JAM Pion PDF Analysis Including Resummation

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Perceiving the Emergence of Hadron Mass through AMBER@CERN

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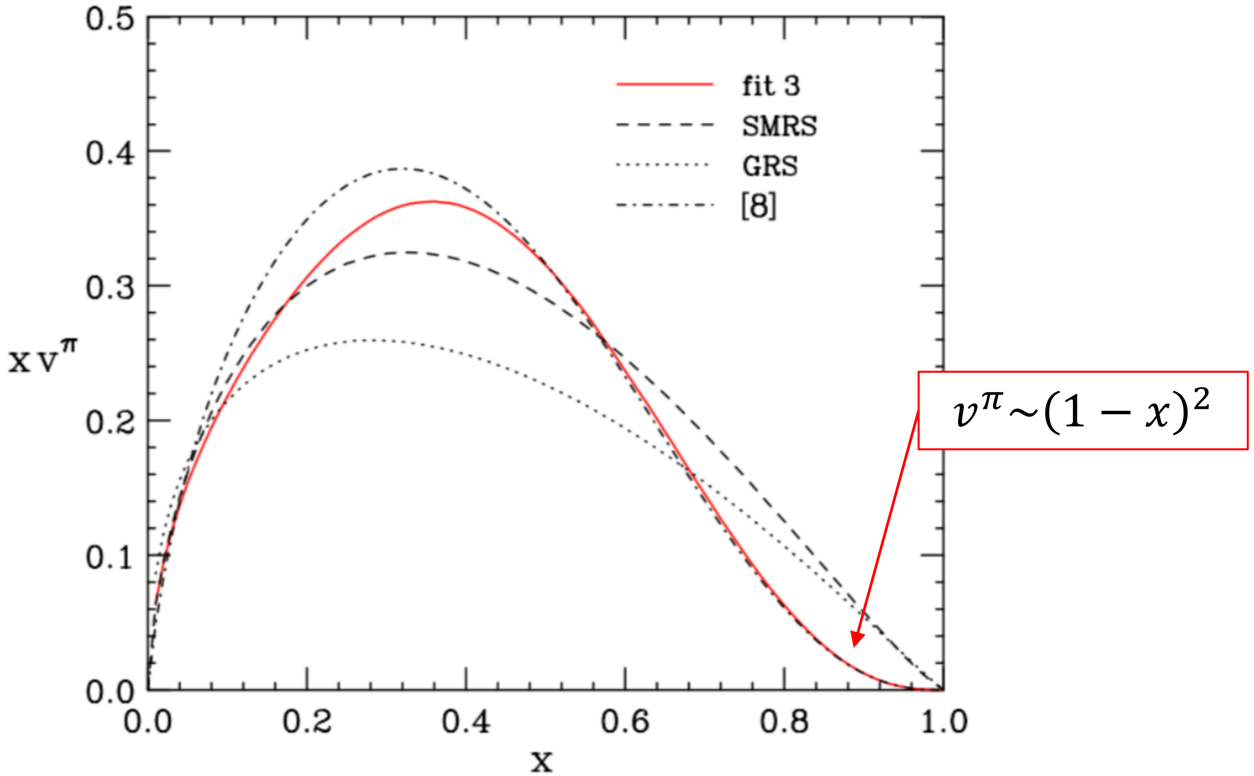
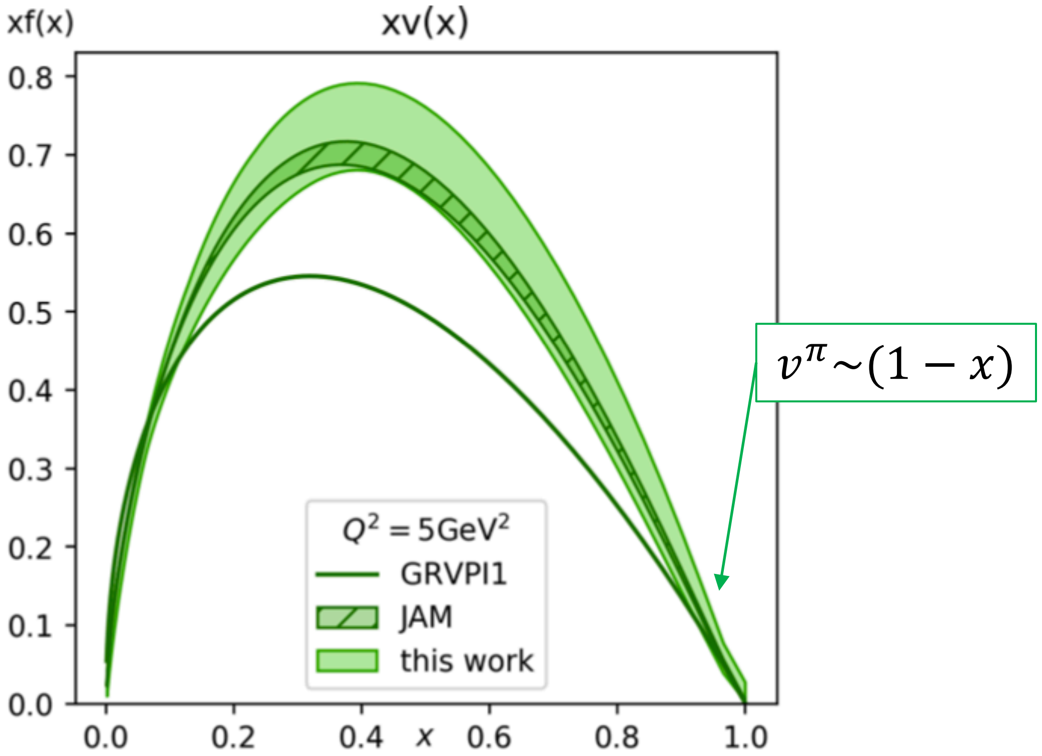
Motivation/Background

Theoretical Interest

- Behavior of PDF as $x_\pi \to 1$ ($\mathrm{v}_\pi \sim (1 - x_\pi)^{2\beta}$) is of theoretical interest
- Recent lattice calculations as well as phenomenologically determined valence quark PDFs using threshold resummation indicate $\beta = 1$ as opposed to fixed order ($\beta = 1/2$)
- This analysis with threshold resummation will have impact on this question

