Moving beyond QCD improved parton model

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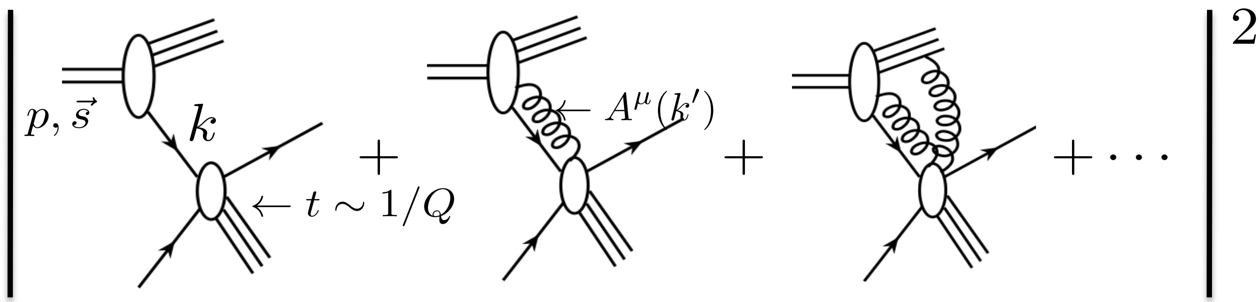
Outline

- Success of QCD improved parton model (PM)
- Hadron properties beyond PDFs
- Potential observables to probe dynamics beyond PM
- One example: single transverse spin asymmetries
- Effect of color Lorentz and magnetic force
- Summary and outlook

Parton, hadron, and cross section

❑ Theorists' view of hadronic cross section:

$\sigma(Q,\vec{s}) \propto$



Any number of partons could participate in the collision

❑ Large momentum transfer simplifies the picture:

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

Single hard scale ➡ Leading power ➡ Collinear factorization

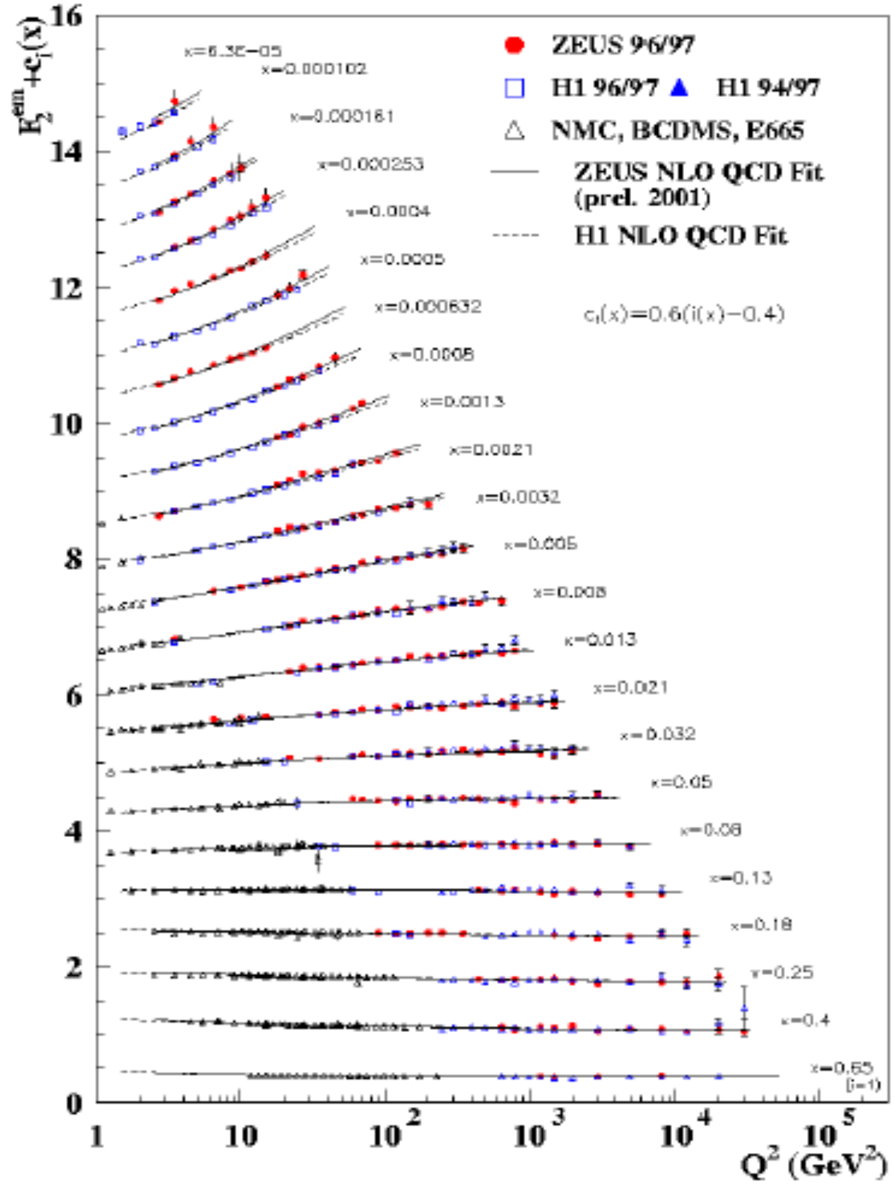
$$\sigma^{(2)}_{AB}(Q,\vec{s}) = \hat{\sigma}_{ab}(x,x',Q) \otimes f_{a/A}(x,Q,\vec{s}) \otimes \left[f_{b/B}(x',Q) \otimes \cdots \right]$$

❑ Predictive power:

Short-distance dynamics, PDFs, and FFs

It worked beautifully – great success of QCD!

