Testing quark counting rules:
a bridge between nonperturbative and perturbative PDF approaches

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Paper in preparation;

All results without specific citations are original and will be available in that paper.

Probing nonperturbative models with QCD scattering data

Many nonperturbative models predict PDFs in a free hadron.

A PDF is not a physical observable.

It enters QCD observables through factorized approximations known up to power suppressed terms.

Our goal is to understand conditions necessary for learning about primordial PDFs from scattering data.

- ⇨ We will use the CT18NNLO global fit of proton PDFs.
- ⇨ We will explore what we can learn about quark counting rules — the classical prediction of QCD theory.
- ⇨ We study proton PDFs because they are well understood.
- ⇨ Some considerations, such as the **functional mimicry** of PDFs, also apply to the **simpler** analysis of pion PDFs.

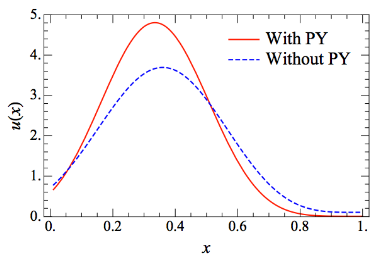
Proton and Pion PDFs (examples)

PDFs in Nonperturbative approaches @ μ_0^2

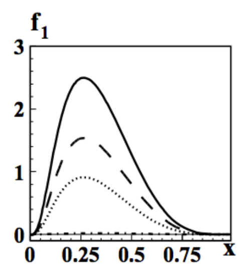
Phenomenological PDFs @ $Q^2 >> \mu_0^2$