Recent Pion Phenomenology



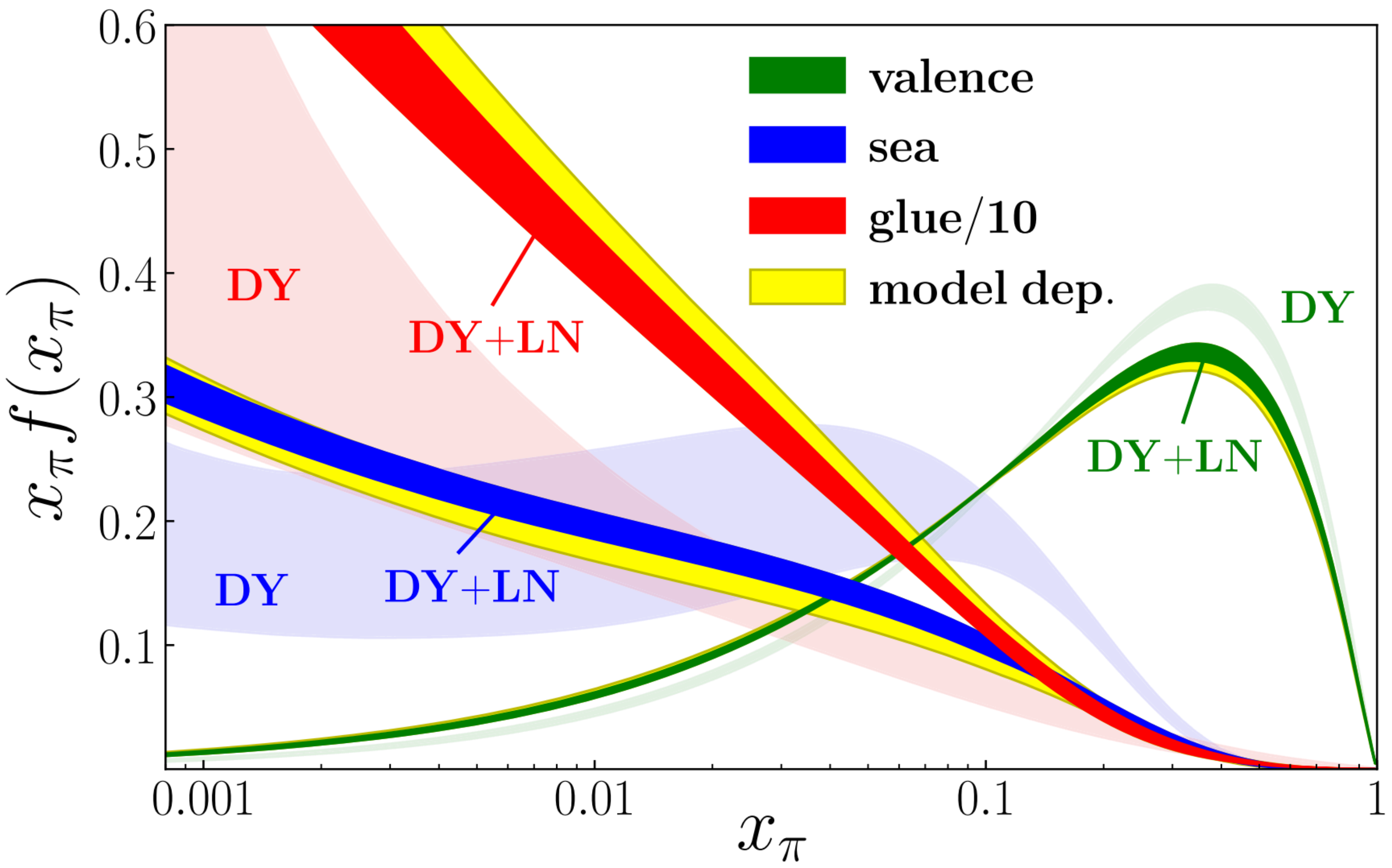


- Recent (M. Aicher, et al, 2010) pion fit to DY data
- Fit uses soft gluon resummation

- Recent (I. Novikov, et al, 2020, xFitter) pion fit to DY and prompt photon data
- Fit uses NLO in α_S

JAM 18 Pion PDFs

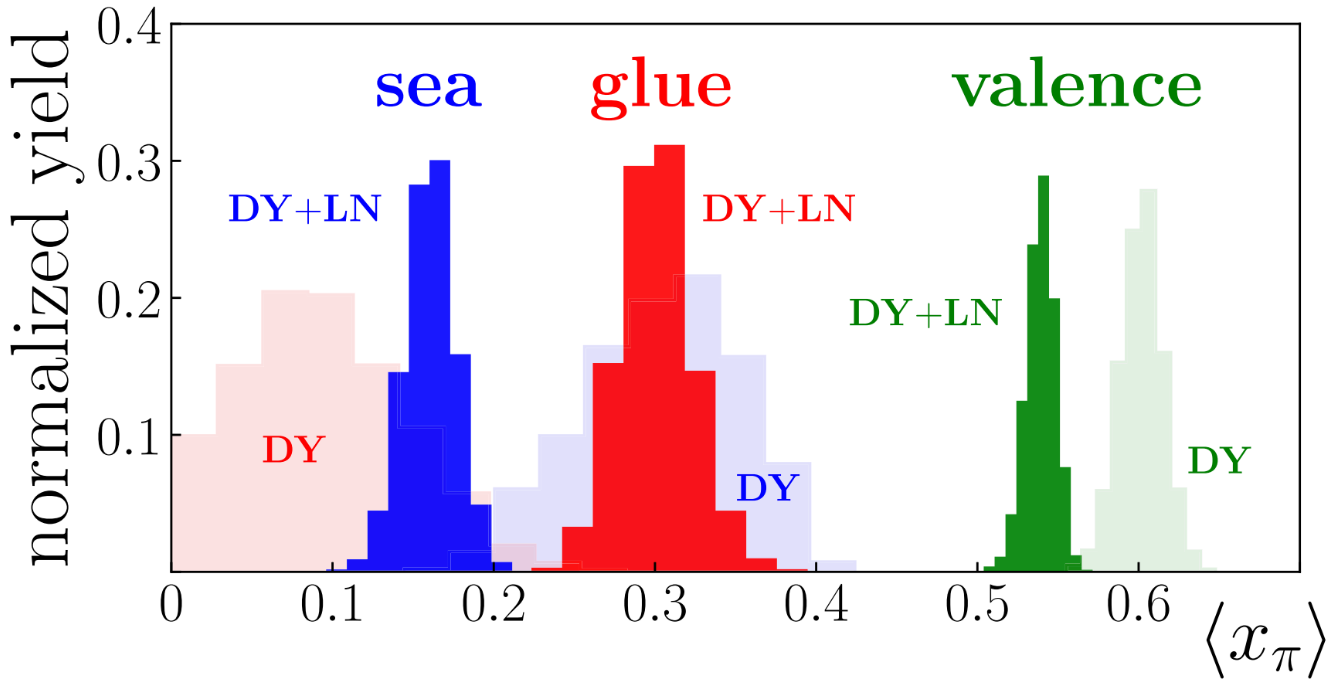
- Valence, sea, and gluon distributions were extracted in an NLO analysis
- Drell-Yan (DY) only fit then include the Leading Neutron (LN)
- Theoretical uncertainty shown only in model dependence for LN treatment



PB, N. Sato, W. Melnitchouk, C. –R. Ji, Phys. Rev. Lett. **121**, 152001 (2018)

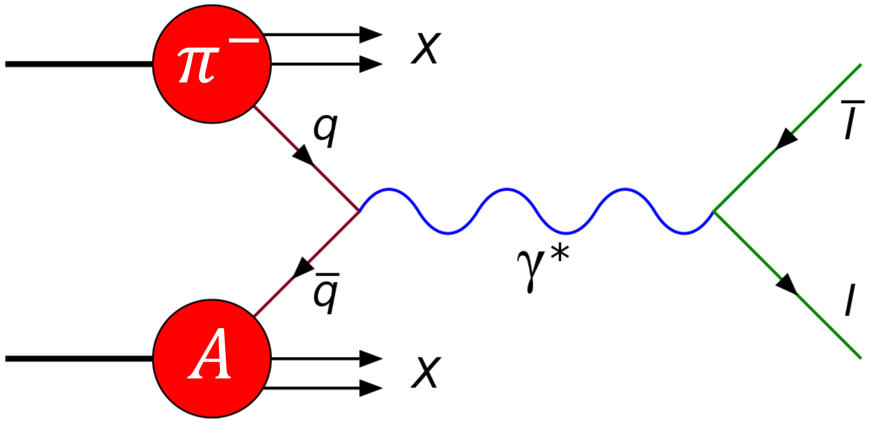
JAM 18 Momentum Fractions

- We also compute the momentum fractions for each flavor
- Large difference in in the gluon and sea $\langle x_\pi \rangle$ from a DY to a DY+LN analysis
- Gluon carries ~30% of the momentum fraction at the initial scale



