MOCa 2020: Materia Oscura en Colombia



Dynamical Symmetry Breaking and Fermion Mass Hierarchy in the Scale-Invariant 3-3-1 Model

PHYSICAL REVIEW D 102, 015021 (2020)

Bruce L. Sánchez–Vega,

UFMG.







SM gauge group

Interesting features of the 331 models:

331 models

 $SU(3)_C \times SU(2)_L \times U(1)_V \rightarrow SU(3)_C \times SU(3)_L \times U(1)_V$

331 gauge group



Electric charge operator

- * dark matter,
- neutrino mass generation and mixing,
- * the strong CP problem, and
- * the number of the fermion families.

Some difficulties in the 331 models

These models:

- * Don't solve the mass hierarchy problem in the SM.
- SU(3) triplets are necessary in the simplest versions.
- predictability.
- * Have more arbitrary energy scale.

* Have a scalar sector with a much larger number of scalar fields. At least more * Have more arbitrary parameters in the model. This provides less



A scale-invariant 331 model: matter content

Charged leptons $\psi_{iL} = (\nu_i, e_i, E_i)_L^T \sim (\mathbf{1}, \mathbf{3}, -2/3), \quad e_{sR} \sim (\mathbf{1}, \mathbf{1}, -1),$

Scalar triplets $\rho = (\rho_1^0, \rho_2^-, \rho_3^-)^T \sim (\mathbf{1}, \mathbf{3}, -2/3),$ $\chi = (\chi_1^+, \chi_2^0, \chi_3^0)^T \sim (\mathbf{1}, \mathbf{3}, 1/3).$

Matter content shared with other 331 models

Quarks

$$Q_{aL} = (d_a, -u_a, U_a)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 1/3),$$

$$Q_{3L} = (u_3, d_3, D)_L^T \sim (\mathbf{3}, \mathbf{3}, 0),$$

$$d_{nR} \sim (\mathbf{3}, \mathbf{1}, -1/3), \qquad u_{mR} \sim (\mathbf{3}, \mathbf{1}, 2/3),$$

A scale-invariant 331 model: additional matter

New charged leptons $\Psi_{iL,R} = (\mathcal{E}_i^+, N_{1i}, N_{2i})_{L,R}^T \sim (\mathbf{1}, \mathbf{3}, 1/3), \qquad \nu_{iR} \sim (\mathbf{1}, \mathbf{1}, 0),$

> Scalar singlet $\varphi \sim (\mathbf{1}, \mathbf{1}, \mathbf{0}),$

 $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \overset{\langle \chi_3^0 \rangle = w/\sqrt{2}}{\longrightarrow} SU(3)_L$

New matter content

New quarks

$$K_{aL,R} = (\mathcal{A}_{a}^{(5/3)}, \mathcal{U}_{1a}, \mathcal{U}_{2a})_{L,R}^{T} \sim (\mathbf{3}, \mathbf{3}, 1),$$

 $K_{3L,R} = (\mathcal{B}^{(-4/3)}, -\mathcal{D}_{1}, \mathcal{D}_{2})_{L,R}^{T} \sim (\mathbf{3}, \mathbf{3}^{*}, -2/3),$

Gauge symmetry breaking pattern

$$\mathcal{B}_C \otimes SU(2)_L \otimes U(1)_Y \overset{\langle \rho_1^0 \rangle = v/\sqrt{2}}{\longrightarrow} SU(3)_C \otimes U(1)_Q,$$



A scale-invariant 331 model: gauge sector

Gauge bosons

Non-hermitian gauge bosons $W_{\mu}^{\pm} = \frac{W_{1\mu} \mp i W_{2\mu}}{\sqrt{2}}, \qquad V_{\mu}^{\pm} = \frac{W_{4\mu} \mp i W_{5\mu}}{\sqrt{2}},$ $V_{\mu}^{0(\dagger)} = \frac{W_{6\mu} \mp i W_{7\mu}}{\sqrt{2}},$

Masses

$$m_{W^{\pm}}^2 = \frac{g^2 v^2}{4}, \qquad m_{V^{\pm}}^2 = \frac{g^2}{4} (v^2 + w^2), \qquad m_{V^0}^2 = \frac{g^2}{4} w^2.$$

