Cosmology III.

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Matter-antimatter asymmetry

characterized in terms of the baryon to photon ratio

$$\eta \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma}$$

~ 6. 10

The great annihilation

10 000 000 001
Matter

10 000 000 000
Anti-matter

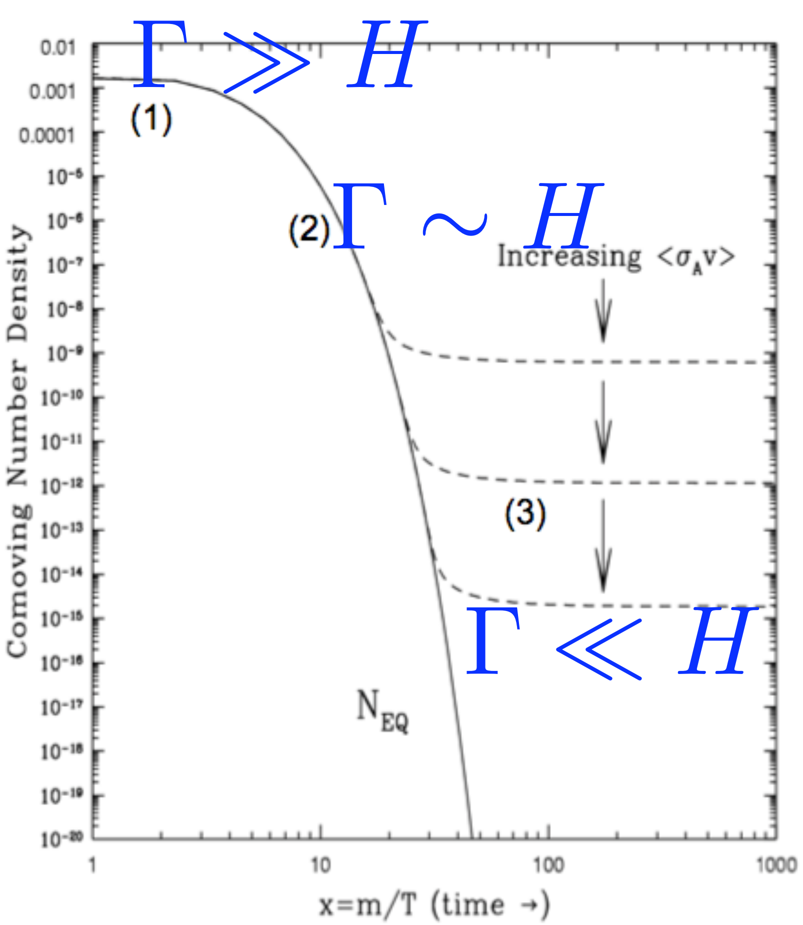
1
(us)

How much baryons would there be in a symmetric universe?

nucleon and anti-nucleon densities are maintained by annihilation processes

$$n + \overline{n} \longleftrightarrow \pi + \pi \longleftrightarrow \gamma + \gamma + \ldots$$

which become ineffective when



$$\Gamma \sim (m_N T)^{3/2} e^{-m_N/T} / m_\pi^2 \sim H \sim \sqrt{g_*} T^2 / m_{Pl}$$

leading to a freeze-out temperature

$$T_F \sim 20 \text{ MeV}$$

$$\frac{n_N}{s} \approx 7 \times 10^{-20}$$

10^9

and there are no antibaryons

-> need to invoke an initial asymmetry

Sakharov's conditions for baryogenesis (1967)

1) Baryon number violation

(we need a process which can turn antimatter into matter)

2) C (charge conjugation) and CP (charge conjugation × Parity) violation

(we need to prefer matter over antimatter)

3) Loss of thermal equilibrium

(we need an irreversible process since in thermal equilibrium, the particle density depends only on the mass of the particle and on temperature --particles & antiparticles have the same mass , so no asymmetry can develop)

$$\Gamma(\Delta B > 0) > \Gamma(\Delta B < 0)$$

Why can't we achieve baryogenesis in the Standard Model?

B is violated

C and CP are violated

but which out-of-equilibrium condition?

no heavy particle which could decay out-of-equilibrium

no strong first-order phase transition

Electroweak phase transition is a smooth cross over

Also, CP violation is too small (suppressed by the small quark masses, remember there is no CP violation if quark masses vanish)

Baryon number violation in the Standard Model

B and L are accidental global symmetries of the Standard Model

$$B = \frac{N_c}{3} \int d^3x \sum_{i=1}^{N_f} (\overline{u}_i \gamma^0 u_i + \overline{d}_i \gamma^0 d_i)$$

$$L_i = \frac{N_c}{3} \int d^3x (\bar{l}_i \gamma^0 l_i + \overline{\nu}_i \gamma^0 (1 - \gamma_5) \nu_i) \qquad i = e, \mu, \tau$$

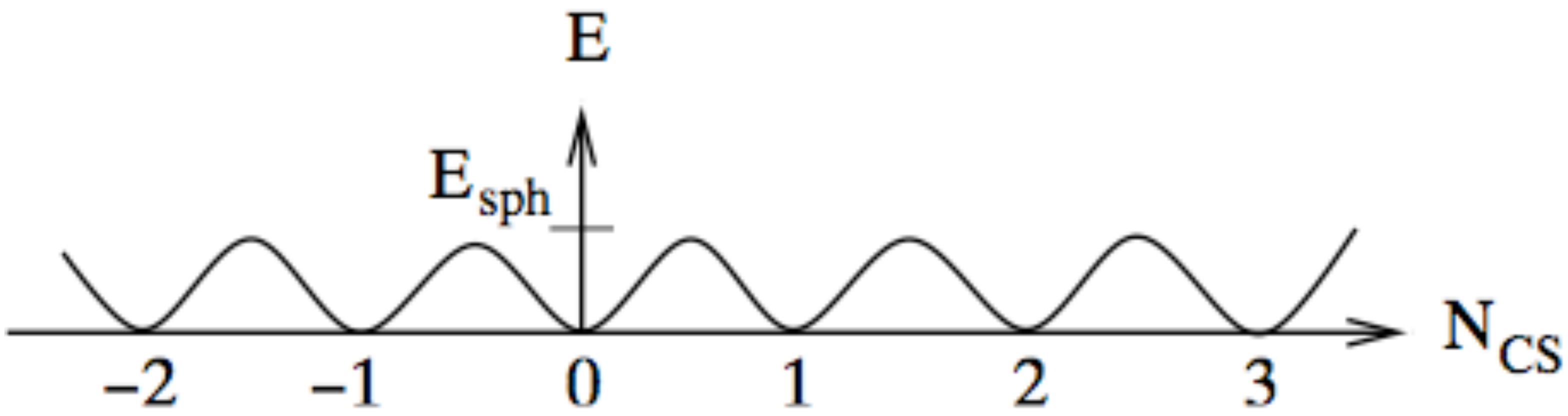
$$L = L_e + L_\mu + L_\tau$$

Non-perturbative (instanton) effects can lead to processes violating (B+L) while (B-L) is conserved. These effects result from:

1) chiral anomaly

2) non trivial topology of the vacuum of the electroweak theory

Baryon number violation in the Standard Model

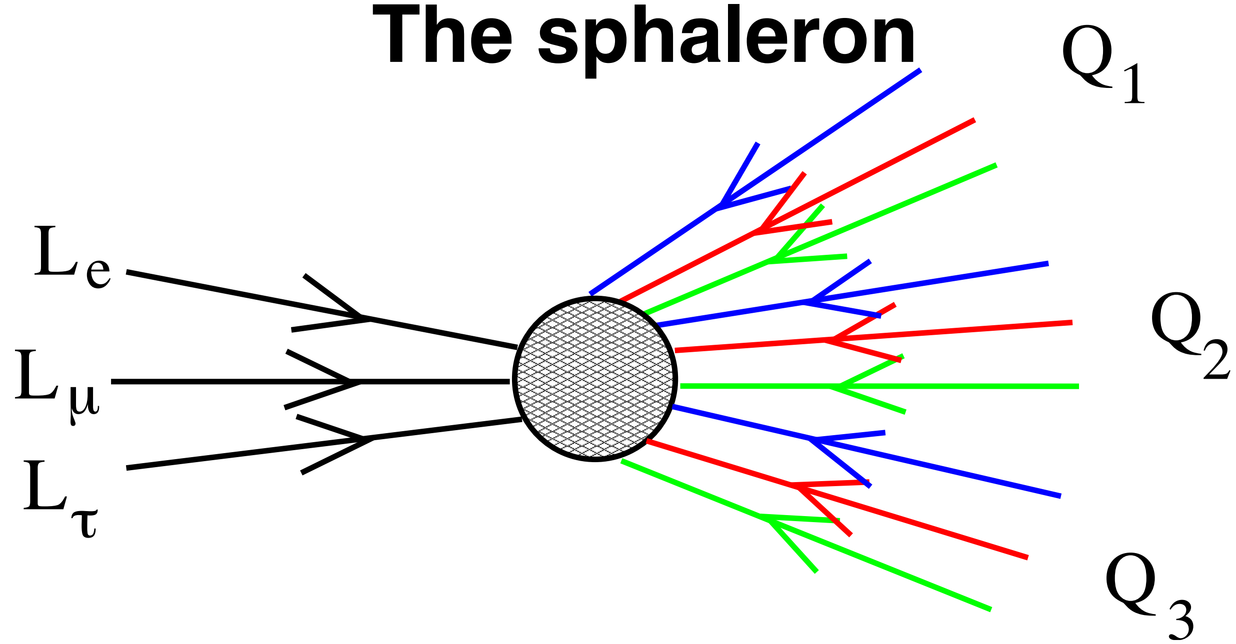


baryons are created by transitions between topologically distinct vacua of the $SU(2)_L$ gauge field

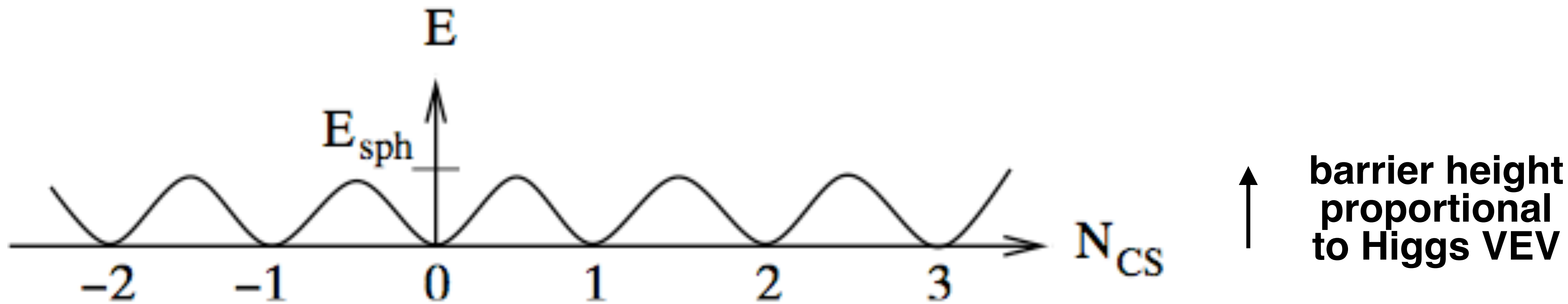
$$\Delta B = \Delta L = N_F \Delta N_{CS}$$

N_{CS}: Chern-Simons number

The sphaleron



Rate of B-violation through sphalerons at T=0



Quantum Tunneling rate: $\Gamma \sim e^{-4\pi/\alpha_W} \sim 10^{-165}$

$\alpha_W \sim 1/30$

⇒ Baryon number violation is totally suppressed in the SM at zero temperature

However, can become sizeable at high temperatures, as it becomes possible to go over the barrier via thermal fluctuations

Rate of Baryon number violation in the Standard Model at finite temperature

Two very ≠ rates depending whether $T>T_c$ or $T<T_c$
Temperature of the EW phase transition: T_c

- **In the EW symmetric phase, $T>T_c$** $\Gamma \sim \alpha_W^4 \, T^4$

out-of-equilibrium condition: $\alpha_W^4 \, T < T^2 / M_{Pl} \rightarrow T > 10^{12}$ GeV

- **In the EW broken phase, $T<T_c$** $\Gamma \sim v^4 \, e^{-c\langle\varphi/T\rangle}$

(more precisely $\frac{\Gamma}{V} = \mathrm{const} \left(\frac{E_{\mathrm{sph}}}{T}\right)^3 \left(\frac{m_W(T)}{T}\right)^4 T^4 e^{-E\mathrm{sph}/T}$)

out-of-equilibrium condition: $\langle\varphi\rangle/T > 1$

$\langle\varphi\rangle$: Higgs vacuum expectation value

Relating Baryon and Lepton number asymmetries at $T>T_c$

$$B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i})$$

$$L = \sum_i L_i, \quad L_i = 2\mu_{\ell_i} + \mu_{e_i}$$

At high temperature in the plasma, all processes at equilibrium give rise to constraints among the chemical potentials of quarks, leptons and Higgs.

(1) 12-fermion non-perturbative interactions induced by sphalerons

$$\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0$$

(2) Vanishing of total hypercharge of plasma

$$\sum_i \left(\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i} + \frac{2}{N_f}\mu_H\right) = 0$$

Relating Baryon and Lepton number asymmetries at T>T$_c$

(3) Yukawa interactions between LH and RH fermions

$$\mu_{q_i} - \mu_H - \mu_{d_j} = 0 \;,$$
$$\mu_{q_i} + \mu_H - \mu_{u_j} = 0 \;,$$
$$\mu_{\ell_i} - \mu_H - \mu_{e_j} = 0 \;.$$

(4) further assume equilibrium between different generations

$$\mu_{\ell_i} \;\equiv\; \mu_\ell \;\text{ and }\; \mu_{q_i} \;\equiv\; \mu_q$$

$$\mu_e = \frac{2N_f+3}{6N_f+3}\mu_\ell, \quad \mu_d = -\frac{6N_f+1}{6N_f+3}\mu_\ell, \quad \mu_u = \frac{2N_f-1}{6N_f+3}\mu_\ell \qquad \mu_q = -\frac{1}{3}\mu_\ell, \quad \mu_H = \frac{4N_f}{6N_f+3}\mu_\ell$$

$$B = \sum_i (2\mu_{q_i} + \mu_{u_i} + \mu_{d_i})$$

$$L = \sum_i L_i, \quad L_i = 2\mu_{\ell_i} + \mu_{e_i}$$

$$\Rightarrow$$

$$B = -\frac{4}{3} N_f \mu_\ell \ ,$$

$$L = \frac{14N_f^2 + 9N_f}{6N_f + 3}\mu_\ell$$

Then, B, L and B-L are related by:

$$\boxed{\mathbf{B} = \frac{8N_f + 4}{22N_f + 13}\ \mathbf{(B\text{-}L)}}$$

c_s=28/79

L= (c_s -1)(B-L)

Therefore, at thermal equilibrium (when sphalerons are active), B relaxes to (B-L). In the SM, B-L=0 so any primordial baryon asymmetry produced at early times gets erased by sphalerons in a theory which conserves B-L.

Consequence:

2 possibilities to achieve baryogenesis

(1) Create B-L ≠0 : e.g through out-of-equilibrium decays producing (B-L charge) which then gets converted into B by sphalerons. Popular example: Leptogenesis

(2) If B-L =0 : In the absence of any B-L breaking , baryogenesis should take place during the EW phase transition: "EW baryogenesis"

Models of Baryogenesis

T

GUT baryogenesis

B washout unless B-L ≠ 0

requires SO(10) → leptogenesis

requires too high reheat temperature to produce enough GUT particles

Thermal leptogenesis

hierarchy pb -> embed in susy-> gravitino pb (can be solved if M_gravitino>100 TeV and DM is neutralino or gravitino is stable)

Affleck-Dine (moduli decay)

Non-thermal leptogenesis (via oscillations)

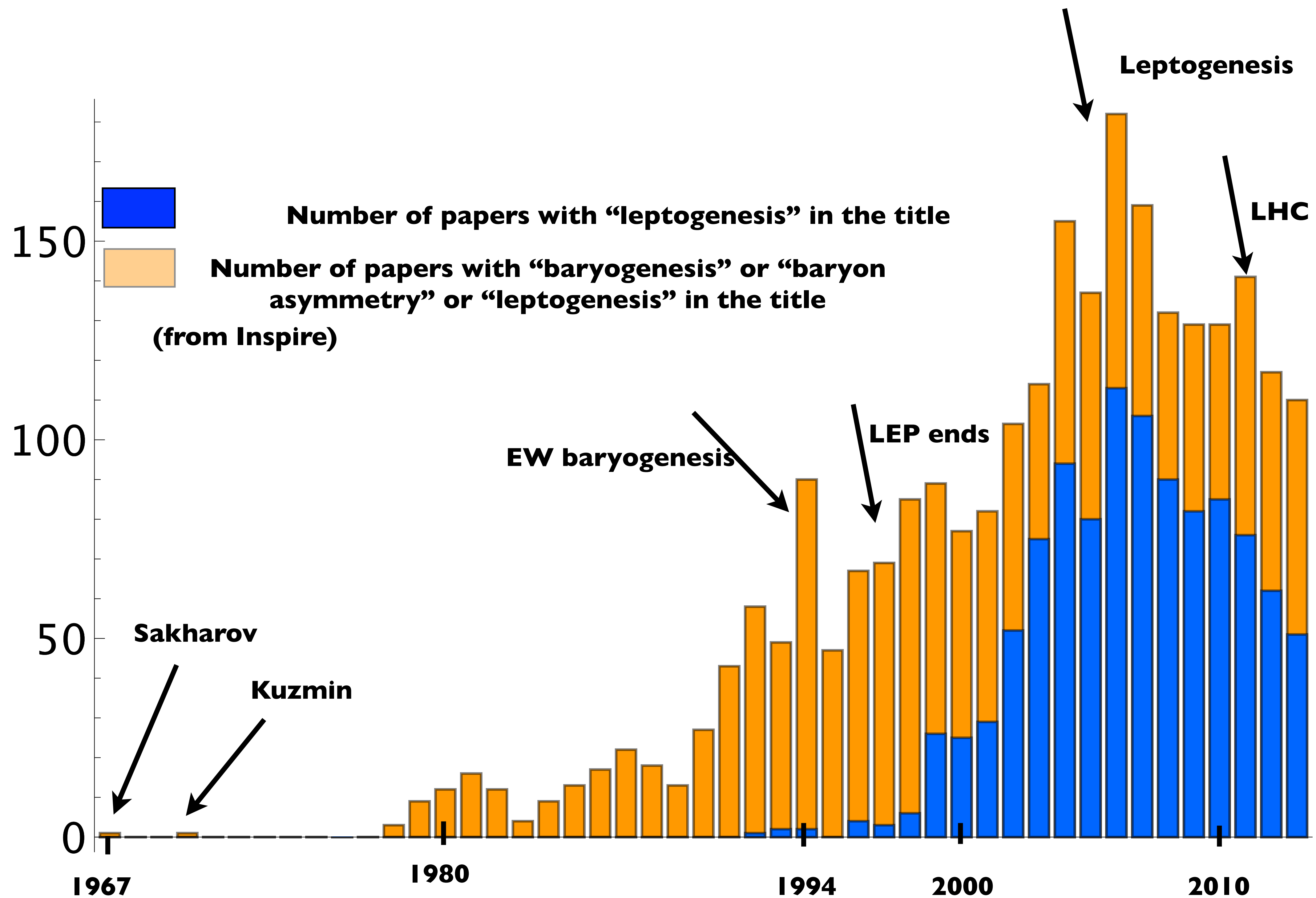
Asymmetric dark matter-cogenesis

EW breaking, sphalerons freese-out

EW (non-local) baryogenesis

EW cold (local) baryogenesis

History of baryogenesis papers



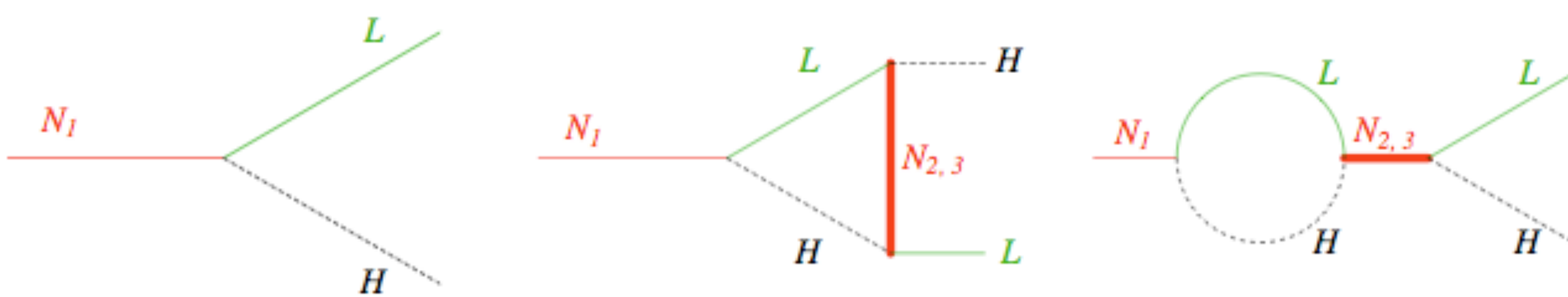
Leptogenesis

Fukugita, Yanagida

nicely connected to the explanation of neutrino masses

Majorana neutrino masses violate L and presumably CP

1) Generate L from the direct CP violation in RH neutrino decay



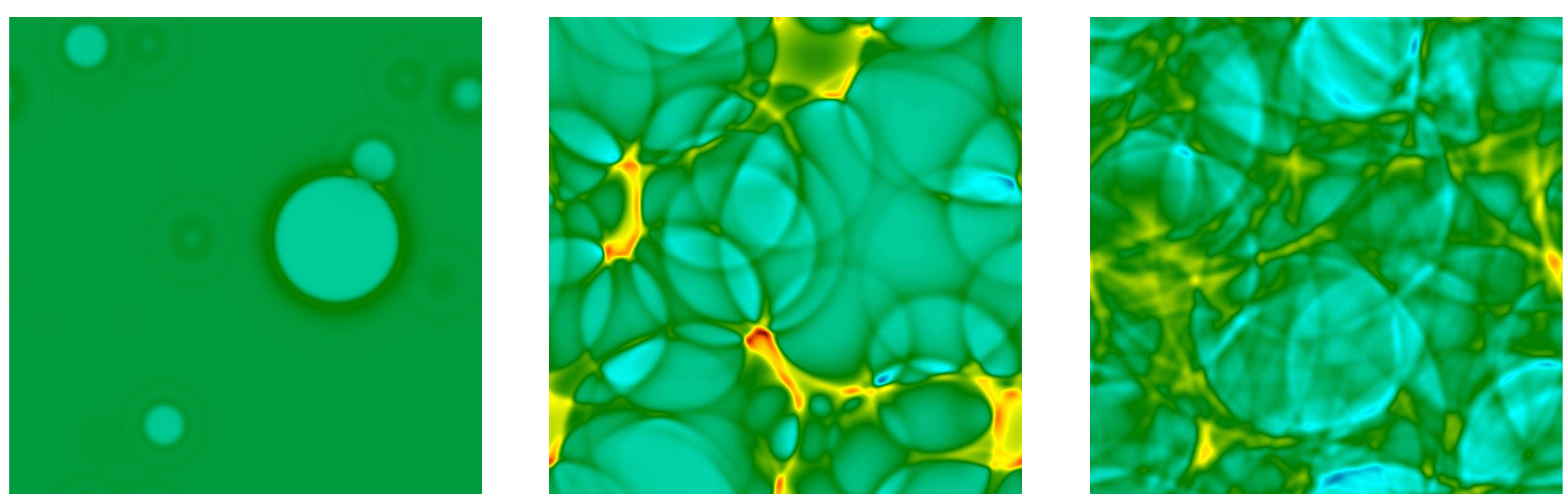
2) L gets converted to B by the electroweak anomaly

Out of equilibrium condition: $H > \Gamma \sim \lambda^2 M_1/(8\pi)$

at $T \sim M_1$, this leads to $\lambda v^2/M_1 < (8\pi)\, v^2/M_{Pl} \sim$ meV

see-saw formula for m_ν

Baryogenesis
at a first-order
EW phase transition



[image credit:1304.2433]

Baryogenesis Recap

GUT Baryogenesis :

-requires too high reheat temperature
-requires (B-L) violation due to washout by sphalerons

-> Leptogenesis as the most viable baryogenesis through out-of equilibrium decays of heavy right-handed Majorana neutrinos (L-violating).
Appealing as it requires hardly any new physics ingredients beyond those needed to explain neutrino masses by the seesaw mechanism.

Drawback: hard to test

—> Only way to achieve baryogenesis in (B-L) conserving theory:
At the electroweak phase transition: Electroweak baryogenesis

EW baryogenesis. Studying procedure

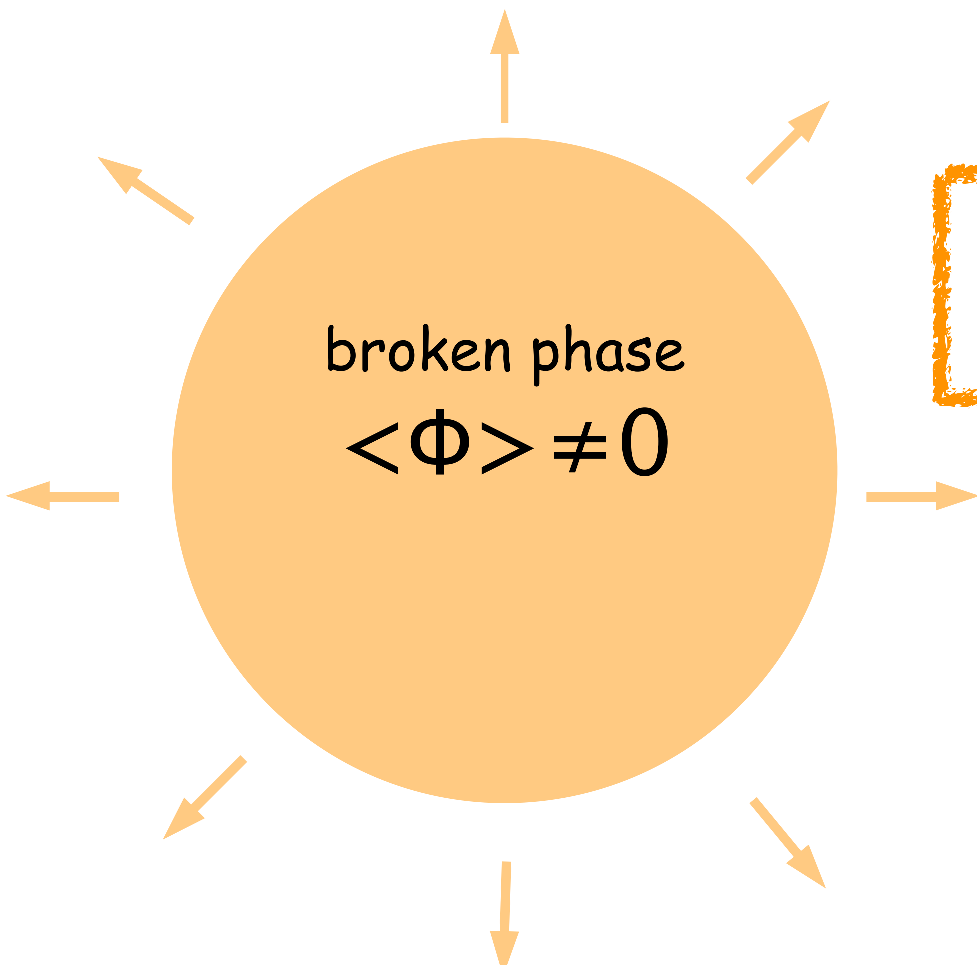
1- The mechanism: Charge transport across bubble wall
2- Nature of the EW phase transition -> computation of Higgs effective potential
3- Calculation of nucleation temperature (bubble action)
4- calculation of baryon asymmetry and CP violation

(I will only discuss 1 and 2).

EW baryogenesis during a first-order EW phase transition .

Kuzmin, Rubakov, Shaposhnikov'85

Cohen, Kaplan, Nelson'91



Baryon asymmetry created at vicinity of CP-violating bubble wall.