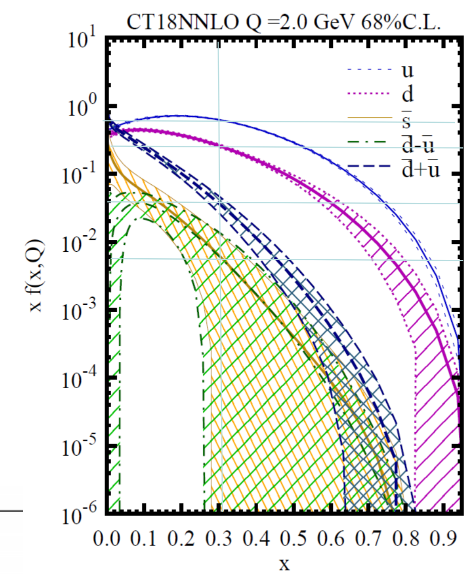
Proton

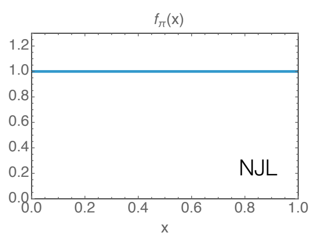
MIT Bag



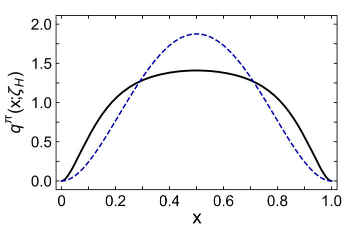
Light-Cone CQM



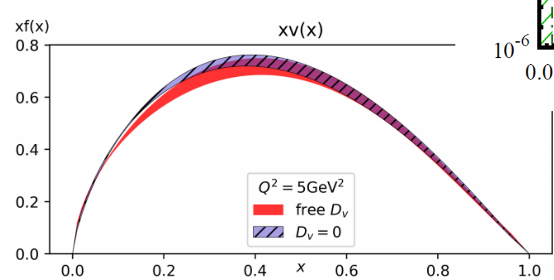




DSE



Pion



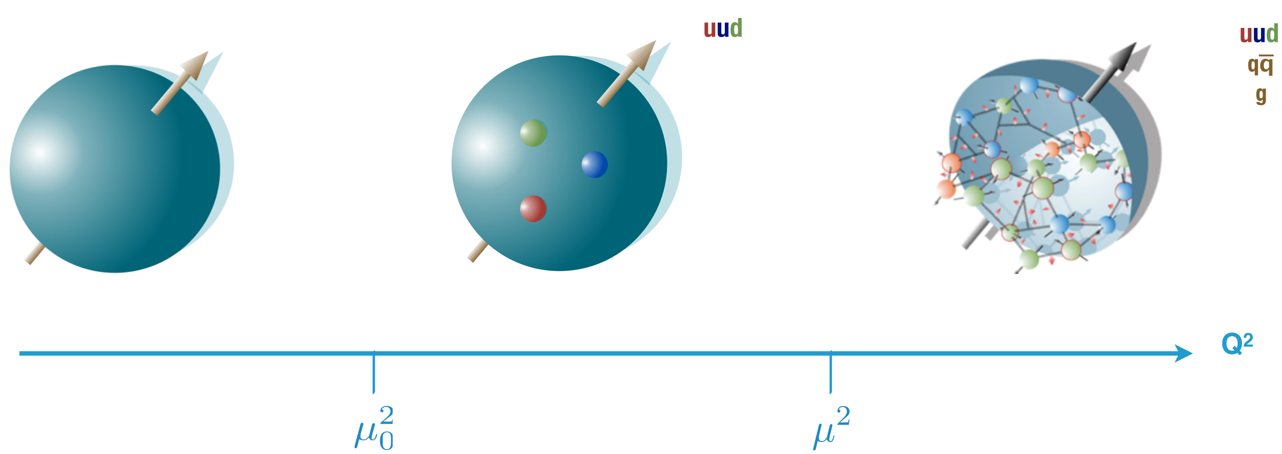
xFitters

PDFs in nonperturbative QCD

- **at hadronic scale μ_0^2 < 1GeV²**
  - ⇨ prefactorization picture
  - ⇨ nonperturbative dynamics
  - ⇨ model's degrees of freedom

Phenomenological PDFs

- **at factorization scale μ^2 > 1GeV²**
  - ⇨ quasi-free partonic degrees of freedom
  - ⇨ defined in the MSbar scheme
  - ⇨ leading-power approximation to full dynamics

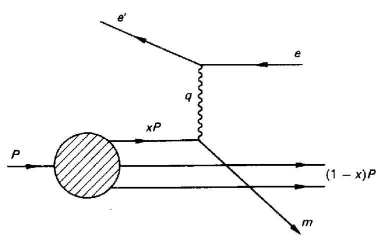


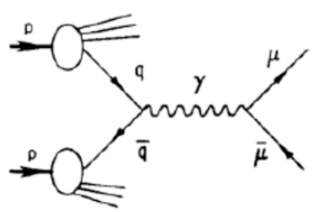
PDFs in nonperturbative QCD

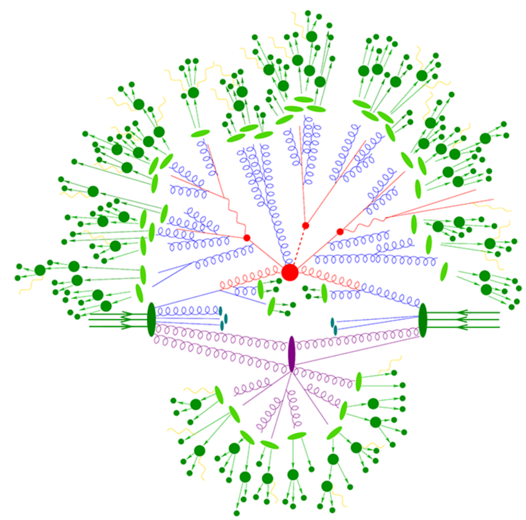
Phenomenological PDFs

How to relate the x dependence of the perturbative and nonperturbative pictures?

⇨ we can learn about nonperturbative dynamics by comparing predictions to data for the simplest scattering processes (DIS and DY)







⇨ pheno PDFs are determined from analyzing many processes with complex scattering dynamics

μ_0^2 $\qquad$ $\mu^2 \sim Q^2$

Definition of PDFs

Quark–quark correlation matrix

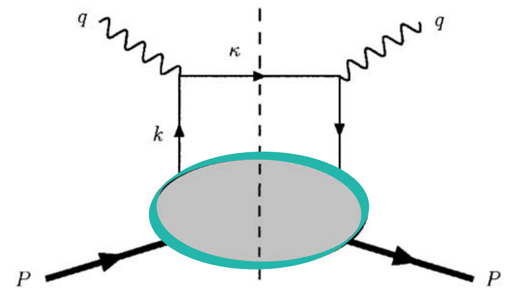
$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3 \boldsymbol{P}_X}{(2\pi)^3 2E_X} (2\pi)^4 \delta^4(P-k-P_X) \langle PS|\psi_j(0)|X\rangle\langle X|\psi_i(0)|PS\rangle.$$

Bjorken regime

$$\langle\gamma^\mu\rangle \equiv \int \frac{d^4k}{(2\pi)^4}\,\delta\left(x-\frac{k^+}{P^+}\right)\mathrm{Tr}(\gamma^\mu\Phi)$$
$$= \int \frac{d\tau}{2\pi} e^{i\tau x}\langle PS|\bar\psi(0)\gamma^\mu\psi(\tau n)|PS\rangle = 2f(x)P^\mu,$$

leading-twist

DIS *handbag* diagram for the structure function



$\overline{\mathrm{MS}}$ Definition of PDFs

Quark–quark correlation matrix

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3 \boldsymbol{P}_X}{(2\pi)^3 2E_X} (2\pi)^4 \delta^4(P-k-P_X) \langle PS|\psi_j(0)|X\rangle\langle X|\psi_i(0)|PS\rangle,$$