Leading power QCD vs DIS data



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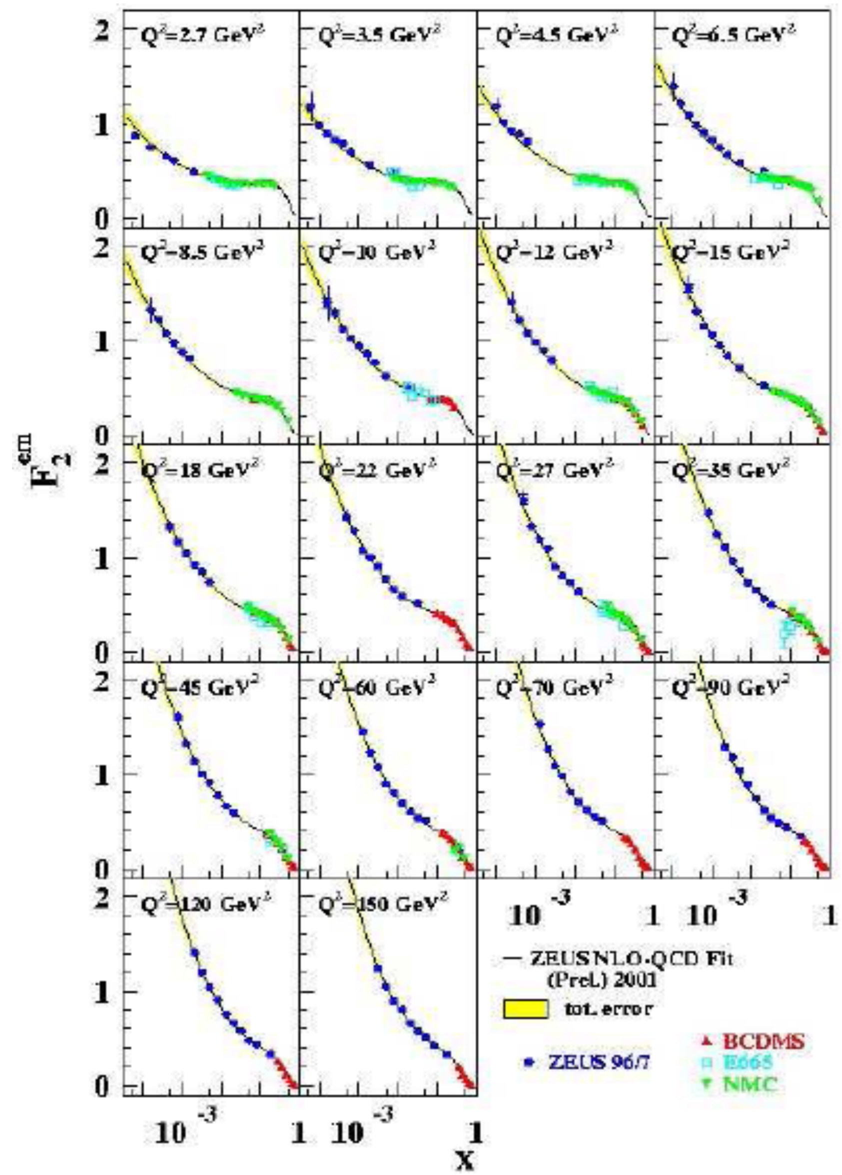
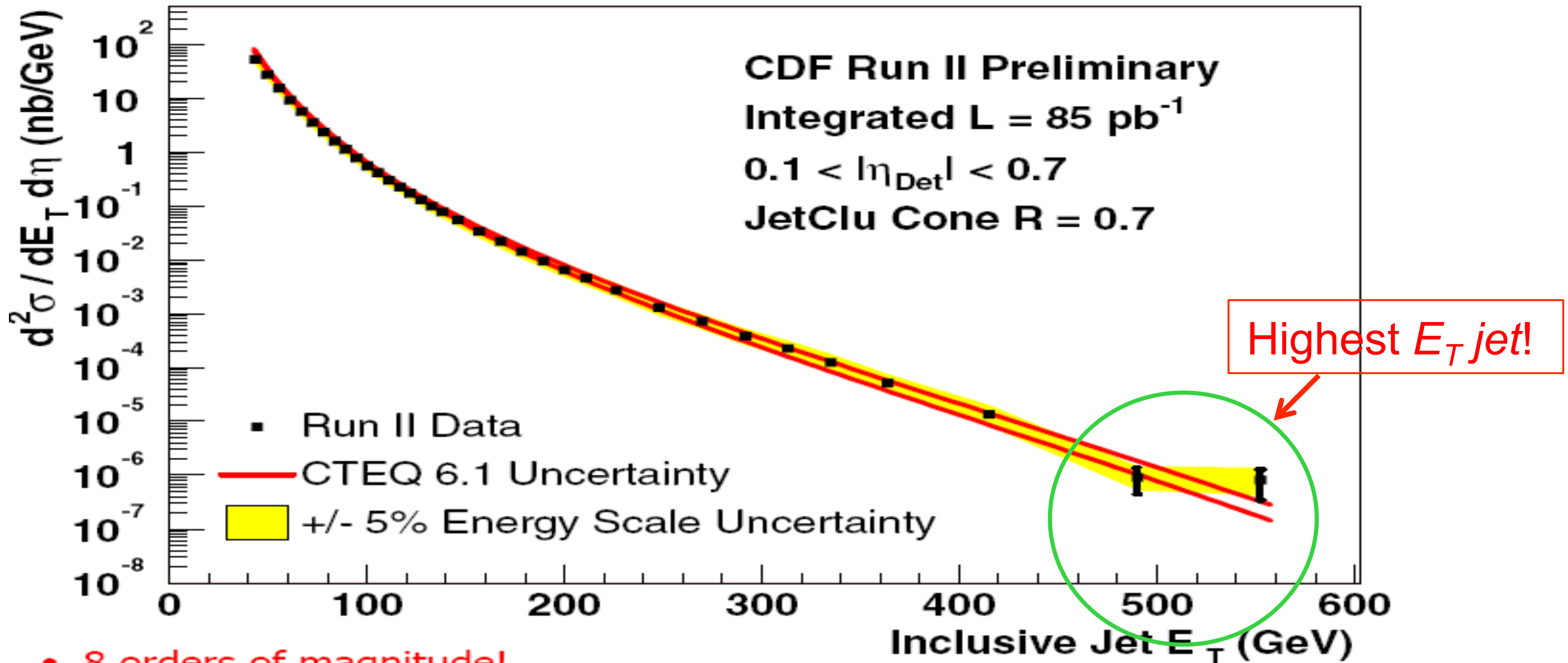
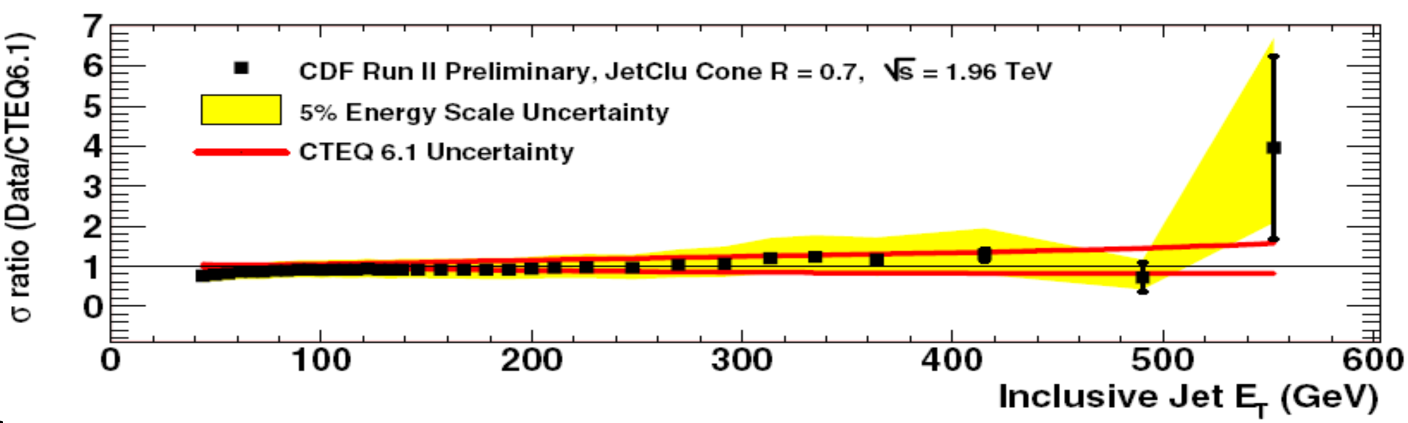


Leading power QCD vs hadronic jet data

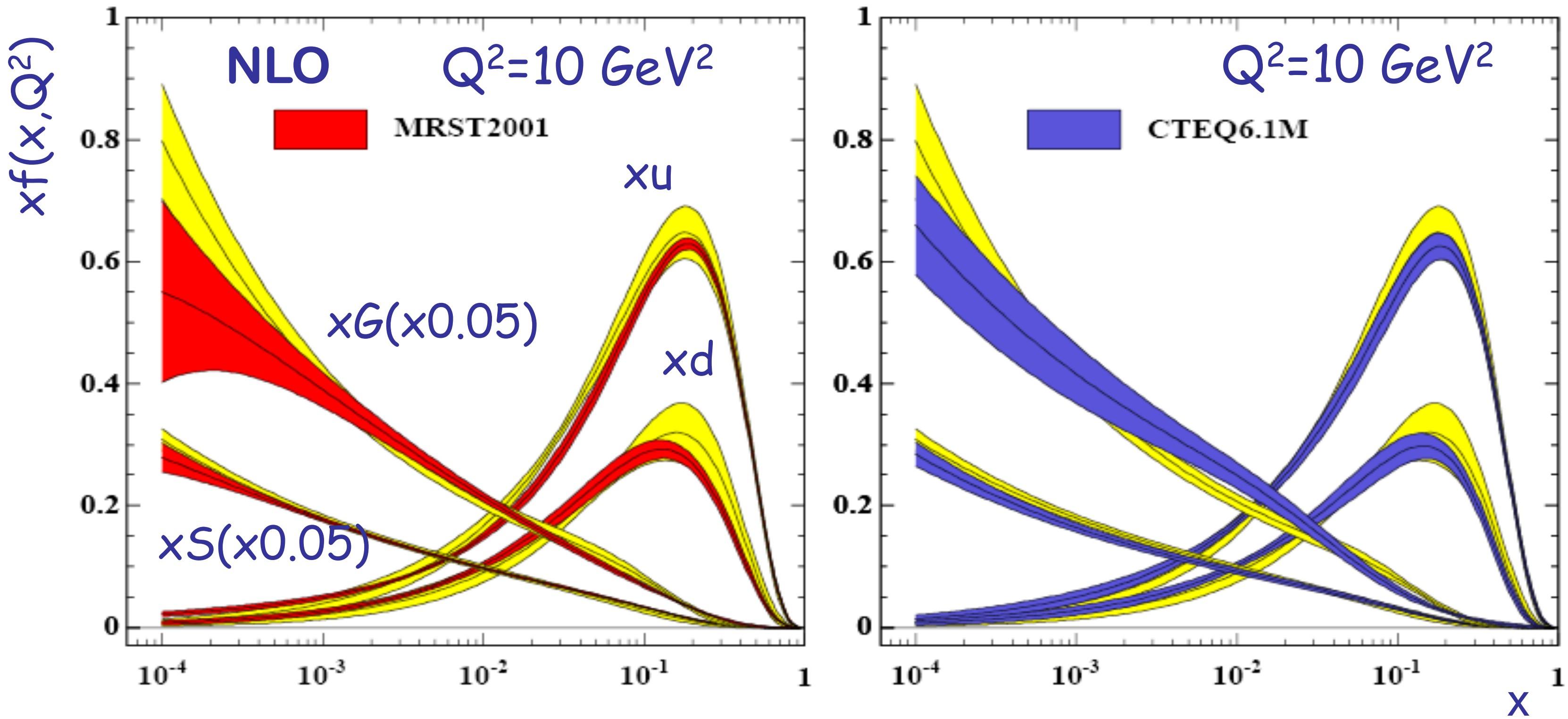


- 8 orders of magnitude!