Strength of EW phase transition $\equiv$ $$\frac{\langle\Phi(T_n)\rangle}{T_n} \gtrsim 1$$

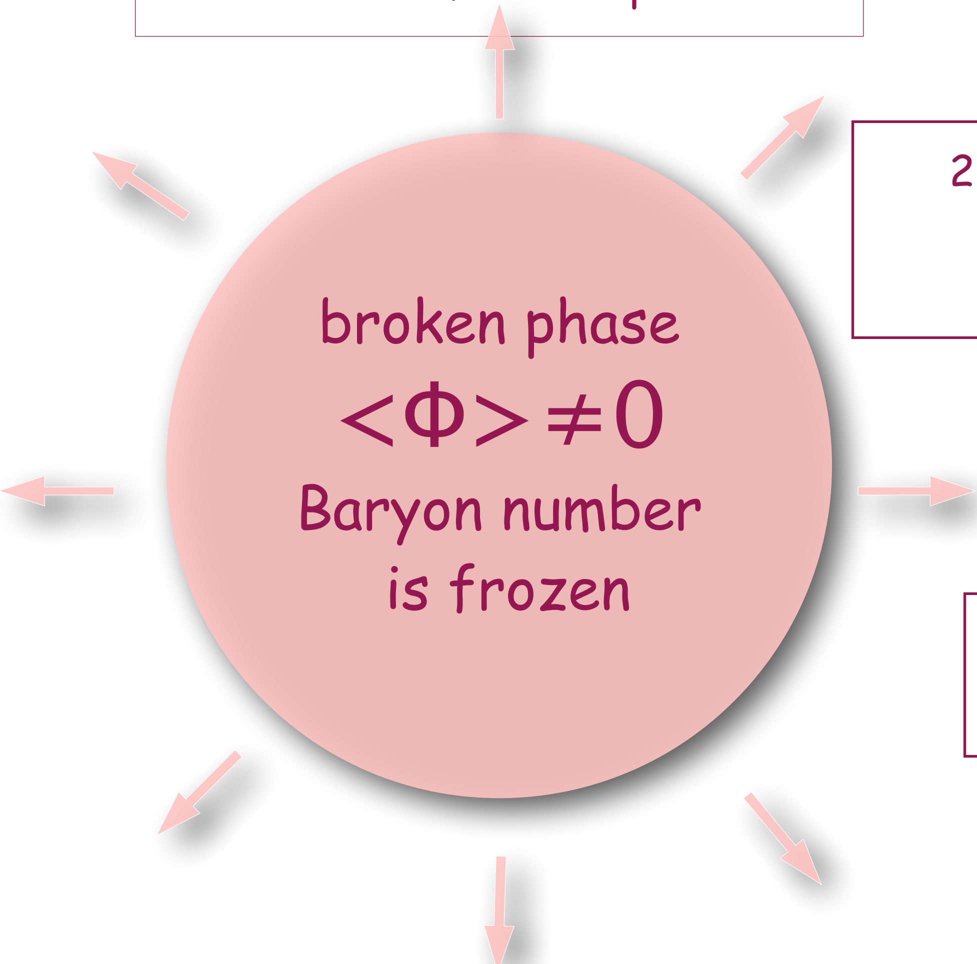
T_n $\equiv$ **nucleation temperature**

Baryon asymmetry and the EW scale

Kuzmin, Rubakov, Shaposhnikov'85

Cohen, Kaplan, Nelson'91

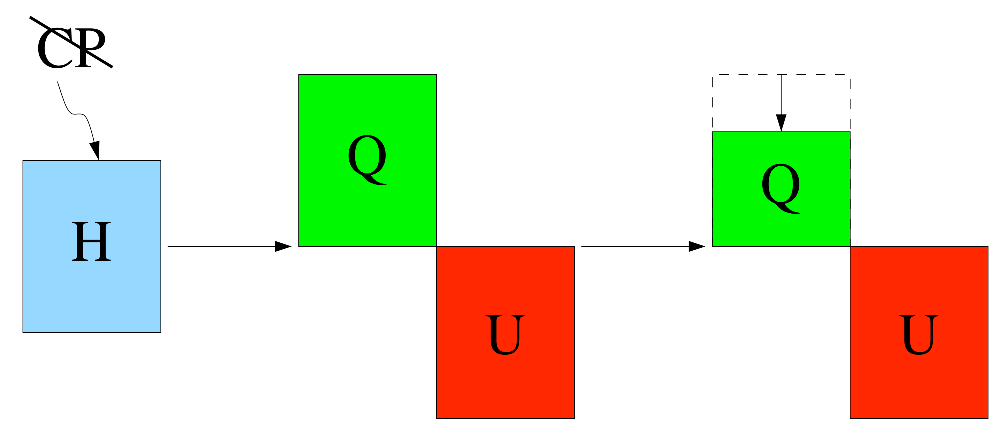
1) nucleation and expansion of bubbles of broken phase



2) CP violation at phase interface responsible for mechanism of charge separation

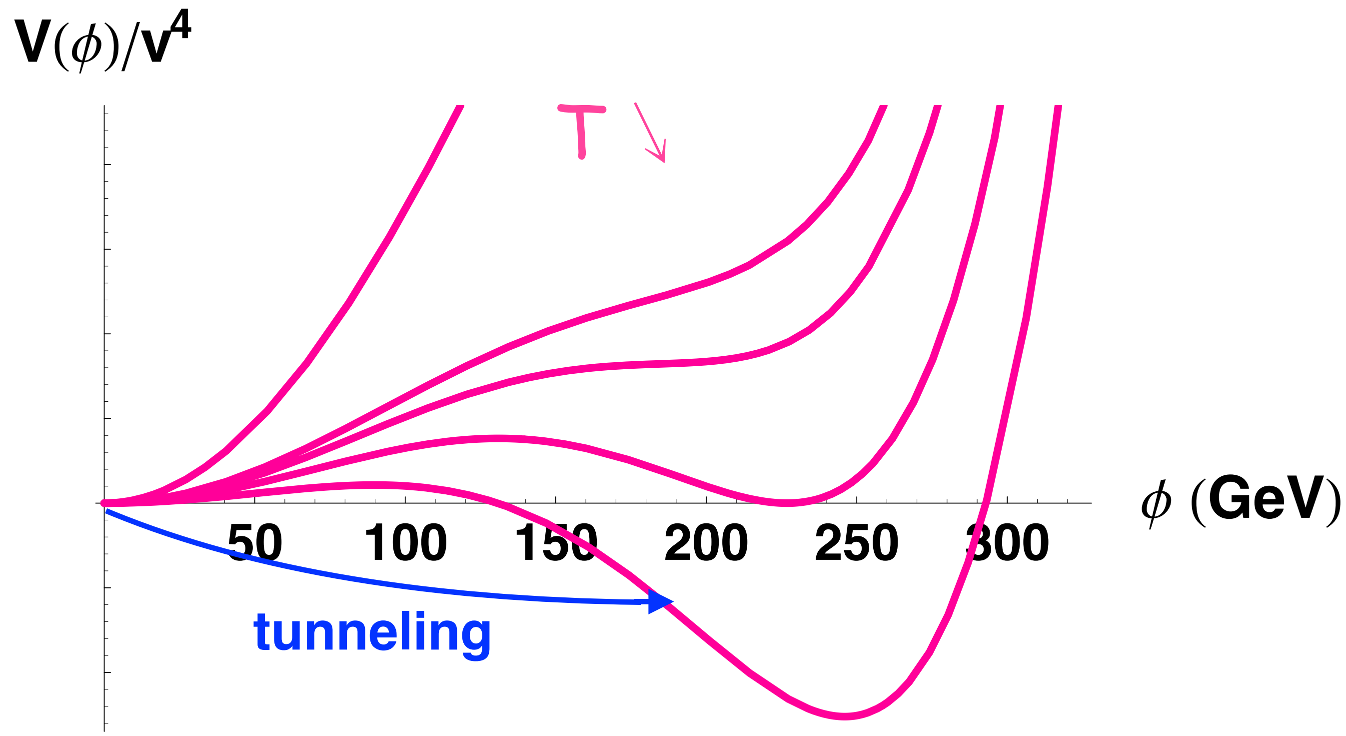
Chirality Flux in front of the wall

3) In symmetric phase, <Φ>=0, very active sphalerons convert chiral asymmetry into baryon asymmetry



Electroweak baryogenesis mechanism relies on a first-order phase transition satisfying $\frac{\langle \Phi(T_n) \rangle}{T_n} \gtrsim 1$

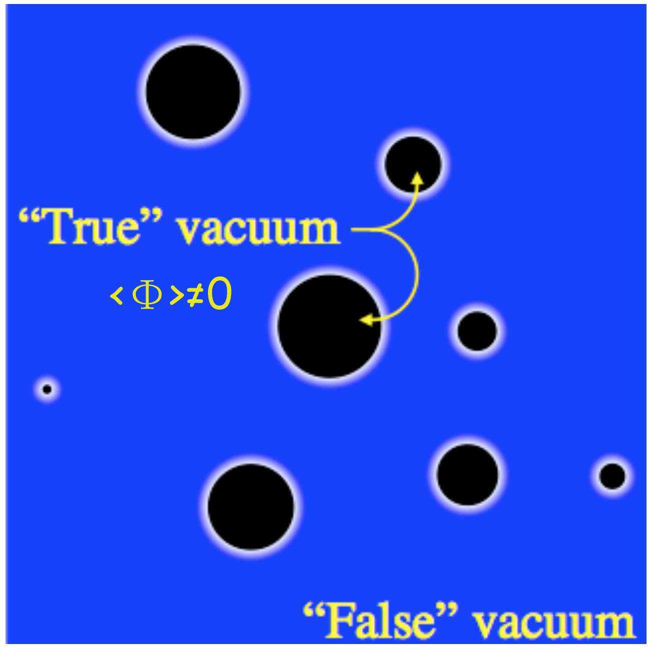
1st-order phase transition described by temperature evolution of scalar potential



Barrier separates 2 degenerate minima

2 phases can coexist

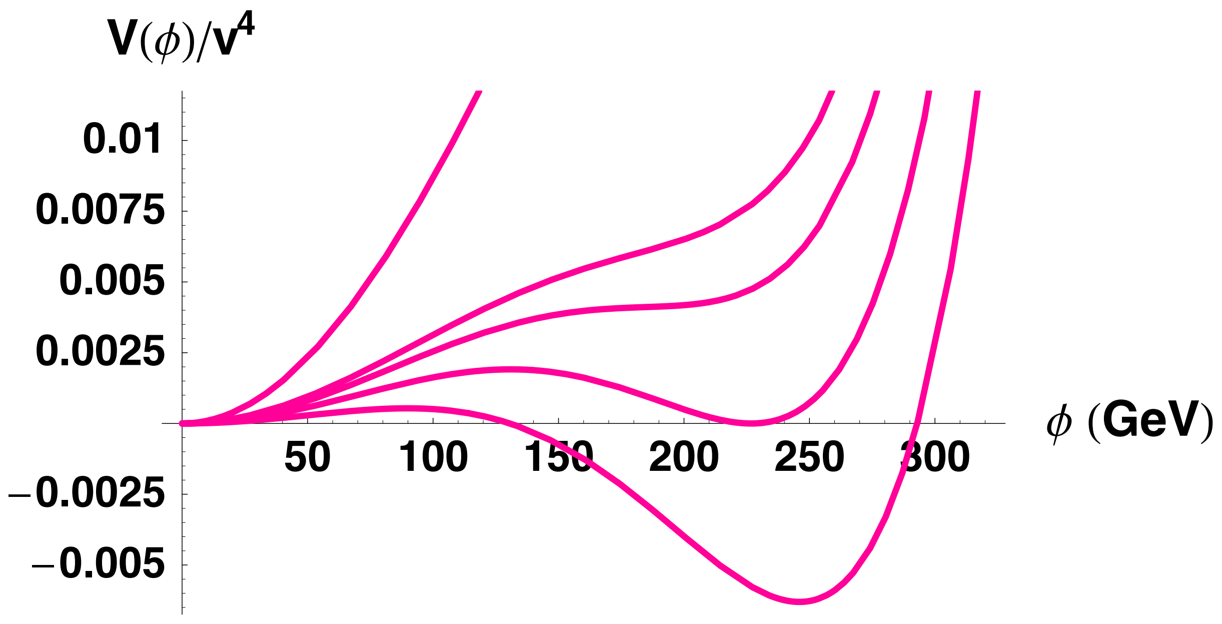
Nucleation, expansion and collision of Higgs bubbles

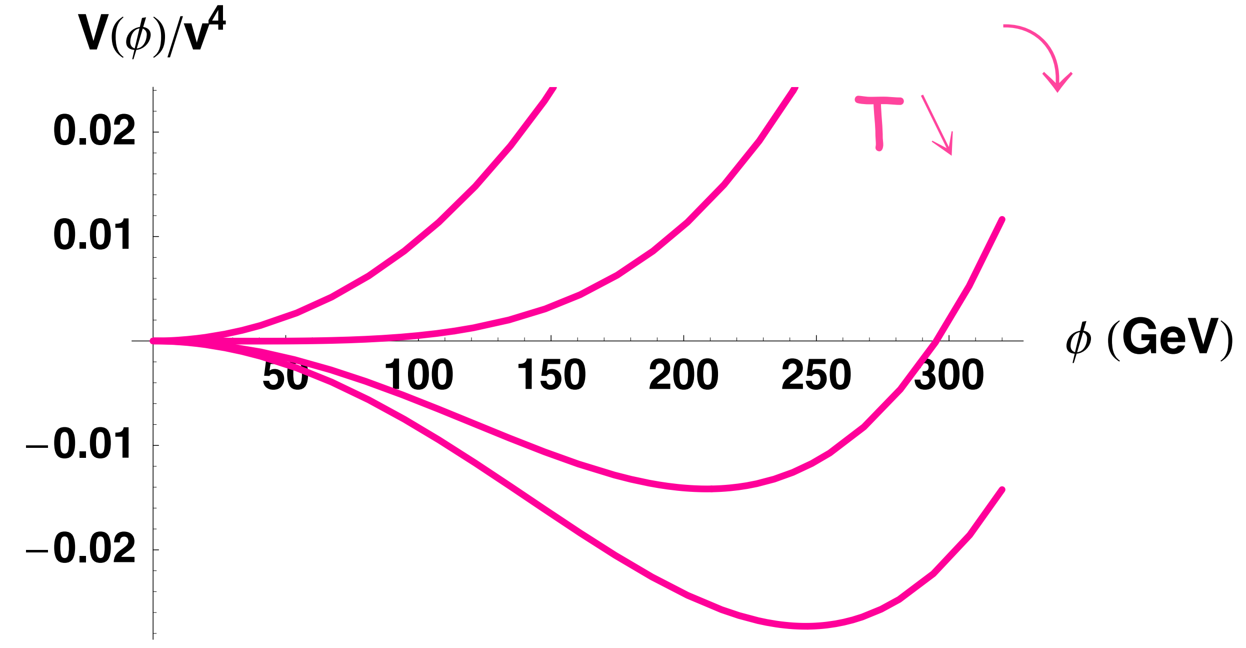


$\langle\Phi\rangle = 0$

Nature of the EW phase transition

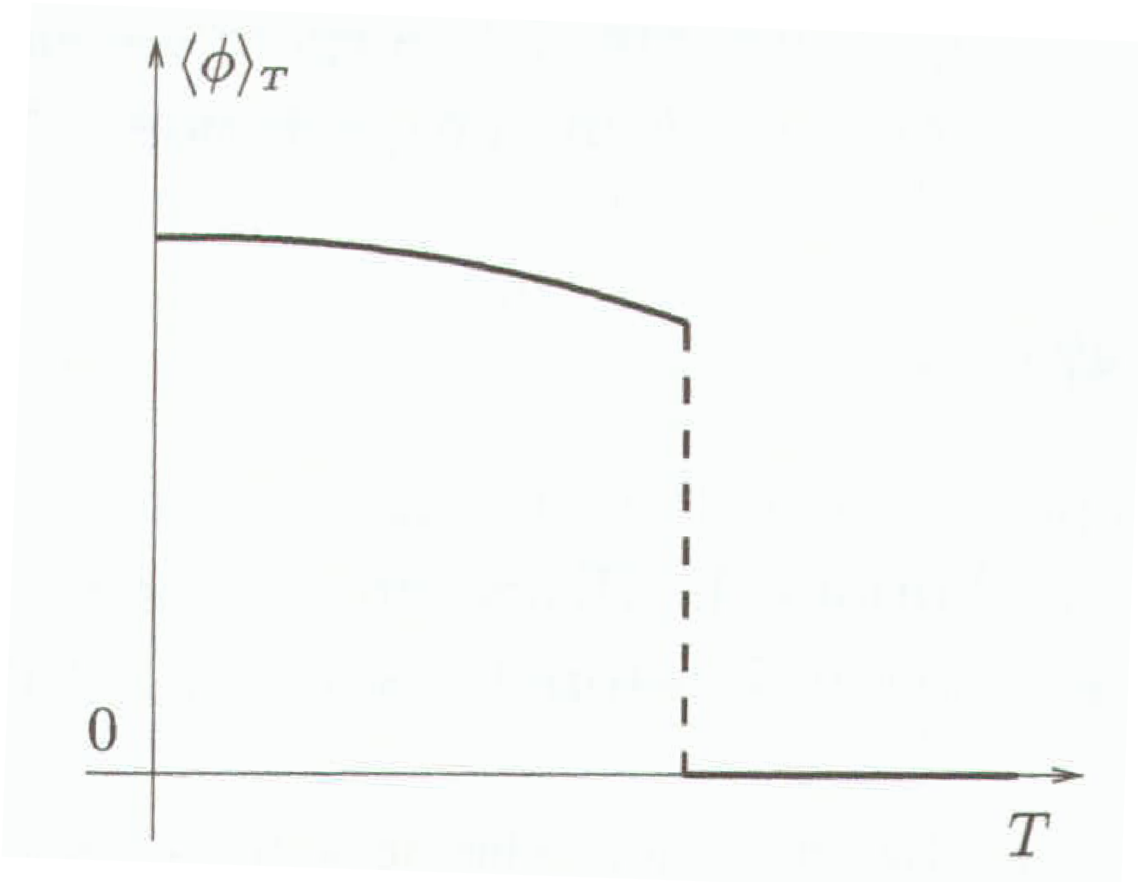
first-order or cross over?

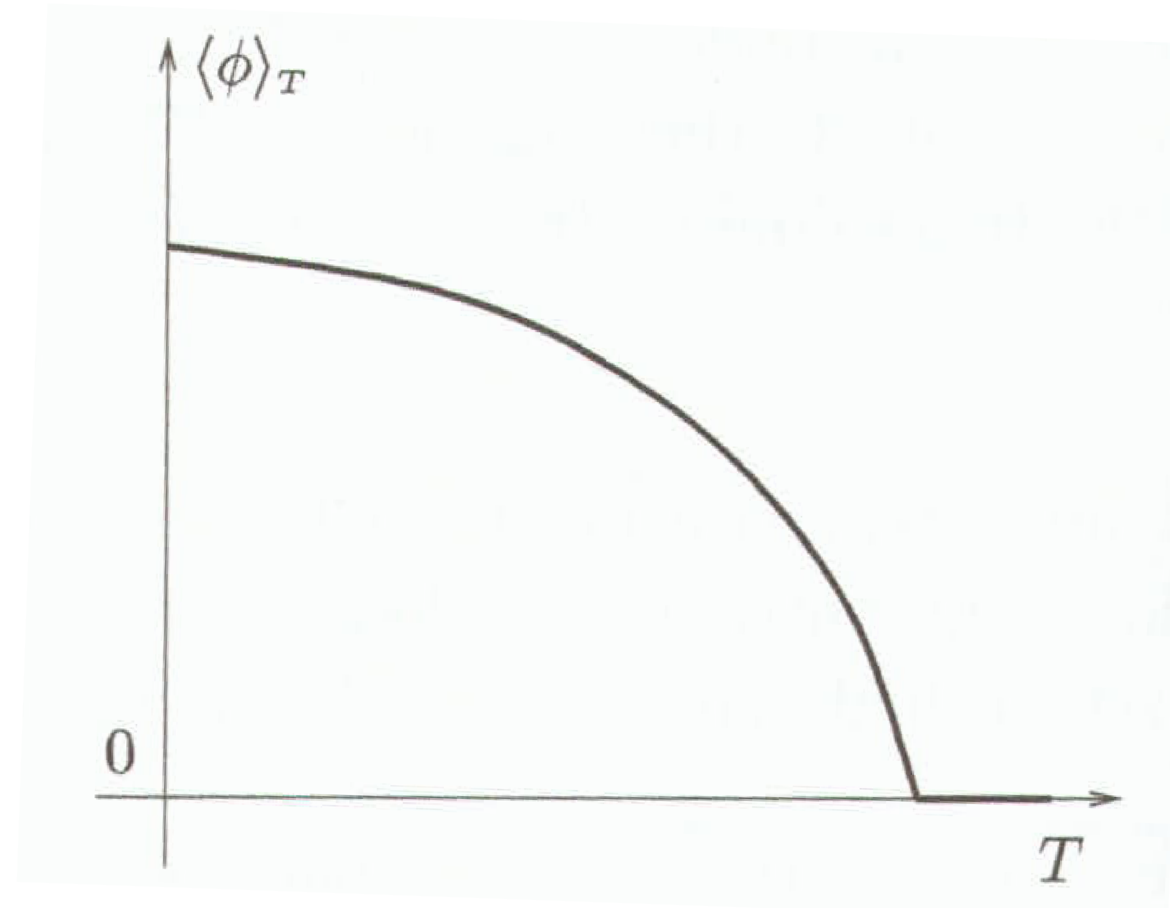




1st-order **versus** **2nd-order**

<φ(T)> discontinuous **1st derivative discontinuous**



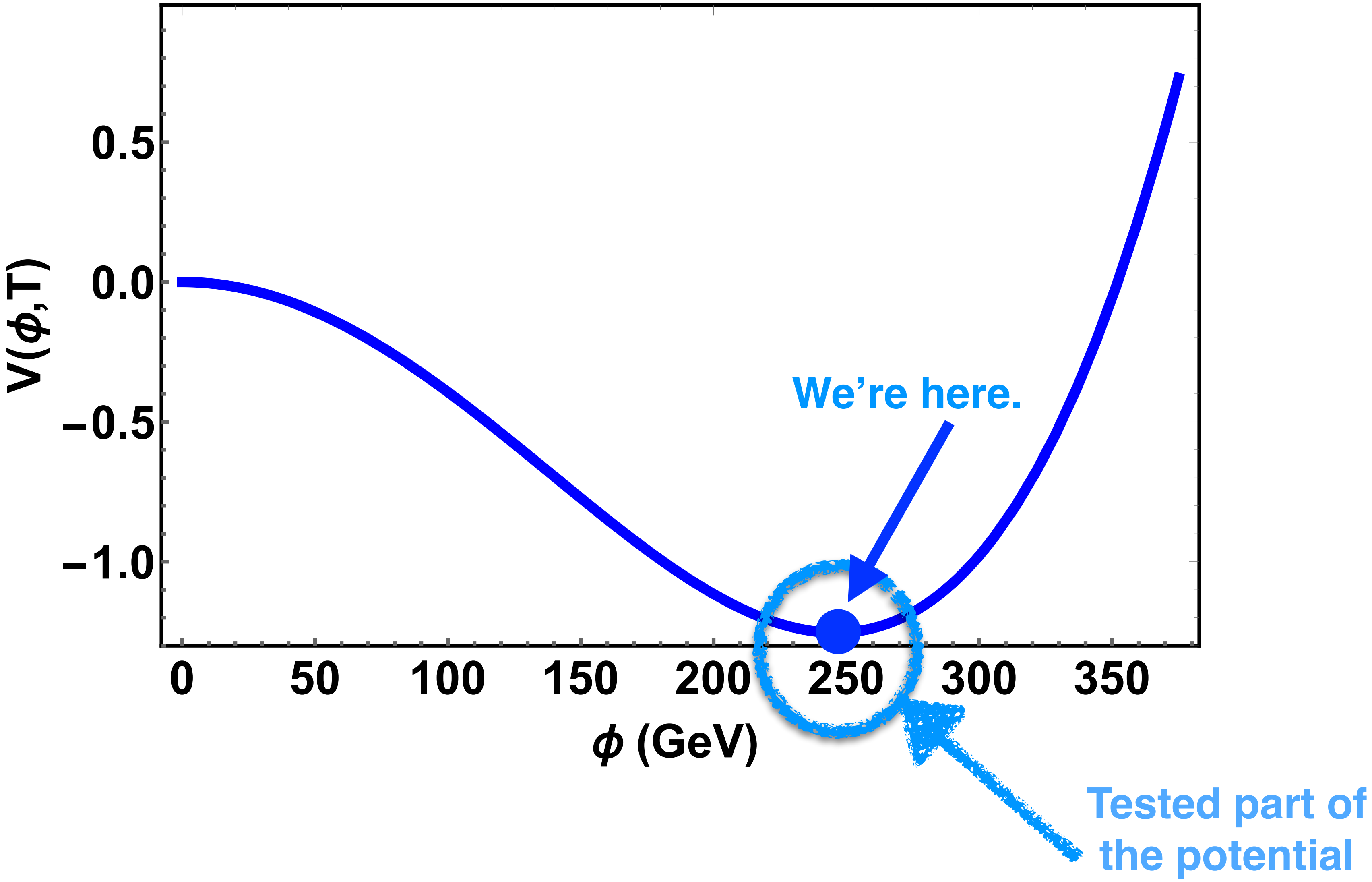


Crossover: no discontinuity in any derivative
impossible to see analytically, only on lattice

THE HIGGS POTENTIAL .

TODAY, T=0

$$V_0(\phi) = -\mu^2\phi^2/2 + \lambda\phi^4/2$$



> How did we end up here ?

Tool to study the nature of the EW phase transition: Higgs effective potential

The thermal 1-loop corrections to the effective potential for the Higgs boson is just the free energy of the Bose-Einstein & Fermi-Dirac distributions of particles getting a mass from φ.

The free energy F=-p , also called the effective potential of a plasma of particles with masses $m_i(\varphi)$ is, in the non-interacting gas approximation:

$$\mathcal{F} = V_0 + T \int \frac{d^3p}{(2\pi)^3} \sum_i \pm g_i \log\left[1 \mp e^{-E_i/T}\right]$$

T=0 potential

internal dof

-/+ : for bosons/fermions

$$E_i^2 = p^2 + m_i^2$$

φ-dependent

$$\mathcal{F} = V_0 + \frac{T^4}{2\pi^2}\sum_i \pm g_i Y_{b/f}(\frac{m_i}{T})$$

T=0 potential **T≠ 0 potential called** $V_1(\phi, T)$

$$Y_{b/f}(x) = \int_0^\infty dy\ y^2 \log\left[1 \mp e^{-\sqrt{x^2+y^2}}\right]$$

For light species compared to T (m/T<<1) -> relativistic gas.
Other species are Boltzmann-suppressed -> neglected

$$Y_{b/f} \to \frac{\pi^4}{45} \text{ for bosons} \qquad\qquad Y_{b/f} \to \frac{7\pi^4}{360} \text{ for fermions}$$

And we recover the usual $$\rho = \frac{\pi^2 g_* T^4}{30} \text{ with } g_* = \sum_i g_{i,b} + \frac{7}{4} g_{i,f}$$

such that $$\mathcal{F} = \frac{\rho}{3}$$

$V_1(\varphi,T)$ can be rewritten as

$$
\begin{aligned}
V_1(\phi,T) \quad = \quad & \sum_f \frac{g_f T^4}{2\pi^2}\left(\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left(\frac{m_f}{T}\right)^2 K_2(n\frac{m_f}{T})\right) \\
& -\sum_b \frac{g_b T^4}{2\pi^2}\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\frac{m_b}{T}\right)^2 K_2(n\frac{m_b}{T})\right)
\end{aligned}
$$

For small masses m/T<<1 in the high-temperature limit, this is (we exclude the terms independent of φ)

$$V_1(\phi,T) = \sum_b g_b\left[\frac{1}{24}m_b^2(\phi)T^2 - \frac{1}{12\pi}m_b^3(\phi)T - \frac{m_b^4(\phi)}{64\pi^2}\log(\frac{m_b^2(\phi)}{c_bT^2})\right] + \sum_f g_f\left[\frac{1}{48}m_f^2(\phi)T^2 + \frac{m_f^4(\phi)}{64\pi^2}\log(\frac{m_f^2(\phi)}{c_fT^2})\right] + \mathcal{O}(\frac{m^6}{T^2})$$

$\log c_b = 5.41$ $\qquad\qquad$ $\log c_f = 2.63$

We see that for m(φ)$\propto\varphi$

— The first term generates a thermal mass term $g^2\varphi^2T^2$
responsible for restoring the electroweak symmetry at high temperature
dominant contribution is from top coupling

— The second term in bosonic contribution generates a term - φ^3T:BARRIER!!
A key term to generate a 1st order phase transition

$$\boxed{V_1(\phi, T) = -\frac{\pi^2 g_* T^4}{90} + D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)\phi^4}{4}}$$

$$D = \frac{1}{8v^2}\left[2m_W^2 + m_Z^2 + 2m_t^2\right] \qquad T_0^2 = \frac{1}{4D}\left[m_H^2 - 8Bv^2\right]$$

$$E = \frac{1}{4\pi v^3}\left[2m_W^3 + m_Z^3\right] \approx 10^{-2}$$ ~ g^3

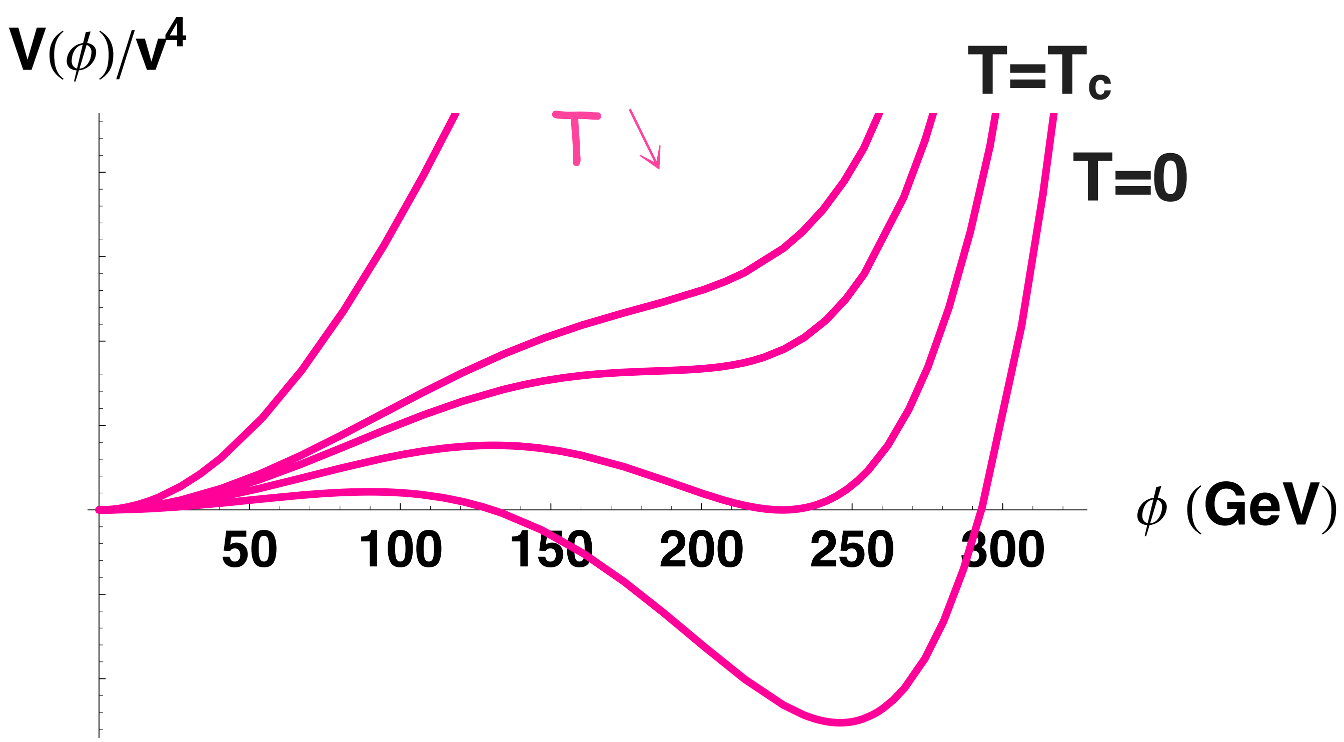
$$B = \frac{3}{64\pi^2 v^4}\left[2m_W^4 + m_Z^4 - 4m_t^2\right]$$

$$\lambda(T) = \lambda - \frac{3}{16\pi^2 v^4}\left(2m_W^4 \log\frac{m_W^2}{a_b T^2} + m_Z^4 \log\frac{m_Z^2}{a_b T^2} - 4m_t^4 \log\frac{m_t^2}{a_f T^2}\right)$$

$$\log a_b = 3.91$$

$$\log a_f = 1.14$$

$$\lambda = \frac{m_H^2}{2v^2}$$



(We ignore the 1-loop T=0 contribution for simplicity)

$V_1(\phi, T)$ **leads to a 1st-order phase transition with critical temperature T_c**

$$T_c^2 = \frac{T_0^2}{1 - \frac{E^2}{\lambda(T_c)D}}$$

Min/Max: $\phi_\pm(T) = \frac{3ET}{2\lambda(T)} \pm \frac{1}{2\lambda(T)}\sqrt{9E^2T^2 - 8\lambda(T)D(T^2 - T_0^2)}$

At T_c

$$V(0, T_c) = V(\phi(T_c), T_c)$$

$$\phi_+^{min}(T_c) = \frac{2ET_c}{\lambda(T_c)}$$

$$\phi_-^{max}(T_c) = \frac{ET_c}{\lambda(T_c)}$$

The order parameter is given by

$$\boxed{\frac{\phi_+(T_c)}{T_c} = \frac{2E}{\lambda(T_c)} \approx 0.146}$$

~ $1/(m_H)^2$

strongly depends on Higgs mass

$$\lambda = \frac{m_H^2}{2v^2}$$

! Perturbation theory not applicable at T_c in the SM for large Higgs mass.