Some predictions

$$m_{V^{\pm}}^2 - m_{V^0}^2 = m_{W^{\pm}}^2$$
 and
 $\frac{m_{Z_2}^2}{m_{V^0}^2} = \frac{\cos^2 \theta_W}{\frac{3}{4} - \sin^2 \theta_W} + \mathcal{O}\left(\frac{v^2}{w^2}\right) \approx 1.48$

Neutral gauge bosons $A^{\mu} = \frac{\sqrt{3}}{\sqrt{3} + 4t^2} \left(tW_3^{\mu} + \frac{t}{\sqrt{3}} W_8^{\mu} + B^{\mu} \right),$ $Z_1^{\mu} = N_{Z_2} (-3m_{Z_2}^2 W_3^{\mu} + \sqrt{3} (3m_{Z_2}^2 - g^2 w^2) W_8^{\mu} + g^2 w^2 t B^{\mu}),$ $Z_2^{\mu} = N_{Z_1} (-3m_{Z_1}^2 W_3^{\mu} + \sqrt{3} (3m_{Z_1}^2 - g^2 w^2) W_8^{\mu} + g^2 w^2 t B^{\mu}),$

$$Masses \\ m_{Z_{1}}^{2} = \frac{\sin^{2}\theta_{W}}{1 - \frac{4}{3}\sin^{2}\theta_{W}}, \qquad m_{Z_{1}}^{2} = \frac{g^{2}v^{2}}{4\cos^{2}\theta_{W}} + \mathcal{O}\left(\frac{v^{2}}{w^{2}}\right), \\ m_{Z_{2}}^{2} = \frac{g^{2}\cos^{2}\theta_{W}w^{2}}{3 - 4\sin^{2}\theta_{W}} + \mathcal{O}\left(\frac{v^{2}}{w^{2}}\right),$$

With $\sin^2 \theta_W \simeq 0.231$.



A scale-invariant 331

Extra symmetries:

* Scale invariance and,

TAB	LE I.	Field	charge	s unde	r the Z_8	₃ symm	etry.												
	ψ_{iL}	e_{iR}	E_{iR}	$ u_{iR}$	Q_{aL}	Q_{3L}	u_{iR}	U_{aR}	d_{iR}	D_R	Ψ_{iL}	Ψ_{iR}	K_{aL}	K_{aR}	K_{3L}	K_{3R}	ρ	χ	φ
Z_8	1	6	0	7	2	3	1	3	4	2	6	4	2	0	3	5	2	1	2

This discrete symmetry allows that fermion mass matrices have a see-saw texture

mod	e	l: extra	symme	tries



Why are the extra fermions necessary?

Without them we have that:

$$M_{\mathbf{E}} = \frac{w}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ y^e & y^E \end{pmatrix},$$

Charged lepton mass matrix.

These fermions are massless at all order of the perturbation theory because their masses are protected by accidental symmetries.

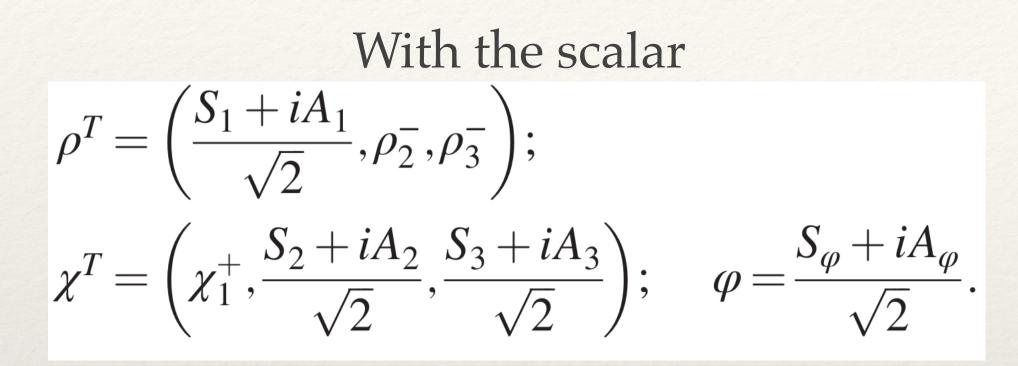
The introduction of the extra fermion fields break these symmetries because of the operators such as