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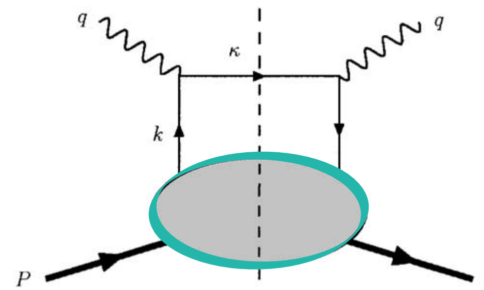
leading-twist

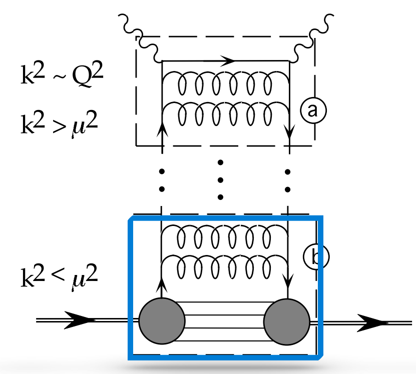
Collinear factorization: *virtuality of the photon gives HARD scale*

$$F(x_B,Q^2) = \sum_a \int_{x_B}^1 \frac{dx}{x} f_{a/A}(x,\mu^2)\,\hat F_{2,a}\left(\frac{x_B}{x},\frac{\mu^2}{Q^2}\right) + \mathcal{O}(M/Q)$$

mathematical object $\longrightarrow$ $f_{a/A}(x,\mu^2) = \overline{MS}$ PDFs

DIS *handbag* diagram for the structure function



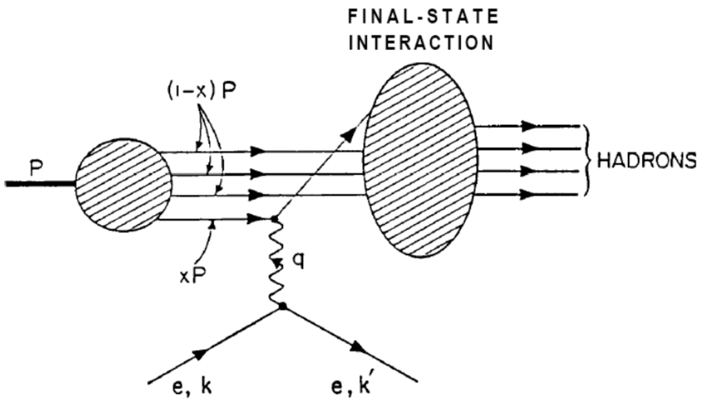


$+\mathcal{O}(M/Q)$

Testing Quark Counting Rules

Complementary testing to first principles: support, endpoints, positivity

QCD-based large-x behavior



Threshold limit in DIS ⇨ x → 1

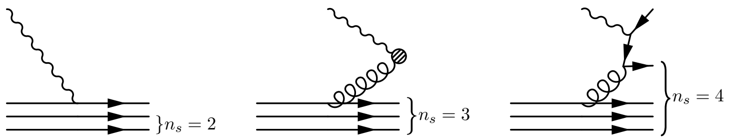
Proved for exclusive and inclusive *processes*

Brodsky and Farrar, PRL31 and PRD11
Ezawa, Nuovo Cim. A23
Berger and Brodsky, PRL42.

$$F_2(x_B) \xrightarrow[x_B \to 1]{} (1-x_B)^{2p-1+2|\lambda_q-\lambda_A|}$$

p=#spectators

extended to PDFs — without λ [Ball et al, Eur.Phys.J.C 76]



$$f(x) \xrightarrow[x_B \to 1]{} (1-x_B)^3 \qquad (1-x_B)^5 \qquad (1-x_B)^7$$

01 Can we see the evidence for QCRs in data described by pQCD?

02 Can we test the effective power?

03 Is the effective power the same

- in $F_2(x, Q^2)$, u_V and d_V?
- for all processes?

Can we see the evidence for QCRs in data on F_2 described by pQCD?