Threshold Resummation in Drell-Yan

Drell-Yan (DY) Definitions



$\hat{S}$ is the center of mass momentum squared of incoming partons

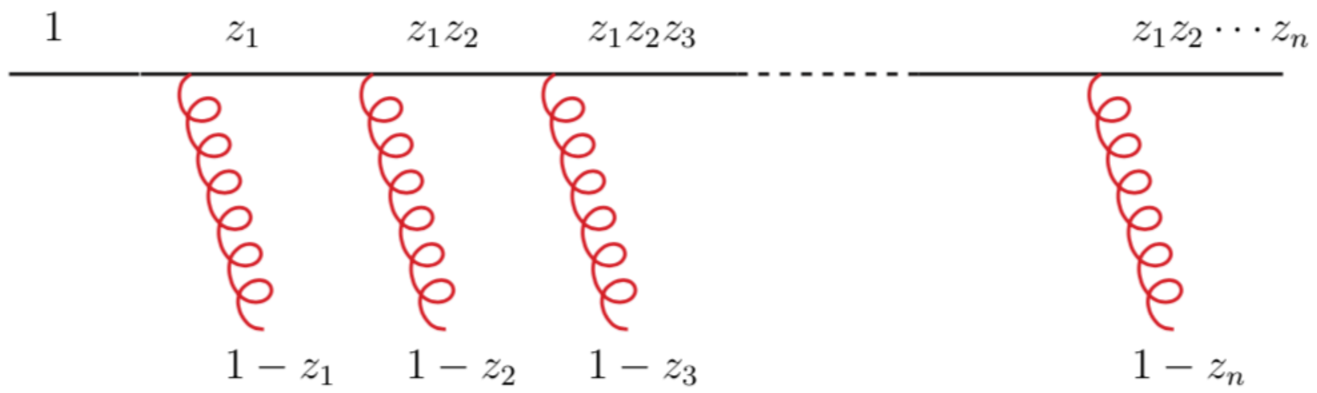
Hadronic variable

$$\tau = \frac{Q^2}{S}$$

Partonic variable

$$z \equiv \frac{Q^2}{\hat{S}} = \frac{\tau}{x_1 x_2}$$

Soft Gluon Resummation



- The goal is to sum the contributions of the soft gluon emissions from the quark line to all orders of α_S
- Can perturbatively calculate these emissions to all orders of α_S
- Here, z_i near 1

Full Hard Kernel to Calculate

$$C^{\mathrm{N}^p\mathrm{LO}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) = \sum_{j=0}^{p} \alpha_s^j\, C^{(j)}(N) + C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) - \sum_{j=0}^{p} \frac{\alpha_s^j}{j!} \left[\frac{d^j C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s)}{d\alpha_s^j} \right]_{\alpha_s=0}$$

Fixed order Kernel (such as NLO)

Resummation Kernel.
Calculate such as Leading Log,
or Next-to-Leading Log

Matching coefficients
Need to subtract in
order to avoid double
counting

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

An NLO calculation gathers the $\mathcal{O}(\alpha_S)$ terms

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_S \log(N)^2$	$\alpha_S \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Add the columns to the rows

	LL	NLL	...	N^pLL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^pLL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Rapidity Distribution

- Formulate resummation in Mellin space for Q^2(or τ) distribution
- For rapidity distribution, a Mellin-Fourier transform can be taken instead of a single Mellin

$$\sigma(N,M) = \int_0^1 d\tau \tau^{N-1} \int_{-\log\frac{1}{\sqrt{\tau}}}^{\log\frac{1}{\sqrt{\tau}}} dY e^{iMY} \frac{d\sigma}{dQ^2 dY}$$

- Where the hard coefficients reduce to

$$C^{\text{res}}(N,M) = \int_0^1 dz z^{N-1} \cos(\frac{M}{2}\log z) C^{\text{res}}(z)$$

Cosine vs Expansion

- Since we focus on the threshold region, that is when $z \to 1$, the log of z will be close to 0, meaning the argument of the cosine will be close to 0
- One can expand to the cosine term such that

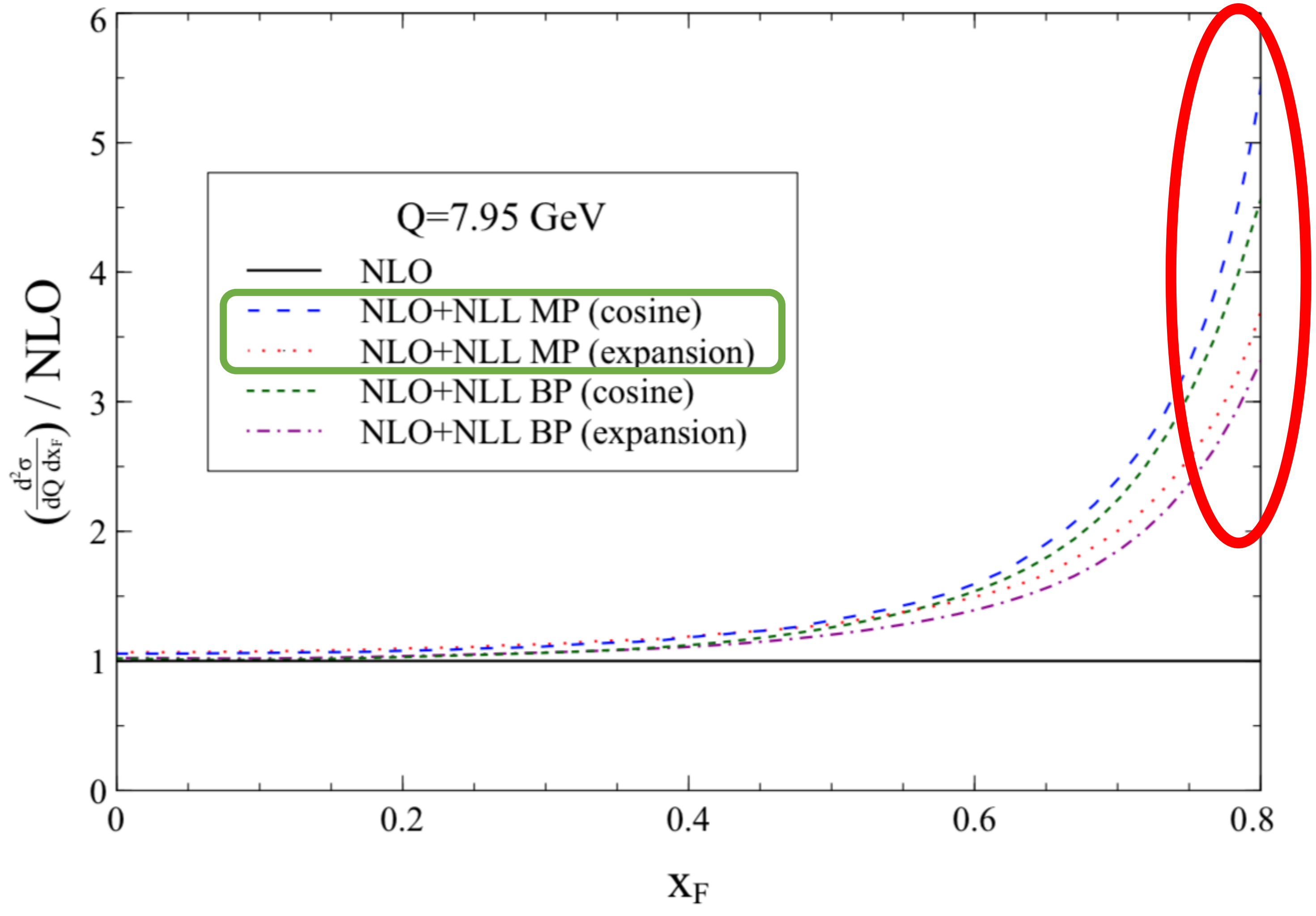
$$\cos(\frac{M}{2}\log z) \approx 1$$

- Or, one can take the cosine exactly using Euler identity

$$\cos(\frac{M}{2}\log z) = \frac{1}{2}(e^{i\frac{M}{2}\log z} + e^{-i\frac{M}{2}\log z}) = \frac{1}{2}(z^{i\frac{M}{2}} + z^{-i\frac{M}{2}})$$

K-factor Aside

- Whether cosine or expansion is used has impact at large x_F
- We will focus on the Minimal Prescription (MP)
- Testing both methods can give a theoretical uncertainty on our pion PDFs

