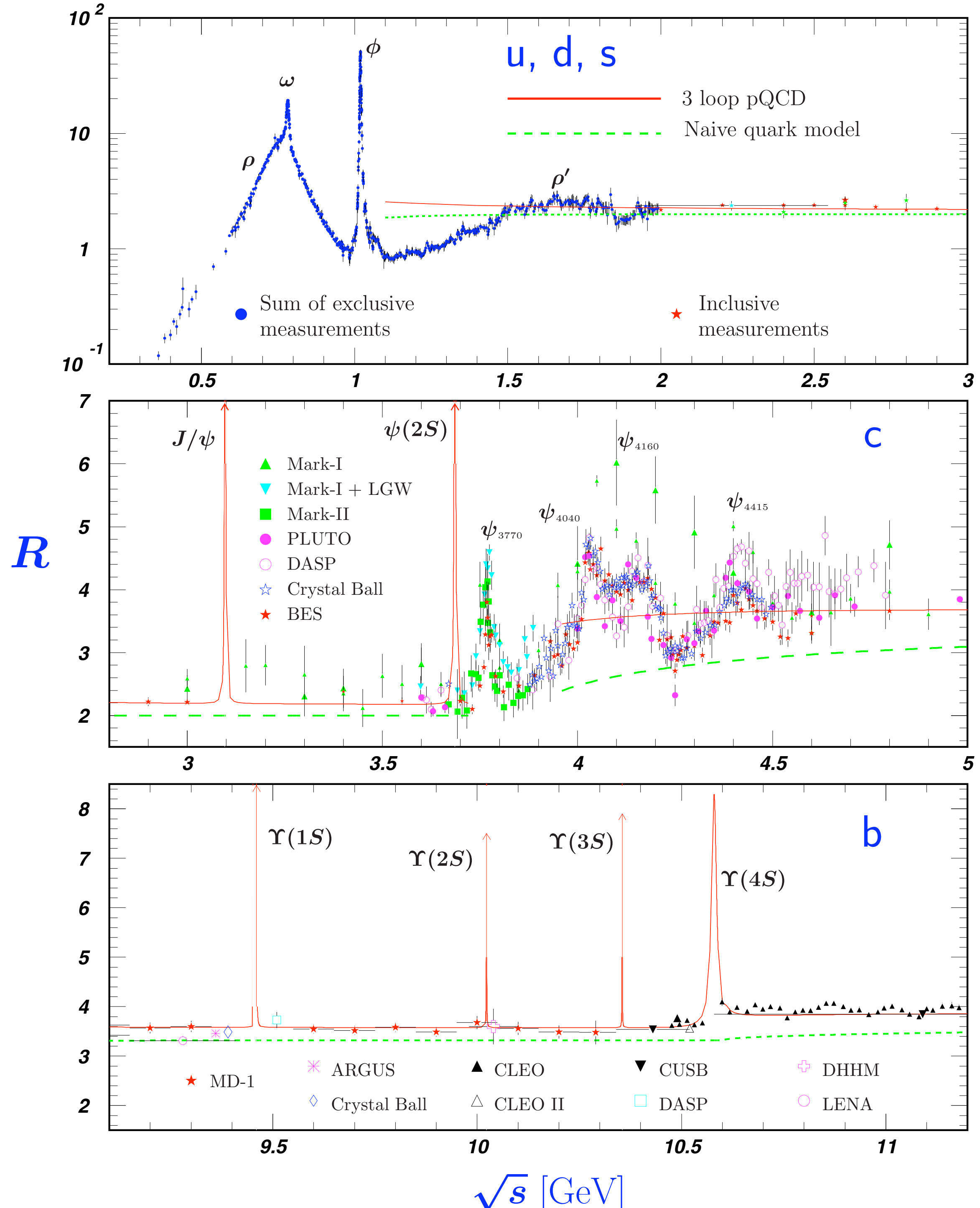




charge 2/3 mass=	up few MeV	charm ~1.6 GeV	top ~172 GeV
charge -1/3 mass =	down few MeV	strange ~100 MeV	bottom ~5 GeV

The R-ratio: comparison to data

R changes crossing quark flavour thresholds $R = N_c \sum_q e_q^2$

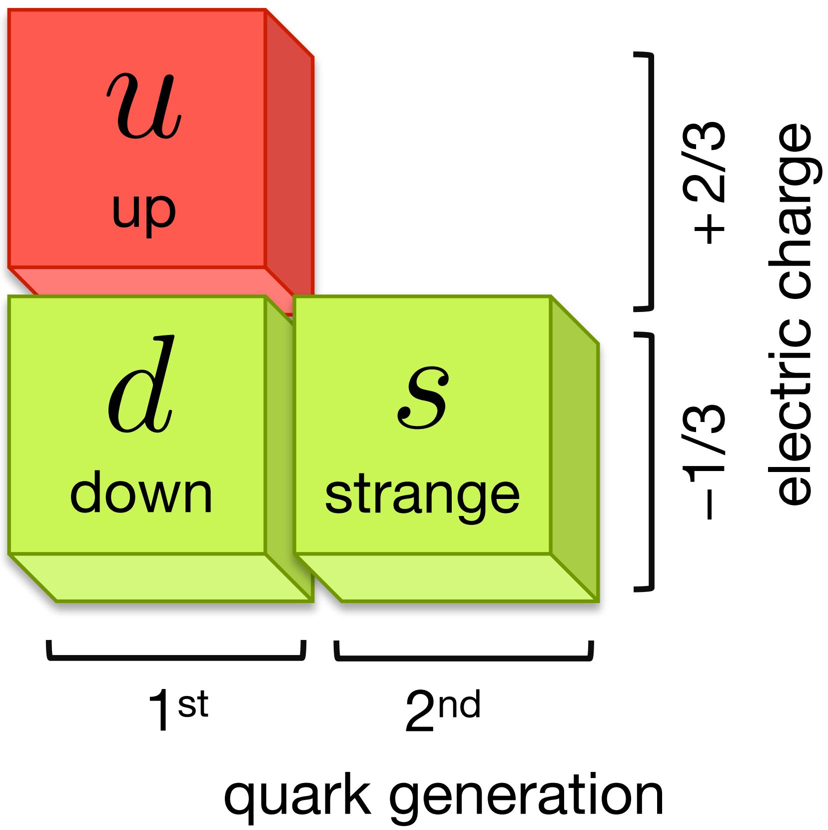


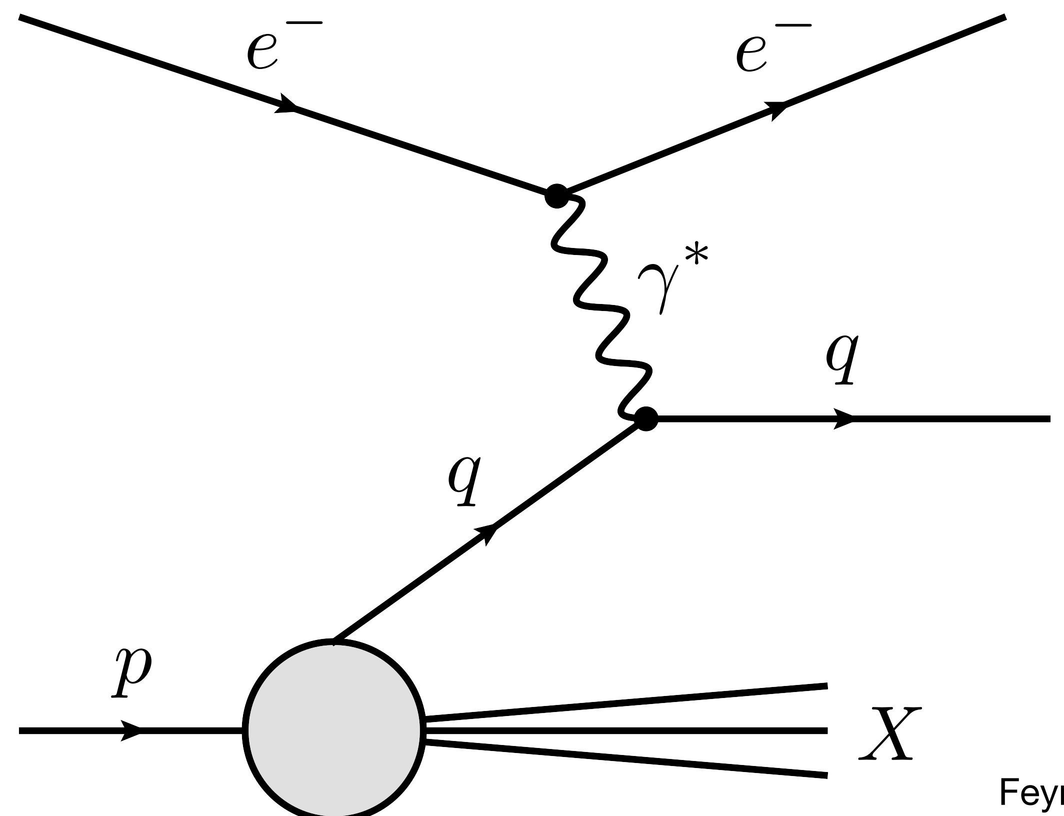
$n_f = 3$ $R = 2$

$n_f = 3 \rightarrow 4$ $R = \frac{10}{3}$

$n_f = 4 \rightarrow 5$ $R = \frac{11}{3}$

QCD matter sector

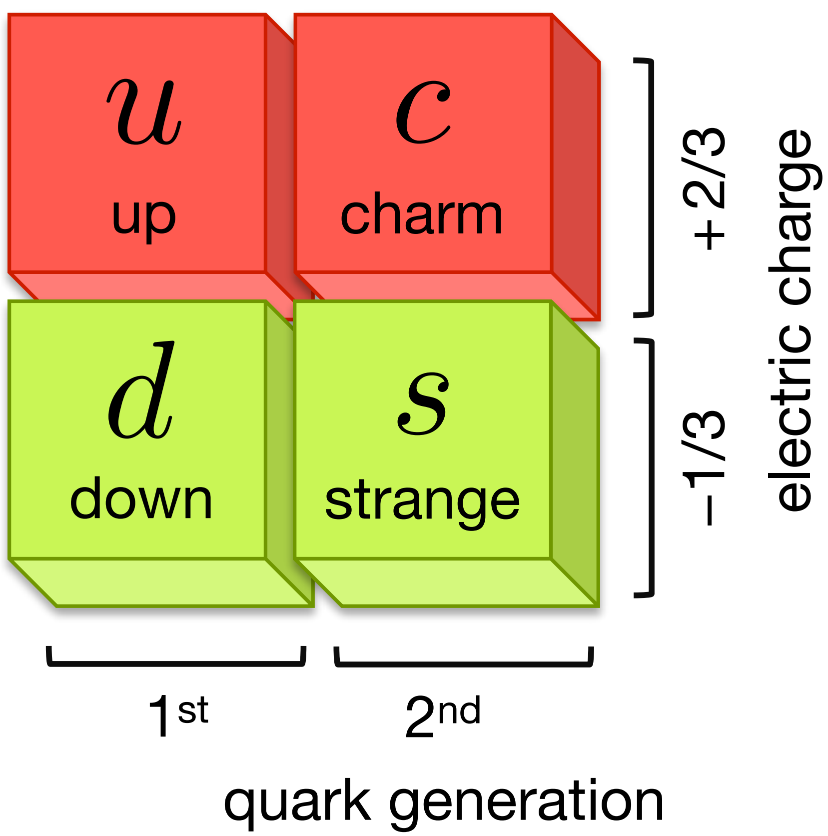




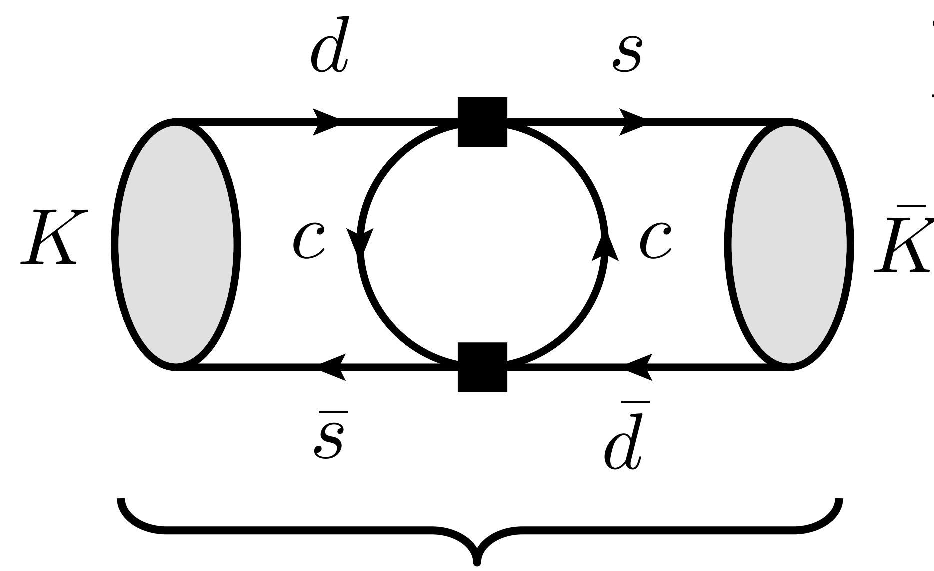
Feynman diagram describing DIS of an electron on a proton

- The light quark's existence was validated by the SLAC's deep inelastic scattering (DIS) experiments in 1968: strange was a necessary component of Gell-Mann and Zweig's three-quark model, it also provided an explanation for the kaon and pion mesons discovered 1947 in cosmic rays

QCD matter sector



Feynman diagram describing the mixing of a kaon into its anti-particle. The black boxes indicate weak effective four-fermion interactions

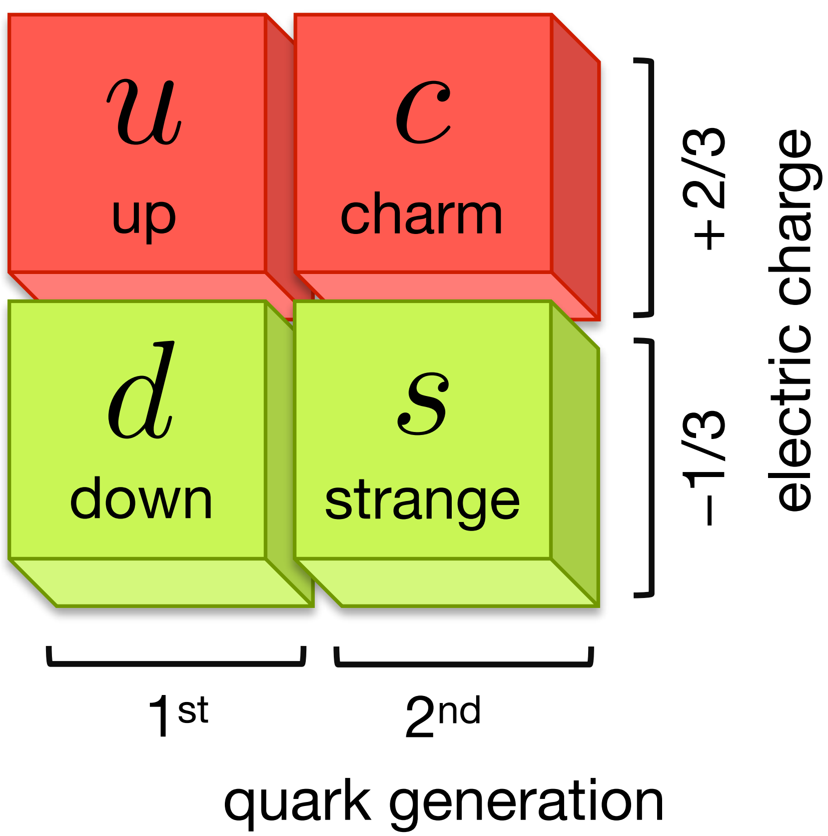


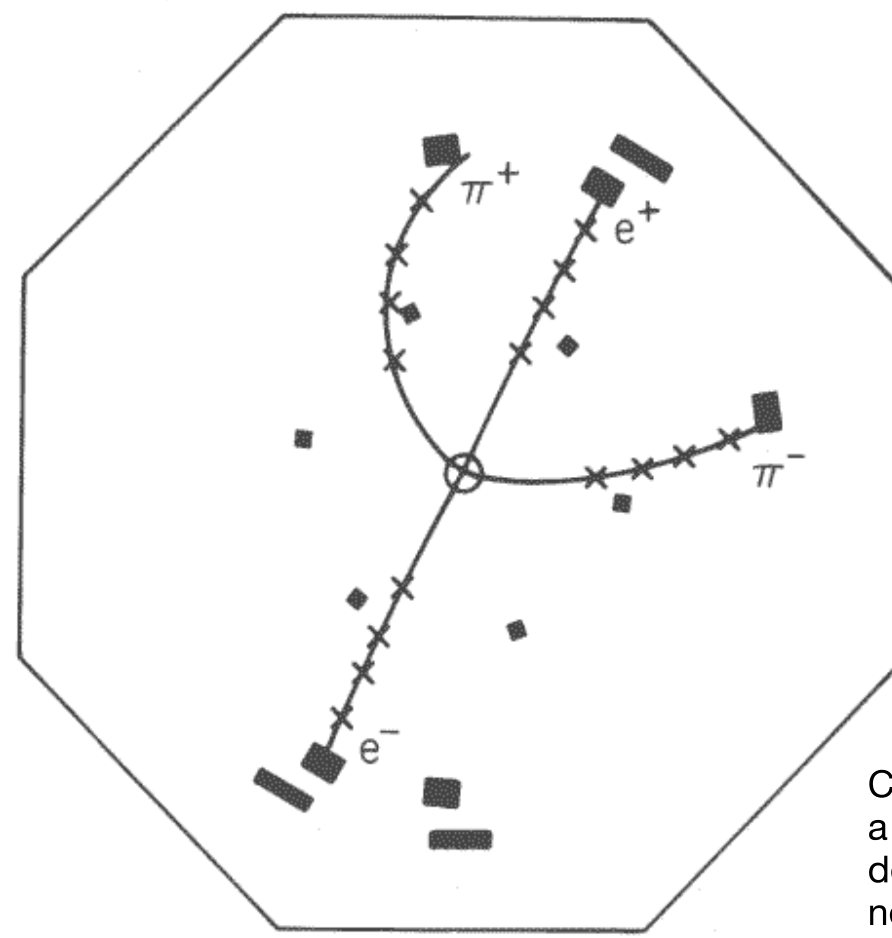
$$\propto G_F^2 \, \sin^2\theta_c \, \frac{m_c^2}{M_W^2}$$