Pheno global fits from CT18NNLO `[Hou et al, 1912.10053]`

Effective exponent for x →1

$$A_2^{\text{eff}}(F_2) \equiv \frac{\partial \ln\left(F_2(x,Q)\right)}{\partial \ln(1-x)}$$

CT18 and all main global fits assume

$$f_{a/A}(x, Q_0^2) = x^{A_{1,a}}(1-x)^{A_{2,a}} \times \Phi_a(x)$$

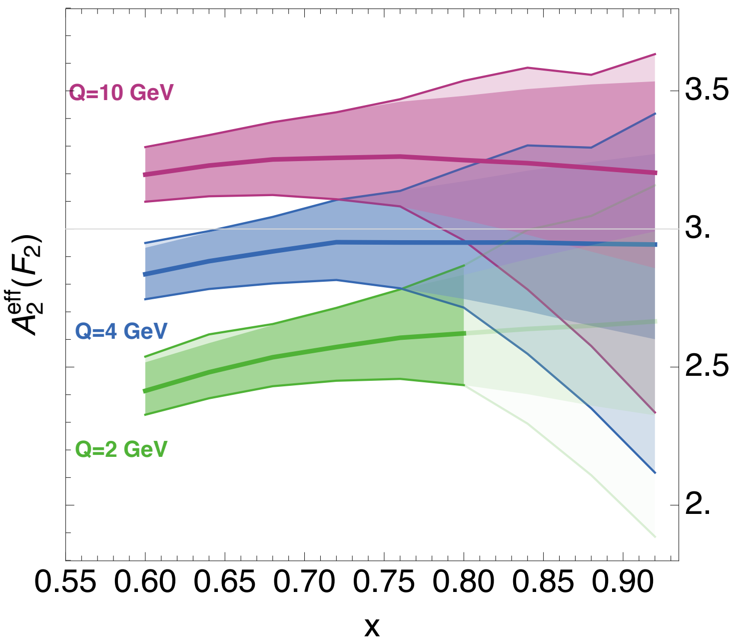
$\Phi_{a,n}$ a Bézier curve for CT18
≡smooth pol. of degree n

Structure Function follows QCRs within uncertainties.

Non-negligible running with Q^2

Bjorken regime defined for particular W^2 and Q^2 regions
($Q^2 \to \infty$ and $W^2 > m_p^2 + (1-x)/x\, Q^2$)

CT18 NNLO, parametrization dependence



Two sources of uncertainties showed:
1. Hessian errors from fit output
2. Parametrization choice

NNLO Pheno PDFs:
experiments compatible with multiple parametric forms at large x