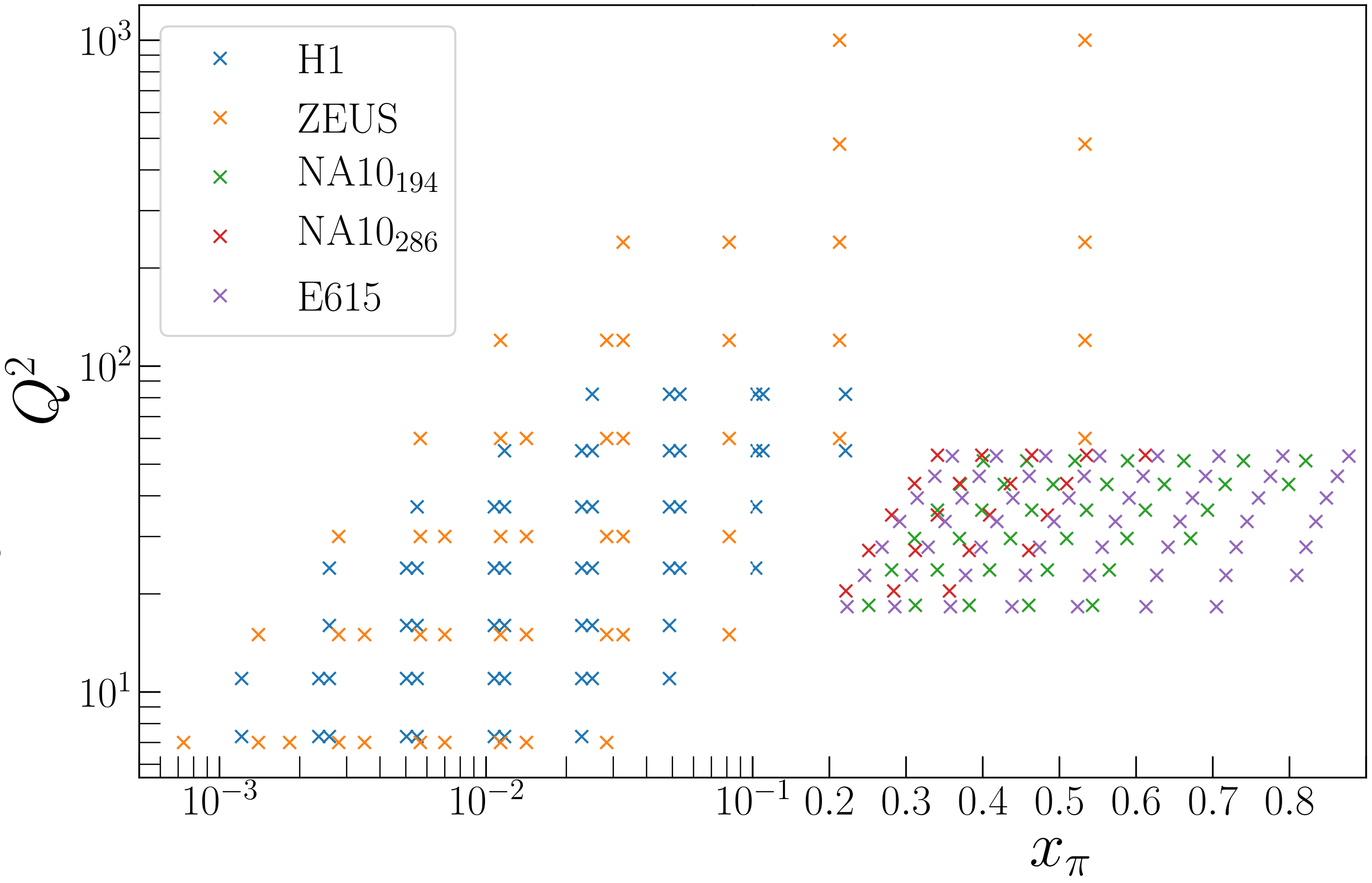


D. Westmark and J. F. Owens, Phys. Rev. D **95**, 056024 (2017).

Extraction Procedure

Kinematic Coverage

- We want to be able to fit simultaneously the Drell-Yan and Leading Neutron data
- We can shape the pion PDFs at both high- and low-x_π with both datasets
- E615, NA10 – DY
- H1, ZEUS – LN



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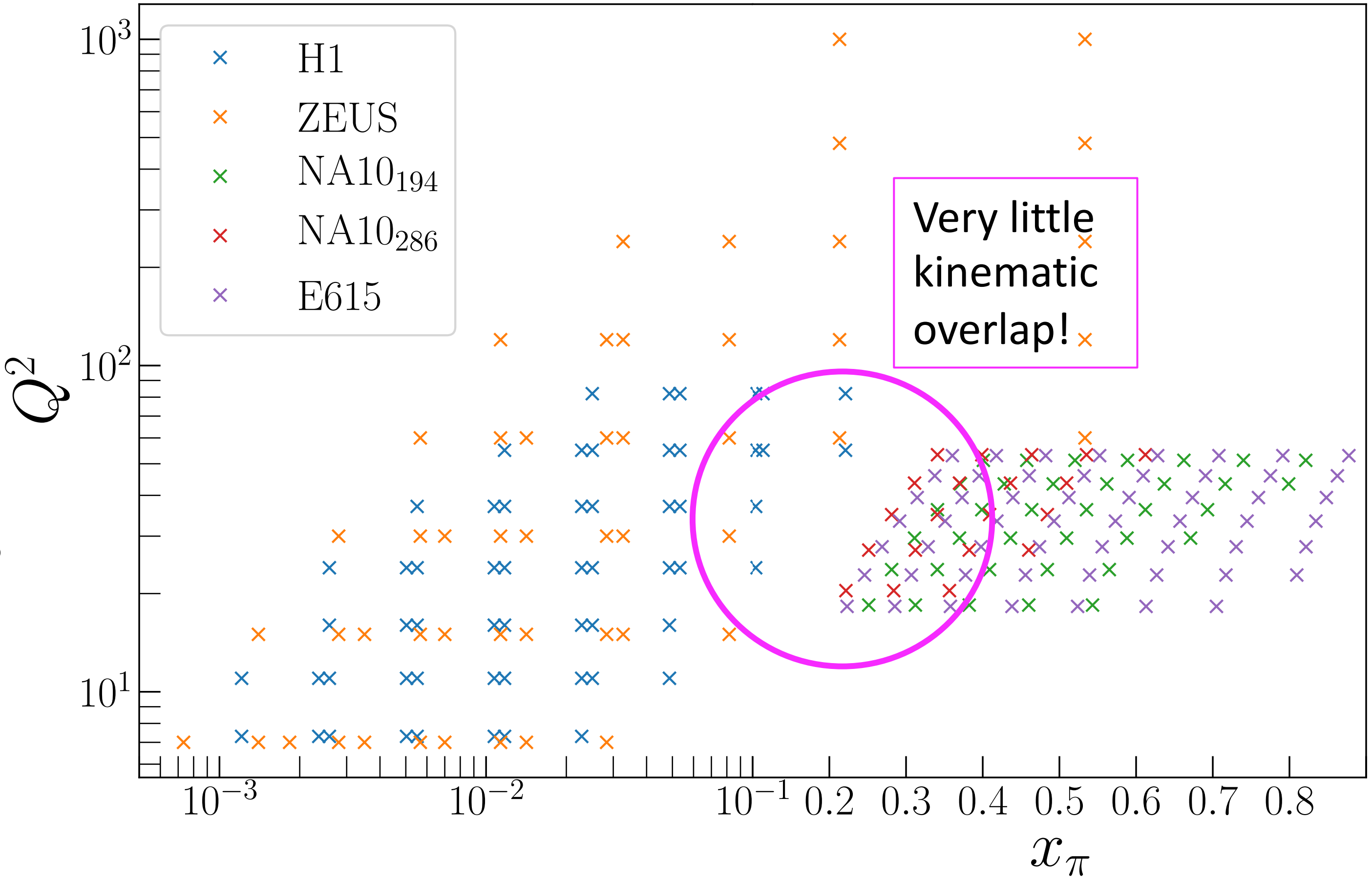


Parametrization of the PDF

- We open the shape up a little for the valence (important for resummation in DY)

$$q_v(x_\pi, Q_0^2, \mathbf{a}) = \frac{N}{N_v'} x_\pi^\alpha (1 - x_\pi)^\beta (1 + \gamma x^2)$$

As was done in Aicher et al.

where

$$N_v' = B(2 + \alpha, \beta + 1) + \gamma B(4 + \alpha, \beta + 1)$$

- And for the sea and the gluon, we parametrize by

$$f(x_\pi, Q_0^2, \mathbf{a}) = \frac{N}{N'} x_\pi^\alpha (1 - x_\pi)^\beta$$

where

$$N' = B(2 + \alpha, \beta + 1)$$

Parameterization of the PDF (in terms of π^-)

- We equate the valence distributions: $\bar{u}_v^{\pi^-} = d_v^{\pi^-}$
- We equate the light sea distributions: $u^{\pi^-} = \bar{d}^{\pi^-} = u_s^{\pi^-} = d_s^{\pi^-} = s = \bar{s}$
- Normalizations of the valence and sea PDFs are fixed by the sum rules

