Cosmology I.

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2022 European school of HEP, Israel
03-12-2022

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Universität Hamburg

Programme of these lectures

Lecture 1: standard cosmology crash course

-FRW metric, Friedmann equation,
-particle decoupling, g_*(T),
-BBN
-hydrogen recombination, photon decoupling,
-qualitative back of the envelope thermal freese-out for a cold relic
-hot relics

classic material,
can be found in many textbooks

Lecture 2: Axion cosmology

-axion-like-particles (ALPs)
-axion dark matter
-relaxion

actual research material,
not yet textbook material but available on ArXiv

Lecture 3: Miscellaneous hot topics

-baryogenesis,
-EW phase transtion,
-primordial gravitational waves

Lecture 1, 03-12-2022

1h20 cosmology crash course:

Important facts you should know about our cosmological history

References

Textbooks:

-Kolb and Turner, The early universe

-Gorbunov and Rubakov, Introduction to the theory of the early universe

-Bailing and Love, Cosmology in gauge field theory and string theory

-Dodelson, Modern cosmology

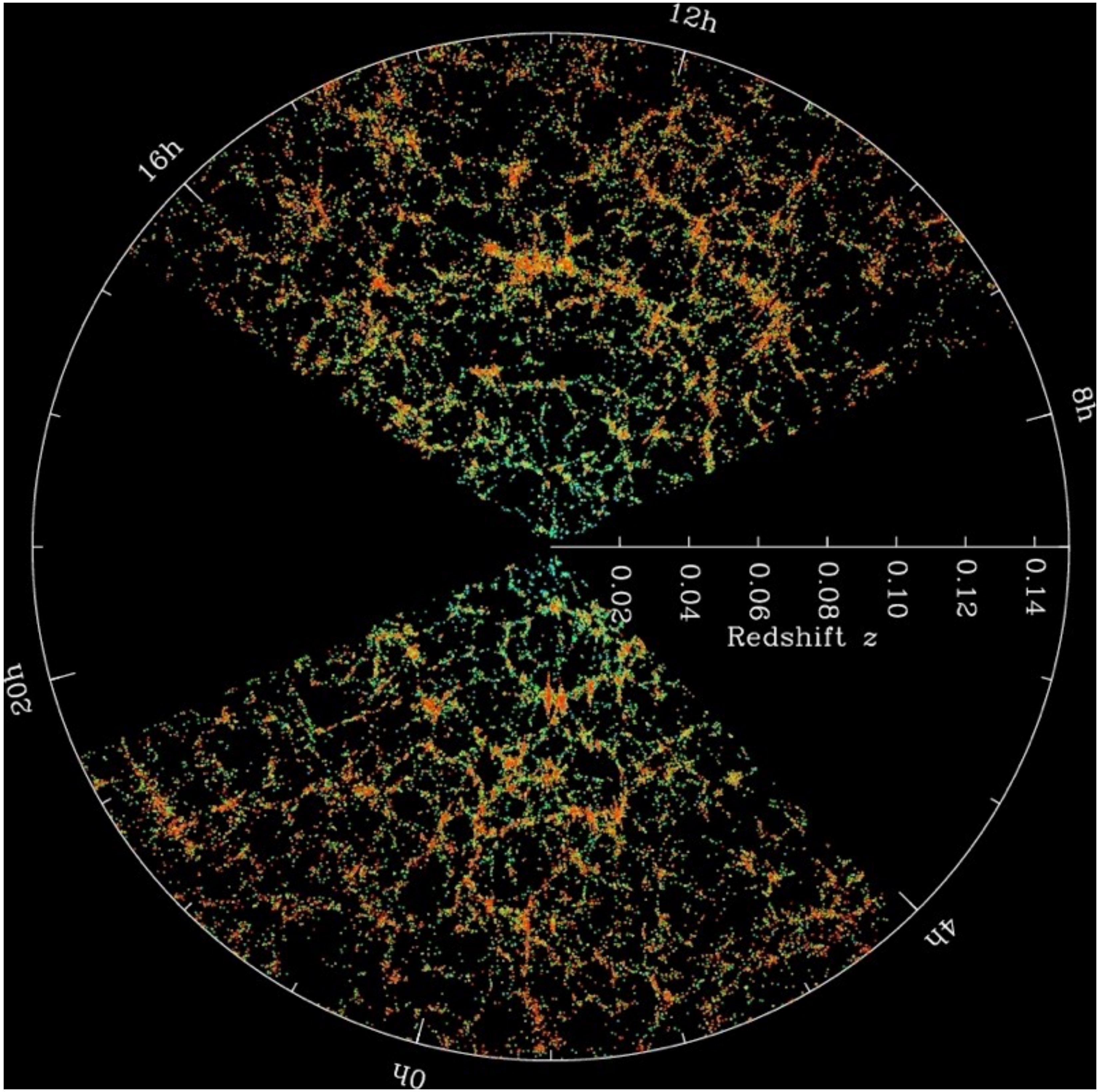
-Weinberg, Gravitation and cosmology

-Weinberg, Cosmology

+ many lecture notes available on the arXiv

recommended : Daniel Baumann's lecture notes

SDSS Galaxy Map



Our universe today

Most important feature: its large-scale homogeneity (no preferred point) and isotropy (no preferred direction)

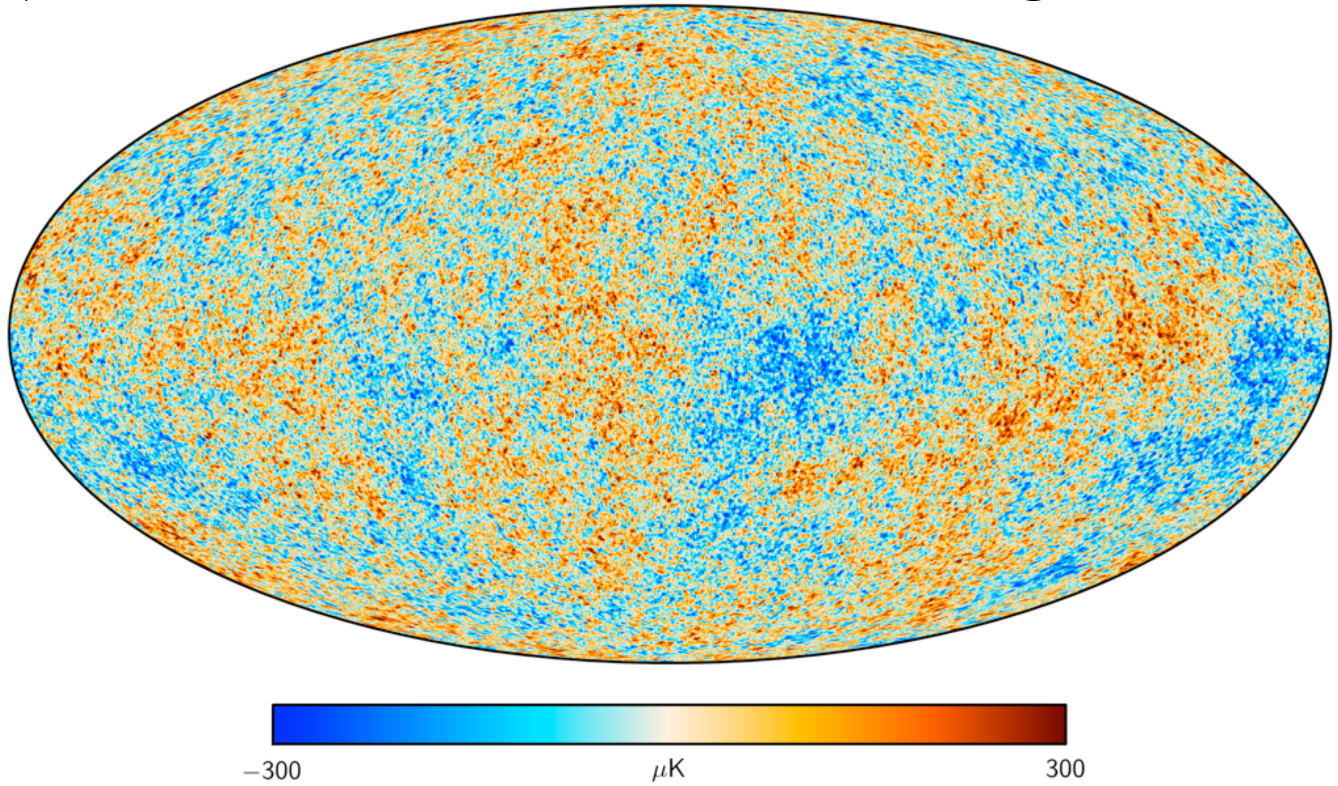
Observable patch of the universe: ~5000 Mpc

- **homogeneous @ large scales (>100 Mpc)**
- **very inhomogeneous @ small scales (<100 Mpc)**

Structures formed by gravitational instability from small initial fluctuations during inflation which set the seeds of future structures. These primordial fluctuations are also imprinted in the Cosmic Microwave Background.

The Cosmic Microwave Background (CMB)

a major success of the standard cosmological model



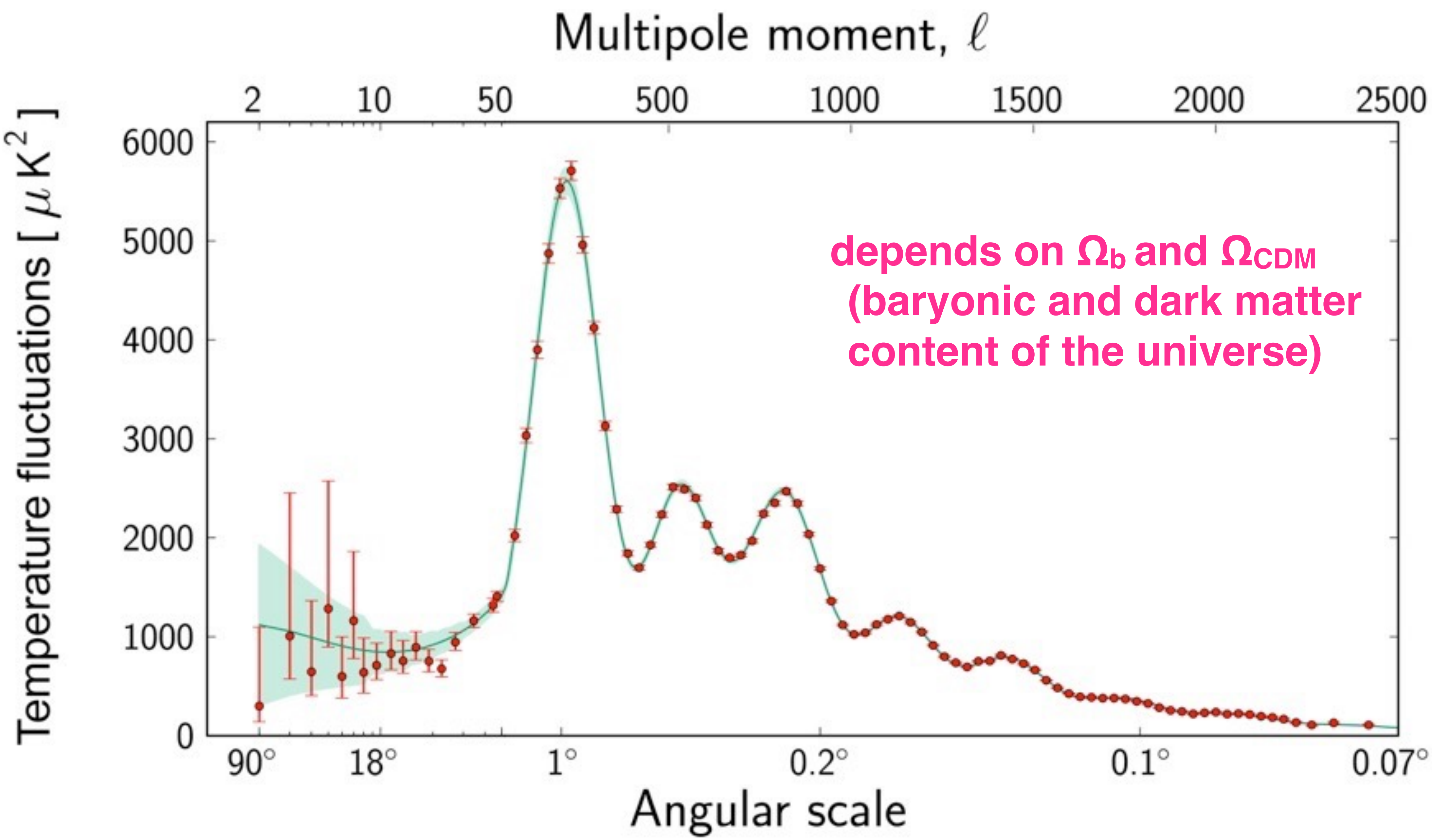
COBE/WMAP/Planck satellites measured CMB temperature fluctuations at the level of $\delta T/T = 5 \times 10^{-5}$ and their angular correlations obtaining an oscillatory pattern.