In fact, at high T, the thermal loop expansion parameter is g^2T/M_W where g is the coupling entering in the loop

At T_c, $g^2T_c/M_W \sim gT_c/\varphi_c \sim g/(E/\lambda) \sim g/(g^3/\lambda) \sim \lambda/g^2 \sim (m_H/m_W)^2$

-> perturbative expansion breaks down at T_c especially at large m_H

In the Standard Model, the EW phase transition is not first-order but is a cross-over.

would be first-order for $m_H <\sim 70$ GeV

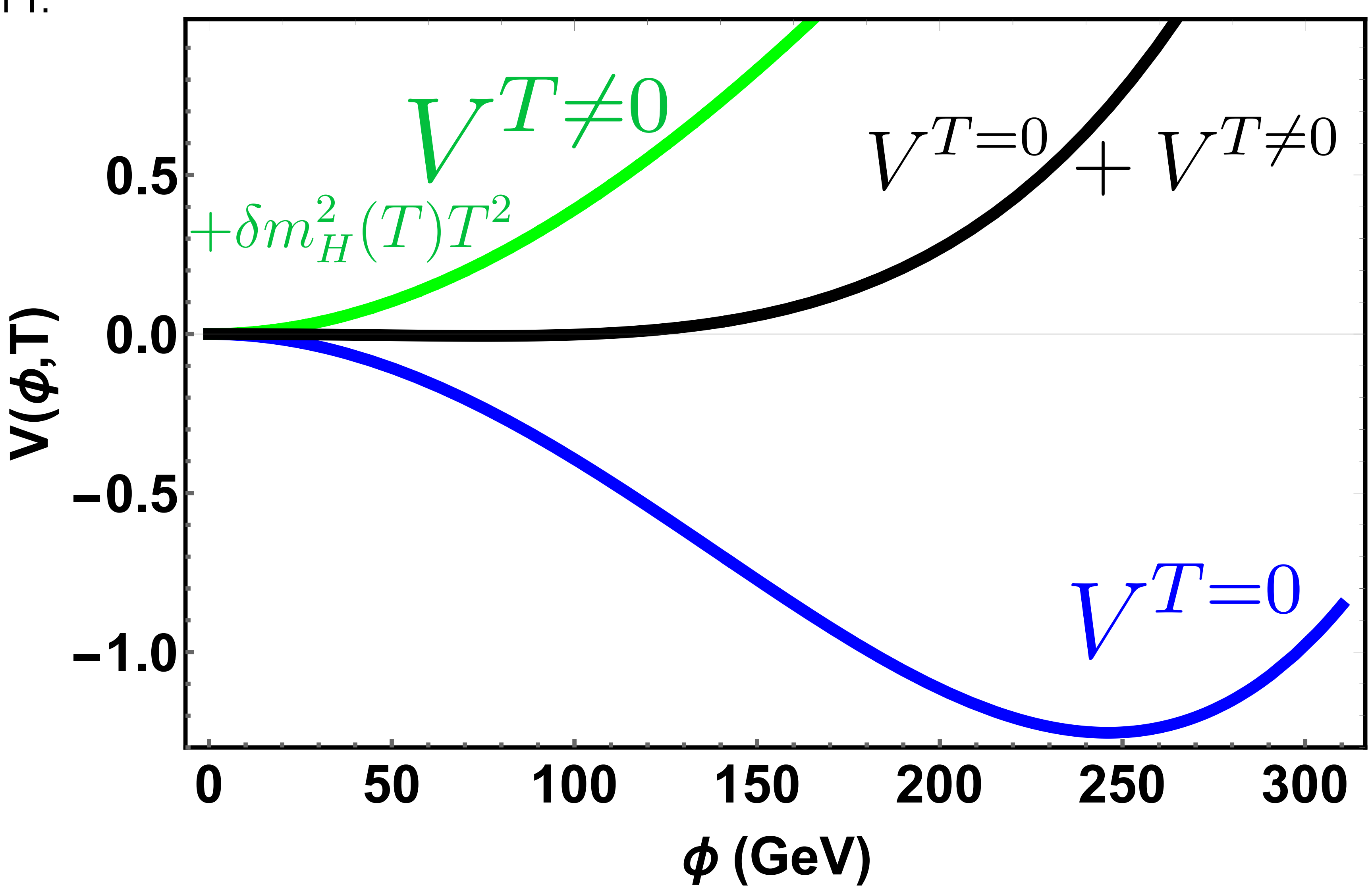
HIGH TEMPERATURE EW SYM. RESTORATION.

At one-loop:

$$V_{\text{eff}} = \boxed{V_{\text{tree}}(\phi) + V_1^0(\phi)} + \boxed{V_1^T(\phi, T) + V_{\text{Daisy}}(\phi, T)}.$$

Tree level — 1-loop T=0 — 1-loop T≠0 — Daisy resummation

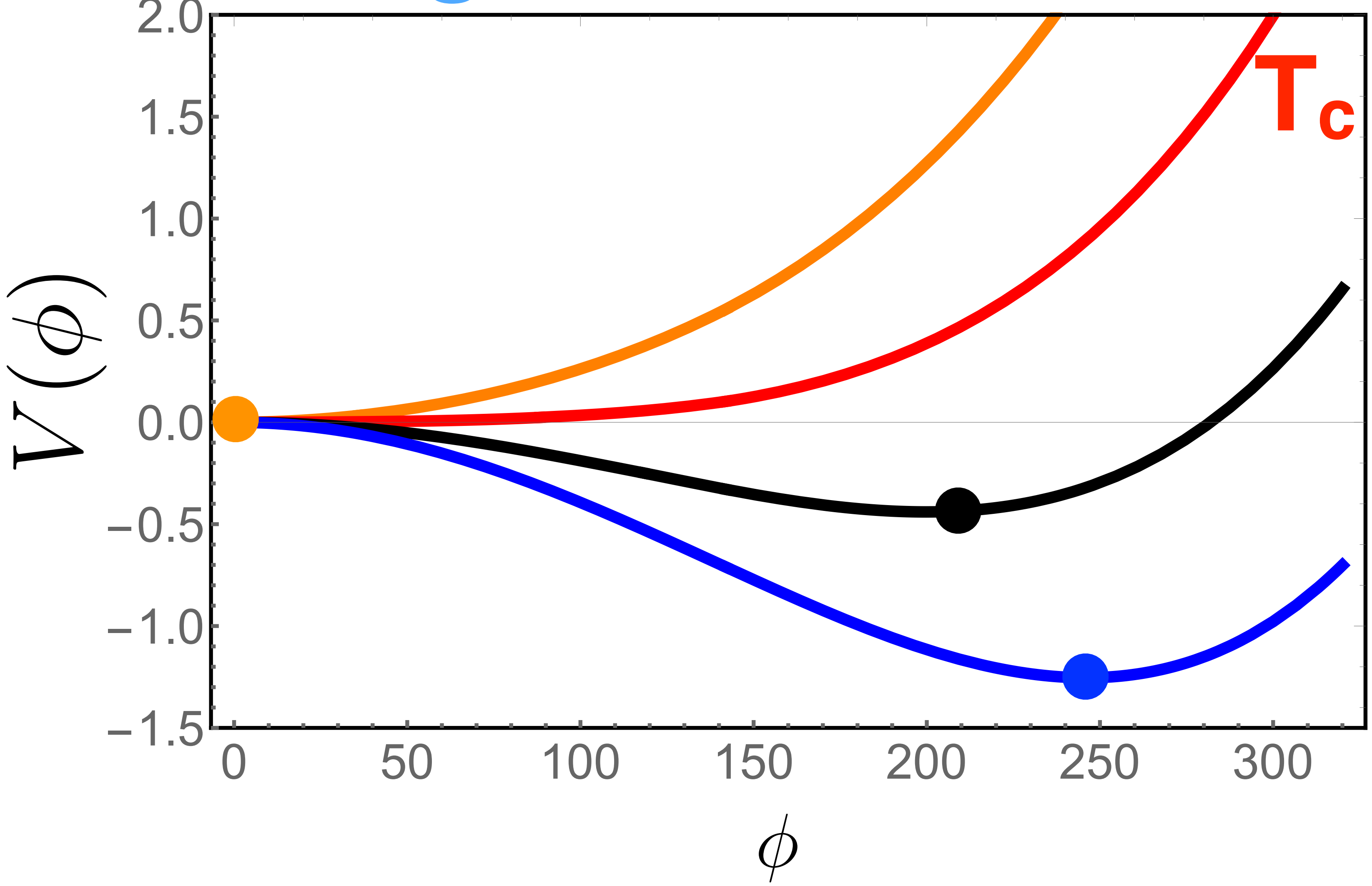
At high T:

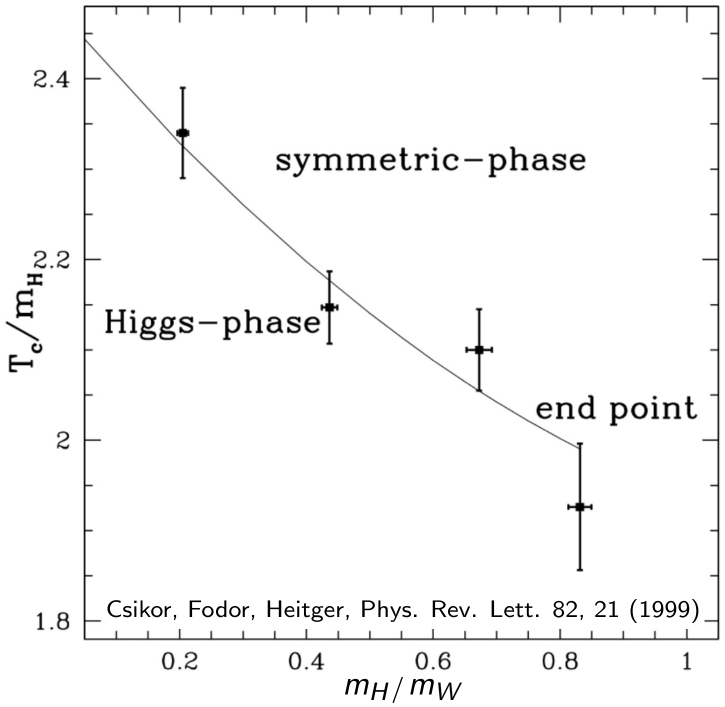


—> **Symmetry restoration at Tc**

EW sym. restored at T≳160 GeV***

through a smooth crossover





No departure from thermal equilibrium

It would have been different if $m_H \lesssim 70$ GeV

***1404.3565

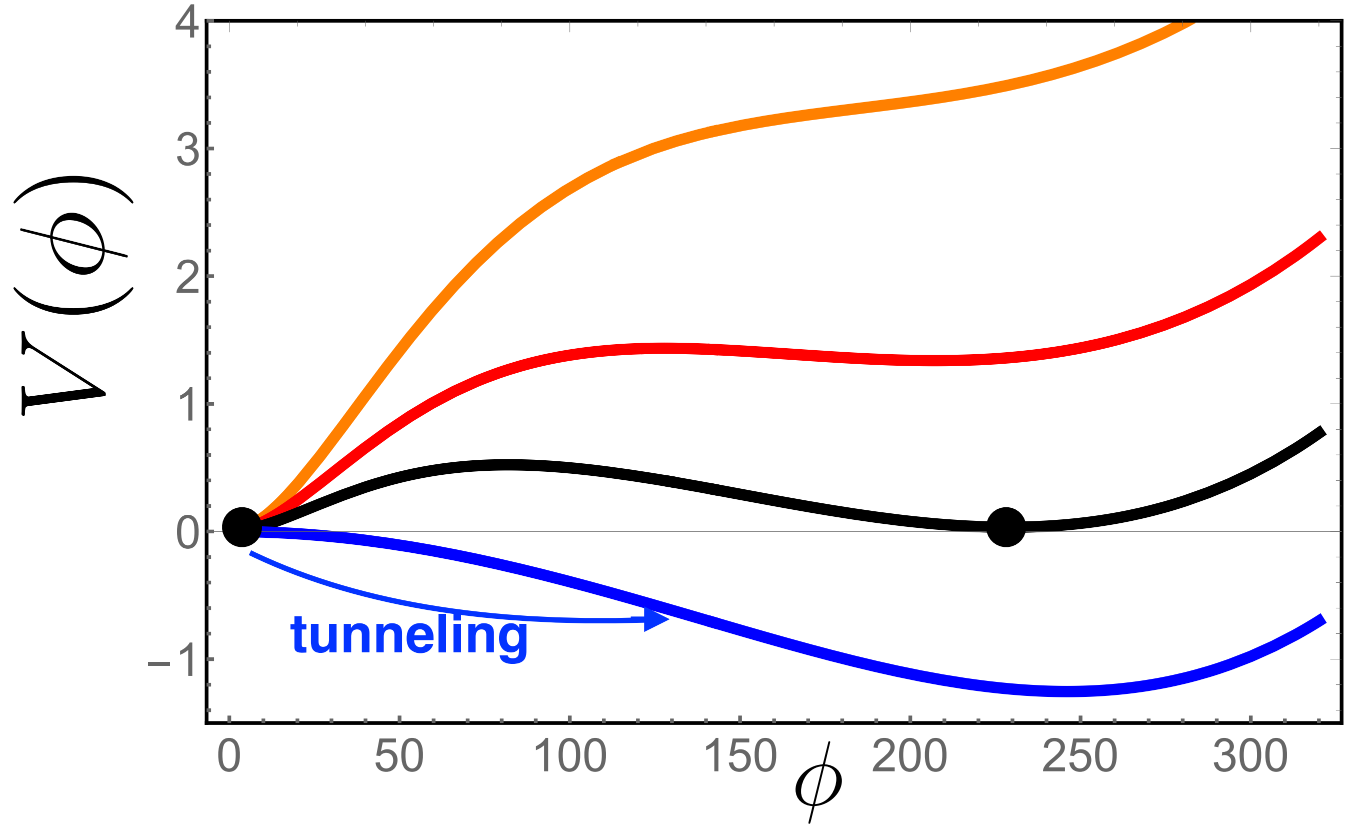
In Standard Model, the EW phase transition is not first-order.

In Minimal Supersymmetric Standard Model:
many new bosonic degrees of freedom with large coupling to the Higgs (e.g “stop”)

$$-ET\phi^3 \subset -\frac{T}{12\pi}\sum_{bosons} m_i^3(\phi)$$

Increases size of barrier, but requires light stop (squarks), now excluded by LHC.

First-order EW phase transition .



Barrier separates 2 degenerate minima

2 phases can coexist

Nucleation, expansion and collision of Higgs bubbles

> Framework for EW baryogenesis !

> Stochastic bgd of gravitational waves detectable at LISA !

What makes the EW phase transition 1st-order ?

> O(1) modifications to the Higgs potential

> Extra **EW-scale** scalar(s) coupled to the Higgs

A hot topic of investigation, search for extra scalars at LHC.

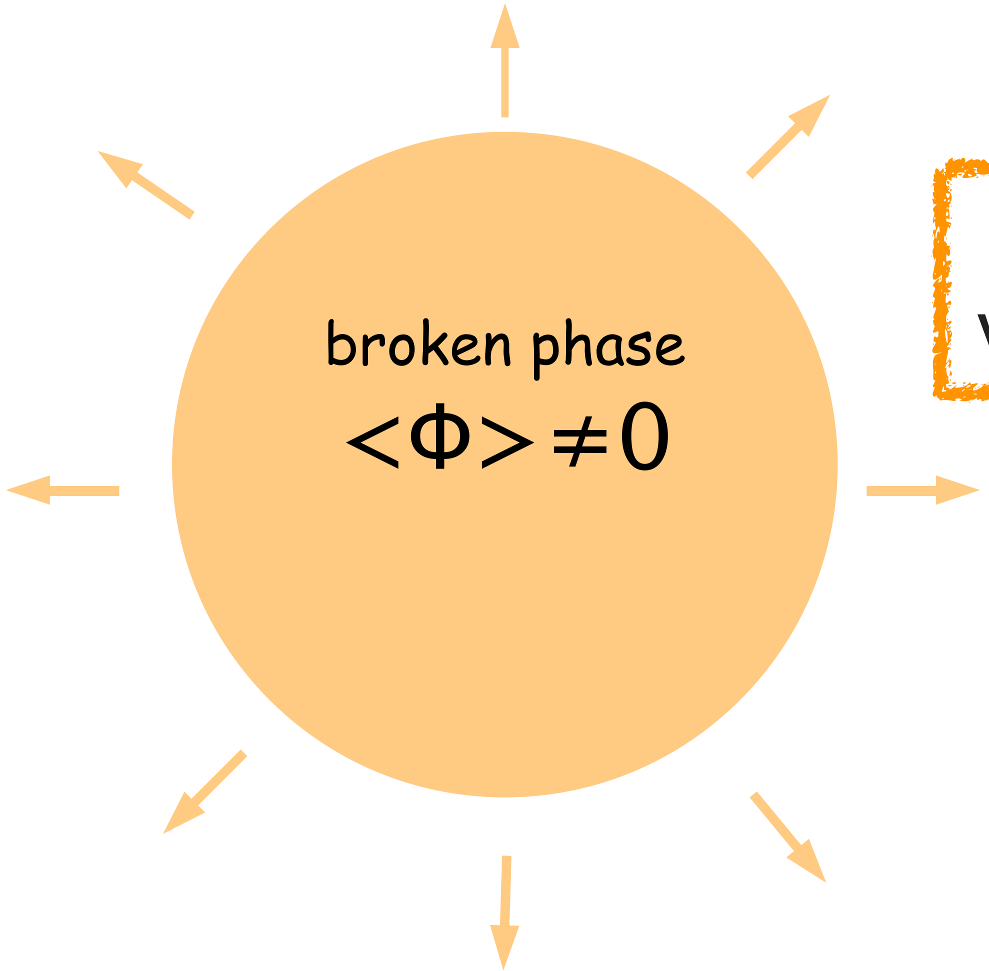
ee e.g. 2212.00056, for a very well-motivated candidate, the dilaton, in composite Higgs models.

Nature of the EW phase transition :

Still many open exotic possibilities regarding what happened when the energy density of the universe was (EW scale)^4.

EW baryogenesis during a first-order EW phase transition

Kuzmin, Rubakov, Shaposhnikov'85
Cohen, Kaplan, Nelson'91



Baryon asymmetry created at vicinity of CP-violating bubble wall.

Strength of EW phase transition $\equiv$ $$\frac{\langle \Phi(T_n) \rangle}{T_n} \gtrsim 1$$

T_n $\equiv$ nucleation temperature

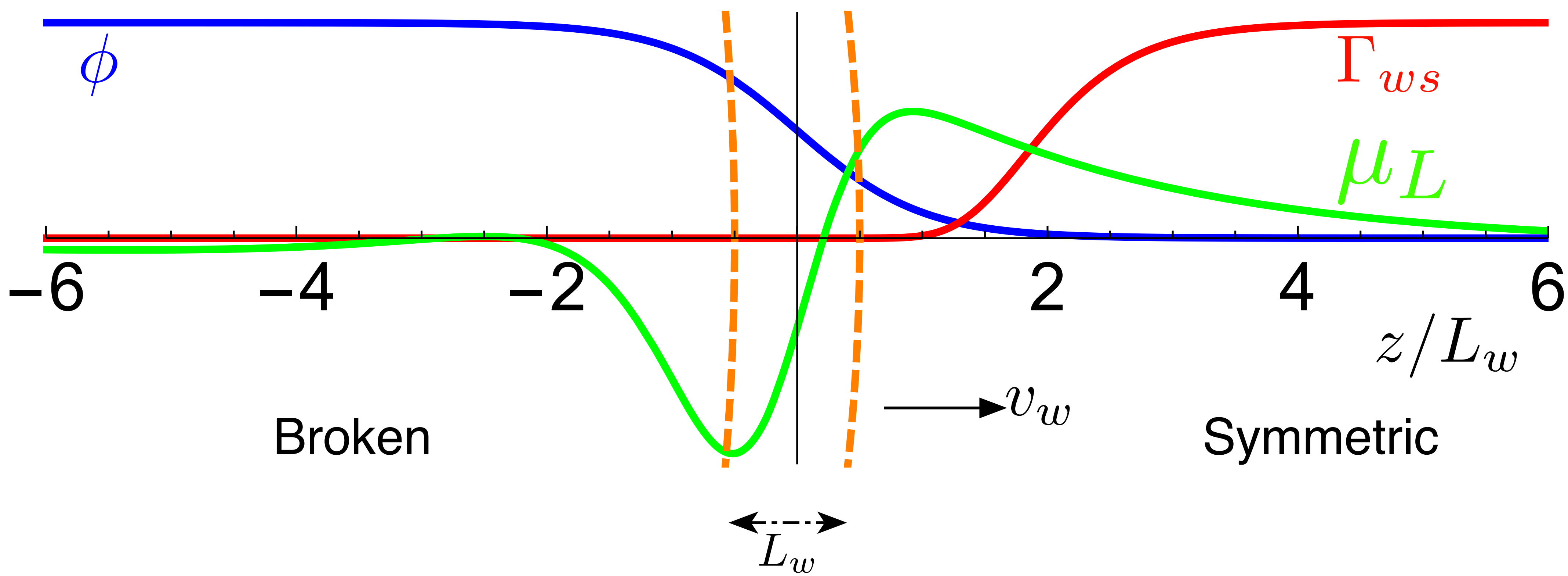
The Electroweak Baryogenesis Miracle:

CP violation

$$\eta_B = \frac{n_B(-\infty)}{s} = \frac{135\ N_c}{4\pi^2 v_w g_* T}\int_{-\infty}^{+\infty} \mathrm{d}z\ \Gamma_{ws}\ \mu_L\ \mathrm{Exp}\left[-\frac{3}{2}A\frac{1}{v_w}\int_{-\infty}^{z}\mathrm{d}z_0\Gamma_{ws}\right]$$

v_w: bubble wall velocity ~ c

sphaleron rate: $\Gamma_{ws} = 10^{-6}\ T\ e^{-\frac{E_{sph}}{T}\frac{\phi(T)}{v}}$



The Electroweak Baryogenesis Miracle:

$$\eta_B = \frac{n_B(-\infty)}{s} = \frac{135\ N_c}{4\pi^2 v_w g_* T} \int_{-\infty}^{+\infty} \mathrm{d}z\ \Gamma_{ws}\ \mu_L\ \mathrm{Exp}\left[-\frac{3}{2} A \frac{1}{v_w} \int_{-\infty}^{z} \mathrm{d}z_0 \Gamma_{ws}\right]$$

$$\Gamma_{ws} = 10^{-6}\ T\ e^{-\frac{E_{sph}}{T}\frac{\phi(T)}{v}}$$

$$\eta_B \sim \frac{\Gamma_{ws}\ \mu_L\ L_w}{g_*\ T}$$

M: fermion mass matrix

$$\mu_L \sim M^{''} M \sim \frac{\delta_{CP}}{L_w^2\ T}$$

$$L_w \sim \frac{1}{T}$$

$$\eta_B \sim \frac{\Gamma_{ws}\ \delta_{CP}}{g_*\ L_w\ T^2} \sim \frac{10^{-6}\ \delta_{CP}}{g_*} \sim 10^{-8} \delta_{CP}$$

All parameters fixed by electroweak physics. If new CP violating source of order 1 then we get just the right baryon asymmetry.

Gravitational Waves from a 1st-order phase transition .

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G a^2 T_{ij}^{(TT)}(k,t)$$

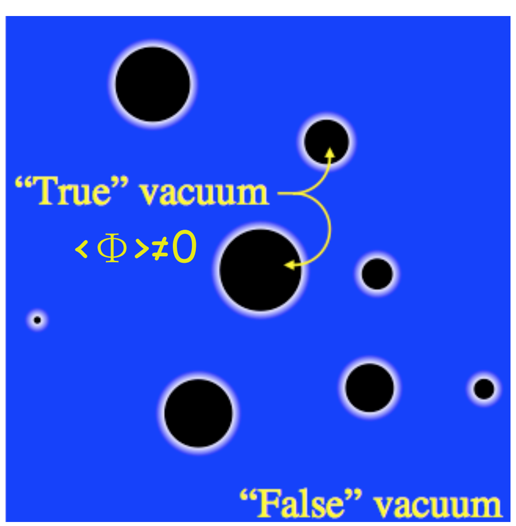
$$T_{ab}(\mathbf{x}) = (\rho + p)\frac{v_a(\mathbf{x})v_b(\mathbf{x})}{1 - v^2(\mathbf{x})}$$

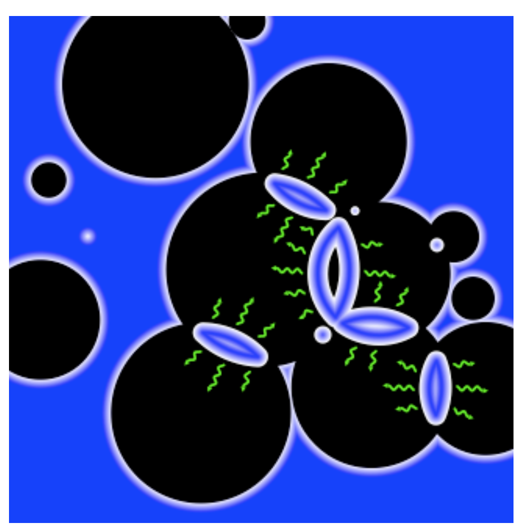
Source of GW: anisotropic stress

Generated mainly from fluid velocities in the vicinity of colliding bubble walls

Bubble nucleation

Bubble percolation





<Φ>=0

violent process if v

Fluid flows

Magnetic fields

Turbulence

Stochastic bgd of gravitational radiation

Sources of GWs during a 1st-order phase transition .

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Fluid (sound waves and turbulence)

$$\Pi_{ij} \sim \gamma^2(\rho + p)\,v_i v_j$$

Electromagnetic field (primordial magnetic fields MHD turbulence)

$$\Pi_{ij} \sim (E^2 + B^2)\frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$$

Scalar field (collision of bubble walls)

$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

In principle entirely determined by the Higgs effective potential.

Frequencies

T_* **temperature of the universe at time of emission**

$$f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*} \sim \frac{T_*}{M_{Pl}} T_0 \sim T_* \times 10^{-18} 10^{-12}$$

If T_* **~ 100 GeV:**

$$f \sim 10^{-28} \text{ GeV} \sim 10^{-28} \times 10^{25} \text{ Hz} \sim \text{mHz}$$

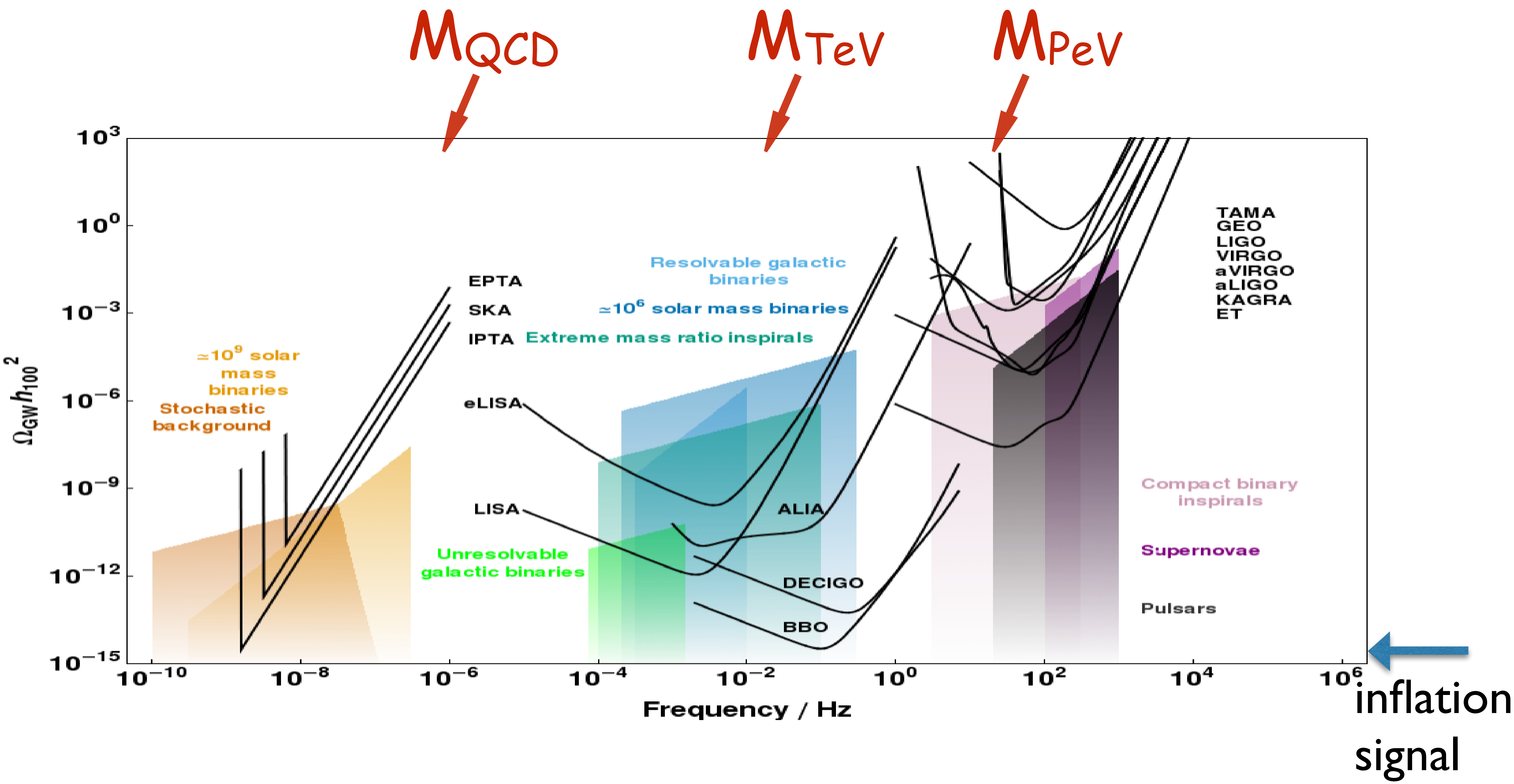
LISA !