Success of leading power QCD

- Universality of PDFs – one set for all data:



- Robust calculation of partonic dynamics in powers of α_s

Consistently fit almost all data with Q > 2 GeV

Question

- What have we learned about QCD from high energy collisions and the leading power formalism?
  - Asymptotic freedom of QCD – short-distance dynamics < 0.1/fm
  - Collinear factorization works beautifully – identified hadron involved

approximation: all hard collisions are between collinear partons

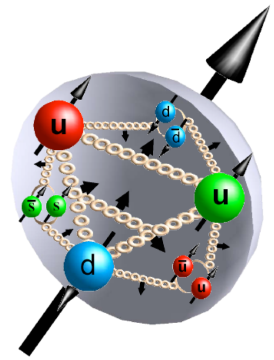
$$\sigma_{AB}^{(2)}(Q,\vec{s}) = \hat{\sigma}_{ab}(x,x',Q) \otimes f_{a/A}(x,Q,\vec{s}) \otimes \left[f_{b/B}(x',Q) \otimes \cdots\right]$$

Bottom line:

We learned enough to be confident to use leading power QCD factorization formalism to calculate and to predict the event structure at the LHC, and to discover the new physics …

More questions

- How much have we learned about the hadron structure from high energy experiments? **NOT much!**
  - PDFs: $q_f(x,Q)$, $g(x,Q)$ – a "probability density" to find a parton of momentum fraction x – probed at a scale Q
  - Helicity distribution functions: $\Delta q_f(x,Q)$, $\Delta g(x,Q)$
  - Hadronization – fragmentation functions: $D_{f\to h}(z,Q), \ldots$



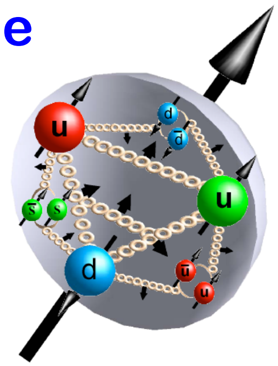
- Hadron structure is much more richer!

  Proton: mass, spin, electric charge, magnetic moment, …

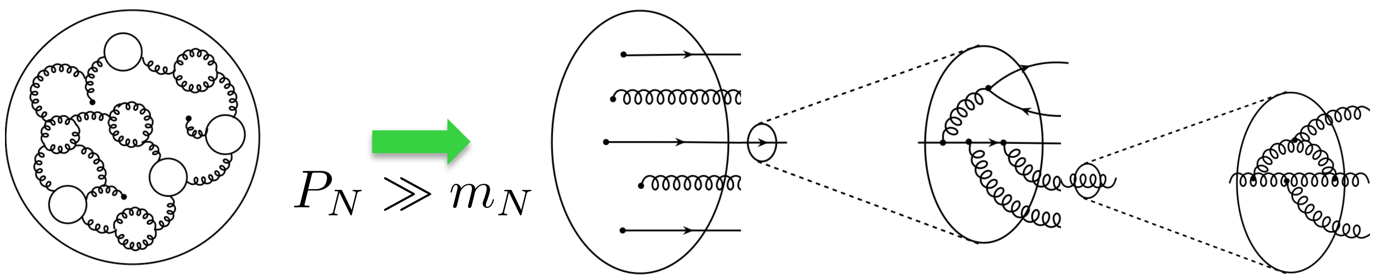
- Explain these properties in terms of QCD: quarks, gluons, and their dynamics? **Too hard a problem?**

Quark-gluon structure of a hadron?

- **Hadron is a dynamical system of quarks and gluons:**
  - **Mass: mainly from energy of quarks and gluons**
  - **Spin: a composite system without localized color charge**
  - **Structure: quantum fluctuations at various time scales**



- **Picture of the structure is "probe" sensitive!**



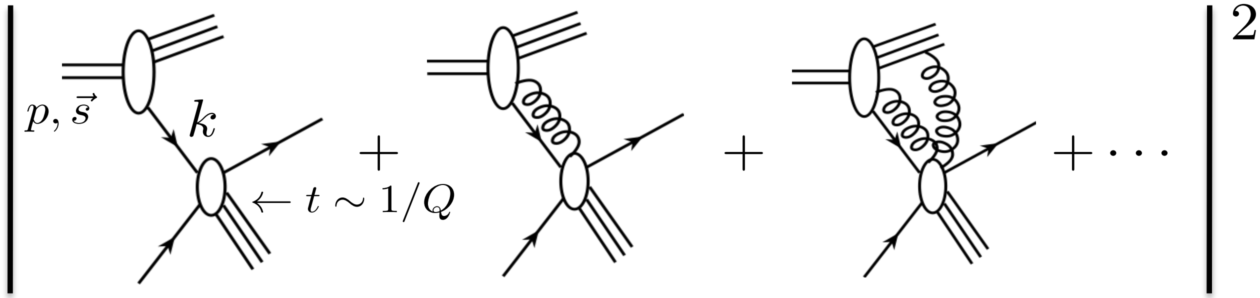
"seen" by a hard probe

- **Localized hard probe: 1/Q >> 1/fm**
  - **More sensitive to short-distance quantum fluctuation**
  - **but, not sensitive to long-range coherence – hadron structure**

Moving beyond the local density?

We measure cross sections:

$$\sigma(Q,\vec{s}) \propto$$



$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

Too large to compete?

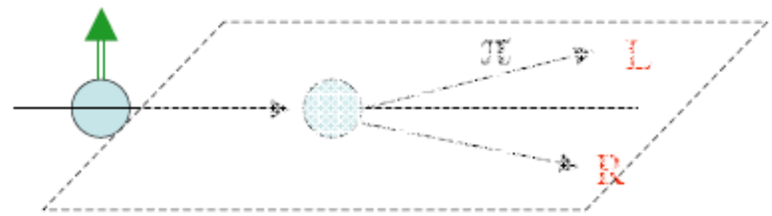
Explore new observables:

- **Spin asymmetry:** $\sigma_{AB}(Q,\vec{s}) - \sigma_{AB}(Q,-\vec{s})$ if the 1st term cancels
- **Small-x probes – hard probe is NOT local – size (or A)-dependence!**
- **Multiple observed scales – TMD, GPD, …** $2R \gg \frac{1}{xp} \gtrsim 2R\,\frac{m}{p}$

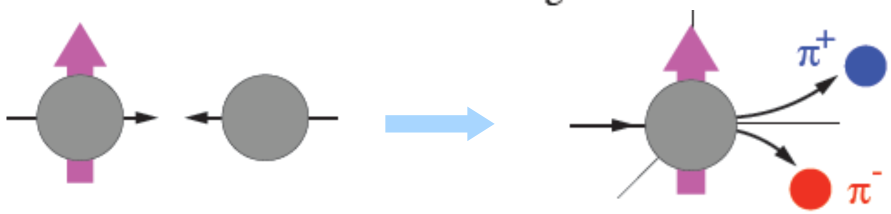
$$Q \gg Q' \gtrsim 1/\mathrm{fm} \sim \Lambda_{\mathrm{QCD}}$$

