LFWFs for the Pion and kaon and some of their implications for the EHM

Universidad de Huelva

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In collaboration with:

C. Lei, D. Binosi, C. Mezrag,

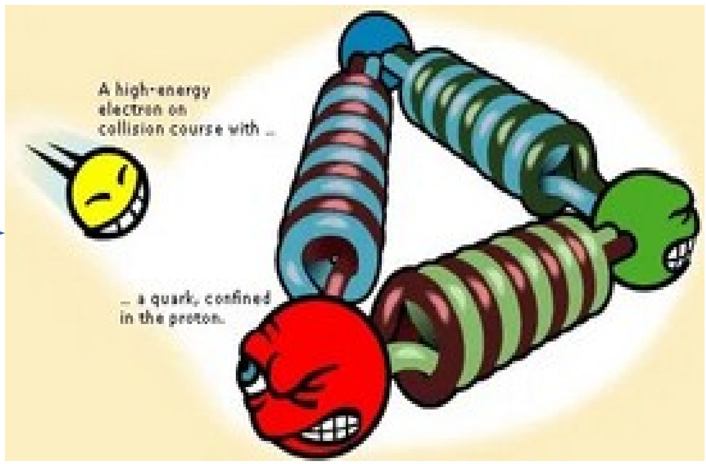
K. Raya, C. D. Roberts, M. Ding ...

Perceiving the Emergence of Hadron Mass, AMBER@CERN, August 6-7, 2020.

QCD and hadron physics

- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

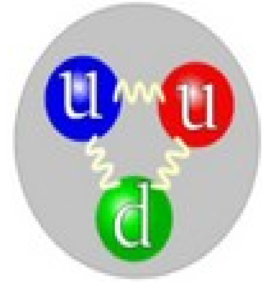
Gluons and quarks have never been seen isolated in nature; only colorless bound states (hadrons) have.



'Higgs' masses

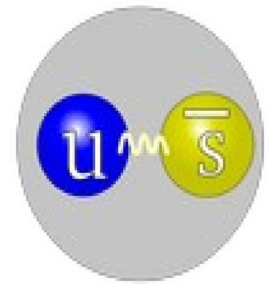
$m_{u/d} \approx 0.004$ GeV

$m_s \approx 0.095$ GeV



$m_p \approx 0.940$ GeV

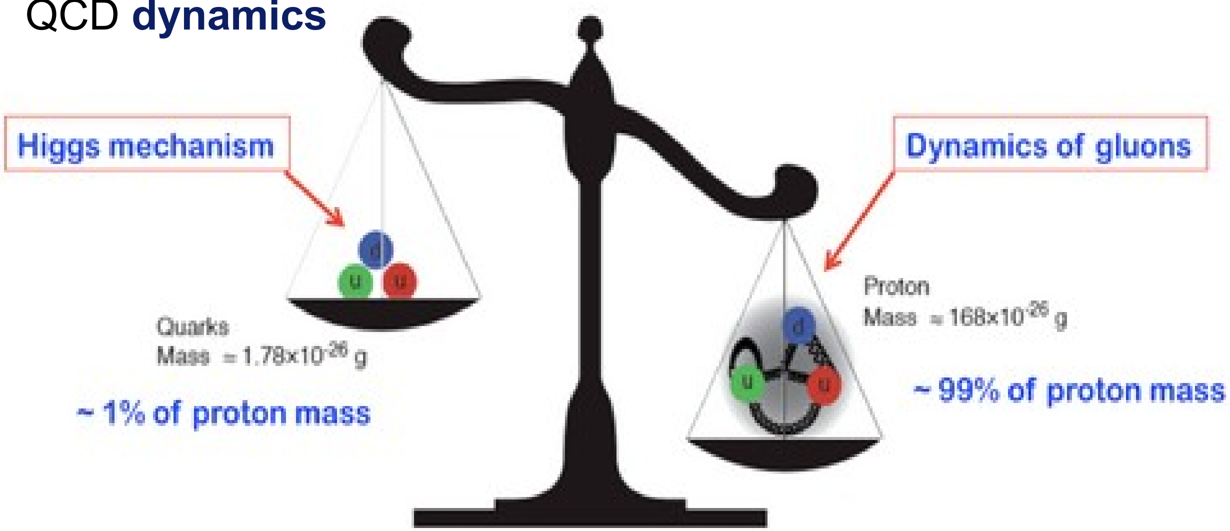
$m_\pi \approx 0.140$ GeV



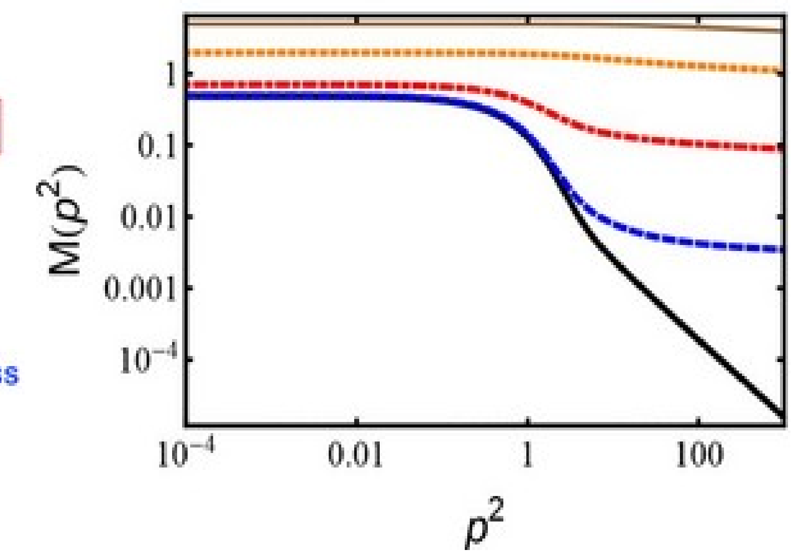
$m_K \approx 0.490$ GeV

Pions and **Kaons** emerge as QCD's (pseudo)-**Goldstone** bosons.

Emergence of hadron masses (EHM) from QCD **dynamics**

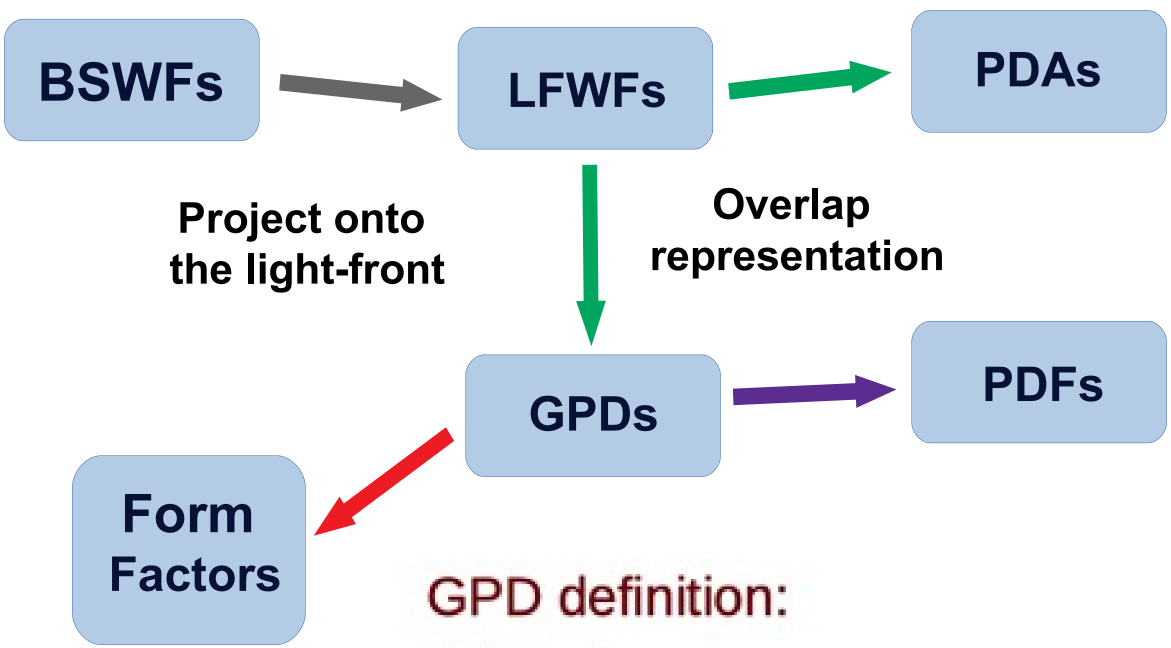


Dynamical Chiral Symmetry Breaking (**DCSB**)



LFWFs, PDFs and PDAs from DSEs

- **Goal:** get a **broad picture** of the pion/Kaon structure.



The approach:
Compute ***everything*** from the **LFWF,** obtained as ***solutions*** from quark **DSE** and meson **BSE**.

- Already on the market:
  PDAs, PDFs, Form factors...
  K. Raya et al., arXiv: 1911.12941 [nucl-th]
  Z-F Cui et al., arXiv: 2006.14075 [hep-ph]

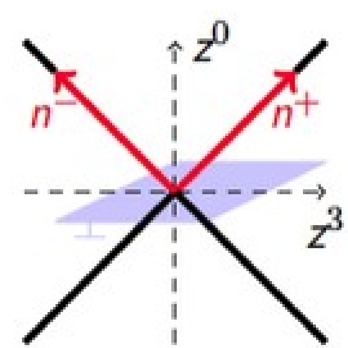
GPD definition:

$$H^q_\pi(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t=\Delta^2$ and $\xi = -\Delta^+/(2P^+)$.

- green: $\int dk_\perp$
- red: $\int dx$
- purple: $t=0, \xi=0$

Muller et al., Fortchr. Phys. 42 (1994) 101
Radyushkin, Phys. Lett. B380 (1996) 417
Ji, Phys. Rev. Lett. 78 (1997) 610



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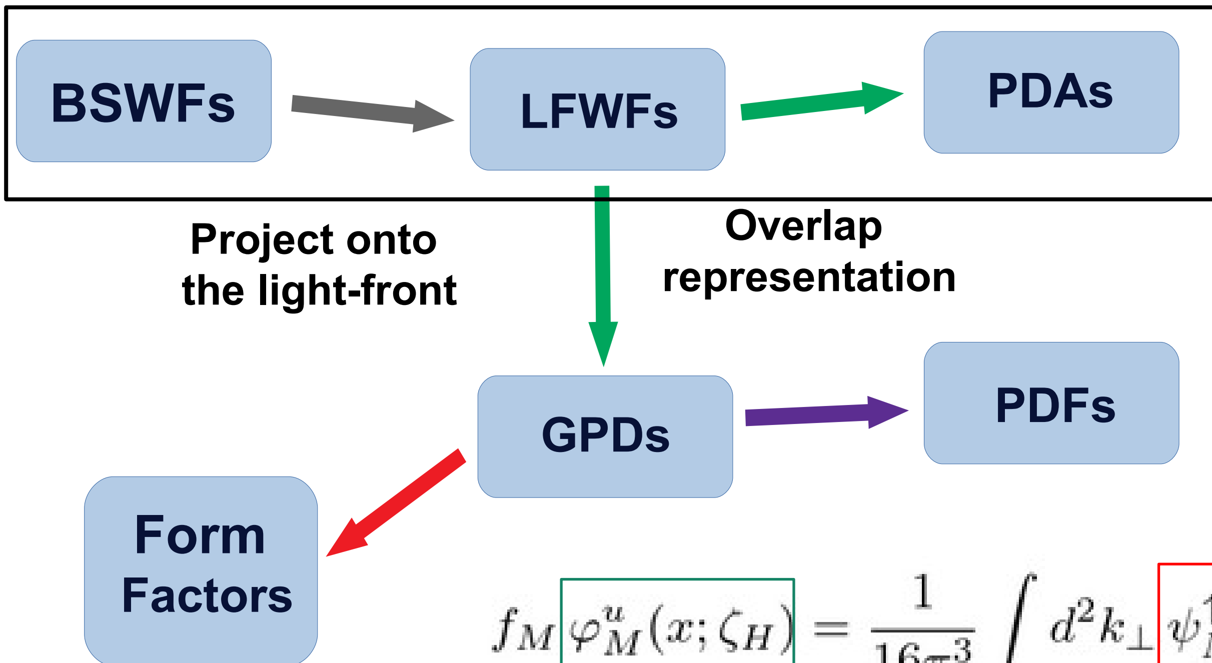

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$$f_M \varphi_M^u(x;\zeta_H) = \frac{1}{16\pi^3}\int d^2k_\perp \, \psi_{M_u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H)$$

$$= N_c \mathrm{tr} Z_2(\zeta_H,\Lambda)\int_{dk}^{\Lambda} \delta_n^x(k_\eta)\gamma_5\gamma\cdot n\,\chi_M(k_{\eta\bar{\eta}};P;\zeta_H)$$

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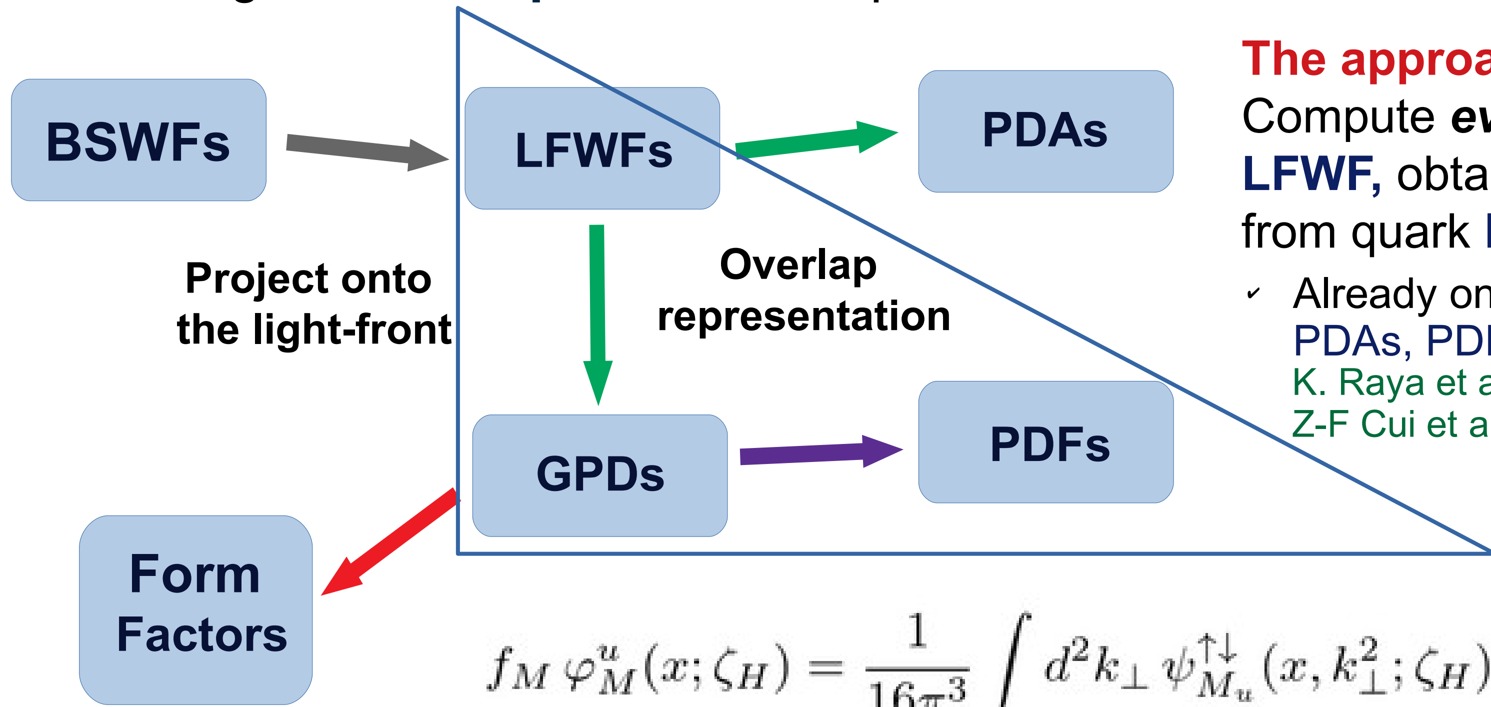

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$$u^M(x;\zeta_H) = H_M^u(x,0,0;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\left|\psi_{Mu}^{\uparrow\downarrow}(x,\mathbf{k}_\perp^2;\zeta_H)\right|^2$$