Stochastic background of Gravitational Waves: isotropic, unpolarized, stationary

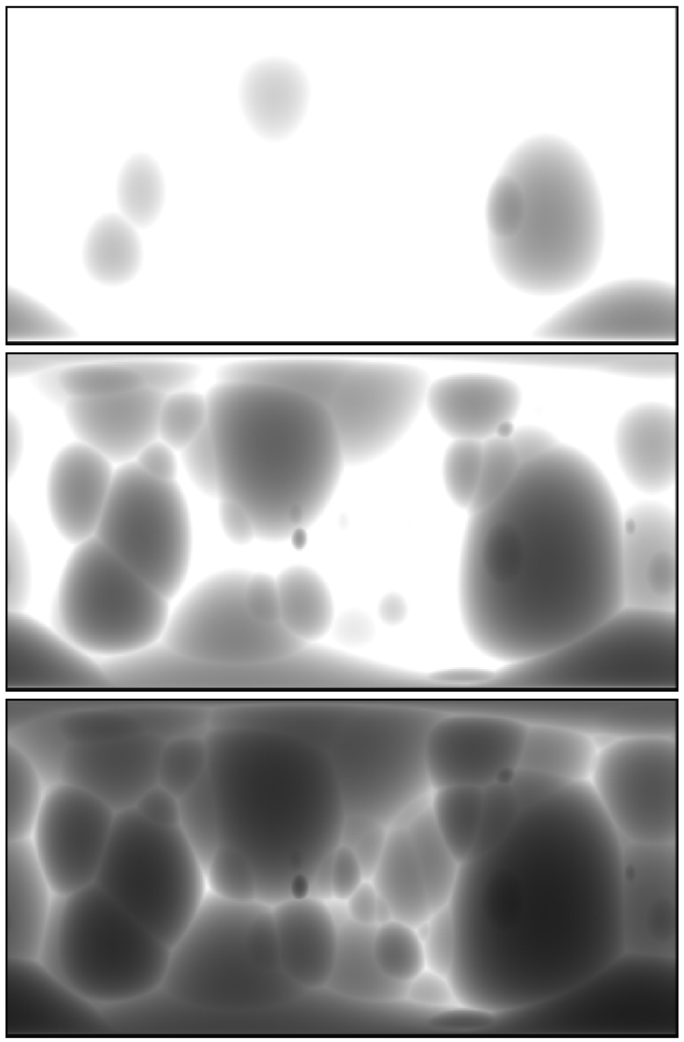
GW energy density:

$$\Omega_G = \frac{\langle \dot{h}_{ij}\dot{h}^{ij}\rangle}{G\rho_c} = \int \frac{dk}{k}\frac{d\Omega_G(k)}{d\log(k)}$$

A huge range of frequencies



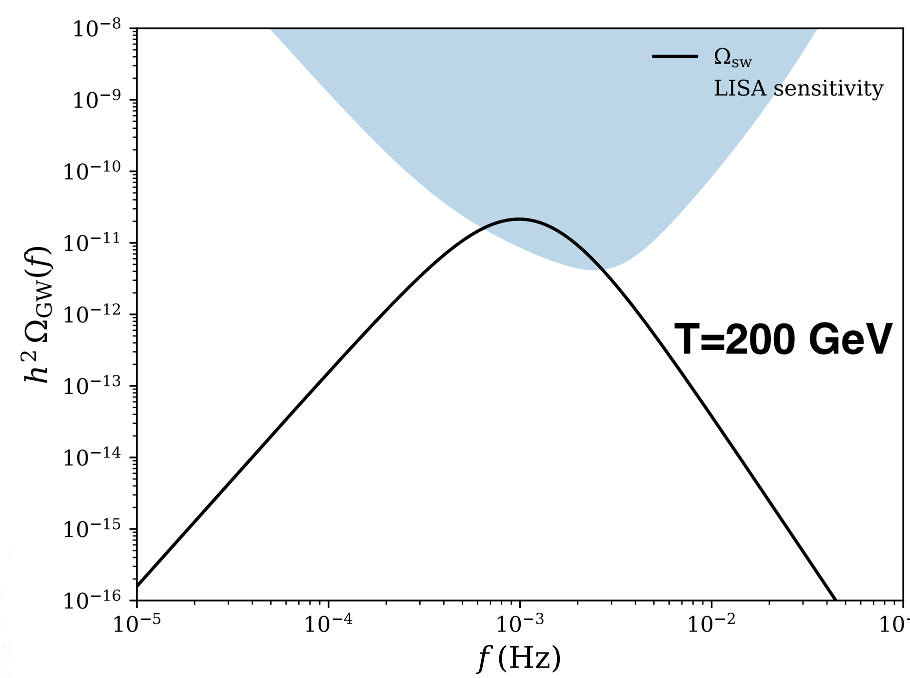
GWs from a 1st-order phase transition .

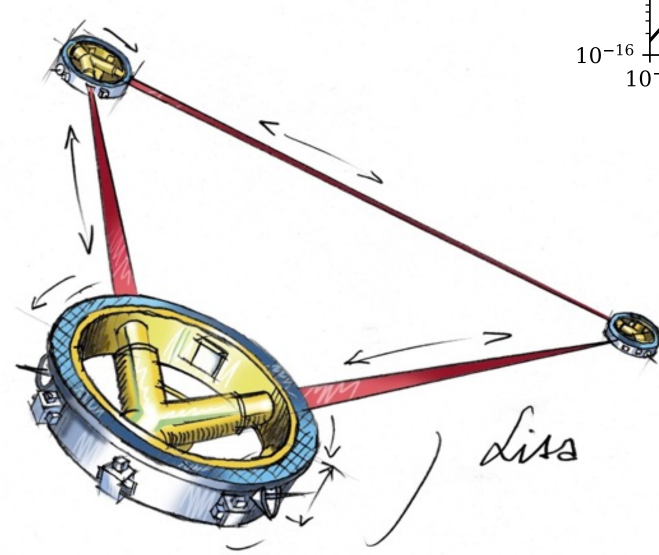


Konstandin'18

Electroweak Phase Transition

-> milli-Hertz —> LISA !





[LISA Cosmology Working group, 1512.06239]
+ update 1910.13125

LISA: a new window on the Weak Scale

hep-ph/0607107

complementary to collider informations

Two Characteristic quantities

- β : inverse duration of the phase transition

set by the tunneling probability $P \propto e^{\beta t} \propto \frac{T^4}{H^4} e^{-S_3/T} \sim 1$ ➜ $\frac{S_3}{T}$ ~ 140

$$\beta \equiv \frac{dS}{dt}\Big|_* = -H_* T_* \frac{dS}{dT}\Big|_*$$

typically $\frac{\beta}{H} \sim \mathcal{O}(10^2 - 10^3)$

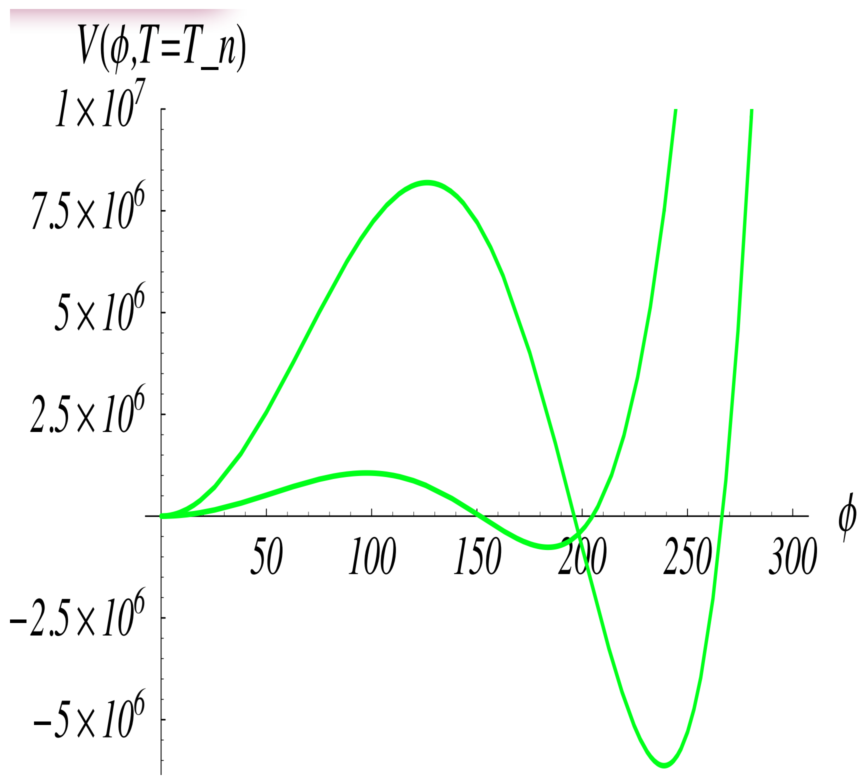
corresponds to the characteristic inverse size of bubbles at time of collisions $R_* = v_w/\beta$

sets the characteristic frequency $f_* \propto \frac{\beta}{v_w}$

$$f_0 = \frac{a_*}{a_0} f_* \approx 20 \frac{1}{v_w} \frac{\beta}{H_*} \frac{T_*}{100GeV} \left(\frac{g_*}{100}\right)^{1/6} \mu\text{Hz}$$

- α : vacuum energy/radiation energy density

α and β : entirely determined by the effective scalar potential at high temperature



Estimate of the GW energy density at the emission time .

$$\Omega_{GW,*} = \frac{\rho_{GW,*}}{\rho_{tot,*}} \sim \frac{G}{\beta^2}\Pi^2_{source} \times \frac{1}{\rho_{tot,*}} \sim \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2$$

$$\Pi_{source} \sim \kappa \rho_{vac}$$

$$\kappa_\phi = \frac{\rho_\phi}{\rho_{\rm vac}} \qquad \kappa_v = \frac{\rho_v}{\rho_{\rm vac}} \qquad \kappa_t = \epsilon\, \kappa_v$$

fractions of vacuum energy that goes into either gradient energy in bubble kinetic energy in the fluid o into turbulent motion.

$$\left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2 \sim \frac{\kappa^2\alpha^2}{(1+\alpha)^2}$$

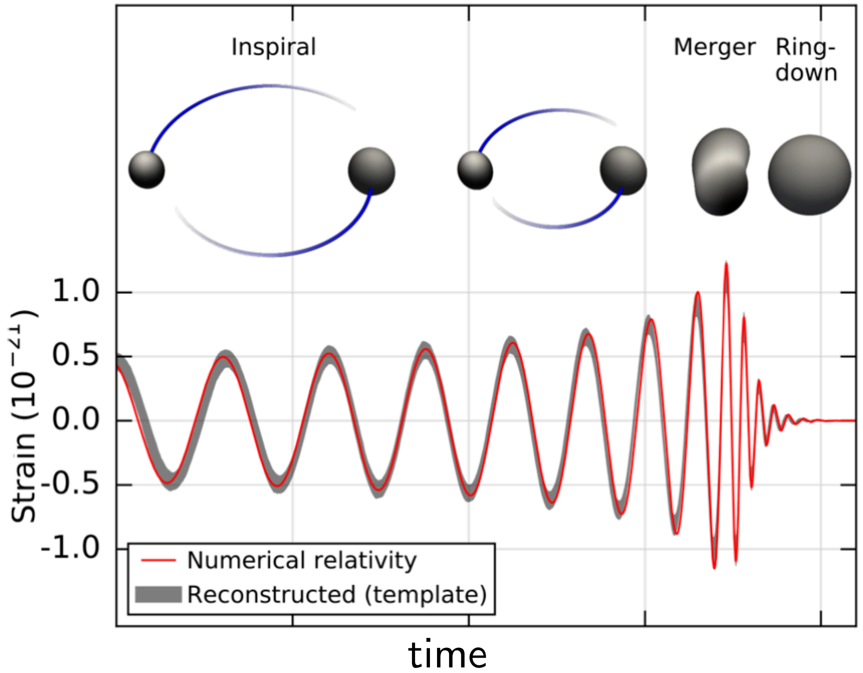
$$\boxed{\Omega_{GW,*} \sim \left(\frac{H_*}{\beta}\right)^2 \times \frac{\kappa^2\alpha^2}{(1+\alpha)^2}}$$

Other primordial gravitational waves

Two types of gravitational-wave (GW) signals

- **Astrophysical signals (in the late universe)**

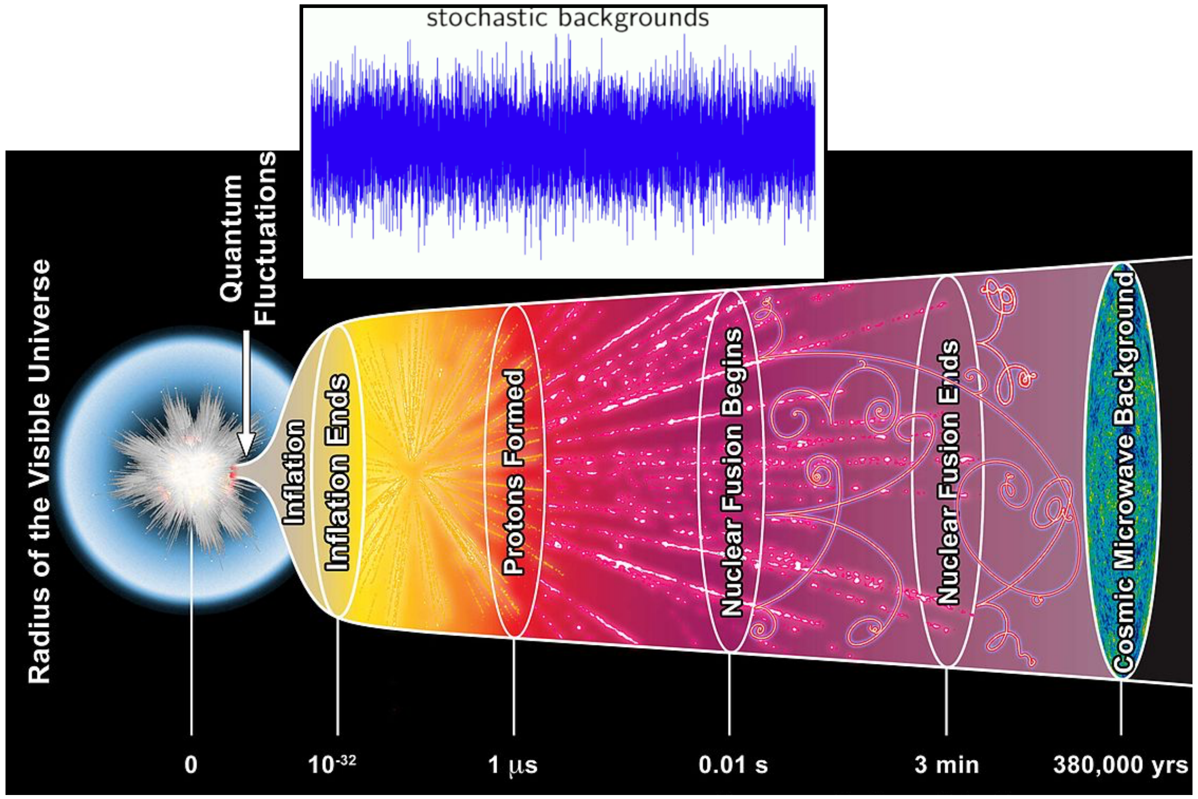
✔ detected



LIGO&Virgo, arXiv:1602.03841

- **Cosmological background filling the whole universe (a relic from the early universe)**

✘ not yet detected

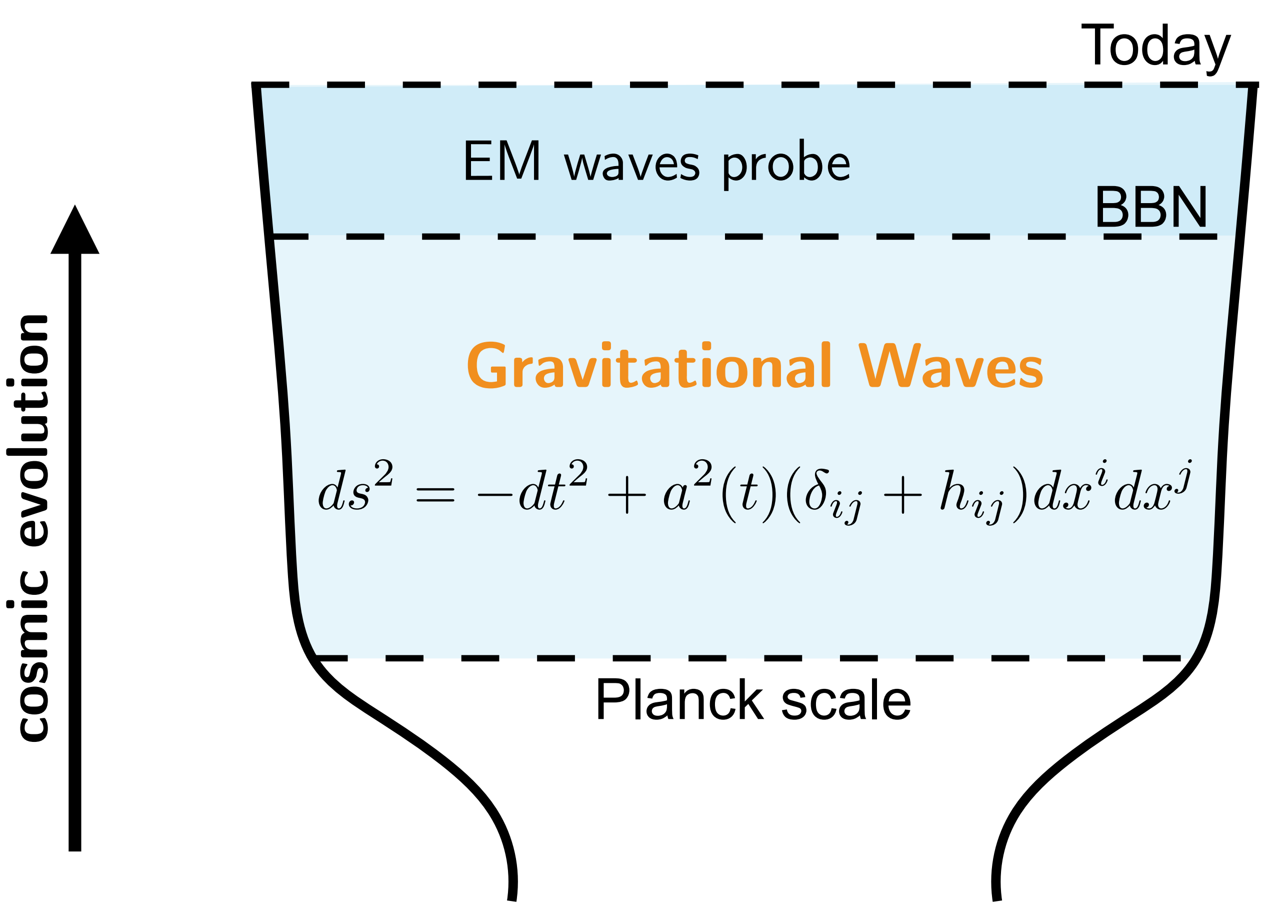


Note: Astrophysical signals can lead to a stochastic background if they cannot be resolved.

Primordial gravitational waves: Fossil radiation .

superposition of GW generated by an enormous number of causally independent sources, arriving at random times and from random directions.

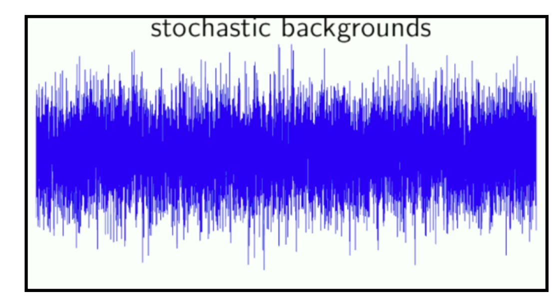
Individual waves are not detectable, sources can not be resolved but instead we can only observe a Stochastic GW Background. For most of the cosmological sources, it is homogeneous, isotropic, gaussian and unpolarized and appears as a noise in the detector.



Random h_{ij}

$\Downarrow$

Stochastic GW background (SGWB) characterized by energy density



$$\rho_{\mathrm{GW}} = \frac{\langle \dot{h}_{ij} \dot{h}^{ij} \rangle}{32\pi G}$$

Ensemble average

$\Updownarrow$

time/space average

Probing high-energy physics with gravitational waves .

Interaction rate of GW

Expansion rate

$$\frac{\Gamma_{\mathrm{GW}}(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{\mathrm{pl}}} = \left(\frac{T}{M_{\mathrm{pl}}}\right)^3 \ll 1$$

GW produced below the Planck scale are decoupled: They propagate freely in the universe until today.

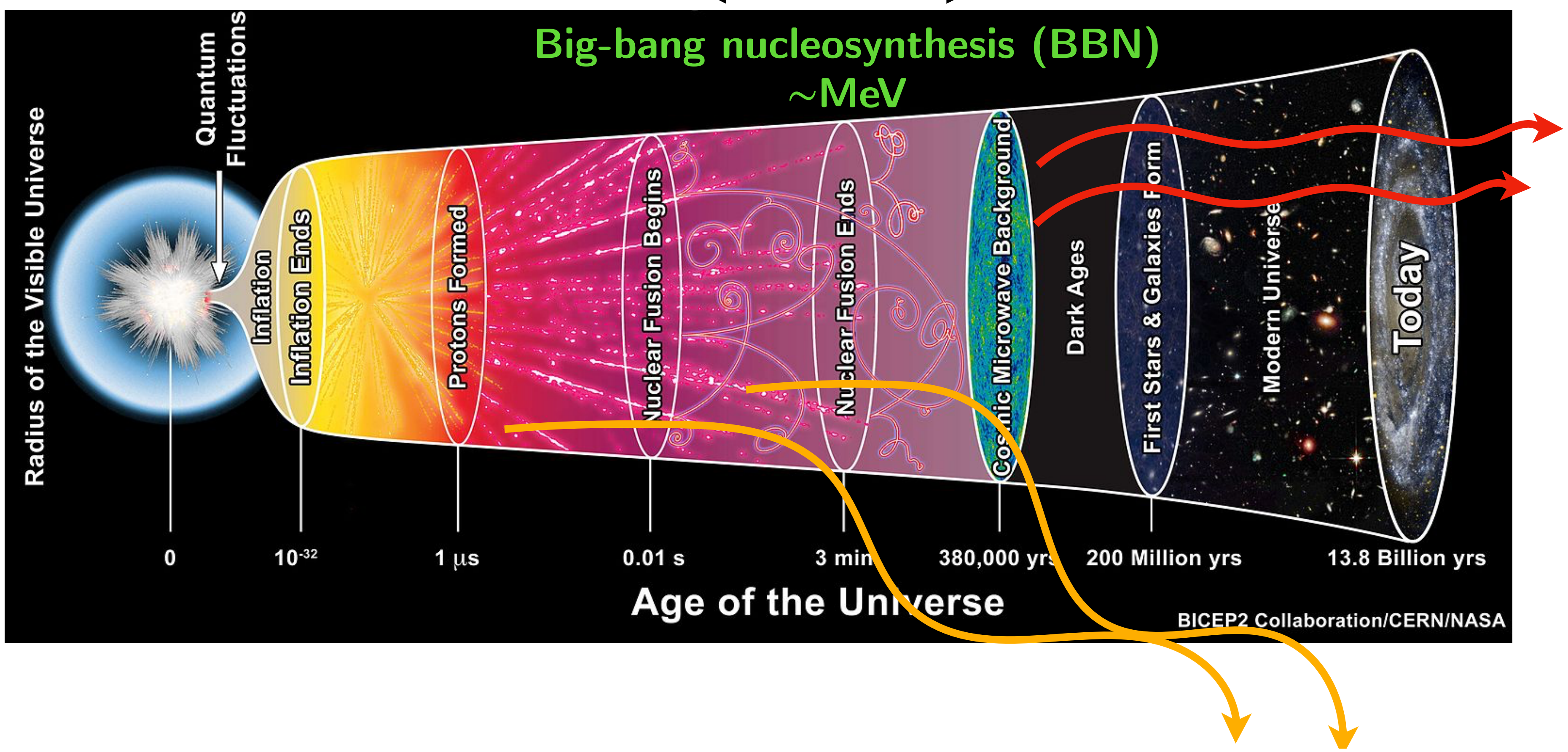
They do not loose memory of conditions when produced.

They retain spectral shape, typical frequency and intensity characteristic of production mechanism, encoding information about particle physics at high-energy scales that cannot be probed by colliders.

Probing high-energy physics with gravitational waves

High energies **Low energies**

unconstrained ← → **well-tested**



Electromagnetic-wave probes

GW

Energy density of GW background: $\rho^{\mathrm{GW}}_{\mathrm{today}} = \rho^{\mathrm{GW}}_{\mathrm{prod}} \left(\dfrac{a_{\mathrm{prod}}}{a_{\mathrm{today}}} \right)^4$

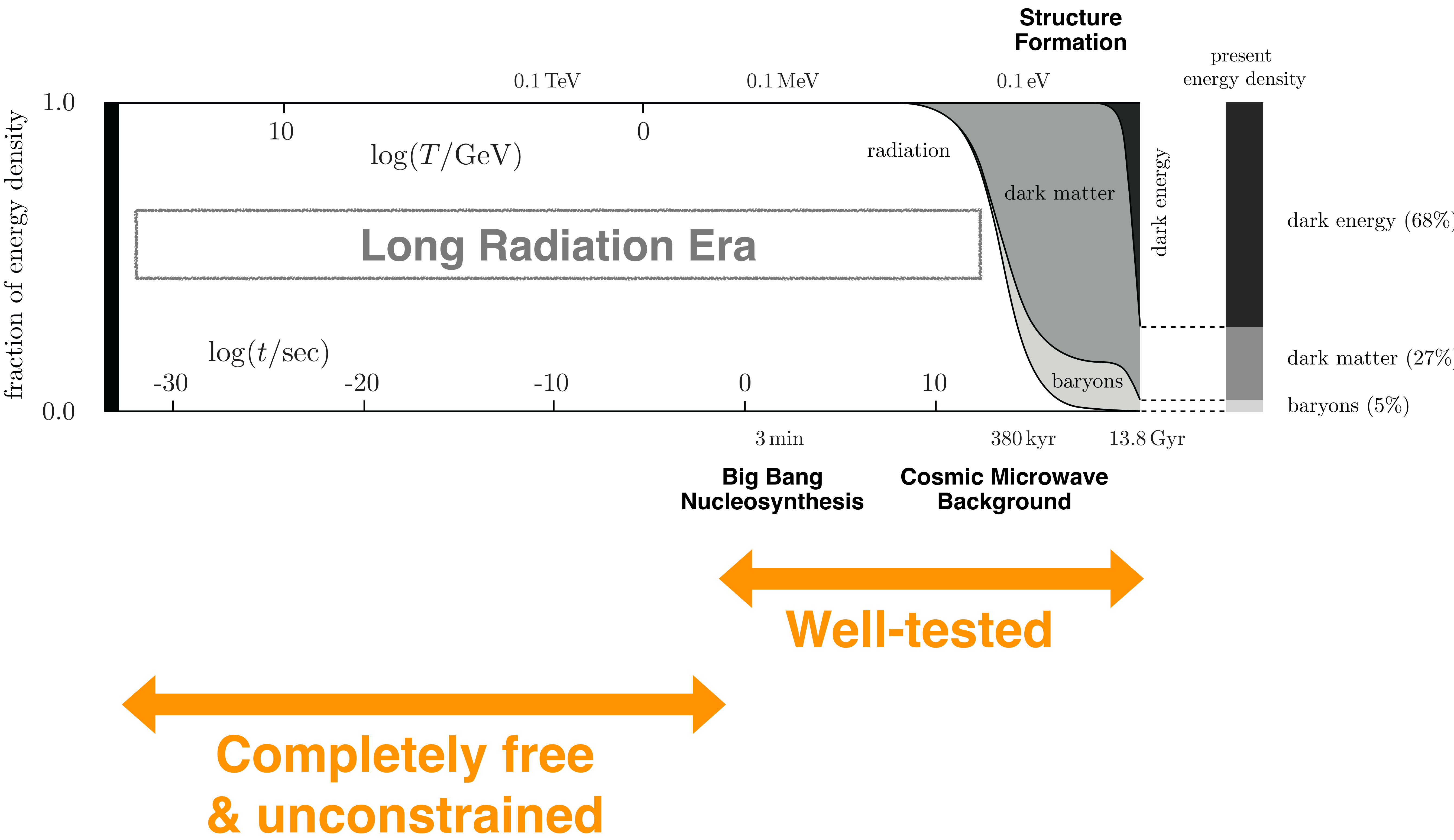
The universe is expanding. ⇨ BSM of cosmology

its production mechanism ⇨ particle physics beyond the Standard Model (BSM)

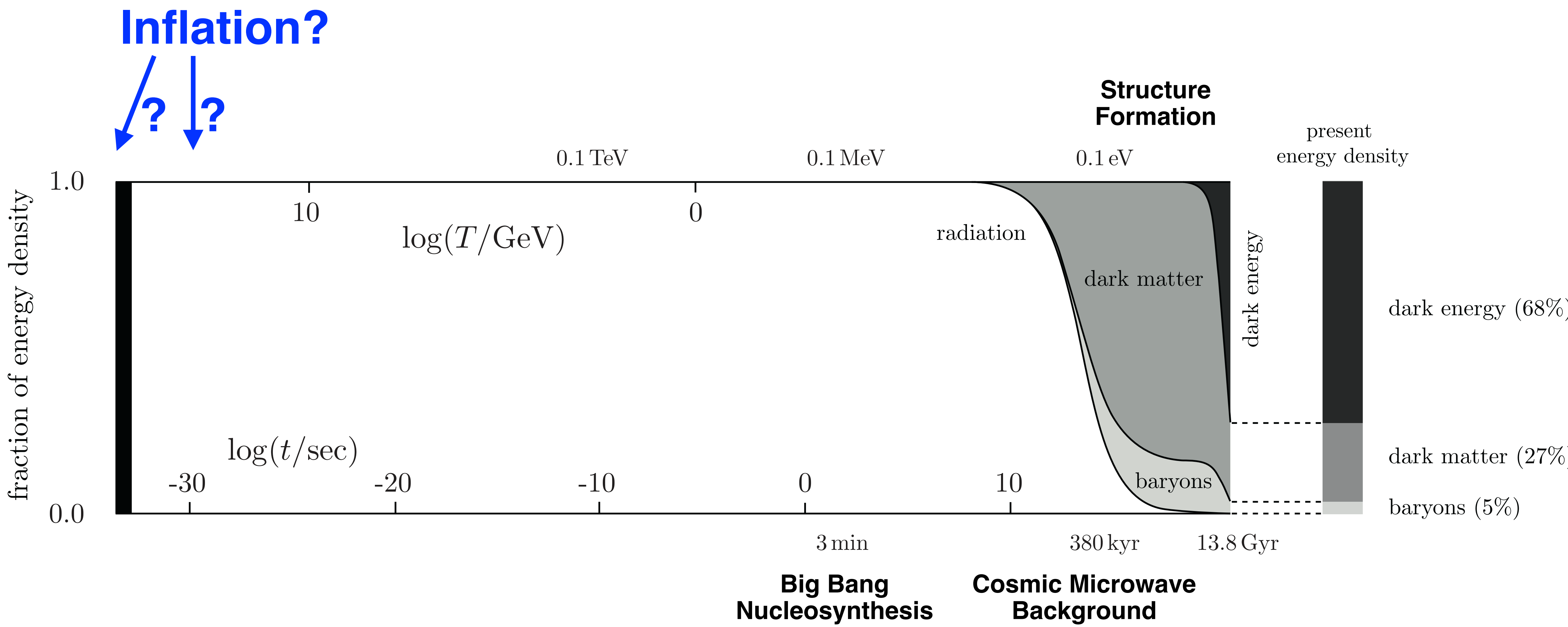
Key question:

What can we learn on particle physics from early universe cosmology?