These oscillations are visualized in l-space by expanding in spherical harmonics,

$$\delta T = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

and plotting $C_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$ **as a function of l**

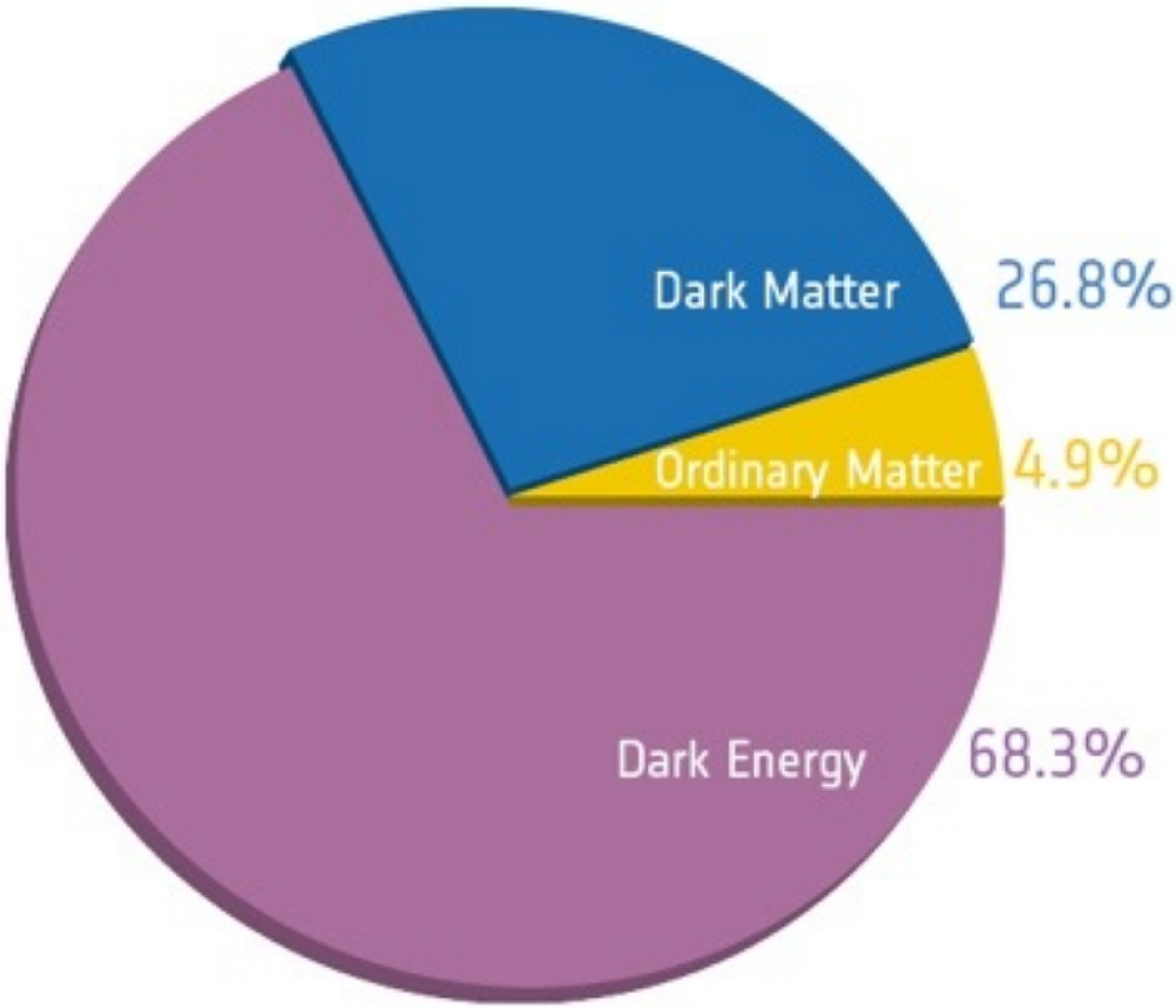
CMB power spectrum

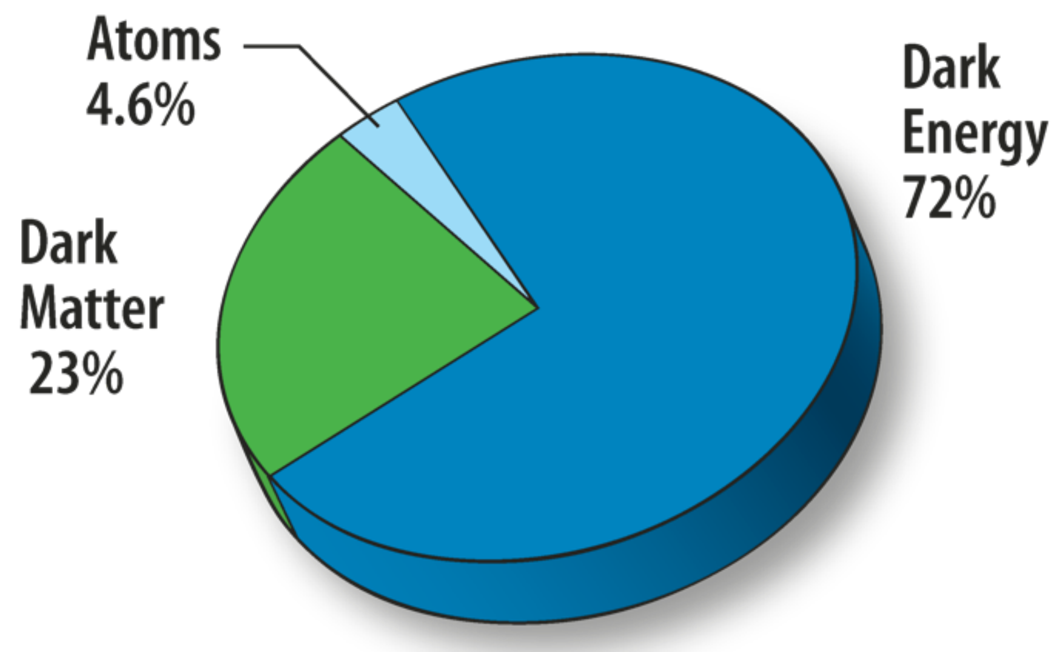


set of peaks -> set of angular scales at which we observe a particularly strong correlation in temperatures.

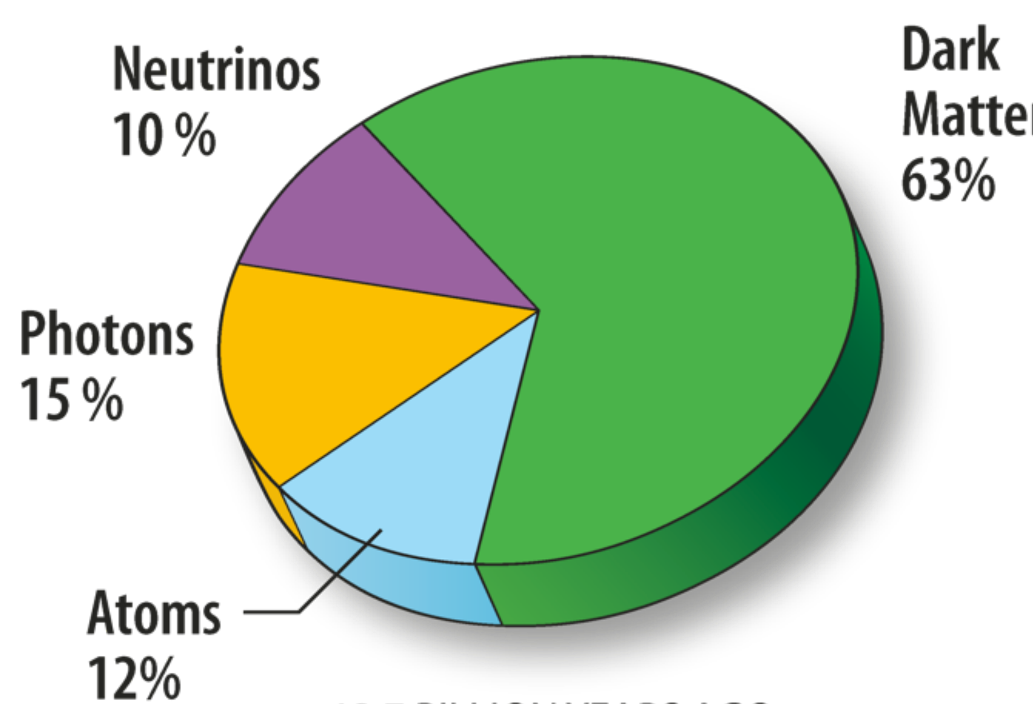
They are generated through the acoustic oscillations.