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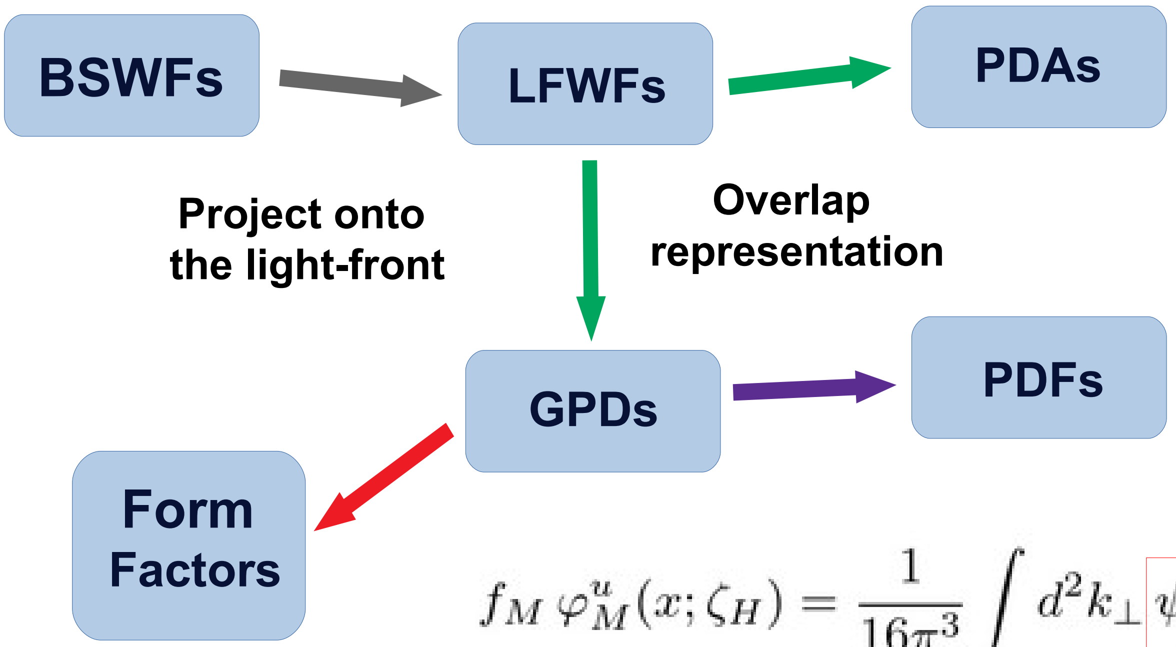

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S.-S. Xu et al,. Phys.Rev.D97094014(2018)

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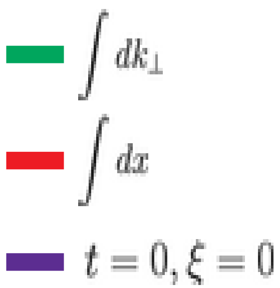

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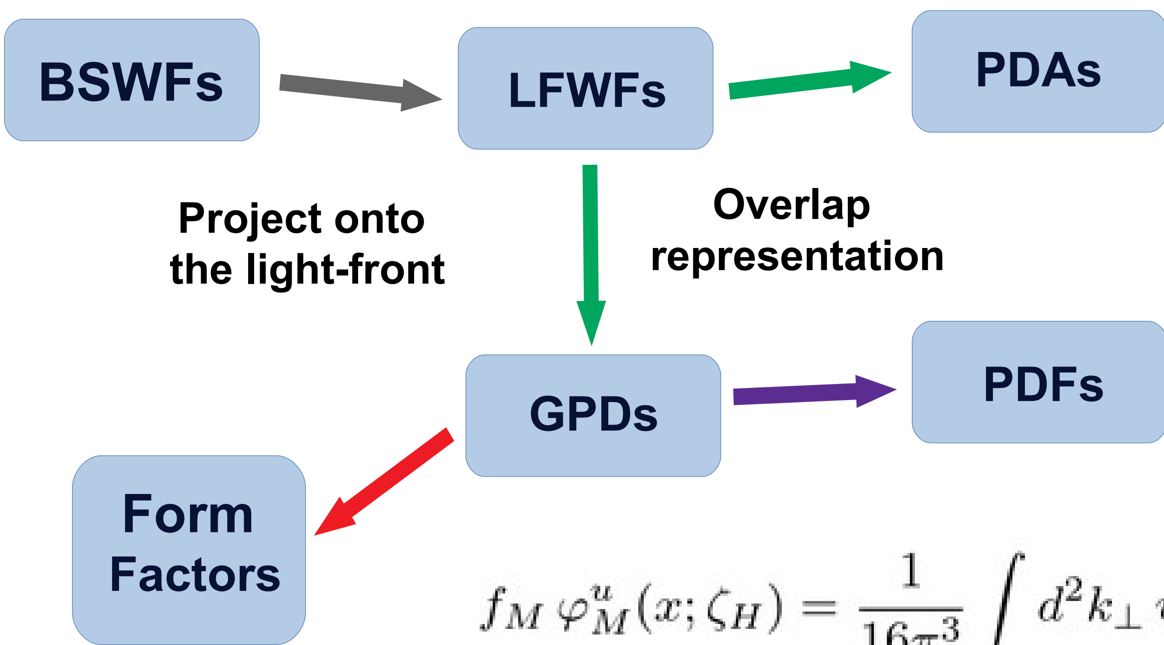

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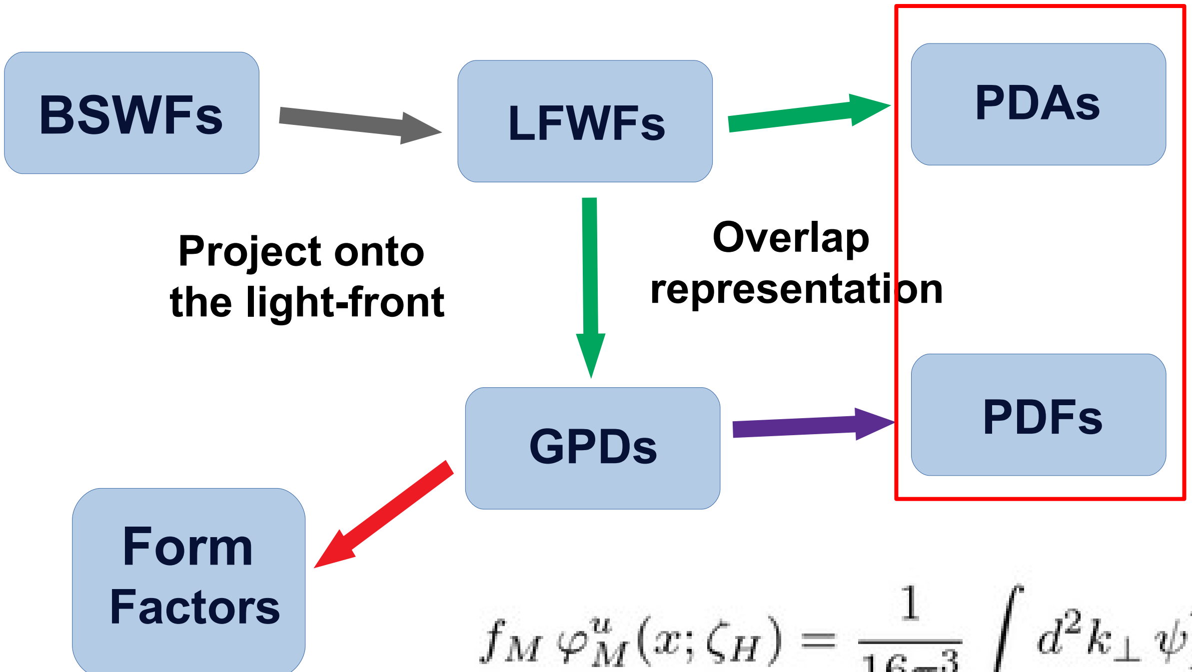

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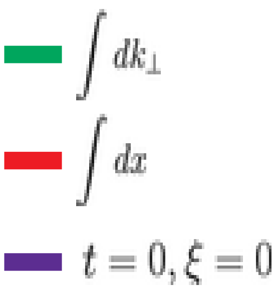


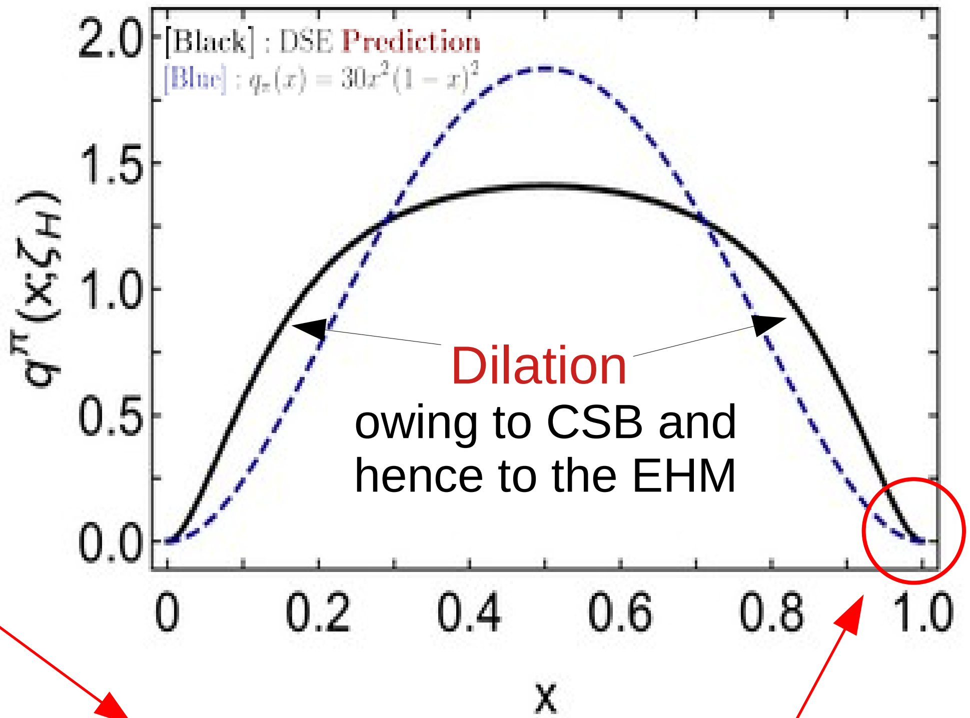
Direct connection between meson PDAs and PDFs at the hadronic scale, ζ_H, grounded on the *factorization approximation*, only valid for integrated quantities and not beyond ζ_H due to parton splitting effects.

LFWFs, PDFs and PDAs from DSEs

Symmetry-preserving DSE computation of the valence-quark PDF:
[L. Chang et al., Phys.Lett.B737(2014)23]
[M. Ding et al., Phys.Rev.D101(2020)054014

$$q^{\pi}(x;\zeta) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta) \Gamma_\pi^P(k_{\bar{\eta}\eta};\zeta)\, S(k_{\bar{\eta}};\zeta) \times \{n \cdot \frac{\partial}{\partial k_\eta} [\Gamma_\pi^{-P}(k_{\eta\bar{\eta}};\zeta) S(k_\eta;\zeta)]\}$$

$$q_{\mathrm{O}}^{\pi}(x;\zeta_H) = 213.32\, x^2(1-x)^2 \times [1 - 2.9342\sqrt{x(1-x)} + 2.2911\, x(1-x)]$$



$$q^M(x;\zeta_H) \overset{x\simeq 1}{=} c(\zeta_H)\,(1-x)^{\beta(\zeta_H)}$$

$$\beta(\zeta_H) = 2$$

Farrar, Jackson, Phys.Rev.Lett 35(1975)1416
Berger, Brodsky, Phys.Rev.Lett 42(1979)940

- The EHM-triggered broadening shortens the extent of the domain of convexity lying on the neighborhood of the endpoints, induced too by the QCD dynamics
- It cannot however spoil the asymptotic QCD behaviour at large-x (and, owing to isospin symmetry, at low-x)

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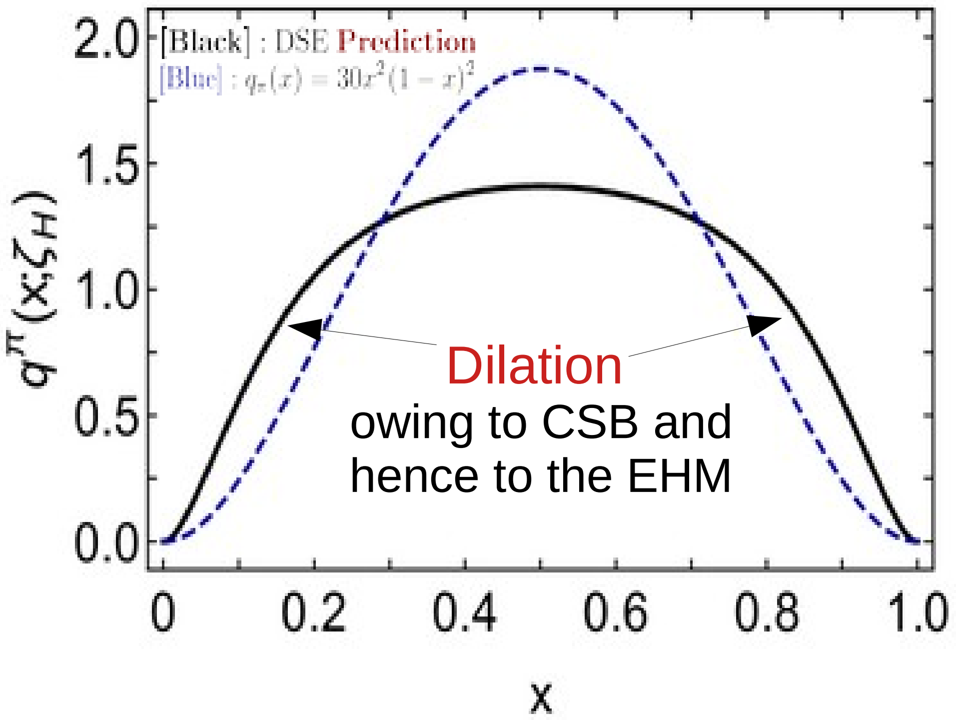

PDA computation using the BSA obtained with the DB kernel:

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$$f_M\, \varphi_M^u(x;\zeta_H) = \frac{1}{16\pi^3} \int d^2k_\perp\, \psi_{M_u}^{\uparrow\downarrow}(x, k_\perp^2;\zeta_H) \quad \cdot n\chi_M(k_{\eta\bar\eta};P;\zeta_H)$$

$$\varphi_\pi^{\mathrm{DB}}(x;\zeta_H) = 20.227\, x(1-x) \times [1 - 2.5088\sqrt{x(1-x)} + 2.0250\, x(1-x)]$$

Both computations scale-independent, albeit describing properly the hadronic degrees of freedom only at the hadronic scale

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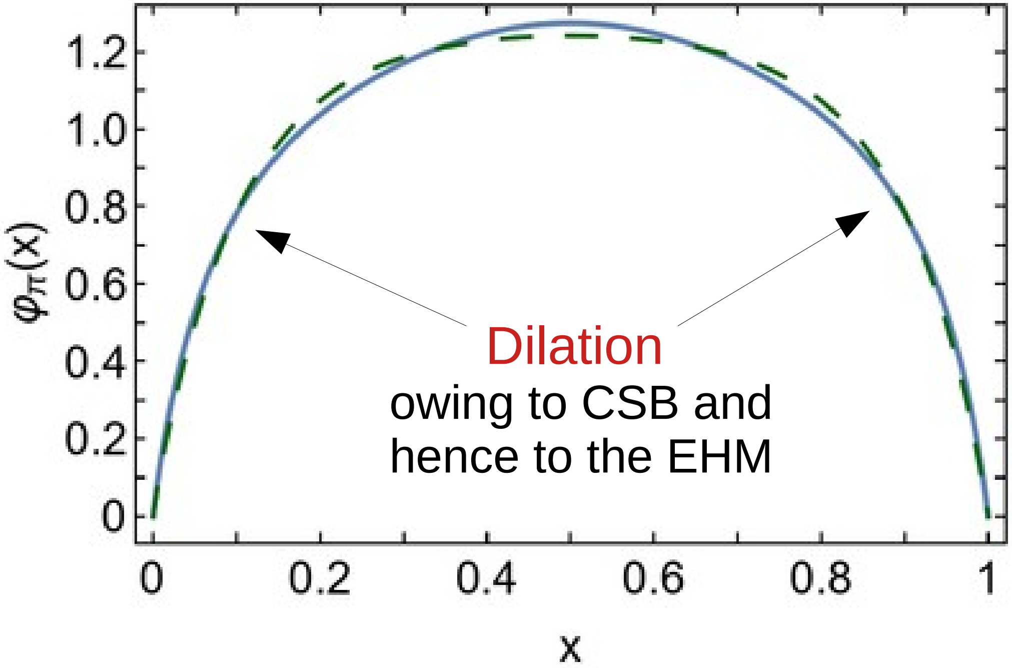

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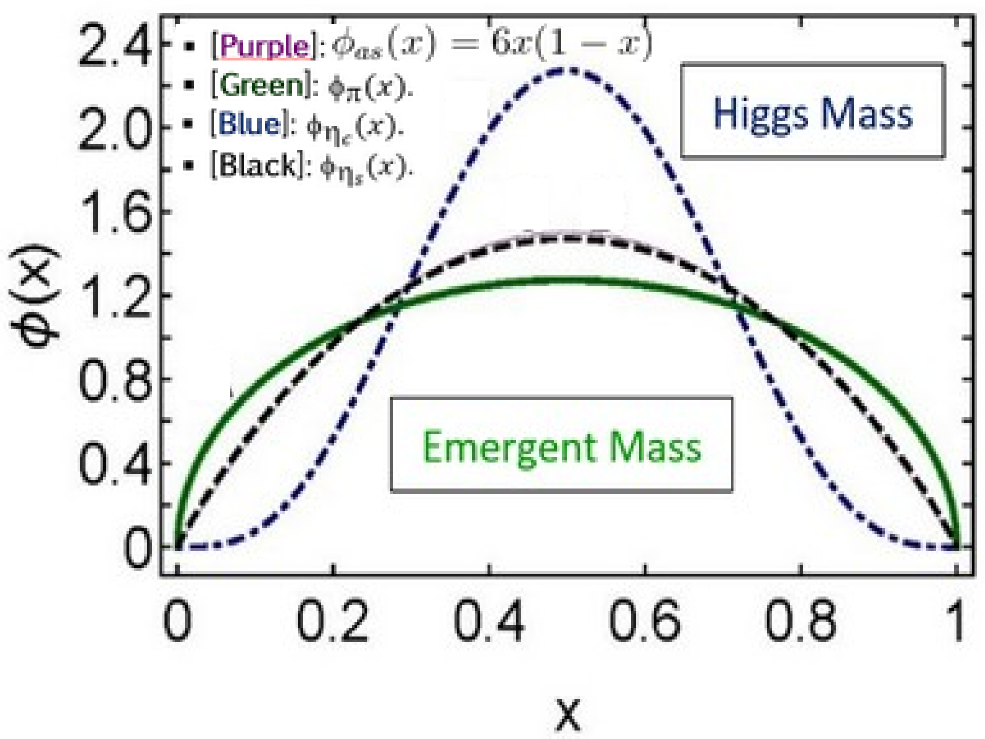
A. Aguilar et al. Eur.Phys.J. A55 (2019) no.10, 190