Quark sum rule $$\int_0^1 dx_\pi q_v^\pi = 1$$

Momentum Sum Rule $$\int_0^1 dx_\pi x_\pi (2q_v^\pi + 6q_s^\pi + g^\pi) = 1$$

Monte Carlo

- Using Bayesian statistics, we describe the probability

$$\mathcal{P}(\mathbf{a}|\mathrm{data}) \propto \mathcal{L}(\mathrm{data}|\mathbf{a})\pi(\mathbf{a})$$

- We quantify the expectation value and variance of our observable $\mathcal{O}$ as a function of the parameter set $\boldsymbol{a}_i$

$$E[\mathcal{O}] = \frac{1}{N}\sum_i \mathcal{O}(\mathbf{a}_i)$$

$$V[\mathcal{O}] = \frac{1}{N}\sum_i \left[\mathcal{O}(\mathbf{a}_i) - E[\mathcal{O}]\right]^2$$

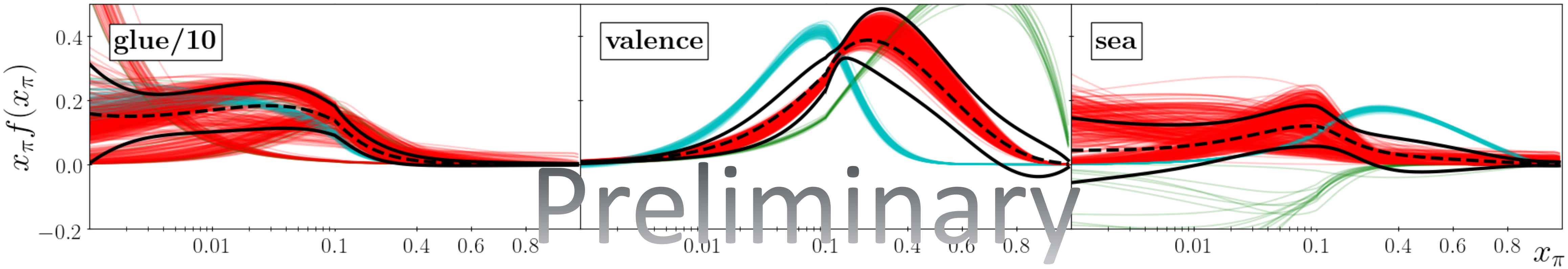
Multi-Step Strategy

- Fitting PDFs to many types of observables all at once is time consuming and slows the fit
- We start with many replicas with flat priors to fit to one observable, the $\pi^{-}W$ DY data
- The posteriors from that fit are used as the priors for the next fit, which includes the LN data

Monte Carlo Results

PDF Results – Cosine

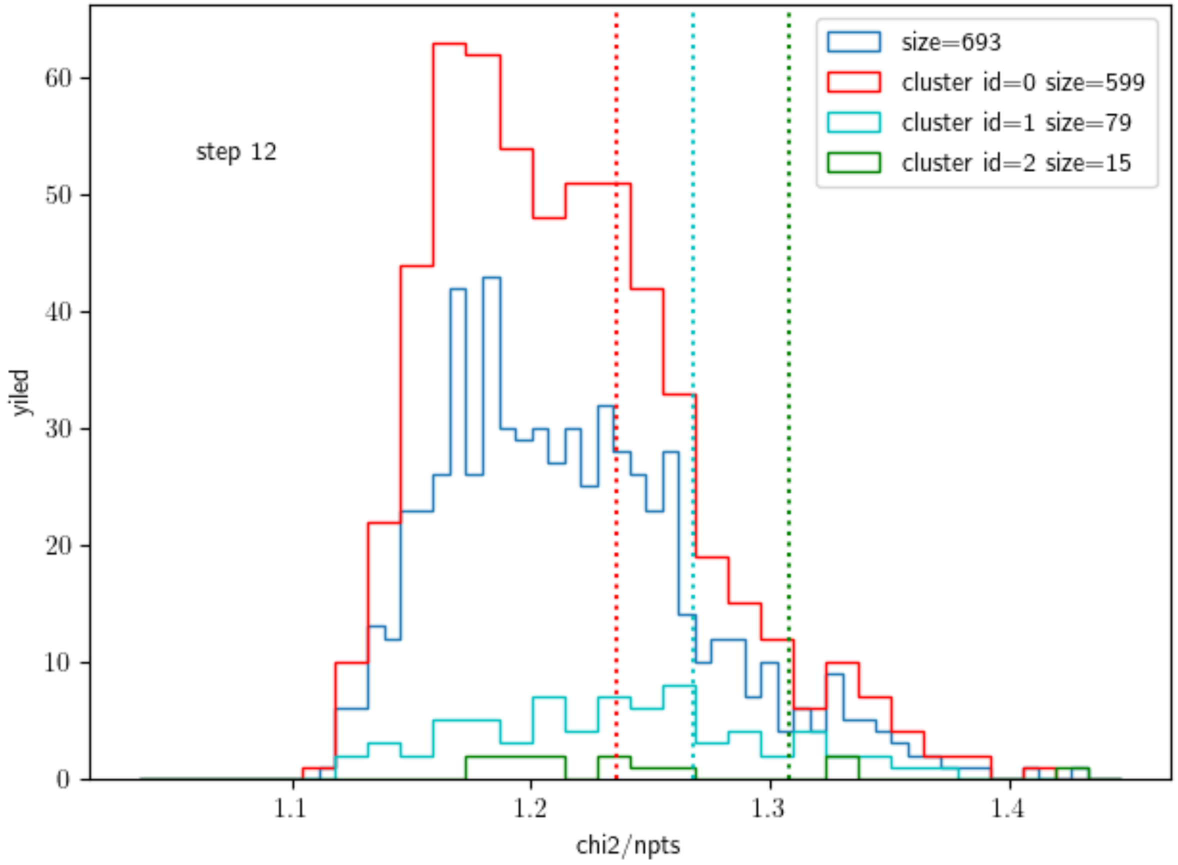
- Fitting to both DY and LN data using the cosine approximation in the minimal prescription



- Clearly there are multiple solutions (evident in the valence)
- Use k-means clustering to distinguish solutions

χ^2 profile of clusters

- Histogram of χ^2 values for the different clusters
- Red is best, but not by much!
- Consider only the red solutions



Comparisons – χ^2

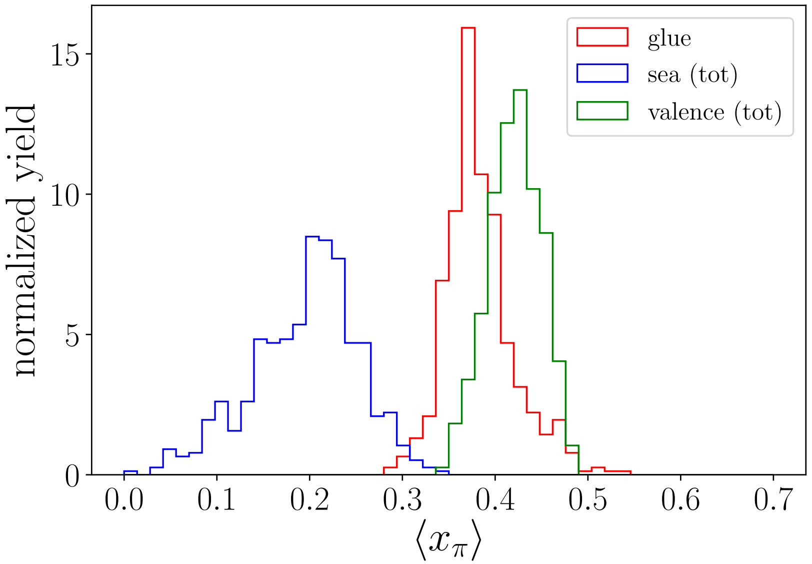
Cosine method

reaction	idx	col	npts	chi2	chi2/npts	norm
ln	1000	H1	58	22.07	0.38	1.26
ln	2000	ZEUS	50	72.75	1.46	0.95
dy-pion	10001	E615	55	72.19	1.31	1.10
dy-pion	10002	NA10	36	48.97	1.36	0.90
dy-pion	10003	NA10	20	27.85	1.39	0.84
			219	243.82	1.11	