The energy budget of the universe today

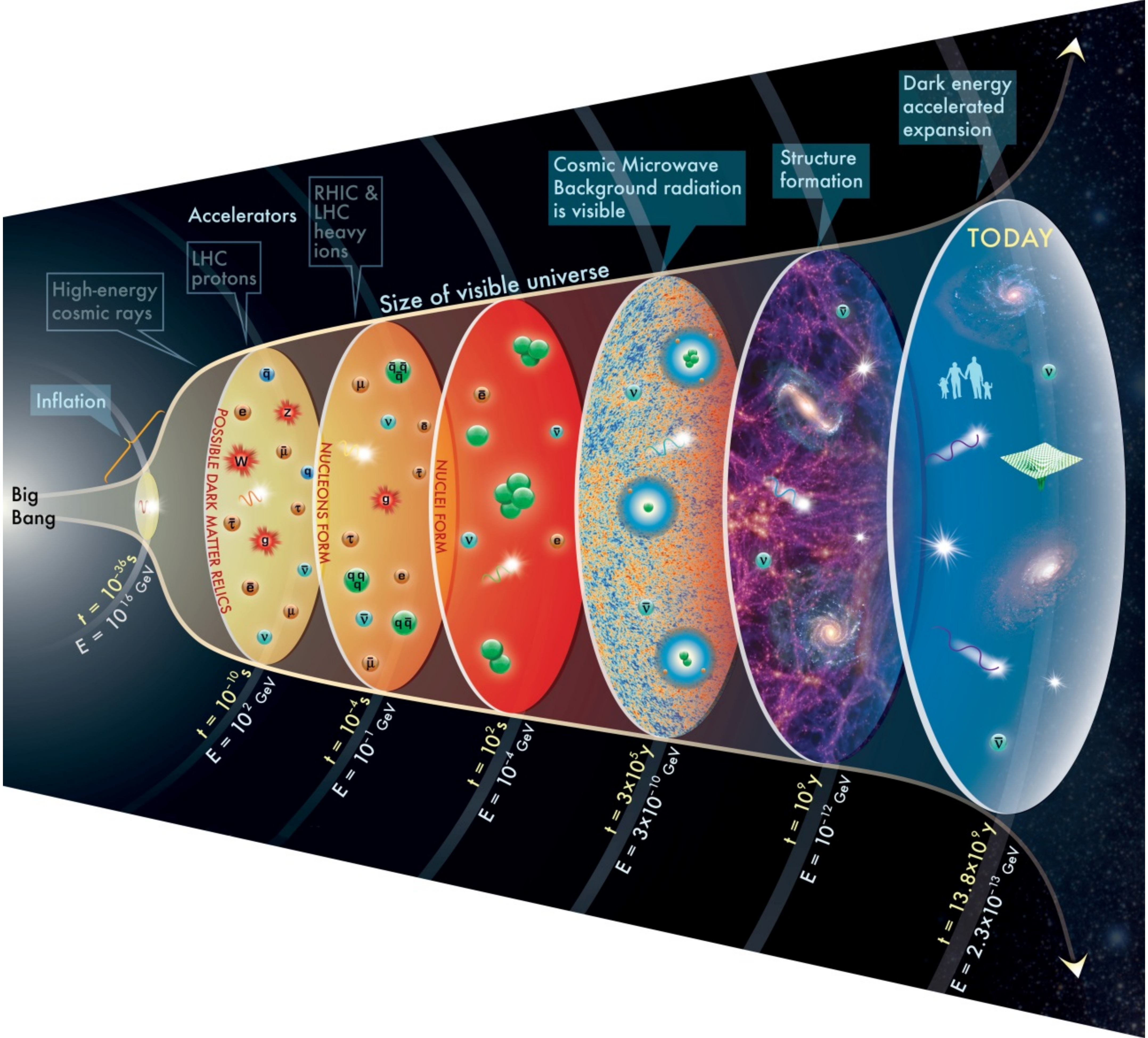




TODAY

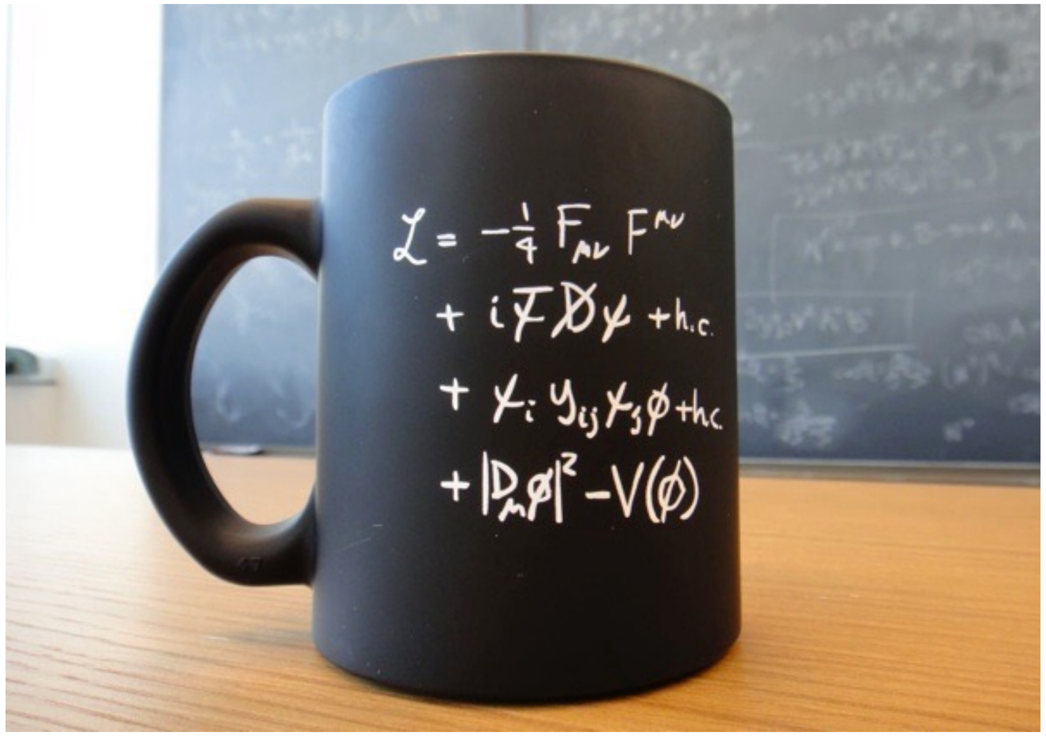


13.7 BILLION YEARS AGO
(Universe 380,000 years old)


Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Size of visible universe
TODAY
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3\times10^{5}$ y
$E = 3\times10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8\times10^{9}$ y
$E = 2.3\times10^{-13}$ GeV

Key events in the thermal history of the universe

Event	time t	redshift z	temperature T	
Inflation	10^{-34} s (?)	–	–	
Baryogenesis	?	?	?	
EW phase transition	20 ps	10^{15}	100 GeV	
QCD phase transition	20 μs	10^{12}	150 MeV	
Dark matter freeze-out	?	?	?	$X + \bar{X} \leftrightarrow \ell + \bar{\ell}\,.$
Neutrino decoupling	1 s	6×10^9	1 MeV	$\nu_e + \bar{\nu}_e \leftrightarrow e^+ + e^-\,,\quad e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e$
Electron-positron annihilation	6 s	2×10^9	500 keV	$e^+ + e^- \leftrightarrow \gamma + \gamma$
Big Bang nucleosynthesis	3 min	4×10^8	100 keV	$n + \nu_e \leftrightarrow p^+ + e^-\,,\quad n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$ $n \leftrightarrow p^+ + \bar{\nu}_e + e^-$ $n + p^+ \leftrightarrow \mathrm{D} + \gamma$ $\mathrm{D} + p^+ \leftrightarrow {}^3\mathrm{He} + \gamma\,,$ $\mathrm{D} + {}^3\mathrm{He} \leftrightarrow {}^4\mathrm{He} + p^+\,.$
Matter-radiation equality	60 kyr	3400	0.75 eV	
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV	$e^- + p^+ \leftrightarrow \mathrm{H} + \gamma$
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV	Thomson $e^- + \gamma \leftrightarrow e^- + \gamma$
Reionization	100–400 Myr	11–30	2.6–7.0 meV	
Dark energy-matter equality	9 Gyr	0.4	0.33 meV	
Present	13.8 Gyr	0	0.24 meV	



The Standard Model of Particle Physics fails to explain:

Matter-antimatter

Dark Matter

Dark Energy

Inflation

Quantum Gravity

All related to physics of the early universe

THEORETICAL COSMOLOGY

Aim: Understanding structure, evolution & origin of the universe

Relies on two "Standard Models"

- of particle physics

- of cosmology (Hubble diagram, BBN, CMB)

Friedmann Robertson Walker (FRW) metric

Mathematical description of homogeneous and isotropic universe

$$\boxed{ds^2 = dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]}$$

SCALE FACTOR

$$k = \begin{cases} +1 & \text{closed universe} \\ 0 & \text{flat universe} \\ -1 & \text{open universe} \end{cases}$$

How do particles evolve in FRW spacetime?

From the geodesic equation $\Rightarrow \quad p \propto \frac{1}{a}$.

Redshifting of photons

$\lambda = h/p$ photon wavelength λ scales as a(t)

Light emitted at time t_1 with wavelength λ_1 will be observed at time t_0 with wavelength

$$\lambda_0 = \frac{a(t_0)}{a(t_1)} \lambda_1$$

Since $a(t_0) > a(t_1)$, the wavelength of the light increases, $\lambda_0 > \lambda_1$.

Redshift parameter $$z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} .$$

$$\boxed{1 + z = \frac{1}{a(t_1)}}$$

convention:

$a(t_0) \equiv 1$

Friedmann equation

Einstein-Hilbert action

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g}\, R$$

R: Ricci scalar
g: metric determinant

Energy-momentum tensor of perfect fluid

$$T_{\mu\nu} = (\rho + P)\, U_\mu U_\nu - P\, g_{\mu\nu}$$

fluid at rest: $U^\mu = (1, 0, 0, 0)$

Einstein Equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

ρ: total energy density
P: pressure

Friedmann Equation

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}.$$

$$H \equiv \frac{\dot{a}}{a}$$

Notation: $8\pi G = \frac{1}{M_{pl}^2}$

$$\boxed{H \sim \frac{\sqrt{\rho}}{M_{pl}}}$$

for k=0

A flat universe (k = 0) corresponds to the following critical density today

$$\rho_{\mathrm{crit},0} = \frac{3H_0^2}{8\pi G}$$

We use the critical density to define dimensionless density parameters

$$\Omega_{a,0} \equiv \frac{\rho_{a,0}}{\rho_{\mathrm{crit},0}}\,, \qquad a = r, m, \Lambda, \ldots$$

i.e. radiation, matter, cosmological constant

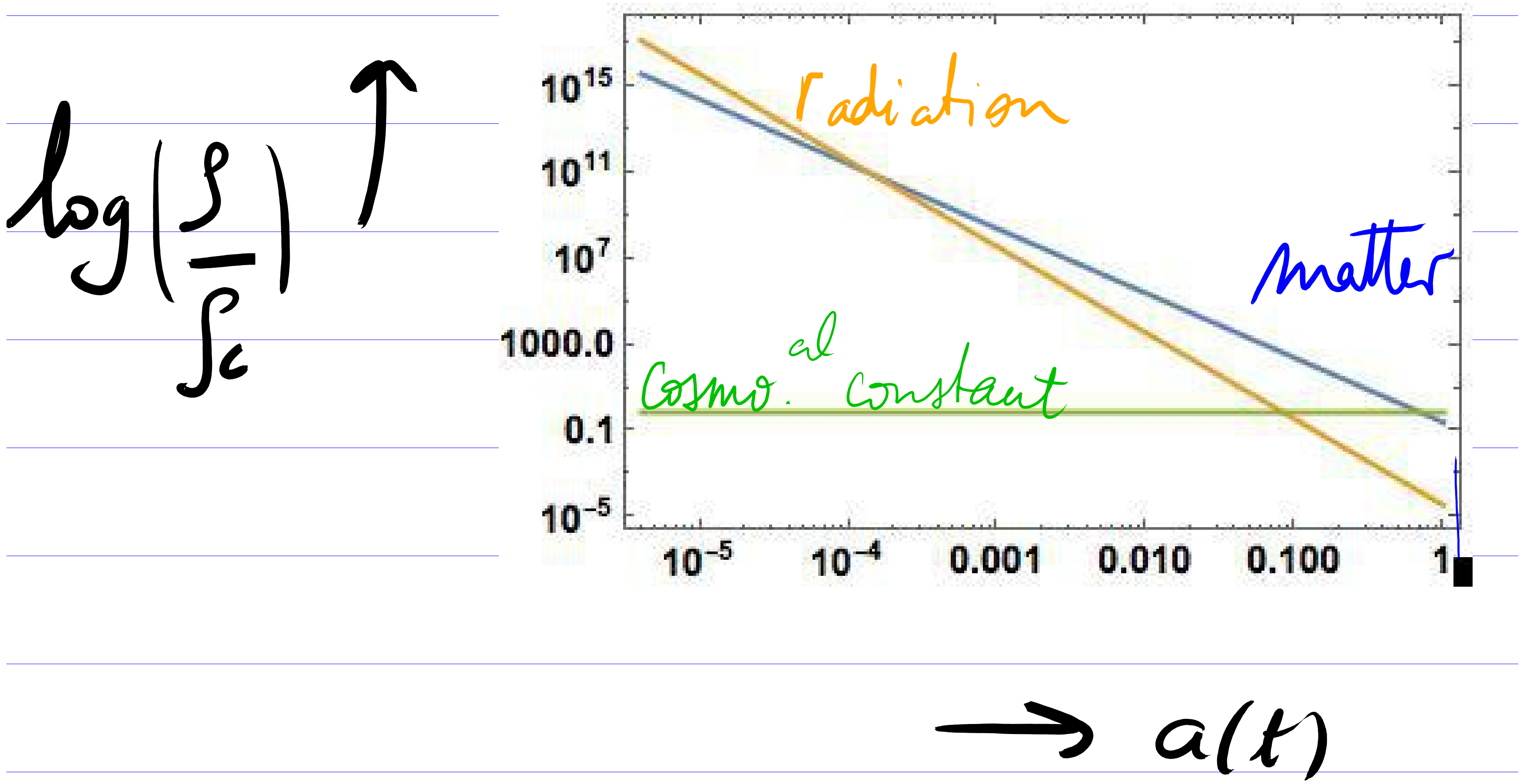
Most cosmological fluids can be parameterised in terms of a constant equation of state:

$$w = P/\rho.$$

This includes cold dark matter (w = 0), radiation (w = 1/3) and vacuum energy (w = -1).