- Dominance of QCD dynamics (EHM) expressed by **broad and concave** PDAs (light sector)
- Dominance of Higgs mass generation (explicit CSB) reflected by **narrow** PDAs (heavy sector)
- **s-quark mass** lies on the boundary where both effects appear **balanced**

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Off-forward extension of PDFs

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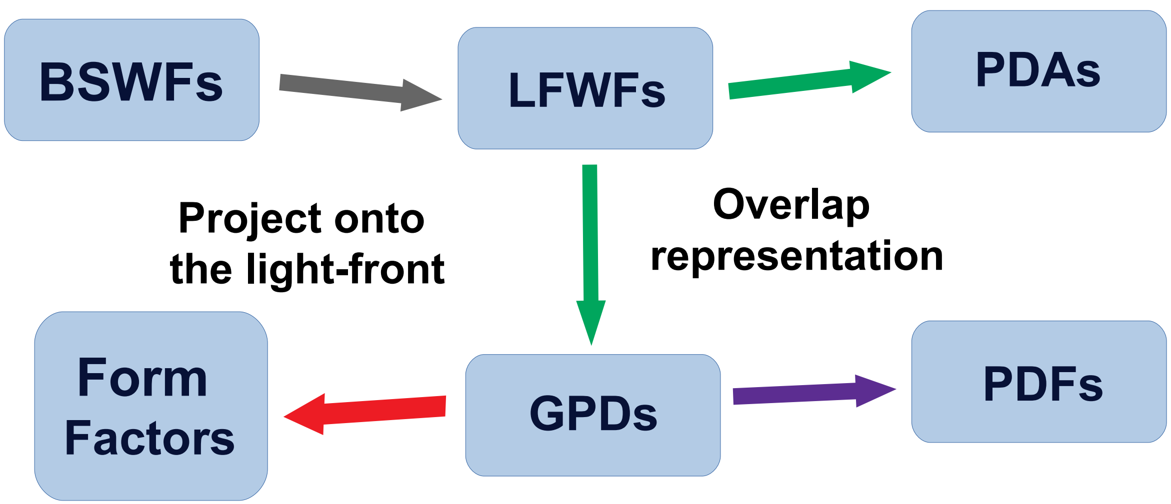

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Let us first apply the factorization approximation:

$$H_M^u(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_{Mu}^{\uparrow\downarrow *}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp + \frac{\mathbf{1-x}}{\mathbf{1}-\xi}\frac{\boldsymbol{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi_{Mu}^{\uparrow\downarrow}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp - \frac{\mathbf{1-x}}{\mathbf{1}+\xi}\frac{\boldsymbol{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$$

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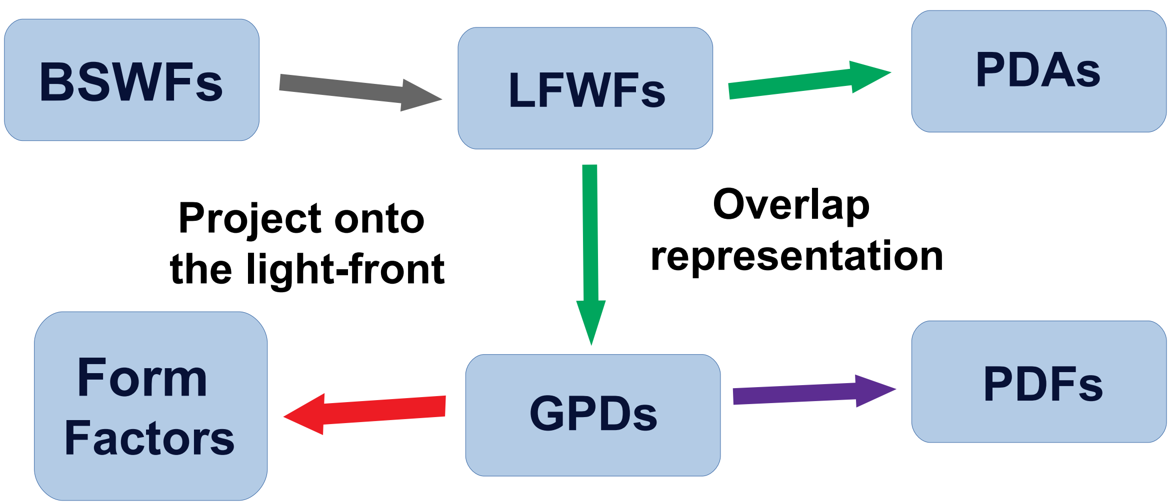

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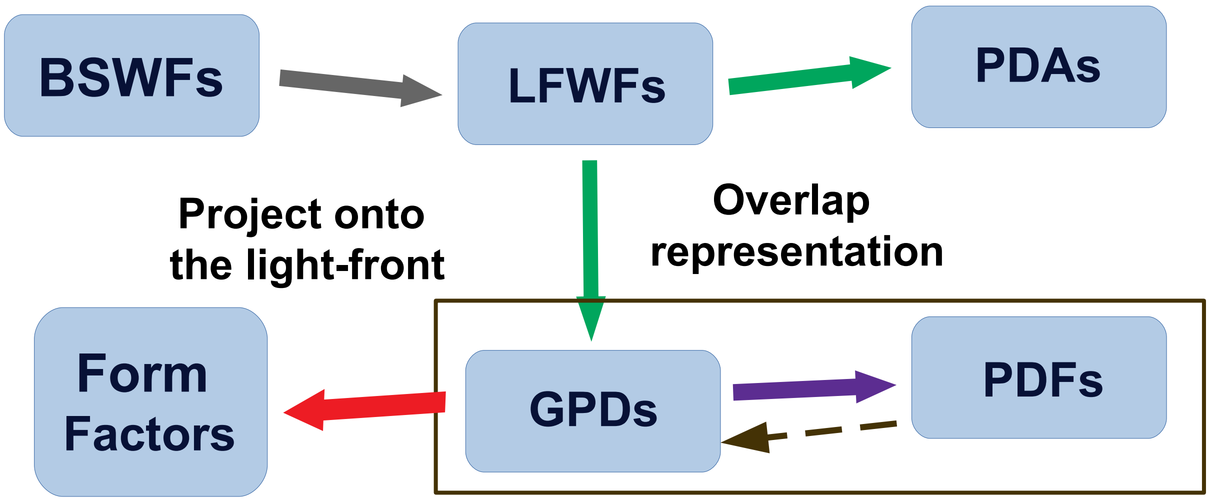

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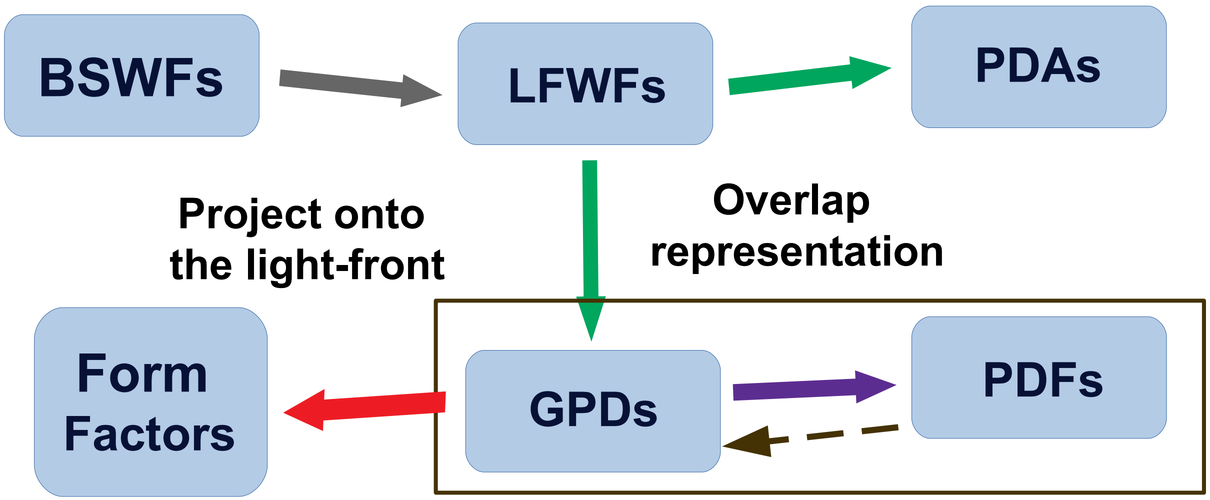

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  Z-F Cui et al., arXiv: 2006.14075 [hep-ph]

Let us first apply the factorization approximation:

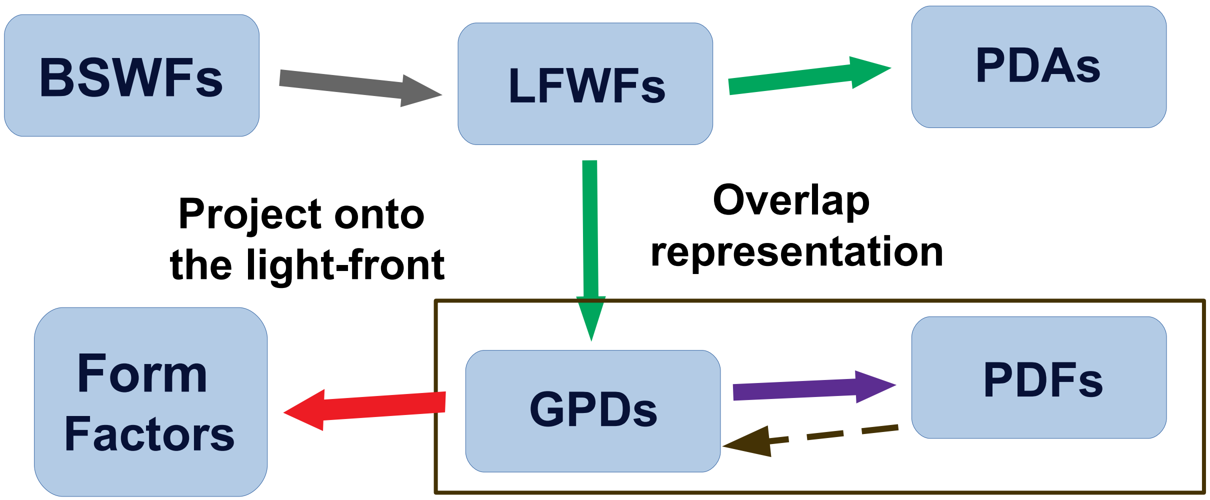
$$H_M^u(x,\xi,t;\zeta_H) = \sqrt{u_M\left(\frac{x-\xi}{1-\xi};\zeta_H\right) u_M\left(\frac{x+\xi}{1+\xi};\zeta_H\right)}\; f_M\left(\frac{(1-x)^2\Delta_\perp^2}{4(1-\xi^2)^2};\zeta_H\right)$$

$$\psi_{M_u}^{\uparrow\downarrow}(x,k_\perp^2;\zeta_H) = \varphi_M^u(x;\zeta_H)\psi_{M_u}^{\uparrow\downarrow}(k_\perp^2;\zeta_H)$$

$$u^M(x;\zeta_H) \propto |\varphi_M^u(x;\zeta_H)|^2$$

Off-forward extension of PDFs

- **Goal:** get a **broad picture** of the pion/Kaon structure.



The approach:
Compute ***everything*** from the **LFWF,** obtained as ***solutions*** from quark **DSE** and meson **BSE**.

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M being a **Goldstone** boson in the **chiral limit.**

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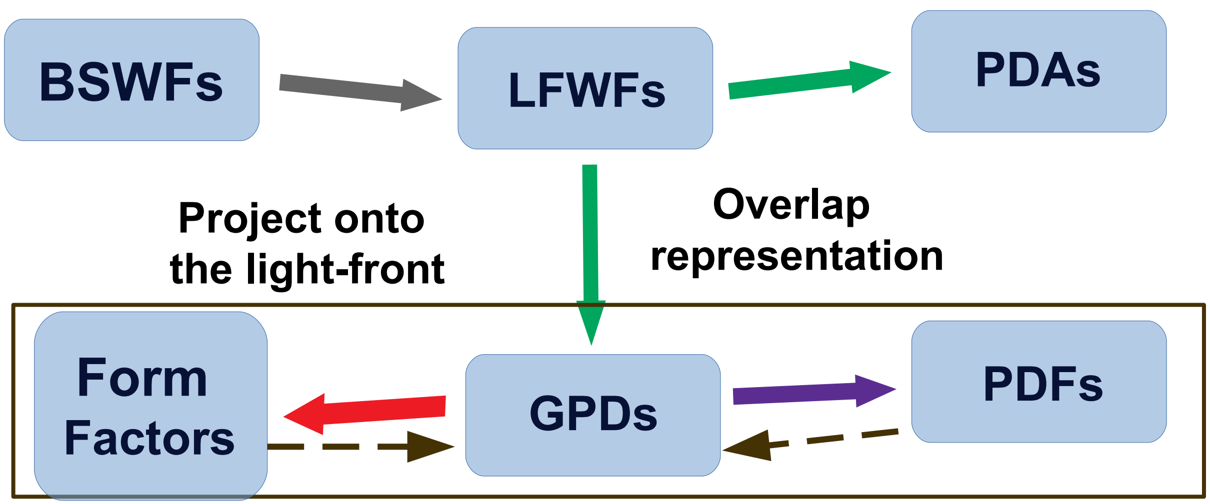

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M being a **Goldstone** boson in the **chiral limit.**

- The **positivity condition** is made manifest $f_M(z) \leq 1$
- It became *saturated* at t=0 $f_M(0) = 1$

$$\psi^{\uparrow\downarrow}_{M_u}(x,k_\perp^2;\zeta_H) = \varphi^u_M(x;\zeta_H)\,\psi^{\uparrow\downarrow}_{M_u}(k_\perp^2;\zeta_H)$$

$$u^M(x;\zeta_H) \propto |\varphi^u_M(x;\zeta_H)|^2$$

$$\int_{-1}^{1} dx\, H^u_M(x,\xi=0,t) = \int_0^1 dx\, u_M(x;\zeta_H)\, f_M\left(-\frac{t}{4}(1-x)^2;\zeta_H\right) = F_M(-t)$$

GPDs from LFWFs

Modeling the LFWF:

S-S Xu et al., PRD 97 (2018) no.9, 094014.

- Considering the Kaon as a example, we employ a Nakanishi-like representation:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k;P_K)}_{1} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k;P_K)}_{3} \,,$$

1: Matrix structure (leading BSA):

$$\mathcal{M}(k;P_K) = -\gamma_5[\gamma\cdot P_K M_u + \gamma\cdot k(M_u - M_s) + \sigma_{\mu\nu}k_\mu P_{K\nu}] \,,$$

2: Sprectral weight: To be described later.

3: Denominators: $\mathcal{D}(k;P_K) = \Delta(k^2, M_u^2)\Delta((k-P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \,,$

$$\text{where: } \Delta(s,t) = [s+t]^{-1}, \ \hat{\Delta}(s,t) = t\Delta(s,t) \,.$$

- **Algebraic** manipulation yields:

$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k;P_K) \int_0^1 d\alpha \, 2\chi_K(\alpha;\sigma^3(\alpha)) \,, \ \sigma = (k-\alpha P_K)^2 + \Omega_K^2 \,,$$

- **$\rho_K(\omega)$** will play a **crucial role** in determining the meson's observables.
- Realisitc **DSE predictions** will help us to shape it.

Scalar function:

$$\chi_K(\alpha;\sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv\right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3}$$

GPDs from LFWFs

Modeling the LFWF:

S-S Xu et al., PRD 97 (2018) no.9, 094014.

- The **pseudoscalar LFWF** can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_\parallel} \delta(n \cdot k - x n \cdot P_K) \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \, .$$

- The **moments** of the distribution:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_\parallel} \left[\frac{n \cdot k}{n \cdot P}\right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$\int_0^1 d\alpha \alpha^m \left[\frac{12}{f_K} \mathcal{Y}_K(\alpha; \sigma^2)\right] , \qquad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha] \mathcal{X}(\alpha; \sigma_\perp^2) \, .$$

Uniqueness of Mellin moments →

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)$$

- ✓ Compactness of this result is a merit of the algebraic model.

- The explicit form of $\rho_K(\omega)$ **controls** the shape of **PDAs, GPDs, PDFs**, etc.