- In 1970 Glashow, Iliopoulos, and Maiani (GIM mechanism) presented strong theoretical arguments for the existence of the as-yet undiscovered charm quark, based on the absence of flavour-changing neutral currents

[S. L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev.* D **2** (1970) 2]

QCD matter sector

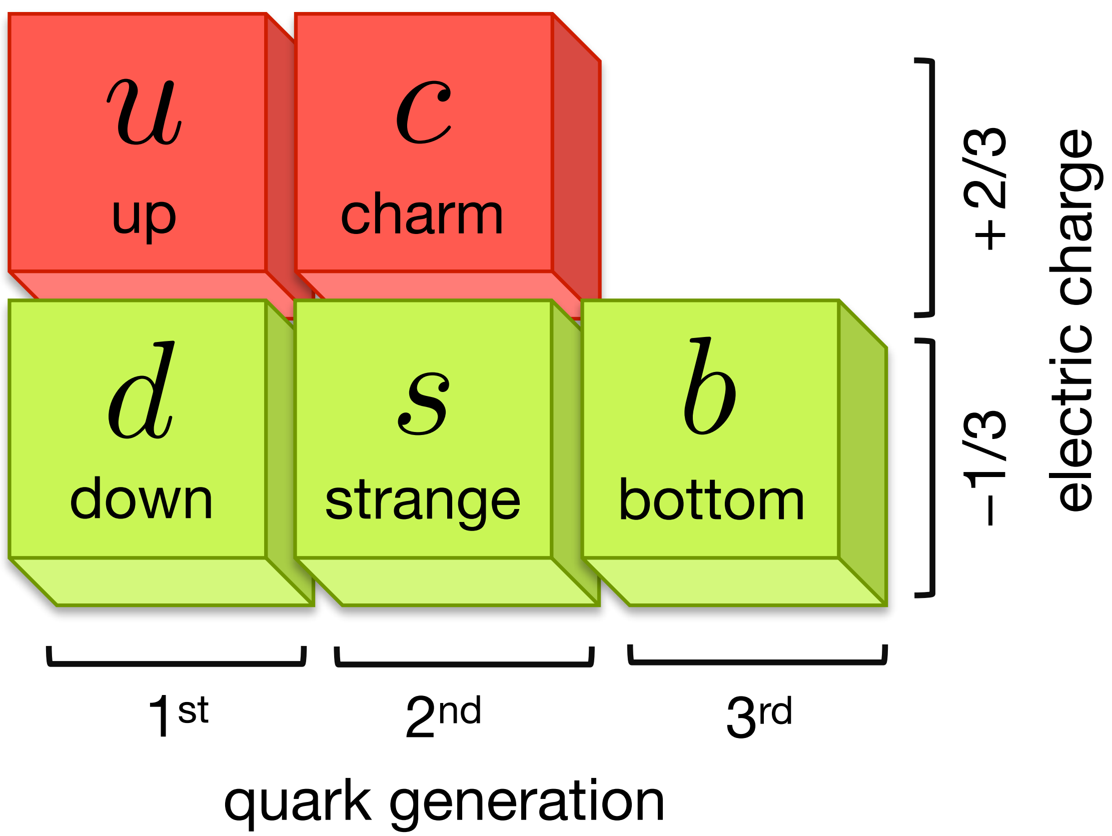


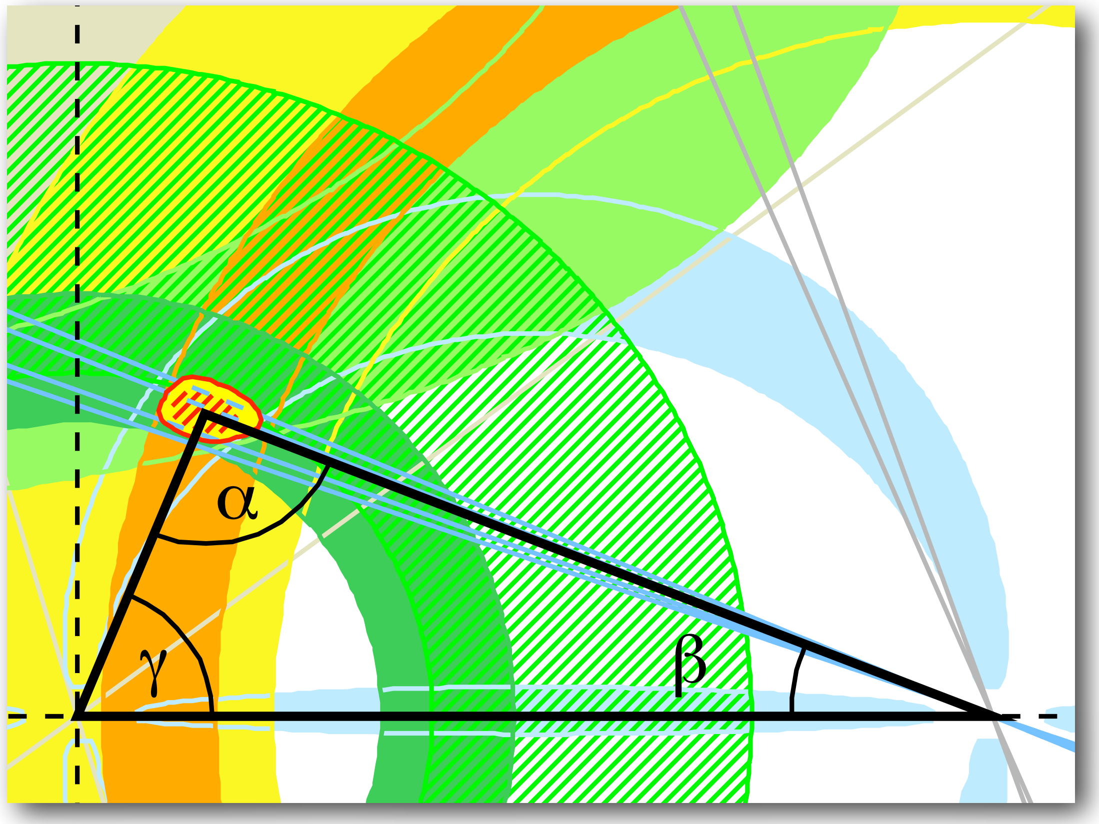


Computer reconstruction of a ψ′ decay in the Mark I detector at SLAC, making a near-perfect image of the Greek letter ψ

- Charm quarks were observed almost simultaneously in November 1974 at SLAC and at BNL as charm anti-charm bound states (charmonium). The two groups had assigned the discovered meson two different symbols, *J* and ψ. Thus, it became known as the *J*/ψ (Nobel Prize 1976)

QCD matter sector



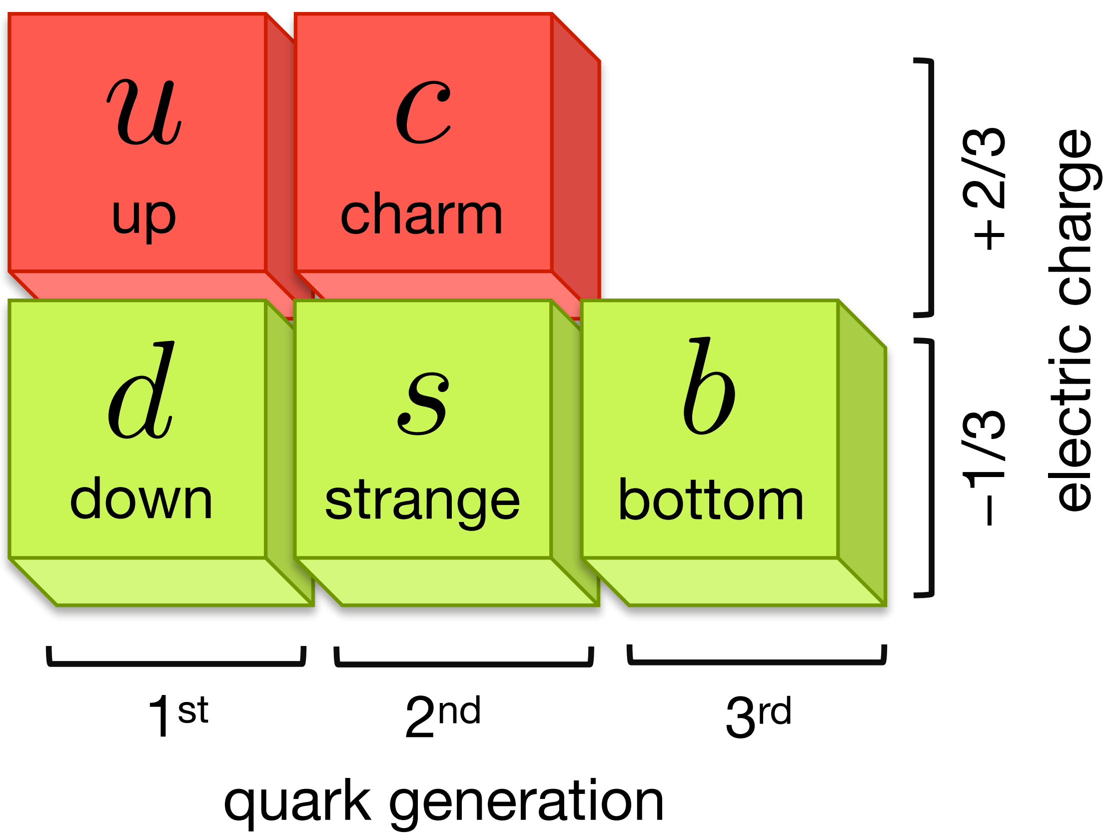


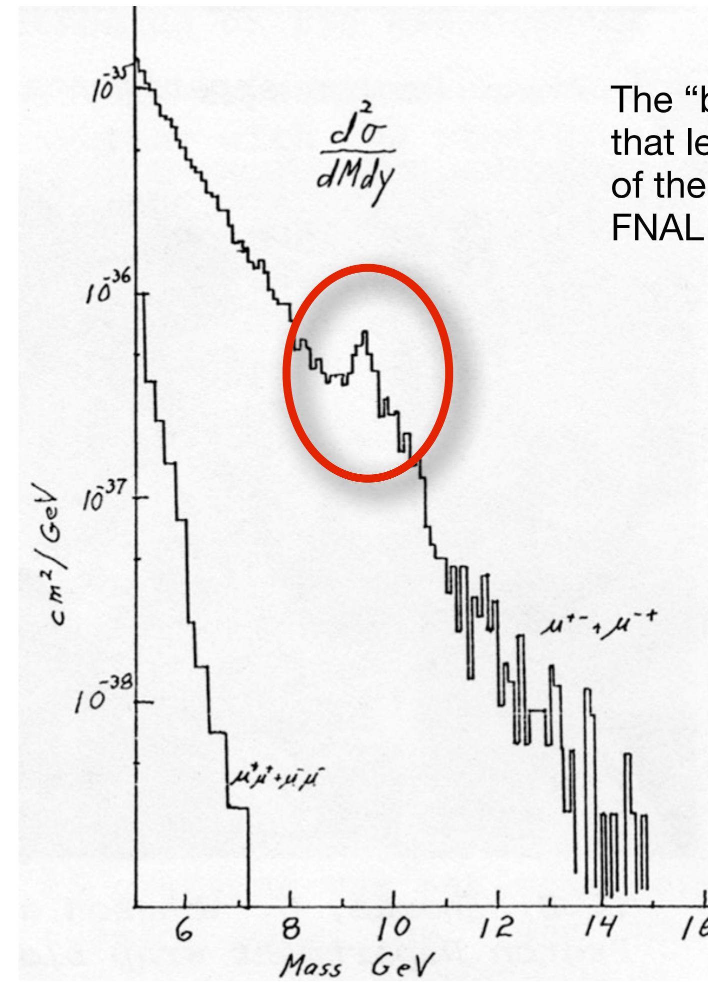
Unitarity triangle measuring the amount of CP violation in the Standard Model

- The bottom quark was postulated in 1973 by Kobayashi and Maskawa to describe the phenomenon of CP violation, which requires the existence of at least three generations of quarks in Nature (Nobel Prize 2008)

[M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652]

QCD matter sector





The "bump" at 9.5 GeV that lead to the discovery of the bottom quark at FNAL in 1977

- In 1977, physicists working at the fixed target experiment E288 at FNAL discovered Υ (Upsilon) meson. This discovery was eventually understood as the bound state of the bottom/anti-bottom (bottomonium)