In that case $\rho \propto a^{-3(1+w)} = \begin{cases} a^{-3} & \text{matter} \\ a^{-4} & \text{radiation} \\ a^{0} & \text{vacuum} \end{cases}$

(follows from the continuity equation $\dot{\rho} = -3(\rho + p)\frac{\dot{a}}{a}.$)



Friedmann equation can then be written as

$$H^2(a) = H_0^2 \left[\Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda,0}\right]$$

Age of universe: t ~ H

Age of universe at electroweak epoch

$$H \sim \frac{\sqrt{s}}{M_{pl}} \sim \frac{(100\,\mathrm{GeV})^2}{10^{19}\,\mathrm{GeV}}$$

$$H \sim 10^{-15}\,\mathrm{GeV}$$

$$t \sim 10^{15}\,\mathrm{GeV}^{-1} \sim 10^{15} \times 10^{-24}\,\mathrm{s} \sim 10^{-9}\,\mathrm{s}$$

Particle decoupling

To understand the universe history, we compare the rate of particle interactions

Whenever

Particles decouple whenever

$$\Gamma = n \, \sigma \, v$$

number density of particles

interaction cross section

relative velocity

$[\Gamma] = 1$

$[n] = 3$

$[\sigma] = -2$

dimensionless

relativistic species $\quad n = \dfrac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 & \text{bosons} \\ \frac{3}{4} & \text{fermions} \end{cases}$

non-relativistic species $\quad n = g \left(\dfrac{mT}{2\pi} \right)^{3/2} e^{-m/T}$

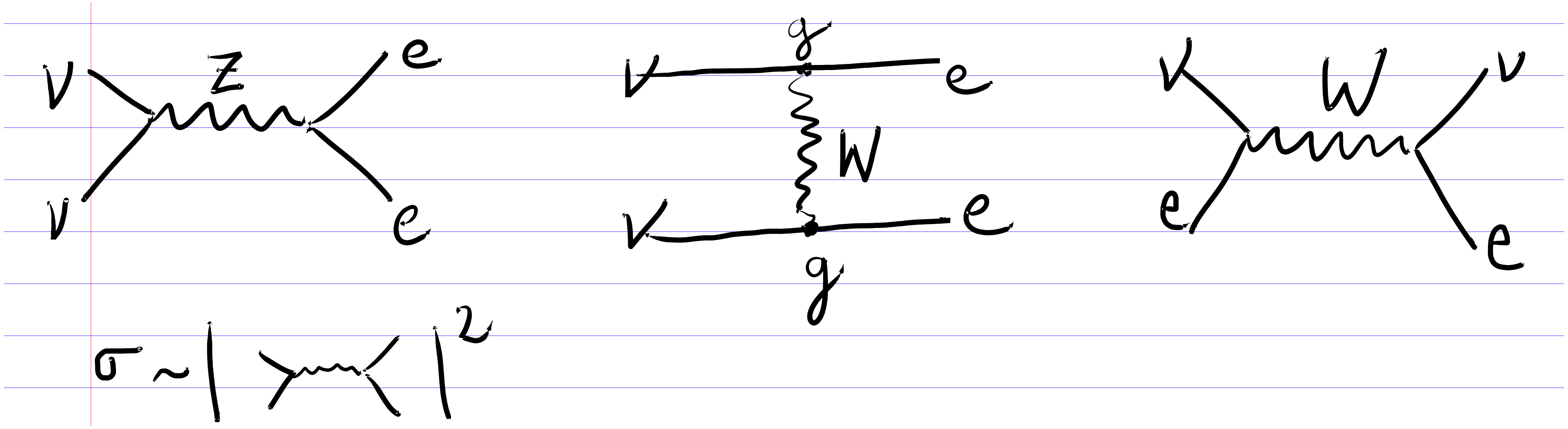
At high temperature T
relativistic: v~1, n~T^3

$$H = \sqrt{\frac{\rho}{3M_{pl}^2}}$$

In a radiation-dominated universe

$$H \sim \frac{T^2}{M_{pl}}$$

Decoupling of electroweak interactions



At T ≳ 100 GeV (massless gauge bosons) $\sigma \sim \dfrac{\alpha^2}{T^2}$

$$\Gamma = n\sigma v \sim T^3 \times \frac{\alpha^2}{T^2} = \alpha^2 T\,.$$

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{\rm pl}}{T} \sim \frac{10^{16}\,{\rm GeV}}{T}$$

all SM particles are in thermal equilibrium

At T ≲ 100 GeV $\sigma \sim G_F^2 T^2$

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{\rm pl} T^3}{M_W^4} \sim \left(\frac{T}{1\,{\rm MeV}}\right)^3$$

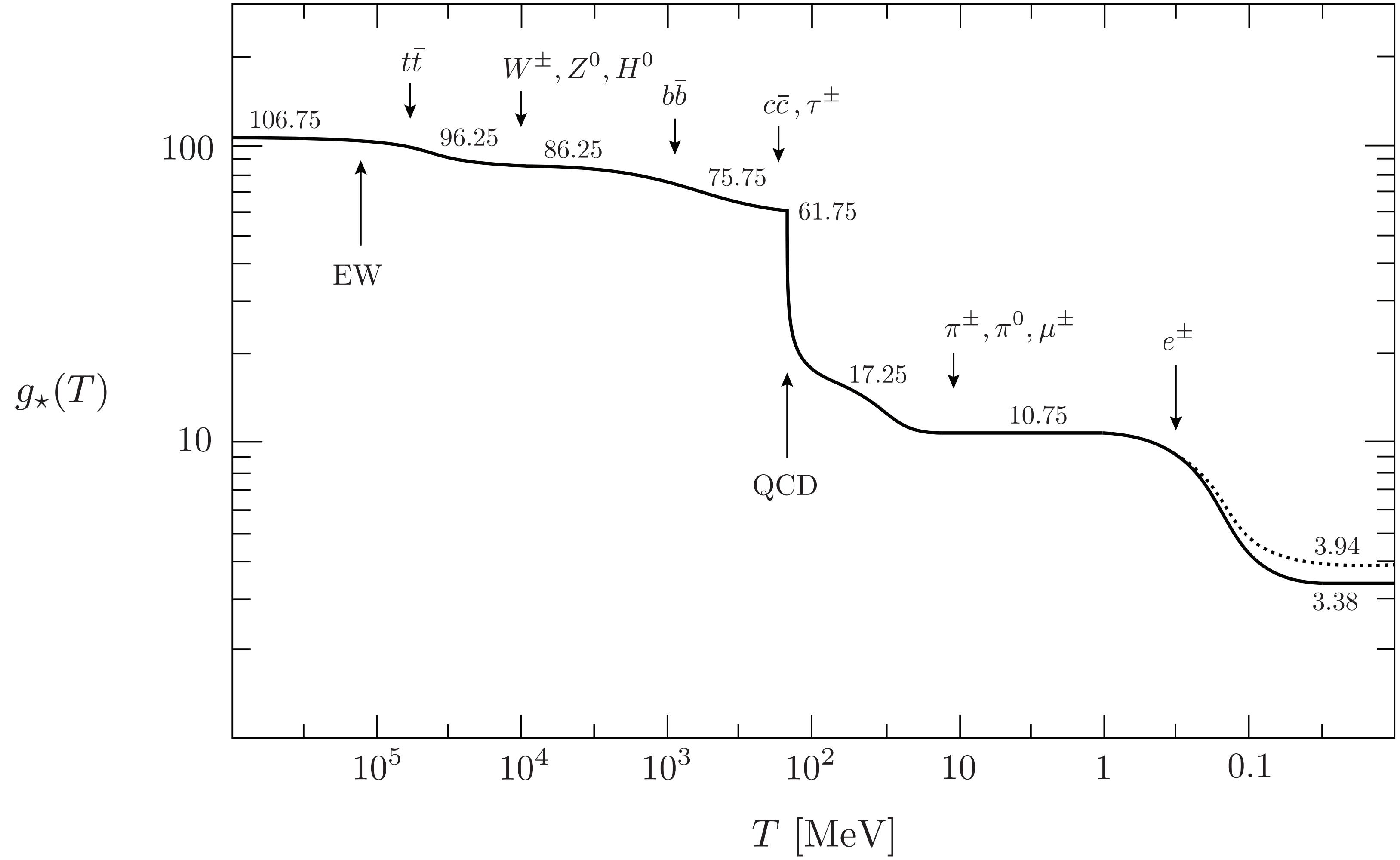
EW interactions decouple at T~1 MeV

Total energy density of the Standard Model plasma at thermal equilibrium

Total radiation energy density of all relativistic species in equilibrium

$$\boxed{\rho_r = \sum_i \rho_i = \frac{\pi^2}{30} g_\star(T)\, T^4}$$

evolution of number of effective relativistic degrees of freedom in the SM



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Big Bang Nucleosynthesis

Big Bang Nucleosynthesis

[Check PDG.LBL.GOV (updated review on BBN)]

Predicts abundances of light elements H, ^{2}H, ^{3}He, ^{4}He, ^{7}Li.

One of the most important tests of the Standard Cosmological Model.

Was the best method to estimate $\Omega_{baryons}$ before accurate CMB measurements by WMAP.

H: 74 % of all baryonic matter energy density
most abundant element in known universe

^{4}He : second most abundant element in known universe (24 %)

Only 2% made of heavier elements and generated by stellar processes

Abundances of C, N, O vary a lot depending on location while ^{4}He is same everywhere -> “primordial”

Abundances produced during BBN: “primordial”

Measurements of abundances: based on spectra (emission lines) of interstellar clouds and stellar surface.

Effect of chemical evolution has to be subtracted to get 'primordial abundance'

^{3}He and ^{7}Li : produced and destroyed in stars so hard to measure primordial abundances.

^{7}Li : measured from population II stars, thought to retain primordial abundances

^{2}H (also noted D): measured from quasar spectra. Necessarily primordial. Cannot be produced/destroyed in stars: too fragile, binding energy is too small .

→ D very sensitive to Ω_{baryons}. Excellent baryometer !

Ω_{baryons} related to matter antimatter asymmetry of the universe

$$\eta \equiv \frac{n_p - n_{\overline{p}}}{n_\gamma}$$

Binding energy of lightest nuclei

^{A}Z	B (in MeV)	g	
^{2}H	2.22	3	boson (spin 1)
^{3}H	8.48	2	fermion (spin 1/2)
^{3}He	7.72	2	fermion (spin 1)
^{4}He	28.3	1	boson (spin 0)
^{12}C	92.2	1	boson

Naively we expect formation of nuclei from the proton-neutron plasma to occur at T ~ 1 MeV.

However it happens at much lower T.