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) \sim \int d\omega \cdots \rho_K(\omega) \cdots$$

GPDs from LFWFs

Modeling the LFWF:

- *Asymptotic* model:

$$\rho_\pi(\omega) \sim (1-\omega^2) \longrightarrow \begin{cases} \phi(x) \sim x(1-x) & \textit{Asymptotic PDA} \\ q(x) \sim [x(1-x)]^2 & \textit{Free-scale PDF} \end{cases}$$

C. Mezrag et al., PLB 741 (2015) 190-196.
C. Mezrag et al., FBS 57 (2016) no.9, 729-772

- Experience and careful analysis lead us to the following **flexible** parametrization intended to a realistic description of meson Dfs:

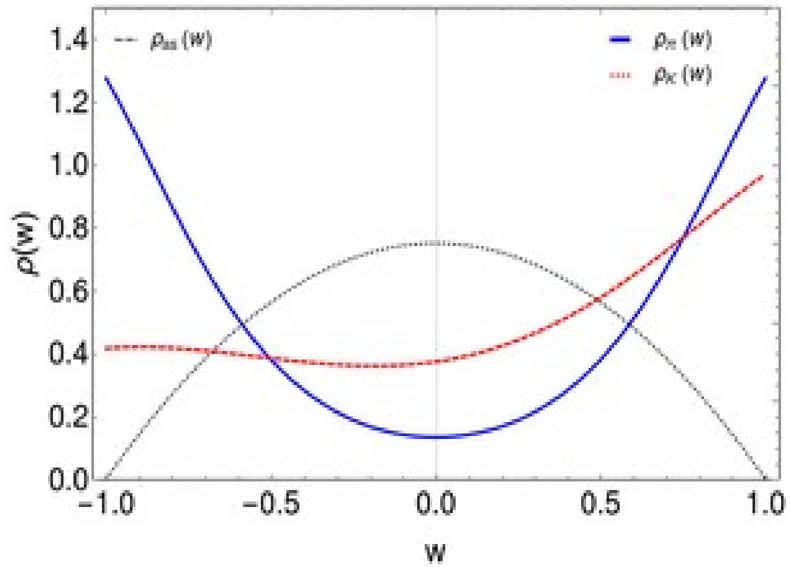
$$u_G \rho_G(\omega) = \frac{1}{2b_0^G}\left[\mathrm{sech}^2\left(\frac{\omega-\omega_0^G}{2b_0^G}\right) + \mathrm{sech}^2\left(\frac{\omega+\omega_0^G}{2b_0^G}\right)\right][1+\omega\, v_G]\ ,$$

- Employing PDFs and PDAs as ***benchmarks:***

Λ_π	b_0^π	ω_0^π	ν_π	Λ_K	b_0^K	ω_0^K	ν_K
M_u	0.275	1.23	0	$2M_s$	0.5	1.3	0.4

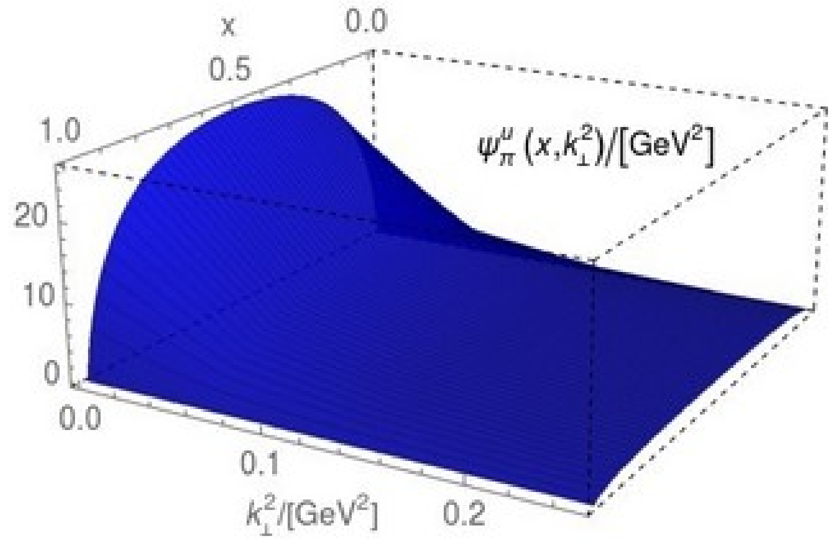
$$m_\pi = 0.140\ \mathrm{GeV}, m_K = 0.49\ \mathrm{GeV}$$
$$M_u = 0.31\ \mathrm{GeV}, M_s = 1.2\ M_u$$

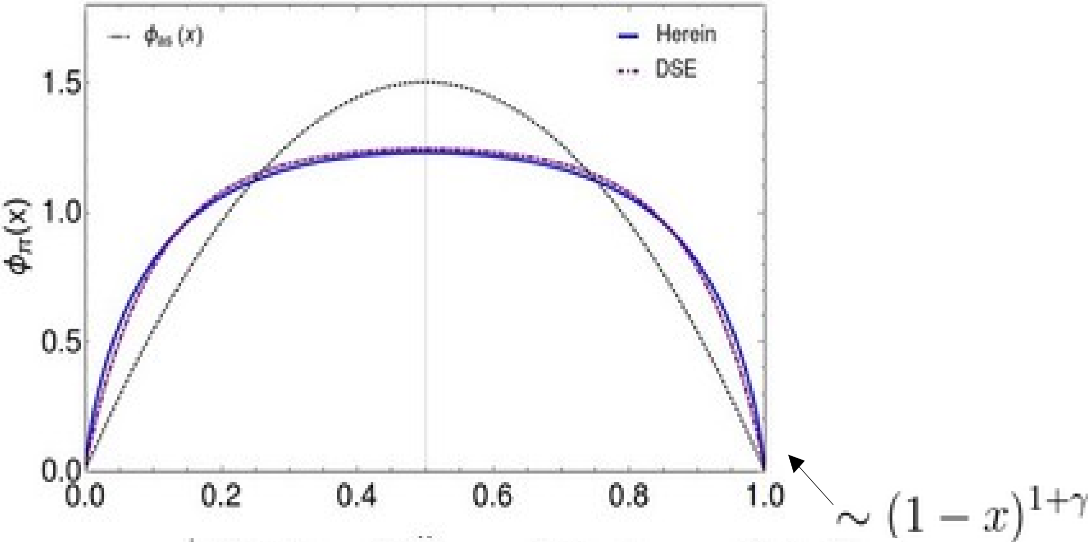
Typical values of **constituent** quark masses, from **realistic** DSEs **solutions**.



GPDs from LFWFs

LFWF and PDAs:





	Herein	DSE	Lat. A	Lat. B
$<(1-2x)^2>$	0.252	0.250	0.234(6)(6)	0.244(30)

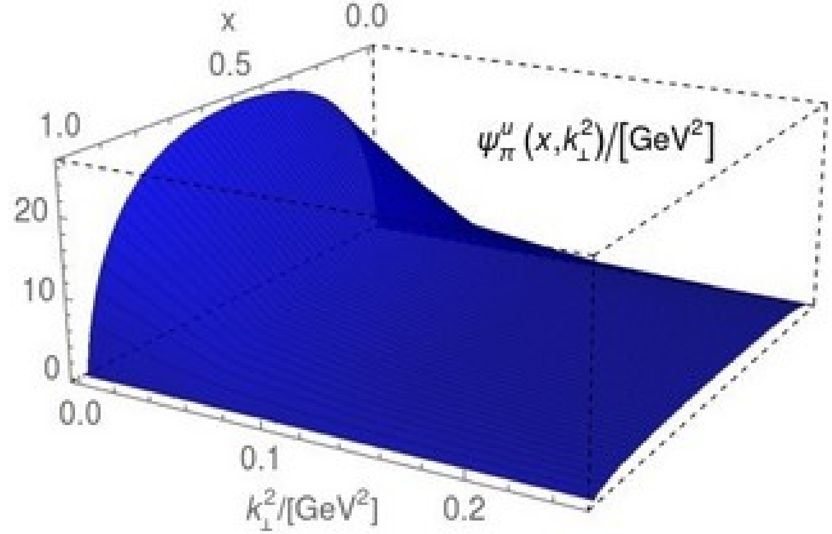
Lattice (A): G.S. Bali et al., JHEP 1908 (2019) 065

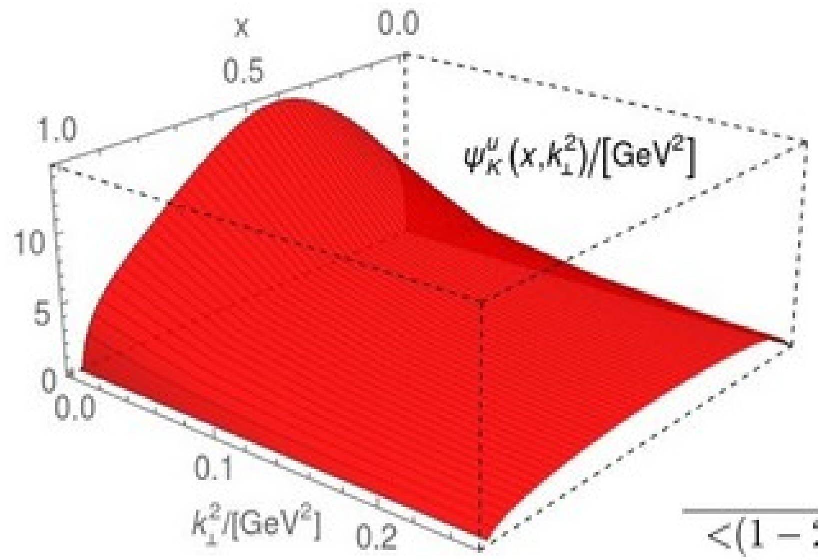
Lattice (B): Rui Zhang et al., arXiv:2005.13955

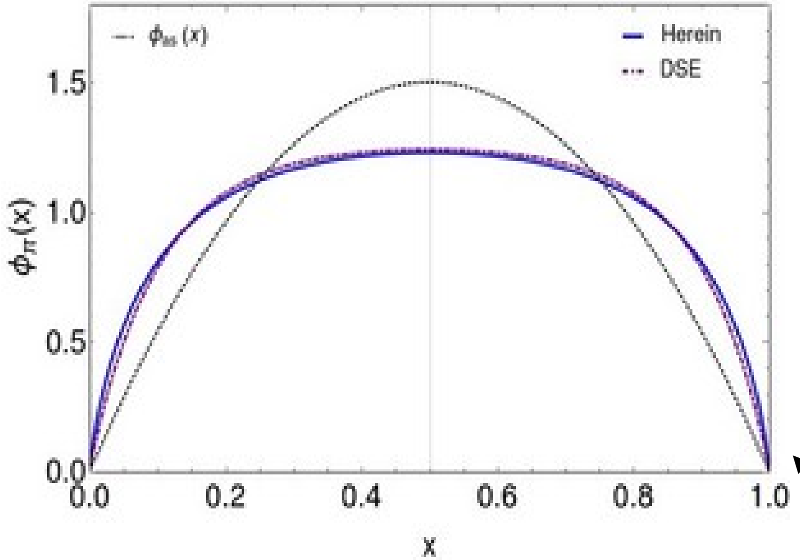
DSE: L. Chang et al., PRL 110 (2013) no.13, 132001

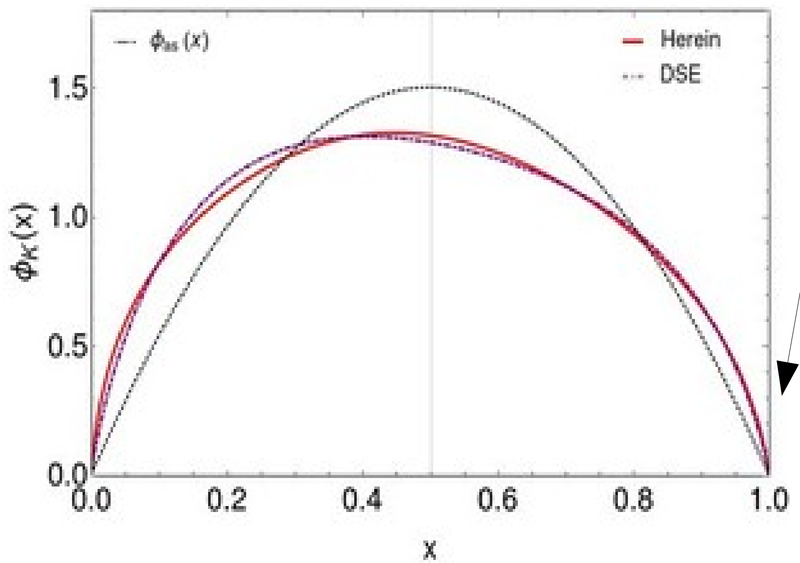
GPDs from LFWFs

LFWF and PDAs:









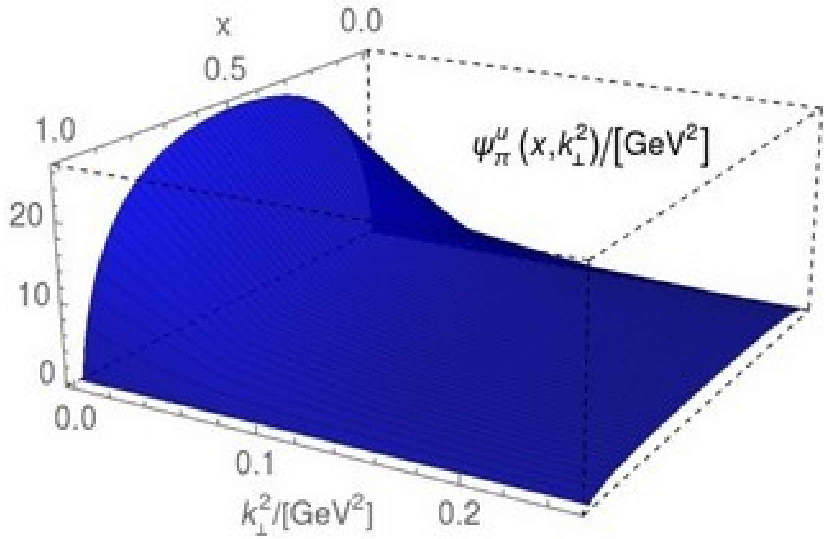
$\sim (1-x)^{1+\gamma}$

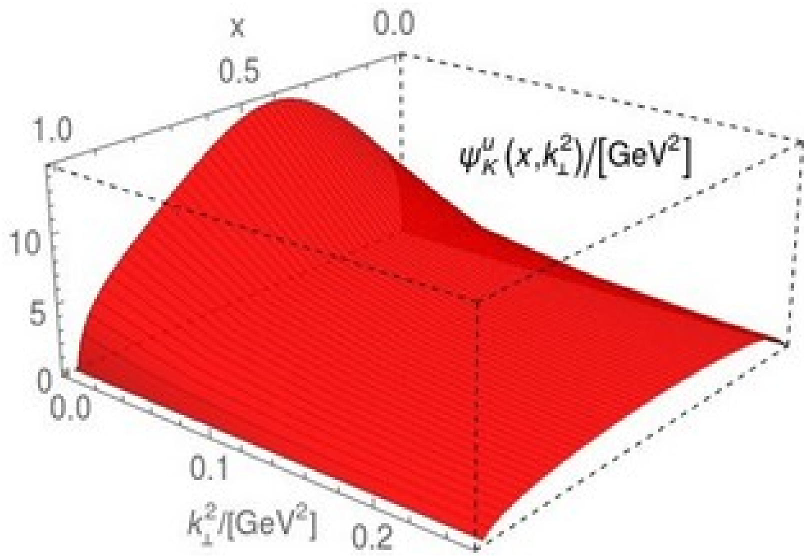
	Herein	DSE	Lattice
$<(1-2x)^1>$	0.033	0.035	0.032(12)
$<(1-2x)^2>$	0.237	0.240	0.231(4)(6)

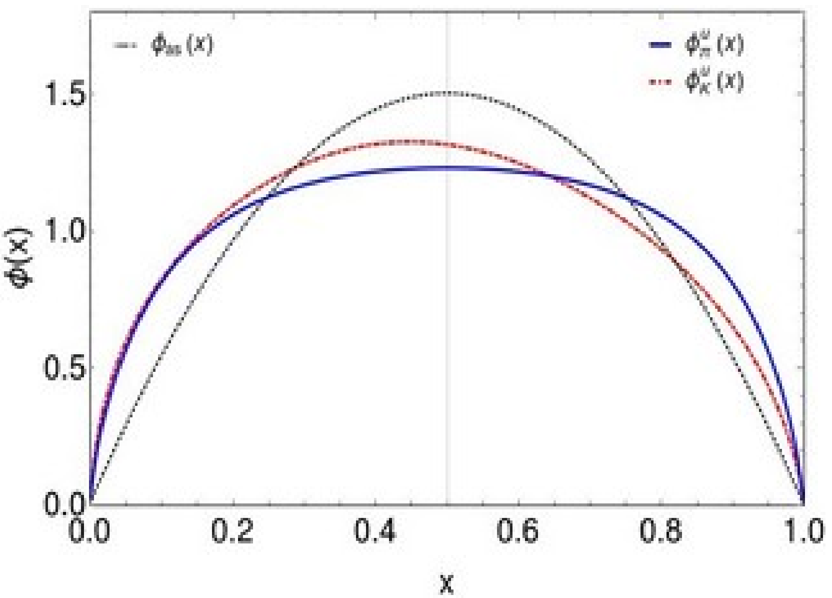
Lattice:
G.S. Bali et al., JHEP 1908 (2019) 065
DSE:
C. Shi et al., PLB 738 (2014) 512-518