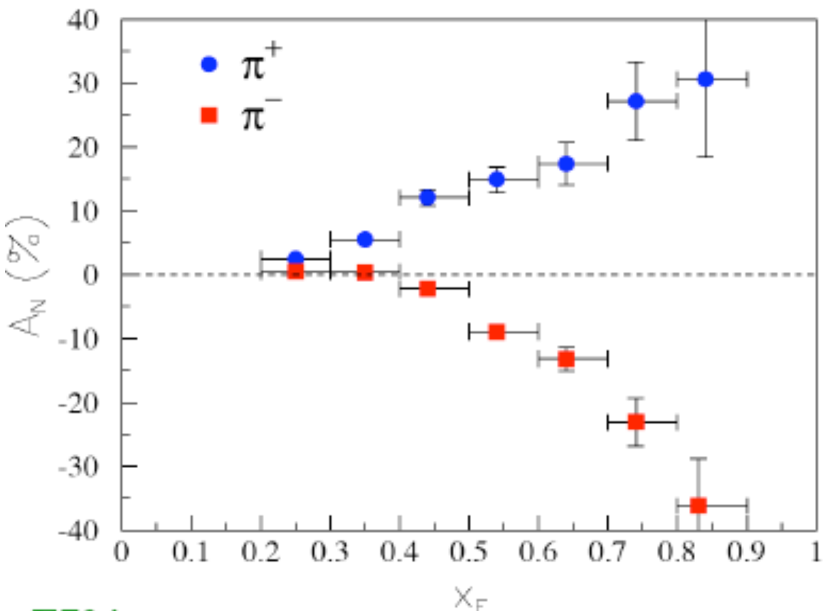
Large SSA in hadronic collisions

- Hadronic $p\uparrow + p \to \pi(l) X$:



$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$





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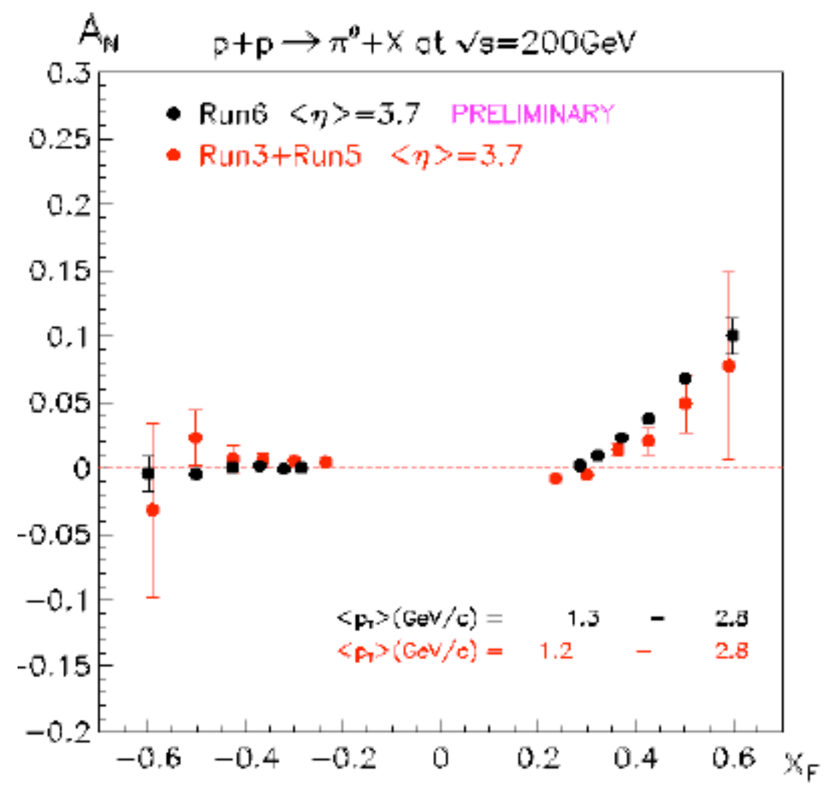
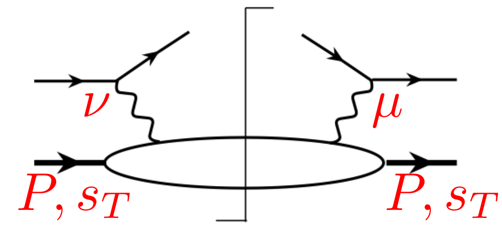


STAR (BRAHMS, too)

Single transverse spin asymmetry - A_N

- A_N = 0 for inclusive DIS – one photon exchange:
- DIS cross section: $\sigma(s_T) \propto L^{\mu\nu} W_{\mu\nu}(s_T)$
- Leptonic tensor is symmetric: $L^{\mu\nu} = L^{\nu\mu}$



- Hadronic tensor: $W_{\mu\nu}(s_T) \propto \langle P, s_T | j^\dagger_\mu(0) j_\nu(y) | P, s_T \rangle$
- Polarized cross section: $\Delta\sigma(s_T) \propto L^{\mu\nu} \left[W_{\mu\nu}(s_T) - W_{\mu\nu}(-s_T) \right]$
- P and T invariance:

$$\langle P, s_T | j^\dagger_\mu(0) j_\nu(y) | P, s_T \rangle = \langle P, -s_T | j^\dagger_\nu(0) j_\mu(y) | P, -s_T \rangle$$

$$\Longleftrightarrow \quad W_{\mu\nu}(-s_T) = W_{\nu\mu}(s_T)$$

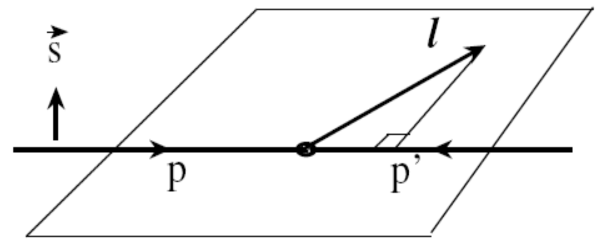
$$\Longrightarrow \quad \Delta\sigma(s_T) \propto L^{\mu\nu} \left[W_{\mu\nu}(s_T) - W_{\mu\nu}(-s_T) \right] = L^{\mu\nu} \left[W_{\mu\nu}(s_T) - W_{\nu\mu}(s_T) \right] = 0$$

Symmetry plays a crucial role in SSAs

Minimum conditions for A_N =\= 0

- SSA corresponds to a naively T-odd triple product:

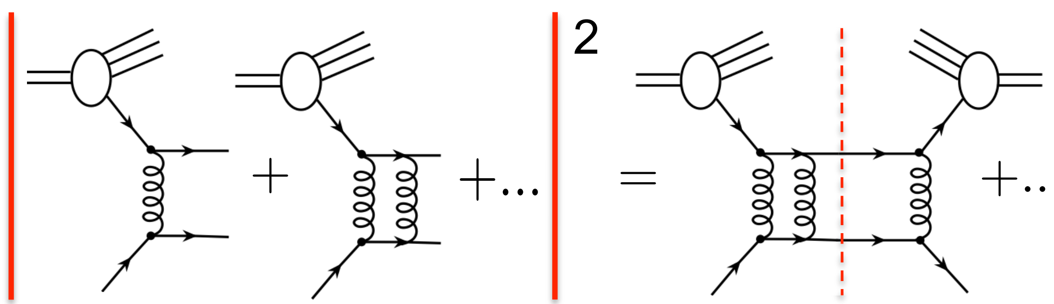
$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \;\Rightarrow\; i\,\epsilon^{\mu\nu\alpha\beta}\,p_\mu s_\nu \ell_\alpha p'_\beta$$



Novanish A_N requires a phase, enough vectors to fix a scattering plan, and a spin flip at the partonic scattering

- Leading power in QCD:

$$\sigma_{AB}(p_T, \vec{s}) \propto$$



$$\propto \alpha_s \frac{m_q}{p_T}$$

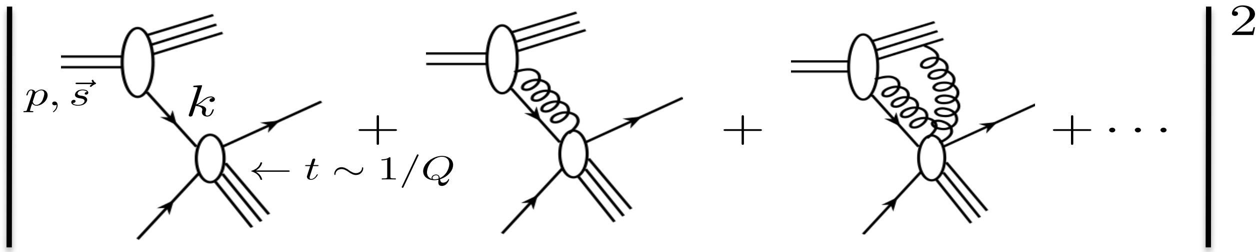
Kane, Pumplin, Repko, PRL, 1978

A_N=\=0 in collinear factorization

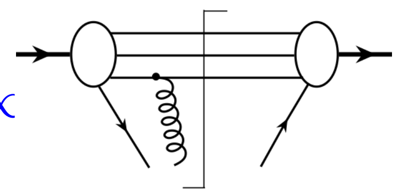
Efremov, Teryaev, 82; Qiu, Sterman, 91

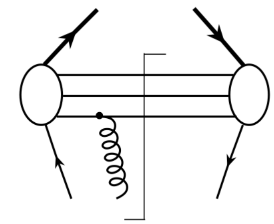
- **A_N – twist-3 effect:**

$\sigma(Q, \vec{s}) \propto$



$$\Delta(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D_f(z) + \delta q_f(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z)$$