Expansion method

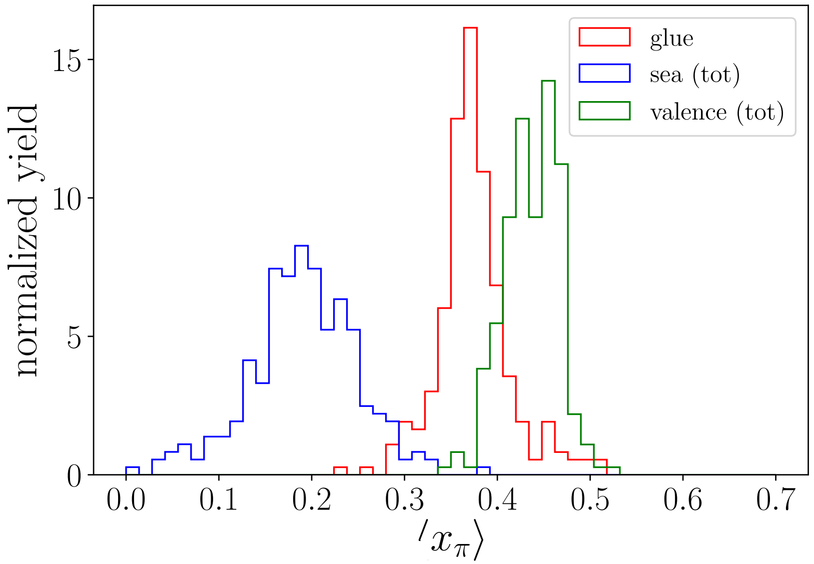
reaction	idx	col	npts	chi2	chi2/npts	norm
ln	1000	H1	58	21.93	0.38	1.25
ln	2000	ZEUS	50	74.39	1.49	0.95
dy-pion	10001	E615	55	57.77	1.05	1.08
dy-pion	10002	NA10	36	26.58	0.74	0.90
dy-pion	10003	NA10	20	15.29	0.76	0.83
			219	195.96	0.89	

