Pion and Kaon Structure using Basis Light-front Quantization

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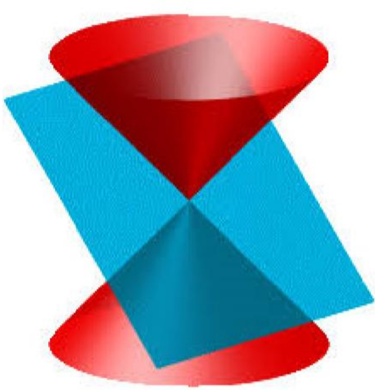


Perceiving the Emergence of Hadron Mass through AMBER@CERN 08/06/2020

Outline

- Basis Light-front Quantization approach

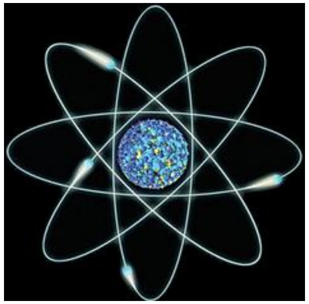
- Application to π and K
  - Leading Fock sector (based on NJL interacton)
  - With one dynamical gluon

- Summary and Future Plan

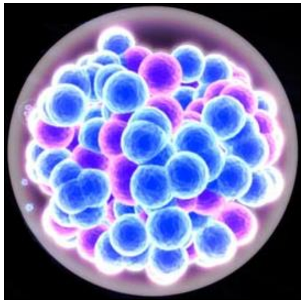
Hamiltonian Formalism

- Schrödinger equation universally describes different physics :

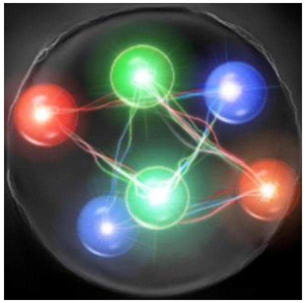
$$H|\psi\rangle = E|\psi\rangle$$



atom



nucleus



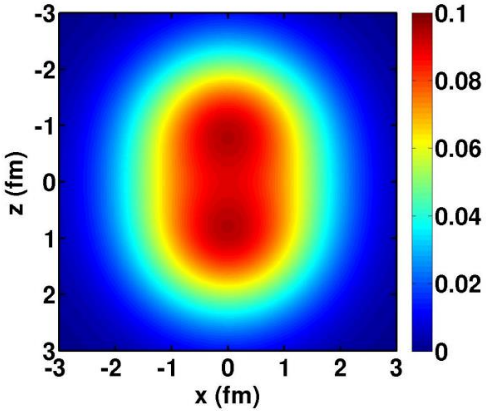
hadron

Nonrelativistic, few-body

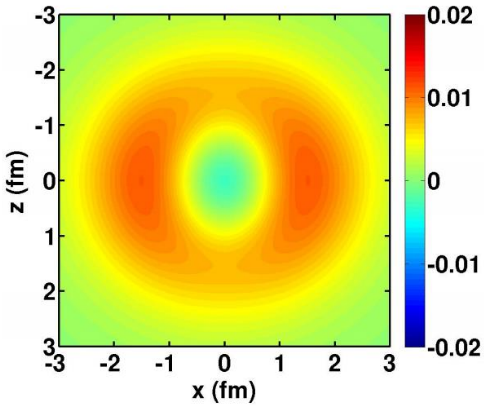
Nonrelativistic, many-body

Relativistic, many-body

- Wave functions encode full information of the system



Proton density



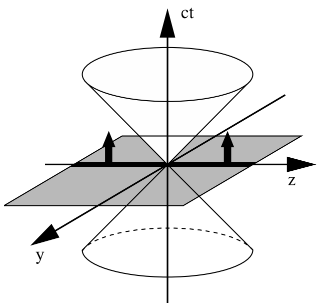
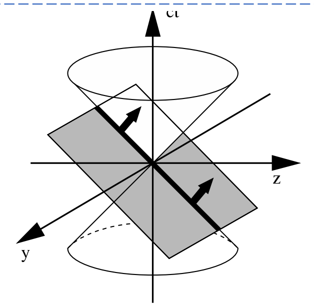
Neutron – proton density

^{9}Be

Adapted from Yang Li's talk

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- Not just a coordinate transformation
- New theory !!!

Advantages:

- Frame-independent wave functions
- Simple vacuum structure
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle$$

  - P^{-}: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P_{β}^{-}: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\boldsymbol{\pi}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}gg\rangle + d|q\bar{q}q\bar{q}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi_{n,m}^{b}(\vec{p}_{\perp})$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i (2n_i + |m_i| + 1) \le N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

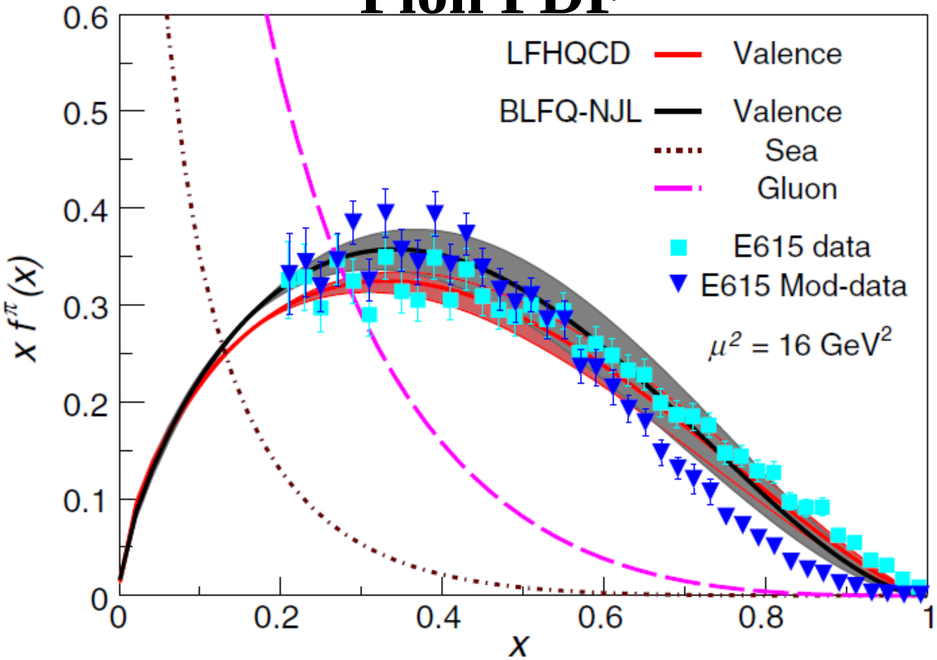
Application to π and K

PDF from BLFQ and QCD Evolution for Light Mesons

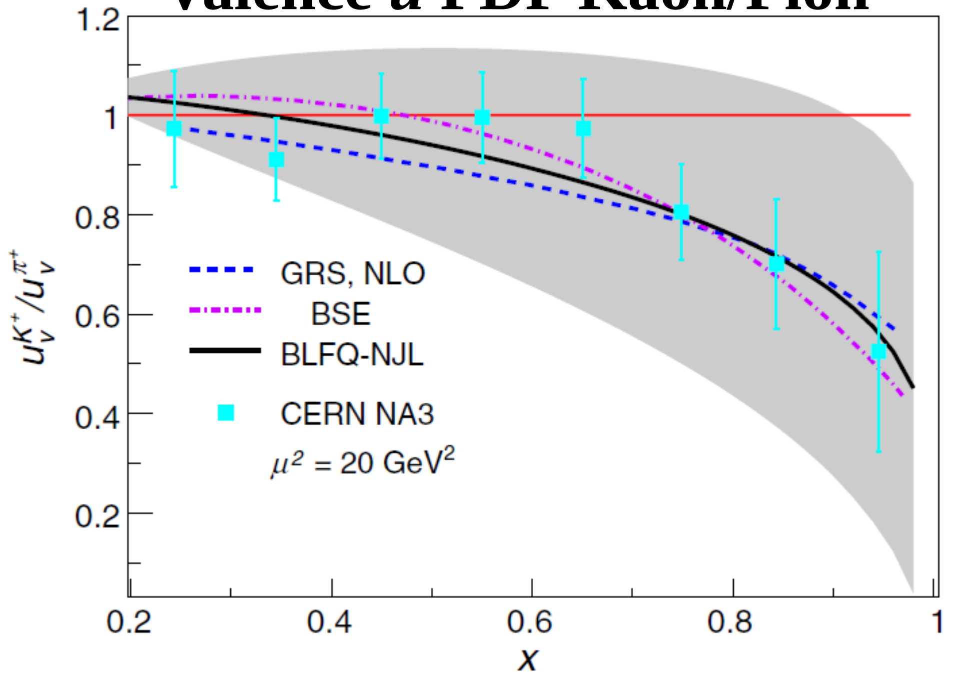
$$H_{\text{eff}} = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big) + H_{\text{eff}}^{\text{NJL}}$$

PDF for the valence quark result from the light-front wave functions obtain by diagonalizing the effective Hamiltonian.