QCD matter sector

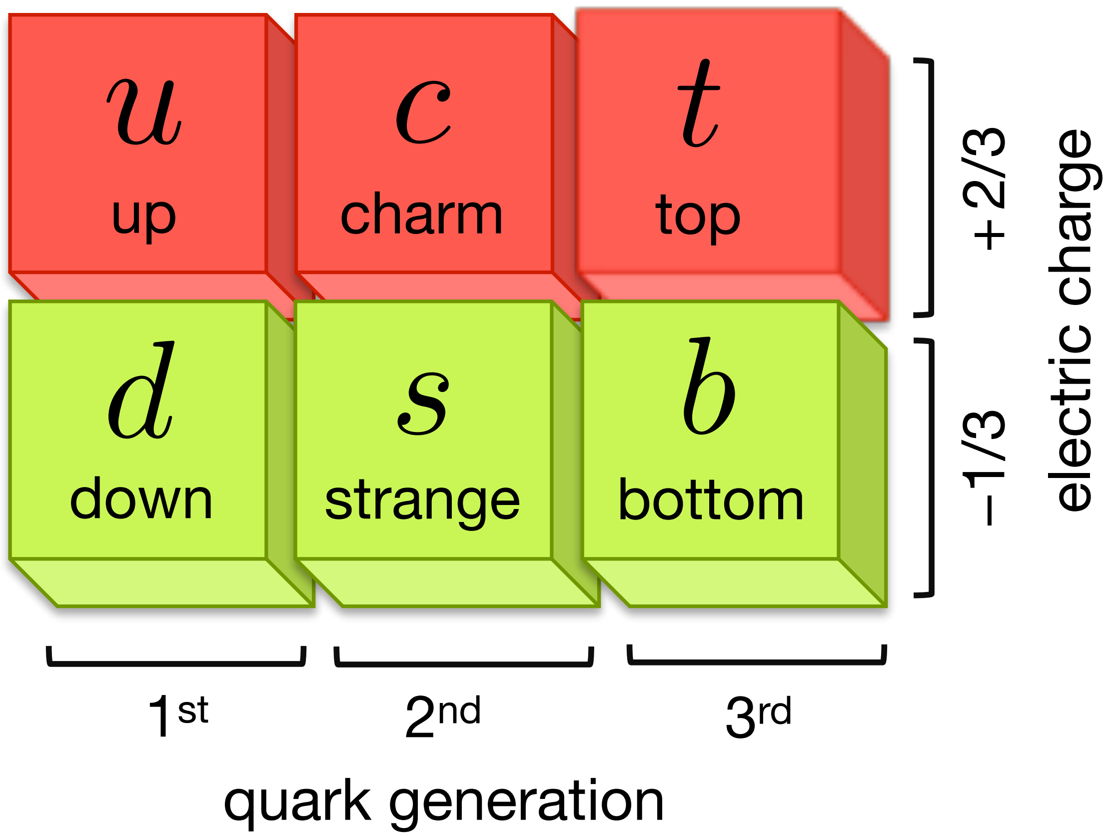
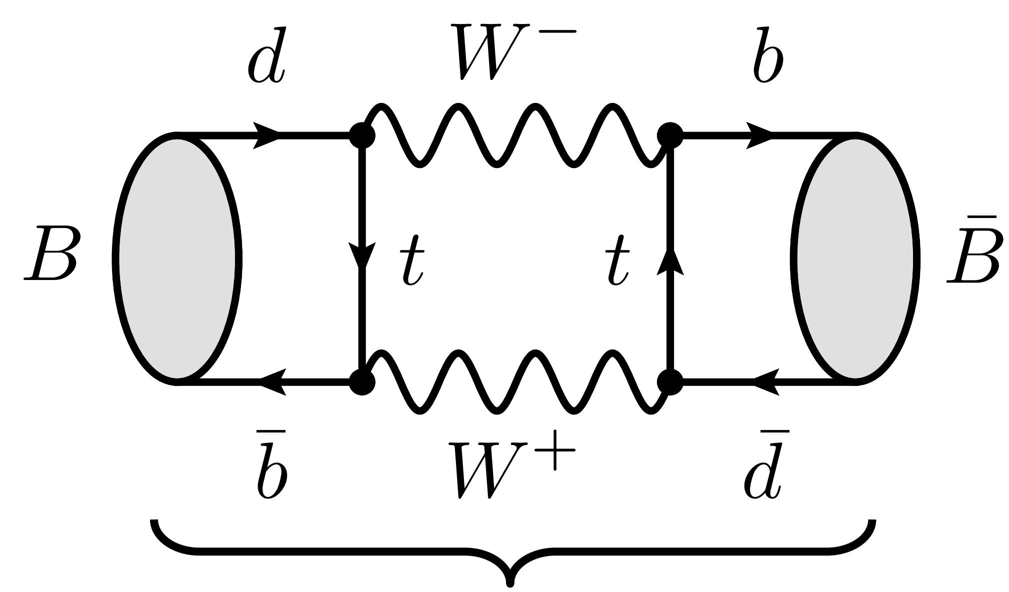


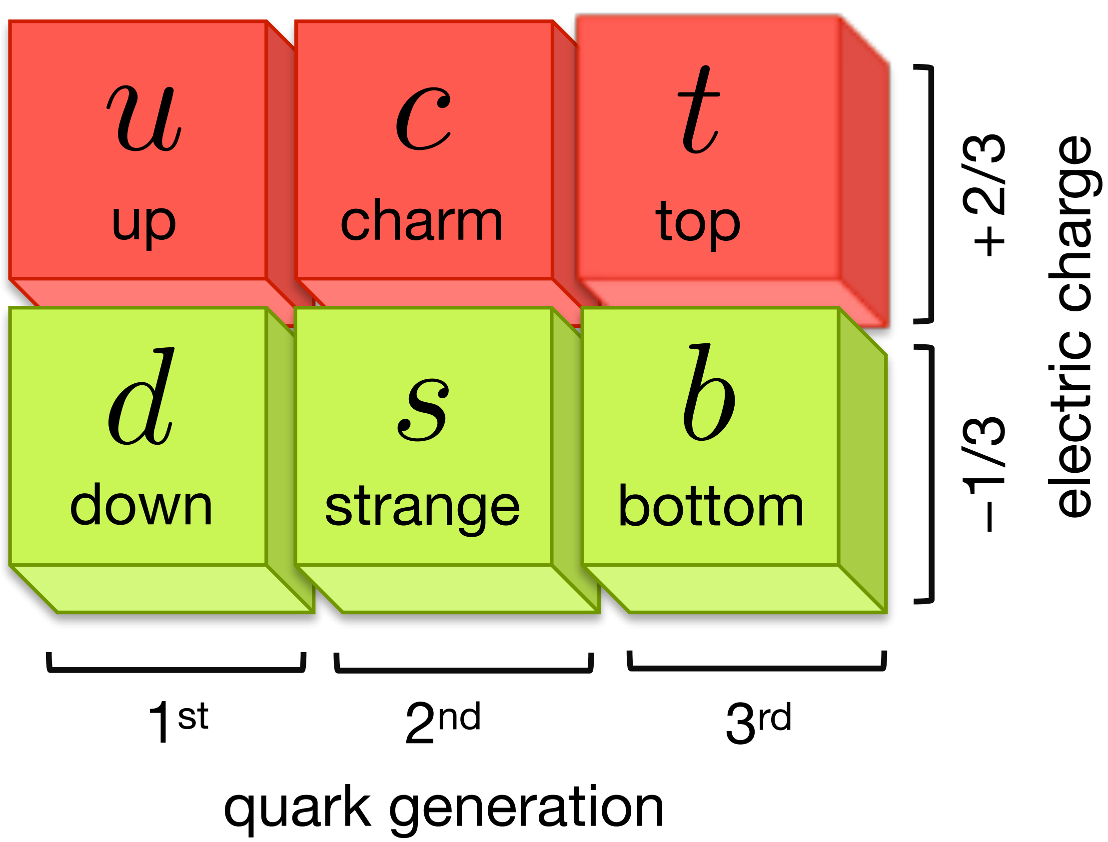
Diagram involving the virtual exchange of top quarks that induces a mass difference in the *B* meson system

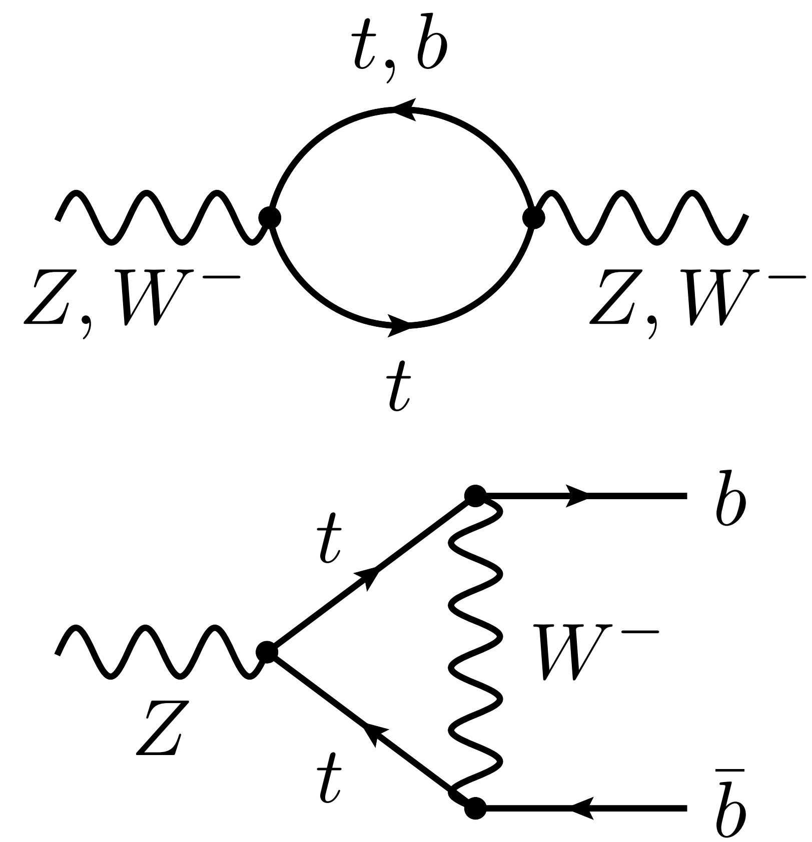


$$\Delta M_B \propto G_F^2 \, m_B \, f_B^2 \, |V_{td}|^2 \, m_t^2$$

- The measurement of the oscillations of *B* mesons into its own anti-particles in 1987 by ARGUS led to the conclusion that the top-quark mass has to be larger than 50 GeV. This was a big surprise at that time, because in 1987 the top quark was generally believed to be much lighter

QCD matter sector

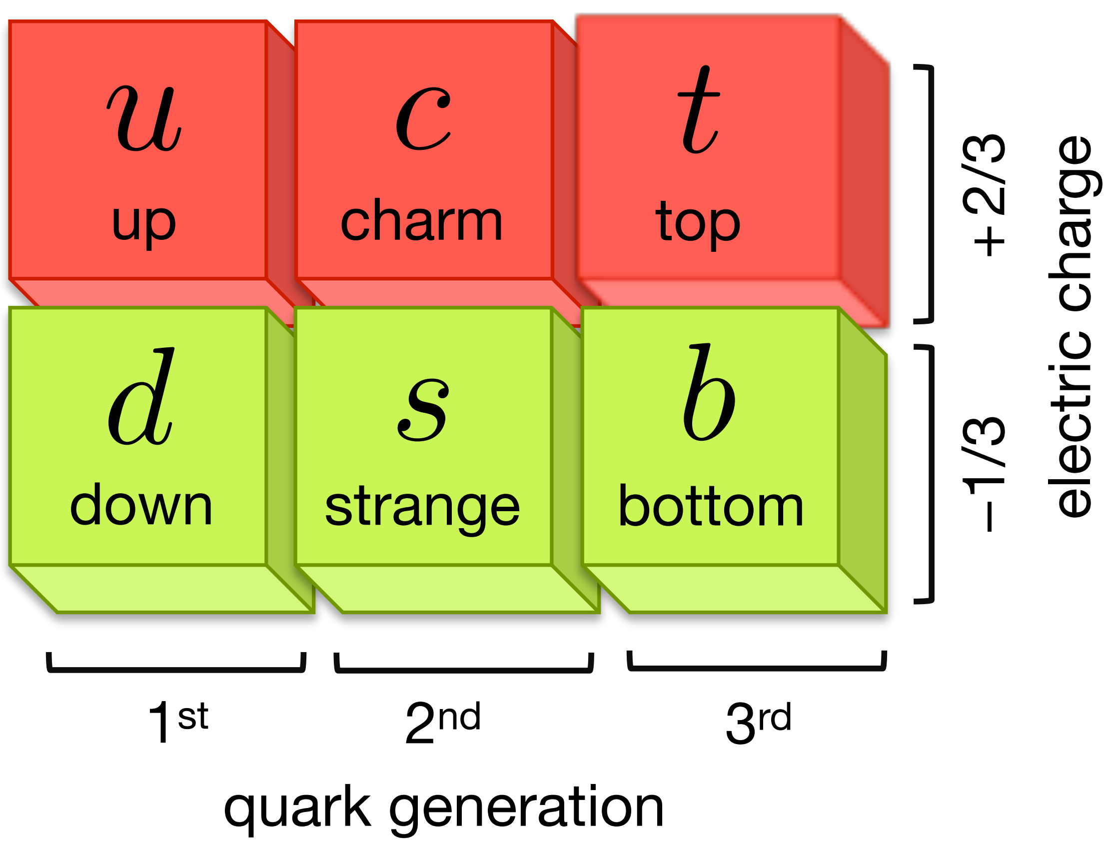


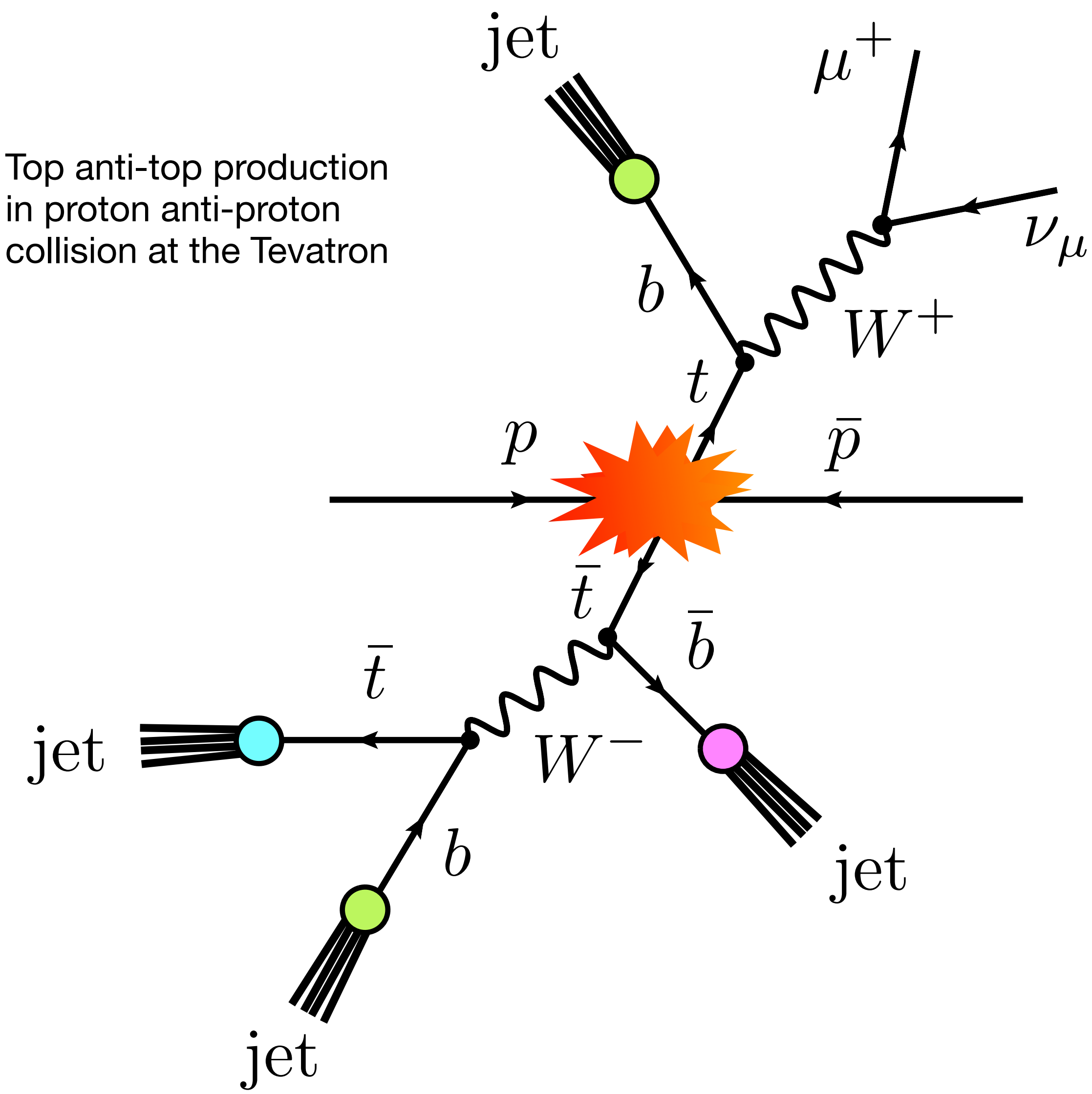


Diagrams that feature a quadratic dependence on the top-quark mass

- It was also realized that certain precision measurements of the EW vector-boson masses and couplings are very sensitive to the mass of the top quark. By 1994 the precision of these indirect measurements led to a prediction of the top-quark mass between 145 GeV and 185 GeV