GPDs from LFWFs

LFWF and PDAs:







- Well behaved at the endpoints
- Broad and concave functions in x, in fully consistency with DCSB

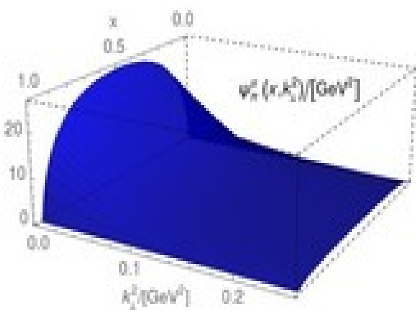
[L. Chang et al., PRL 110 (2013) no.13, 132001]

- Kaon asymmetry led by the difference M_s - M_u

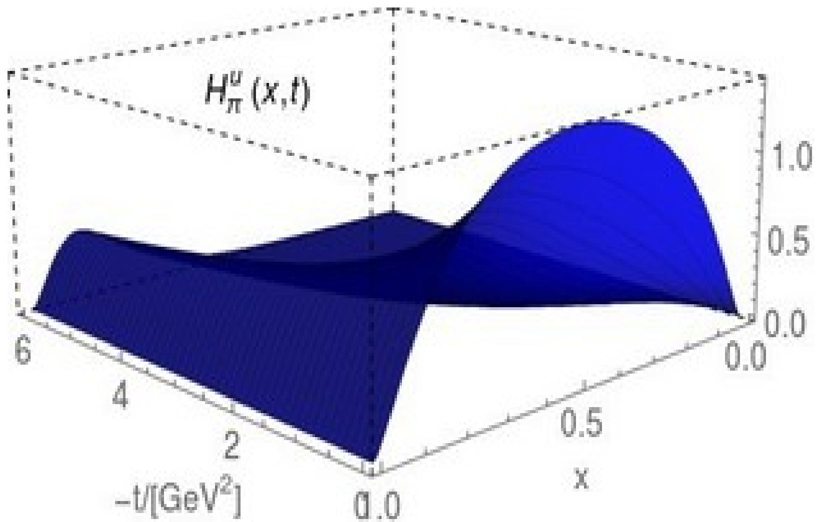
GPDs from LFWFs

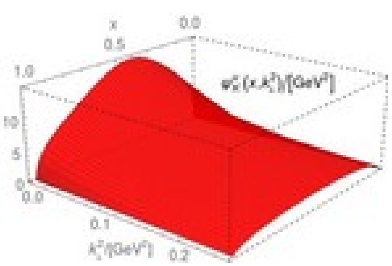
- In the overlap representation, the valence-quark GPD reads

$$H_M^u(x,\xi,t;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi_{Mu}^{\uparrow\downarrow *}\left(\frac{x-\xi}{1-\xi},\left(\mathbf{k}_\perp + \frac{\mathbf{1-x}}{\mathbf{1-\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)\psi_{Mu}^{\uparrow\downarrow}\left(\frac{x+\xi}{1+\xi},\left(\mathbf{k}_\perp - \frac{\mathbf{1-x}}{\mathbf{1+\xi}}\frac{\mathbf{\Delta}_\perp}{\mathbf{2}}\right)^2;\zeta_H\right)$$

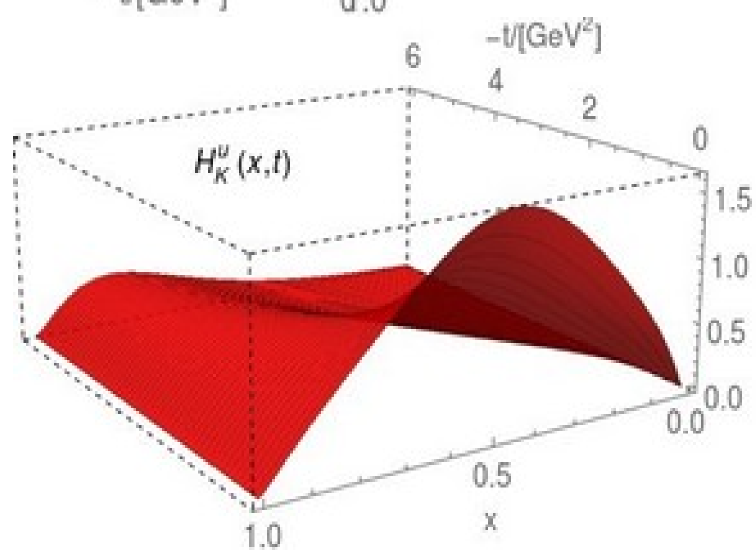


Pion LFWF





Kaon LFWF



- ✓ Valid in the **DGLAP** region
- ✓ Can be extended to the **ERBL** region: $|x| \leq \xi$
- ✓ ***Compatible*** with diagram approach

N. Chouika et al.,
PLB 780 (2018) 287-293.

C. Mezrag et al.,
FBS 57 (2016) no.9, 729-772

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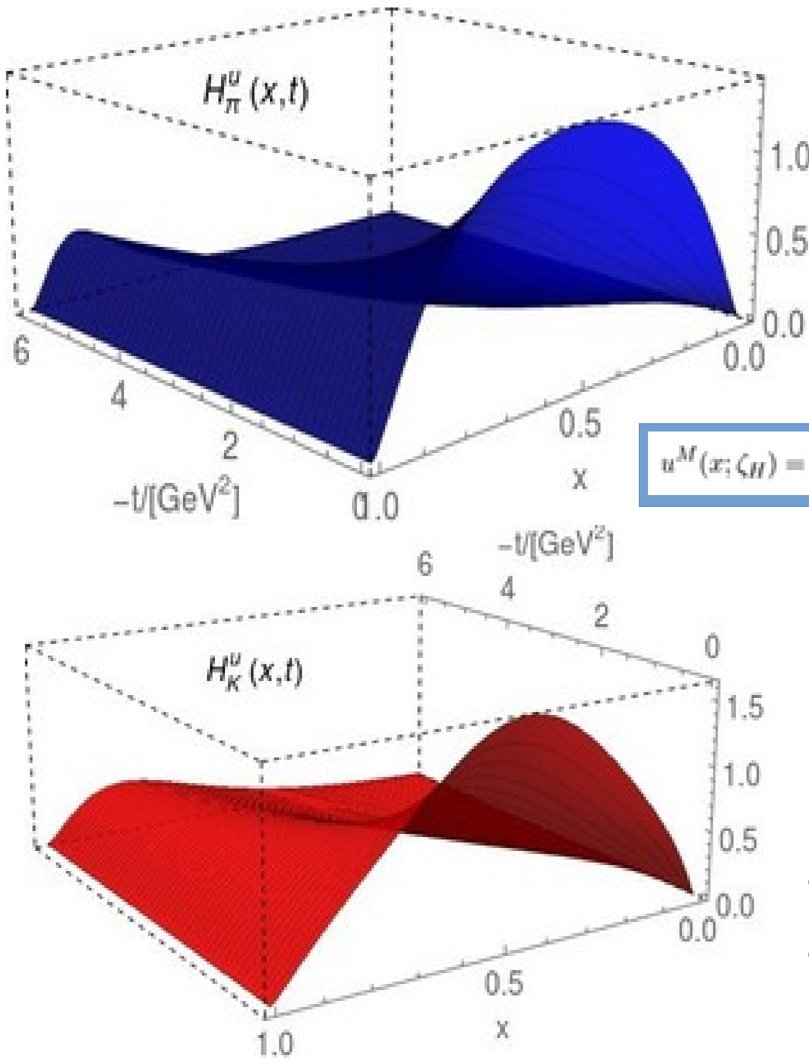
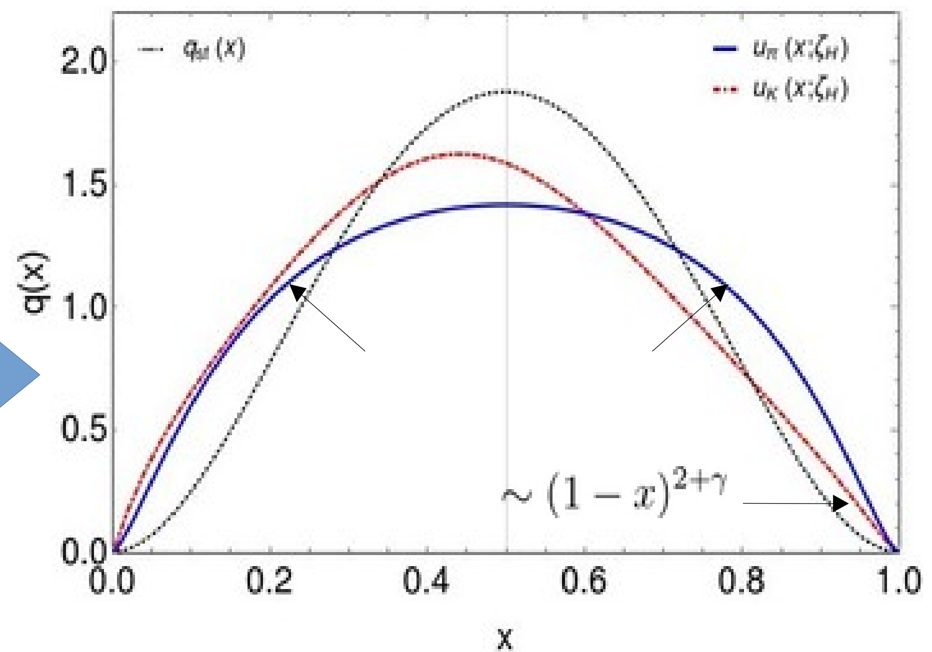


$$u^M(x;\zeta_H) = H_M^u(x,0,0;\zeta_H)$$

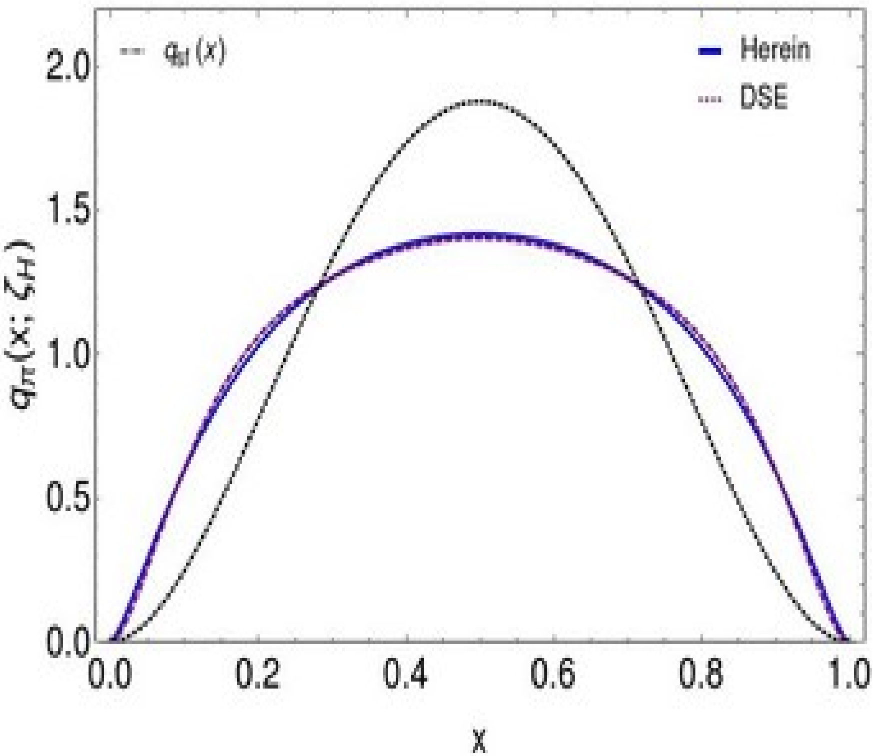


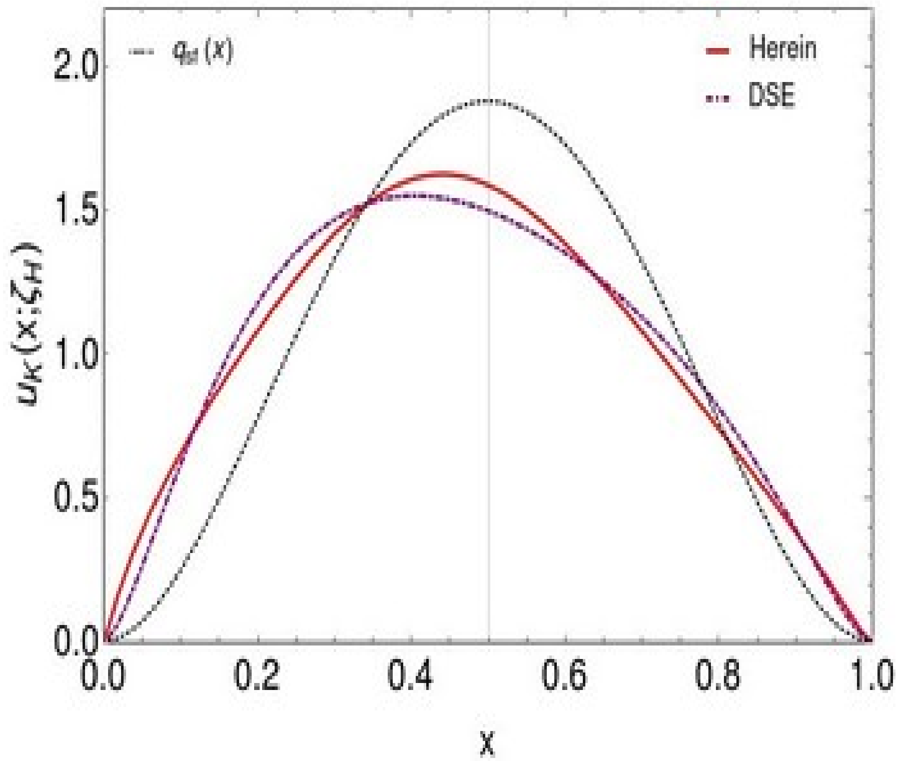
$<x>_\pi^u = 0.50$ $<x>_K^u = 0.47$

- ζ_H: **all** the momentum is carried by the valence-quarks.
- **Defined** from the **PI** effective charge.

GPDs from LFWFs

Valence-quark PDFs:





ζ_H	$\langle x^2\rangle_\pi$	$\langle x^4\rangle_\pi$	$\langle x^6\rangle_\pi$
DSE	0.3020	0.1460	0.0857
Herein	0.3020	0.1461	0.0861

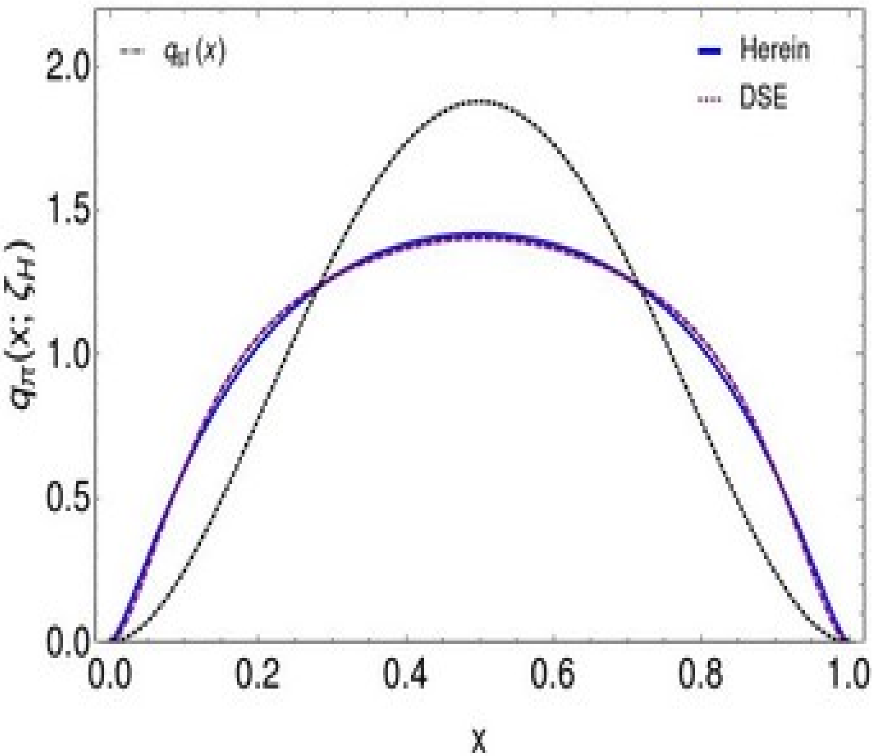
ζ_H	$\langle x\rangle_K^u$	$\langle x^2\rangle_K^u$	$\langle x^3\rangle_K^u$
DSE	0.471	0.270	0.173
Herein	0.468	0.266	0.170

DSE: M. Ding et al. Chin.Phys. 44 (2020) no.3, 031002.
PRD 101 (2020) no.5, 054014

DSE: Z-F Cui et al. arXiv: 2006.14075.