Pion PDF



Valence *u* PDF Kaon/Pion

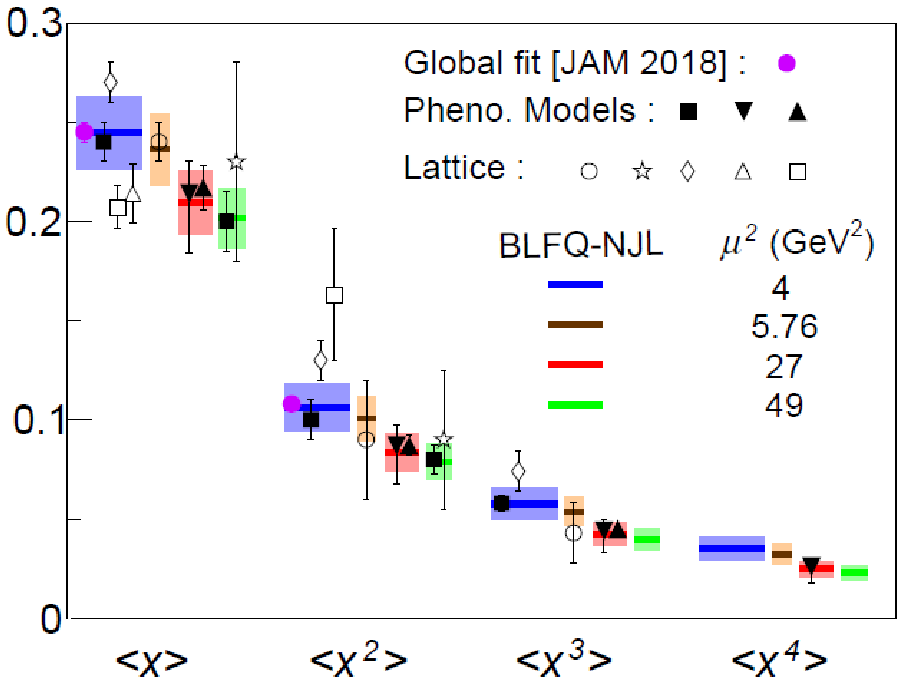


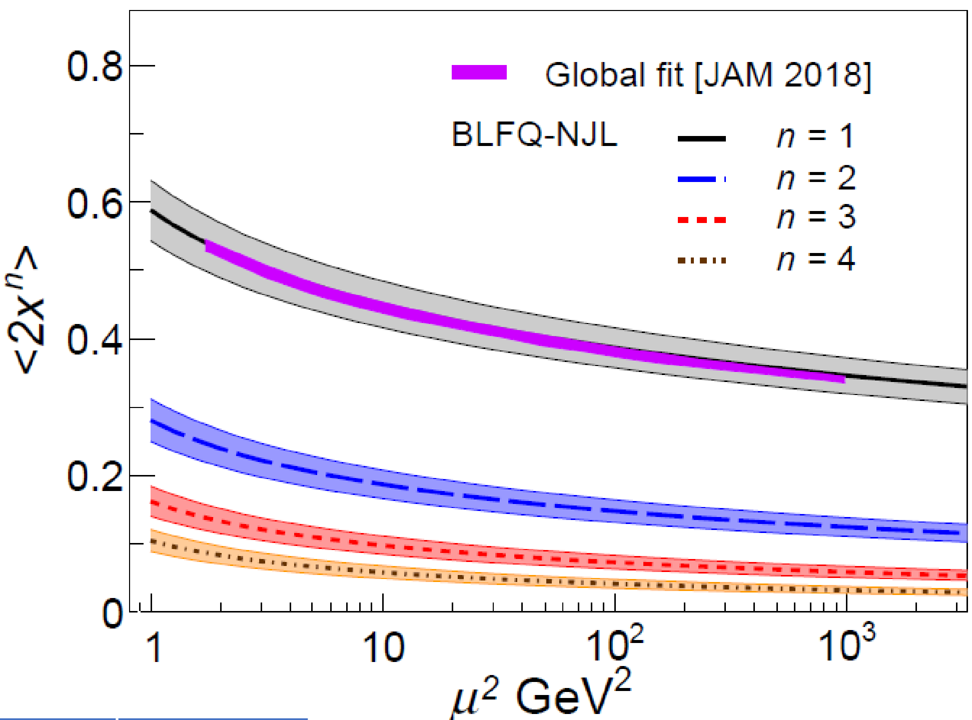
[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

Agree with experimental results

The moments of pion valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x, \mu^2),\ \ n = 1, 2, 3, 4.$$





$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019’]	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\pi\rangle = |q\bar{q}\rangle + \cdots$$

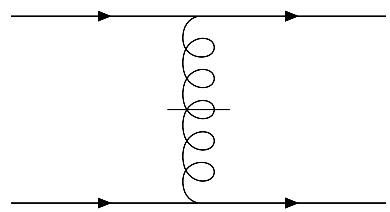
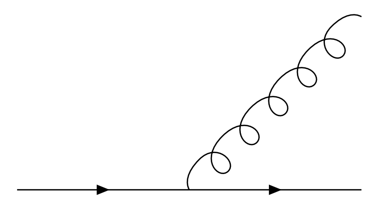
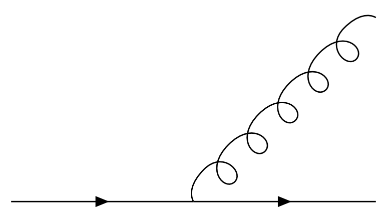
Jiangshan Lan

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

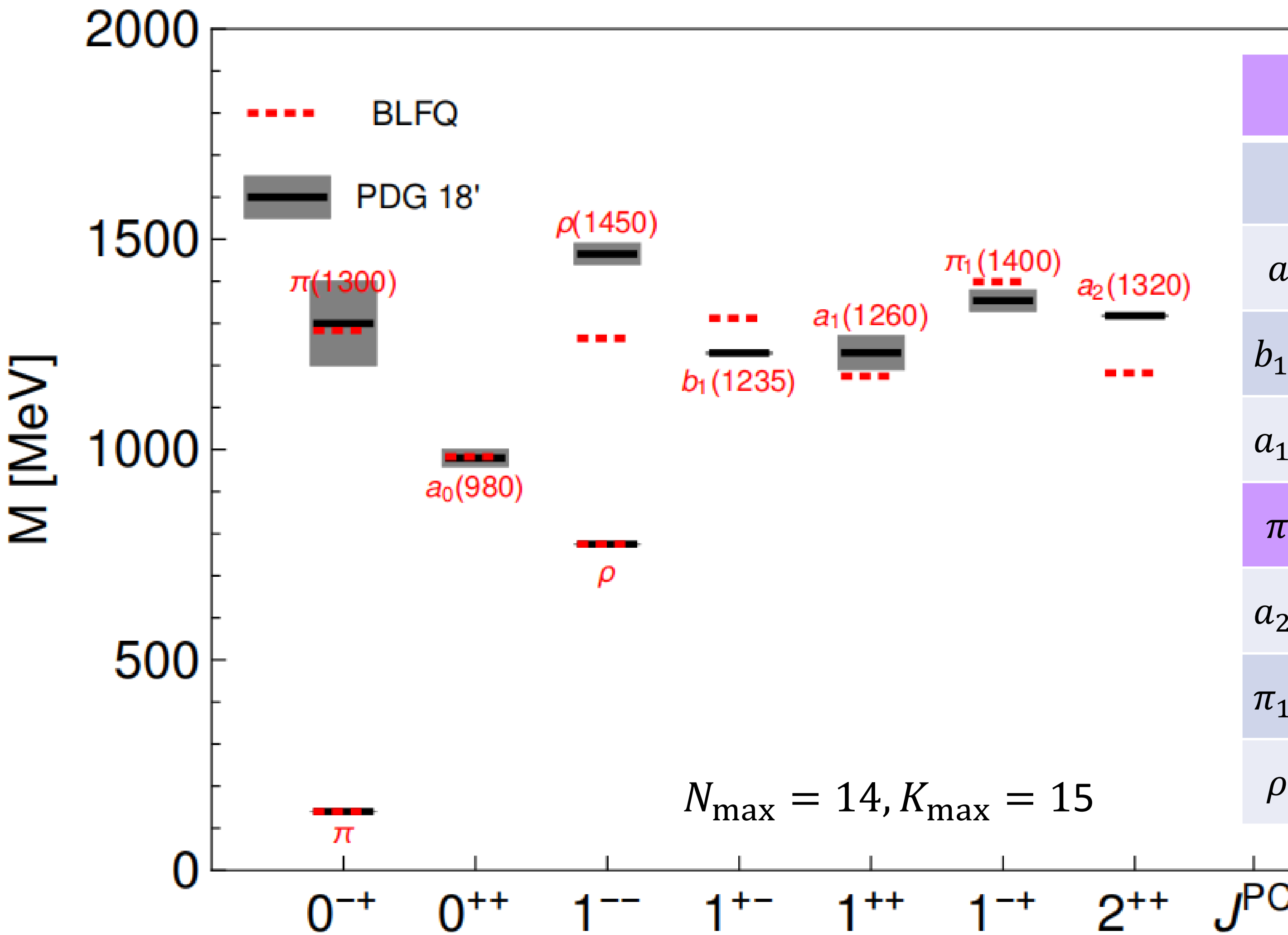
Structure of Hamiltonian

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$P^{-} = \frac{\vec{k}_{\perp}^{2}+m_{q}^{2}}{x} + \frac{\vec{k}_{\perp}^{2}+m_{\bar{q}}^{2}}{1-x} + \kappa^{4}x(1-x)\vec{r}_{\perp}^{2} - \frac{\kappa^{4}}{(m_{q}+m_{\bar{q}})^{2}}\partial_{x}(x(1-x)\partial_{x}) + \boldsymbol{H}_{\mathbf{int}}$$

$\boldsymbol{H}_{\mathbf{int}}$	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