Neutron to proton ratio

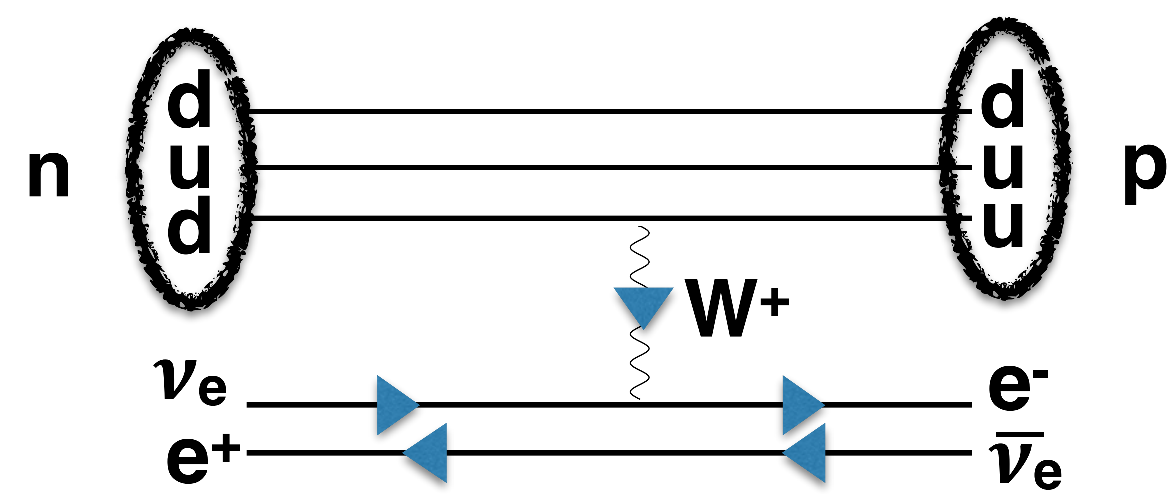
Start at $m_p \gtrsim T \gtrsim 10$ MeV   **-after nucleon-antinucleon annihilation**

-when ν's are at equilibrium

Total number of nucleons is then constant due to baryon # conservation.

p and n are non-relativistic. They are converted to each other by weak interactions.

- $n + \nu_e \leftrightarrow p + e^-$
- $n + e^+ \leftrightarrow p + \overline{\nu}_e$
- $n \leftrightarrow p + e^- + \overline{\nu}_e$



Define $$Q \equiv m_n - m_p = 1.293 \text{ MeV}$$

$$\left.\frac{n_n}{n_p}\right|_{eq} = e^{-Q/T}$$ **($m_n \approx m_p$)**

Fortunately Q ~ O(T_F)
Amazing coincidence

T_F freese-out of weak interactions
which depends on G_F , M_{Pl} , m_u , m_d

So depletion of neutrons will be avoided. CRUCIAL INPUT FOR BBN!
And we are lucky n has not yet decayed (τ_n = 886.7 s)

Remember T_F estimated when $$\underset{(\Gamma_{n\leftrightarrow p})}{G_F^2 T^5} \sim H \sim g_* T^2/M_{Pl}$$

T_F ~ 1 MeV (This depends on g*!) —> BBN constrains g*

$$e^{-Q/T_F} \approx \frac{1}{6}$$

Deuterium production

$$n + p \leftrightarrow D + \gamma$$

$m_D \approx 2m_n \approx 2m_p \approx 1.9$ GeV

$B_D = m_n + m_p - m_D = 2.22$ MeV crucial!

$$\left.\frac{n_D}{n_p}\right|_{eq} \approx \eta \left(\frac{T}{m_p}\right)^{3/2} e^{B_D/T}$$

"Deuterium Bottleneck"

when $n_D/n_p \sim O(1)$: start of BBN

$$\left.\frac{n_D}{n_p}\right|_{eq} \approx \eta \left(\frac{T}{m_p}\right)^{3/2} e^{B_D/T}$$

η inhibits production of Deuterium until T drops well below B_D

Even if T is well below B_D , the photons of the high energy tail of the photon distribution efficiently destroy D

Only when T ≲0.07 MeV, D becomes important

t_{nuc} ~ 330 s

At that time neutrons have started to decay

Neutron Decay

$$X_n \equiv \frac{n_n}{n_n + n_p} \approx 1/6 \approx 0.17$$

$$X_n(t_{nuc}) = \frac{1}{6} \times e^{-t/\tau_n} = \frac{1}{6} e^{-330s/886.7s}$$ **~ 1/8**

$$\tau_n = 886s$$

Helium production

(Only when there is sufficient deuterium)

- $D + p \to {}^3He + \gamma$
- $D + D \to {}^3He + n$
- $D + {}^3He \to {}^4He + p$

$B_{He} > B_D$ so Helium produced immediately after Deuterium

So most of the remaining neutrons are processed into ^{4}He

Since 2 neutrons go into one nucleus of ^{4}H: $n_{^4He} = \frac{1}{2} n_n(t_{nuc})$

$$\frac{n_{He}}{n_H} = \frac{n_{He}}{n_p} \sim \frac{\frac{1}{2} X_n(t_{nuc})}{1 - X_n(t_{nuc})} \sim \frac{1}{2} X_n(t_{nuc}) \sim \frac{1}{16}$$

result is usually expressed as mass fraction of Helium

$$\frac{4 n_{He}}{n_H} \sim \frac{1}{4}$$

Note: ^{3}H and D decrease with η because they fuse

Small η delays BBN —> smaller fraction of ^{4}He

Lithium production $\quad {}^4He + {}^3He \rightarrow {}^7Be + \gamma$

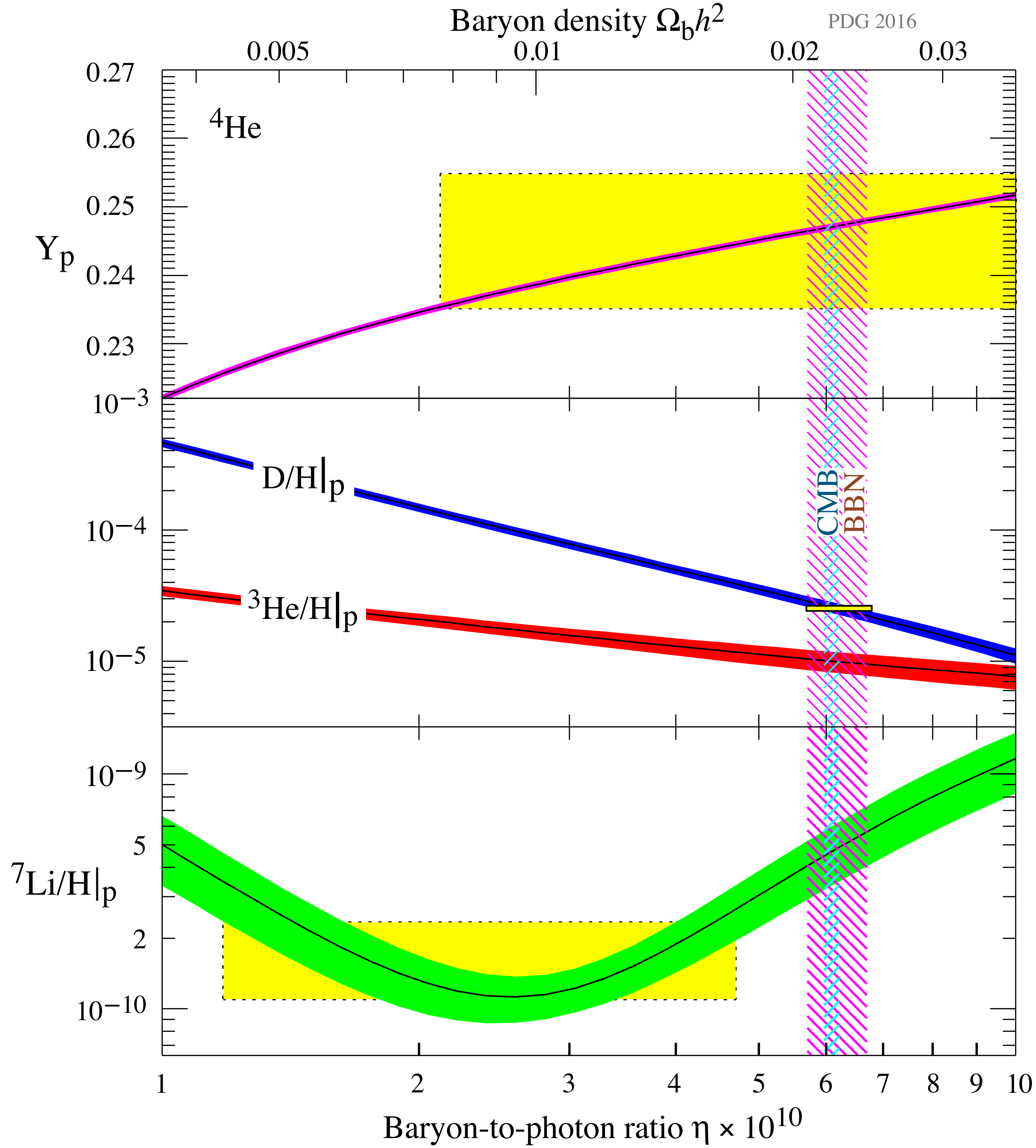
$$^7Be + e^- \rightarrow {}^7Li + \nu_e$$

Lithium destruction $\quad {}^7Li + p \rightarrow {}^4He + {}^4He$

No heavier elements: Coulomb barrier shuts off nuclear reaction at T $\lesssim$30 keV before there is time for heavier elements to form

BBN

Comparaison between theory and observations.



The primordial abundances of ^{4}He, D, ^{3}He, and ^{7}Li as predicted by the standard model of Big-Bang nucleosynthesis—the bands show the 95% CL range Boxes indicate the observed light element abundances. The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN concordance range (both at 95% CL).