$$\overline{\Psi_R^c}\rho^* e_R, \ \overline{\psi_L}\Psi_L^c\chi^*, \ \overline{Q_L}K_R\rho, \qquad \overline{Q_{3L}}K_{3R}\rho^*,$$

$$Up-quark \text{ mass matrix.}$$
$$M_{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0_{[2\times2]} & 0_{[2\times1]} & 0_{[2\times2]} \\ h_{[1\times2]}^{u} v & h^{u_{3}} v & h_{[1\times2]}^{U} v \\ y_{[2\times2]}^{a} w & y_{[2\times1]}^{u_{3}} w & y_{[2\times2]}^{U} w \end{pmatrix},$$
$$Down-quark \text{ mass matrix.}$$
$$M_{D} = \frac{1}{\sqrt{2}} \begin{pmatrix} h_{[2\times2]}^{d} v & h_{[2\times1]}^{d_{3}} v & h_{[2\times1]}^{D} v \\ 0_{[1\times2]} & 0 & 0 \\ y_{[1\times2]}^{d} w & y^{d_{3}} w & y^{D} w \end{pmatrix},$$



Scalar sector



Applying the copositivity criterium, the positivity of the hermitian matrix and the scalar masses we have

 $\begin{aligned} \lambda_{\rho} \geq 0, & \lambda_{\chi} \geq 0, \\ -2\sqrt{\lambda_{\rho}\lambda_{\chi}} \leq \lambda_{\rho\chi} \leq 2\sqrt{\lambda_{\rho}\lambda_{\chi}}, & -2\sqrt{\lambda_{\rho}\lambda_{\chi}}, \\ & -2\sqrt{\lambda_{\chi}(\lambda_{\varphi}-2|\lambda_{\varphi}'|)} \end{aligned}$

Tree-level scalar potential

$$\begin{split} V_{0} &= \lambda_{\rho} (\rho^{\dagger} \rho)^{2} + \lambda_{\chi} (\chi^{\dagger} \chi)^{2} + \lambda_{\rho \chi} \rho^{\dagger} \rho \chi^{\dagger} \chi \\ &+ \lambda_{\rho \chi}^{\prime} \rho^{\dagger} \chi \chi^{\dagger} \rho + \lambda_{\rho \varphi} \rho^{\dagger} \rho \varphi^{*} \varphi + \lambda_{\chi \varphi} \chi^{\dagger} \chi \varphi^{*} \varphi \\ &+ \lambda_{\varphi} (\varphi^{*} \varphi)^{2} - |\lambda_{\varphi}^{\prime}| (\varphi^{4} + \varphi^{*4}). \end{split}$$

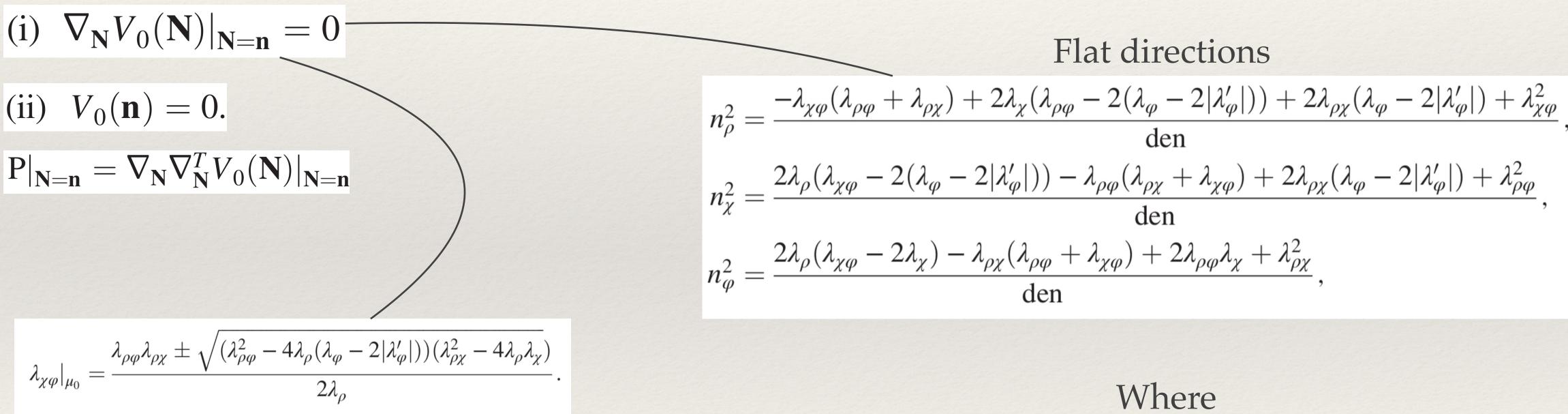
$$\begin{split} \lambda_{\varphi} - 2|\lambda_{\varphi}'| &\geq 0, \qquad \det \Lambda_{0} \geq 0, \\ \sqrt{\lambda_{\rho}(\lambda_{\varphi} - 2|\lambda_{\varphi}'|)} &\leq \lambda_{\rho\varphi} \leq 2\sqrt{\lambda_{\rho}(\lambda_{\varphi} - 2|\lambda_{\varphi}'|)}, \\ &\leq \lambda_{\chi\varphi} \leq 2\sqrt{\lambda_{\chi}(\lambda_{\varphi} - 2|\lambda_{\varphi}'|)}. \end{split}$$