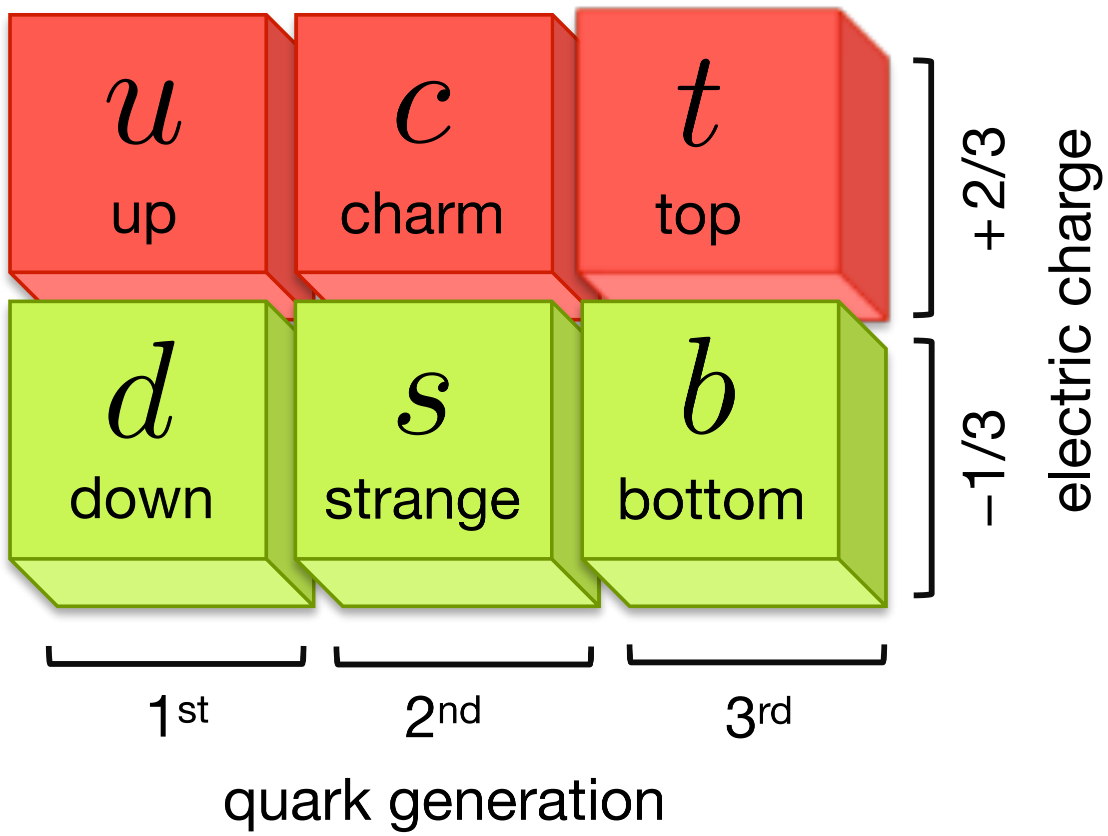
QCD matter sector

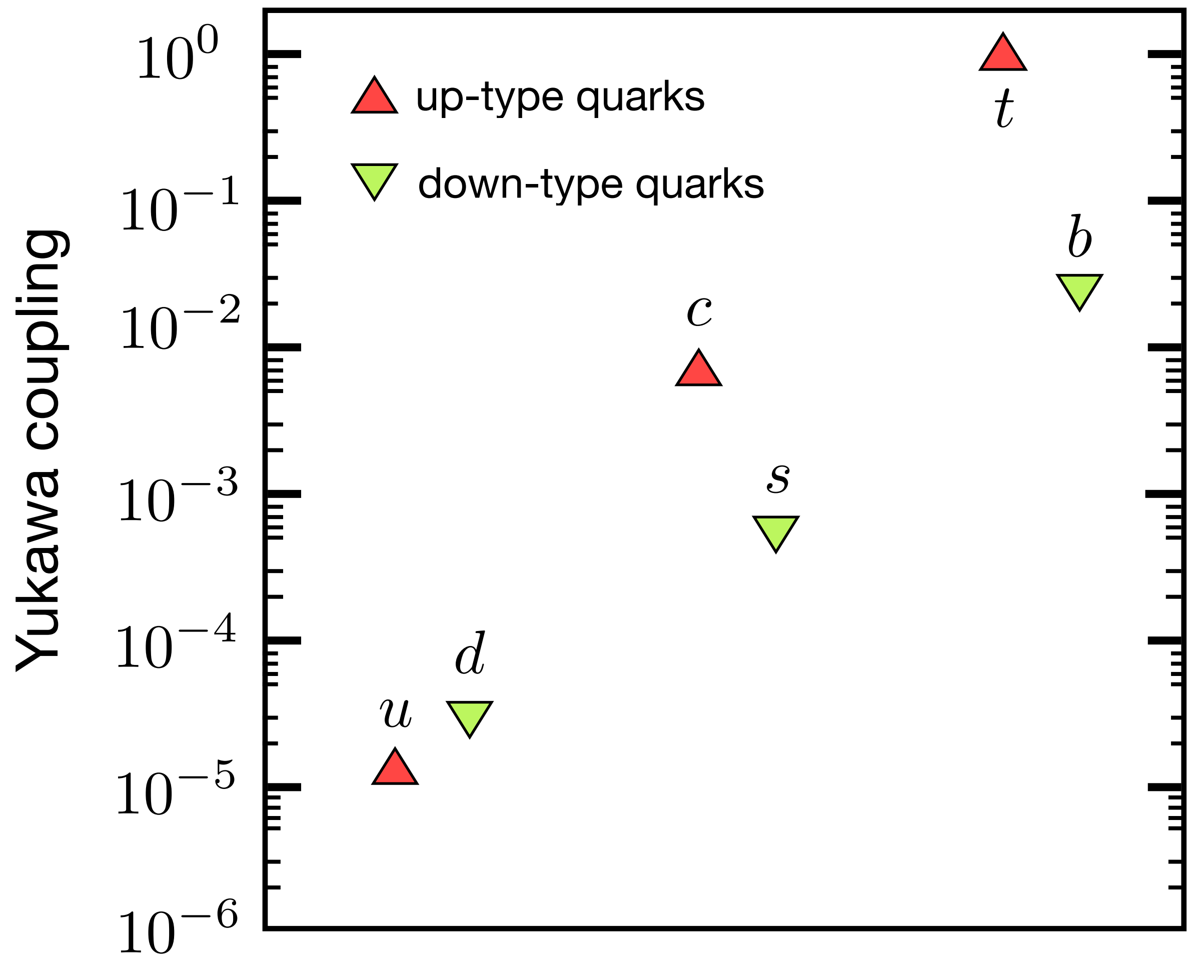




- The top quark was finally discovered in 1995 by CDF and D0 at FNAL. It's mass is today measured to be m_t = (173.1 ± 0.6) GeV

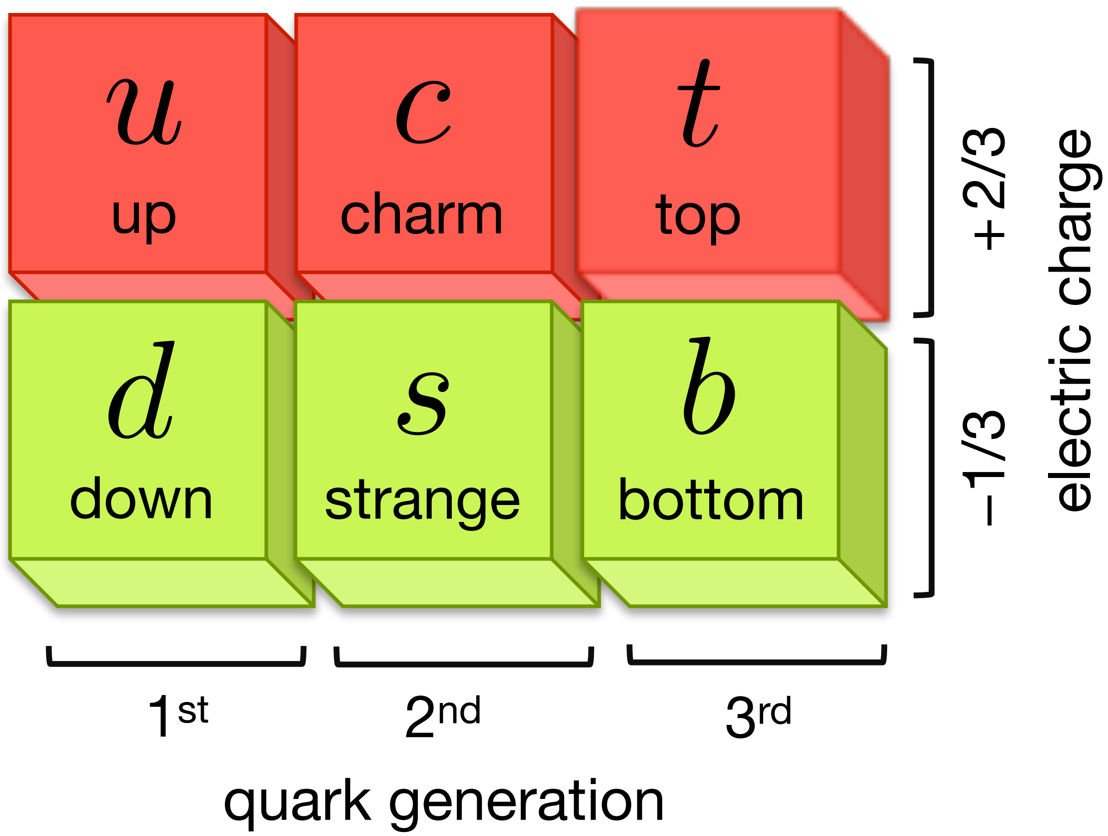
QCD matter sector

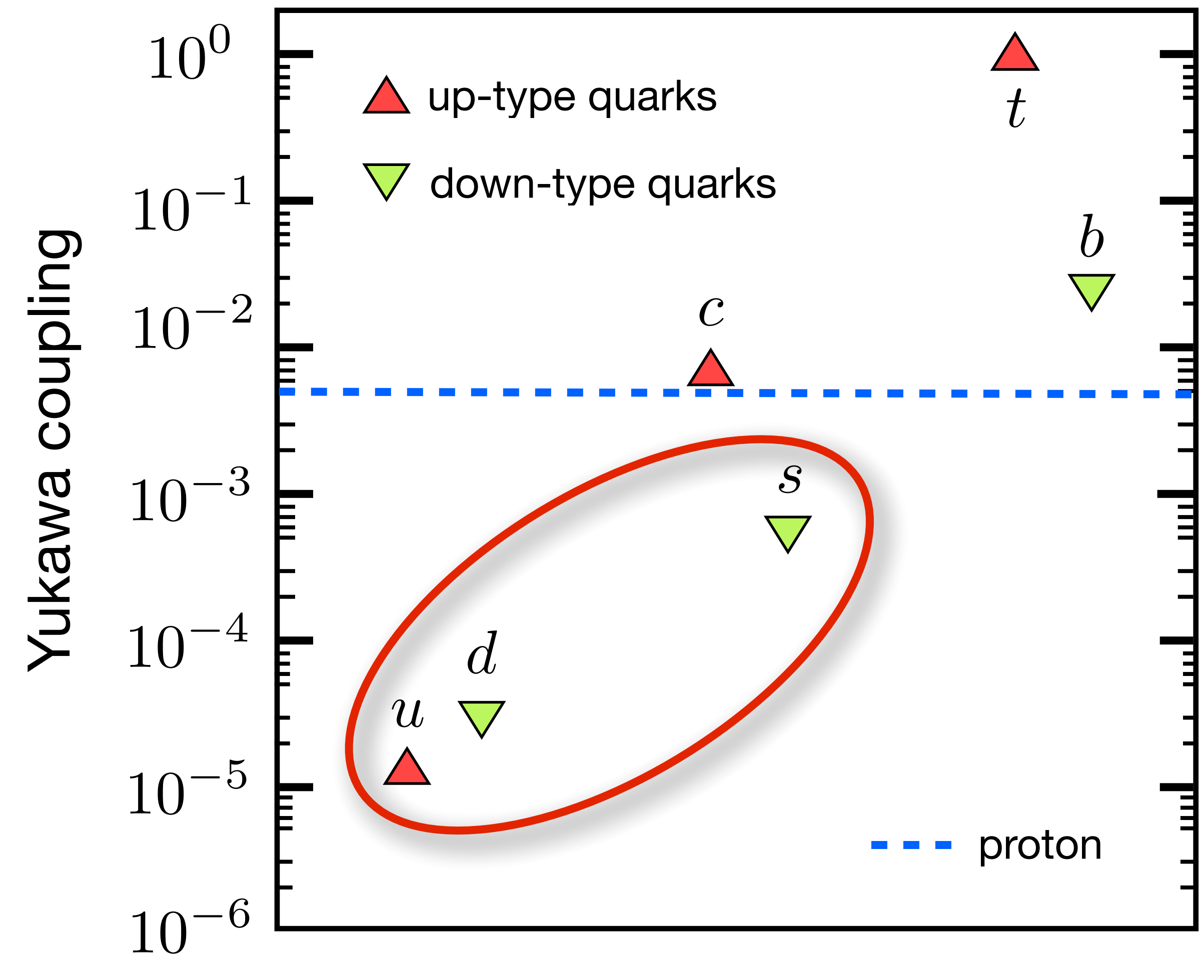




- The masses of the six different quarks range from 2 MeV for the up quark to 172 GeV for the top quark. Why these masses are split by almost six orders of magnitude is one of the big mysteries of particle physics

QCD matter sector





- The up, down, and strange quark are much lighter than the proton. If one takes them to have an identical mass, the quarks become indistinguishable under QCD, and one obtains an effective $SU(3)_f$ symmetry

QED and QCD

- QED and QCD are very similar, yet very different theories
- quarks are a bit like leptons, but there are three of each
- gluons are a bit like photons, but there are eight of them
- gluons interact with themselves
- the QCD coupling is also small at collider energies, but larger then the QED one
- the similarities and differences are evident from the two Lagrangians

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So, let's start by looking at the QED Lagrangian

The QED Lagrangian

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Maxwell}} + \mathcal{L}_{\text{int}}$$

$$= \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi - \frac{1}{2}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu$$

$$= \bar{\psi}\,(iD\!\!\!\!/ - m)\,\psi - \frac{1}{2}(F_{\mu\nu})^2$$

- electromagnetic vector potential A_μ
- field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- covariant derivative $D_\mu = \partial_\mu + ieA_\mu$

QED Feynman rules

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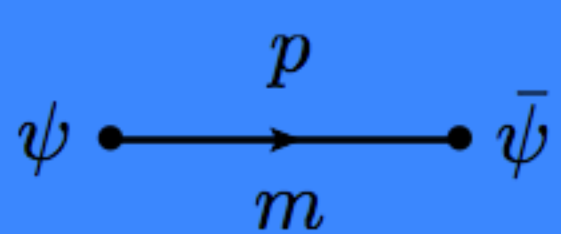
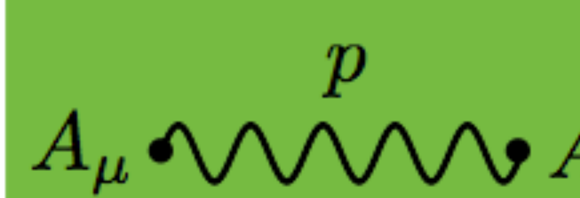
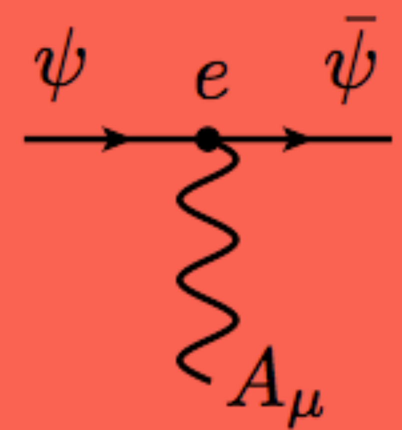


$$= \frac{i\,(p\!\!\!/ + m)}{p^2 - m^2}$$



$$= \frac{-ig^{\mu\nu}}{p^2}$$



$$= ie\gamma^\mu$$

QED gauge invariance

$$\mathcal{L}_{\rm QED} \;=\; \bar{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{2}(F_{\mu\nu})^2$$