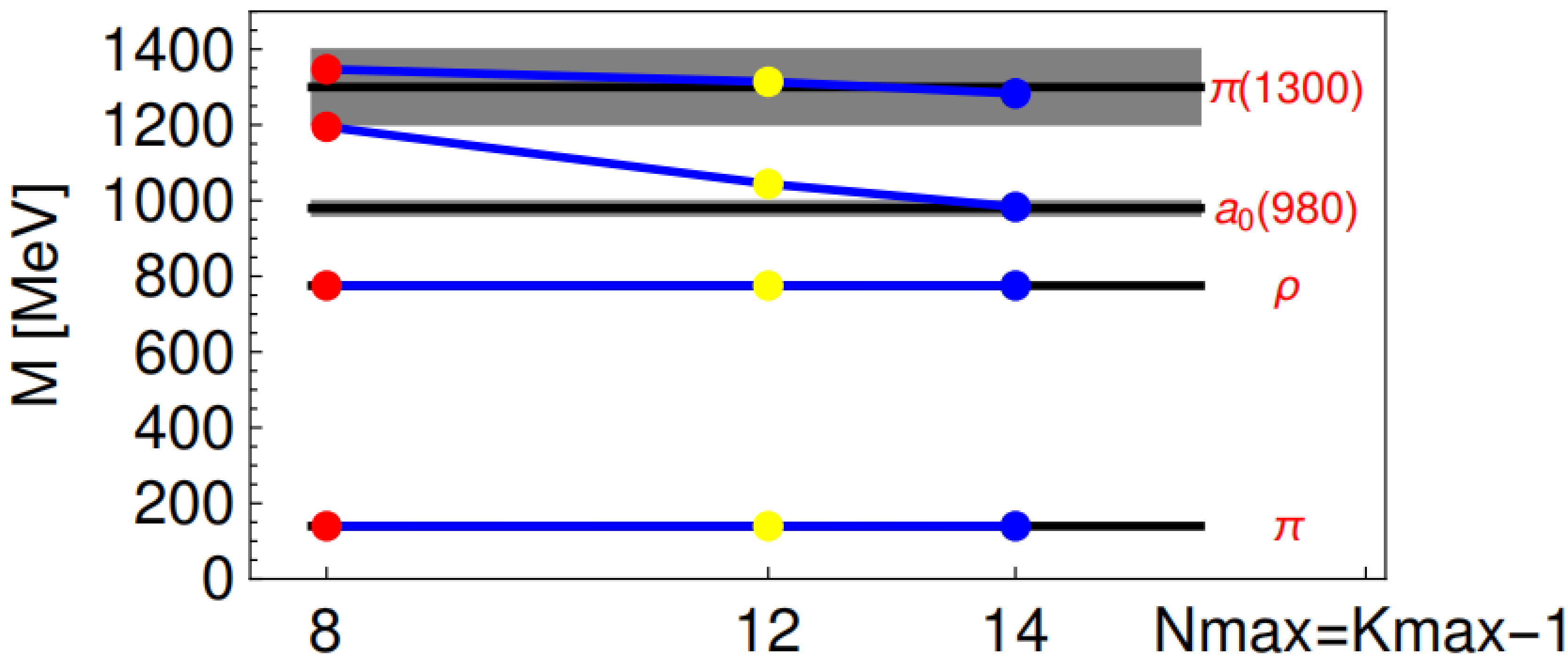
Mass spectrum



	norm$q\bar{q}$	**DC[MeV]**
π	0.492	138
ρ	0.486	129
$a_0(980)$	0.370	0
b_1 (1235)	0.306	0
a_1 (1260)	0.324	11
$\pi(1300)$	0.284	53
a_2 (1320)	0.320	0
π_1 (1400)	0.002	0
$\rho(1450)$	0.312	46

Preliminary

[Lan, et al, in preparation]



M [MeV]
0
200
400
600
800
1000
1200
1400
8
12
14
Nmax=Kmax−1
π(1300)
a0(980)
ρ
π

Pion mass, DC, Radii

$$\langle r_c^2 \rangle = -6 \frac{\partial}{\partial Q^2} F(Q^2)|_{Q^2 \to 0}$$

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

$$\langle 0|\bar{\psi}(0)\gamma^+\gamma_5\psi(0)|P(p)\rangle = \mathrm{i}p^+ f_P,$$

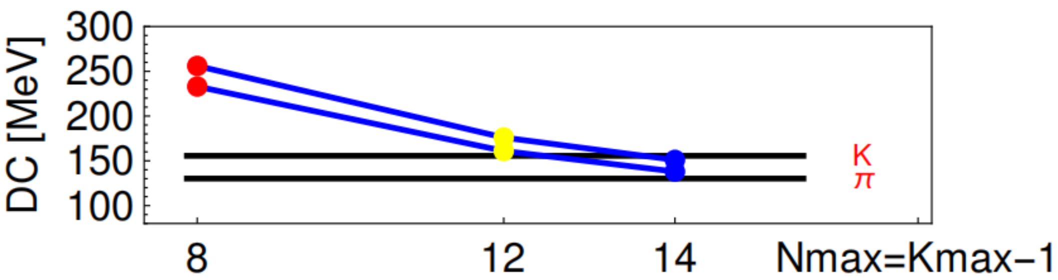
$$\langle 0|\bar{\psi}(0)\gamma^+\psi(0)|V(p,\lambda)\rangle = e_\lambda^+ M_V f_V.$$

	m_{π^+} [MeV]	m_{ρ^+} [MeV]	f_{π^+} [MeV]	f_{ρ^+} [MeV]	$\sqrt{\langle r_c^2 \rangle}\vert_{\pi^+}$ [fm]	norm $q\bar{q}$
BLFQ	**139.57**	**775.26**	**138.2**	**129.0**	**0.516~?**	**0.492**
PDG [Tanabashi, et al, PRD(2018)]	***139.57***	***775.26±0.25***	***130.2±1.7***	***221±2***	***0.672±0.008***	
BLFQ-NJL [Jia, Vary, PRC(2018)]	*139.57*	*775.23±0.04*	*202.10*	*100.12*	*0.68±0.05*	

Preliminary

BLFQ

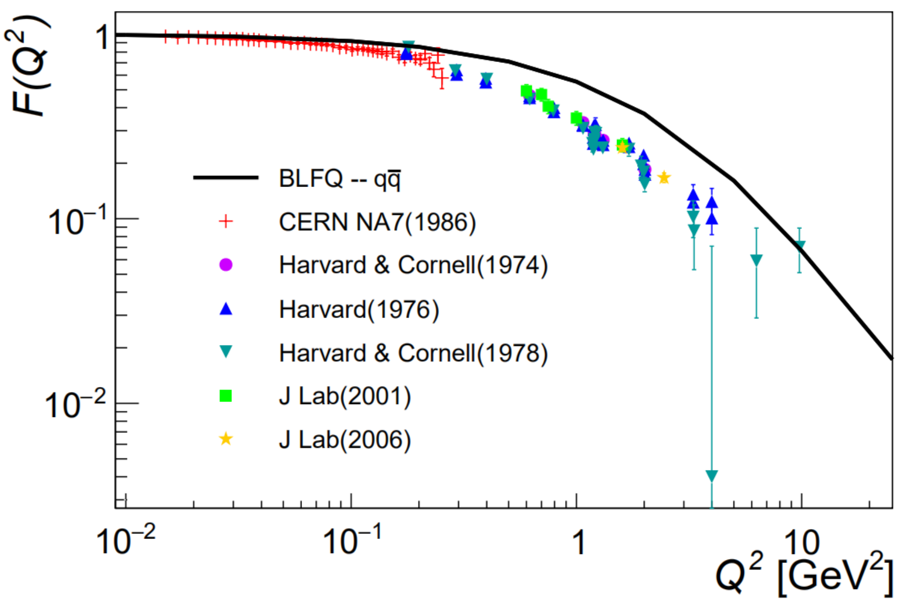
$N_{\max} = 14, K_{\max} = 15, M_J = 0$
$m_q = 0.39$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

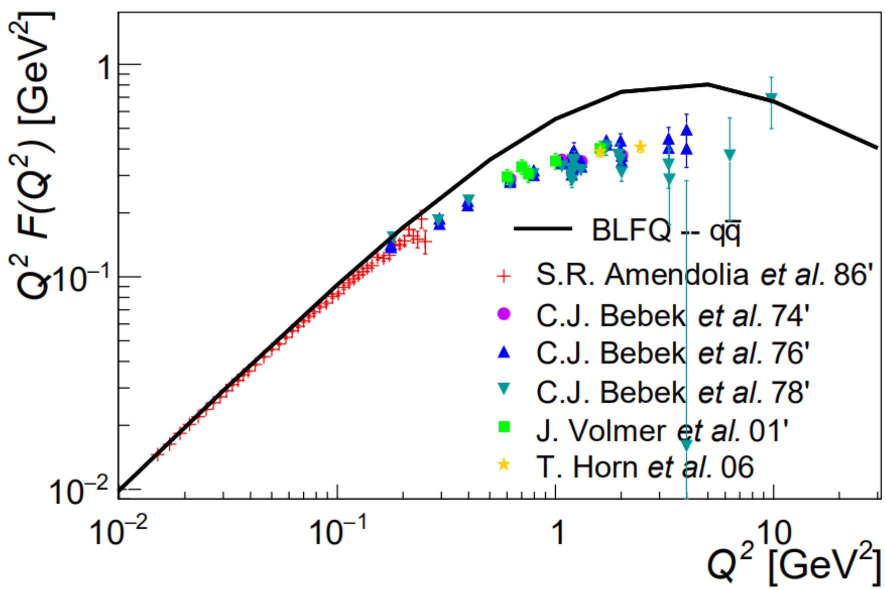


[Lan, et al, in preparation]

Pion Form Factor

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

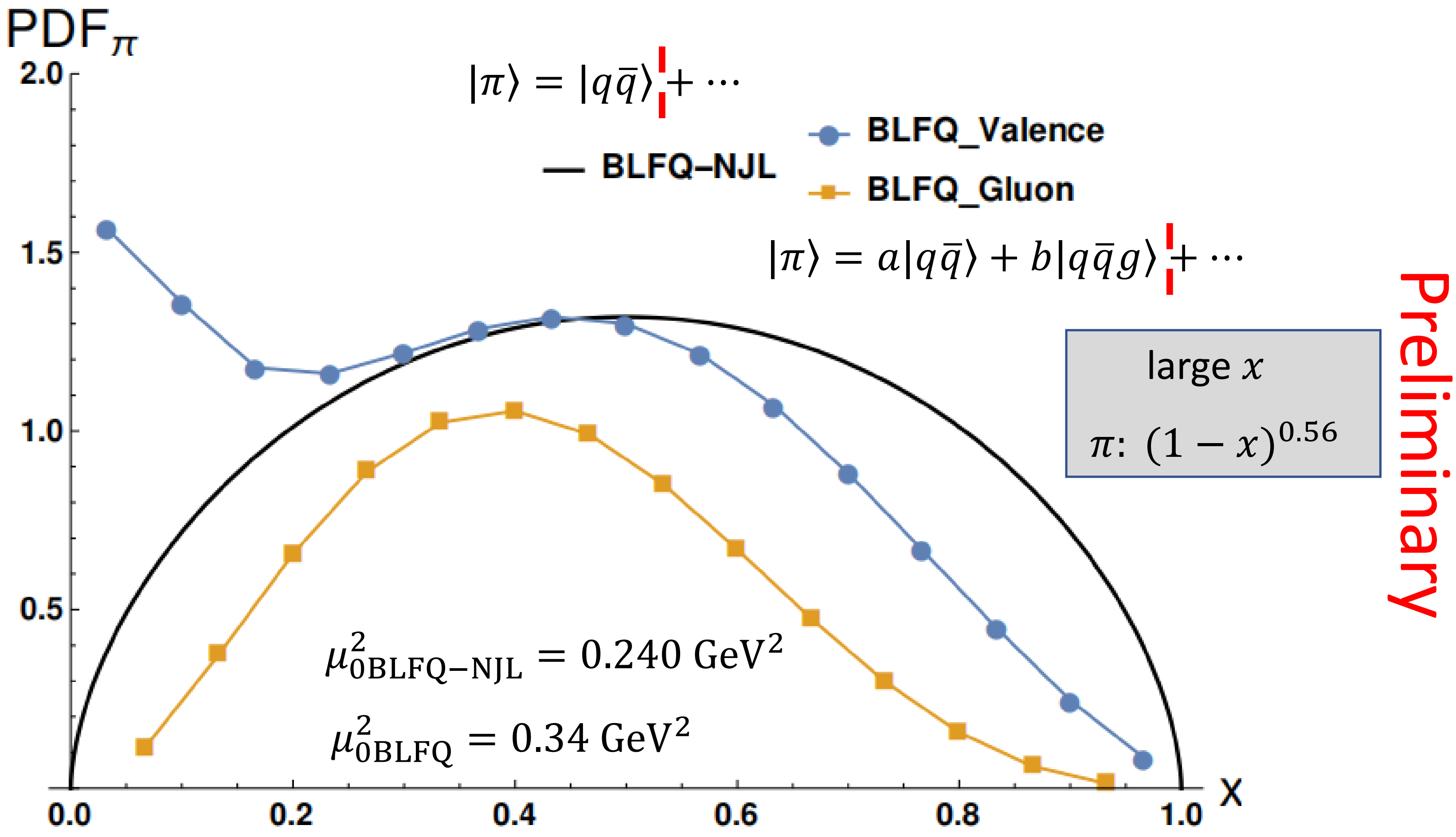




Preliminary: based on leading Fock Sector WF

[Lan, et al, in preparation]