Flat direction

Following the Gildener-Weinberg method, we find, first, the flat direction

Condition for the flat direction



$$den \equiv -4\lambda_{\rho}(\lambda_{\varphi} + \lambda_{\chi} - \lambda_{\chi\varphi} - 2|\lambda_{\varphi}'|) - 2\lambda_{\rho\chi}(\lambda_{\rho\varphi} + \lambda_{\chi\varphi} + 4|\lambda_{\varphi}'|) - 4\lambda_{\chi}(-\lambda_{\rho\varphi} + \lambda_{\varphi} - 2|\lambda_{\varphi}'|) + (\lambda_{\rho\varphi} - \lambda_{\chi\varphi})^{2} + \lambda_{\rho\chi}^{2} + 4\lambda_{\rho\chi}\lambda_{\varphi}.$$



Scalar sector

Charged scalar
$$H^{\pm} = \frac{1}{\sqrt{v^2 + w^2}} (w \rho_3^{\pm} + v \chi_1^{\pm}),$$

$$m_{H^{\pm}}^2 = rac{\lambda'_{
ho\chi}}{2}(v^2 + w^2),$$

 $\begin{aligned} \text{CP-odd} \\ m_{A_{\varphi}}^2 &= 8 |\lambda_{\varphi}'| v_{\varphi}^2. \end{aligned}$

CP -even scalars

$$h \simeq \frac{1}{N_h} \left[S_1 + \frac{\lambda_{\rho\varphi}}{\lambda_{\rho\chi} - \lambda_{\rho\varphi}} \frac{v}{w} S_3 - \frac{v}{v_{\varphi}} S_{\varphi} \right],$$

$$H \simeq \frac{1}{N_H} \left[\frac{\lambda_{\chi}}{\lambda_{\rho\chi} - \lambda_{\rho\varphi}} \frac{v}{w} S_1 + S_3 - \frac{w}{v_{\varphi}} S_{\varphi} \right],$$

$$\begin{split} m_h^2 &= \lambda_\rho v^2 + (\lambda_\varphi - 2|\lambda'_\varphi|) v_\varphi^2 + \lambda_\chi w^2 - m_\Delta^2, \\ m_H^2 &= \lambda_\rho v^2 + (\lambda_\varphi - 2|\lambda'_\varphi|) v_\varphi^2 + \lambda_\chi w^2 + m_\Delta^2, \end{split}$$

Scalon: NG boson of the scale-invariance symmetry

$$S = \frac{1}{\sqrt{v^2 + w^2 + v_{\varphi}^2}} [vS_1 + wS_3 + v_{\varphi}S_{\varphi}].$$

$$m_S^2 = 8B\langle \phi_r \rangle^2,$$

One-loop mass

 $V_{1-\text{loop}}(\boldsymbol{\phi}_r \mathbf{n}) =$

 $A = \frac{1}{64\pi^2 \langle \phi_r \rangle^4} \left[\sum_{\mathcal{S}} n_{\mathcal{S}} m_{\mathcal{S}}^4 \left(\ln \frac{m_{\mathcal{S}}^2}{\langle \phi_r \rangle^2} - \frac{3}{2} \right) \right]$ $+3\sum_{\mathcal{V}} n_{\mathcal{V}} m_{\mathcal{V}}^4 \left(\ln \frac{m_{\mathcal{V}}^2}{\langle \phi_r \rangle^2} - \frac{5}{6} \right)$ $-4\sum_{\mathcal{F}} n_{\mathcal{C}} n_{\mathcal{M}} \operatorname{Tr} \left[M_{\mathcal{F}}^{4} \left(\ln \frac{M_{\mathcal{F}}^{2}}{\langle \phi_{r} \rangle^{2}} - 1 \right) \right] \right],$