GPDs from LFWFs

Valence-quark PDFs:



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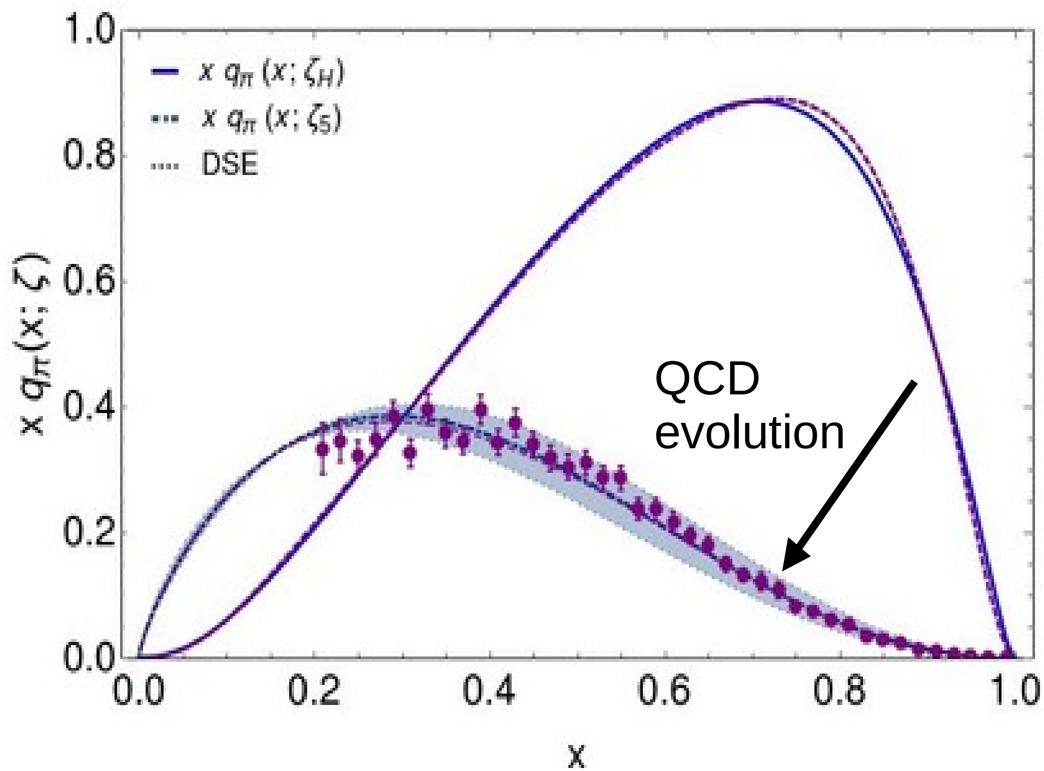
- At ζ_H there are **small deviation** from the *realistic* PDFs from DSEs (factorization approximation effects?), but they are irrelevant after QCD DGLAP evolution.



DSE: M. Ding et al. Chin.Phys. 44 (2020) no.3, 031002.
PRD 101 (2020) no.5, 054014

Form Factors

- **Electromagnetic** form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

Can safely take **ξ = 0**

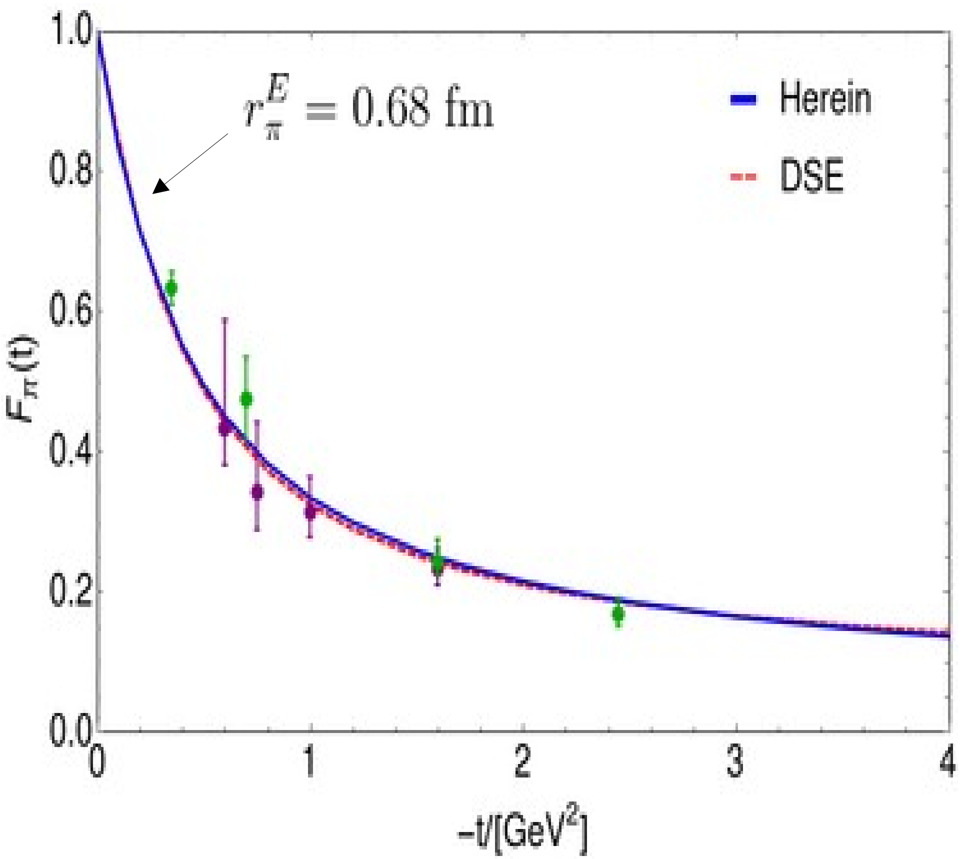
"Polinomiality"

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_{\bar{f}} F_M^{\bar{f}}(\Delta^2)$$

Weighted by electric charges

→ **Isospin symmetry**

$$\Rightarrow \quad F_{\pi^+}(-t) = F_{\pi^+}^u(-t)$$



Data: G.M. Huber et al. PRC 78 (2008) 045202

DSE: L. Chang et al. PRL 111 (2013) 14, 141802

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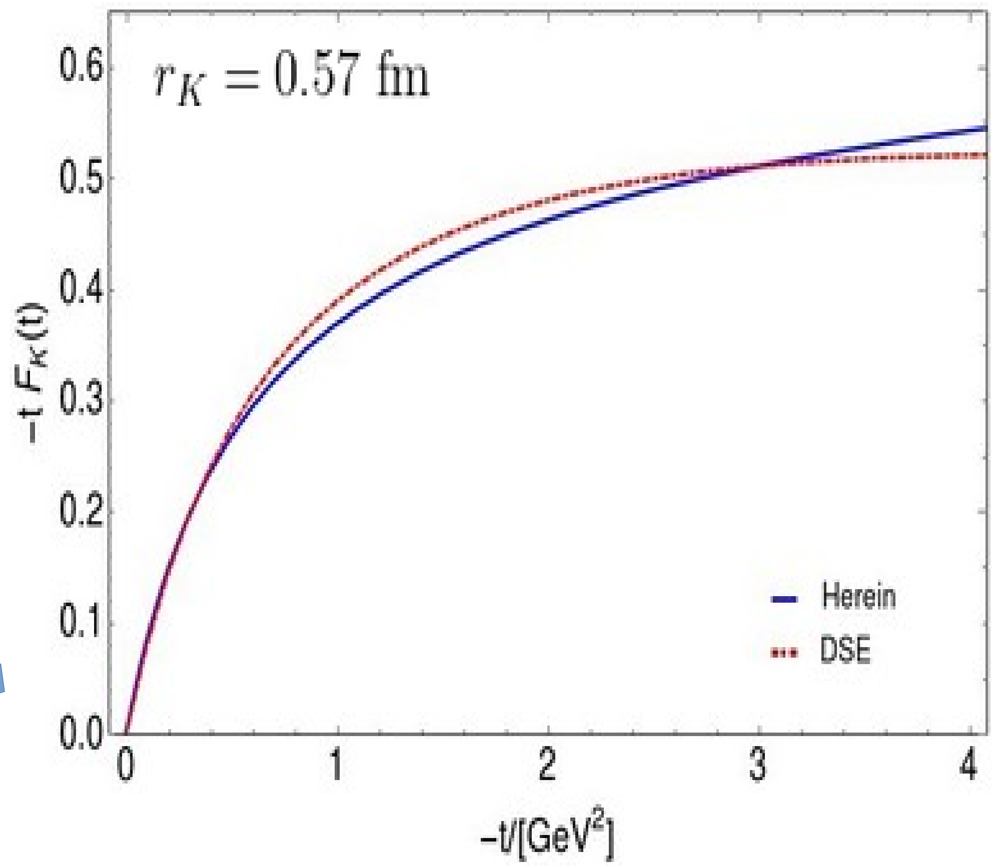
Weighted by electric charges



- A more **notorious** difference, but still *acceptable*.

- Also compatible with:

G. Eichmann et al. PRD 101 (2020) 5, 054015

DSE: F. Gao et al. PRD 96 (2017) 3, 034024

Form Factors

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

  - Directly obtained if **ξ = 0**
    - Only **DGLAP** evolution is needed

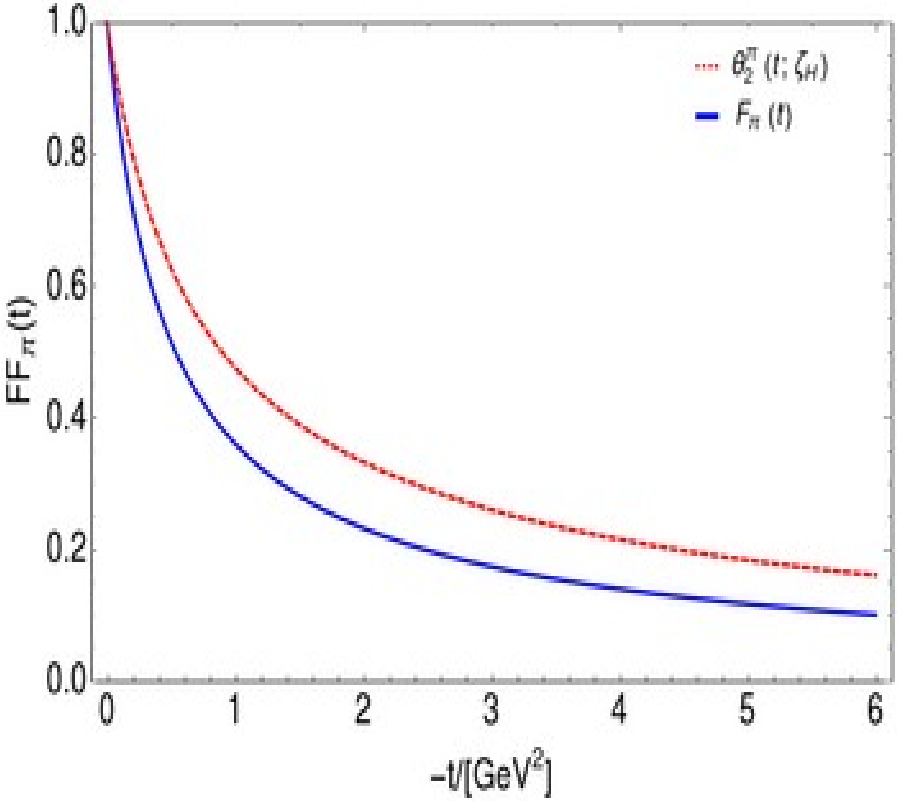
  → **Isospin symmetry**

$$\Rightarrow \quad \Theta_2^\pi(t) = \int_0^1 dx\, 2x H_\pi^u(x,0,t)$$

- **Forward limit:** momentum fractions (**PDF**)

  (t = 0)

- **GFFs** connect with Energy-momentum tensor.



$$r_\pi^E = 0.68 \text{ fm}\,, \quad r_\pi^{\theta_2} = 0.56 \text{ fm}$$

(charge radius) (mass radius)

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$$= c_0^{(1)}(t) + \xi^2 \int_{-1}^{1} dz\, zD(z,t)$$

"D-term"

(Polyakov-Weiss scheme)

- **ERBL** extension is **possible...** But insufficient

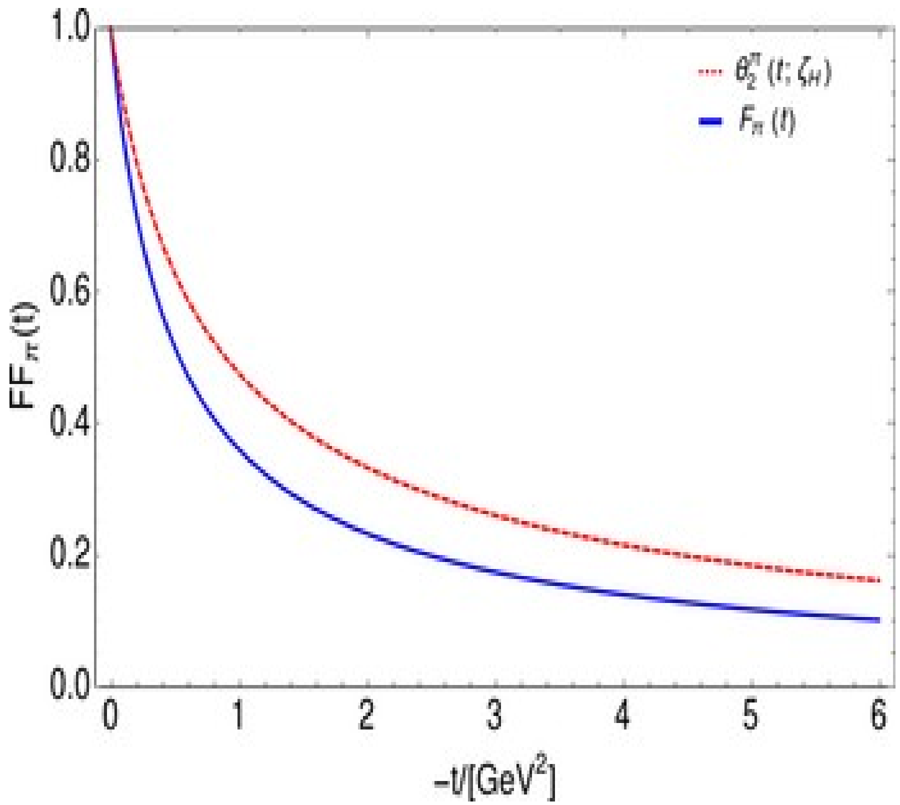
- 'Soft-pion theorem implies:

$$\int_{-1}^{1} dz zD(z,t=0) = -c_0^{(1)}(0)$$

All LFWFs tell us: $\theta_1^\pi(0) = \theta_2^\pi(0)$

→ *External* **inputs** needed in the **LFWF** approach.

(for example, dispersive evaluations:
B. Pasquini et al., PLB 739 (2014) 133-138)



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(charge radius) (mass radius)

QCD evolution

DGLAP "at al orders" and effective coupling

- **Aprroach:** Define an effective coupling such that the equations below are exact.

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y;t) P\left(\frac{x}{y}\right)$$

- or -

$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

i.e. no LO, NLO, etc:
all orders are there

… and identify, not tune, the (*initial*) **hadron scale ζ_H.**
Fully dressed quasiparticles are the correct degrees of freedom.