Comparisons – Momentum Fractions

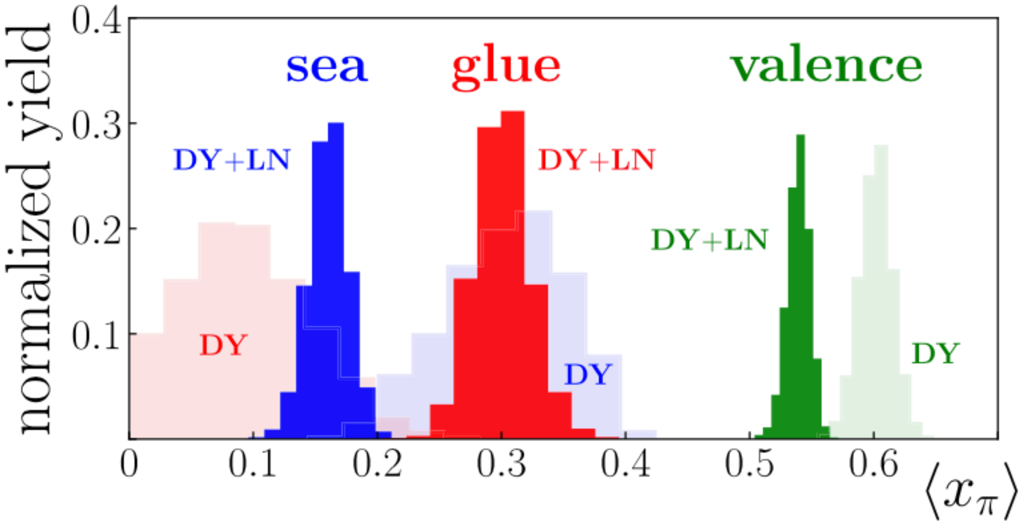


Cosine method

Expansion method

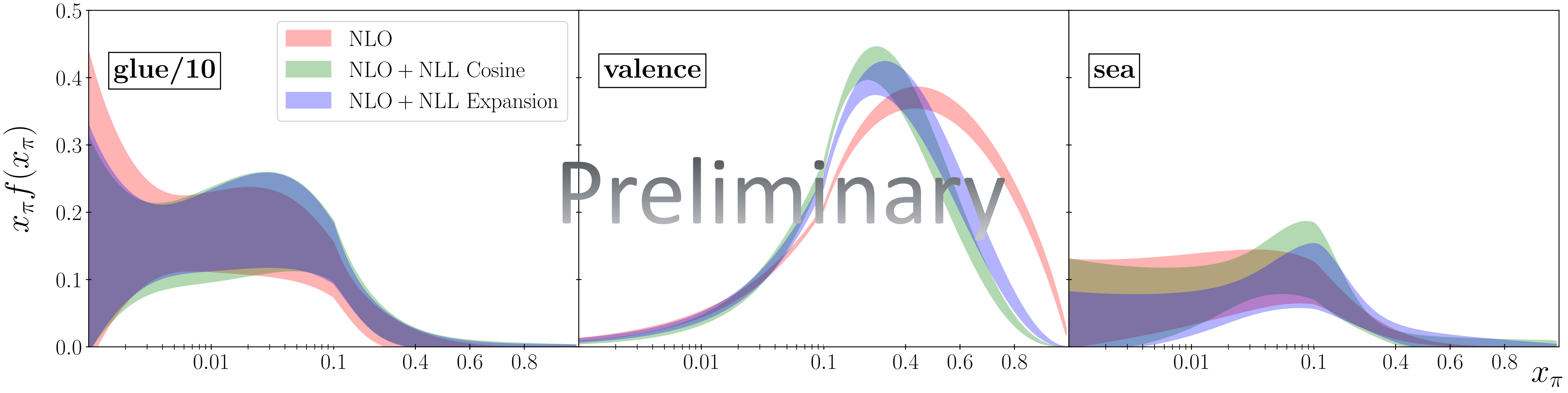


Fixed order result from 2018 PRL



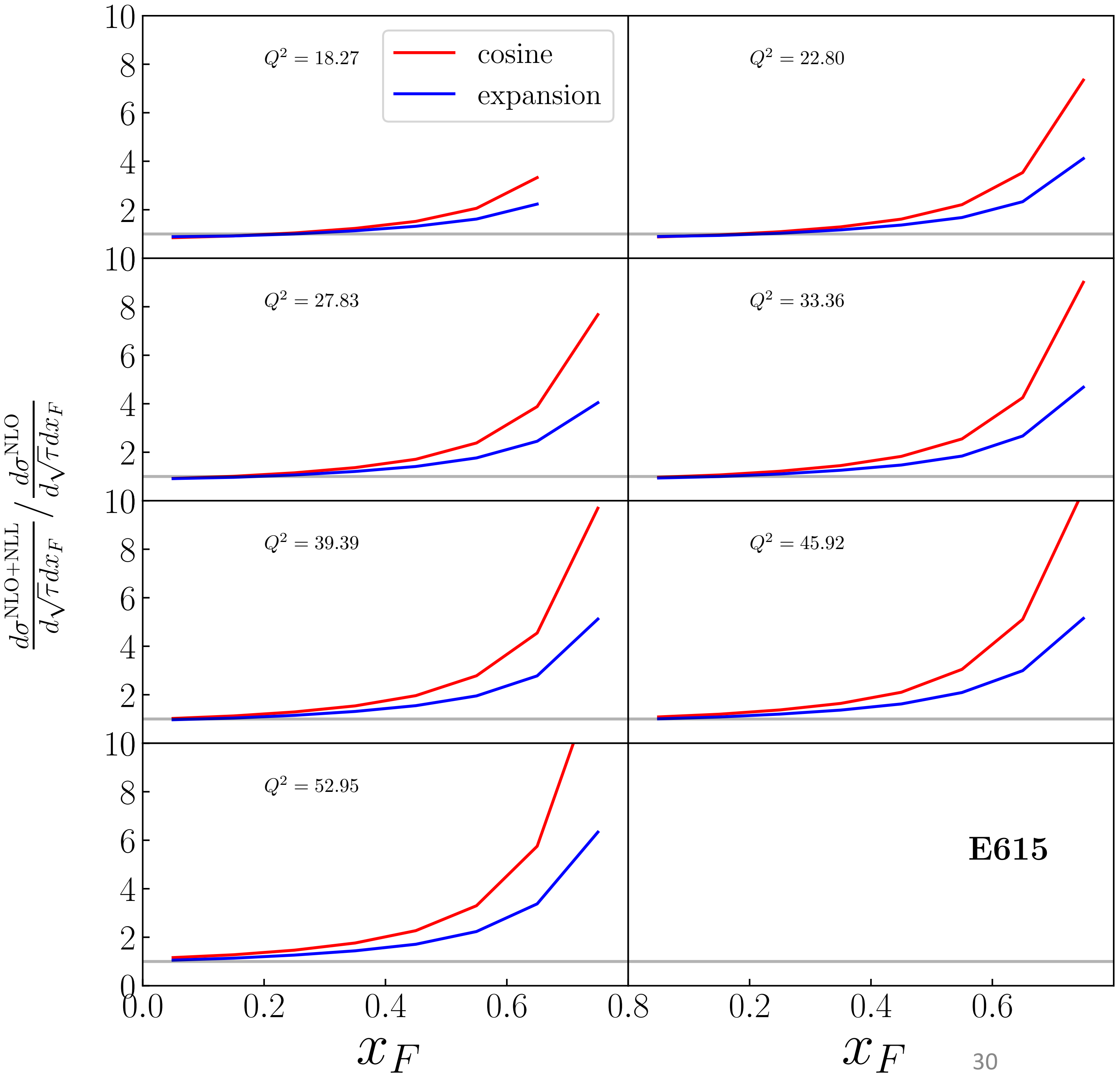
Comparisons – PDFs

- Comparison of the PDFs at the initial scale with fixed order



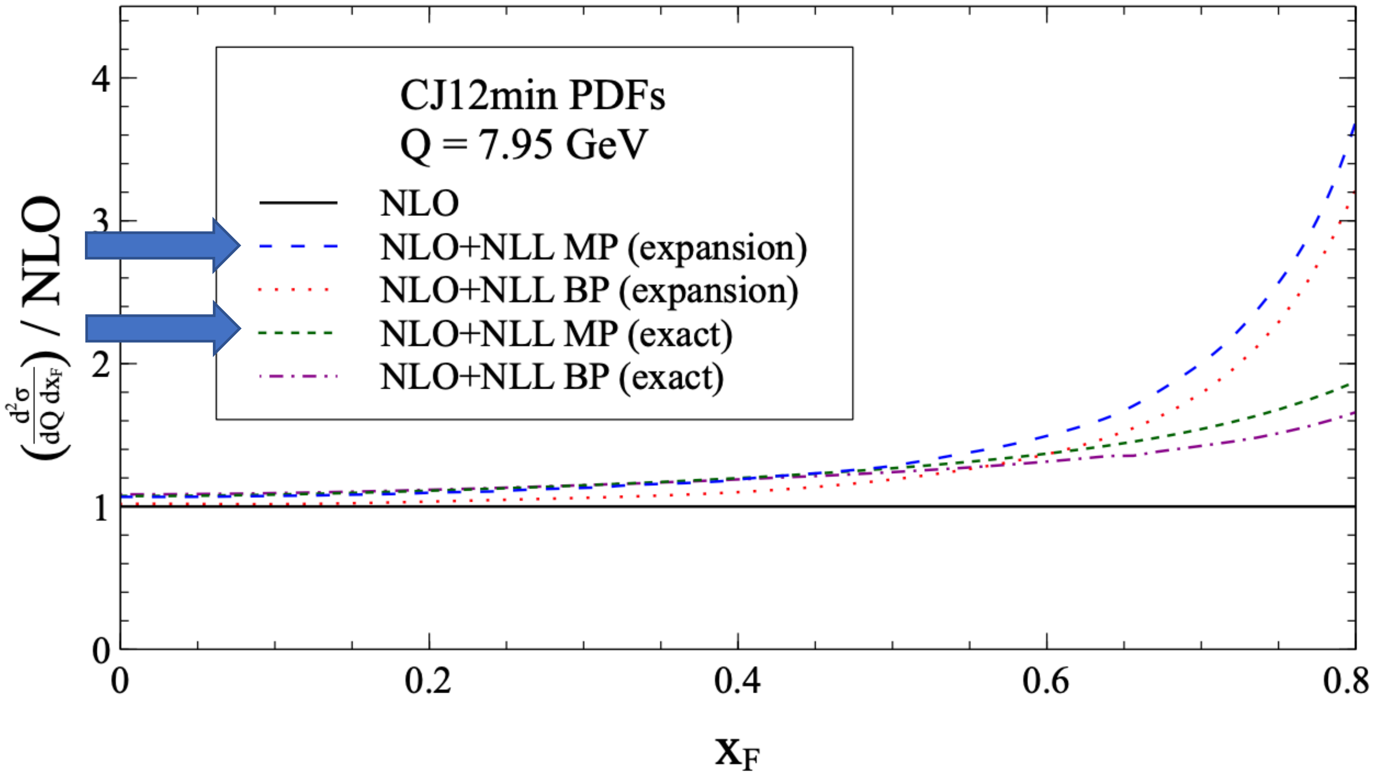
K Factor

- K factor for kinematics associated with the E615 dataset
- PDFs are consistent with each curve
- Cosine seems to gather more terms at higher orders than expansion



K Factor – More calculations in MP

- Invokes an "exact" resummation prescription with Double Mellin transforms for rapidity distribution
- Lowers the K factor from expansion to near 1 for all x_F
- Large range in K factor for 3 MP methods effectively captures various resummation effects



D. Westmark and J. F. Owens, Phys. Rev. D **95**, 056024 (2017).

Conclusions

Conclusions

- The analysis of the resummation focuses on the Minimal Prescription
- With resummation, the valence quark distribution is softer as $x_\pi \to 1$ than in the fixed order case
- Fits using the Double Mellin "exact" method are needed
- Expectation is that the K factor for the "exact" method will be closer to 1 and the PDFs will be closer to the NLO calculation

Backup

Setting it up

- Because of the Eikonal approximation, in the soft limit, matrix elements of large numbers of emitted gluons can be factorized as such:

$$\mathcal{M}_n(z_1, \ldots, z_n) \overset{\text{soft}}{\simeq} \frac{1}{n!} \prod_{i=1}^{n} \mathcal{M}_1(z_i)$$

- Even though the amplitudes factorize in z-space in that way, the phase space does not because of the presence of a delta function for conservation of momentum

$$\delta(z - z_1 z_2 \ldots z_n).$$

Setting it up

- In Mellin space, however, we do have factorization of the phase space,

$$\int_0^1 dz z^{N-1} \delta(z - z_1 z_2 ... z_n) = z_1^{N-1} z_2^{N-1} ... z_n^{N-1}$$

- So for hard kernels, for each order of α_S, we have:

$$C^{(n)}(N) \overset{\text{soft}}{\simeq} \frac{1}{n!} \left[C^{(1)}_{\text{soft}}(N) \right]^n$$

- Where $C^1_{soft}(N)$ is the hard kernel for one soft gluon emitted from the quark line

Exponentiation in Mellin space

- The matrix elements of emitted soft gluons that carry large logarithms are factorized in the Eikonal approximation
- Phase space only factorizes in Mellin space
- Summing over all orders of α_S leads to exponentiation of the Mellin space coefficients

$$\sum_{n=1}^{\infty} C^{(n)}(N) = \sum_{n} \frac{1}{n!} [C_{\text{soft}}^{(1)}(N)]^n$$
$$= \exp\left(C_{\text{soft}}^{(1)}(N)\right)$$

Computing the Expressions

- Specifically for the DY case, we need to use the following for each initial state parton (2 for DY)

$$\log \Delta(N) = \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dk_\perp^2}{k_\perp^2} A_q(\alpha_S(k_\perp))$$

where

$$A_q(\alpha_S) = \sum_{i=1}^{\infty} \alpha_S^i A_q^{(i)}$$

and

$$A_q^{(1)} = \frac{C_F}{\pi}, \qquad A_q^{(2)} = \frac{C_F}{2\pi^2} \Big[C_A \Big(\frac{67}{18} - \frac{\pi^2}{6} \Big) - \frac{5}{9} N_f \Big].$$