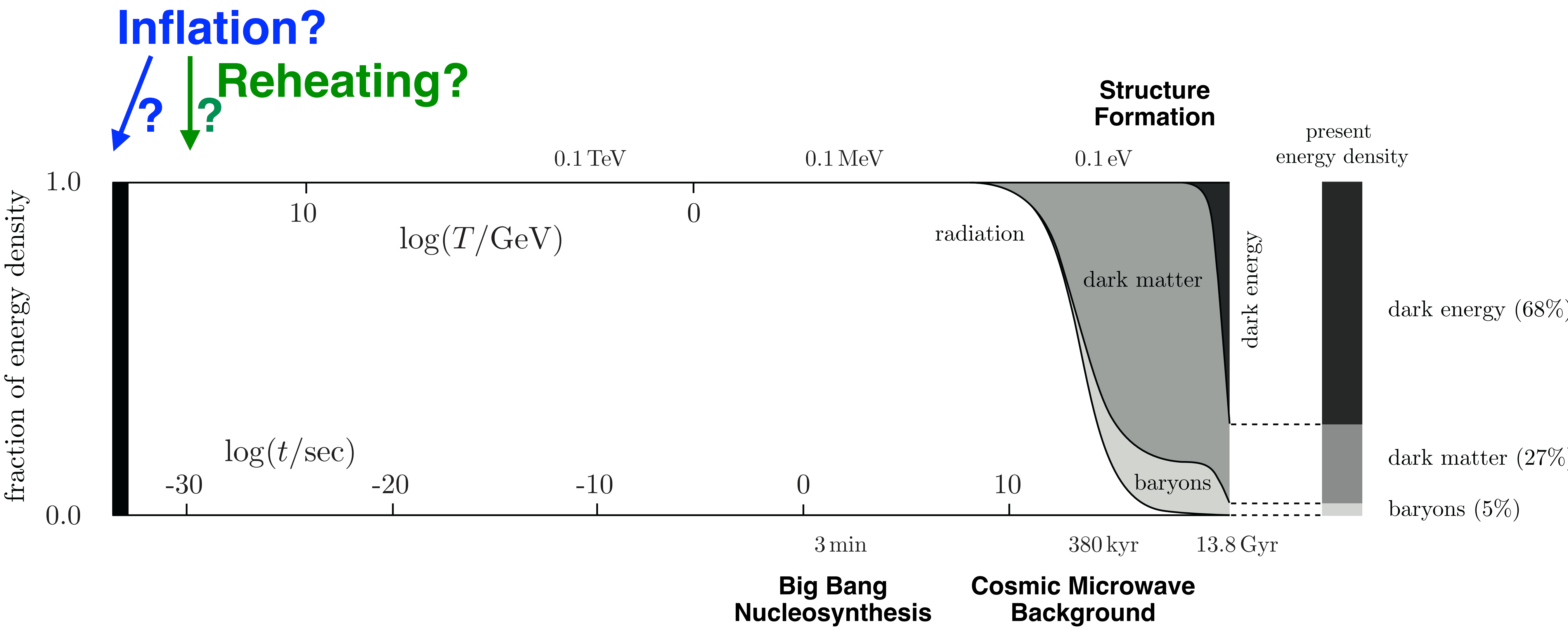
Standard Cosmological History



Cosmological History

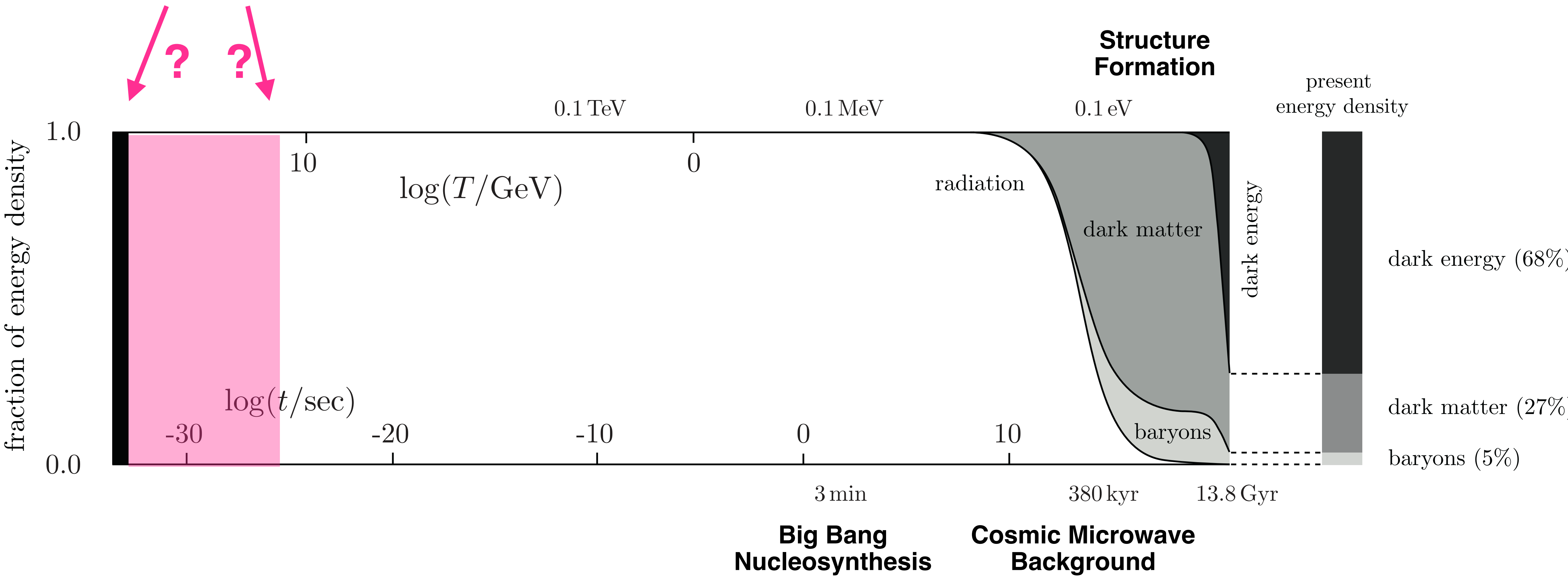


Cosmological History



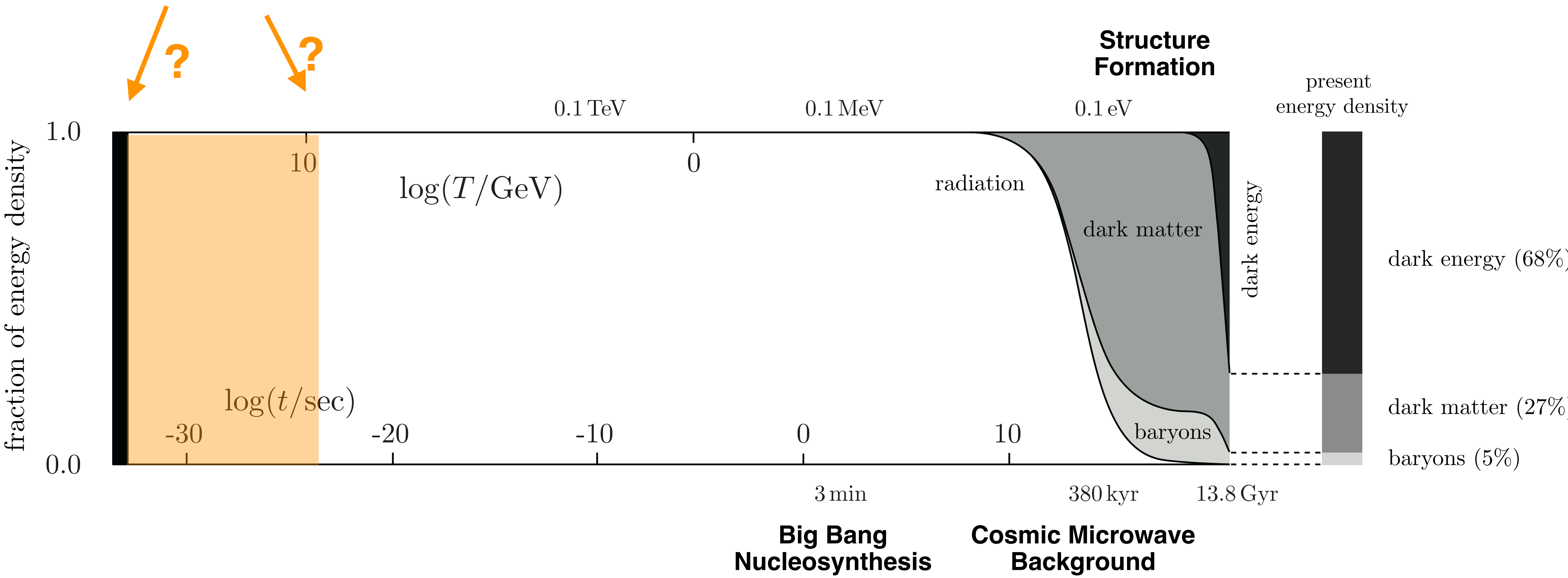
Cosmological History

Kination after inflation?



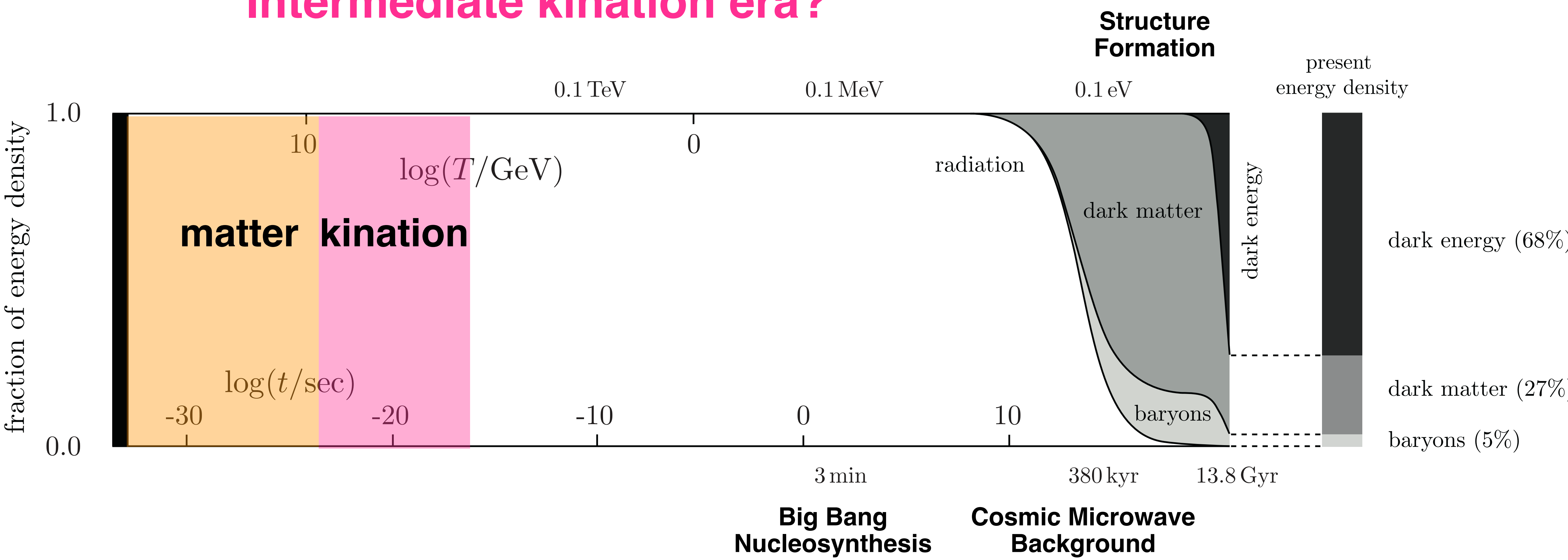
Cosmological History

Early Matter era after inflation?



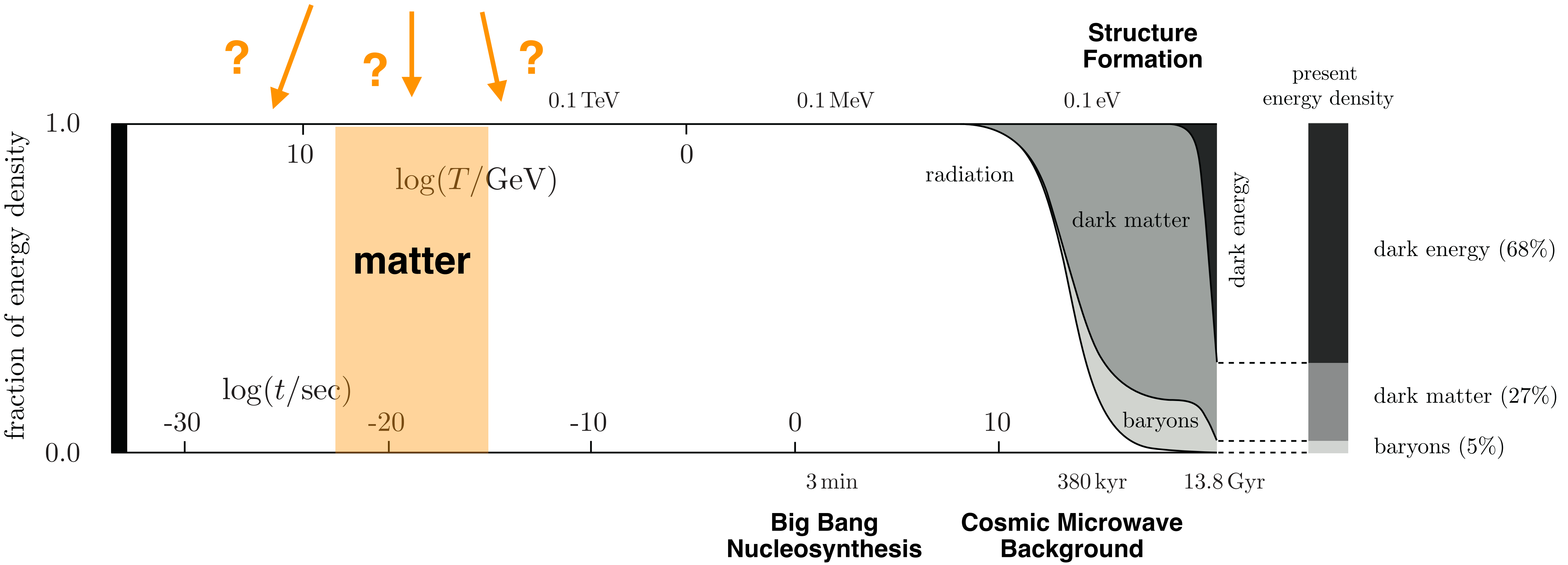
Cosmological History

Intermediate kination era?



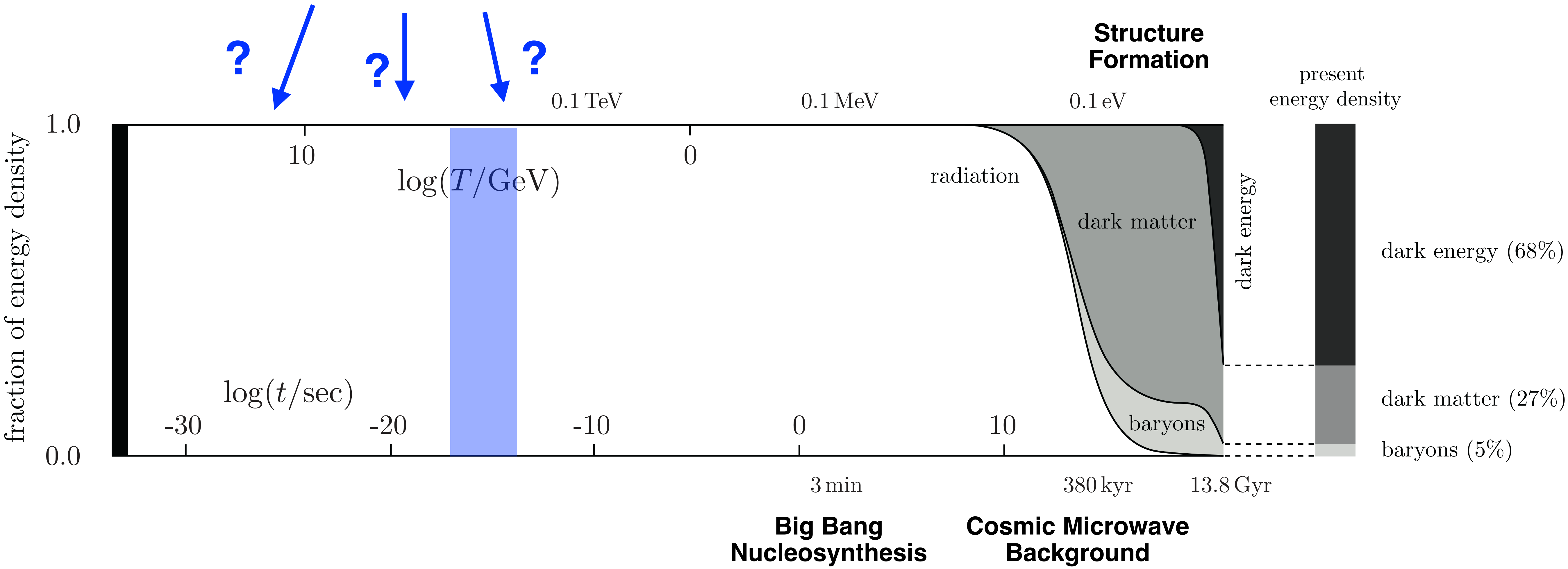
Cosmological History .

Intermediate matter eras?

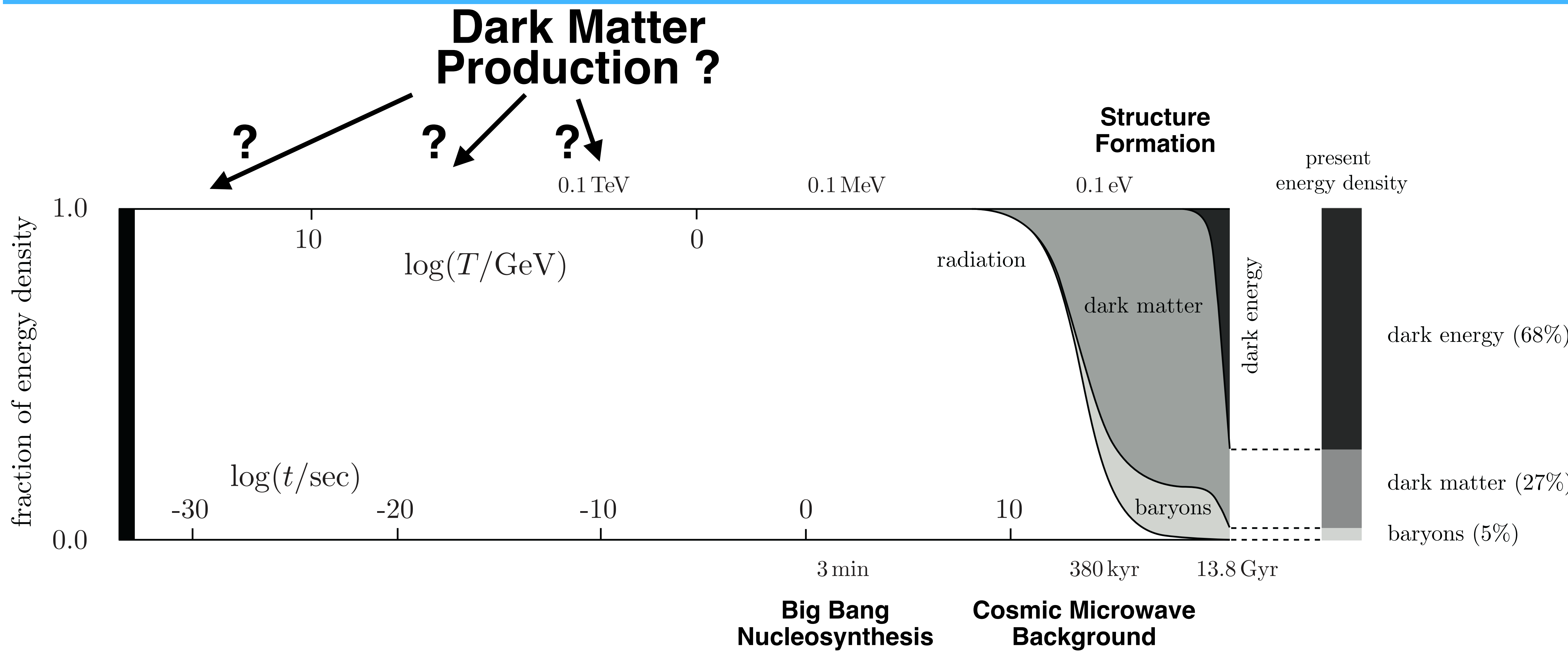


Cosmological History

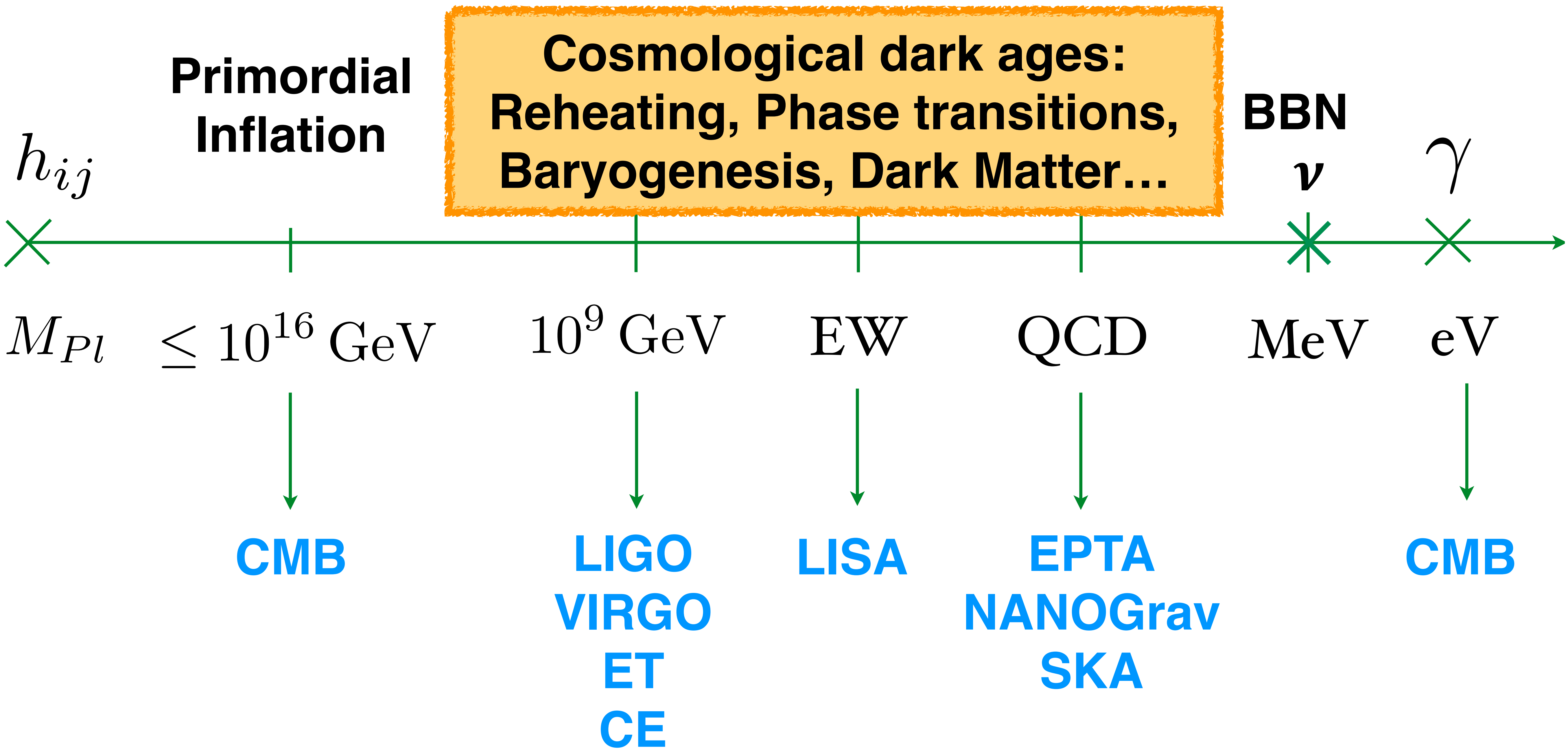
Secondary intermediate late inflation eras?