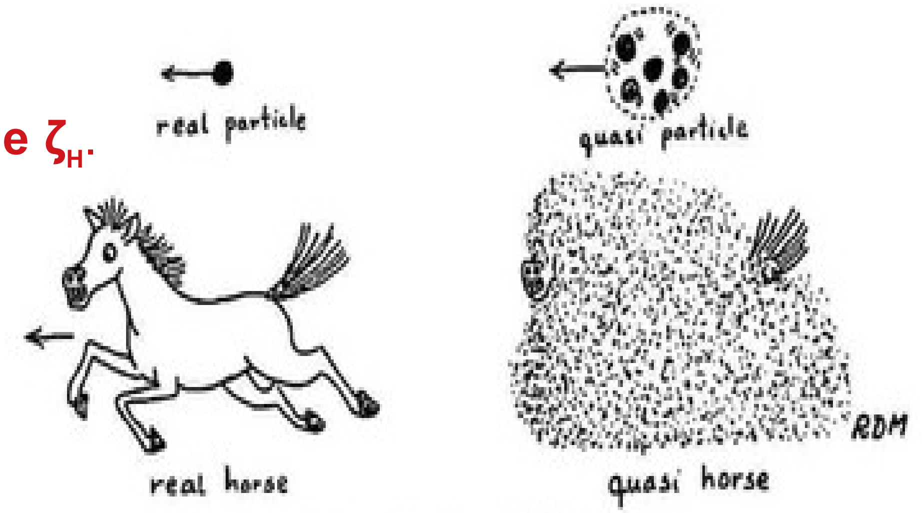

Fig. 0.4 Quasi Particle Concept

QCD evolution

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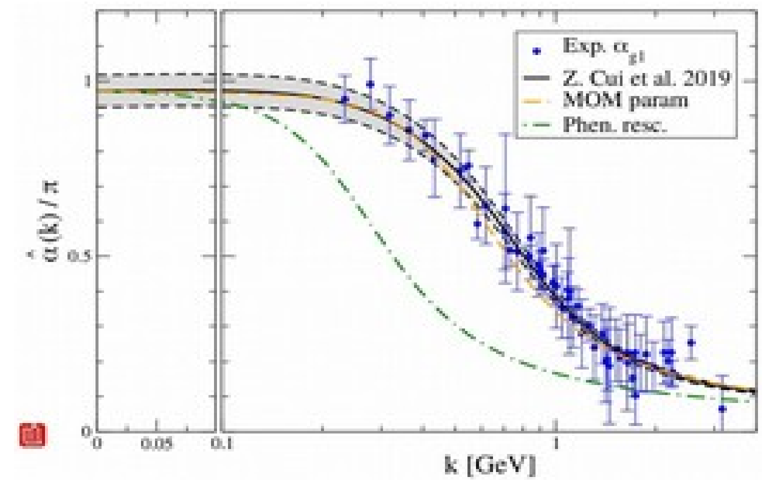
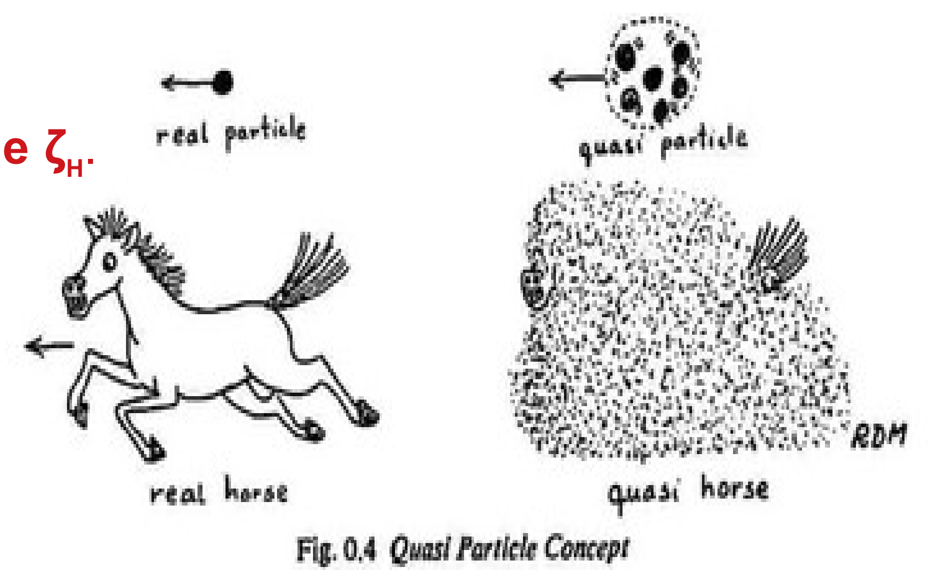
$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

i.e. no LO, NLO, etc: **all orders are there**



… and identify, not tune, the (*initial*) **hadron scale ζ_H**.
Fully dressed quasiparticles are the correct degrees of freedom.

➤ Features of the **PI effective** charge lead to the **answer**.



J. R-Q et al., arXiv: 1909.13802; D. B. et al., PRD 96 (2017) no.5, 054026. J. R-Q. et al., FBS 59 (2018) no.6, 121; Z-F Cui et al., CPC 44 (2020) 8, 083102

QCD evolution

The IR fixed point from PI effective charge:

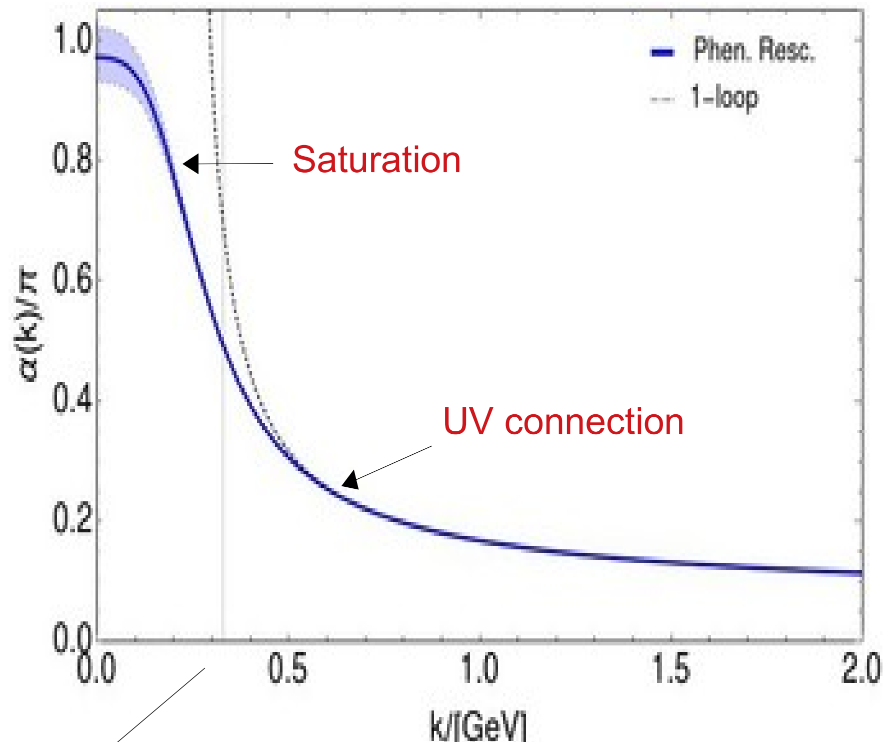
- **Approach:** Define an effective coupling such that:

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y;t) P\left(\frac{x}{y}\right)$$

"All orders hypothesis"

- The ***coupling***:

$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]} \; ; \; \alpha(0) = 0.97(4)$$



Where $\mathcal{M}(k^2 = \Lambda^2_{\rm QCD}) := m_G = 0.331(2)\ {\rm GeV}$ defines a *screening* mass.

→We identify: $\zeta_H := m_G(1 \pm 0.1)$ ⇐ 10% uncertainty

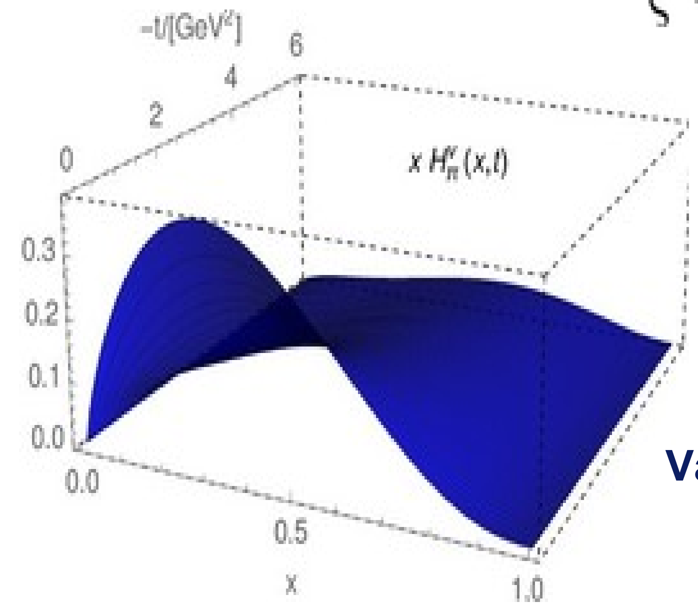
QCD evolution

Evolved GPDs:

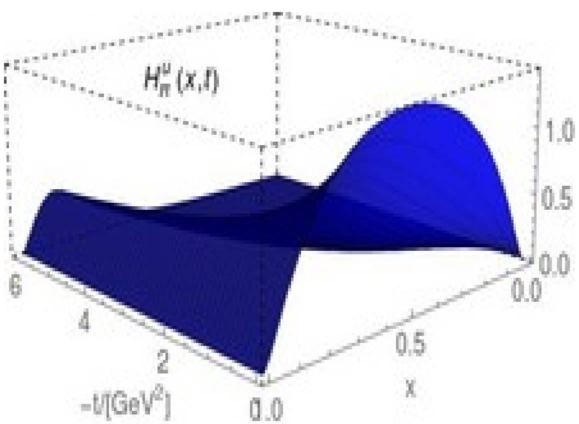
- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.

- Thus **gluon** and **sea** **GPDs** are obtained.

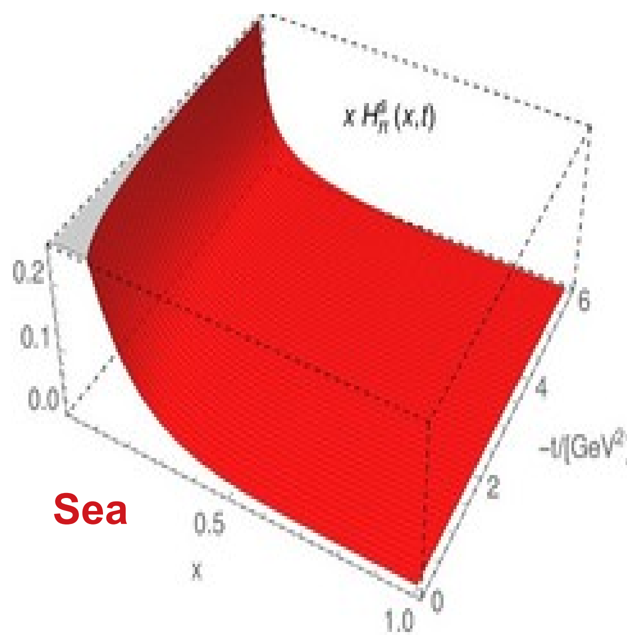
$\zeta = 5.2\ \mathrm{GeV}$



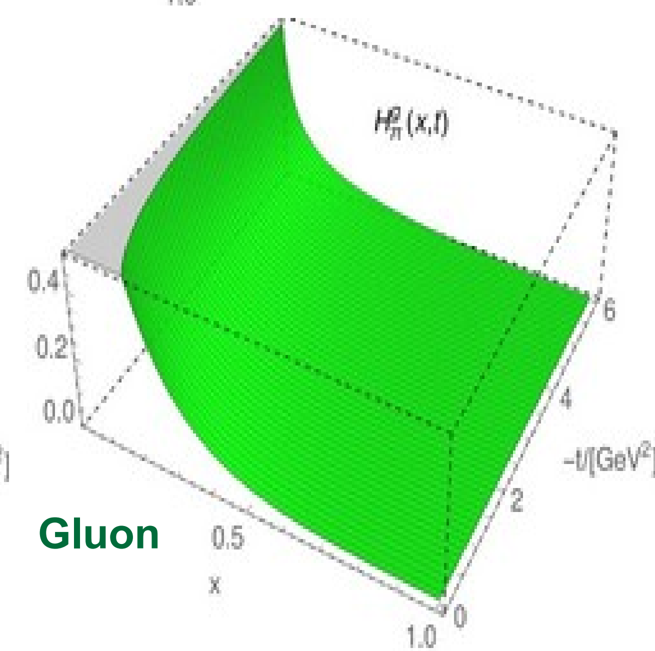
Valence



$\zeta_H = 0.331\ \mathrm{GeV}$



Sea



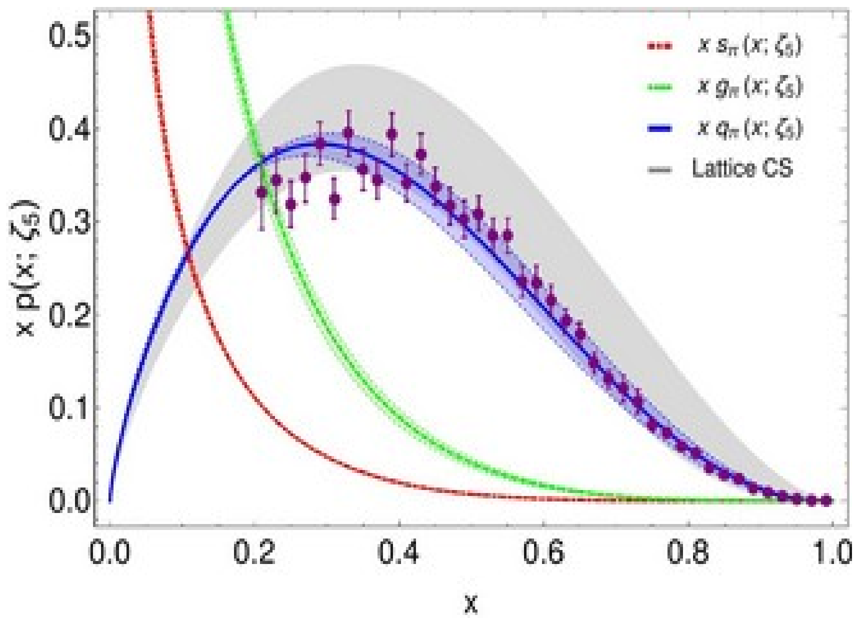
Gluon

QCD evolution

Evolved PDFs:

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.

$\zeta = 5.2$ GeV



ζ	$<x>_V$	$<x>_G$	$<x>_S$
2 GeV	0.483(42)	0.411(24)	0.106(18)
5.2 GeV	0.412(36)	0.449(19)	0.138(17)

Data: M. Aicher et al., PRL 105 (2010) 252003

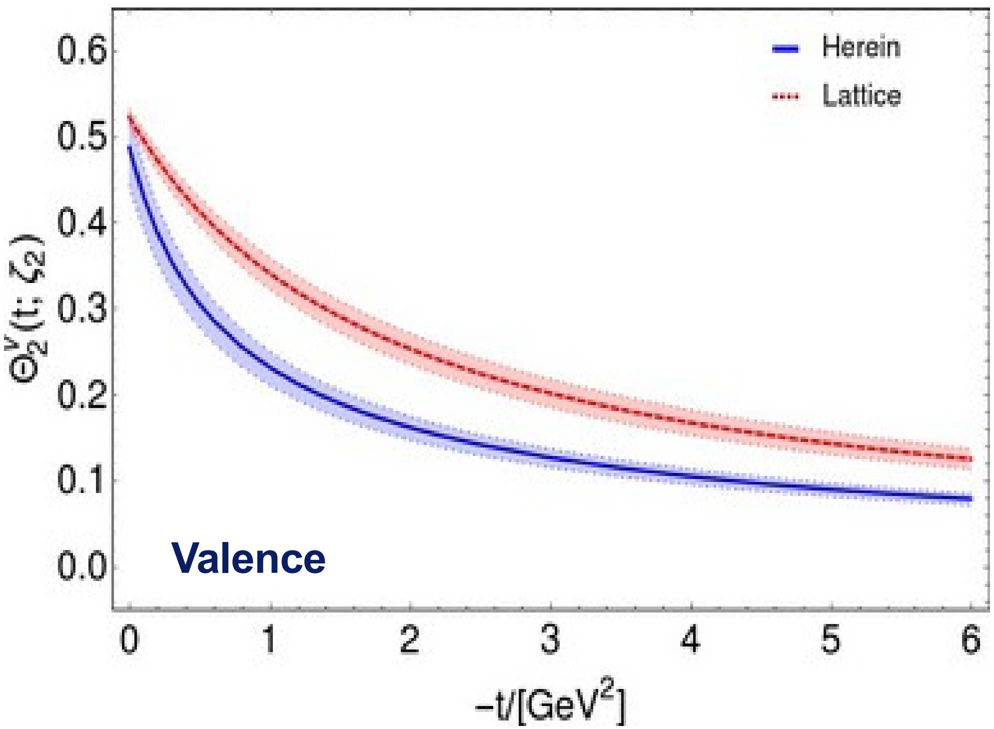
Lattice: R.S. Sufian et al., arXiv:2001.04960

QCD evolution

Evolved GFFs:

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
- **GFF** $\theta_2(t)$ comes from the *off-forward* <x>.

$\zeta = 2$ GeV



Herein: softer GFF,

Lattice: D. Brommel, PhD. Thesis 2007

$$\Theta_2(0)/2 = 0.26(4) \quad (m_\pi^2 > 0.3\ \mathrm{GeV}^2)$$

QCD evolution

Evolved GFFs:

$\zeta = 2\ \text{GeV}$

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
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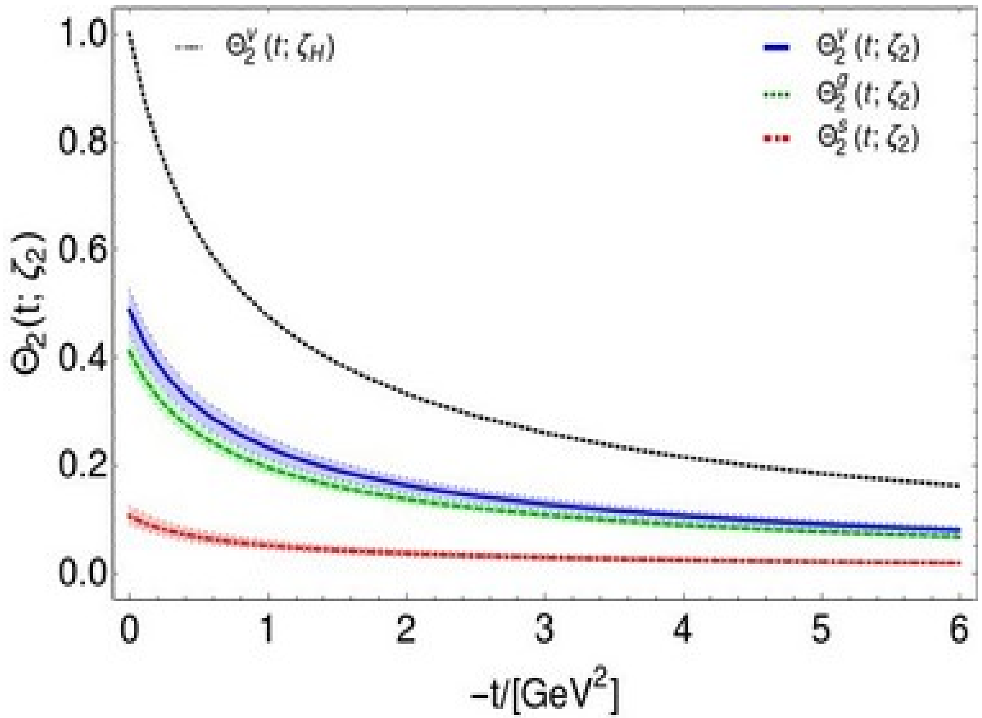

QCD evolution

Evolved GFFs:

$\zeta = 2$ GeV

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.