A crucial property of the QED Lagrangian is that it is invariant under

$$\psi(x) \to e^{i\alpha(x)}\,\psi(x)\,, \qquad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

which acts on the Dirac field as a local phase transformation

Exercise:
Check that the QED Lagrangian is invariant under the above transformations

QED gauge invariance

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Yang and Mills (1954) proposed that the local phase rotation in QED could be generalised to invariance under a continuous symmetry

[C. N. Yang and R. L. Mills, Phys. Rep. 96 (1954) 191]

The QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)}$$

$$D^\mu_{ij} \equiv \partial^\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a \,, \qquad F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

⇒ covariant derivative ⇒ field strength

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⇒ covariant derivative ⇒ field strength

- only one QCD parameter g_s regulating the strength of the interaction (quark masses have EW origin)

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- setting $g_s = 0$ one obtains the free Lagrangian (free propagation of quarks and gluons without interaction)

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⇒ covariant derivative ⇒ field strength

- only one QCD parameter g_s regulating the strength of the interaction (quark masses have EW origin)
- setting $g_s = 0$ one obtains the free Lagrangian (free propagation of quarks and gluons without interaction)
- terms proportional to g_s in the field strength cause self-interaction between gluons (makes the difference w.r.t. QED)

The QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \sum_f \bar{\psi}_i^{(f)} \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)}$$

$$D^\mu_{ij} \equiv \partial^\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a \,, \qquad F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

⇒ covariant derivative ⇒ field strength

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- color matrices t^a_{ij} are the generators of SU(3)
- QCD flavour blind (differences only due to EW)

The generators of SU(N)

The gauge group of QCD is SU(N) with N =3

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So, the fundamental representation of SU(N) has N^2-1 generators t^a :
N×N traceless hermitian matrices ⇒ N^2-1 gluons

$$U = e^{i\theta_a(x)t^a} \qquad a = 1, \cdots N^2 - 1$$

The Gell-Mann matrices

One explicit representation: $t^A = \frac{1}{2}\lambda^A$

λ^A are the Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Standard normalization: $\mathrm{Tr}(t^a t^b) = T_R\, \delta^{ab} \quad T_R = \frac{1}{2}$

Notice that the first three Gell-Mann matrices contain the three Pauli matrices in the upper-left corner

The generators of SU(N)

Infinitesimal transformations (close to the identity) give complete information about the group structure. The most important characteristic of a group is the commutator of two transformations:

$$
\begin{aligned}
[U(\delta_1), U(\delta_2)] \equiv U(\delta_1)U(\delta_2) - U(\delta_2)U(\delta_1) \\
= (i\delta_1^a)\,(i\delta_2^b)\,[t^a, t^b] + \mathcal{O}(\delta^3)
\end{aligned}
$$

The two matrices to not commute, therefore the transformations don't. Such a group is called non-abelian.

- Familiar abelian groups: translations, phase transformations U(1) ...
- Familiar non-abelian groups: 3D-rotations

The generators of SU(N)

Consider the commutator

$$Tr([t_a, t_b]) = 0 \quad \Rightarrow \quad [t_a, t_b] = i f_{abc} t^c$$

The Lie algebra of the group is defined by the commutation relation of the generators if the group. f_{abc} are the structure constants of the SU(N_c) algebra, they generate the adjoint representation of the algebra

Clearly, f_{abc} is anti-symmetric in (ab). It is easy to show that it is fully antisymmetric

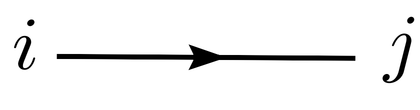
$$i f_{abc} = 2\,\mathrm{Tr}\left([t_a, t_b] t_c\right)$$

and that hence it is fully antisymmetric

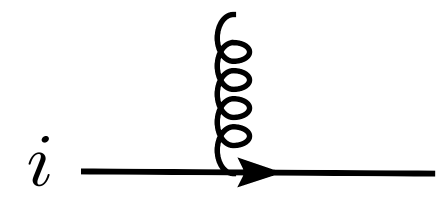
$$f_{abc} = -f_{bac} = -f_{acb}$$

Color algebra: fundamental identities

Fundamental representation 3:

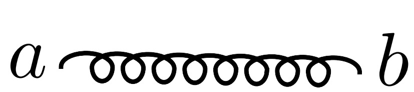


$= \delta_{ij}$

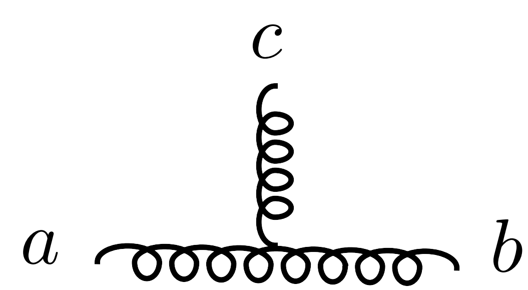


$= t^a_{ij}$

Adjoint representation 8:

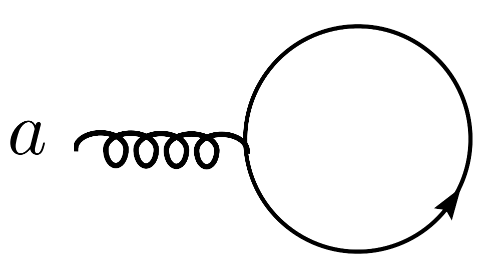


$= \delta_{ab}$



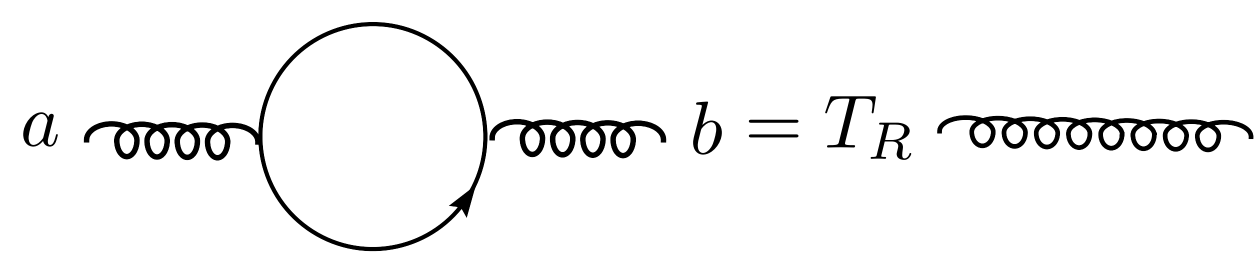
$= i f_{abc}$

Trace identities:



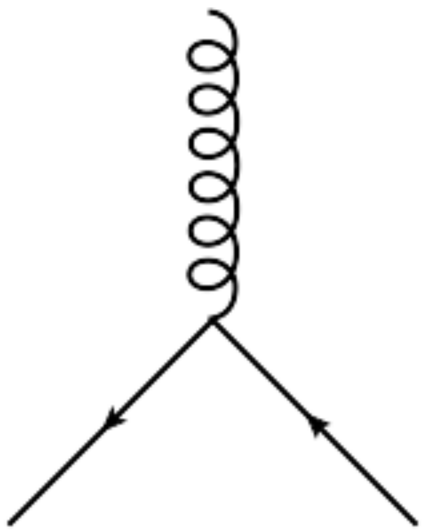
$= 0$

$\mathrm{Tr}(t^a) = 0$



$\mathrm{Tr}(t^a\, t^b) = \mathrm{T_R}\delta^{ab}$

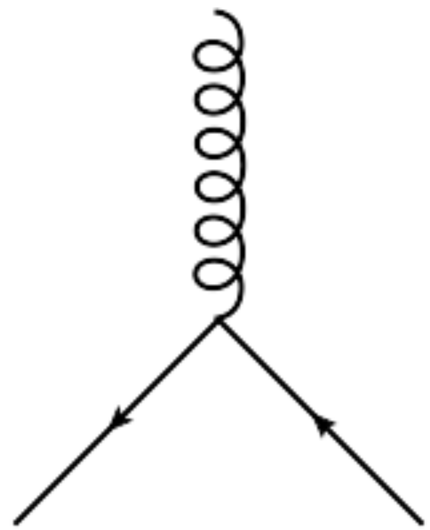
What do color identities mean physically



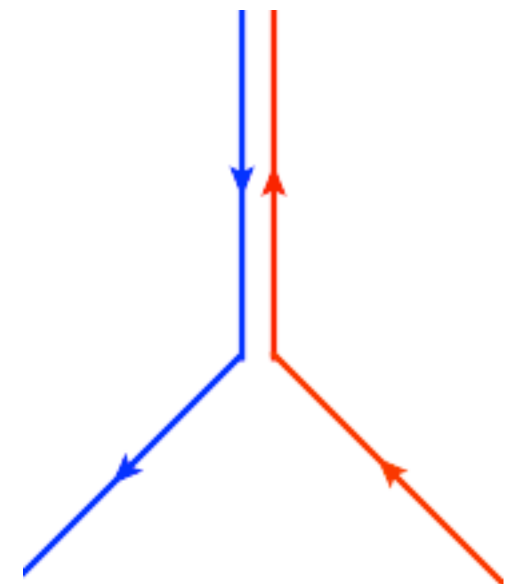
$$\bar{\psi}_i \, t^A_{ij} \, \psi_j$$

What does this really mean?

What do color identities mean physically



$\bar{\psi}_i \, t^A_{ij} \, \psi_j$



$$(1, 0, 0) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

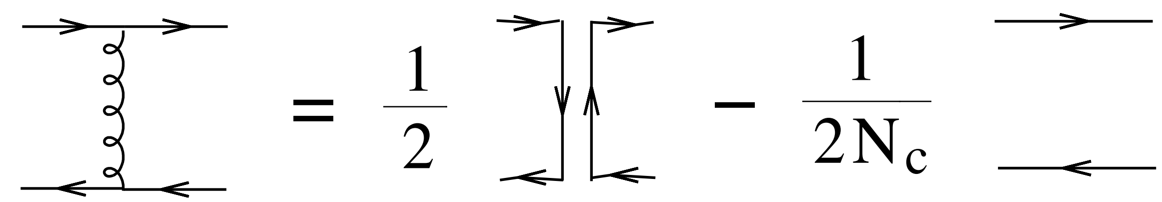
What does this really mean? $\bar{\psi}_i$ t^1_{ij} ψ_j

Gluons carry color and anti-color. They repaint quarks and other gluons.

Color algebra: Casimirs & Fierz identity

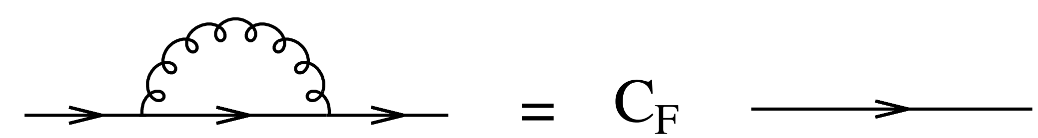
Fierz identity:

$$(t^a)^i_k (t^a)^l_j = \frac{1}{2}\delta^i_j\delta^l_k - \frac{1}{2N_c}\delta^i_k\delta^l_j$$



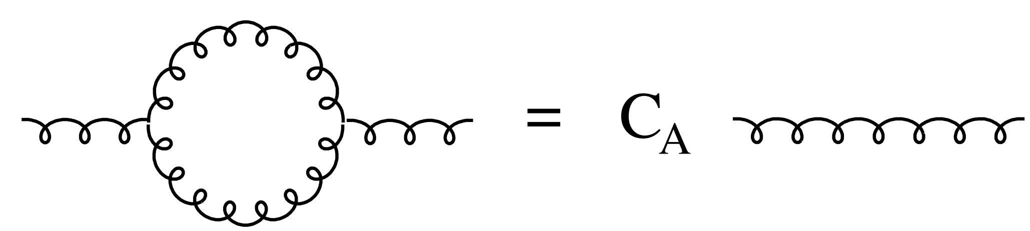
Fundamental representation 3:

$$\sum_a (t^a_{ij})(t^a_{kj}) = C_F\delta_{ij} \qquad C_F = \frac{N_c^2-1}{2N_c}$$



Adjoint representation 8:

$$\sum_{cd} f^{acd} f^{bdc} = C_A\delta^{ab} \qquad C_A = N_c$$



Exercises:

1) derive Fierz identity

2) use the Fierz identity to derive the value of C_F and C_A

Gauge invariance

The QCD Lagrangian is invariant under local gauge transformations, i.e. one can redefine the quark and gluon fields independently at every point in space and time without changing the physical content of the theory

- Gauge transformation for the quark field

$$\psi \to \psi' = U(x)\psi$$

- The covariant derivative $(D_\mu)_{ij} = \partial_\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a$ must transform as (covariant = transforms "with" the field)

$$D_\mu \psi \to D'_\mu \psi' = U(x) D_\mu \psi$$

- From which one derives the transformation property of the gluon field

$$t^a A_a \to t^a A'_a = U(x) t^a A_a U^{-1}(x) + \frac{i}{g_s} \left(\partial U(x)\right) U^{-1}(x)$$

Gauge invariance

The QCD Lagrangian is invariant under local gauge transformations, i.e. one can redefine the quark and gluon fields independently at every point in space and time without changing the physical content of the theory

- It follows that

$$\bar\psi \to \bar\psi' = \bar\psi U^\dagger(x)$$

$$t^a F^a_{\mu\nu} \to t^a F^{a'}_{\mu\nu} = U(x) t^a F^a_{\mu\nu} U^{-1}(x)$$

e.g. because $\quad i\, g_s\, t^a F^a_{\mu\nu} = [D_\mu, D_\nu]$

- Therefore the QCD Lagrangian is indeed gauge invariant

$$-\frac{1}{4} F'^{\mu\nu}_a F'^a_{\mu\nu} = -\frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu}$$

$$\sum_f \bar\psi'^{(f)}_i \left(i\not{D}'_{ij} - m_f \delta_{ij}\right) \psi'^{(f)}_j = \sum_f \bar\psi^{(f)}_i \left(i\not{D}_{ij} - m_f \delta_{ij}\right) \psi^{(f)}_j$$

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The QCD Lagrangian is invariant under local gauge transformations, i.e. one can redefine the quark and gluon fields independently at every point in space and time without changing the physical content of the theory

<u>Remarks:</u>

- the field strength alone is not gauge invariant in QCD (unlike in QED) because of self interacting gluons (carries of the force carry colour, unlike the photon)

- a gluon mass term violate gauge invariance and is therefore forbidden (as for the photon). On the other hand quark mass terms are gauge invariant.

$$\cancel{m^2 A_\mu A^\mu}$$

Isospin symmetry

Isospin SU(2) symmetry: relates the content in terms of up and down quarks. Isospin invariance means invariance under u ↔ d

$$I_3 = \frac{1}{2}\left[(n_u - n_{\bar{u}}) - (n_d - n_{\bar{d}})\right]$$

- Proton has isospin I_3=1/2, while the neutron I_3=-1/2. Protons and neutrons form an iso-spin multiplet
- Pions have I_3=1 (π^+ = ud), I_3=0 (π_0 = 1/√2(uu+dd), I_3=0 (π^- = ud). They are also in an iso-spin multiplet

Particles in the same isospin multiplet have very similar masses (proton and neutron, neutral and charged pions)

Isospin symmetry

Isospin arises because QCD interactions are flavour blind and the accidental fact that up and down have very close masses

Check: the QCD Lagrangian has isospin symmetry if $m_u = m_d$ or $m_u, m_d \to 0$

In this limit of vanishing m_u, m_d masses, one can separate the fermion fields into left and right chiralities

$$\psi_L = P_L \psi \,, \quad \psi_R = P_R \psi \,, \quad P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$$

The QCD Lagrangian in terms of left and right states becomes

$$\mathcal{L}_F = \sum_f \left(\bar{\psi}_L^{(f)} \not{D} \psi_L^{(f)} + \bar{\psi}_R^{(f)} \not{D} \psi_R^{(f)} \right) - \sum_f m_f \left(\bar{\psi}_R^{(f)} \psi_L^{(f)} + \bar{\psi}_L^{(f)} \psi_R^{(f)} \right)$$

So neglecting fermion masses the Lagrangian has the larger symmetry

$$SU_L(N_f) \times SU_R(N_f) \times U_L(1) \times U_R(1)$$

Isospin symmetry

The fact that left-handed and right-handed terms in the Lagrangian are separately invariant is a direct consequence of the fact that the chirality of massless fermions is conserved

This symmetry of the QCD is known as chiral symmetry and it is spontaneous broken by the QCD vacuum (much as the Higgs mechanism). The (approximately massless) pions are the Goldstone boson of the broken symmetry

This happens when the vacuum state of the theory is not invariant under the same symmetries as the Lagrangian. In the case of QCD it is known that

$$\langle 0|q\bar{q}|0\rangle = \langle 0|u\bar{u} + d\bar{d}|0\rangle \approx (250MeV)^3$$

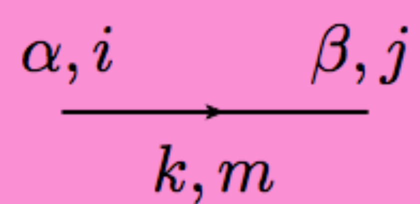
Similar mechanisms to the one that break the chiral symmetry in QCD have been proposed to explain why the Higgs boson so is light in composite Higgs scenarios … but this is a topic for a different lecture!