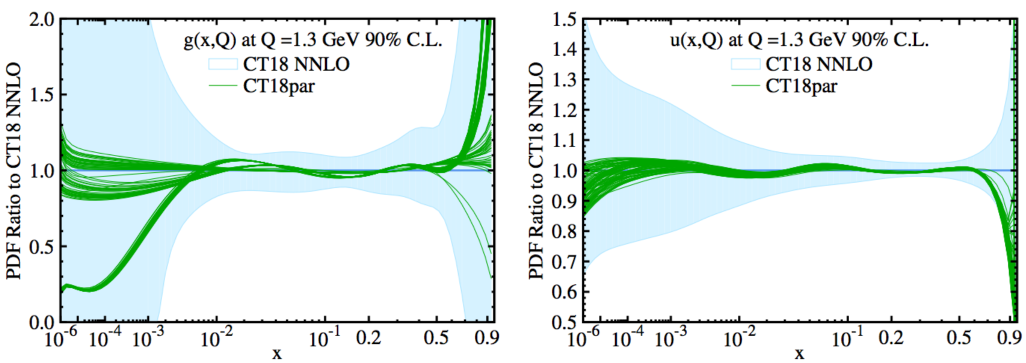
Data do not constrain the x> 0.8 region

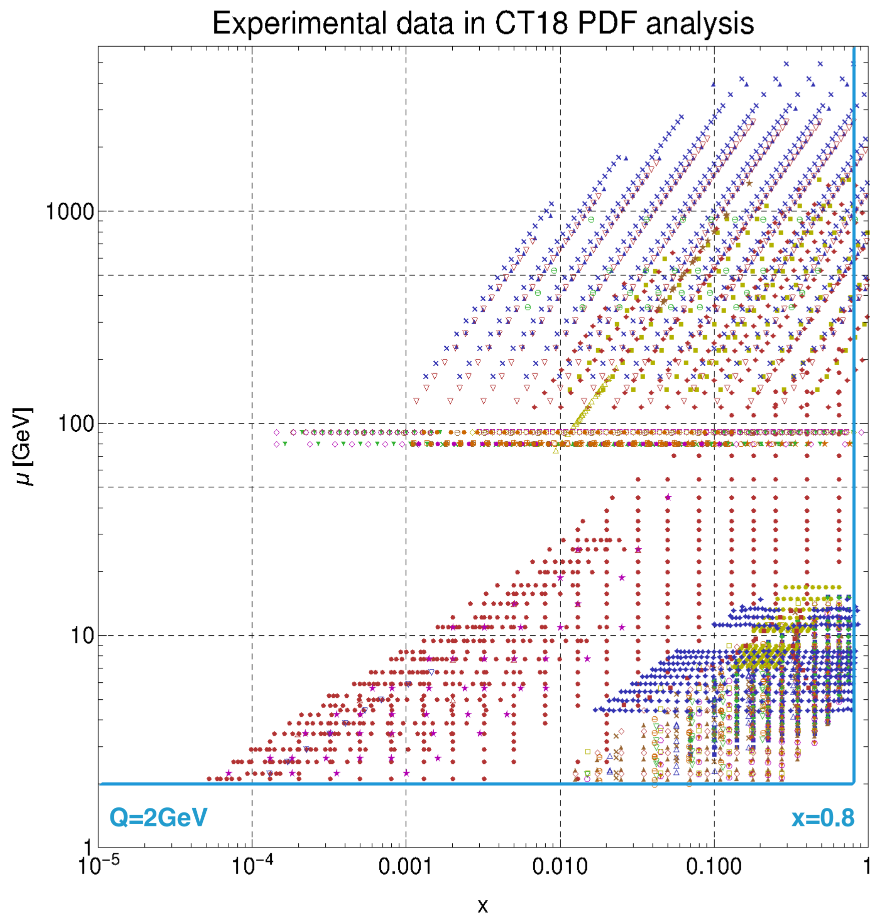
Q_0=1.3 GeV is the starting evolution scale

1. Hessian error propagation from fit output
2. Choice of functional form:
   - there is more than one representation with equally good agreement
   - dominant error for valence quarks

CT18: W^2>15GeV^2 , fixed order, Q^2>4GeV^2

[Hou et al, 1912.10053]





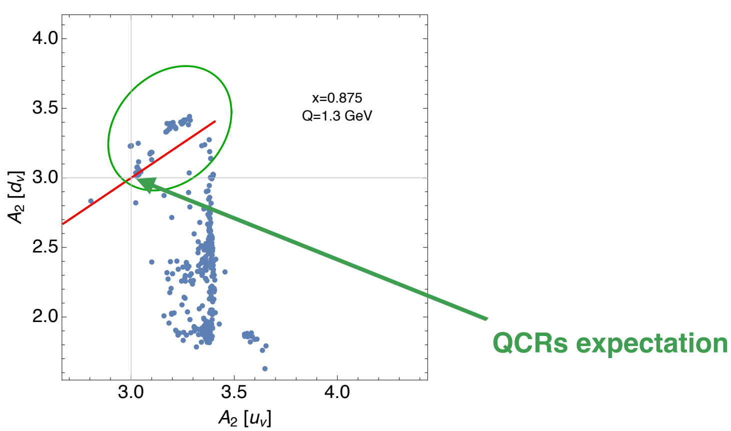
Can we see the evidence for QCRs in u_V and d_V?