- Thus **gluon** and **sea** **GPDs** are obtained.

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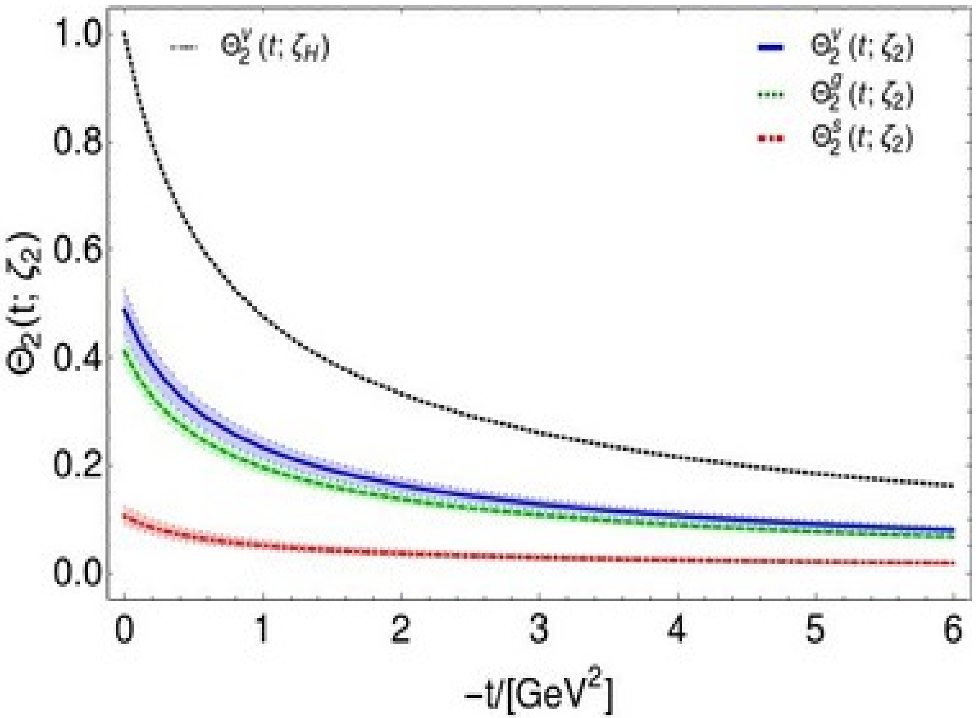
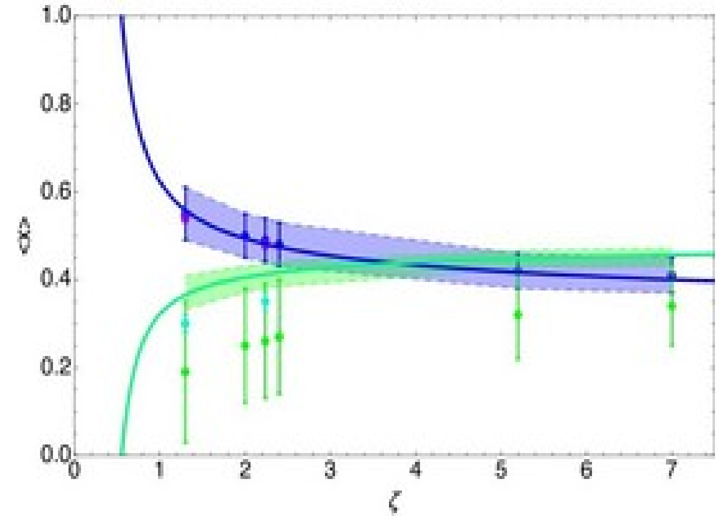
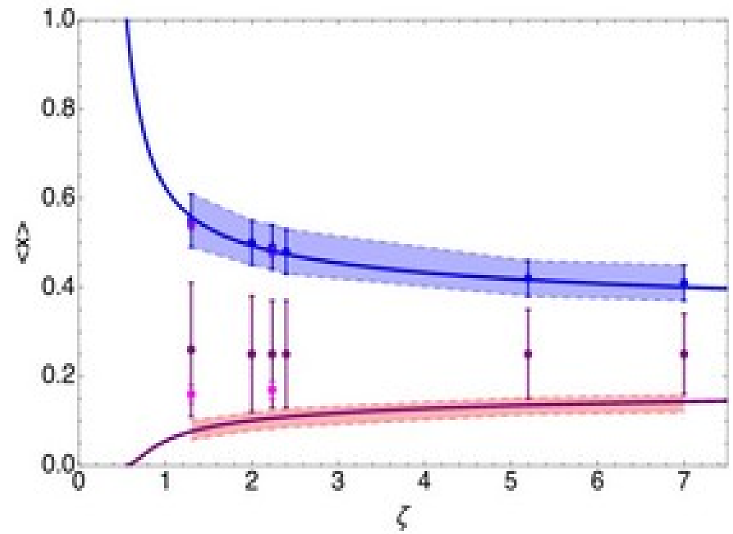


QCD evolution

Evolved GFFs:

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
- **GFF** $\theta_2(t)$ comes from the *off-forward* <x>
- One can also test the evolution with the scale, for instance for the momentum fraction





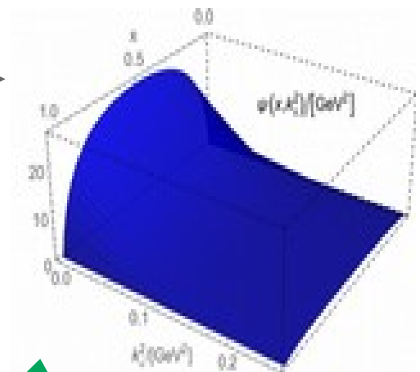
I. Novikov et al, arXiv: 2002.02902 "**xFitter**"

Summary: Pion

- Using our **DSE prediction** of pion PDF as **benchmark**, we modeled the pion **BSWF**.

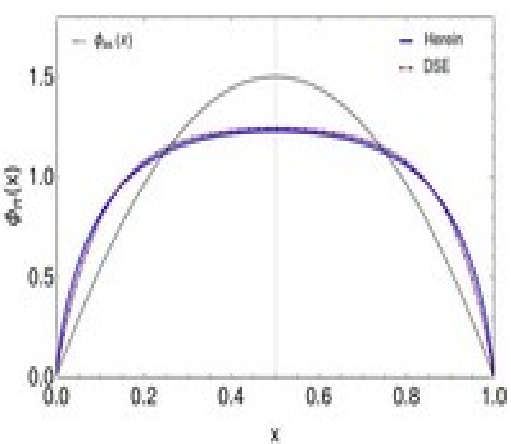
DSE: CPC 44 (2020) no.3, 031002, PRD 101 (2020) no.5, 054014

BSWF →

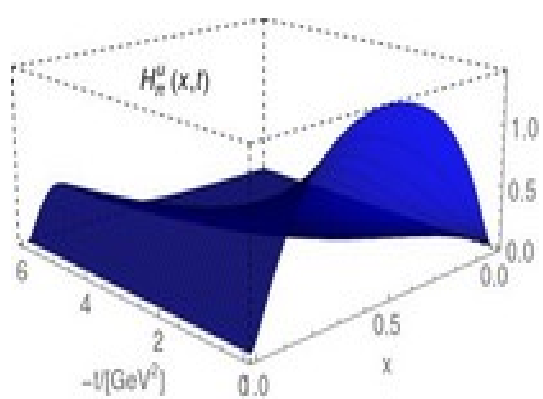


$\int dk_\perp$

LFWF



PDA



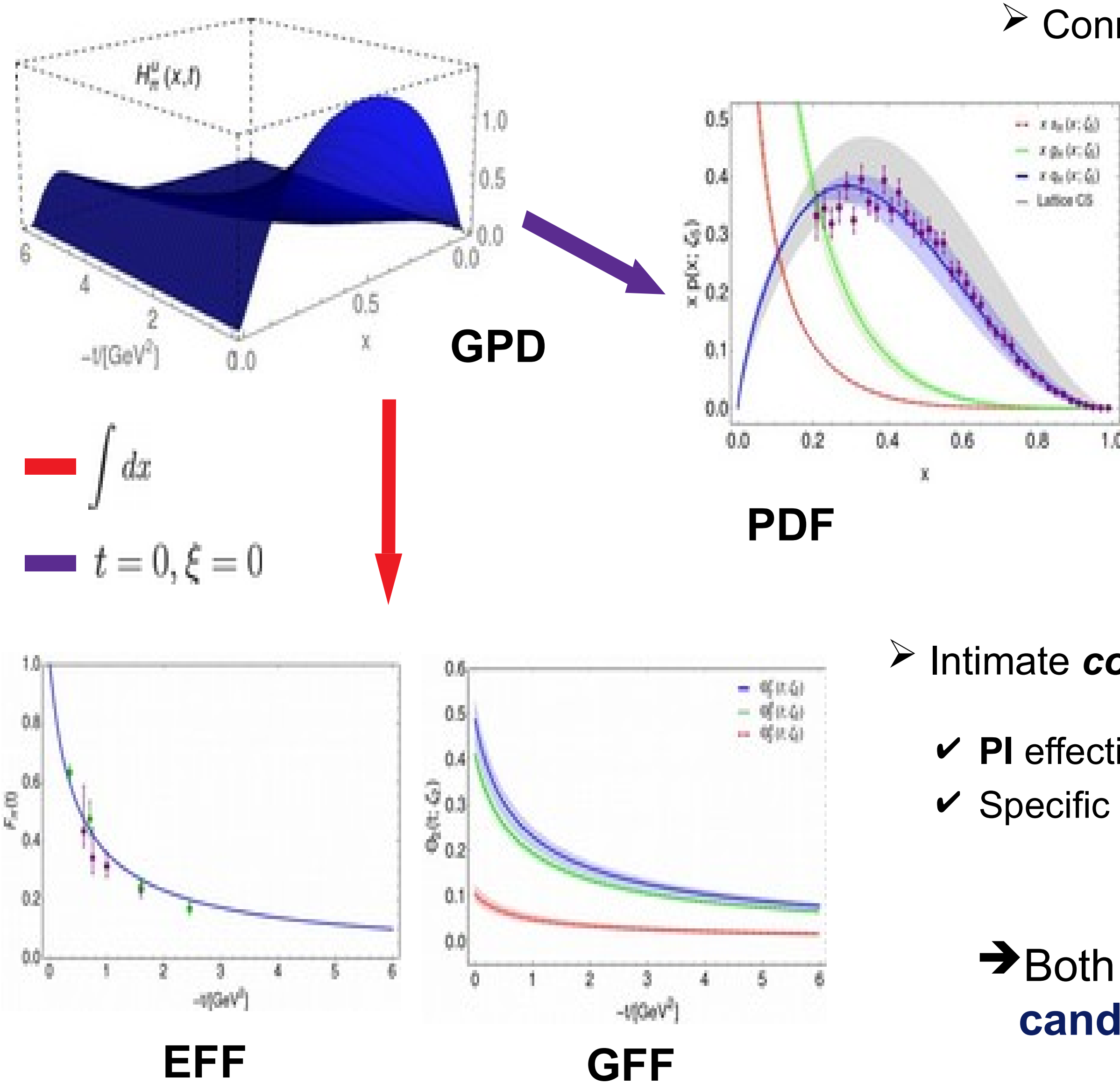
GPD

- **Consistent** features of the **PDA**:
  - ✔ Broad and concave at real world scales.
  - ✔ Correct endpoint behavior.
  - ✔ Agreement with **Lattice** and **DSE** results.

- The valence **GPD** is obtained from the **overlap** representation.
  - ✔ Limited to the **DGLAP** region.
  - ✔ **Gluon** and **sea** obtained from *evolution* equations.
  - ✔ **Extension** to ERBL region is possible.

(but insufficient)

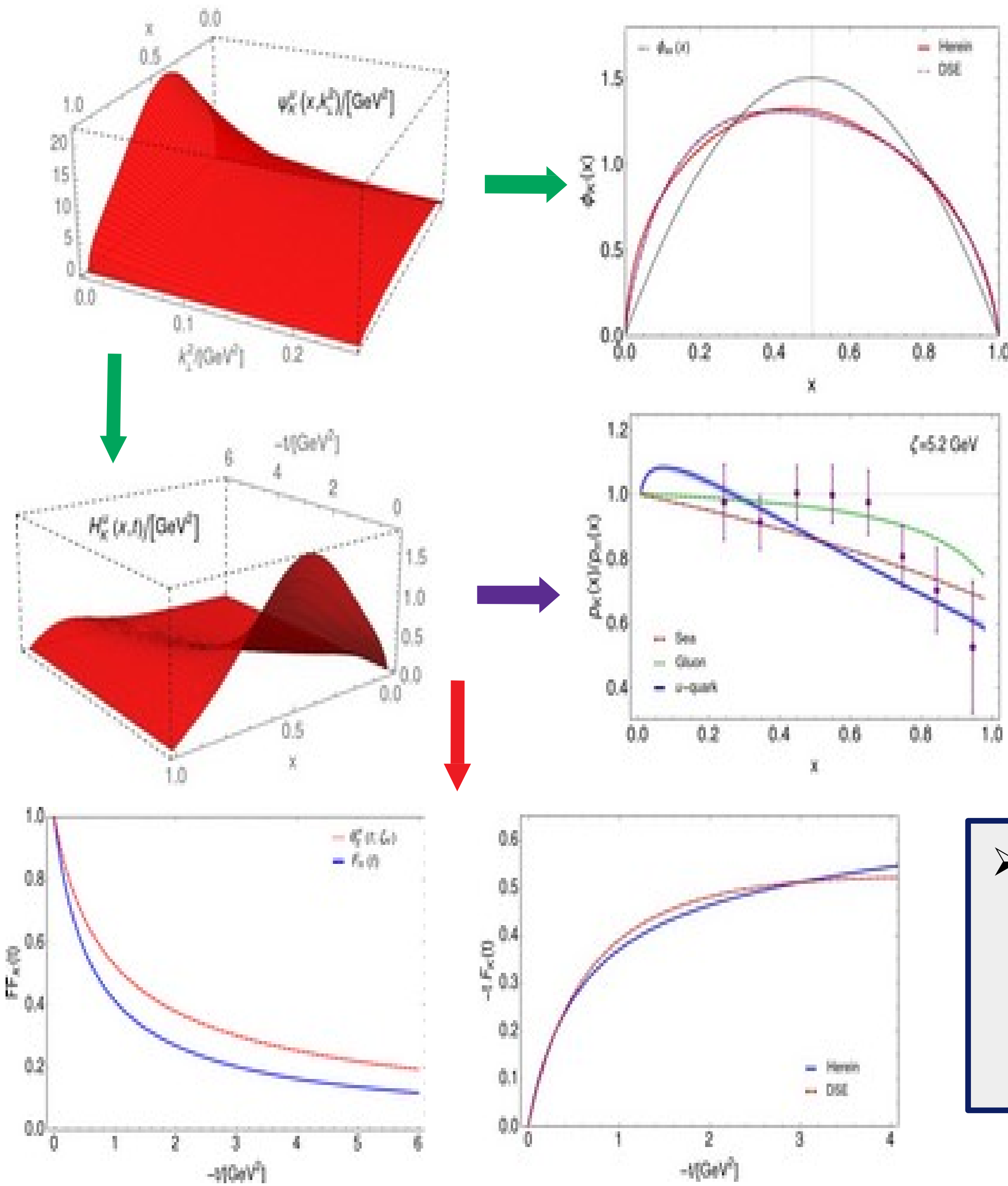
Summary: Pion



- Connection **PDF** with **DSE predictions** implies:
  - ✔ Keen agreement with reanalyzed data.
  - ✔ Large-x behavior as predicted by **pQCD**.
  - ✔ Compatible with novel **Lattice** results.
- **EFF *consistent*** with empirical data.
  - ✔ One can trust the off-forward quantities.
  - ✗ ERBL region + D-term **needed.**
- Intimate ***connection*** with the **running coupling**:
  - ✔ **PI** effective charge → effective coupling for **evolution**.
  - ✔ Specific ***definition*** of the hadron scale.

➔Both **LFWF** and **GPD** are **promising candidates** to be the real objects.

Summary: Kaon



➢ Connection with **DSE predictions** implies:

- ✔ **Qualitiative** features of the distributions are properly captured.
- ✔ *Large-x* behavior of the **PDA** and **PDF** as predicted by **pQCD**.
- ✔ **K/pi PDF** ratio in agreement with data.

We still need new experiments !!!

✔ Computed **Gluon** and **Sea** Kaon **PDFs**

the **GPDs** are available too !!!

➢ Next steps:

- ✔ **Impact** parameter distributions
- ✔ **Transverse** momentum distributions **(TMDs)**