Feynman rules: propagators

Obtain quark/gluon propagators from free piece of the Lagrangian

<u>Quark propagator</u>: replace i∂ → k and take the i × inverse

$$\mathcal{L}_{\mathrm{q,free}} = \sum_f \bar{\psi}_i^{(f)} \left(i\partial\!\!\!/ - m_f \right) \delta_{ij} \psi_j^{(f)}$$



$$= \left(\frac{i}{k\!\!\!/ - m} \right)_{\alpha\beta} \delta_{ij}$$

<u>Gluon propagator</u>: replace i∂ → k and take the i × inverse ?

$$\mathcal{L}_{\mathrm{g,free}} = \frac{1}{2} A^{\mu} \left(\Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) A^{\nu}$$

➥ inverse does not exist, since $\left(\Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) \partial_\mu = \Box \partial_\nu - \Box \partial_\nu = 0$

How can one to define the propagator ?

Gauge fixing

Solution:
add to the Lagrangian a gauge fixing term which depends on an arbitrary parameter ξ

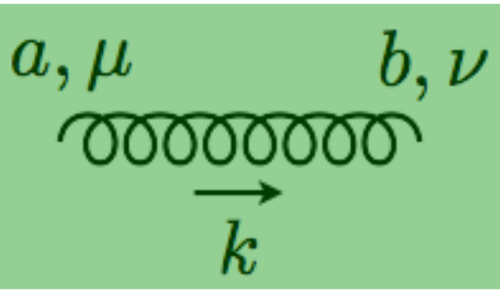
In covariant gauges:

$$\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{\xi}\left(\partial^{\mu} A_{\mu}^{A}\right)^2$$

ξ=1 Feynman gauge
ξ=0 Landau gauge

Gluon propagator:

$$\frac{-i}{k^2}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab} =$$

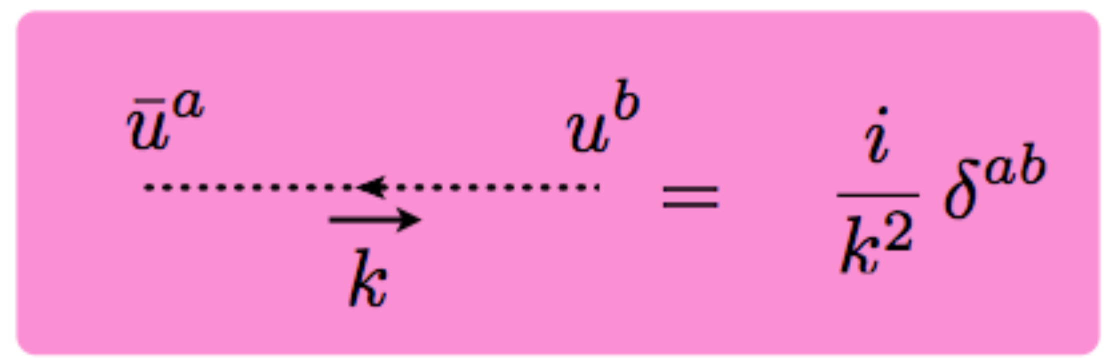


Gauge fixing explicitly breaks gauge invariance. However, in the end physical results are independent of the gauge choice. Powerful check of higher order calculations: verify that the ξ dependence fully cancels in the final result

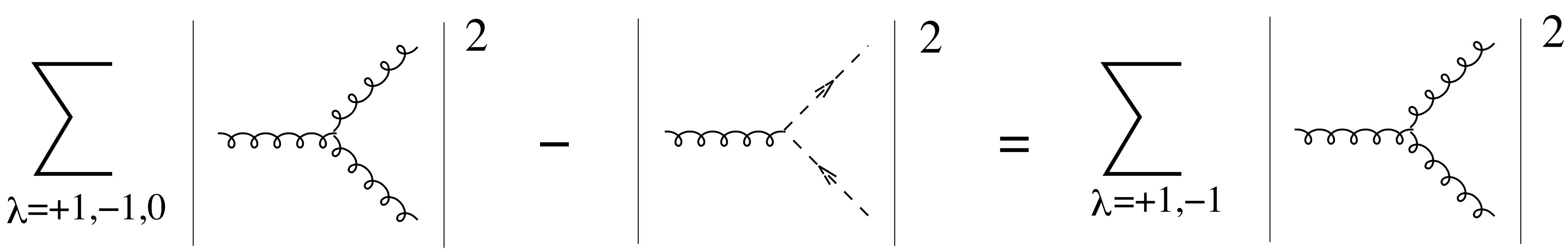
Ghosts

In covariant gauges gauge fixing term must be supplemented with ghost term to cancel unphysical longitudinal degrees of freedom which should not propagate

$$\mathcal{L}_{\text{ghost}} = \partial_\mu \eta^{a\dagger} D^\mu_{ab} \eta^b$$



η: complex scalar field which obeys Fermi statistics



Axial gauges

Alternative: choose an axial gauge (introduce an arbitrary direction n)

$$\mathcal{L}_{\text{axial gauge}} = -\frac{1}{\xi}\left(n^{\mu} A_{\mu}^{A}\right)^2$$

The gluon propagator becomes

$$d_{\mu\nu} = \frac{i}{k^2}\left(-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} + \frac{(n^2+\xi k^2)k_\mu k_\nu}{(n\cdot k)^2}\right)\delta_{ab}$$

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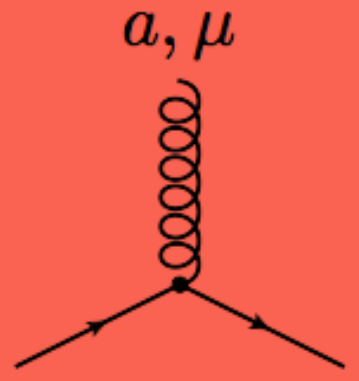
Light cone gauge: $n^2 = 0$ and $\xi = 0$

Axial gauges for $k^2 \to 0$

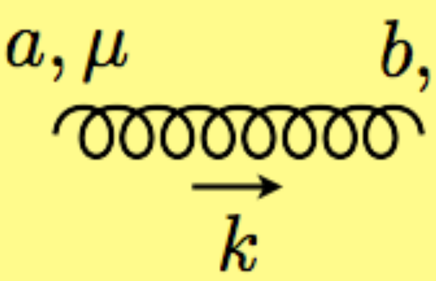
$$d_{\mu\nu}k^\mu = d_{\mu\nu}n^\mu = 0$$

i.e. only two physical polarizations propagate, that's why often the term physical gauge is used

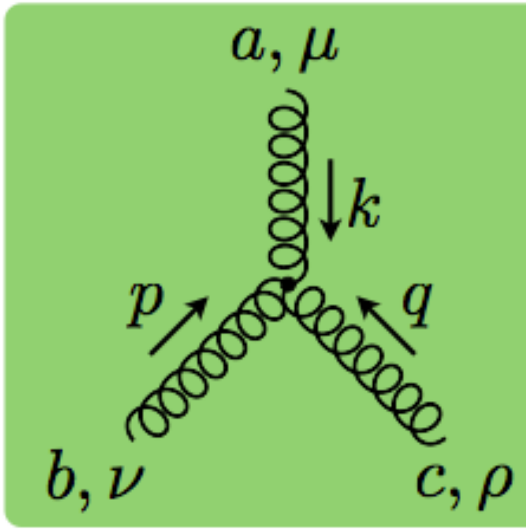
QCD Feynman rules: the vertices



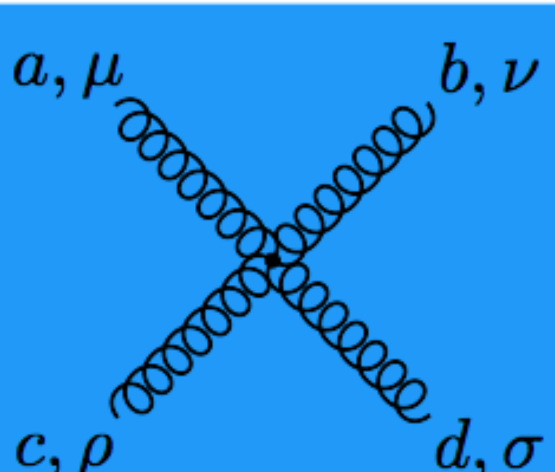
$$= \quad i g_s \gamma^\mu t^a$$



$$= \quad \left(\frac{-i g_{\mu\nu}}{k^2} \right) \delta^{ab}$$



$$= \quad g_s\, f^{abc} \Big[g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu \Big]$$

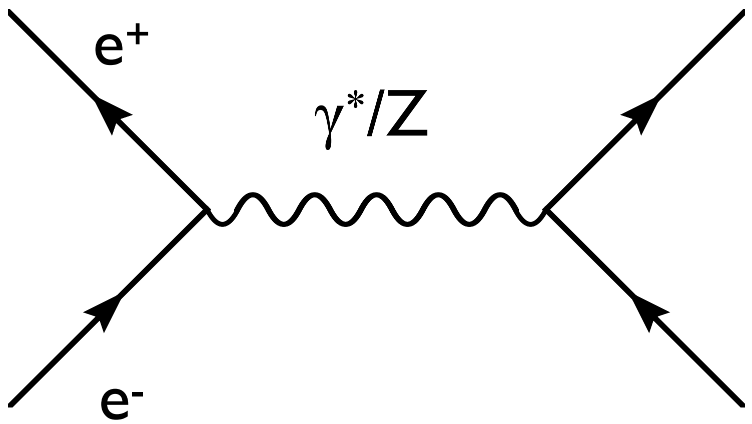


$$= \quad -i g_s^2 \Big[f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ace} f^{bde} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + f^{ade} f^{bce} \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} \right) \Big]$$

Perturbative expansion of the R-ratio

The R-ratio is defined as

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



At lowest order in perturbation theory (PT)

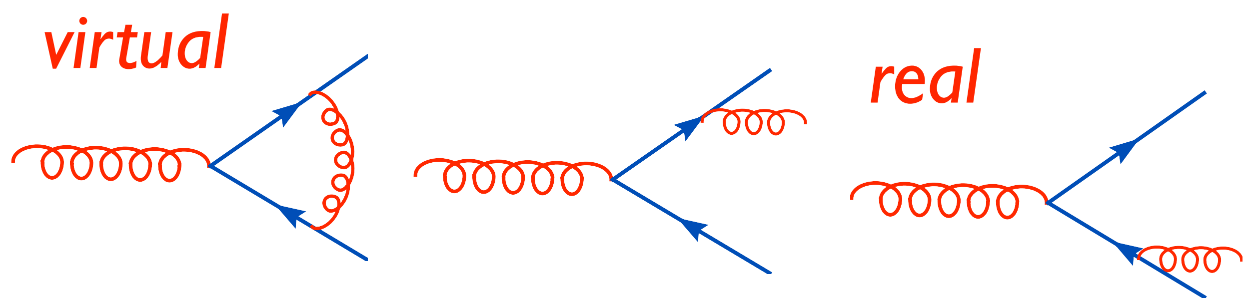
$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_0(e^+e^- \to q\bar{q})$$

Since common factors cancel in numerator/denominator, to lowest order one finds

$$R_0 = \frac{\sigma_0(\gamma^* \to \text{hadrons})}{\sigma_0(\gamma^* \to \mu^+\mu^-)} = N_c \sum_f q_f^2$$

The R-ratio: perturbative expansion

First order correction



Real and virtual do not interfere since they have a different # of particles.
The amplitude squared becomes

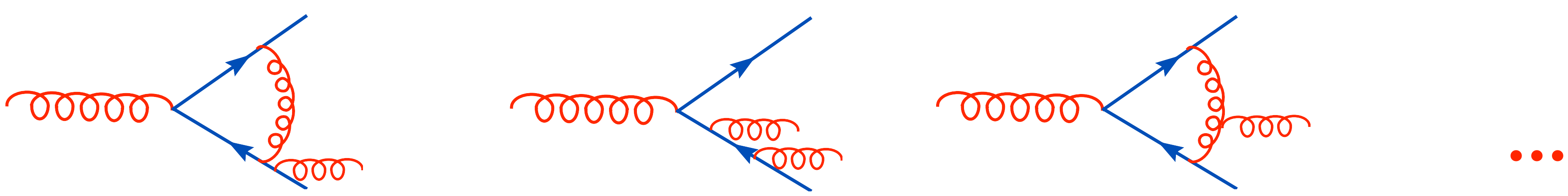
$$|A_1|^2 = |A_0|^2 + \alpha_s \left(|A_{1,r}|^2 + 2\mathrm{Re}\{A_0 A_{1,v}^*\}\right) + \mathcal{O}(\alpha_s^2) \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

Integrating over phase space, the first order result reads

$$R_1 = R_0 \left(1 + \frac{\alpha_s}{\pi}\right)$$

R-ratio and UV divergences

To compute the second order correction one has to compute diagrams like these and many more



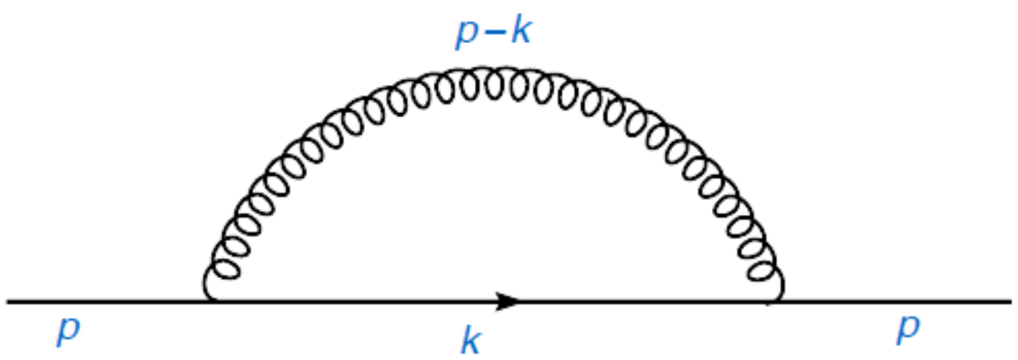
One gets

$$R_2 = R_0 \left(1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(c + \pi b_0 \ln \frac{M_{\mathrm{UV}}^2}{Q^2}\right)\right) \qquad b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

Ultra-violet divergences do not cancel. Result depends on UV cut-off.