Pion initial PDF

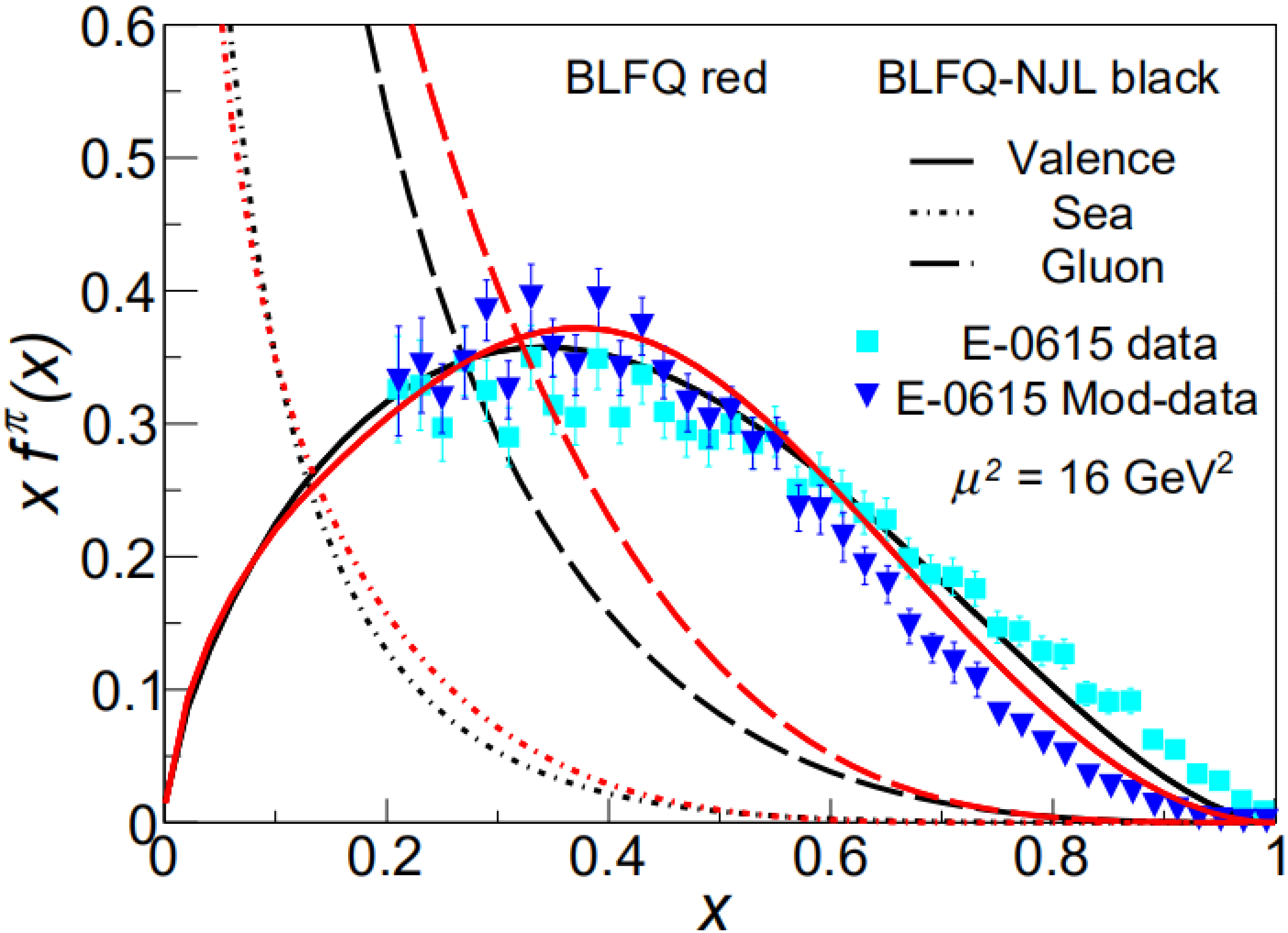


Valence close to BLFQ-NJL result at large x, more than BLFQ-NJL result at small x; we have gluon in initial PDF.

[Lan, et al, in preparation]

Pion PDF

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



Preliminary

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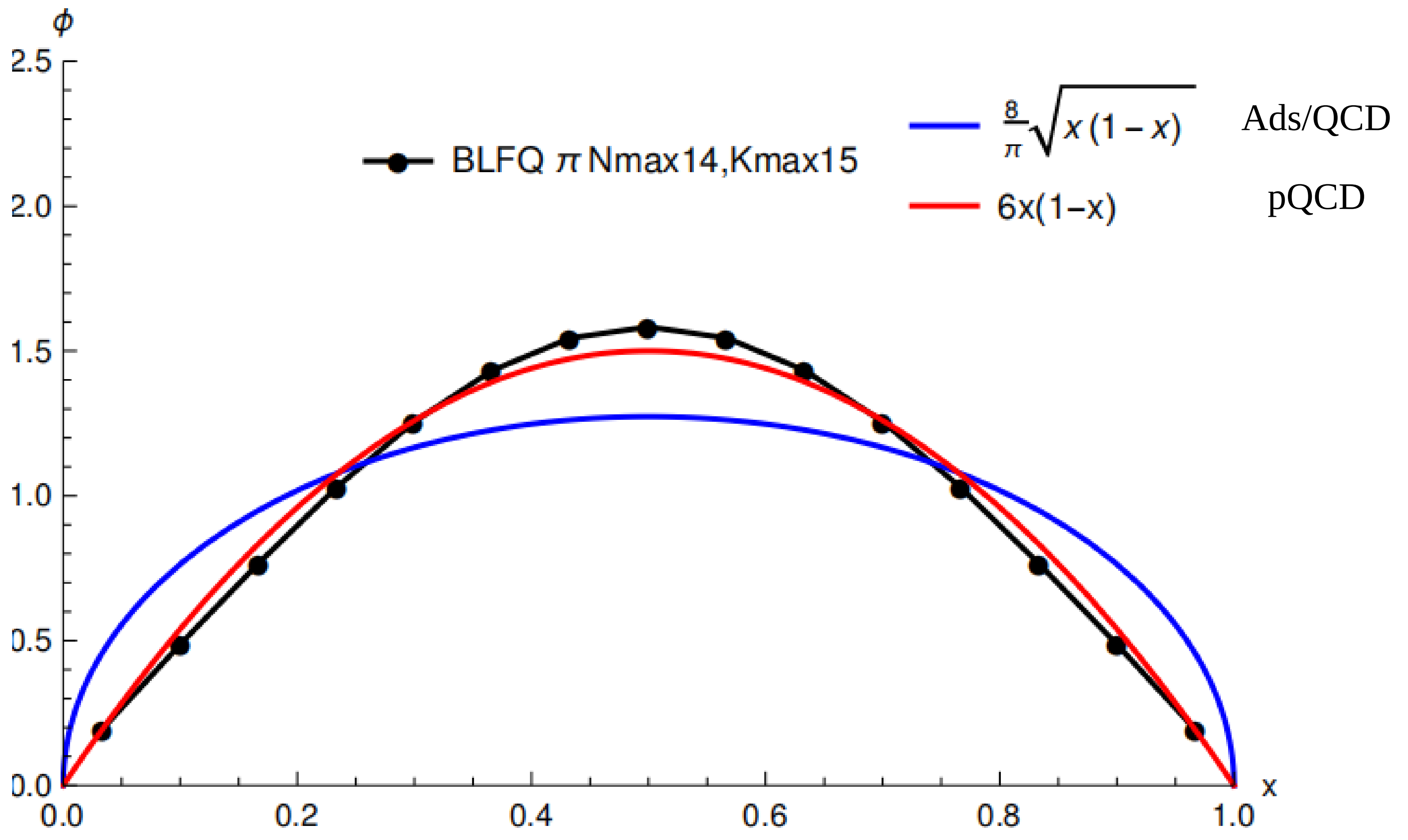
BLFQ	**0.483**	**0.421**	**0.096**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

[Lan, et al, in preparation]