Repeat the fit with N~300 functional forms

Variations of less than .5% in χ^2

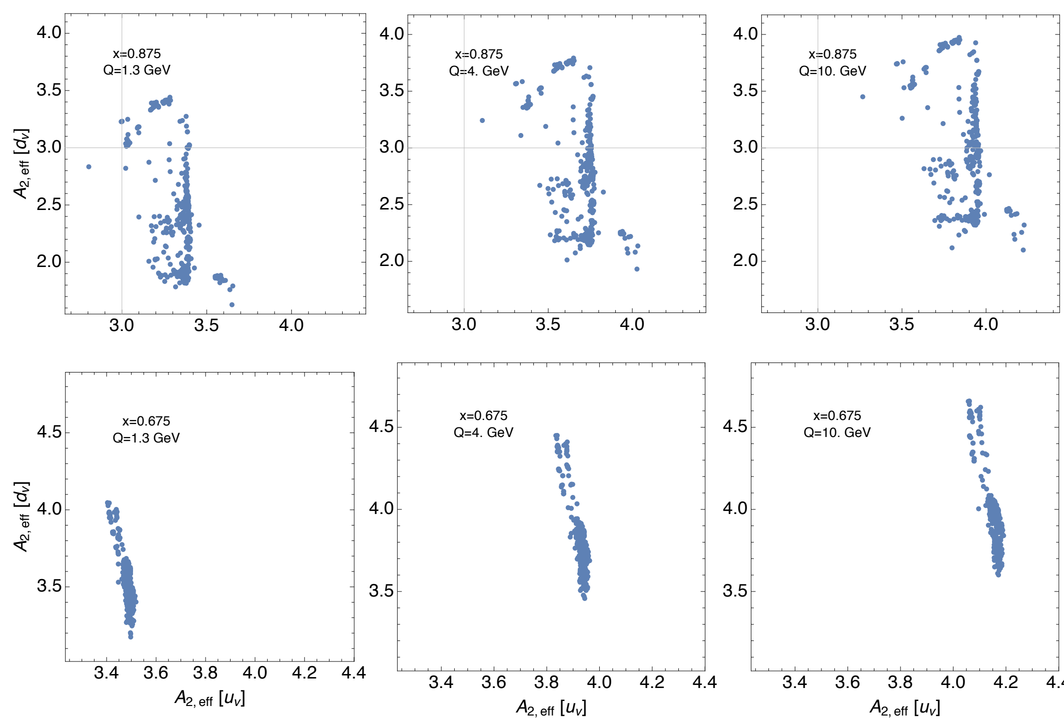
Extrapolation in the region x>0.75-0.8

1. Hessian error propagation from tabulated A_2
2. Hessian error propagation for $A_{2,eff}$ from CT18NNLO
3. **Scatter plot for the central fits for N parametrizations**



Running expected in pQCD

(e.g. Eur.Phys.J.C 76, Phys. Lett. B112)



Can we see the evidence for QCRs in gluon and $\bar{u}+\bar{d}$?

Repeat the fit with N~300 functional forms

Variations of less than .5% in χ^2

Extrapolation in the region x>0.75-0.8

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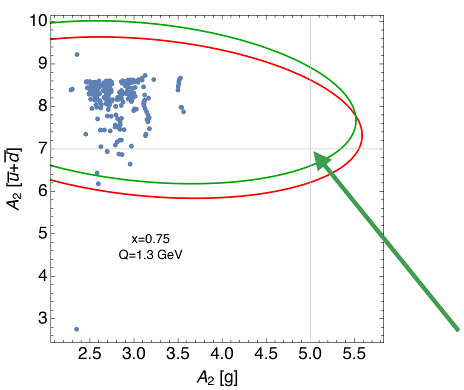
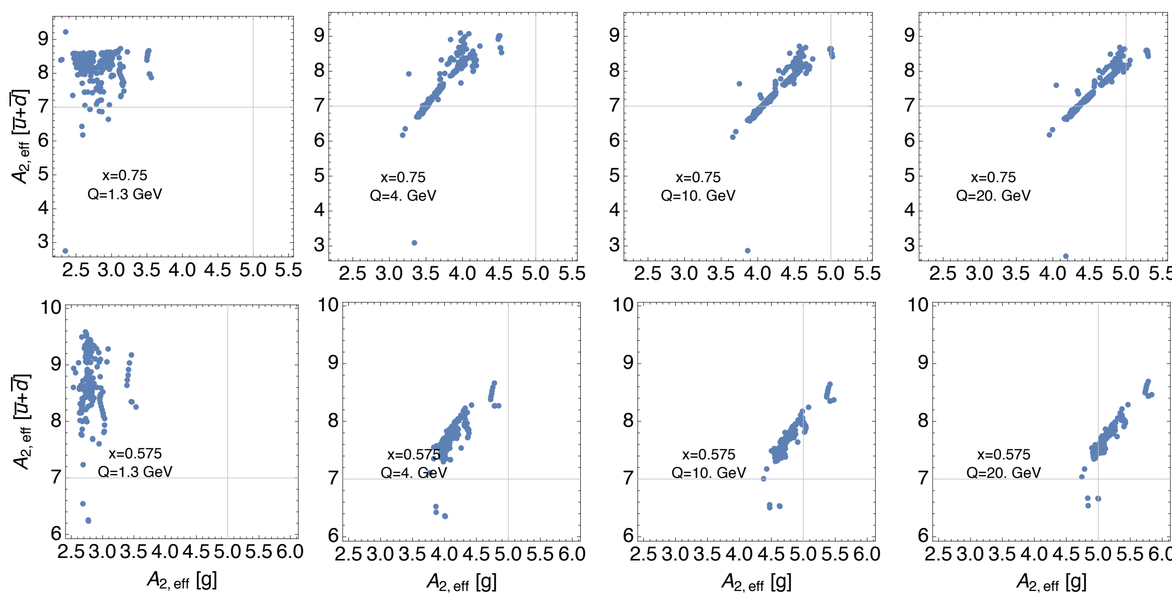
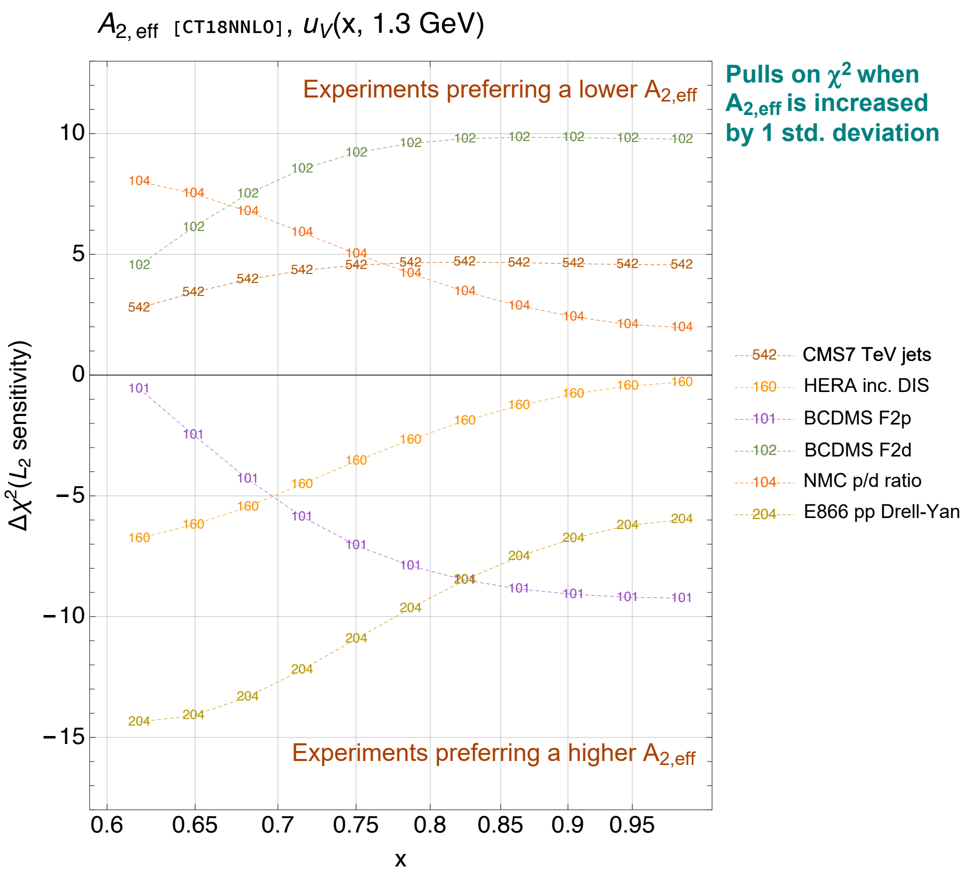