Renormalization

Loop corrections in QCD are (often) divergent. Divergences originate from regions of very large momenta



$$\sim g^2 \int_{p^2}^{\infty} d^4k \, \frac{1}{k^2} \frac{1}{(p-k)^2} \to \infty$$

QCD is a renormalizable theory. This means that that one can

1. regularize the divergence (e.g. using dimensional regularization)

$$d^4k \to \mu^{2\epsilon} d^{4-2\epsilon}k$$

2. absorbe all UV divergences into a universal redefinition of a finite number of the bare parameters of QCD

Renormalization and running coupling

For the R-ratio, the divergence is dealt with by renormalization of the coupling constant

$$\alpha_s(\mu) = \alpha_s^{\text{bare}} + b_0 \ln \frac{M_{UV}^2}{\mu^2} \left(\alpha_s^{\text{bare}}\right)^2$$

R expressed in terms of the renormalized coupling is finite

$$R = R_0 \left(1 + \frac{\alpha_s(\mu)}{\pi} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(c + \pi b_0 \ln \frac{\mu^2}{Q^2}\right) + \mathcal{O}(\alpha_s^3(\mu))\right)$$

Renormalizability of the theory guarantees that *the same redefinition of the coupling removes all UV divergences from all physical quantities (massless case)*

Renormalization achieved by replacing bare masses and the bare coupling with renormalized ones. Masses and coupling become dependent on the renormalization scale. The dependence is fully predicted in pQCD

- the coupling ⇒ β function
- the masses ⇒ anomalous dimensions γ_m

The beta-function

$$\beta(\alpha_s^{\text{ren}}) \equiv \mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2}$$

The renormalized coupling is

$$\alpha_s(\mu) = \alpha_s^{\text{bare}} + b_0 \ln \frac{M_{UV}^2}{\mu^2} \left(\alpha_s^{\text{bare}}\right)^2$$

So, one immediately gets

$$\beta = -b_0 \alpha_s^2(\mu) + \ldots$$

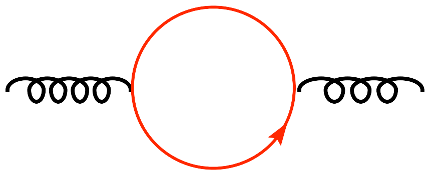
Integrating the differential equation one finds at lowest order

$$\frac{1}{\alpha_s(\mu)} = b_0 \ln \frac{\mu^2}{\mu_0^2} + \frac{1}{\alpha_s(\mu_0)} \qquad \Rightarrow \qquad \alpha_s(\mu) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}}$$

More on the beta-function

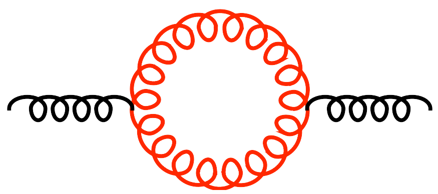
Roughly speaking:

(a) quark loop vacuum polarization diagram gives a negative contribution to $b_0 \sim - 2n_f / 12\pi$

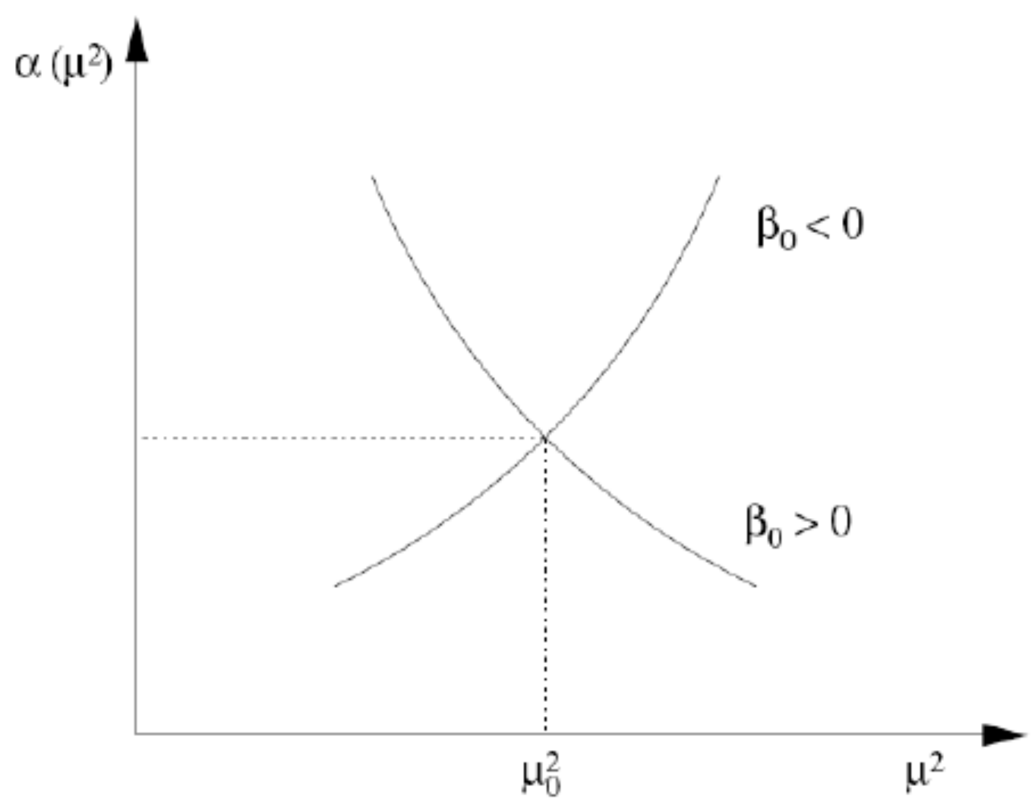


(a)

(b) gluon loop gives a positive contribution to $b_0 \sim 11N_c / 12\pi$



(b)



Since (b) > (a) $\Rightarrow$ $b_{0,QCD} > 0$

$\Rightarrow$ overall negative beta-function in QCD

While in QED (b) = 0 $\Rightarrow$ $b_{0,QED} < 0$

$$\beta_{\rm QED} = \frac{1}{3\pi}\alpha^2 + \ldots$$

More on the beta-function

- QCD: perturbative picture valid for scales $\mu >> \Lambda_{QCD}$ (about 300 MeV)
- QED: perturbative picture valid for scales $\mu << \Lambda_{QED}$

Question: why does nobody talk about Λ_{QED}?

More on the beta-function

- QCD: perturbative picture valid for scales μ >> Λ_{QCD} (about 300 MeV)
- QED: perturbative picture valid for scales μ << Λ_{QED}

Question: why does nobody talk about Λ_{QED}?

Answer:

$$\Lambda_{\text{QED}} = m_e \exp\left\{-\frac{1}{2b_0\alpha(m_e)}\right\} \sim 10^{90}\text{GeV} >> M_{\text{Planck}}$$

(Note that the fact that QED is not a consistent theory up to very high scales implies that it must be an effective theory)

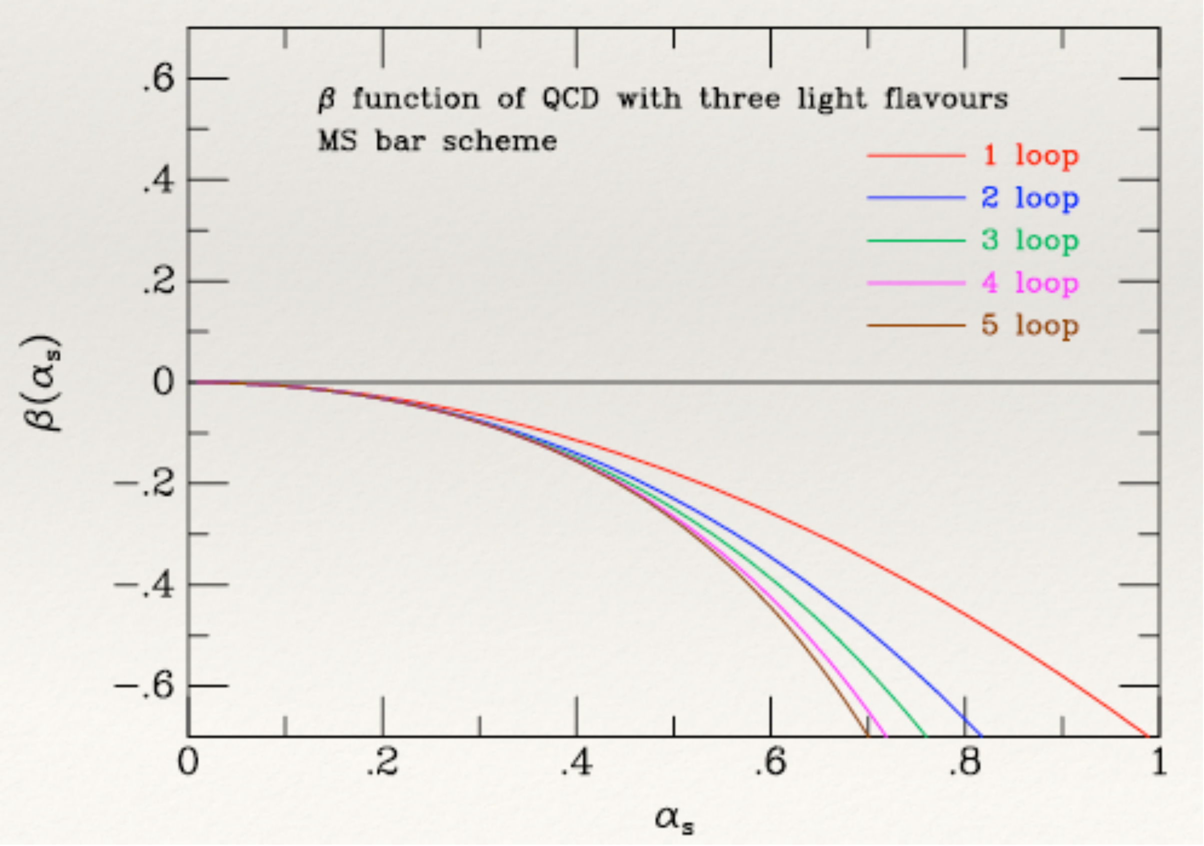
Back to the QCD beta-function

Perturbative expansion of the beta-function:

$$\beta = -\alpha_s^2(\mu) \sum_i b_i \alpha_s^i(\mu)$$

$$b_0 = \frac{11N_c - 4n_f T_R}{12\pi}$$

$$b_1 = \frac{17N_c^2 - 5N_c n_f - 3C_F n_f}{24\pi^2}$$

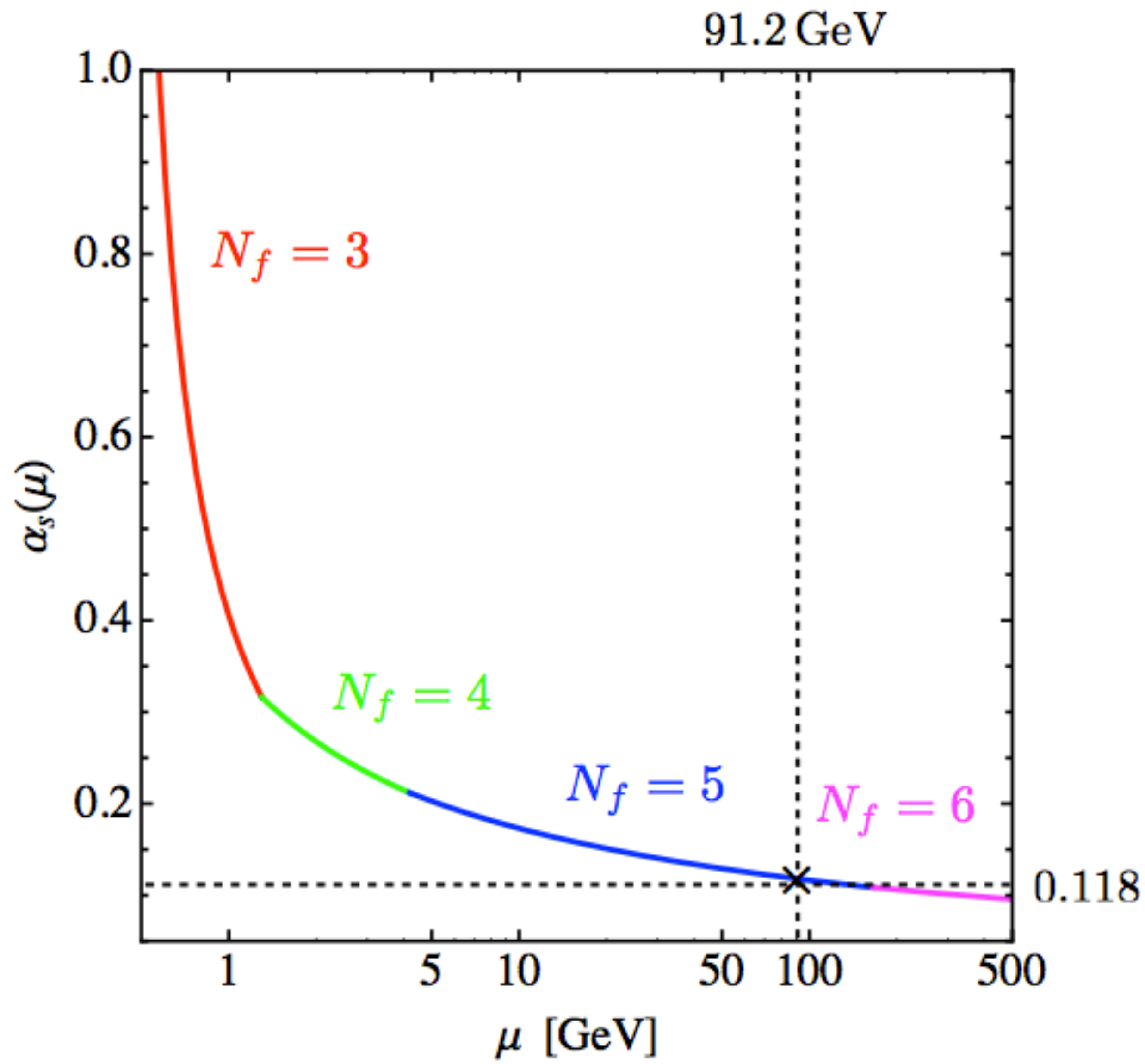


- n_f is the number of active flavours (depends on the scale)
- today, the beta-function known up to five loops, but only first two coefficients are independent of the renormalization scheme

Exercise: proof the above statement [hint: use the fact that at $O(\alpha_s)$ the coupling in two different schemes is related by a finite change]

Active flavours & running coupling

The active field content of a theory modifies the running of the couplings



Constrain New Physics by measuring the running at high scales?