One-loop effective potential

$$=A\phi_r^4 + B\phi_r^4 \ln\left(\frac{\phi_r^2}{\mu_0^2}\right),$$

$$B = \frac{1}{64\pi^2 \langle \phi_r \rangle^4} \left[\sum_{\mathcal{S}} n_{\mathcal{S}} m_{\mathcal{S}}^4 + 3 \sum_{\mathcal{V}} n_{\mathcal{V}} m_{\mathcal{V}}^4 - 4 \sum_{\mathcal{F}} n_{\mathcal{C}} n_{\mathcal{M}} \operatorname{Tr}[M_{\mathcal{F}}^4] \right],$$

One-loop effective potential

Using

$$0 = \left[\frac{\partial V_{1-\text{loop}}(\phi_r \mathbf{n})}{\partial \phi_r} \right]_{\langle \phi_r \rangle}$$

 $V_{1-\text{loop}}(\boldsymbol{\phi}_r \mathbf{n}) =$

Working in the approximation

 $v \ll w \ll v_{\varphi}(\simeq \langle \phi_r \rangle).$

$$\langle \phi_r \rangle = \mu_0 \exp\left[-\frac{1}{4} - \frac{A}{2B}\right],$$

$$= B\phi_r^4 \left[\ln\left(\frac{\phi_r^2}{\langle \phi_r \rangle^2}\right) - \frac{1}{2} \right],$$

$$B = \frac{1}{64\pi^2 \langle \phi_r \rangle^4} \left[\sum_{\mathcal{S}} n_{\mathcal{S}} m_{\mathcal{S}}^4 + 3 \sum_{\mathcal{V}} n_{\mathcal{V}} m_{\mathcal{V}}^4 \right] - 4 \sum_{\mathcal{F}} n_{\mathcal{C}} n_{\mathcal{M}} \operatorname{Tr}[M_{\mathcal{F}}^4],$$

Fermion spectrum: neutrinos

Yukawa Lagrangian of the leptons

$$\mathcal{L}_{l} = y_{ij}^{E} \overline{\psi}_{iL} \chi E_{jR} + h_{ij}^{\nu} \overline{\psi}_{iL} \rho$$
$$+ \frac{f_{ij}^{\nu}}{2} \varphi \overline{\nu}_{iR}^{c} \nu_{jR} + f_{ij}^{\Psi} \varphi \overline{\Psi}$$

Basis

 $\tilde{\mathbf{N}}_L \equiv (\nu_L, \nu_R^c, N_{1L}, N_{1R}^c)^T$

Type-I See-saw texture

 $M_{\tilde{\mathbf{N}}} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_{\varphi} \end{pmatrix}$

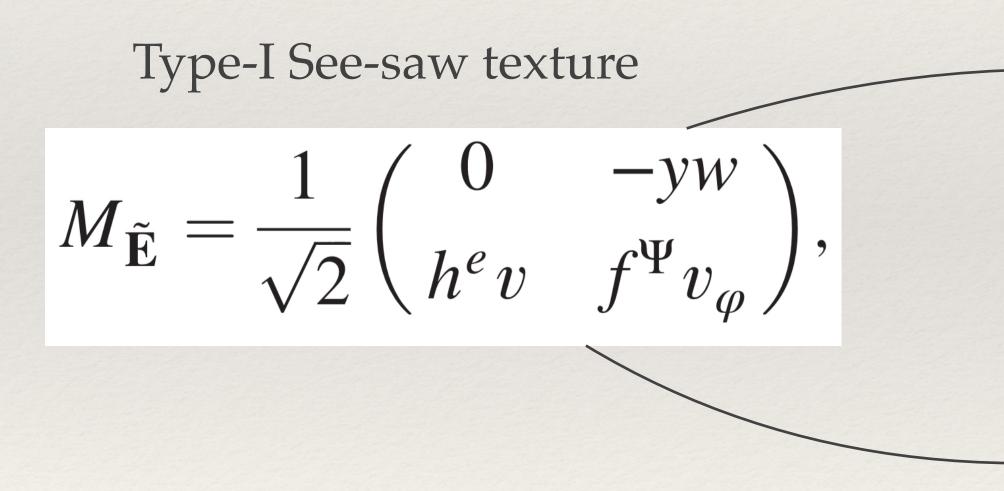
 $M_{N_2} = \frac{v_{\varphi}}{\sqrt{2}} f^{\Psi}.$