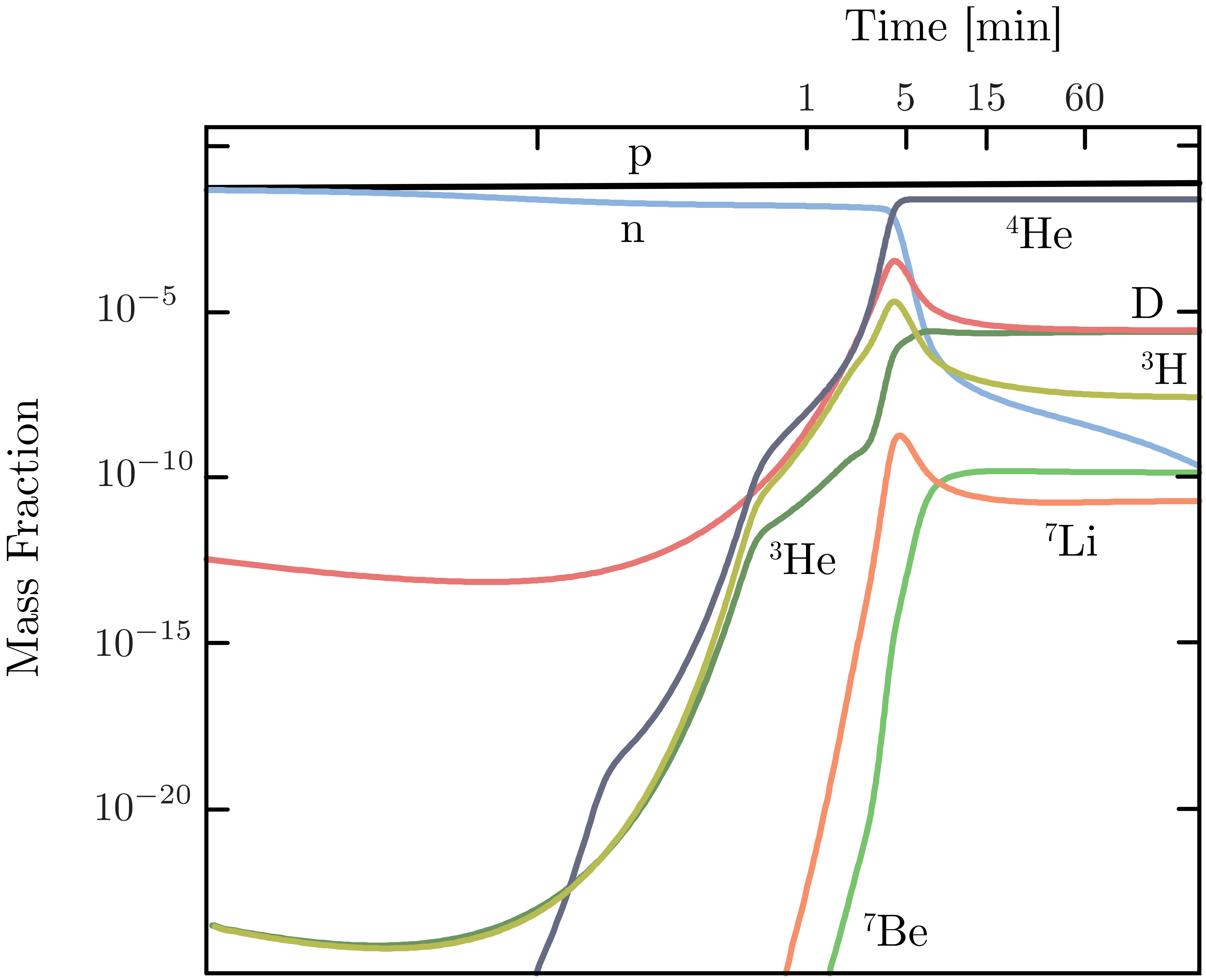


Figure 3.11: *Numerical results for the evolution of light element abundances.*

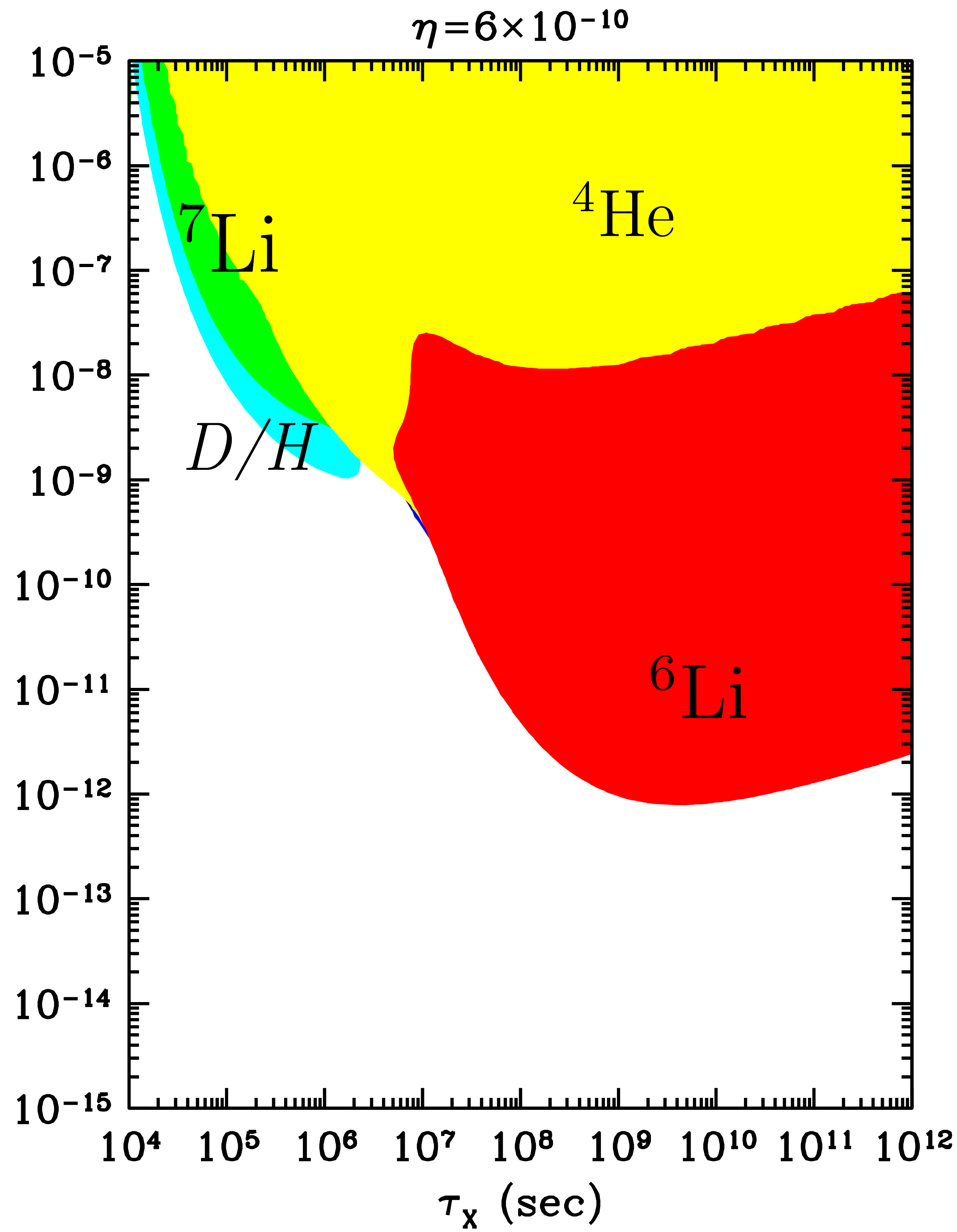
picture from D. Baumann's lectures.

BBN constraints on Unstable Relic Particles

astro-ph/0211258

$$\zeta_X \equiv \frac{n_X^0}{n_\gamma^0} M_X$$

would-be number density of X today if it had not decayed



Exercise

Effect of proton-neutron mass difference on Big Bang Nucleosynthesis

Suppose the difference in rest energy of the neutron and proton were $Q = (m_n - m_p) = 0.129$ MeV instead of 1.29 MeV, with all other physical parameters unchanged. What is the proton to neutron abundance at the time of neutron freese-out?

Estimate Y_{max} the maximum possible mass fraction of ^{4}He, assuming that all available neutrons are incorporated into ^{4}He nuclei , i.e. that there were no neutron decays (they would in fact be stable as Q is less than the electron mass).

Can you think of the conse- quences in this case?

Note about BBN

Baryon to photon ratio $$\eta \equiv \frac{n_b}{n_\gamma} = \frac{n_{b,0}\ a^{-3}}{n_{\gamma,0}(T/T_0)^3} = \left(\frac{T_0}{aT}\right)^3 \eta_0$$

as long as T ~ 1/a → **η=cst**

$\eta_{BBN} = \eta_{CMB} = \eta_0$!

From CMB and BBN: η ~ 6 . 10^{-10} <<1

Huge number of ɣ's per baryon

but $$\frac{\rho_b}{\rho_\gamma} \sim \frac{m_b n_b}{T n_\gamma} \sim \eta \frac{m_b}{T} \neq \text{const}$$

POST-BBN ERA

Recombination & photon decoupling

POST-BBN ERA

After BBN, we are left with radiation (photons) and matter (e^-,protons and nuclei)

In radiation era t=H/2 $$T = \frac{1.56}{g_*^{1/4}}\sqrt{\frac{1\ \mathrm{s}}{t}}\ \mathrm{MeV} = 1.15\sqrt{\frac{1\ \mathrm{s}}{t}}\ \mathrm{MeV}$$ **after e^+e^- annihilation g_*=3.36**

T= 100 keV @ 2 minutes
T= 10 keV @ 4 hours
T= 1 keV @ 2 weeks
T= 100 eV @ 4 years
T= 10 eV @ 400 years
T= 1 eV @ 40 Kyears

Later (T~0.7 eV) matter-radiation equality, see exercise sheet #3.

Hydrogen recombination $p + e^- \rightarrow H + \gamma$

when universe becomes transparent

will lead to release of CMB (when ɣ mean free path>Horizon)

Formation of first atom: $p + e^- \rightarrow H + \gamma$

Baryons and ɣ at equilibrium through electromagnetic reactions

At the same time, rapid interactions between ɣ and e⁻

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

keep ɣ and matter in interaction -> opaque universe

$\Gamma_\gamma \approx n_e \sigma_T$ **Thomson cross section**

e⁻ and ɣ decouple when $H \sim \Gamma_\gamma$

Evolution of the e^- fraction

Consider T> 1 eV

e^-, p and H are all non-relativistic

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-(m_i-\mu_i)/T}$$

After BBN, there are no free neutrons, they have either decayed or combined with p to form He, so

$$n_b \approx n_p + n_H = n_e + n_H$$

Binding energy of Hydrogen $B_H = m_p + m_e - m_H = 13.6$ eV

$n_p = n_e$ (electrical neutrality of universe)

$$\left.\frac{n_H}{n_e^2}\right|_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T}$$

Define the fractional ionisation (free e$^-$ fraction) : $$X_e = \frac{n_e}{n_b} = \frac{n_p}{n_p + n_H}$$

$$\boxed{\left.\frac{1 - X_e}{X_e^2}\right|_{eq} = \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}\ \eta\ \frac{2\zeta(3)}{\pi^2}}$$

Saha equation

When $B_H \ll T \ll m_e$, the RHS is $\ll 1$ so $X_e \approx 1$

At large T almost all protons and e$^-$ are free

Because $\eta \ll 1$ and $T/m_e \ll 1$, T needs to fall $\ll$ B before RHS gets large.

Time of recombination

Defined as $X_e|_{rec} = 10^{-1}$ **i.e. 10% of free electrons and $(1-X_e)/X_e^2 = 90\%$**

$$T_{rec} \approx B/40 \approx 0.3 \text{ eV} \ll B$$

(matter era)

$\rightarrow$ **$t_{rec} \approx$290 000 years**

Photon decoupling

photon "last scattering"

takes place when X_e is small enough such that $\Gamma_\gamma \lesssim H$

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\Gamma_\gamma \approx n_e \sigma_T \approx n_b \sigma_T X_e$$

$$\sigma_T = 6.65 \times 19^{-25}\ \mathrm{cm}^2 = 2 \times 10^{-3}\ \mathrm{MeV}^{-2}$$

$$T_{dec} \sim 0.27\ \mathrm{eV}$$

$$X_e(T_{dec}) = 0.01$$

$$t_{dec} \sim 380000\ \mathrm{years}$$

—> CMB emission

Key events in the thermal history of the universe

Event	time t	redshift z	temperature T	
Inflation	10^{-34} s (?)	–	–	
Baryogenesis	?	?	?	
EW phase transition	20 ps	10^{15}	100 GeV	
QCD phase transition	20 μs	10^{12}	150 MeV	
Dark matter freeze-out	?	?	?	$X + \bar{X} \leftrightarrow \ell + \bar{\ell}\,.$
Neutrino decoupling	1 s	6×10^9	1 MeV	$\nu_e + \bar{\nu}_e \leftrightarrow e^+ + e^-\,, \quad e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e$
Electron-positron annihilation	6 s	2×10^9	500 keV	$e^+ + e^- \leftrightarrow \gamma + \gamma$
Big Bang nucleosynthesis	3 min	4×10^8	100 keV	$n + \nu_e \leftrightarrow p^+ + e^-\,, \quad n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$; $n \leftrightarrow p^+ + \bar{\nu}_e + e^-$; $n + p^+ \leftrightarrow \mathrm{D} + \gamma$; $\mathrm{D} + p^+ \leftrightarrow {}^3\mathrm{He} + \gamma\,,$ $\mathrm{D} + {}^3\mathrm{He} \leftrightarrow {}^4\mathrm{He} + p^+\,.$
Matter-radiation equality	60 kyr	3400	0.75 eV	
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV	$e^- + p^+ \leftrightarrow \mathrm{H} + \gamma$
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV	Thomson $e^- + \gamma \leftrightarrow e^- + \gamma$
Reionization	100–400 Myr	11–30	2.6–7.0 meV	
Dark energy-matter equality	9 Gyr	0.4	0.33 meV	
Present	13.8 Gyr	0	0.24 meV	

From BBN, we have determined that $\eta \sim 6 \,.\, 10^{-10}$
(that is independently confirmed by CMB measurements)

What would η be in a symmetric universe?