Ball et al QCRs expectation



Are the effective powers the same for all processes?

- ⇨ **Tool to estimate the preferences for the value of $A_{2,eff}$ per data set**
  - At x~1
    - Proton BCDMS and DY E866 favors a larger value of $A_{2,eff}[u_V]$
    - Deuteron BCDMS favors a smaller value of $A_{2,eff}[u_V]$



- ⇨ **Sensitivity**
  - PDFSense:

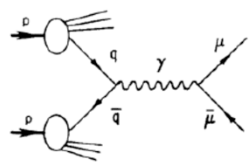
    correlation between observable and objective function of a given fit.
  - [Phys.Rev. D98 (2018) & Phys.Rev.D 100 (2019)]

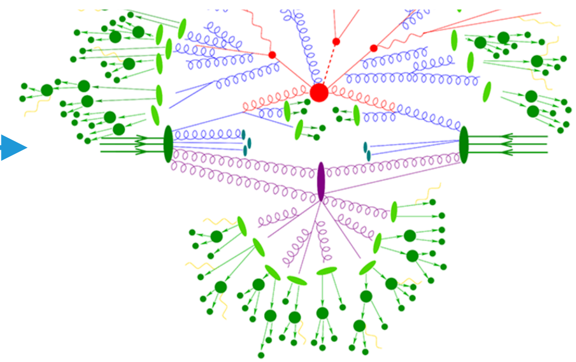
What is the error on the effective power that accounts for realistic measurement effects?

1. **Free hadron vs. modification of hadron before hard scattering**

   ⇨ affects O(M/Q) corrections in a process dependent way J. Collins (Cambridge University Press, 2013)

   Proton-proton collision: Increase in underlying hadronic activity with energy





e.g. PYTHIA [0710.3820]

2. **At threshold, soft gluon resummation modifies the hard cross section.**

   and, for DIS, the resonance region requires a nonpert. treatment too!

   - CJ: large-x PDF with TMC and *higher-twist* Accardi et al, PRD93
   - NNPDF with threshold resummation Bonvini et al, JHEP 09

Does $A_{2,\mathrm{eff}}$ capture the true leading (1-x)-power?

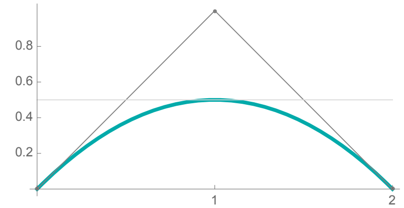
$A_{2,\mathrm{eff}}$ depends on x and Q^2

The same curve can be described to polynomials of different orders.