Pion PDA



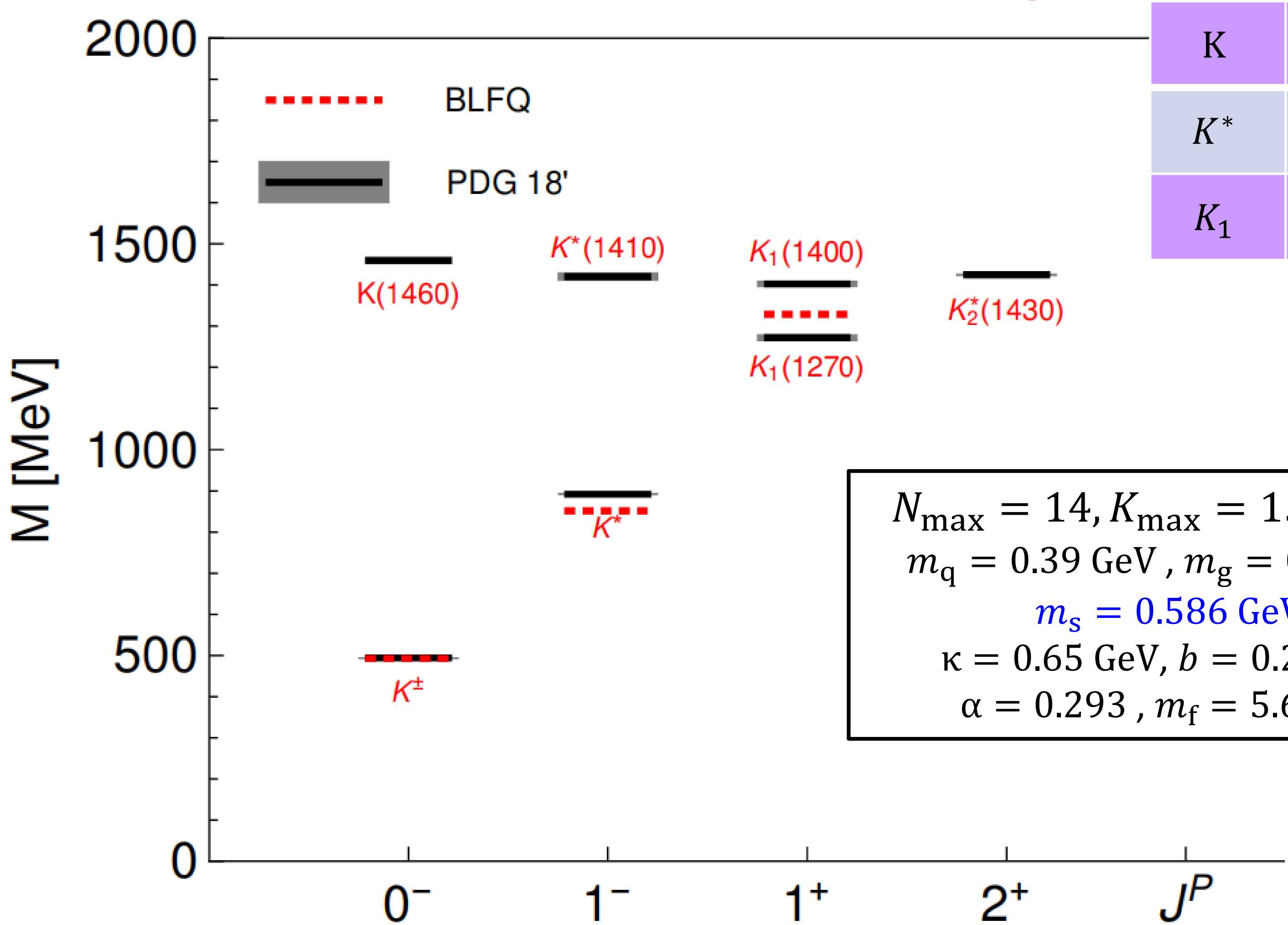
ϕ
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
BLFQ π Nmax14,Kmax15
$\frac{8}{\pi}\sqrt{x(1-x)}$
Ads/QCD
6x(1−x)
pQCD

Light meson *in progress*

Kaon *Spectrum*

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$

	Norm1	DC[MeV]
K	0.588	151
K^*	0.602	142
K_1	0.419	13



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$m_q = 0.39$ GeV , $m_g = 0.60$ GeV,
$m_s = 0.586$ GeV,
$\kappa = 0.65$ GeV, $b = 0.29$ GeV,
$\alpha = 0.293$, $m_f = 5.69$ GeV

Preliminary

[Lan, et al, in preparation]

Kaon Form Factor

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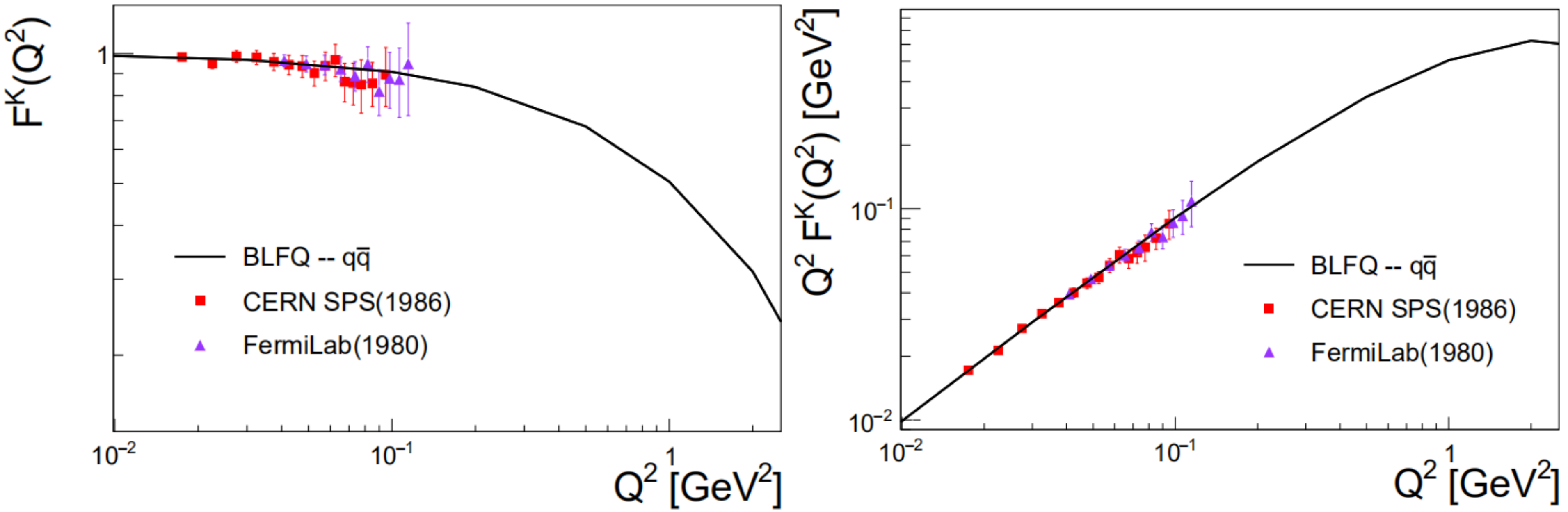


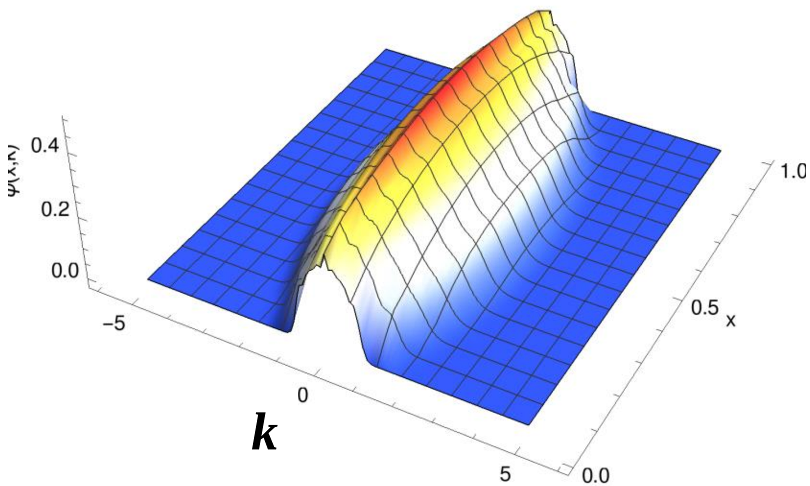
Preliminary: based on leading Fock Sector WF

[Lan, et al, in preparation]

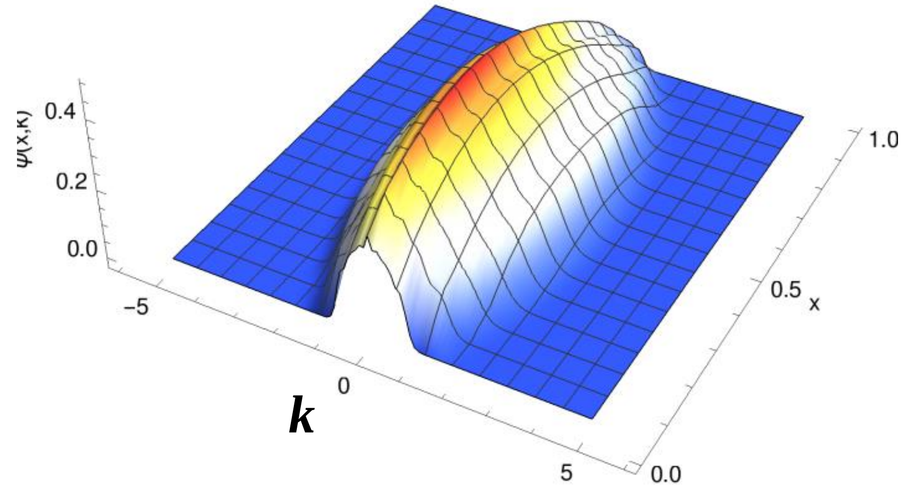
Wave function

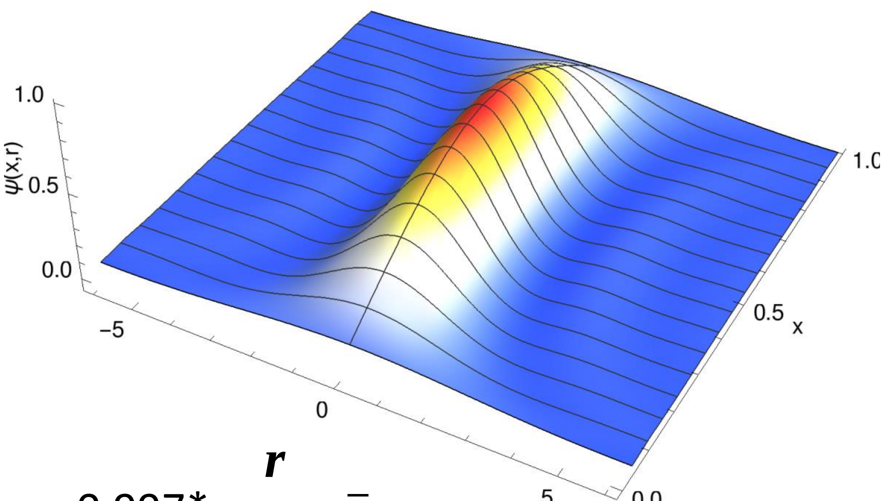
$\uparrow\downarrow - \downarrow\uparrow$