Baryon to photon ratio $\eta \equiv \dfrac{n_b}{n_\gamma}$

$\Omega_{baryons} = n_b \, m_b / \rho_c$

Notation: $Y \equiv n/s$ comoving number density

s~T^3: entropy density

Since $s \sim a^{-3}$ $Y \sim n \, a^3$

If no entropy is produced $Y_{today} = Y_{freese\text{-}out}$

Calculation of the relic abundance of cold relics

Freese-out of a stable massive particle
Back-of-the-envelope calculation

Cold relic $$n \sim (mT)^{3/2}\, e^{-m/T}$$

Freese-out : $$n\sigma \sim H \longrightarrow \boxed{n_{f.o} \sim \frac{T_{f.o}^2}{\sigma M_{Pl}}}$$

$$x \equiv \frac{m}{T}$$

$$\left(\frac{m^2}{x}\right)^{3/2} e^{-x} \sim \frac{m^2}{x^2}\frac{1}{M_{Pl}\sigma}$$

$$\longrightarrow \quad \sqrt{x}e^{-x} \sim \frac{1}{mM_{Pl}\sigma} \longrightarrow x \sim \log[\sigma m M_{Pl}]$$

x~ 20 … 30

$$\Omega_X = \frac{m_X}{\rho_c} n_X(T=T_0) = \frac{m_X}{\rho_c} T_0^3 \boxed{\frac{n_0}{T_0^3}} = \frac{m_X}{\rho_c} T_0^3 \boxed{\frac{n_{f.o}}{T_{f.o}^3}}$$

$$= \frac{m_X}{\rho_c} T_0^3 \frac{1}{T_{f.o} M_{Pl}\sigma} = \frac{x_{f.o}T_0^3}{\rho_c M_{Pl}\sigma}$$

$$\Omega_X = \frac{m_X}{\rho_c} T_0^3 \frac{1}{T_{f.o} M_{Pl} \sigma} = \frac{x_{f.o} T_0^3}{\rho_c M_{Pl} \sigma}$$

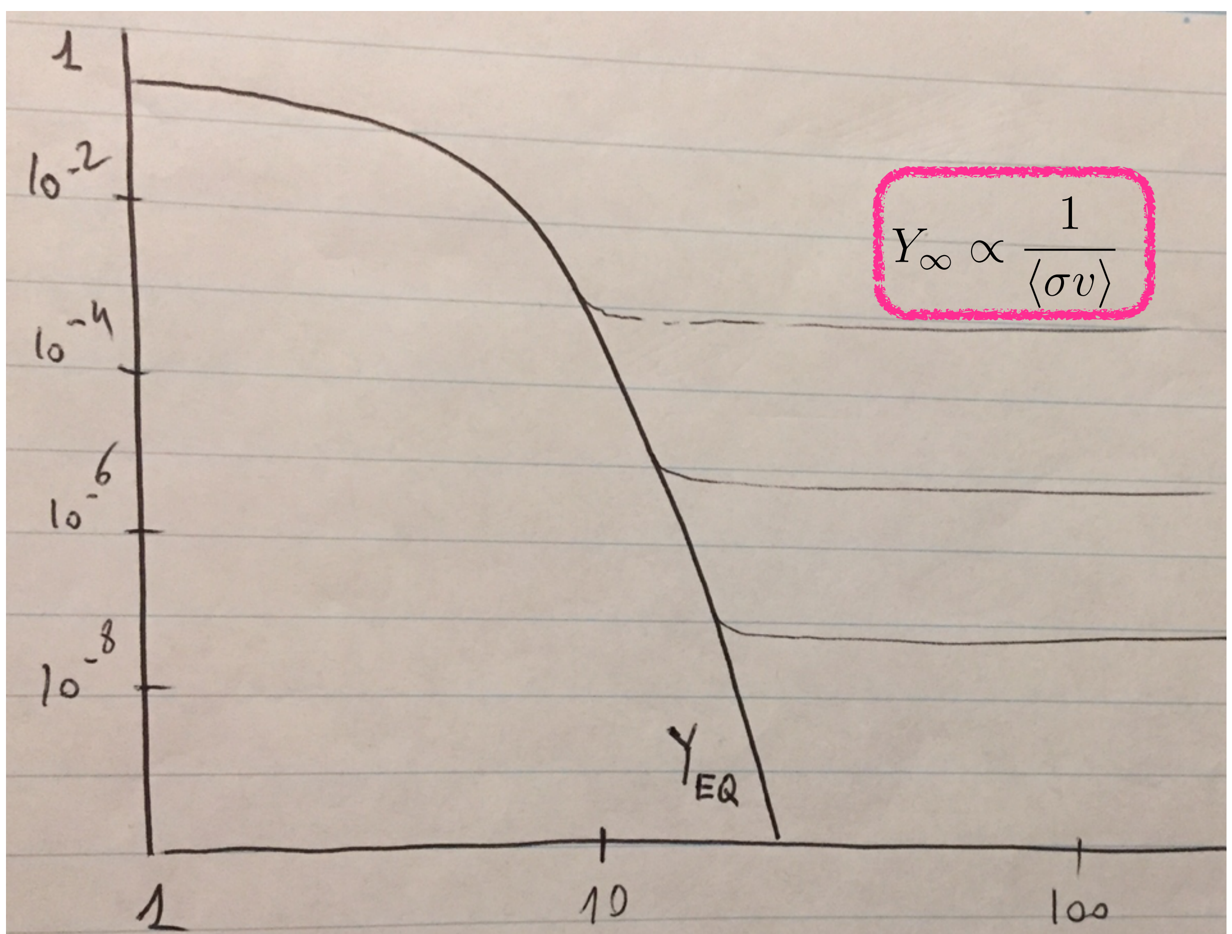
$$\rho_c = 8.098 \times 10^{-47} \mathrm{h}^2 \mathrm{GeV}^4$$

$$T_0 = 2.36 \times 10^{-13} \ \mathrm{GeV}$$

$$\boxed{\frac{\Omega_X}{0.2} \sim \frac{x_{f.o}}{20} \times \left(\frac{10^{-8} \ \mathrm{GeV}^{-2}}{\sigma} \right)}$$

`WIMP miracle’

The famous ‘freese-out plot’



increasing σ

Example of a cold relic: protons

X=p: $\sigma \sim \sigma_\pi \sim m_\pi^{-2} \sim 10^{10}\sigma_{EW}$ **m_π = 135 MeV**

$x_F \sim 40$ $T_F \sim 22$ MeV $Y_\infty \sim 7\times10^{-20}$

but we know from BBN that $Y_\infty \sim \eta/7 \sim 10^{-10}$

our freese-out calculation predicts a baryon number density that is ~ 9 orders of magnitude too small than the measured one.

$\Omega_b \sim 10^{-10}$ in symmetric universe !

~ 10^{-9} smaller than measured

$\rightarrow$ **Existence of a primordial asymmetry**

to prevent the annihilation catastrophe in a symmetric universe

Theory of baryogenesis required

Relic abundance of hot relics

Decoupling of relativistic species

Hot relic: freezes out when species is still relativistic.

The final value of the relic abundance is very insensitive to the details of freese-out.

The abundance is of the same order as that of photons

$$n_\infty \approx 800 \frac{g_{eff}}{g_*(x_F)} \mathrm{cm}^{-3}$$

$$g_{eff} = \begin{cases} g \text{ (bosons)} \\ 3g/4 \text{ (fermions)} \end{cases}$$

Hot relic

$$\rho_X = m_X n_X \approx 800 \frac{g_{eff}}{g_*(x_F)} \left(\frac{m_X}{\text{eV}}\right) \ \text{eV cm}^{-3}$$

$$\rho_c = \ 1.05 h^2 \times 10^4 \text{eV cm}^{-3}$$

$$\boxed{\Omega_X h^2 \approx 0.076 \frac{g_{eff}}{g_*(x_F)} \left(\frac{m_X}{\text{eV}}\right)}$$

proportional to mass & insensitive to interaction cross section

The case of SM neutrinos: hot relics

Neutrinos decouple when T ~ MeV and $g_{*s}=g_*=10.75$

For a single 2-component neutrino species $g_{eff}=2 \times 3/4=3/2$

$g_{eff}/g_{*s}(x_F)=0.140$ $\boxed{\Omega_{\nu,\overline{\nu}} h^2 \approx \frac{m_\nu}{93 \text{ eV}}}$ **(energy density per flavour)**

3 flavours: $n_\nu = 336 \text{ cm}^{-3}$

$\Omega_{\nu,\overline{\nu}} h^2 \approx 0.11$
dark matter → **we would need m_ν~10 eV !**

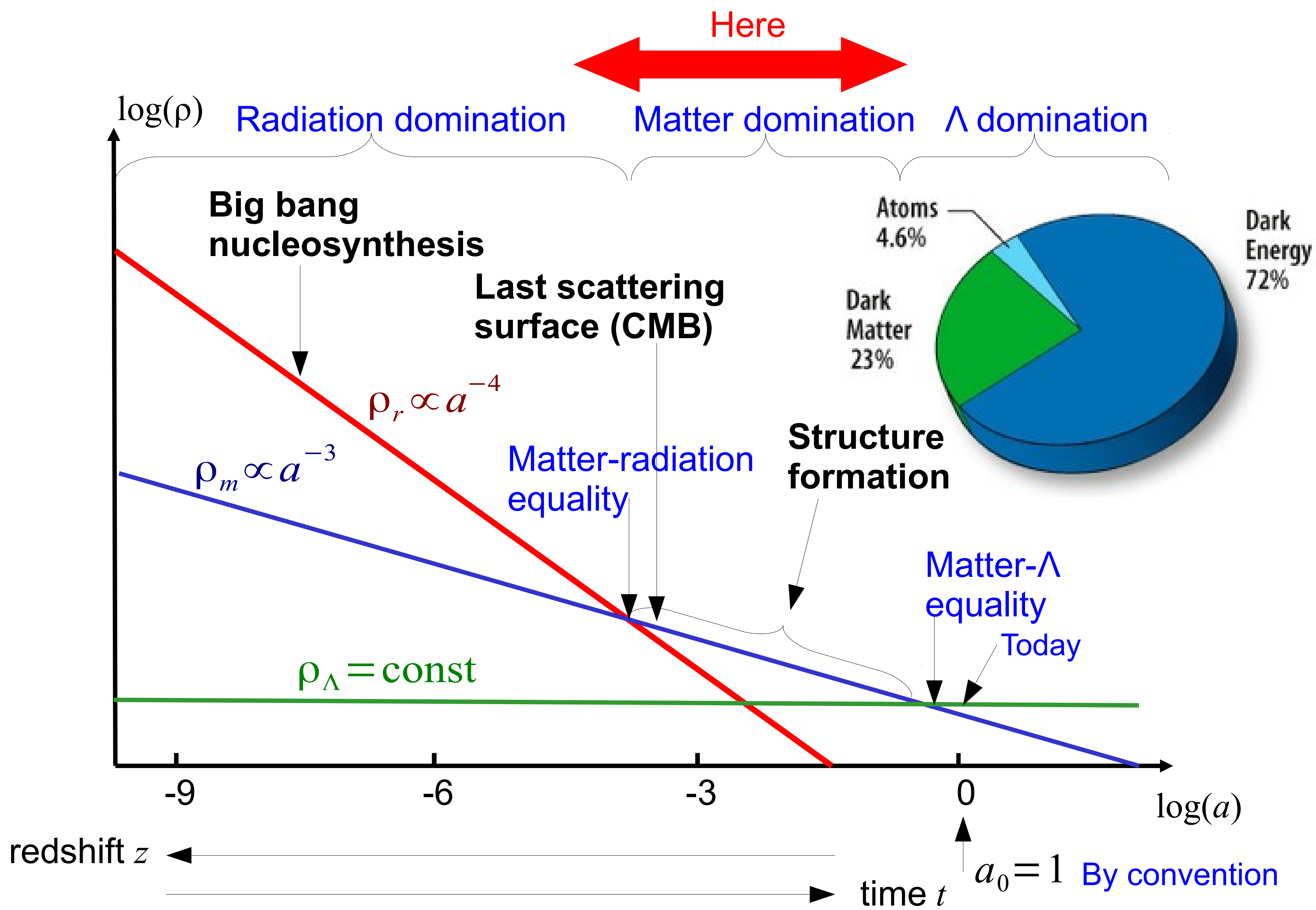
while $m_\nu \lesssim$ 2 eV from Tritium beta-decay → $\Omega_{\nu,\overline{\nu}} h^2 \lesssim 0.02$

Cosmology (CMB): $\sum_i m_{\nu_i} \lesssim 0.12 \text{ eV}$ → $\Omega_{\nu,\overline{\nu}} h^2 \lesssim 0.001$

Cosmological constraints on neutrino mass

From structure formation

Hot Dark Matter prevents formation of structures and excludes light neutrinos as main component of dark matter.