Computing the Expressions

- We also need a closed form for α_S, in which case, we use the two-loop (needed for up to NLL accuracy)

$$\alpha_S(k_T^2) = \frac{\alpha_S(\mu^2)}{1 + b_0\alpha_S(\mu^2)\log(\frac{k_T^2}{\mu^2})}\left[1 - \frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log\left(1 + b_0\alpha_S(\mu^2)\log\left(\frac{k_T^2}{\mu^2}\right)\right.}{1 + b_0\alpha_S(\mu^2)\log\left(\frac{k_T^2}{\mu^2}\right)}\right]$$

Computing the Expressions

- Plugging those in, we get the following

$$\log \Delta(N) = \int_0^1 dz \frac{z^{N-1}-1}{1-z} \int_{\mu^2}^{(1-z)^2 Q^2} \frac{dk_\perp^2}{k_\perp^2} \Bigg\{ A_q^{(1)} \frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \times \Big[\frac{b_1}{b_0} \frac{\alpha_S(\mu^2) \log(1 + b_0 \alpha_S \log \frac{k_\perp^2}{\mu^2})}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \Big] + A_q^{(2)} \Big(\frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \Big[\frac{b_1}{b_0} \frac{\alpha_S(\mu^2) \log(1 + b_0 \alpha_S \log \frac{k_\perp^2}{\mu^2})}{1 + b_0 \alpha_S(\mu^2) \log \frac{k_\perp^2}{\mu^2}} \Big] \Big)^2 \Bigg\}.$$

Large N Approximation

- The z integral on the previous slide is difficult, but not impossible
- Recall, our aim is the soft limit, *i.e.* when $z \to 1$
- In Mellin space, soft limit is $N \to \infty$
- In the large N limit, we may use the approximation

$$z^{N-1} - 1 \approx -\Theta(1 - \frac{1}{\bar{N}} - z)$$

where

$$\bar{N} = N e^{\gamma_E}$$

Plugging it in

- We can use the large N approximation to compute the following

$$\log\Delta(N) = -\int_0^{1-\frac{1}{N}} \frac{dz}{1-z}\int_{\mu^2}^{(1-z)^2Q^2} \frac{dk_\perp^2}{k_\perp^2}\Bigg\{A_q^{(1)}\frac{\alpha_S(\mu^2)}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}} \times\Big[\frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log(1+b_0\alpha_S\log\frac{k_\perp^2}{\mu^2})}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big] + A_q^{(2)}\Big(\frac{\alpha_S(\mu^2)}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big[\frac{b_1}{b_0}\frac{\alpha_S(\mu^2)\log(1+b_0\alpha_S\log\frac{k_\perp^2}{\mu^2})}{1+b_0\alpha_S(\mu^2)\log\frac{k_\perp^2}{\mu^2}}\Big]\Big)^2\Bigg\}.$$

Form for exponent

- Performing for the case of DY, we have

$$\log \Delta(N) = 2h^{(1)} \log(\bar{N}) + 2h^{(2)}(\lambda, Q^2/\mu^2)$$

$$\lambda = b_0 \alpha_s(\mu^2) \ln \bar{N}.$$

$$h^{(1)} = \frac{A_q^{(1)}}{2b_0\lambda}[2\lambda + (1-2\lambda)\log(1-2\lambda)]$$

$$h^{(2)} = (A_q^{(1)} b_1 - b_0 A_q^{(2)}) \frac{2\lambda + \log(1-2\lambda)}{2b_0^3} + \frac{A_q^{(1)} b_1}{4b_0^3} \log^2(1-2\lambda) + \frac{A_q^{(1)}}{2b_0} \log(1-2\lambda) \log \frac{Q^2}{\mu^2}$$

Expansion

- If we use the expansion method, then

$$C^{\text{res}}(N,M) = \int_0^1 dz z^{N-1} \cos(\frac{M}{2} \log z) C^{\text{res}}(z)$$

- Goes to

$$C^{\text{res}}(N,M) = \int_0^1 dz z^{N-1} C^{\text{res}}(z) = C^{\text{res}}(N)$$

- Note the independence of *C* on *M*

Cosine

- If we use the cosine method, then

$$C^{\text{res}}(N,M) = \int_0^1 dz z^{N-1} \cos(\frac{M}{2}\log z) C^{\text{res}}(z)$$

- Goes to

$$\begin{aligned} C^{\text{res}}(N,M) &= \int_0^1 dz z^{N-1} \left[\frac{1}{2}\left(z^{iM/2} + z^{-iM/2}\right)\right] C^{\text{res}}(z) \\ &= \int_0^1 dz \frac{1}{2}(z^{(N+iM/2)-1} + z^{(N-iM/2)-1}) C^{\text{res}}(z) \end{aligned}$$

Note the average of Mellin moments of C with Mellin variables $N \pm \frac{iM}{2}$

The need for prescriptions

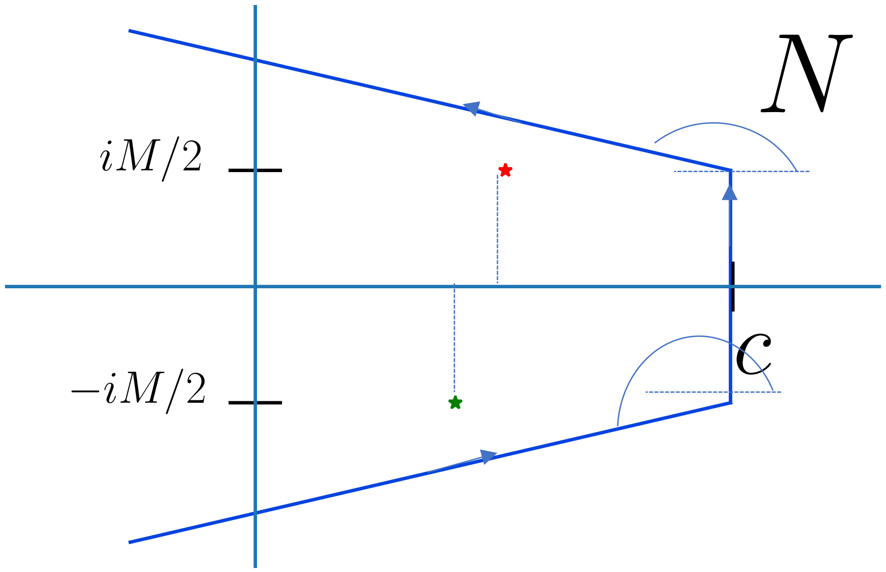
- To compare with data, one must Mellin invert so that the formulas are in momentum-fraction space and not moment space
- The Mellin inversion of the hard kernel appears order-by-order, but it is divergent because of the divergence of α_S
- One can locate the divergences and avoid them as in the Minimal Prescription (main focus)
- Or one can manipulate the summation to make it convergent as in the Borel prescription (out of the scope of this talk)

Minimal Prescription

- In principle, one can just do the Mellin inversion exactly
- However, the ambiguity appears in the Landau pole
- We can locate the Landau and avoid it
- By looking at *e.g.* the $h^1(\lambda)$ term, we can see where the arguments of the logarithms go to 0 and become negative
- This location is the Landau pole

$$1 - 2\lambda > 0 \implies \bar{N} < \exp\left(1/2\alpha_S b_0\right)$$

MELLIN CONTOUR – Expansion and Cosine



- Here, c is to the right of the PDFs' rightmost poles
- Because the PDF moments are evaluated at $N \pm i\frac{M}{2}$ instead of the usual N, the poles are also located $\pm i\frac{M}{2}$ from the real axis (red and green stars)
- Contour is misshapen to ensure poles are encapsulated

$$N_1 = c - i\frac{M}{2} + z_1\, e^{\phi_1}$$
$$0 < z_1 < \infty$$

$$N_2 = c - i\frac{M}{2} + z_2\, i\, M$$
$$0 < z_2 < 1$$

$$N_3 = c + i\frac{M}{2} + z_3\, e^{\phi_3}$$
$$0 < z_3 < \infty$$