Pion

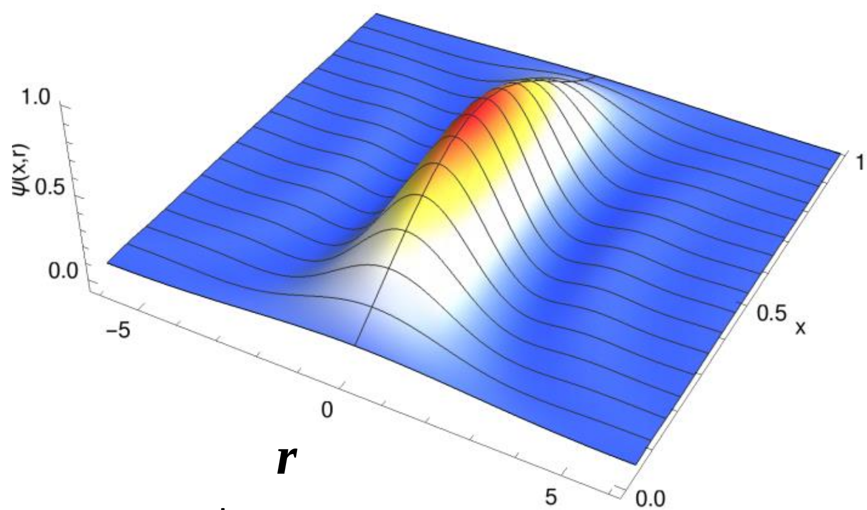


Kaon





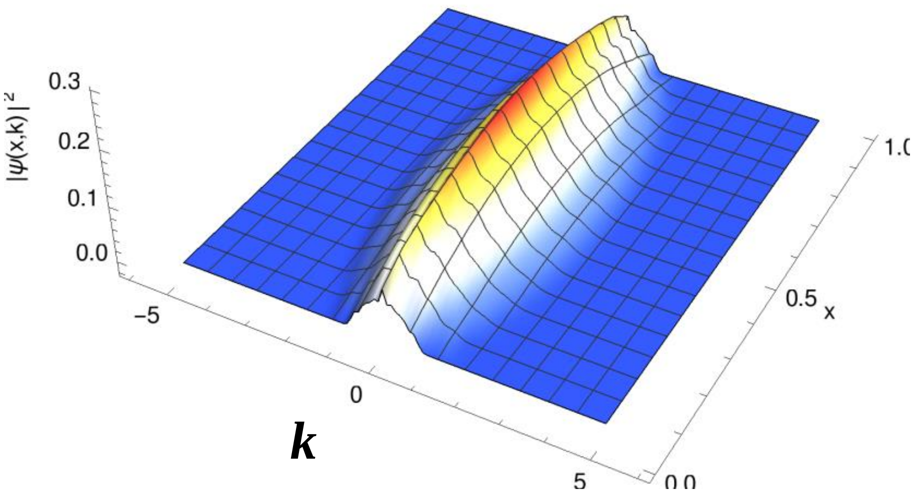
0.997*norm$q\bar{q}$

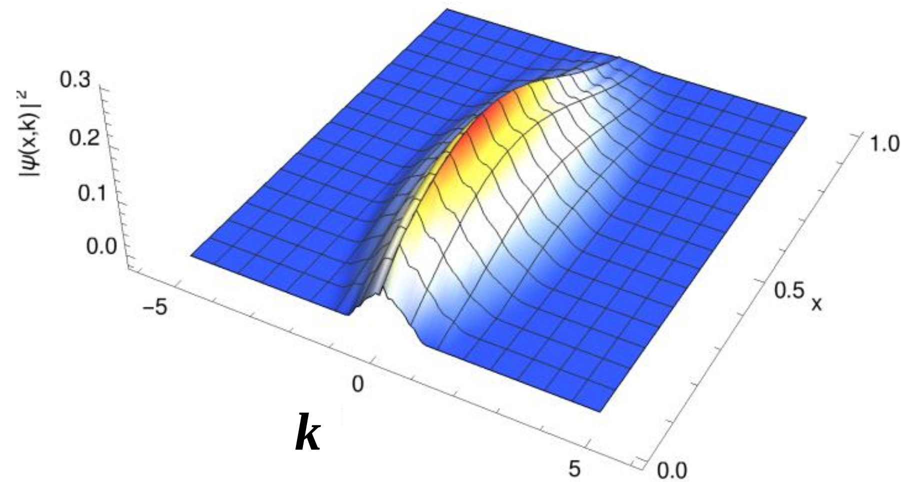


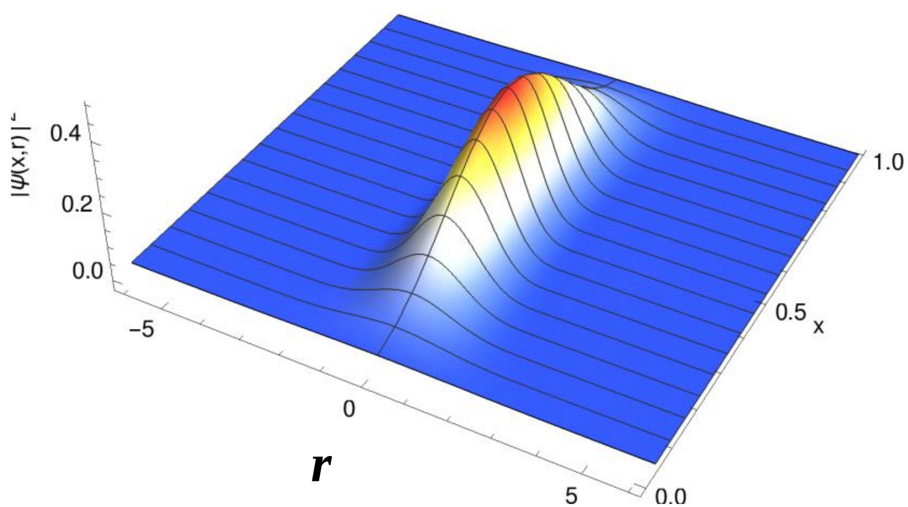
0.998*norm$q\bar{q}$

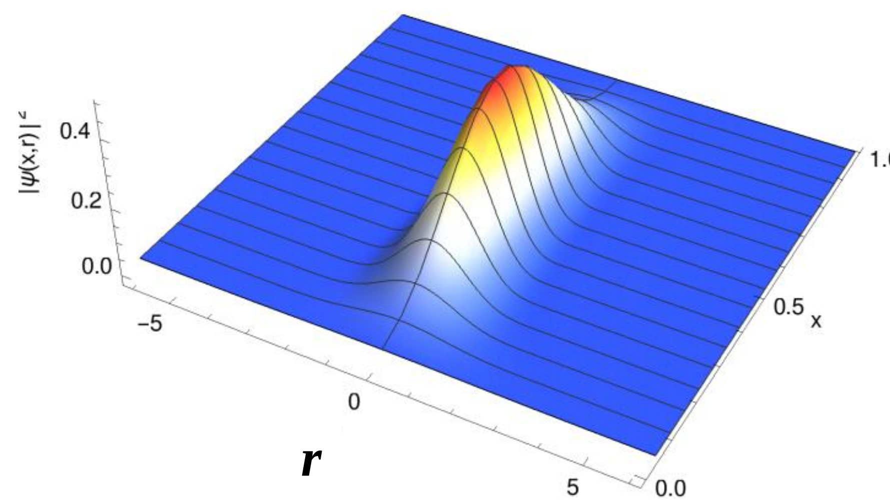
Wave function $|\psi|^2$

Pion **Kaon**

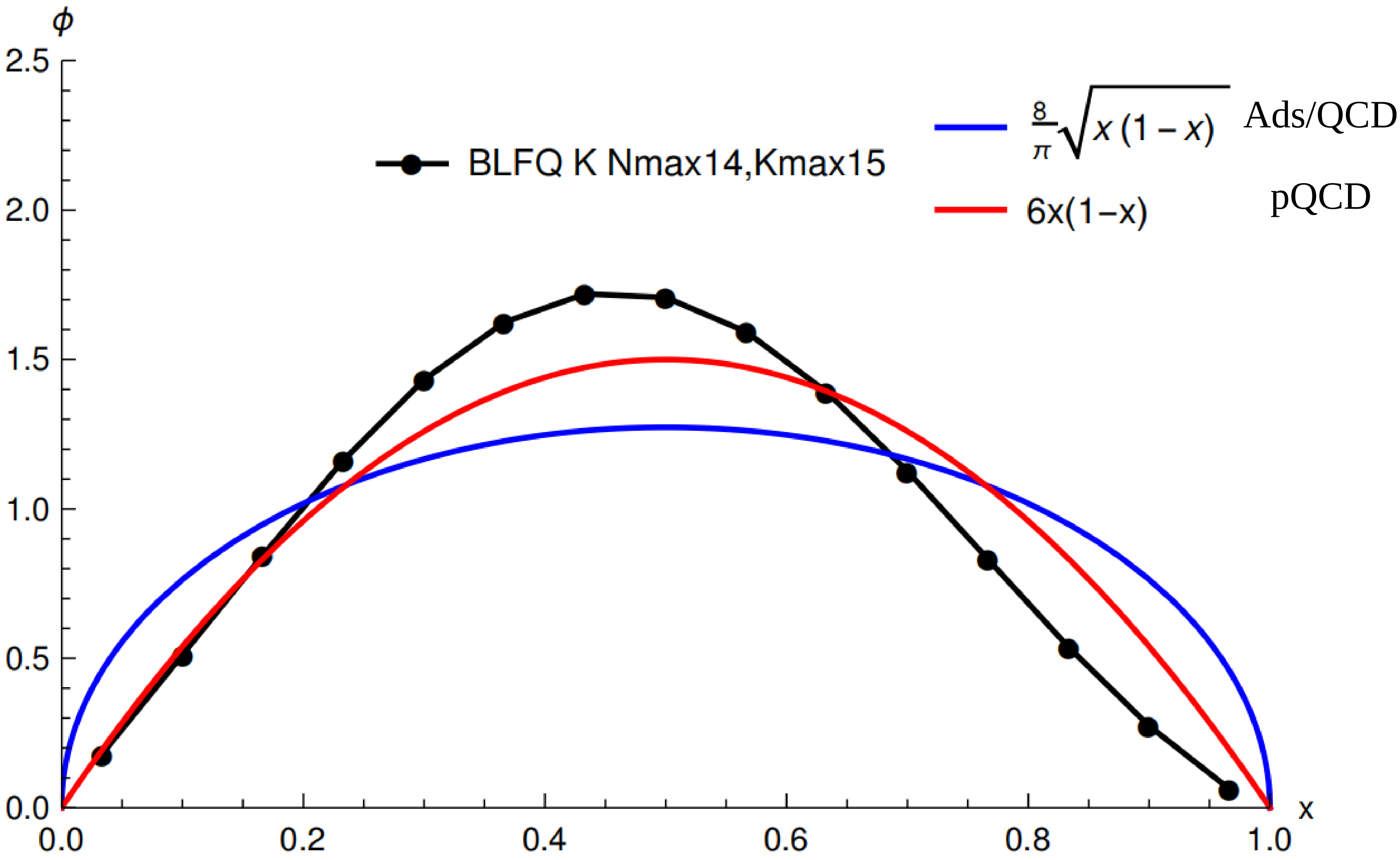








Kaon PDA



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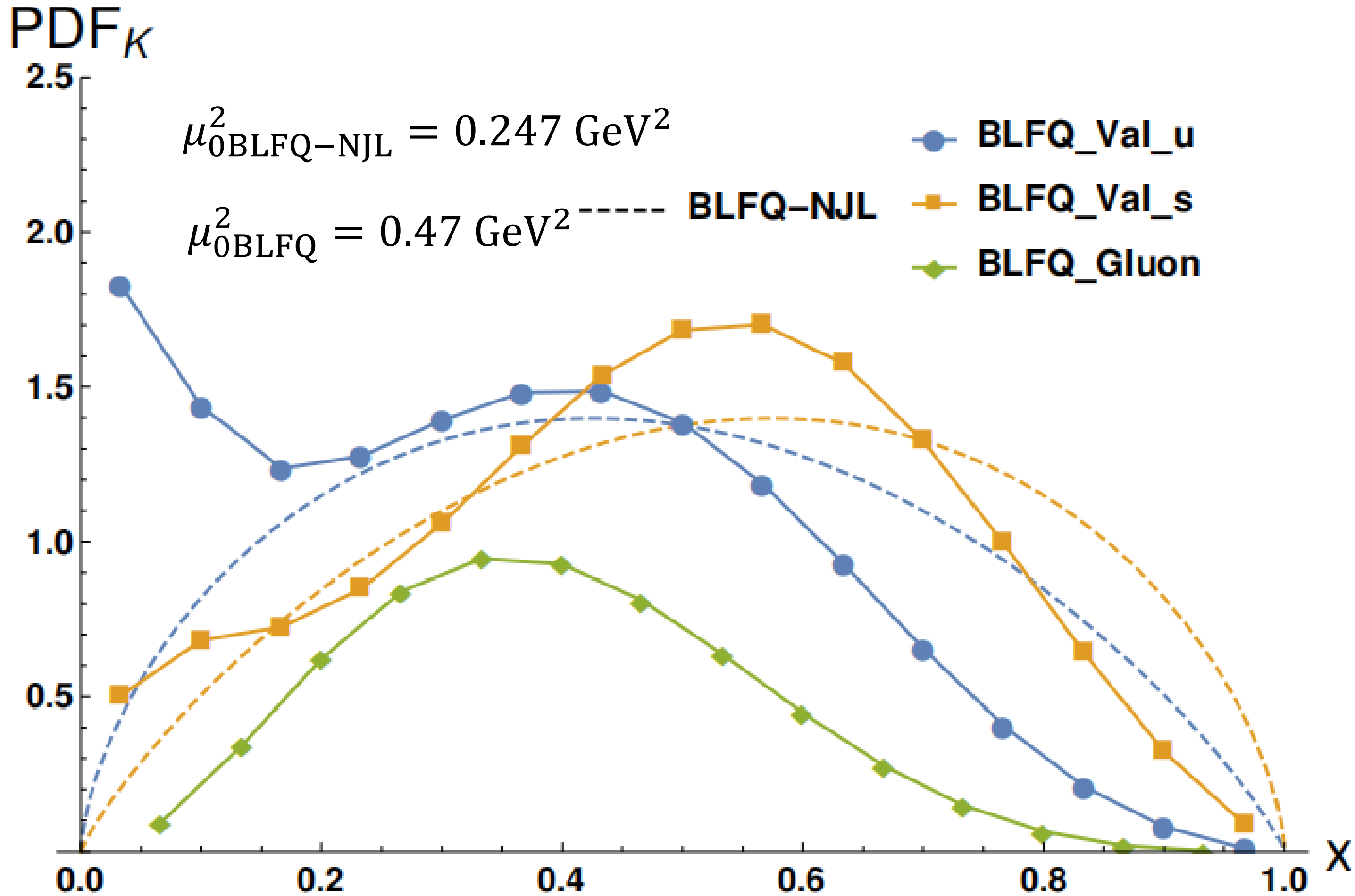