$T^{(3)}(x,x) \propto$ 

$D^{(3)}(z,z) \propto$ 

Qiu, Sterman, 1991

Kang, Yuan, Zhou, 2010

- **Spin flip:**
  - Interference of single parton and a two-parton composite state
- **The phase:**
  - Interference of Real and Imaginary part of scattering amplitude
  - gluonic pole: $\propto T^{(3)}(x,x)$
  - fermionic pole contribution $\propto T^{(3)}(x,0)$ or $T^{(3)}(0,x)$

Features of A_N in collinear factorization

Qiu, Sterman, 91

❑ Factorization is valid (as good as leading power):

$$\Delta\sigma_{AB\to h}(p_T,\vec{s}_T)=\sum_{abc}T^{(3)}_{a/A}(x,\vec{s}_T)\otimes f_{b/B}(x')\otimes\hat{\sigma}_{ab\to c}(p_T,\vec{s}_T)\otimes D_{c\to h}(z)$$

Qiu, Sterman, 1991,98

$$+\sum_{abc}\delta q^{(2)}_{a/A}(x,\vec{s}_T)\otimes f_{b/B}(x')\otimes\hat{\sigma}'_{ab\to c}(p_T,\vec{s}_T)\otimes D^{(3)}_{c\to h}(z)$$

Kang, Yuan, Zhou, 2010

Kanazawa, Koike, 2000

$$+\sum_{abc}\delta q^{(2)}_{a/A}(x,\vec{s}_T)\otimes f^{(3)}_{b/B}(x')\otimes\hat{\sigma}''_{ab\to c}(p_T,\vec{s}_T)\otimes D_{c\to h}(z)$$

❑ Generic features:

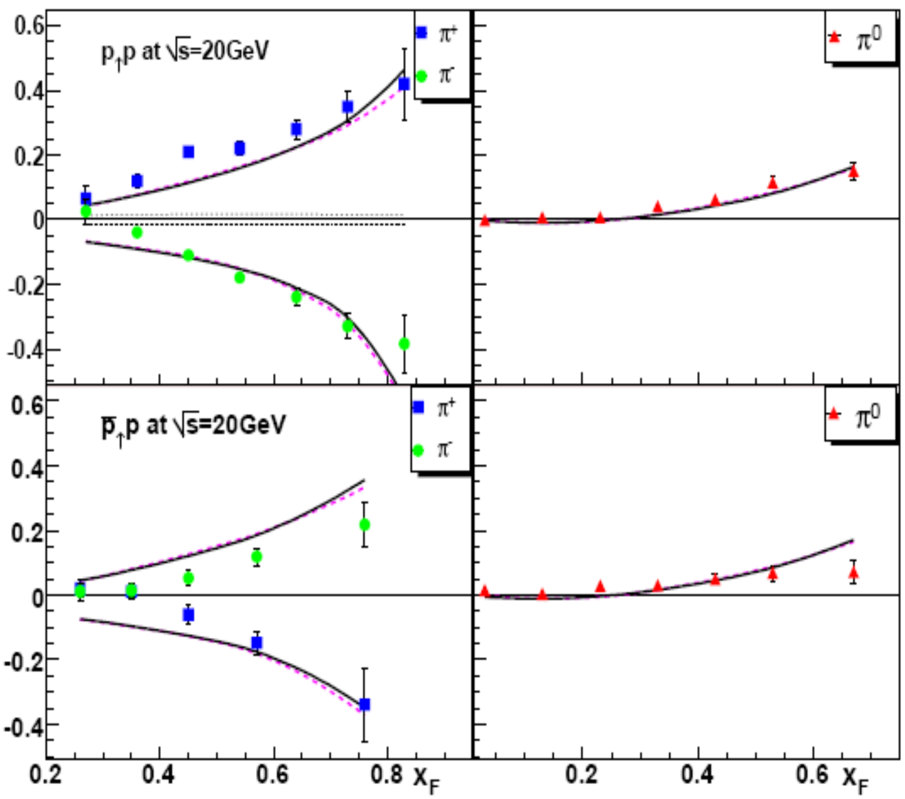
$$A_N\propto\frac{\epsilon_\perp^{\alpha\beta}s_\alpha p_{T\beta}}{-\hat{t}}\left[-x\frac{d}{dx}T^{(3)}(x,x)\right]\propto\frac{\epsilon_\perp^{\alpha\beta}s_\alpha p_{T\beta}}{p_T^2}\left[\frac{n}{1-x}\right]$$

$$\text{if }T^{(3)}(x,x)\propto q(x)\propto(1-x)^n$$

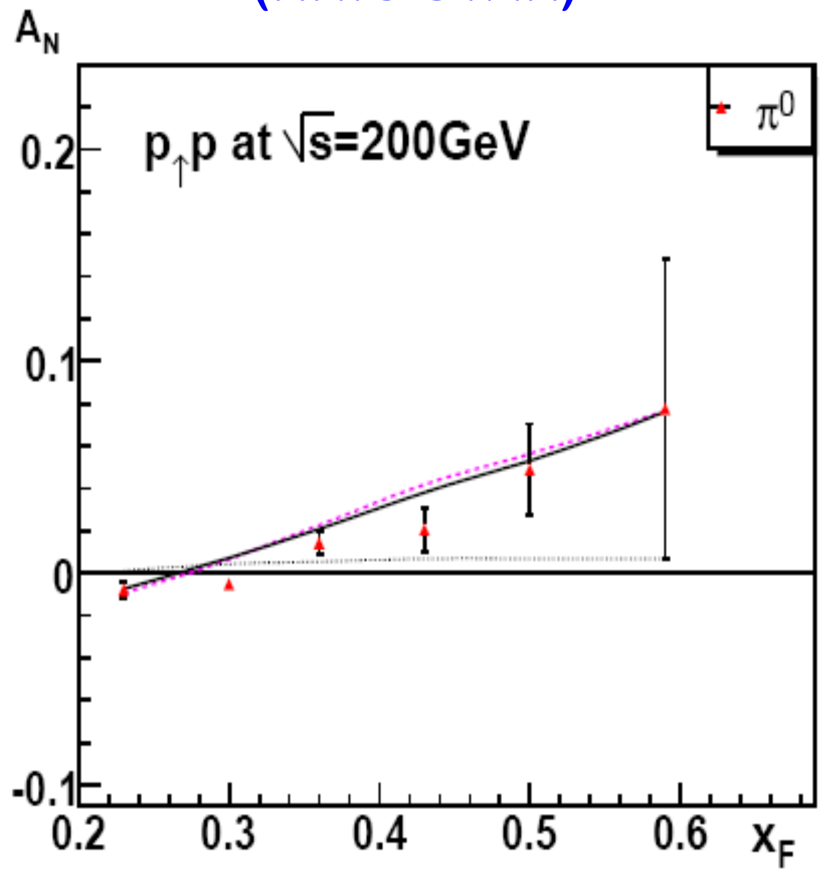
- A_N falls as $1/p_T$ if p_T is large
- A_N increases as x_F if x_F is large

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



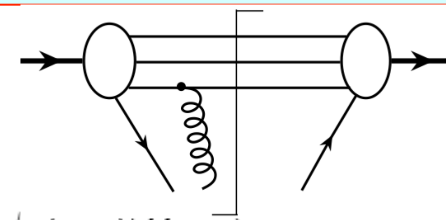
(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function ➡ Nonvanish transverse motion

Twist-3 distributions relevant to SSA



❑ Two-sets Twist-3 correlation functions:

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\, \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle\, (i\epsilon_{\perp\rho\lambda})$$

No probability interpretation!