Renormalization Group Equation

Consider a dimensionless quantity A, function of a single scale Q. The dimensionless quantity should be independent of Q. However in quantum field theory this is not true, as renormalization introduces a second scale μ

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So, for any observable A one can write a renormalization group equation

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s}\right] A\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

$$\alpha_s = \alpha_s(\mu^2) \qquad \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2}$$

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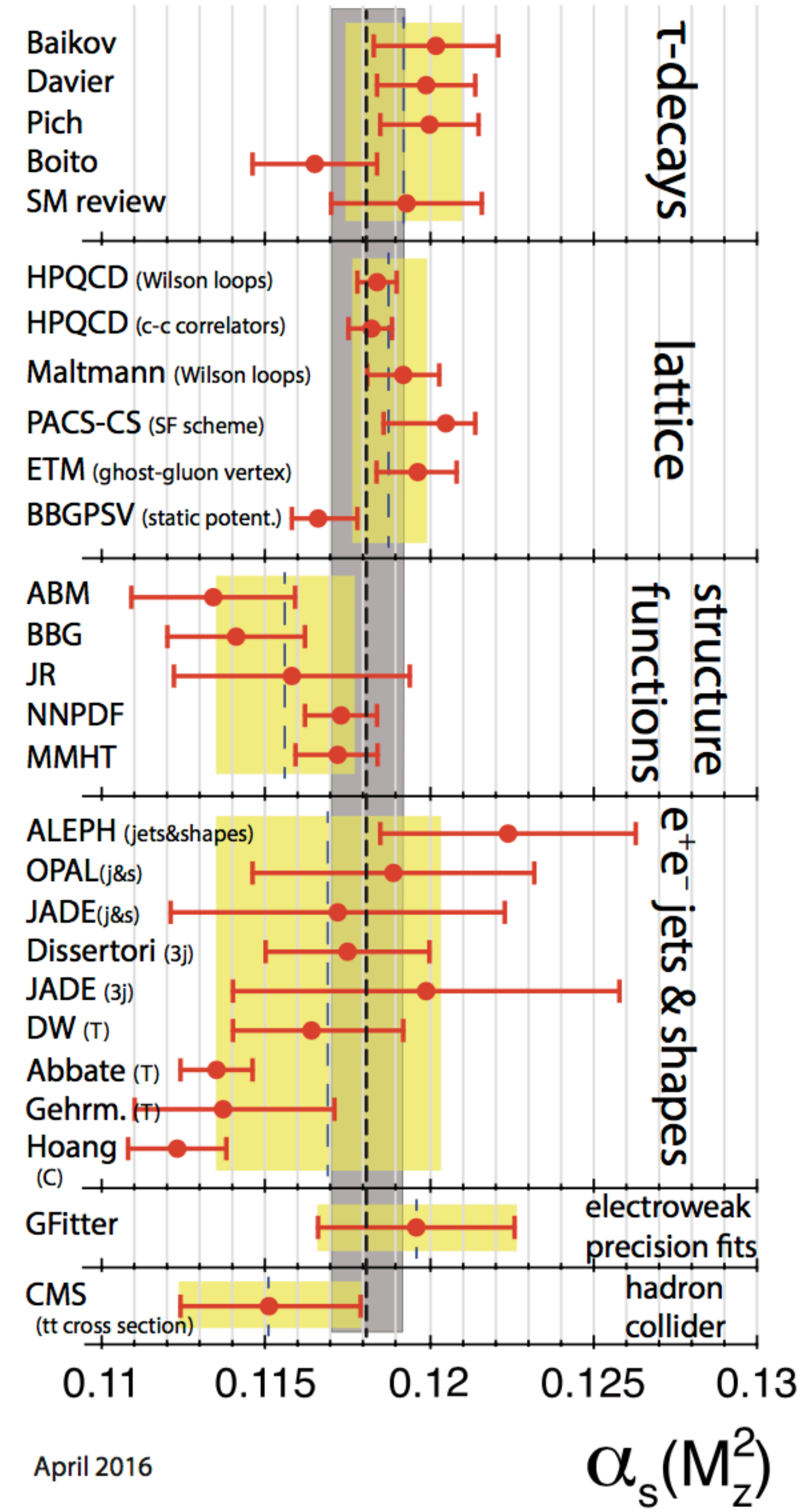
Scale dependence of A enters through the running of the coupling: knowledge of $A(1, \alpha_s(Q^2))$ allows one to compute the variation of A with Q given the beta-function

Measurements of the running coupling

Summarizing:

- overall consistent picture: α_s from very different observables compatible
- α_s is not so small at current scales
- α_s decreases slowly at higher energies (logarithmic only)
- higher order corrections are and will remain important



World average

$$\alpha_s(M_Z) = 0.1181 \pm 0.0011$$

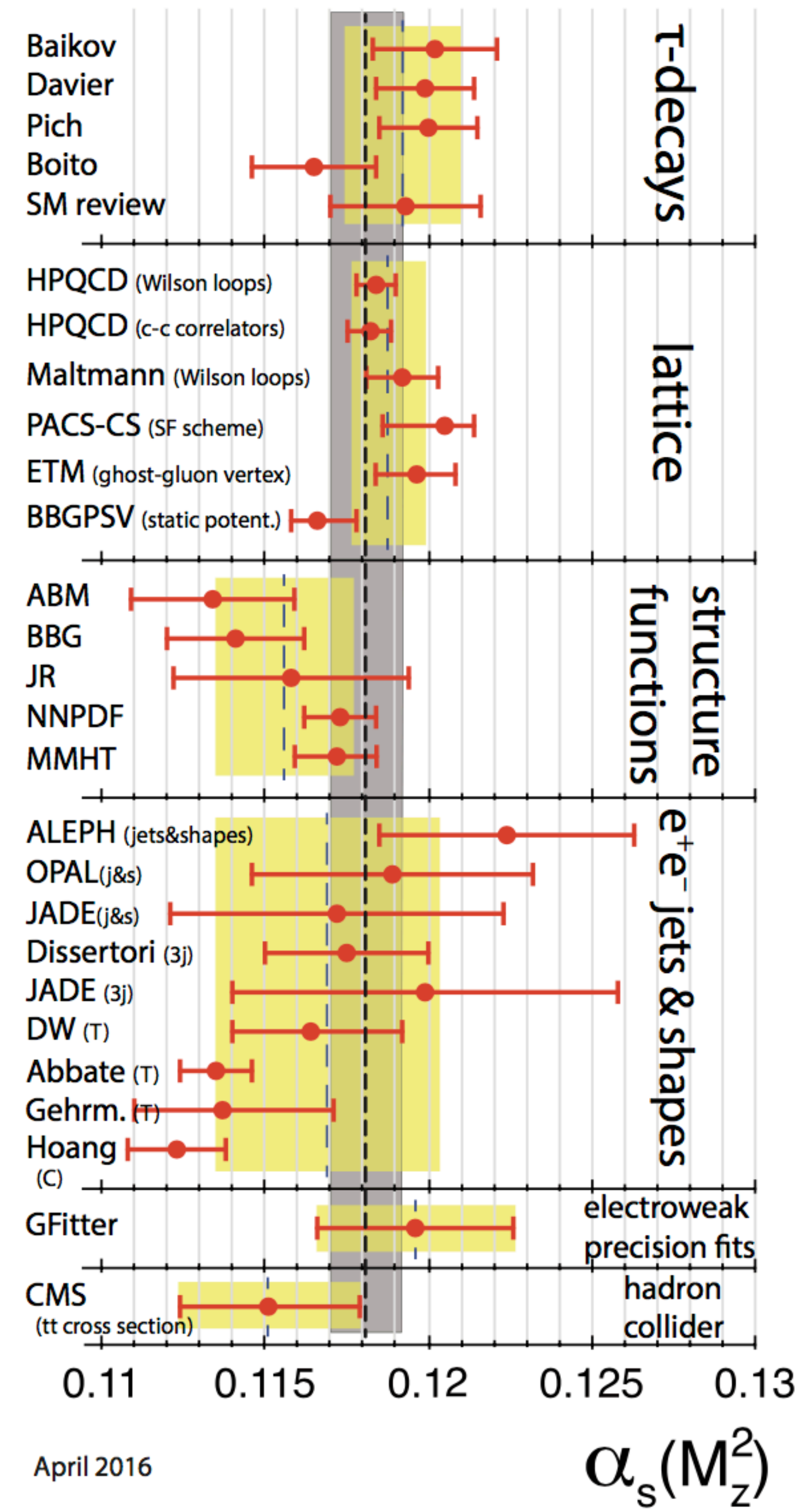
Measurements of the running coupling

Questions:

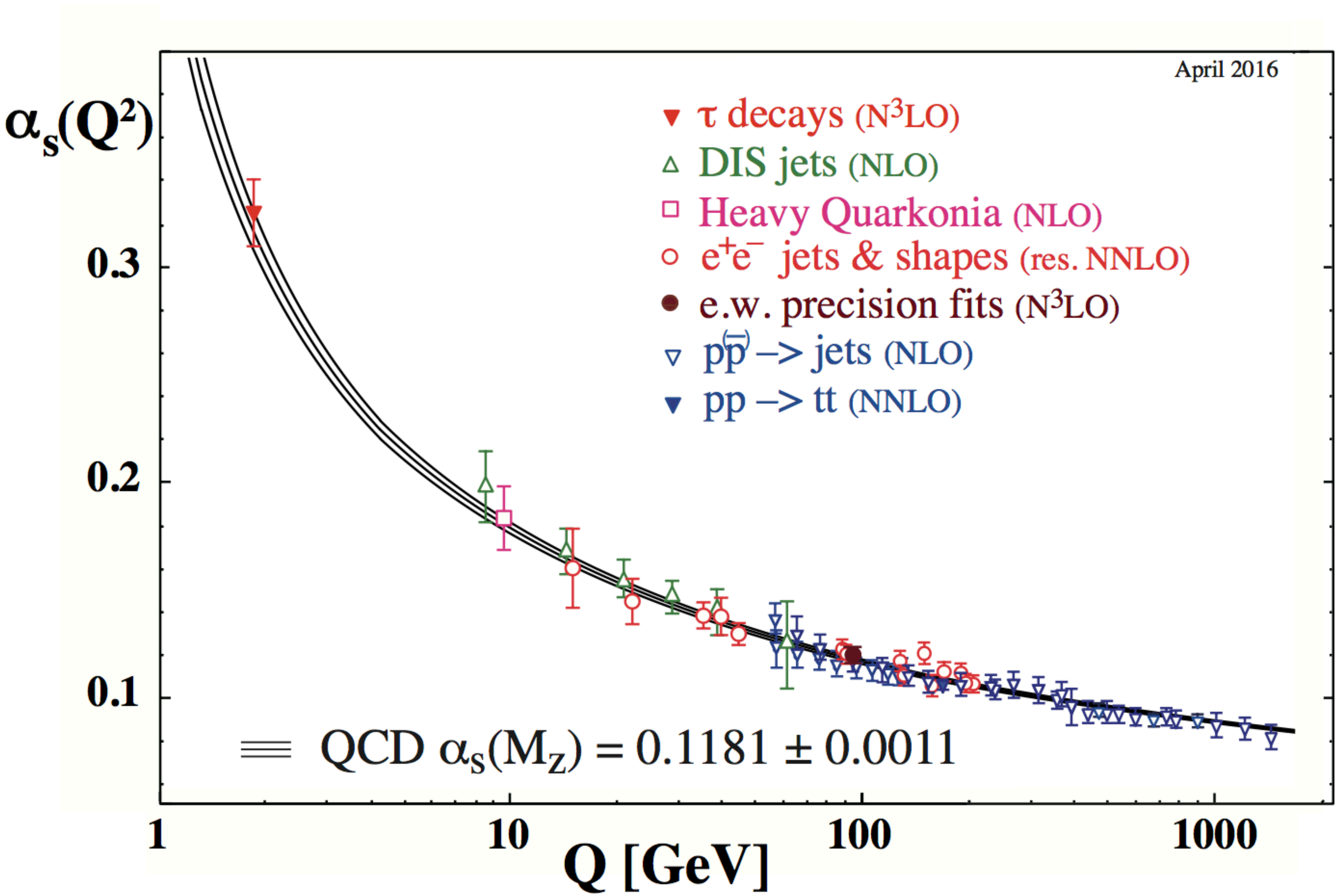
- Why is the determination of α_s from τ-decays so accurate?
- Why is the determination of α_s from the four-jet rate so accurate?

World average

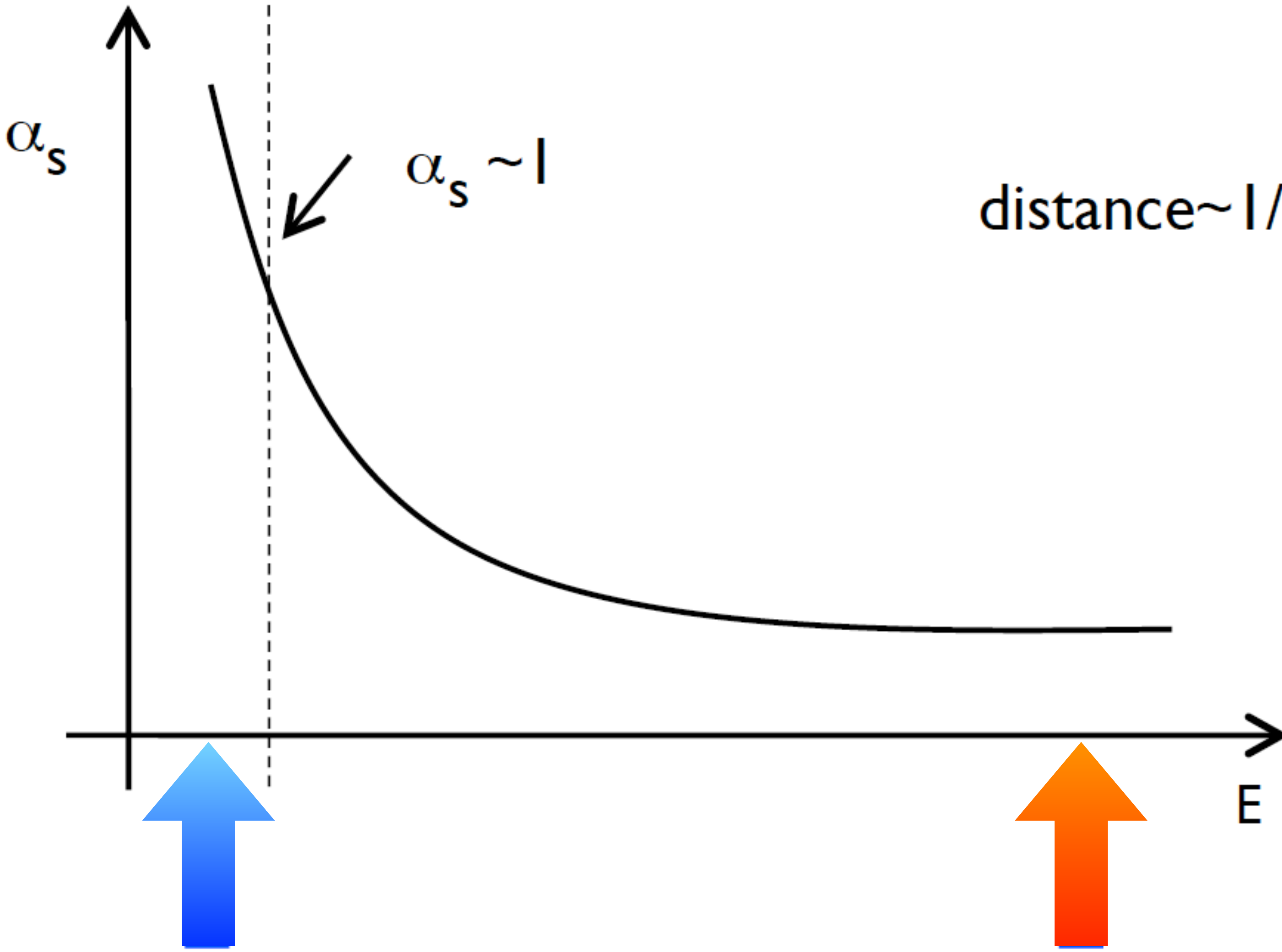
$$\alpha_s(M_Z) = 0.1181 \pm 0.0011$$



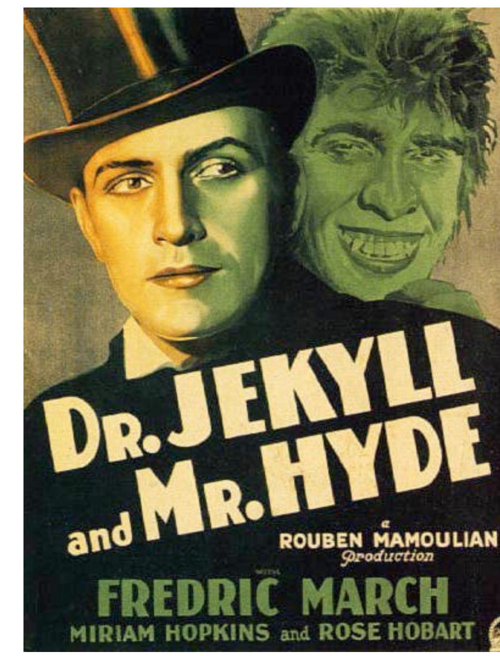
Measurements of the running coupling



The two faces of QCD



distance~1/energy



Confinement
(large distance)

asymptotic freedom
(short distance)

NB: no proof of confinement. We simply never observed quarks as free particles

Recap

The formulation of QCD as a non-abelian Quantum Field Theory allows to

- Describe the hadron spectrum
- Explain experimentally the observed symmetries in the strong iteration
- Avoid mixing between strong and weak interactions
- Obtain a field-theoretical description of the strong force, opening the path to a unified formalism of all fundamental interactions

We have then discussed the UV behaviour of QCD

- discussed renormalisation of UV divergences
- introduced the running of the coupling constant and the beta-function
- discussed measurements of the coupling constant

As we will see, the perturbative description of QCD is very predictive but we understand much less the regime governed by strong dynamics.

Next

Next we'll discuss generic properties of QCD amplitudes

- Soft-collinear divergences (and how they are dealt with)
- Kinoshita-Lee-Nauenberg theorem
- The concept of infrared finiteness
- Sterman Weinberg jets