The case of SM neutrinos: hot relics

Neutrinos become non-relativistic at T_{nr} when $m_\nu = \langle p \rangle$

For Fermi-Dirac distribution in relativistic limit $\langle p \rangle = 3.15\ T$

$$\rho = \frac{7}{8}\frac{\pi^2}{30} g_i T^4$$

$$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_i T^3$$

$$\langle p \rangle = \frac{\rho}{n} = \frac{7\pi^4}{180\zeta(3)} T \approx 3.15T$$

$$\mathbf{T_{nr} \sim m_\nu / 3}$$

$$\mathbf{T_{nr} / T_0^\nu = a_0 / a_{nr}}$$

$$T_\nu^0 \simeq 1.7 \times 10^{-4}\ \text{eV} \simeq 1.9\ \text{K}$$

$$\mathbf{T_{nr} = T_0^\nu / a_{nr} \sim m_\nu / 3}$$

$$|\Delta m_{31}^2|^{1/2} > |\Delta m_{21}^2|^{1/2} > T_\nu^0$$

At least two of the neutrino mass eigen states are non-relativistic today

Matter era : $T < T_{eq} \sim 0.75$ eV **$T_{nr} < T_{eq}$** (T_{eq}:matter-radiation equality)

Neutrinos become non-relativistic deep in matter era

Once neutrinos have decoupled from the plasma, they simply travel in free fall in the expanding universe.

Energetic motion of neutrinos destroys formation of small structures and prevents formation of first structures.

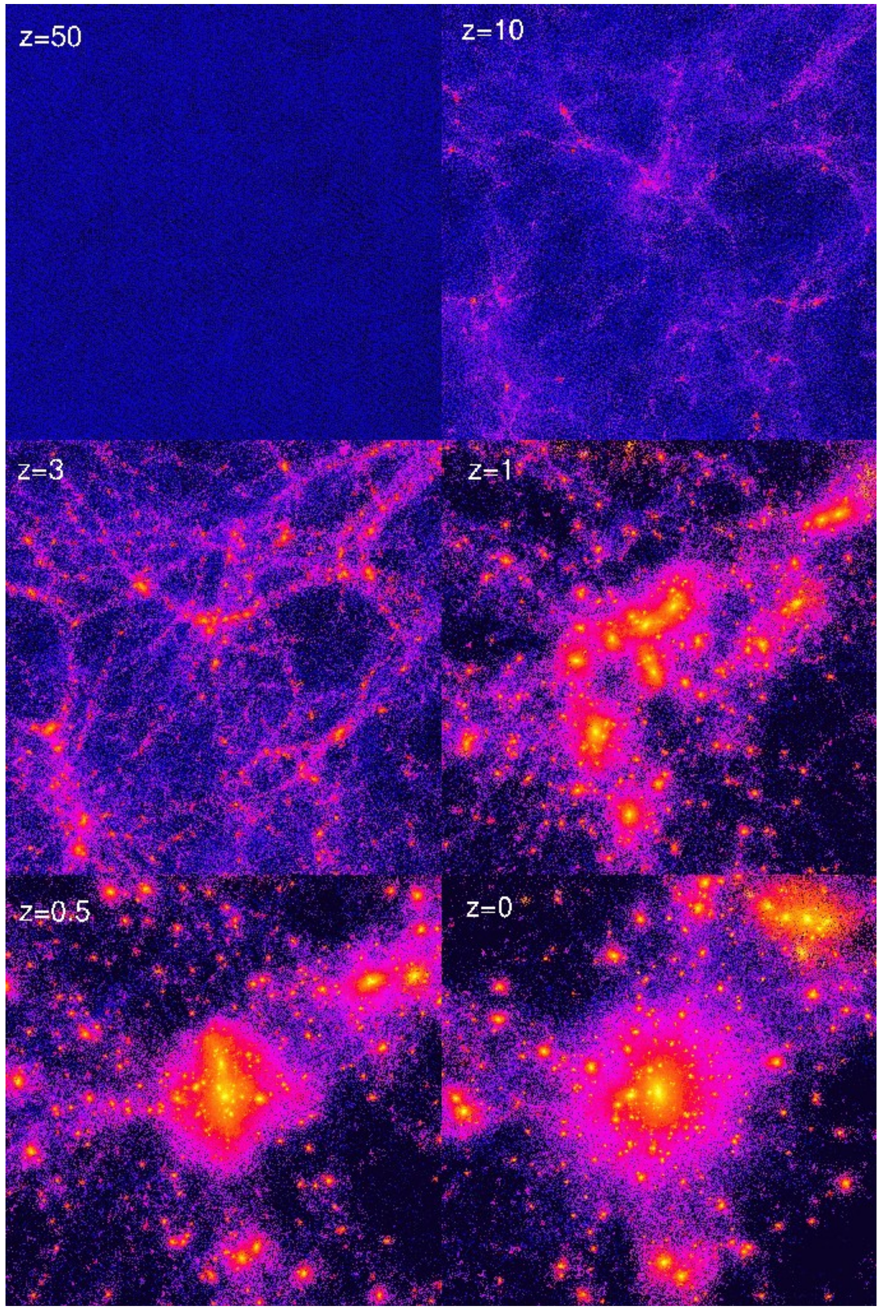
Free streaming of fast-moving neutrinos washes out any inhomogeneities in their spatial distributions that will later become galaxies.

How structures form

The early universe is filled with an almost homogeneous matter density field with tiny random fluctuations.

Perturbations grow via gravitational instability, and eventually form galaxies and galaxy clusters, etc.

Leading theory for the origin of small fluctuations is inflation. (Quantum fluctuations on the inflaton field.)



Free streaming length λ_{FS} : sets minimum scale for structure formation

The distance traversed by a free-streaming particle at time t is

$$\lambda_{FS}(t) = \int_0^t dr = \int_0^t \frac{v(t')}{a(t')} dt'$$

$$a(t)dr = v(t)dt$$

in free fall

Initially v~c, later v ~1/a

Primordial density fluctuations smaller than λ_{FS} get washed out as particles move from overdense to underdense regions, while fluctuations larger than λ_{FS} are unaffected.

Free-streaming ends when neutrinos become nonrelativistic:

$$\begin{aligned} \lambda_{FS} &= \int_0^{t_{NR}} \frac{dt'}{a(t')} + \int_{t_{NR}}^{t_{eq}} \frac{v(t')}{a(t')} dt' \\ &\approx \frac{t_{NR}}{a_{NR}} + \int_{t_{NR}}^{t_{eq}} \frac{a_{NR}}{a^2(t')} dt' \end{aligned}$$

t_{eq} : when structures start to form

$t_{NR} \sim l_H$ **Horizon size when relic becomes non-relativistic:**

$l_H \sim 1/H(T=m) \sim M_{Pl} / m^2$

Corresponding present size

$l_{H,0} = l_H \times (T/T_0) \sim M_{Pl} / (T_0 m)$

= Present maximum size of suppressed density perturbations

$m \sim 1$ keV -> $l_{H,0} \sim 0.1$ Mpc
$m \sim 1$ eV -> $l_{H,0} \sim 100$ Mpc

(1 Mpc= 10^{38} /GeV)

$\lambda_{FS,max}$: **maximum size of objects that could not have been formed in a neutrino dark matter-only universe.**

$$\lambda_{FS}^{\infty} = \int_0^t dr \approx 70 \text{ Mpc} \frac{1 \text{ eV}}{T_{nr}} \approx \frac{210 \text{ Mpc } 1 \text{ eV}}{m_\nu}$$

Structures smaller than ~ 210 Mpc should have been destroyed by a neutrino of mass < 1 eV if they were the main constituents of Dark Matter.

The limit m_ν =1 keV : warm dark matter limit -> free streaming around 0.1 Mpc which is typical size for perturbations that developed into small structures like dwarf galaxies.

CDM:	**objects with**	**λ_{FS} <<**	**protogalaxy**
Warm DM:	**…**	**λ_{FS} ~**	**…**
Hot DM:	**…**	**λ_{FS} >>**	**…**

For hot dark matter:

Larger structures (clusters of galaxies) form first and then fragment into smaller structures.

This sequence of events is in disagreement with observations.

Conclusion: Dark Matter must be cold.

Energy density from thermal relics

("thermal"=that were once in thermal equilibrium)

$$\frac{\Omega_X}{0.2} \sim \frac{x_{f.o}}{20} \times \left(\frac{10^{-8}\ \mathrm{GeV}^{-2}}{\sigma} \right)$$

cold relic

$$\Omega_X h^2 \approx 0.076 \frac{g_{eff}}{g_*(x_F)} \left(\frac{m_X}{\mathrm{eV}} \right)$$

hot relic

Lower bound on Dark Matter Mass

Dark Matter must behave classically to be confined on galaxy scales. DM with De Broglie wavelength > size of dwarf galaxies ~ kpc will prevent their formation

We demand $\lambda <$ kpc -> m v > 1/ kpc

$1\ \mathrm{pc} = 3\times10^{18}\ \mathrm{cm} = 3\times10^{18} / (2\times10^{-14}\ \mathrm{GeV}) = 10^{32}\ \mathrm{GeV}^{-1} = (10^{-32}\ \mathrm{GeV})^{-1}$

$1\ \mathrm{kpc}^{-1} = 10^{-35}\ \mathrm{GeV} = 10^{-26}\ \mathrm{eV}$

$v \sim 10^{-3}$

$mv \sim m\ 10^{-3}$

$m_{DM} \gtrsim 10^{-23}\ \mathrm{eV}$

More stringent bound for fermionic Dark Matter

Pauli exclusion principle.
Phase space density for fermions has a maximum value,

$$M_{\rm halo} = mV \int d^3p f(p) < mV \int d^3p < mV(mv)^3$$

$$v \sim \sqrt{\frac{GM_{\rm halo}}{r_{\rm halo}}}$$

$$M_{\rm halo} < R^3_{\rm halo} m^4 \left(\frac{GM_{\rm halo}}{R_{\rm halo}}\right)^{3/2}$$

$$m > \frac{1}{(G^3 R^3_{\rm halo} M_{\rm halo})^{1/8}}$$

for dwarf galaxies: m> 0.7 keV