Cosmological History

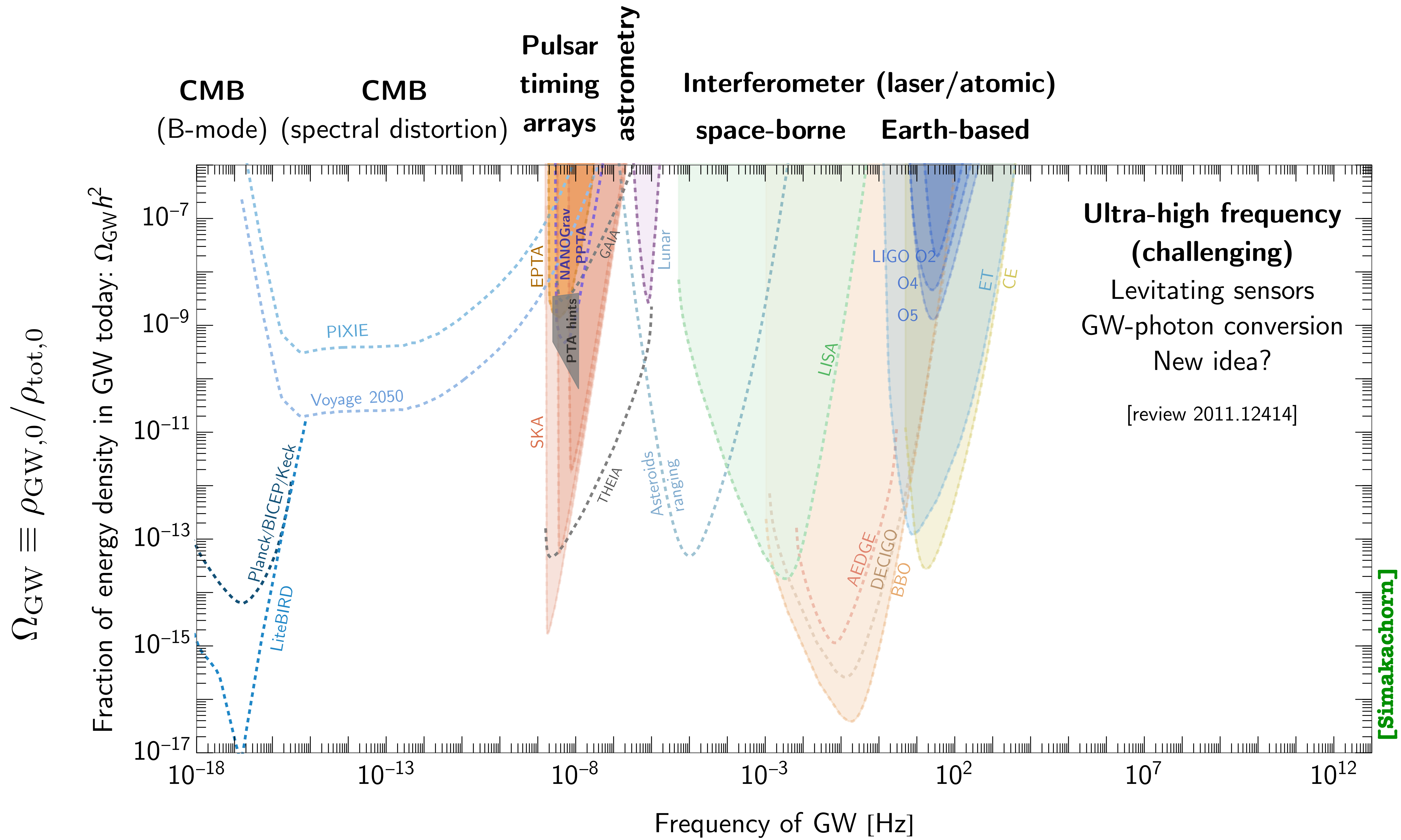


Probing the cosmological history with Gravitational Waves:



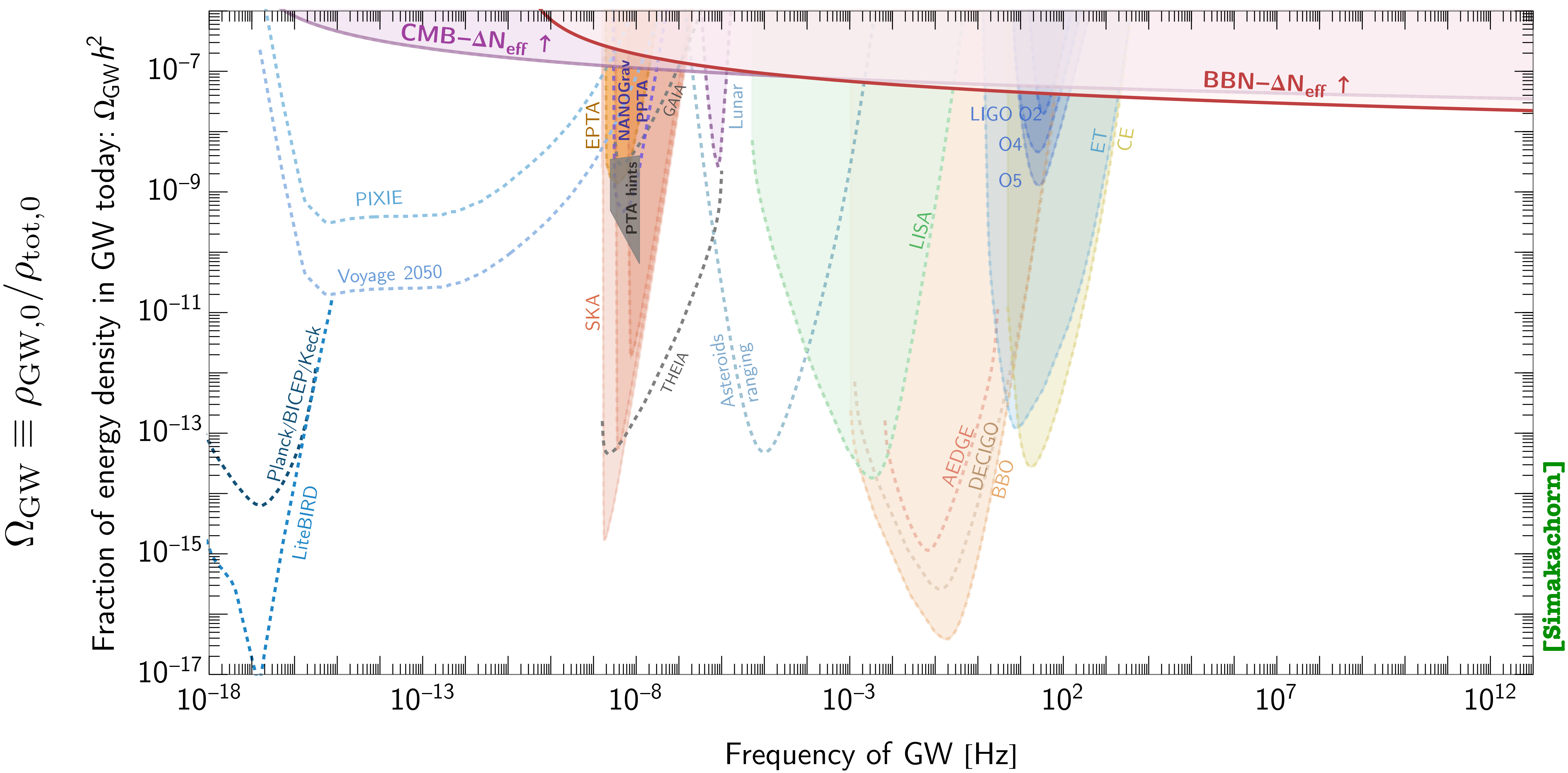
Current and future GW experiments constitute a new avenue of investigation in particle physics and cosmology.

The landscape of current & future GW experiments.



Upper theoretical bound.

GW as extra radiation: $\int_{f_{\min}}^{f_{\max}} \frac{df}{f}\, \Omega_{\rm GW}(f) \lesssim 0.23\, \Omega_{\rm rad,0} \Delta N_{\rm eff}$ where $\Delta N_{\rm eff}^{\rm BBN,CMB} \lesssim 0.2$



Primordial GW

Tensor perturbations of Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

Wave equation:

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Source:

Tensor anisotropic stress

=Transverse Traceless component
of the energy-momentum tensor of the source $= (P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})T_{lm}$

$P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Well-known cosmological sources .

-> Cosmological Phase Transitions

-> Cosmic Strings

-> Inflation

-> Reheating of the universe

see review 1801.04268

Characteristic Frequencies for causal (and short-lasting) sources .

T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

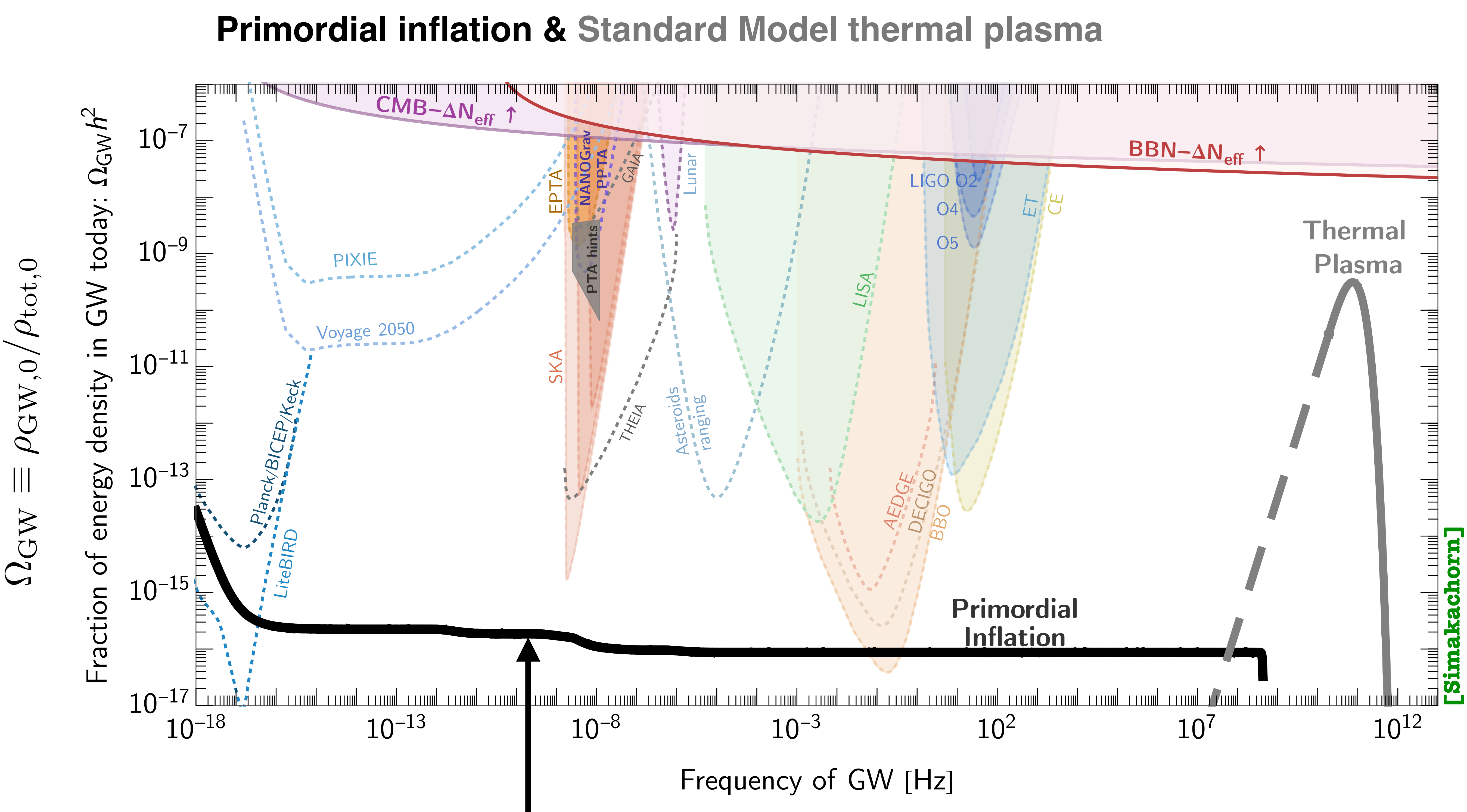
observed frequency: $f \sim f_* \dfrac{T_0}{T_*} \sim \mathcal{O}(H_*) \dfrac{T_0}{T_*} \sim \dfrac{T_*}{M_{Pl}} T_0 \sim T_* \times 10^{-18} 10^{-12}$ **GeV**

If T_* **~ 100 GeV:**

$$f \sim 10^{-28}\ \text{GeV} \sim 10^{-28} \times 10^{25}\ \text{Hz} \sim \text{mHz}$$

LISA !

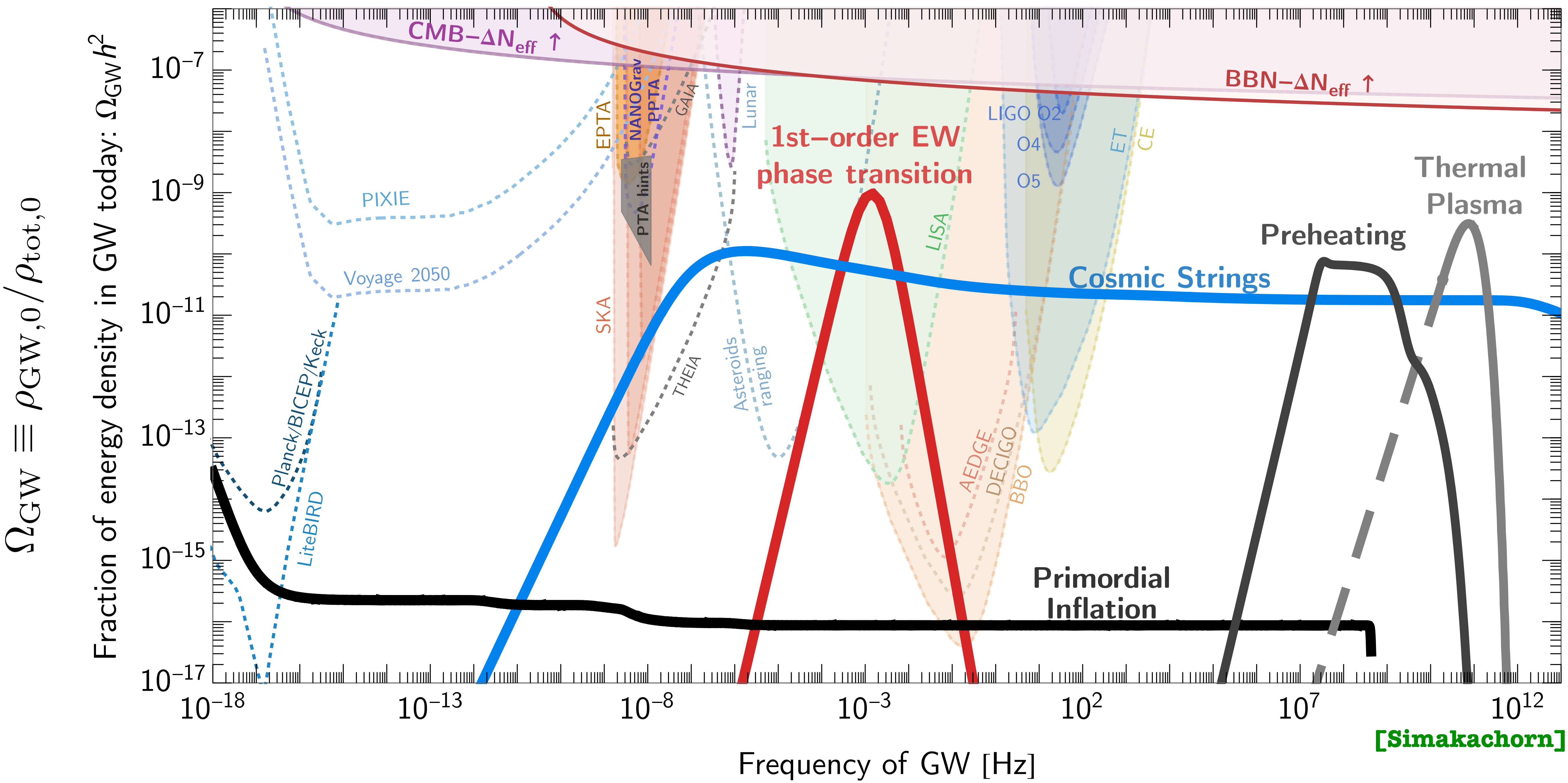
Standard Model sources of primordial GW.



Irreducible GW background from amplification of initial quantum fluctuations of the gravitational field during inflation

Beyond-the-Standard Model sources.

Preheating, first-order phase transitions, cosmic strings



Reading the history of the universe.

GW frequency $$f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)$$

Low-freq. limit

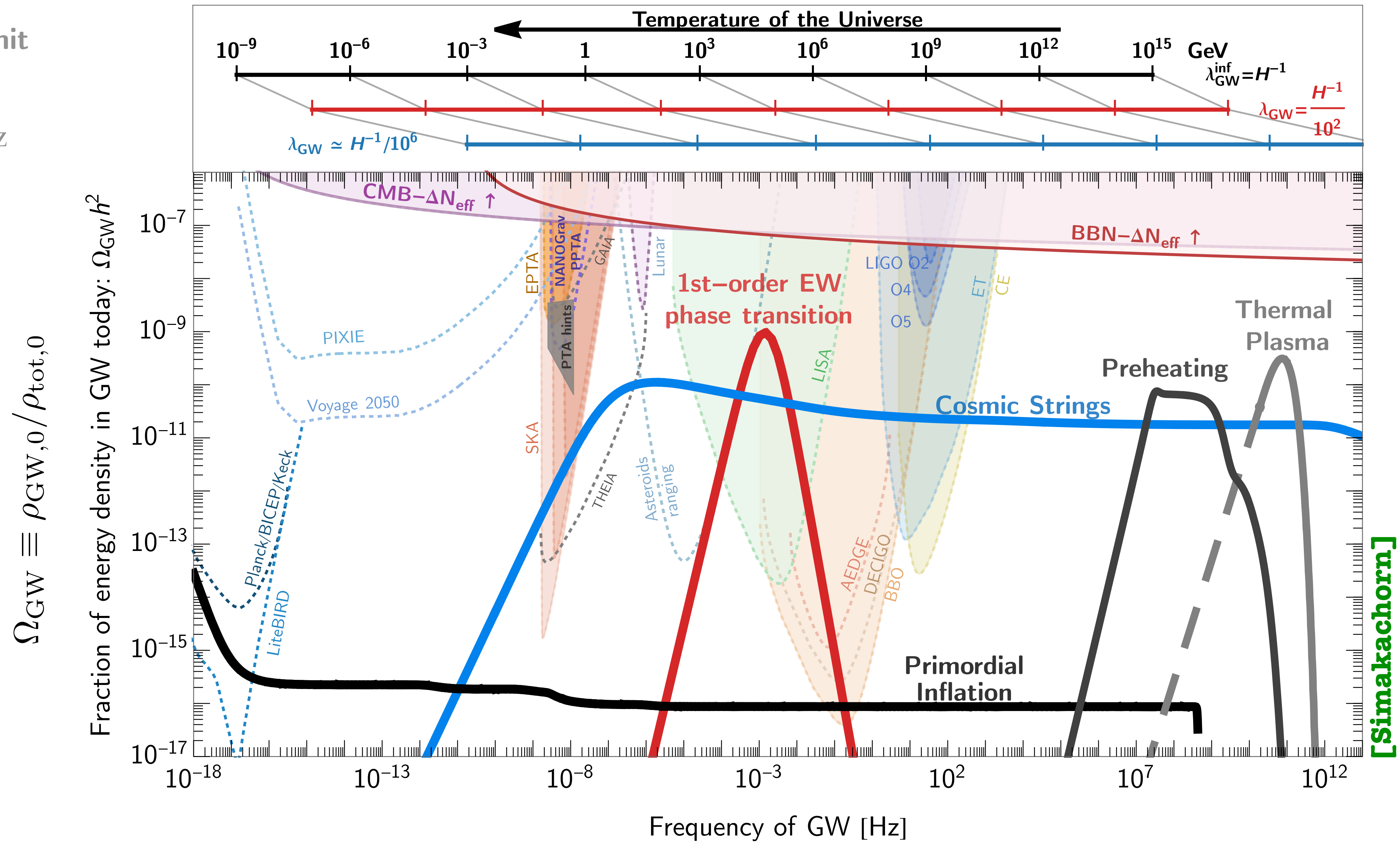
$f_{\mathrm{GW}}^{\min} \simeq H_0^{-1}$

$\simeq 10^{-18}$ Hz

High-freq. limit

$f_{\mathrm{GW}}^{\max} \simeq 10^{13}$ Hz

$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$



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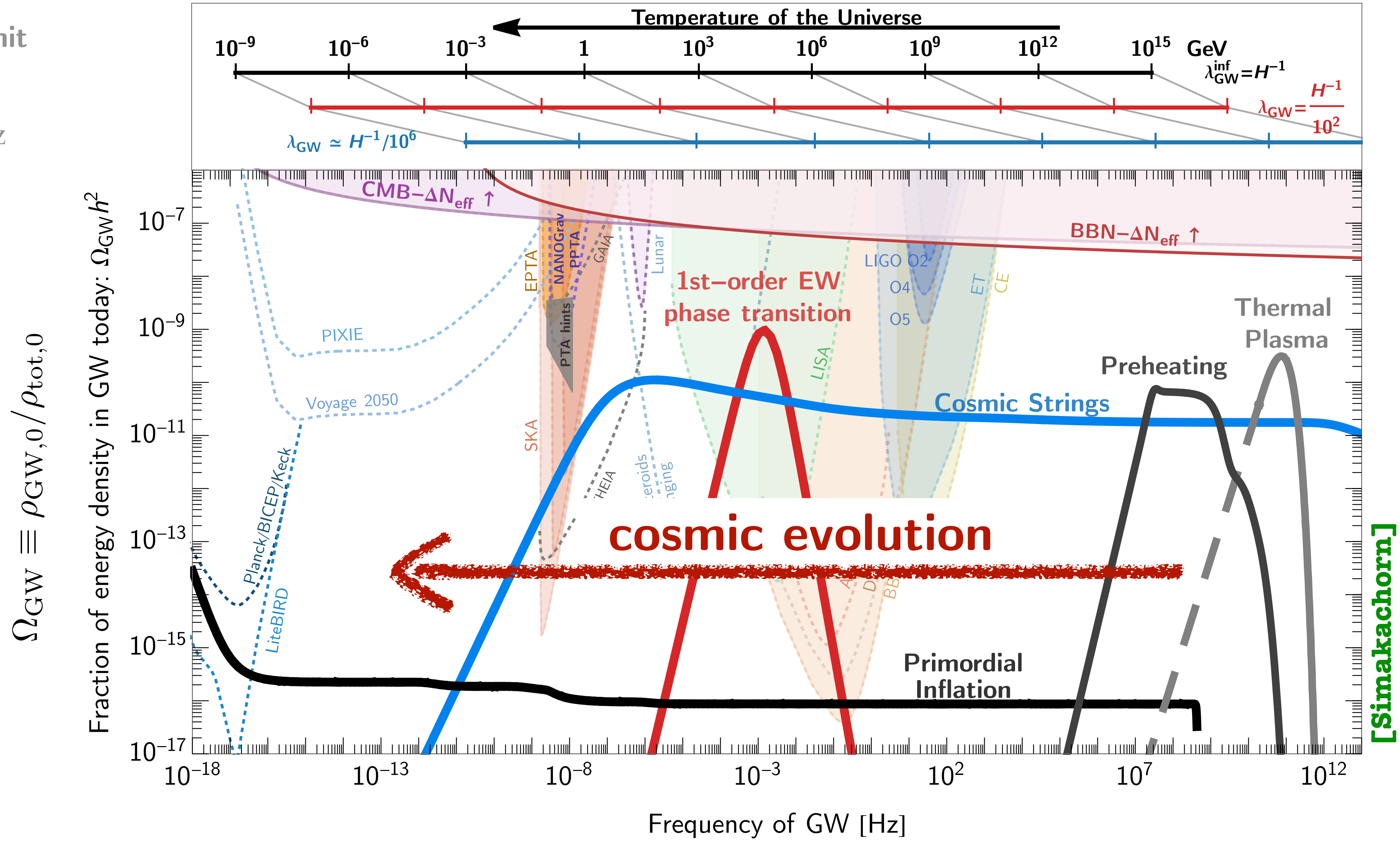

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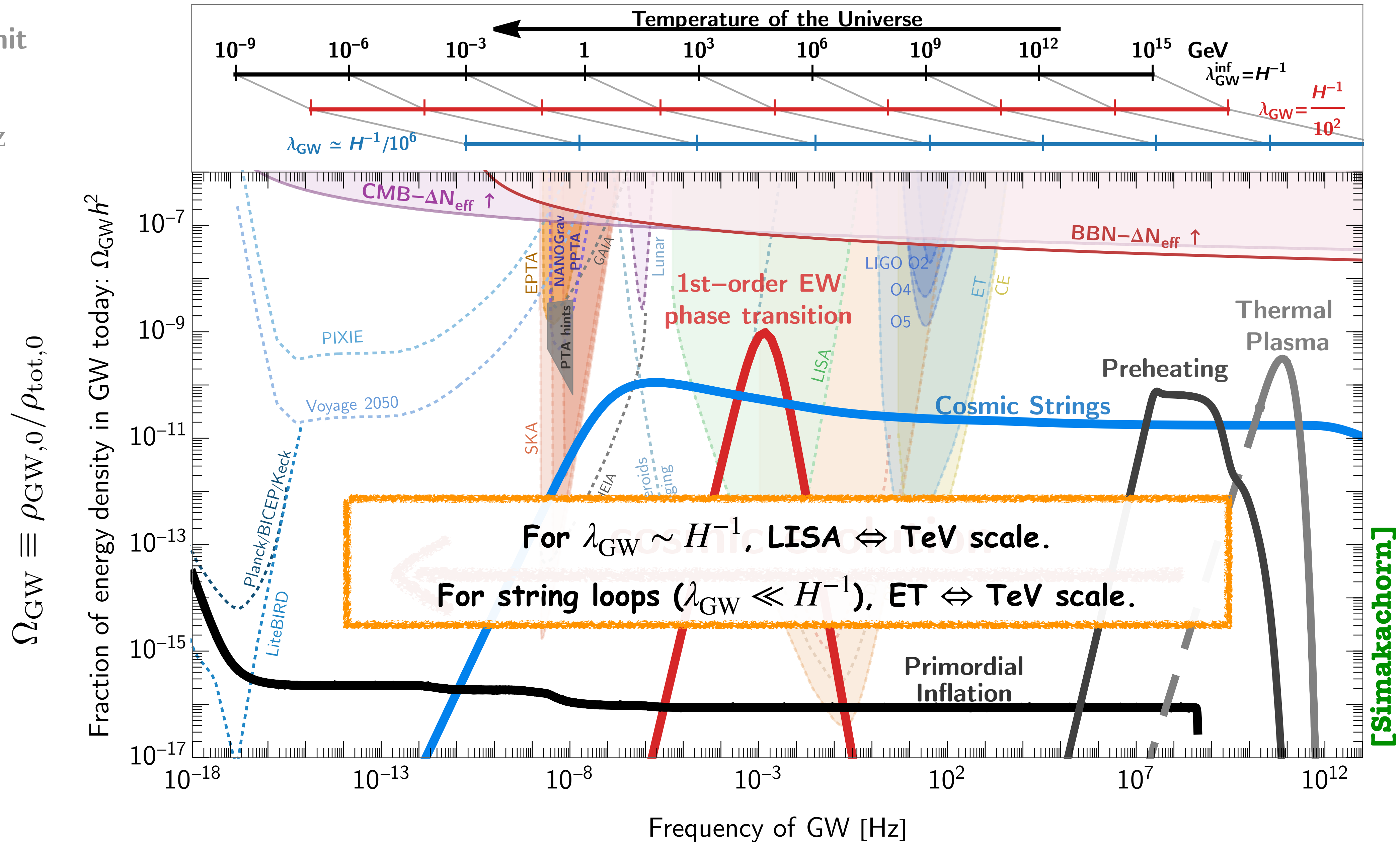


For $\lambda_{\mathrm{GW}} \sim H^{-1}$, LISA ⇔ TeV scale.

For string loops ($\lambda_{\mathrm{GW}} \ll H^{-1}$), ET ⇔ TeV scale.

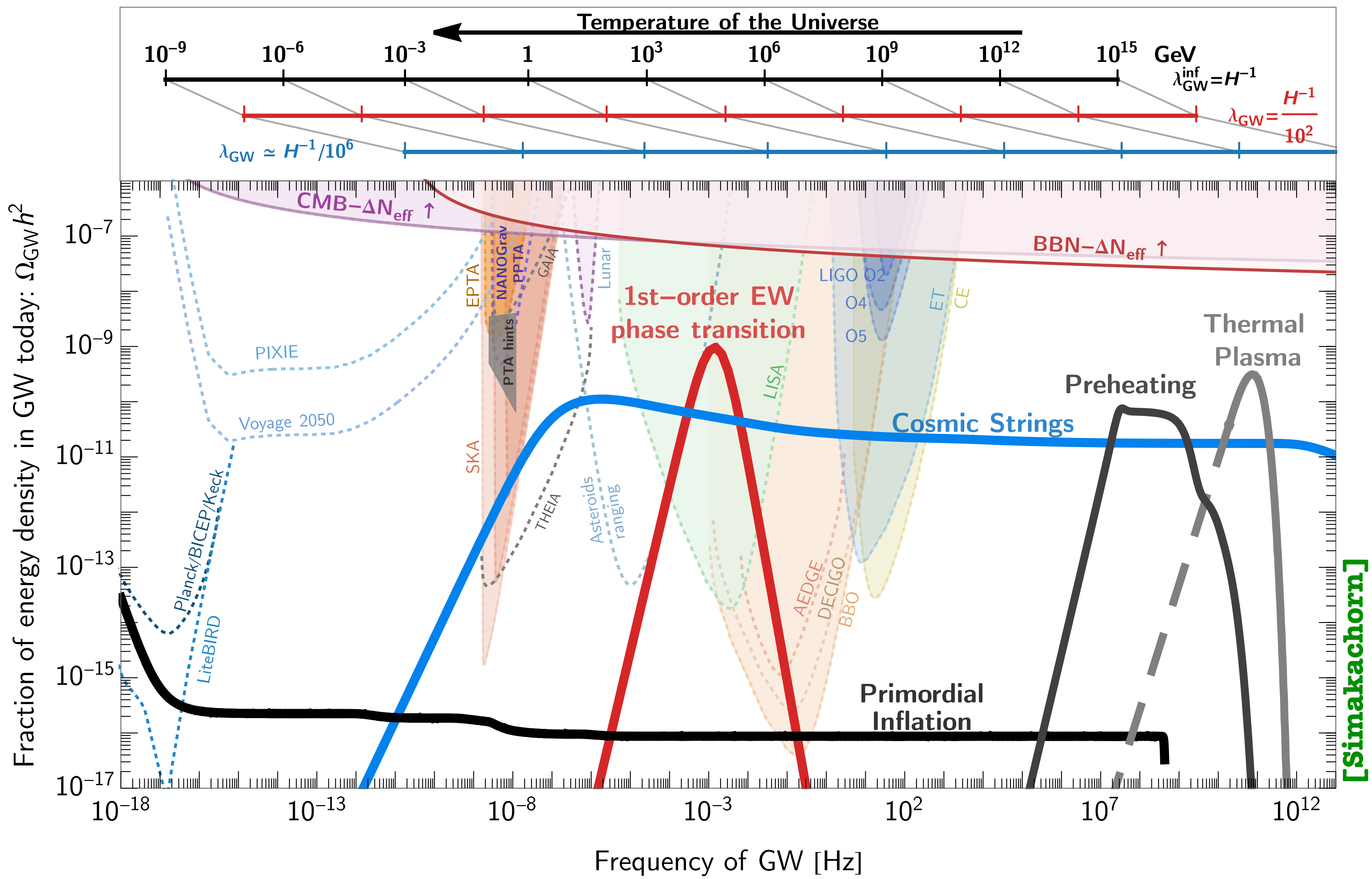
High-freq. limit

$f_{\mathrm{GW}}^{\mathrm{max}} \simeq 10^{13}$ Hz

$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$

GW spectra are sensitive to the cosmological history.