Kang, Qiu, 2009
Braun, et al 2009

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

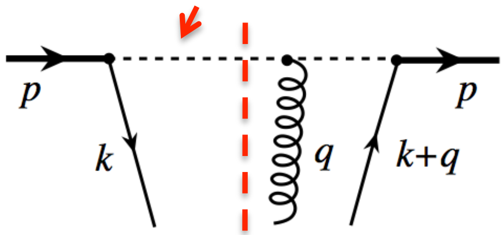
$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

Model calculation for twist-3 distributions

Kang, Qiu, Zhang, 2010

- **Quark-diquark model of nucleon:**

Scalar or axial-vector spectator



Cut-vertex

$$\mathcal{V}^{\rm LC}_{q,F} = \frac{\gamma^+}{2p^+} 2\pi g \delta\left(x - \frac{k^+}{p^+}\right) y\, \delta\left(y - \frac{q^+}{p^+}\right) \left(i\, \epsilon^{s_T \mu n \bar{n}}\right) [-g_{\mu\sigma}]\, \mathcal{C}_q$$

$$\mathcal{V}^{\rm LC}_{\Delta q,F} = \frac{\gamma^+\gamma^5}{2p^+} 2\pi g \delta\left(x - \frac{k^+}{p^+}\right) y\, \delta\left(y - \frac{q^+}{p^+}\right) \left(-s_T^\mu\right) [-g_{\mu\sigma}]\, \mathcal{C}_q$$

- **Only diagonal quark-gluon distribution is finite:**

At this order:

$$T_{q,F}(x,0) = T_{q,F}(0,x) = 0$$

$$T_{\Delta q,F}(x,0) = T_{\Delta q,F}(0,x) = 0$$

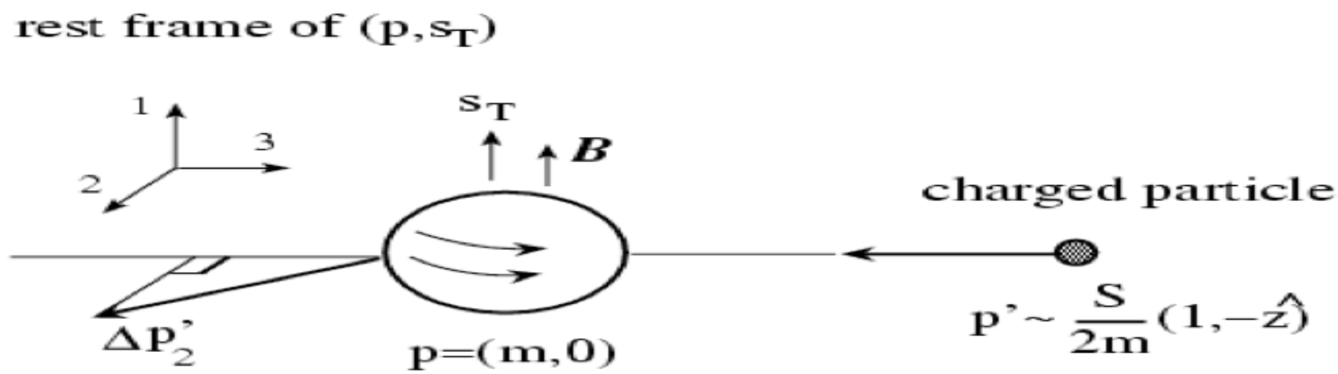
$$T_{\Delta q,F}(x,x) = 0$$

$$T^{(s)}_{q,F}(x,x)\Big|_{\rm dipol\cdot} = \frac{N_c C_F g \lambda_s^2 g_s}{16(2\pi)^3}(1-x)^3(m+xM)\left(\frac{\Lambda_s^2}{L_s^2(\Lambda_s^2)}\right)^2$$

$$T^{(v)}_{q,F}(x,x)\Big|_{\rm dipolar} = \frac{N_c C_F g \lambda_v^2 g_v}{16(2\pi)^3}\, x(1-x)^2(m+xM)\left(\frac{\Lambda_s^2}{L_s^2(\Lambda_s^2)}\right)^2$$

What the twist-3 distribution can tell us?

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_{\sigma}{}^{+}$$

- The total change: $\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_{\sigma}{}^{+}(y^-)}$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Evolution equations and evolution kernels

- **Evolution is a prediction of QCD:**

  Like twist-2 PDFs, both collinear and UV divergence are logarithmic, and share the same slope

  Kang, Qiu, 2009

  → **Evolution equation for factorization scale dependence = renormalization group equation for UV renormalization**

- **Evolution kernels are process independent:**

  - **Calculate directly from the variation of process independent twist-3 distributions**

    Kang, Qiu, 2009
    Yuan, Zhou, 2009

  - **Extract from the scale dependence of the NLO hard part of any physical process**

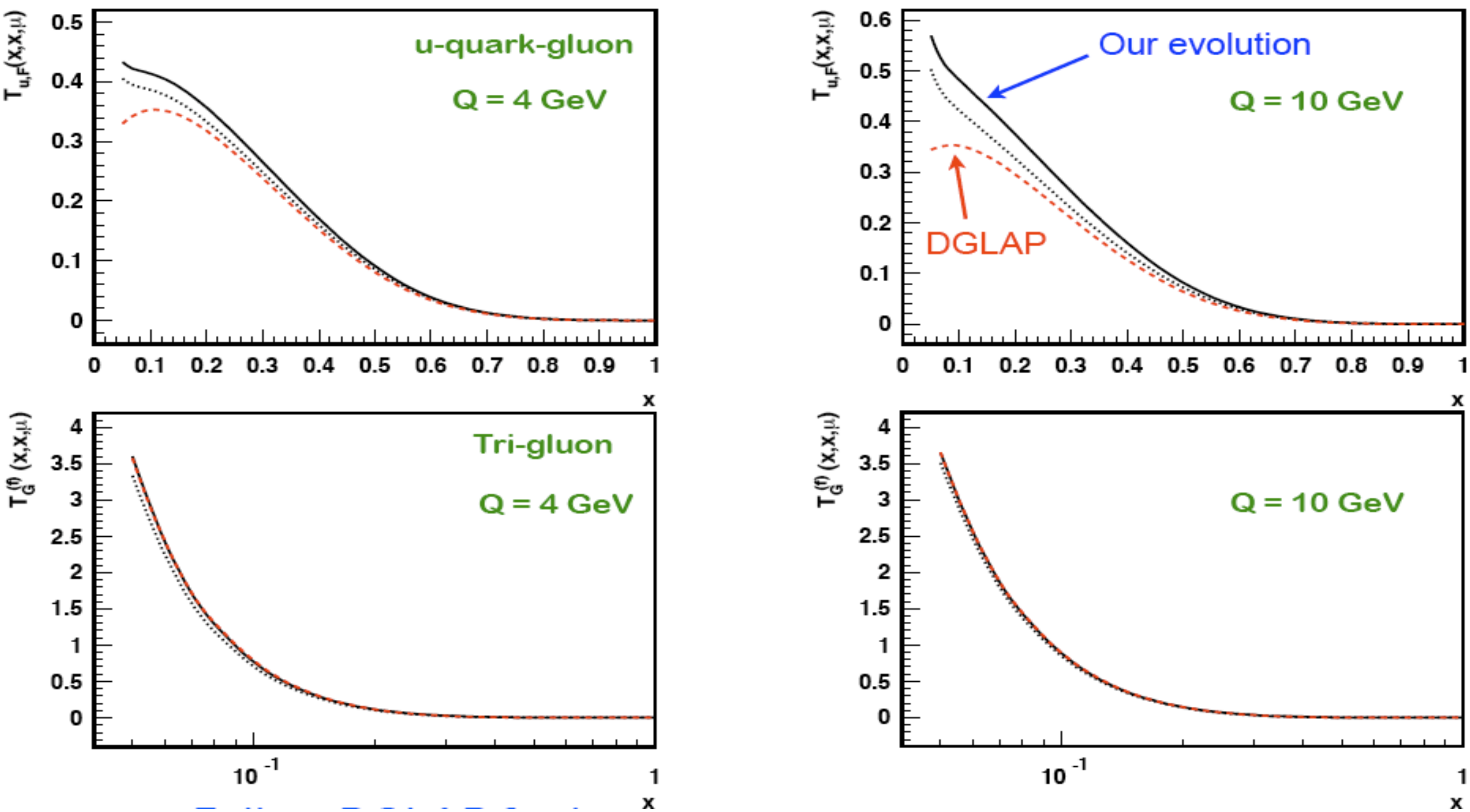
    Vogelsang, Yuan, 2009

  - **UV renormalization of the twist-3 operators**

    Braun et al, 2009

  - **All approaches are equivalent and should give the same kernel**

Scaling violation of twist-3 correlations



- Follow DGLAP at large x
- Large deviation at low x (stronger correlation)

Kang, Qiu, PRD, 2009

Multi-gluon correlation functions

❑ Diagonal tri-gluon correlations:

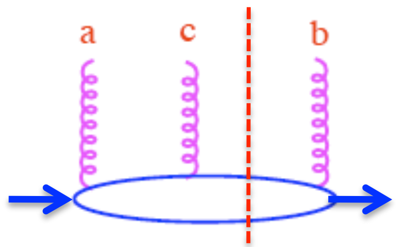
Ji, PLB289 (1992)