 $\overline{\Psi_{iL}} \Psi_{jR} + h_{ij}^e \overline{\Psi_{iR}^c} \rho^* e_{jR} + y_{ij} \overline{\Psi_{iL}} \Psi_{jL}^c \chi^*$ $\overline{\Psi_{iL}} \Psi_{jR} + \text{H.c.}, \qquad (38)$

 $\begin{array}{l} \mathcal{W}_{1L}, N_{1R}^c)^T \\ \mathcal{W}_{1L}, N_{1R}^c)^T \\ \mathcal{W}_{1L}, N_{1R}^c)^T \\ \mathcal{W}_{2L} \\ \mathcal{W}_{2$

Fermion spectrum: charged leptons

Basis
$$(e, \mathcal{E}^{+c})_{L(R)}^T$$



All of the fermion mass matrices have the same type-I see-saw texture

Masses $M_E = \frac{w}{\sqrt{2}} y^E,$ $M_{E'}^2 \simeq \frac{v_{\varphi}^2}{2} f^{\Psi} f^{\Psi^{\dagger}}$

$$M_{e'}^2 \simeq \frac{v^2 w^2}{2 v_{\varphi}^2} y(f^{\Psi})^{-1} h^e [y(f^{\Psi})^{-1} h^e]^{\dagger}$$

 $\mathcal{G} = -4T$

TABLE II. symmetries.

	ψ_{iL}	e_{iR}	E_{iR}	Q_{aL}	Q_{3L}	u_{iR}	U_{aR}	d_{iR}	D_R	$\Psi_{iL,R}$	$K_{aL,R}$	$K_{3L,R}$	ρ	χ	W^+_μ	V^+_μ	V^0_μ
											1						
$U(1)_{\mathcal{G}}$	$\begin{pmatrix} 0\\ 4\\ -1 \end{pmatrix}$	4	-1	$\begin{pmatrix} 3\\-1\\4 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$	-1	4	3	-2	$\begin{pmatrix} -4 \\ 0 \\ -5 \end{pmatrix}$	$\begin{pmatrix} 0\\ 4\\ -1 \end{pmatrix}$	$\begin{pmatrix} 2\\ -2\\ 3 \end{pmatrix}$	$\begin{pmatrix} 0\\ 4\\ -1 \end{pmatrix}$	$\begin{pmatrix} 1\\5\\0 \end{pmatrix}$	-4	1	5

U(1) generator

$$T_3 + 2\sqrt{3}T_8 + N,$$

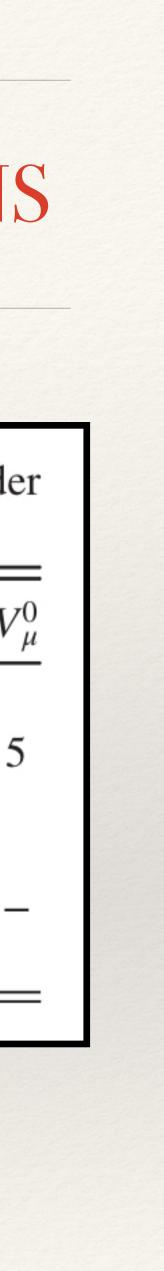
Field charges under the $U(1)_N$ and $U(1)_G$ symmetries. The fields not shown above do not carry charges under these



these symmetries.