0.8
1.0
x
BLFQ K Nmax14,Kmax15
$\frac{8}{\pi}\sqrt{x(1-x)}$ Ads/QCD
6x(1−x) pQCD

Kaon initial PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **vs** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



Preliminary

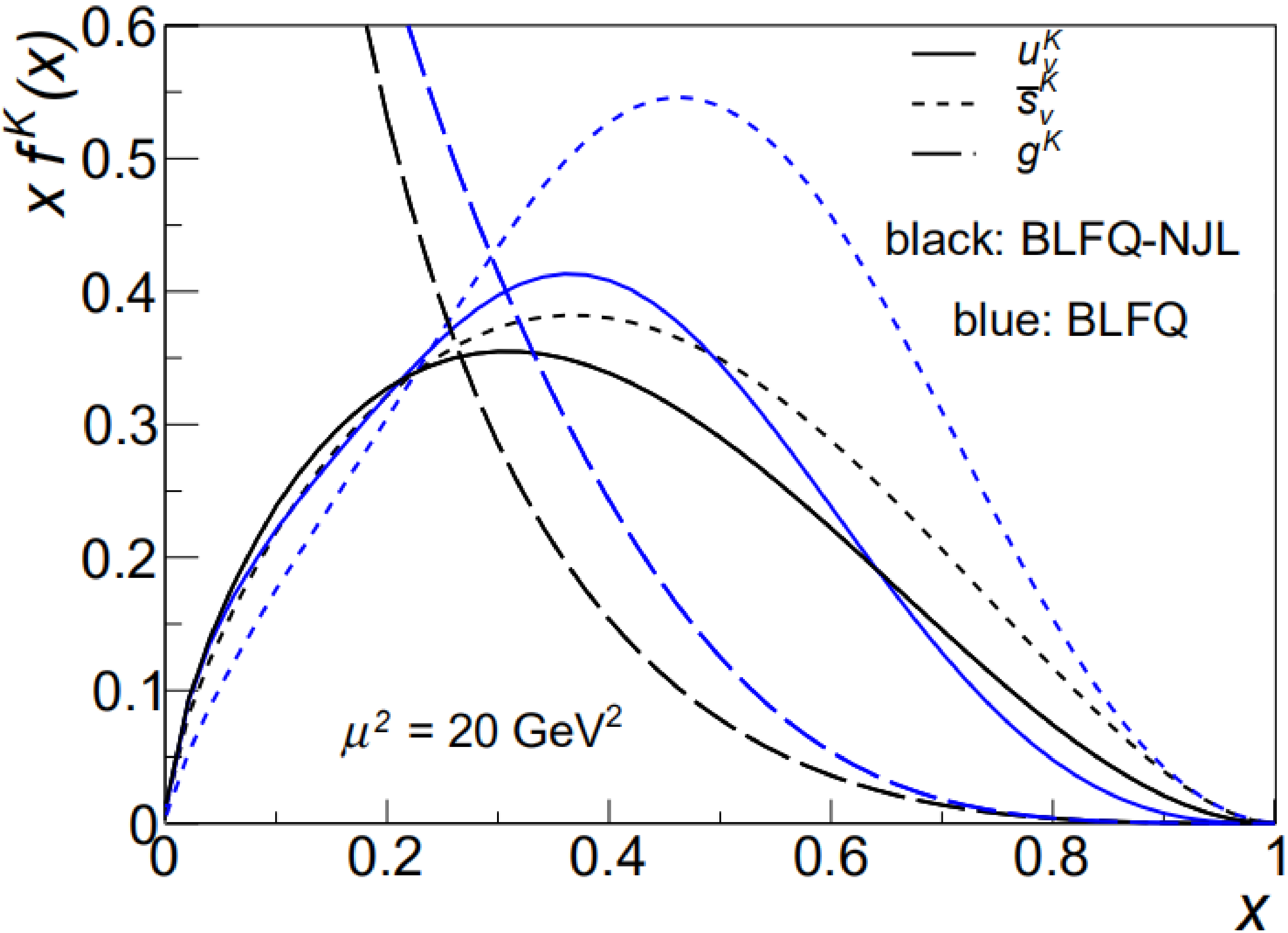
large x

$K: (1-x)^{0.96}$

[Lan, et al, in preparation]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **vs** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$