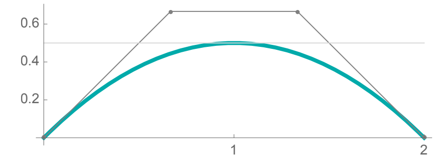
⇨ polynomial mimicry

1. Bézier curves give an example of mathematical equivalence of polynomials of different orders

defined on Bernstein polynomial basis:



$$f(x) = \alpha(1-x)^2 + 2\beta(1-x)x + \gamma x^2$$



$$f(x) = \alpha(1-x)^3 + 3\beta'(1-x)^2 x + 3\beta''(1-x)x^2 + \gamma x^3$$

$$\beta', \beta'' \equiv F[\alpha, \beta, \gamma]$$

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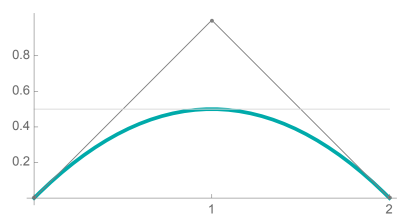

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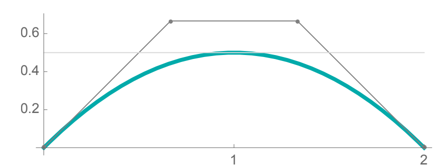

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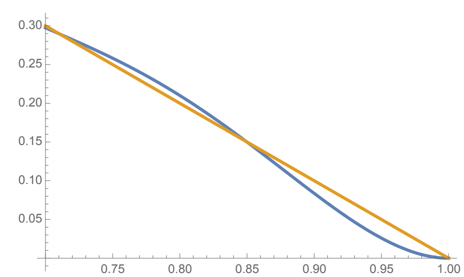
$$\beta', \beta'' \equiv F[\alpha, \beta, \gamma]$$

2. Global/local degree of polynomial

A sum of (1-x)-power contributions can *globally* fit to a lower power

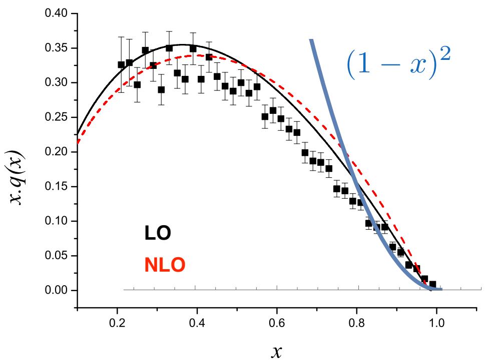
$$\sum_{n=2}^{4} \alpha_n (1-x)^n = (1-x)$$



The shape of PDFs and manifestations of low-energy dynamics

Relevant for the upcoming pion analyses at JLab and AMBER

Nonperturbative theoretical uncertainties also get in the way of the comparisons.



E615 “extraction”

Know your model:

- What are the characteristics of the underlying dynamics present/missing?
- Is DGLAP valid at μ_0^2 ?

⇨ **sources of uncertainty in the comparison with QCRs at a scale** μ^2

Mimicry:

- Multi-parameter analysis: impact of each nonpert. manifestation on objective function?
- $A_{2,eff}$ can be determined, A_2 cannot.

Efforts needed to mindfully compare both pictures.

Conclusions

We have analyzed the **quark counting rules** for the CT18NNLO global fit for the proton PDFs.

We have addressed the question of their **universality** for processes, flavors as well as Structure Function vs. PDFs.

- ⇨ The Q^2 dependence of the (1-x)-power is not negligible — supported by other global fits and by pQCD.
- ⇨ Global analyses rely on complex processes: underlying hadronic activity —not only scaling violations or resummation.
- ⇨ The universality of Quark Counting Rules for PDFs depends on the validity of factorization —O(M/Q) terms.
- ⇨ **Mimicry** reconciles many parametrizations of PDFs with measurements.
- ⇨ The **uncertainties** must be estimated from both the nonperturbative and the pheno side.

How do we cast nonperturbative manifestations into measurable observables?

We advocate for interpretative **effective** (1-x)-exponent.

“ It is worth emphasizing that, as long as the basic theoretical requirements are satisfied, all GPD representations present the same field theoretical object. Therefore, in principle, it should be possible to map a GPD within one representation to that in a different representation […]. This generally makes the popular question “Which GPD representation is better?” meaningless. Instead, one may hope to get an additional insight of GPDs and their physical interpretation by comparing the manifestation of GPD properties within different representations. ”

Müller, Polyakov & Semenov