	ψ_{iL}	e_{iR}	E_{iR}	Q_{aL}	Q_{3L}	<i>u</i> _{iR}	U_{aR}	d_{iR}	D_R	$\Psi_{iL,R}$	$K_{aL,R}$	$K_{3L,R}$	ρ	χ	W^+_μ	V^+_μ	V_{i}^{\prime}
$U(1)_{\mathcal{G}'}$													$\begin{pmatrix} 0\\ 4\\ -1 \end{pmatrix}$				
\mathcal{P}	$\begin{pmatrix} + \\ + \\ - \end{pmatrix}$	+		$\begin{pmatrix} + \\ + \\ - \end{pmatrix}$	$\begin{pmatrix} + \\ + \\ - \end{pmatrix}$	+		+		$\begin{pmatrix} + \\ + \\ - \end{pmatrix}$	$\begin{pmatrix} -\\ -\\ + \end{pmatrix}$	$\begin{pmatrix} -\\ -\\ + \end{pmatrix}$	$\begin{pmatrix} + \\ + \\ - \end{pmatrix}$	$\begin{pmatrix} -\\ -\\ + \end{pmatrix}$	+	_	_
								-Ger	nerat	ors							
				<i>G</i> ′ =	$= \mathcal{G} - \mathcal{I}$	3 B ,					\mathcal{P}	= (-1)	1) ^G				

TABLE III. Field charges under $U(1)_{\mathcal{G}}$ and its parity subgroup $\mathcal{P} = (-1)^{\mathcal{G}}$. The fields not displayed here transform trivially under



Dark matter candidates N_2 and V^0

 $\langle \sigma v_{\rm rel} \rangle_{v \to 0} \simeq \frac{5 \alpha^2 m_X^2}{8 s_W^4 m_W^4}$

But,
$$m_{N_2}(v_{\varphi}) \gg m_{V^0}(w)$$

So, V^0 is the lightest \mathcal{P} -odd particle.

$$\Omega_X h^2 \simeq \frac{0.1 \text{ pb}}{\langle \sigma_{\text{tot}} v_{\text{rel}} \rangle} < \frac{0.1 \text{ pb}}{\langle \sigma v_{\text{rel}} \rangle}$$
$$\simeq 0.0024 \times \left(\frac{m_W}{m_X}\right)^2 < 0.00008,$$



Some signal in colliders

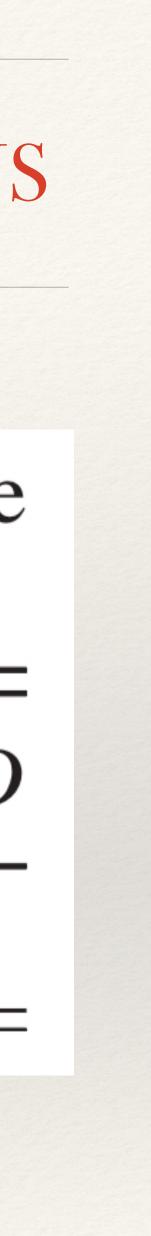
 $pp \rightarrow D\bar{D} \rightarrow bV^0\bar{b}V^{0\dagger}$. $\rightarrow bV^0 \overline{t}V^+ \rightarrow bV^0 \overline{t}t \overline{b}V^{0\dagger},$ $\rightarrow tV^-\bar{b}V^{0\dagger} \rightarrow t\bar{t}bV^0\bar{b}V^{0\dagger},$ $\rightarrow tV^-\bar{t}V^+ \rightarrow t\bar{t}bV^0\bar{t}t\bar{b}V^{0\dagger},$



TABLE IV. Mass benchmarks for the particles at the 3-3-1 scale w = 10 TeV. See the text for details.

Mass (GeV) 800 3500

V^0_μ	V^{\pm}_{μ}	$Z_{2\mu}$	E_i, U_a, D
3264	3265	3974	3535



Conclusions

- this type of models.
- to v/v_{φ} , $w * v/v_{\varphi}^2$.
- explanation for the mass hierarchy between the third and first two families.

* We have proposed the minimal scale invariant 3-3-1 model, based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ symmetry and scale invariance with the simplest scalar potential for

* Together with the gauge and scale symmetries, the Z_8 symmetry makes evident the seesaw texture in most of the fermion mass matrices provided that $v_{\varphi} > > w > > v$. This point is useful to mitigate possible phenomenological issues associated with flavor changing neutral currents because, in this case, the suppressed mixing between light and heavy fermions are proportional

* Interestingly, once the seesaw mechanism takes place, the heavy masses of the extra fermions, proportional to v_{ω} , suppress the masses of some of the standard ones providing thus an