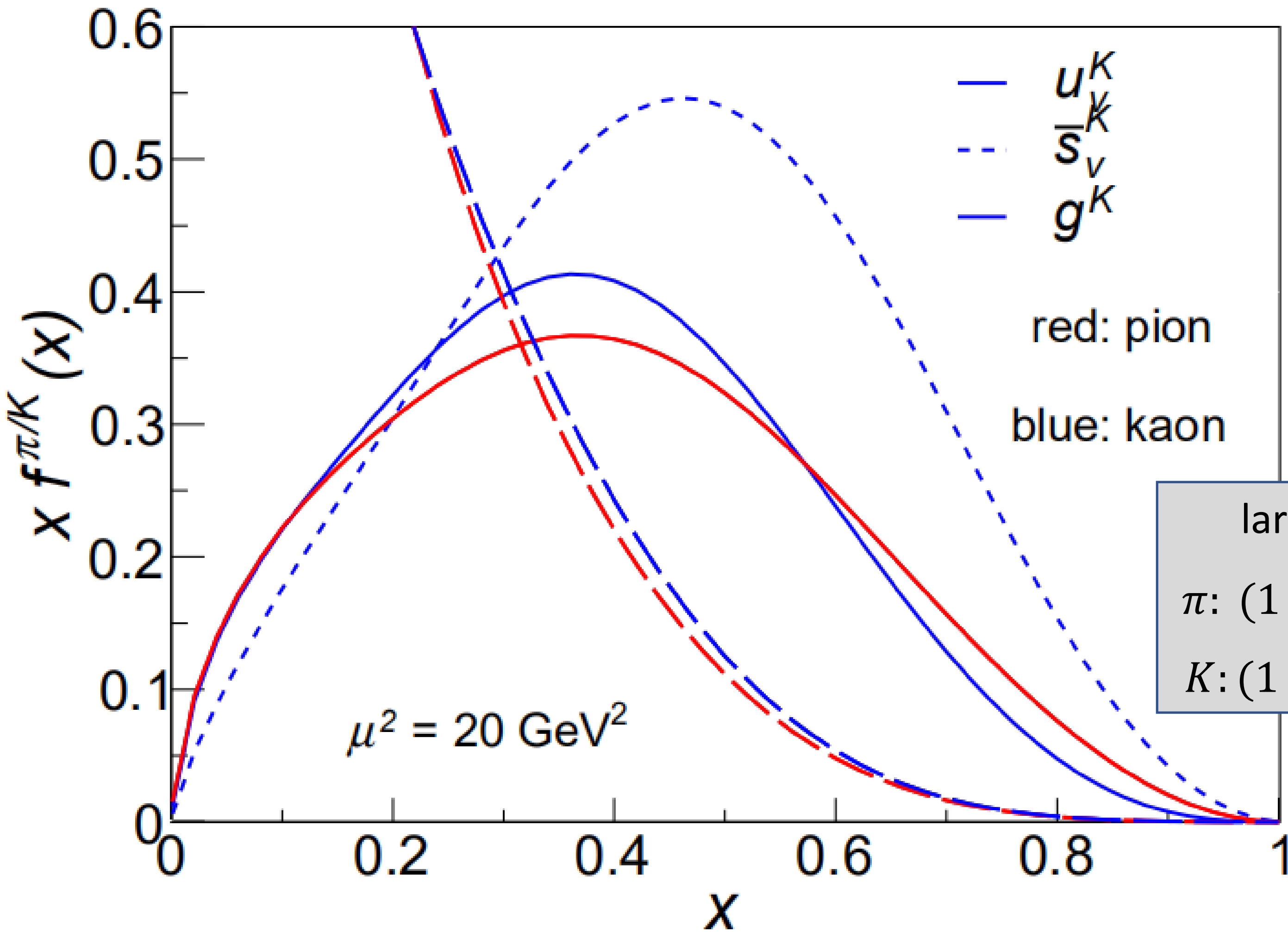


Preliminary

[Lan, et al, in preparation]

Kaon PDF

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$



large x

π: $(1-x)^{1.67}$

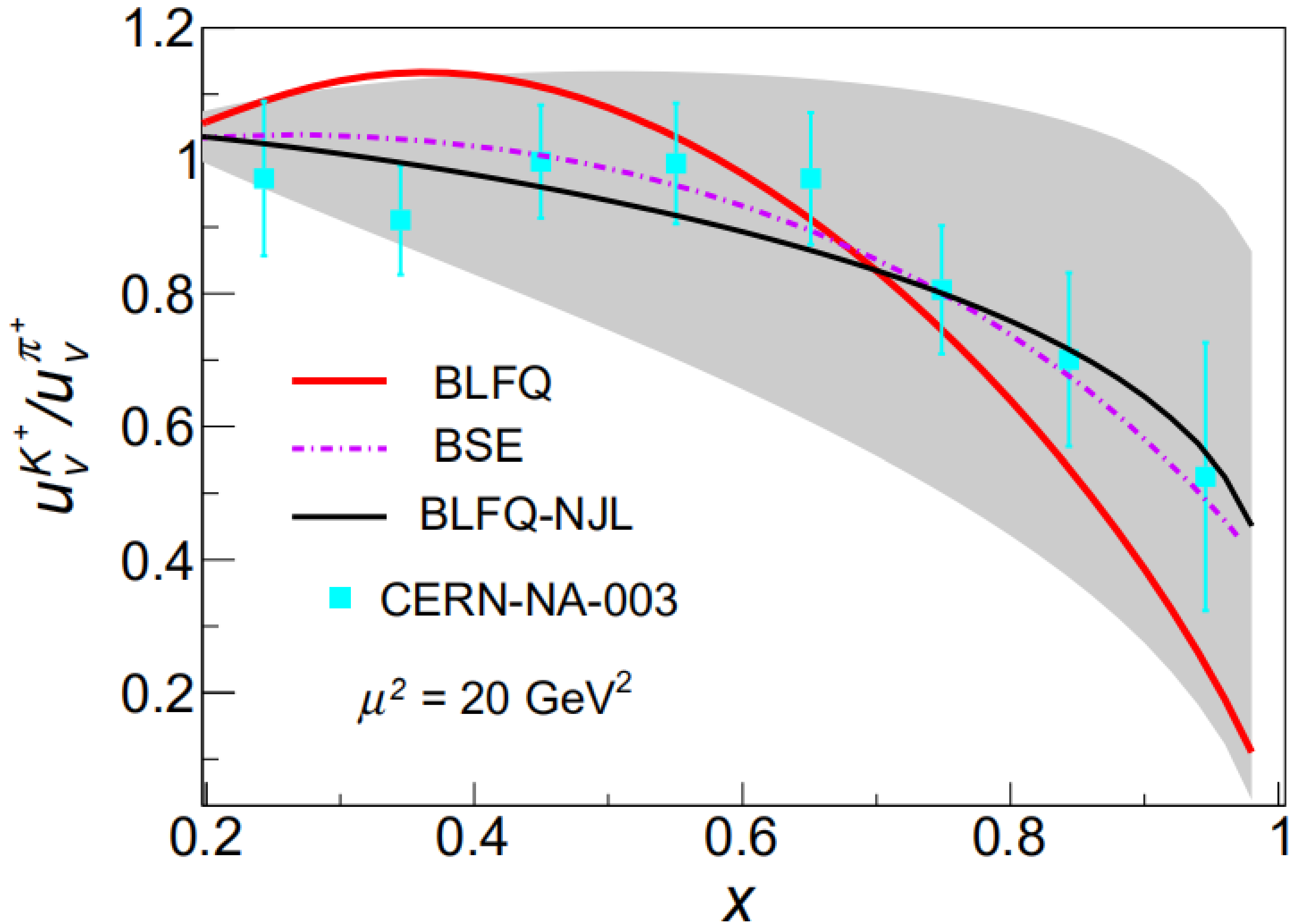
K: $(1-x)^{2.02}$

Preliminary

[Lan, et al, in preparation]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



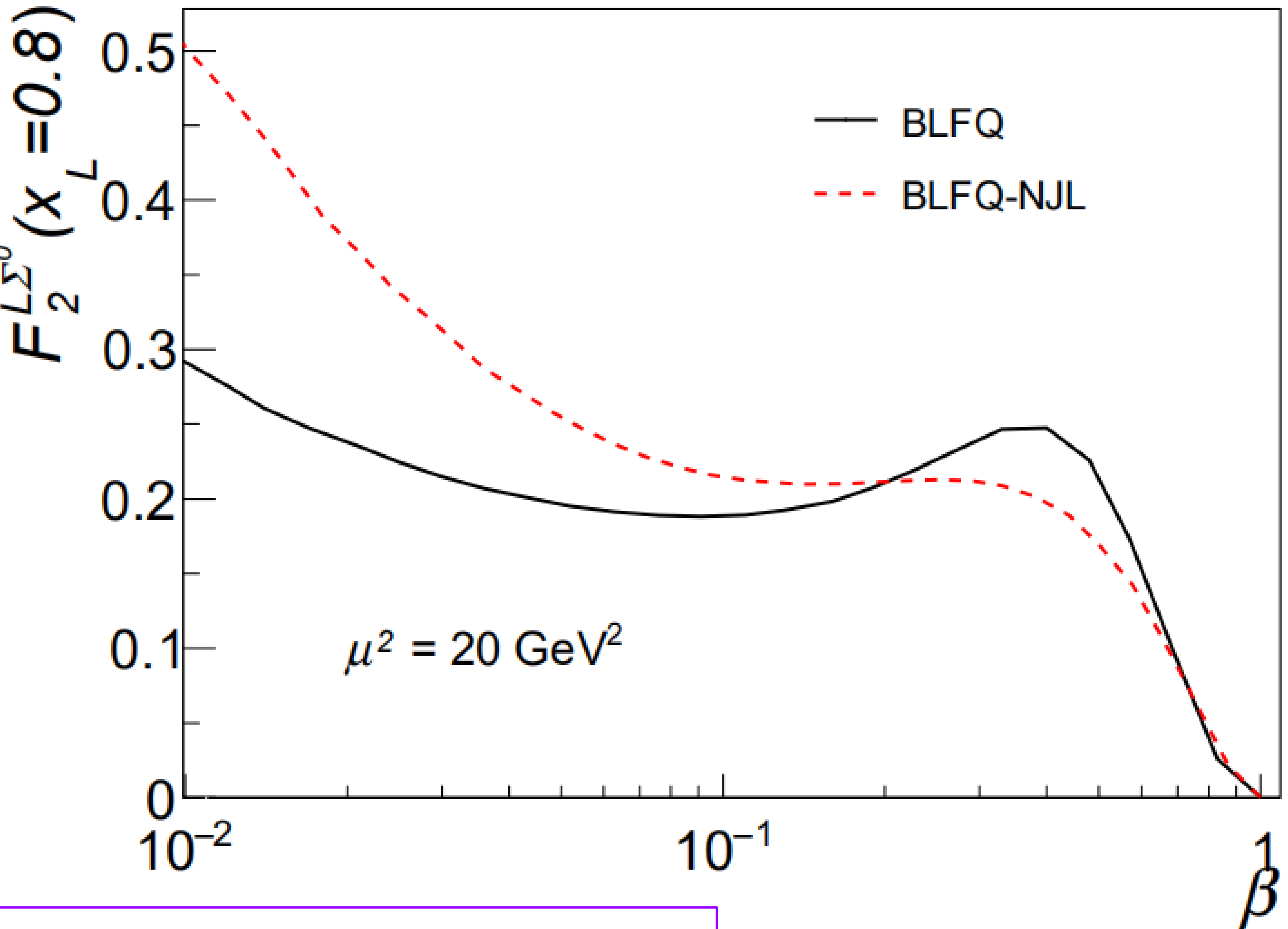
Preliminary

$\chi^2/_{\text{d.o.f}} \sim 3.4$

[Lan, et al, in preparation]

Kaon

$$ep \to e' X \Sigma^0$$



$$F_2(x,\mu^2) = \sum_i e_i^2 x f_i^K(x,\mu^2),$$

EicC ?

Conclusions

- Basis Light-front Quantization:
  - Nonperturbative approach to relativistic many-body bound states
- Light-front Hamiltonian ⟶ Wavefunction ⟶ Observables
  - Mass spectrum ⟷ structure
- Systematically expandable by including higher Fock sectors

  -$|\mathrm{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \cdots$

Thank you !

Questions/suggestions: xbzhao@impcas.ac.cn