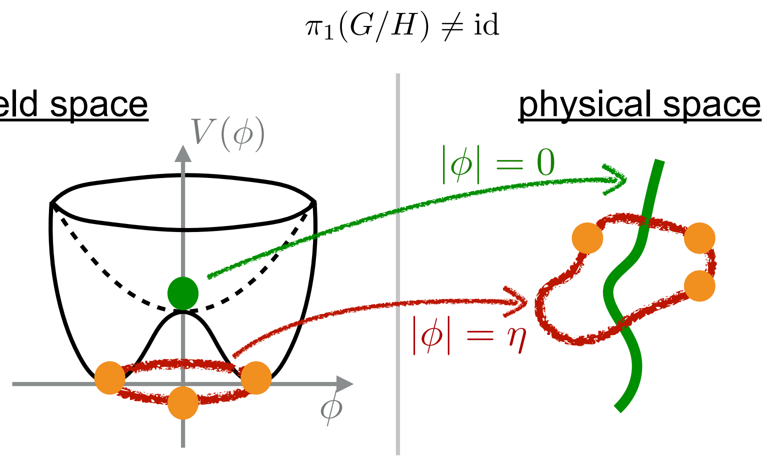
frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1} \left(\frac{a_{\mathrm{prod}}}{a_0} \right)$ **energy density** $\rho_{\mathrm{GW},0} \simeq \rho_{\mathrm{GW}}^{\mathrm{prod}} \left(\frac{a_{\mathrm{prod}}}{a_0} \right)^4$

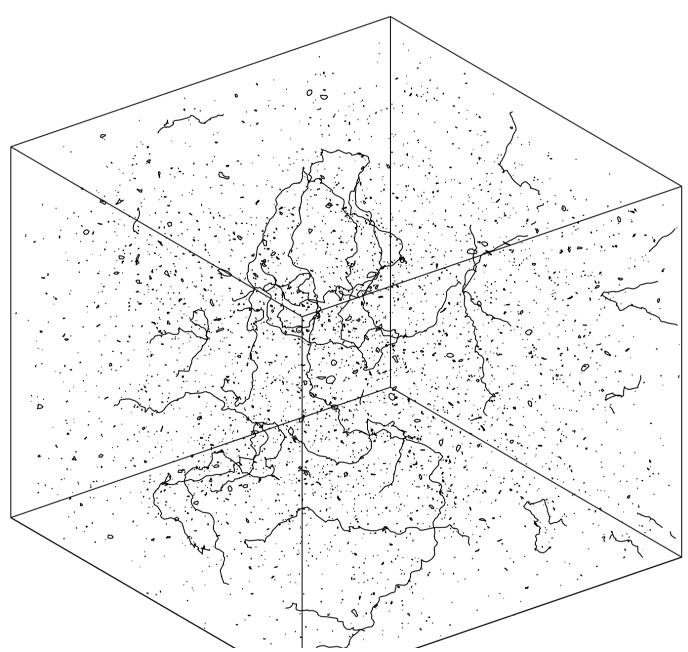


Standard Model radiation era at high energies

What if the universe is not radiation-dominated at high energies?

Gravitational Waves from cosmic strings.

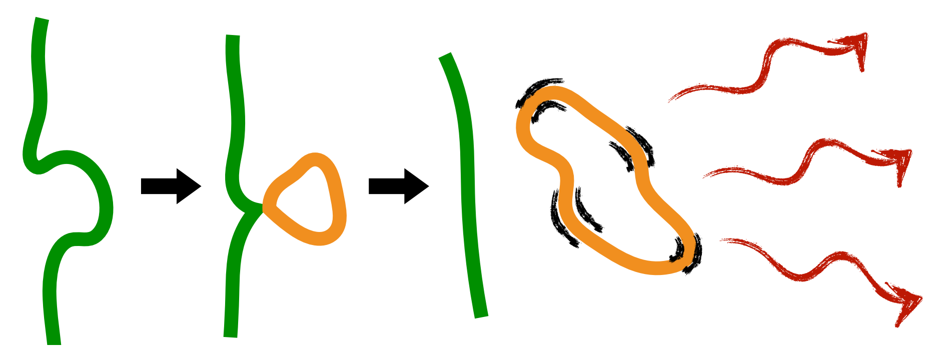




Network of cosmic strings
[Allen & Shellard, 1990]

Cosmic strings:

Long-lasting source of GW



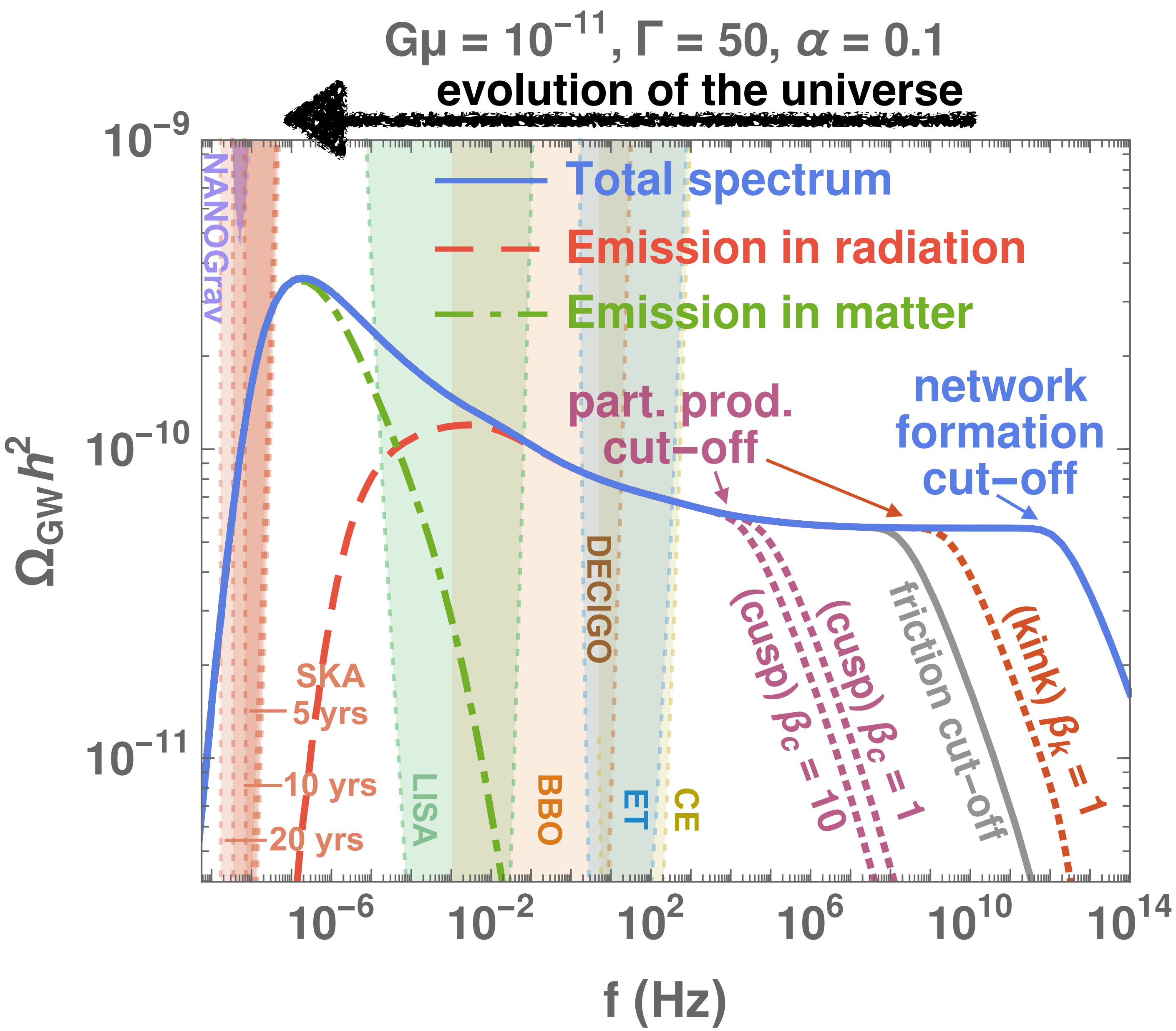
string tension: $\mu \sim \eta^2$

recent review:

JCAP 07 (2020) 032, [1912.02569].

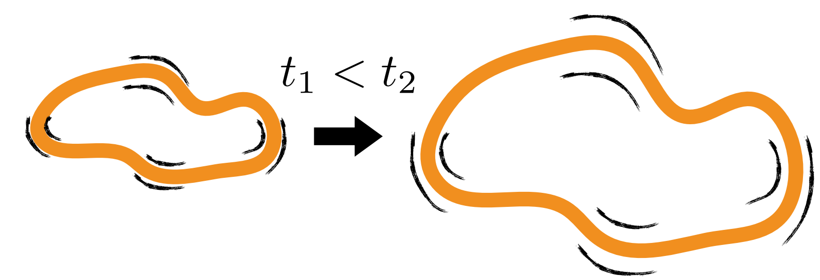
Gravitational Waves from Cosmic strings.

(long-lasting sources).



[1912.02569]

Higher f ⇔ Earlier emission



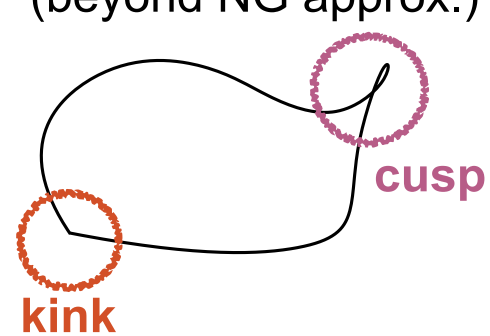
smaller loop ⇔ higher oscillation f

@ earlier t_i

more GW from more loops
but more red-shift

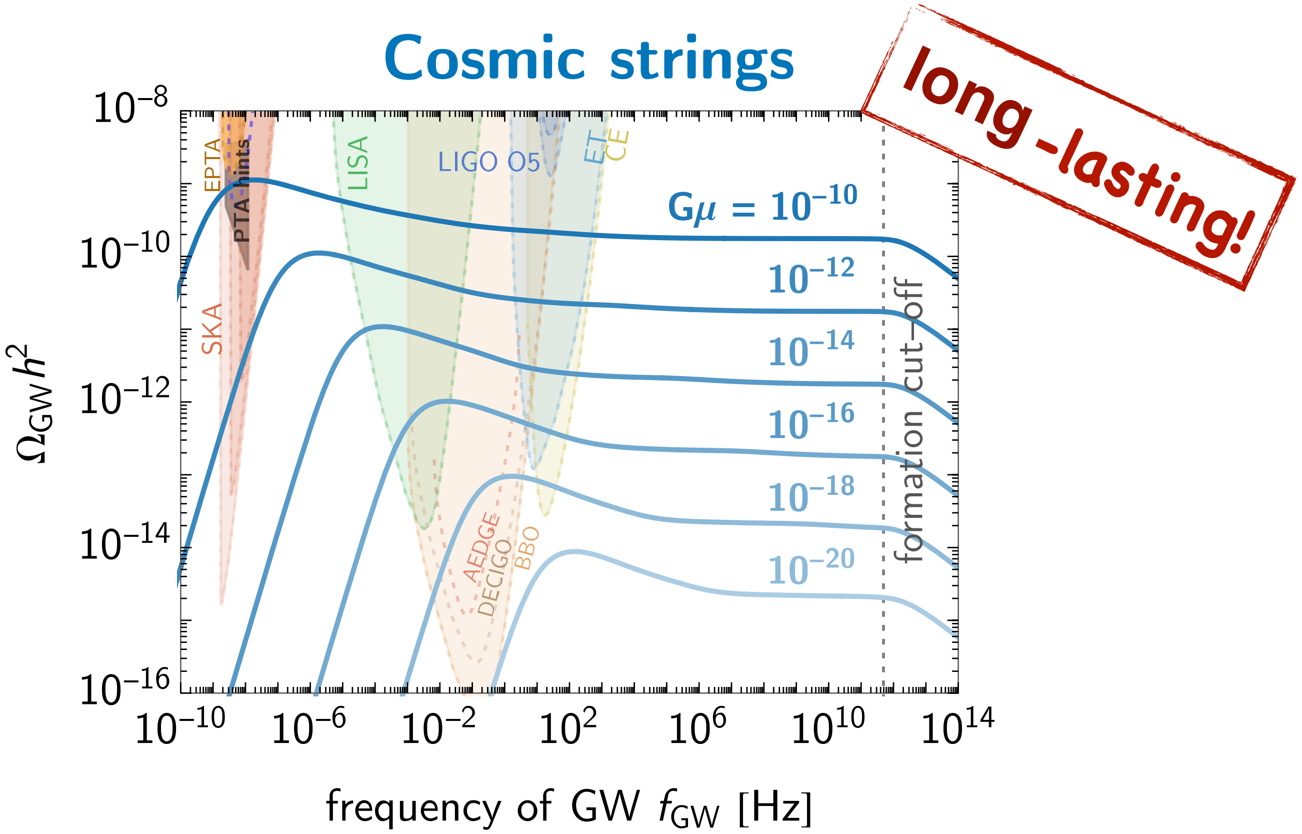
⇨ Flat during radiation

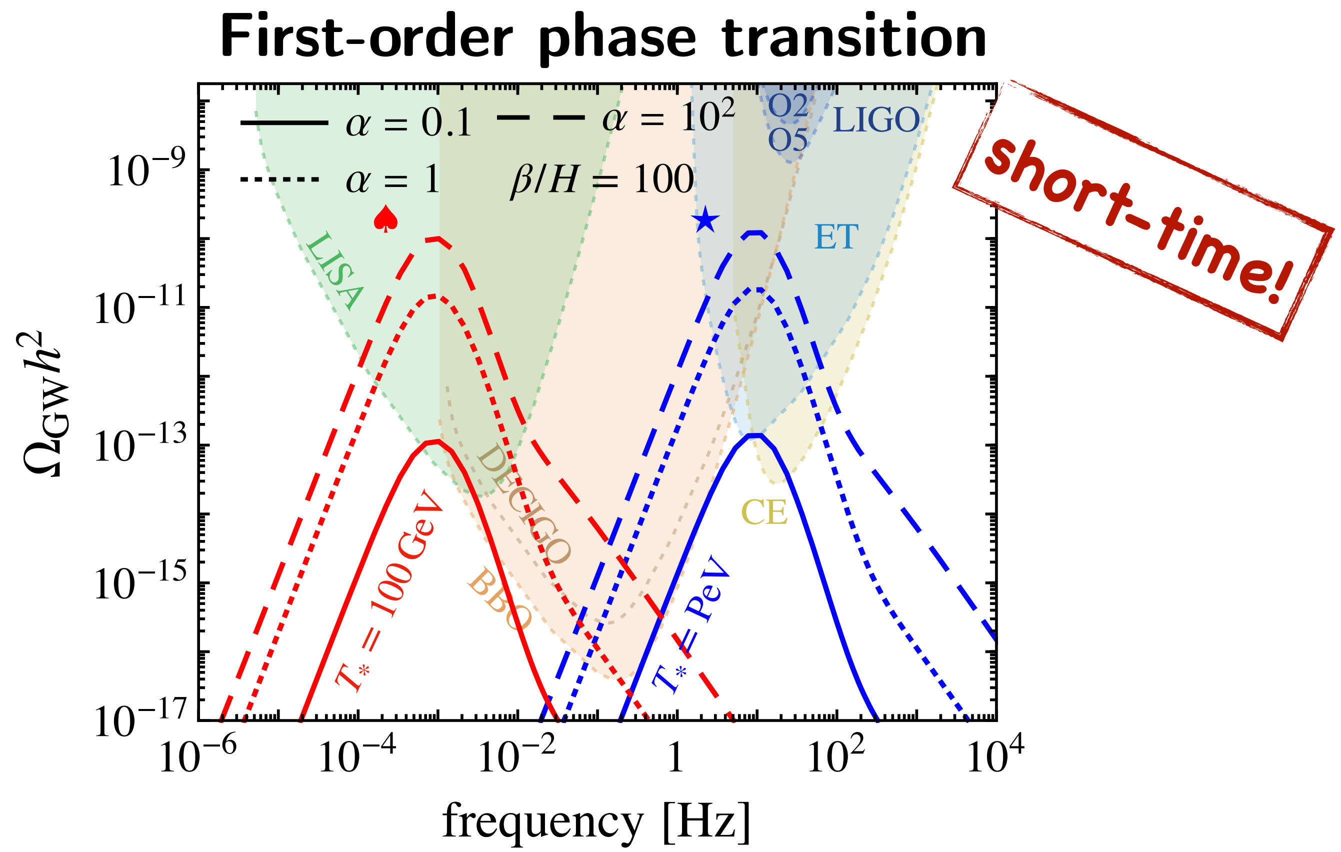
singular structures on loop
(beyond NG approx.)



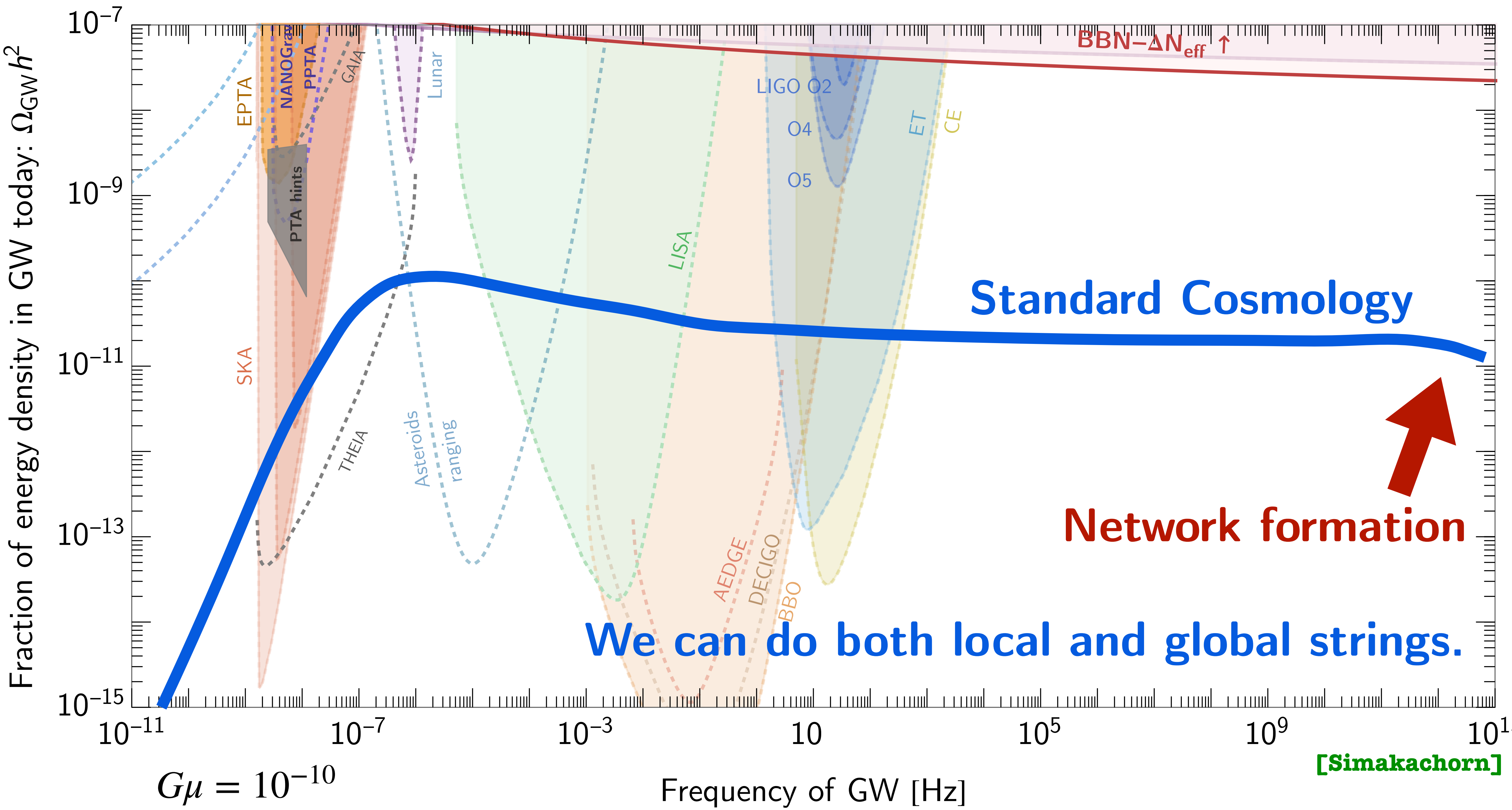
lead to particle emission

Short-lasting vs long-lasting primordial sources.

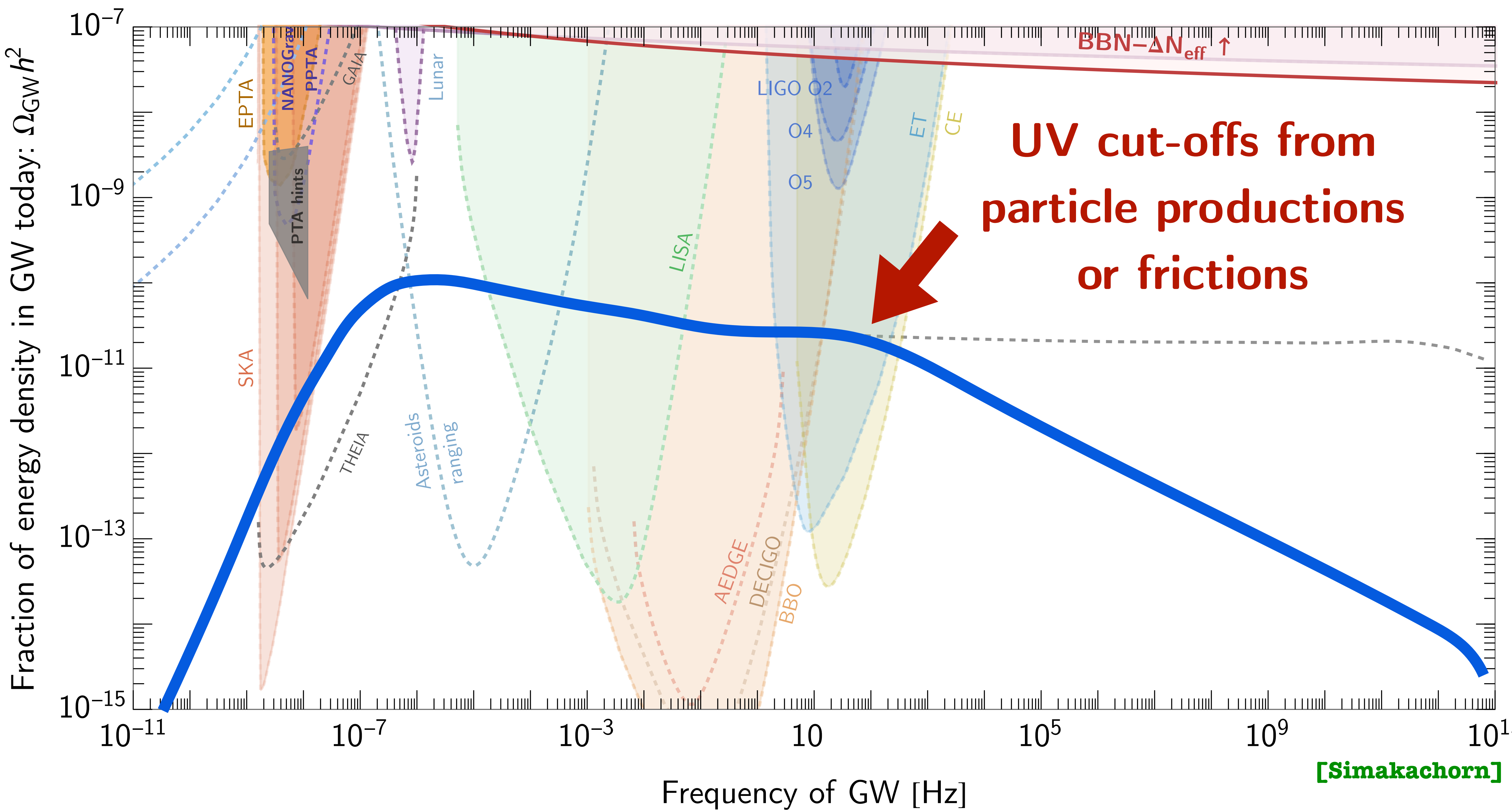




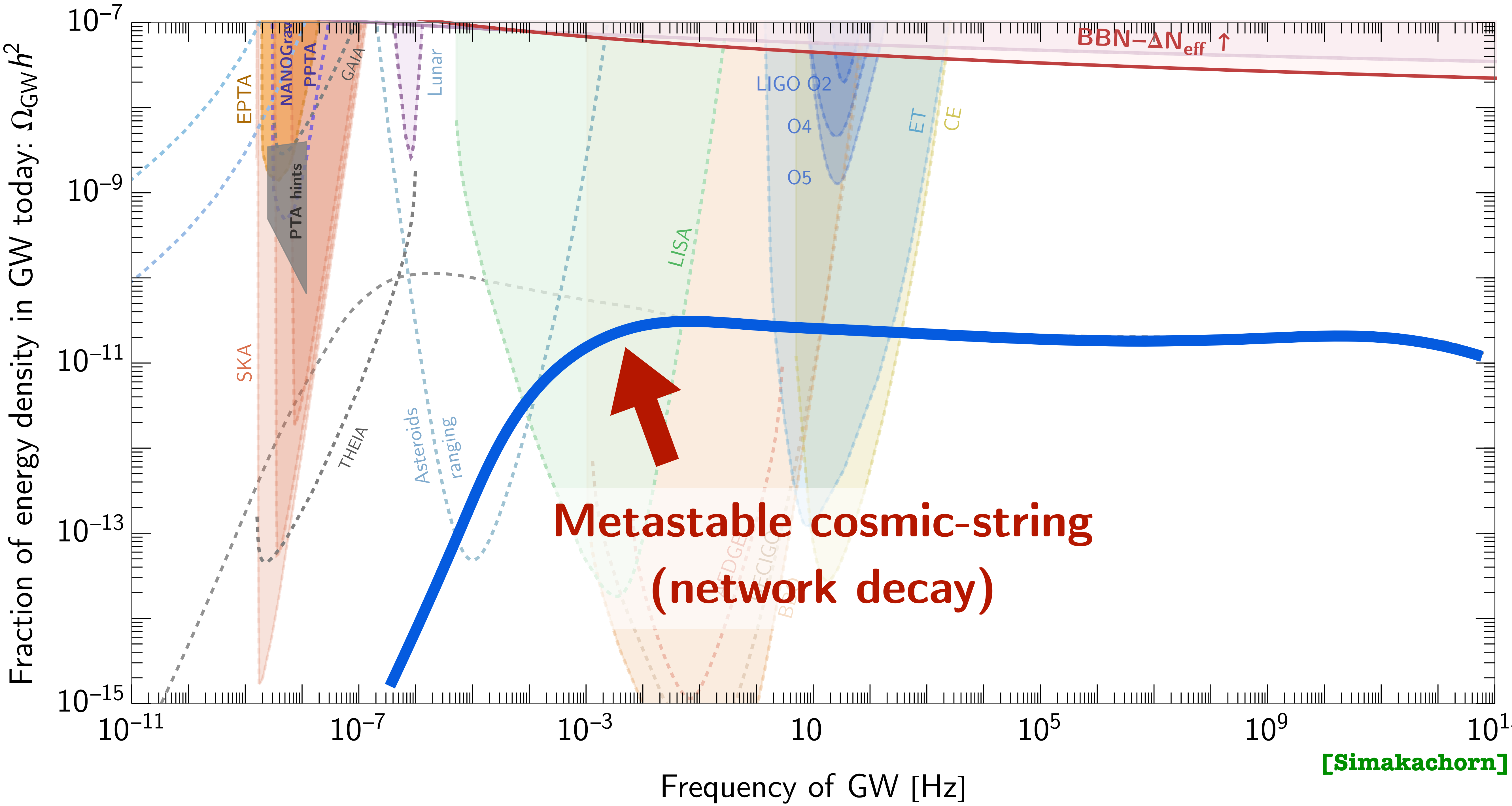
Gravitational Waves from cosmic strings.



Gravitational Waves from cosmic strings.

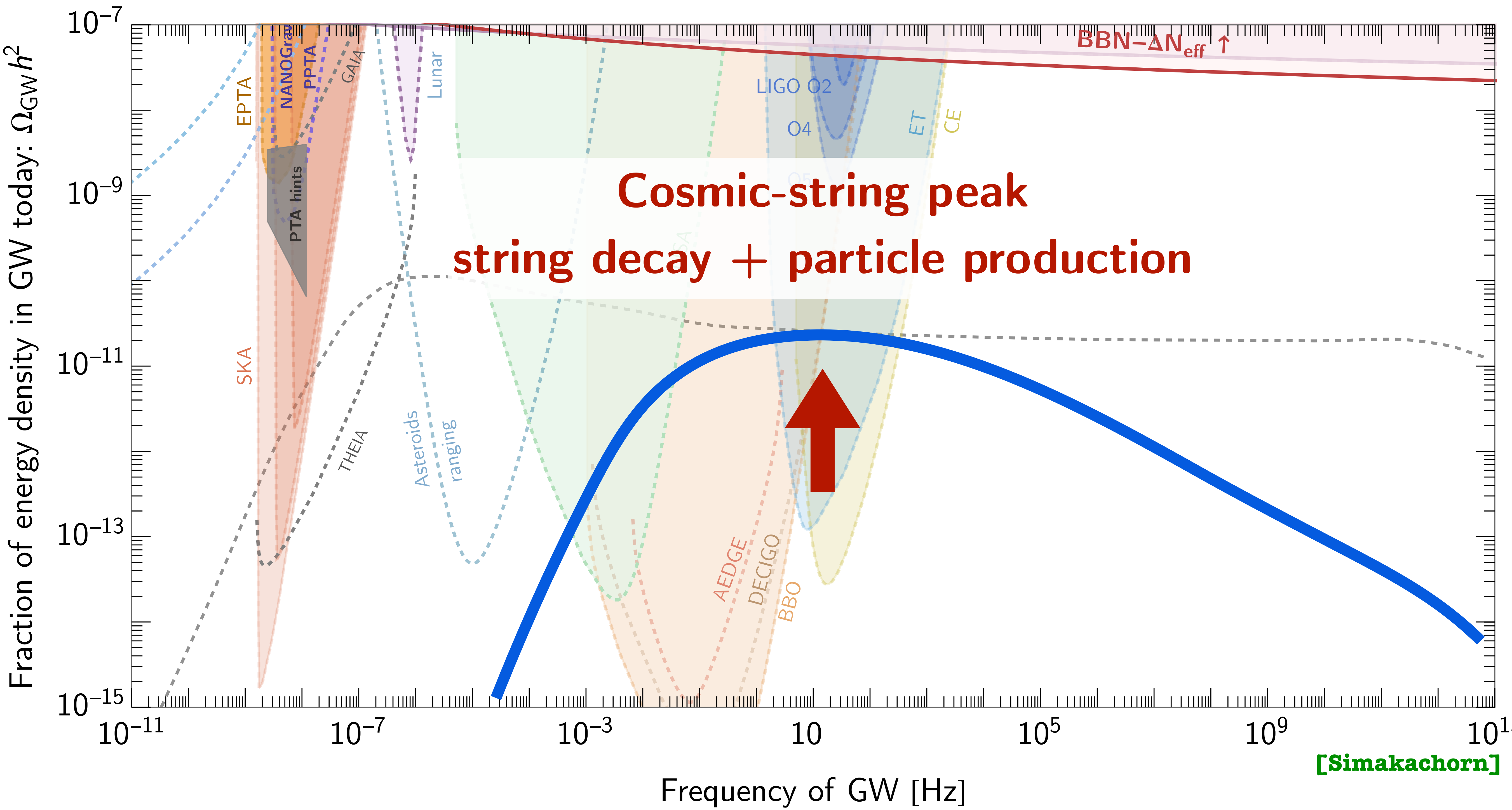


Gravitational Waves from cosmic strings.