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y_1^-}$$
$$\times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-)\right] F^{\alpha+}(y_1^-) | P, s_\perp \rangle$$

❑ Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

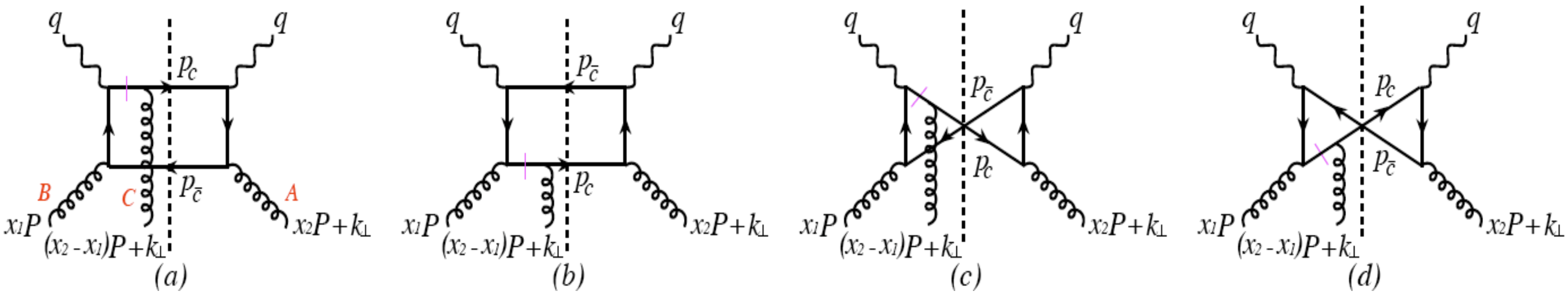
❑ D-meson production at EIC:

- ❖ Clean probe for gluonic twist-3 correlation functions
- ❖ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at future EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- Active parton momentum fraction cannot be too large
- Intrinsic charm contribution is not important
- Sufficient production rate

❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

- SSA is directly proportional to tri-gluon correlation functions
- Any small A_N discovers the tri-gluon correlation!

Features of the SSA in D-production at EIC

- **Dependence on tri-gluon correlation functions:**

$$D - \text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D} - \text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- **Model for tri-gluon correlation functions:**

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- **Kinematic constraints:**

$$x_{min} = \begin{cases} x_B \left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B \left[1 + \frac{2m_c^2}{Q^2}\left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

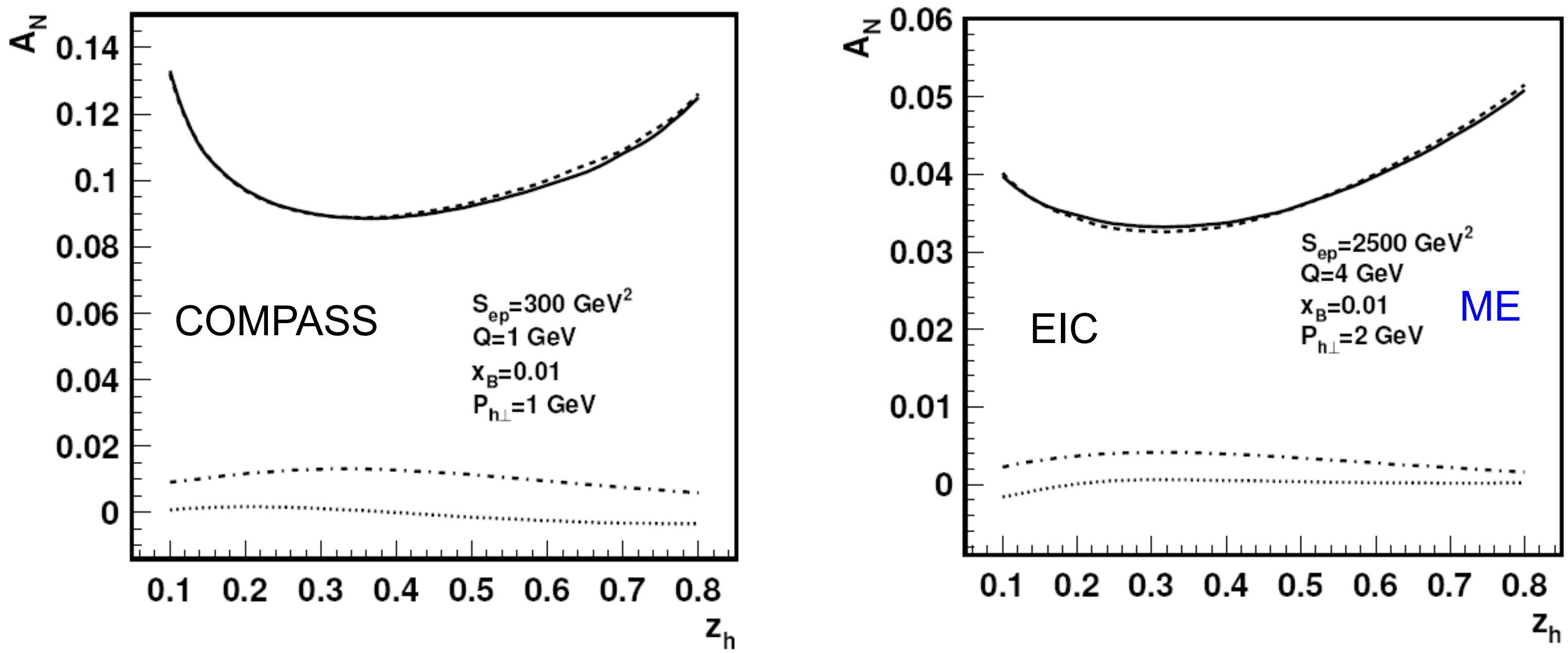
Note: The $z_h(1-z_h)$ has a maximum

SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- SSA for D^0 production (λ_f only):



- Derivative term dominates, and small φ dependence
- Asymmetry is twice if $T_G^{(f)} = +T_G^{(d)}$, or zero if $T_G^{(f)} = -T_G^{(d)}$
- Opposite for the $\bar{D}$ meson
- Asymmetry has a minimum ~ z_h ~ 0.5

TMD vs collinear factorization

- **TMD factorization and collinear factorization cover different regions of kinematics:**

  Collinear: $Q_1 \dots Q_n \gg \Lambda_{QCD}$

  TMD: $Q_1 \gg Q_2 \sim \Lambda_{QCD}$

  ✧ **One complements the other, but, cannot replace the other!**

  ✧ **Predictive power of both formalisms relies on the validity of their own factorization**

  Consistency check – overlap region – perturbative region

- **"Formal" operator relation between TMD distributions and collinear factorized distributions:**

  spin-averaged: $$\int d^2k_\perp \Phi_f^{\rm SIDIS}(x, k_\perp) + {\rm UVCT}(\mu_F^2) = \phi_f(x, \mu_F^2)$$

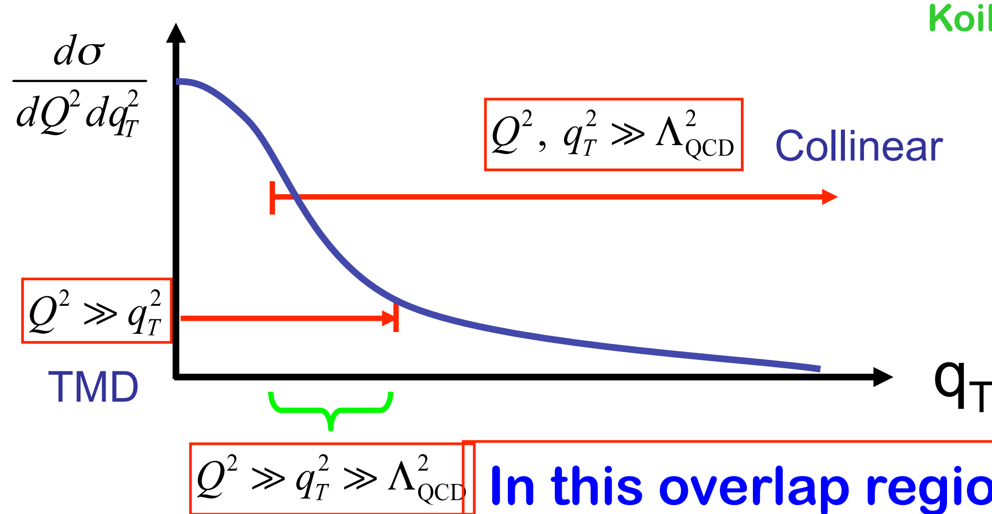
  Transverse-spin: $$\frac{1}{M_P}\int d^2k_\perp\, \vec{k}_\perp^2\, q_T(x, k_\perp) + {\rm UVCT}(\mu_F^2) = T_F(x, x, \mu_F^2)$$

  But, TMD factorization is only valid for low k_T – TMD PDFs at low k_T

The consistency check

- **IF both factorizations are proved to be valid,**
  - ✧ **both formalisms should yield the same result in overlap region**
  - ✧ **Case studies – Drell-Yan/SIDIS**

Ji, Qiu, Vogelsang, and Yuan
Koike, Vogelsang, and Yuan



In this overlap region, both formalisms indeed give the same result

- **TMD factorization fails for processes involving three or more identified hadrons!**

New challenges!