[Simakachorn]

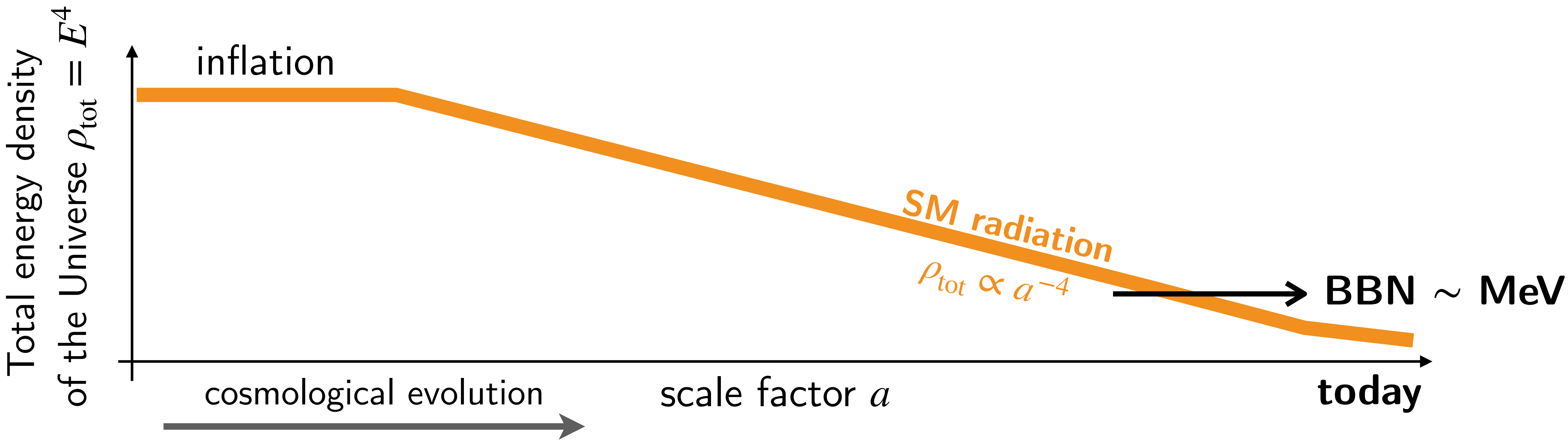
Gravitational Waves from cosmic strings.



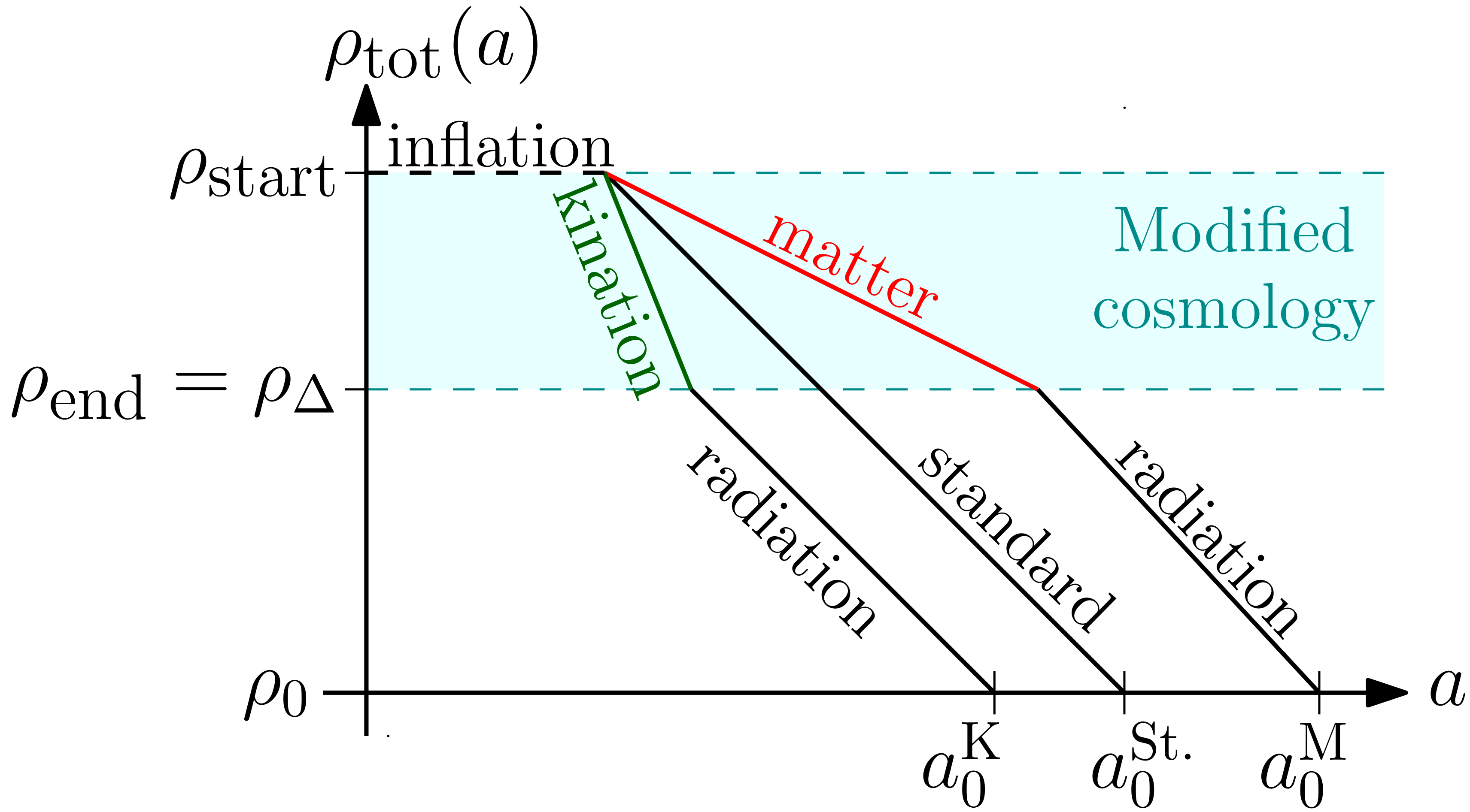
[Simakachorn]

Effect of non-standard cosmology on the GW spectrum.

Standard cosmological history



Impact of the cosmological history on Gravitational Waves:



Energy density of GW background:

$$\rho^{\rm GW}_{\rm today} = \rho^{\rm GW}_{\rm prod}\left(\frac{a_{\rm prod}}{a_{\rm today}}\right)^4$$

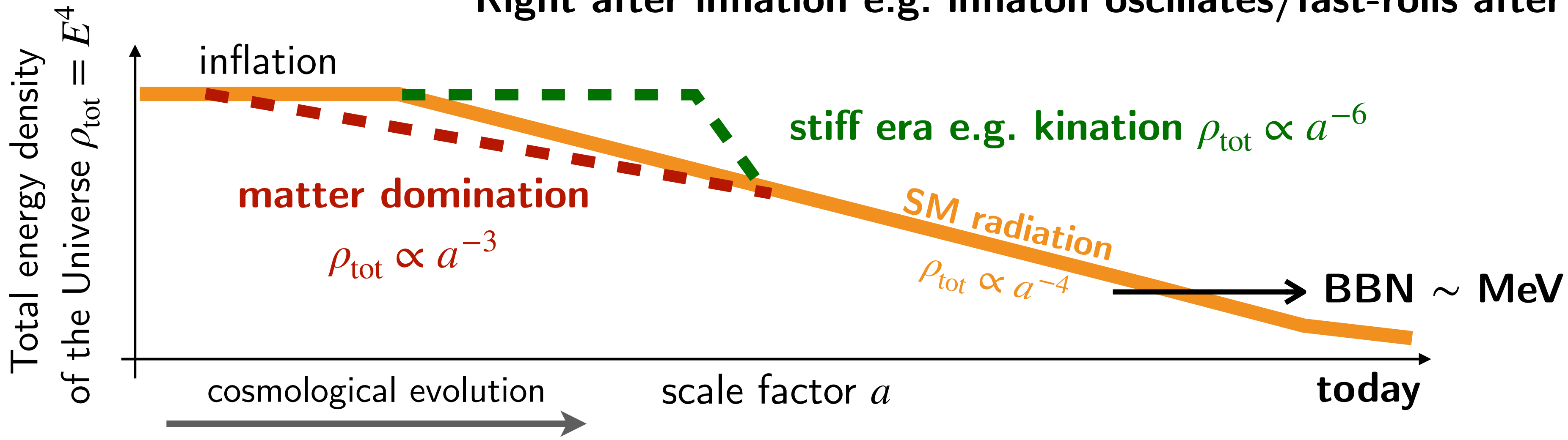
The universe is expanding. ⇨ **BSM of cosmology**

its production mechanism ⇨ **particle physics beyond the Standard Model (BSM)**

[1912.02569]
[2111.01150]

Non-Standard cosmological history

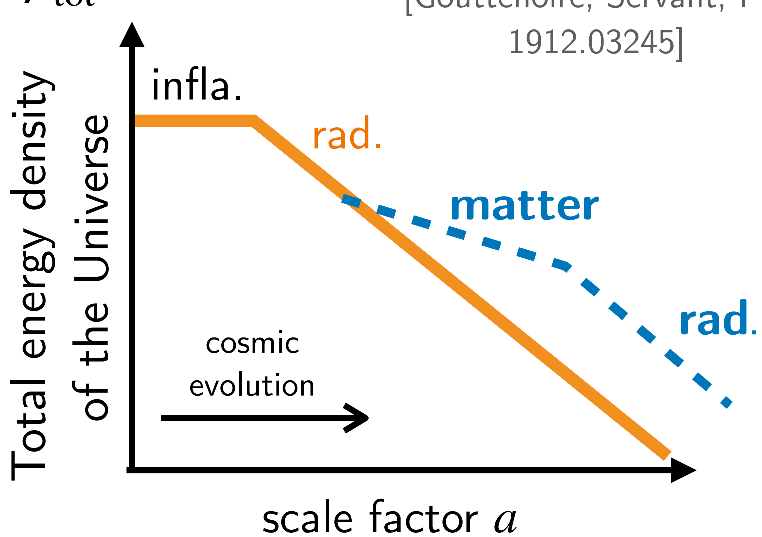
Right after inflation e.g. inflaton oscillates/fast-rolls after inflation ends.



Inside radiation era after reheating (decoupled from inflaton dynamics)

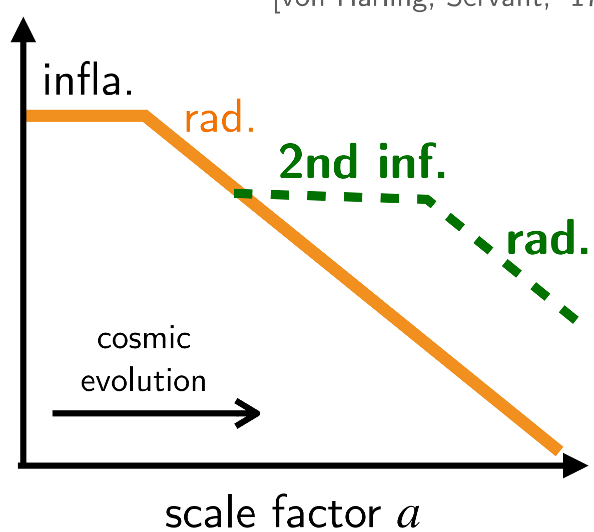
Intermediate matter era
e.g., heavy-unstable particles

[Gouttenoire, Servant, PS, 1912.03245]



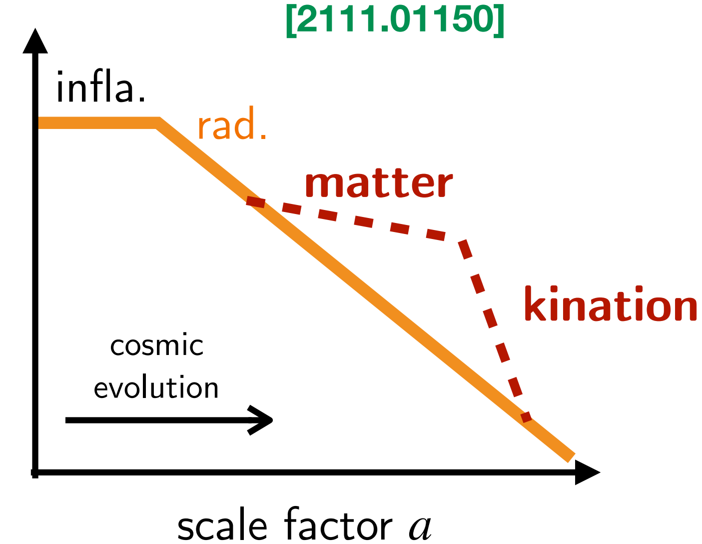
Secondary inflation
e.g., supercool 1st order PT

[von Harling, Servant, '17]

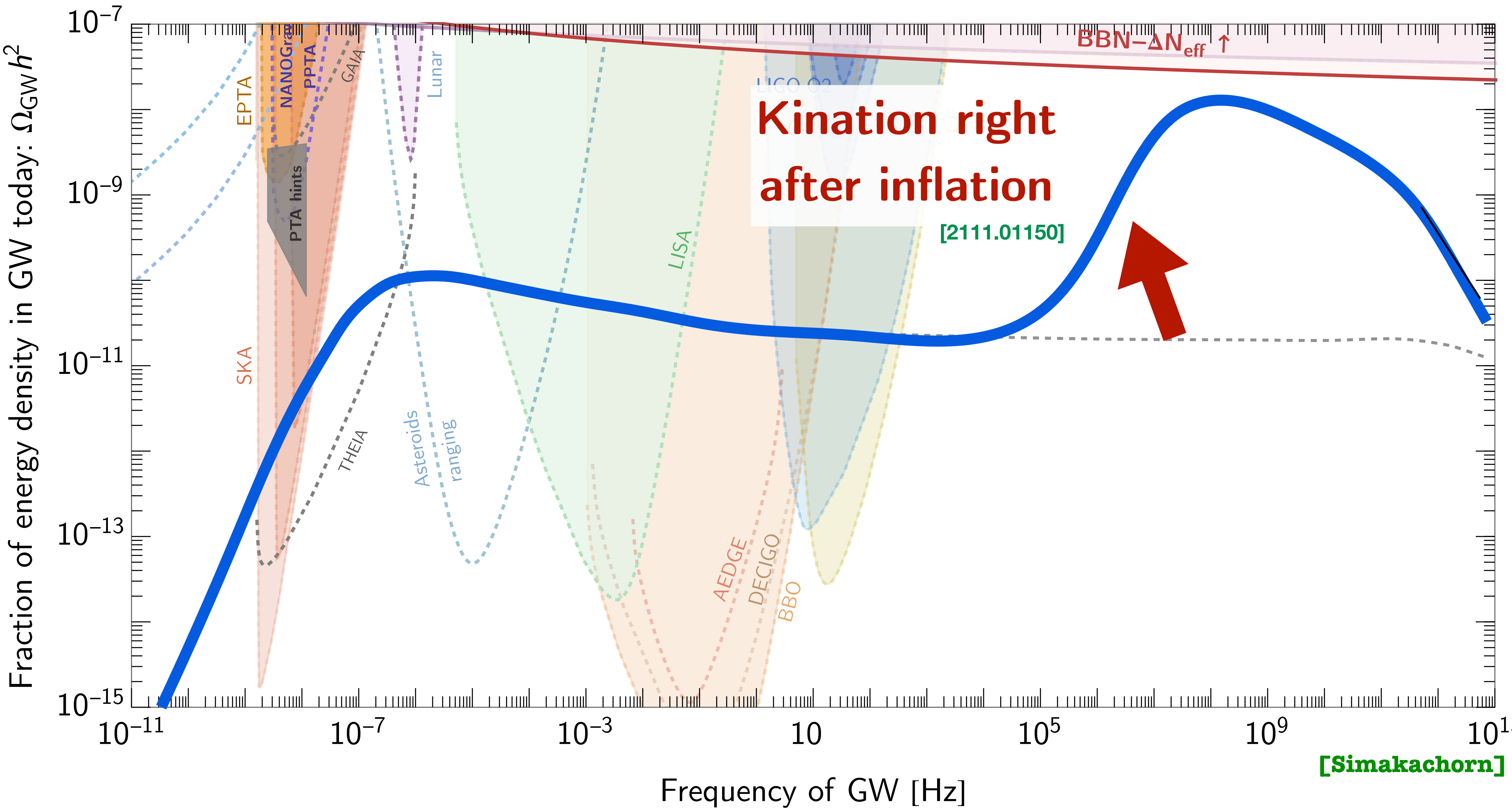


Intermediate
kination

[2111.01150]

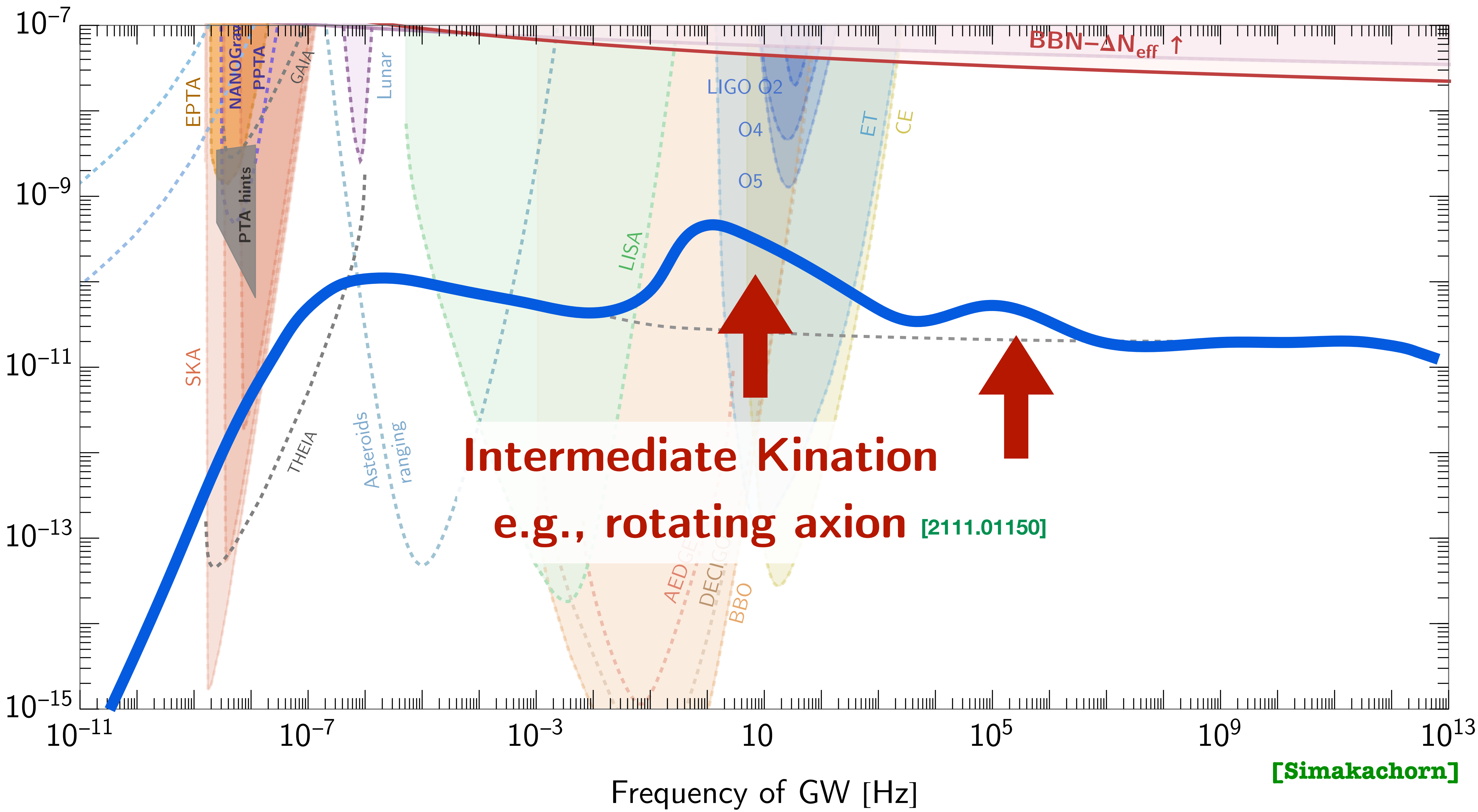


Gravitational Waves from cosmic strings in non-standard cosmology.

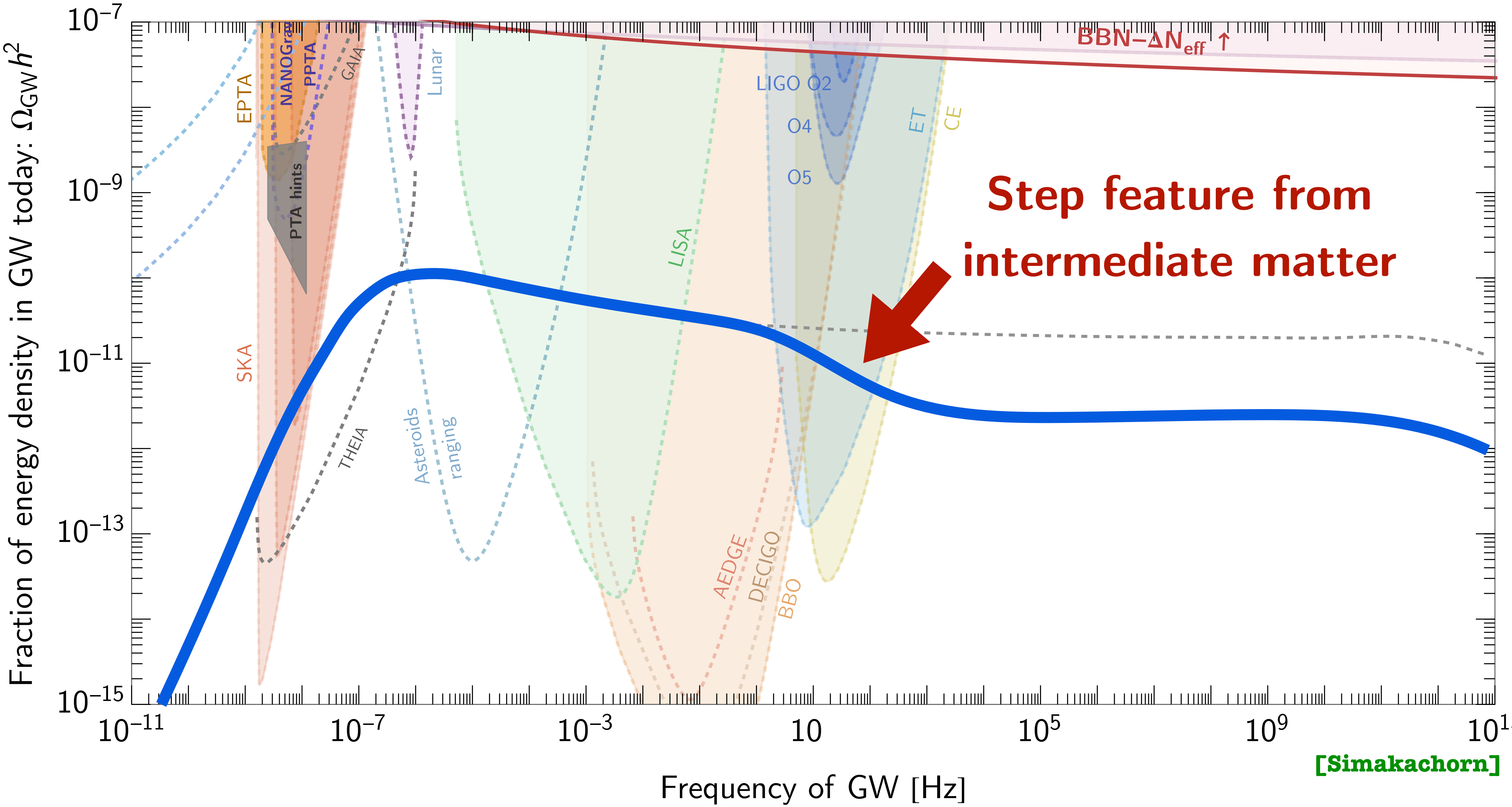


[Simakachorn]

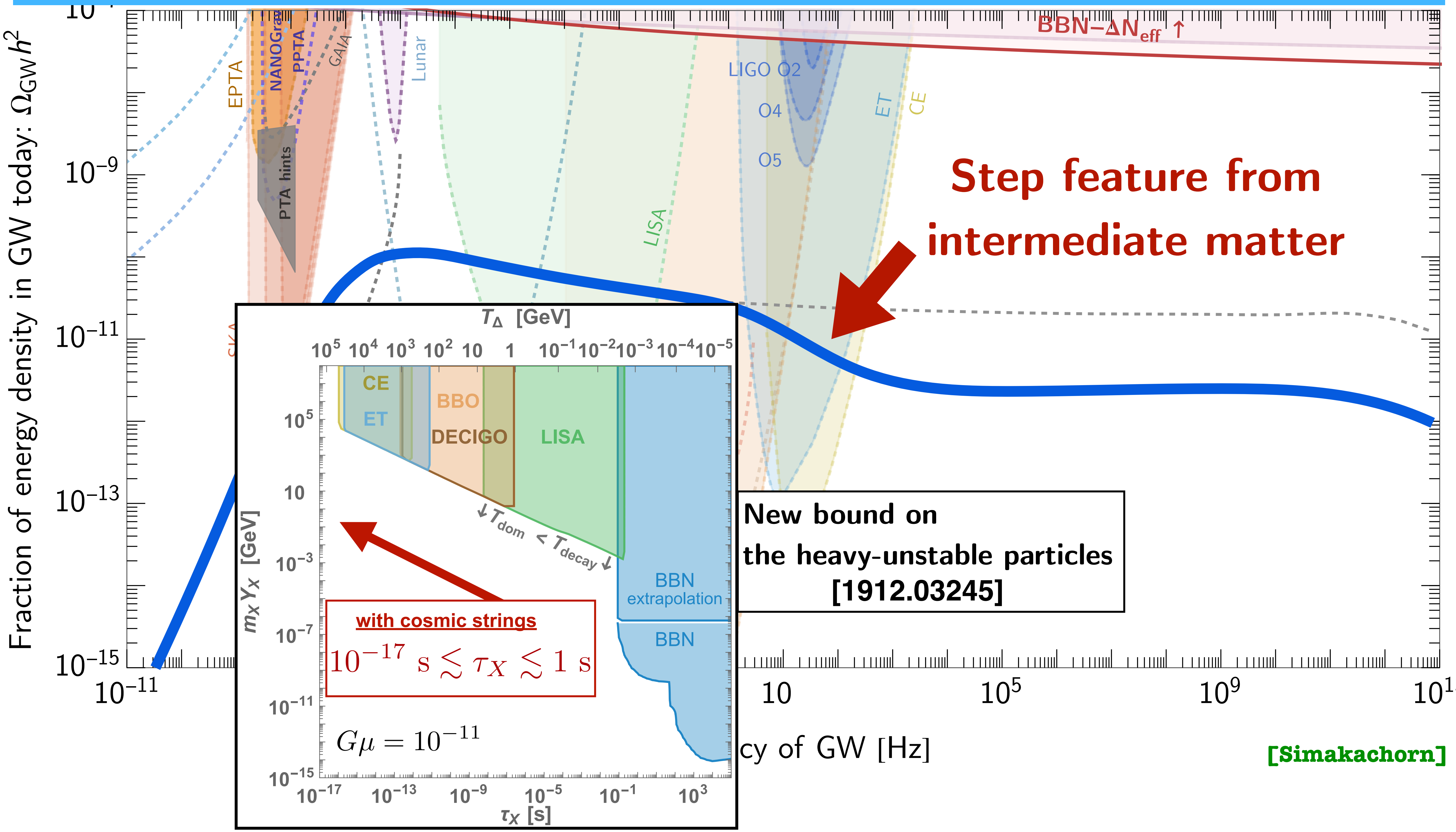
Gravitational Waves from cosmic strings in non-standard cosmology.



Gravitational Waves from cosmic strings in non-standard cosmology.



Gravitational Waves from cosmic strings in non-standard cosmology.



Conclusion.

Gravitational waves: complementary probes of

-Higgs physics (electroweak phase transition)

-Early equation of state of the universe

-Axion physics (its early universe dynamics, before/during/after inflation)

-Dark Matter production mechanism