Collins, Qiu, 2007
Vogelsang, Yuan, 2007, Collins, 2007
Rogers, Mulders, 2010

Summery and outlook

- QCD has been very successful in interpreting high energy data from collisions with hadron(s)
- Beyond the leading power (twist) QCD:
  - QCD at high temperature and density
  - QCD and hadron structure at zero temperature
- Single transverse spin asymmetry opens up many opportunities to explore the parton's transverse motion and test QCD in a completely new domain
- Future Electron-Ion Collider could be a QCD machine

Thank you!

Backup slices

QCD and hadrons

- **For condensed matter physicists, chemists, …**

  Protons, neutrons, …, and hadrons are simple objects with mass, charge, spin, magnetic moment, …

- **For us: particle and nuclear physicists, …**

  Protons, neutrons, …, and hadrons are complicate bound states of quarks and gluons, though we have not seen them directly

- **The challenge:**

  Explain the properties of hadrons in terms of quarks, gluons, and their dynamics – QCD – the theory we believe!

Scale dependence of SSA

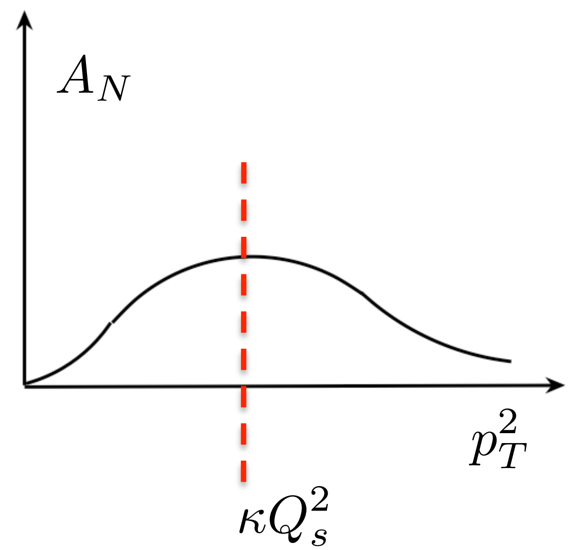
- **Almost all existing calculations of SSA are at LO:**
  - Strong dependence on renormalization and factorization scales
  - Artifact of the lowest order calculation
- **Improve QCD predictions:**
  - Complete set of twist-3 correlation functions relevant to SSA
  - LO evolution for the universal twist-3 correlation functions
  - NLO partonic hard parts for various observables
  - NLO evolution for the correlation functions, …
- **Current status:**
  - Two sets of twist-3 correlation functions
  - LO evolution kernel for $T_{q,F}(x,x)$ and $T_{G,F}^{(f,d)}(x,x)$ — Kang, Qiu, 2009; Braun et al, 2009
  - NLO hard part for SSA of p_T weighted Drell-Yan — Vogelsang, Yuan, 2009

A_N at low p_T

- **Collinear factorization does not work at low p_T:**

$$A_N^{(3)} \propto \frac{\epsilon_\perp^{\alpha\beta} s_\alpha p_{T\beta}}{p_T^2} \longrightarrow \frac{1}{p_T} \longrightarrow \infty \quad \text{as} \quad p_T \to 0$$

Should not apply for $p_T < Q_s$



- **Symmetry requirement:**

$$A_N \longrightarrow 0 \quad \text{as} \quad p_T \to 0$$

- **Role of Q_s:**

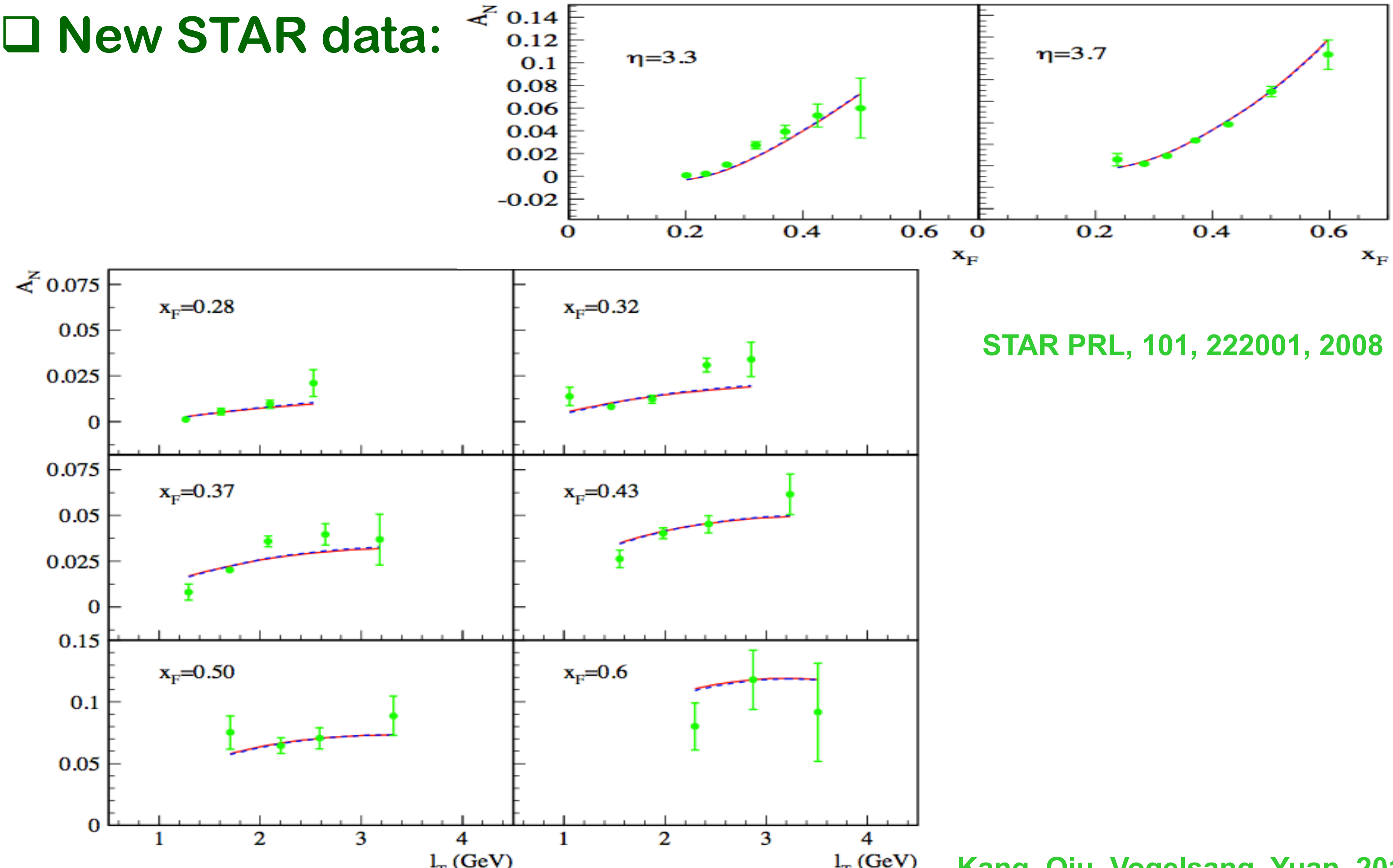
$$A_N^{(3)} \propto \frac{\epsilon_\perp^{\alpha\beta} s_\alpha p_{T\beta}}{p_T^2} \longrightarrow \frac{\epsilon_\perp^{\alpha\beta} s_\alpha p_{T\beta}}{p_T^2 + \kappa Q_s^2} \longrightarrow 0 \quad \text{as} \quad p_T \to 0$$

- **Transition region:**

  – **probe the scale where the fixed order pQCD fails!**

Consistency Check!

- New STAR data:



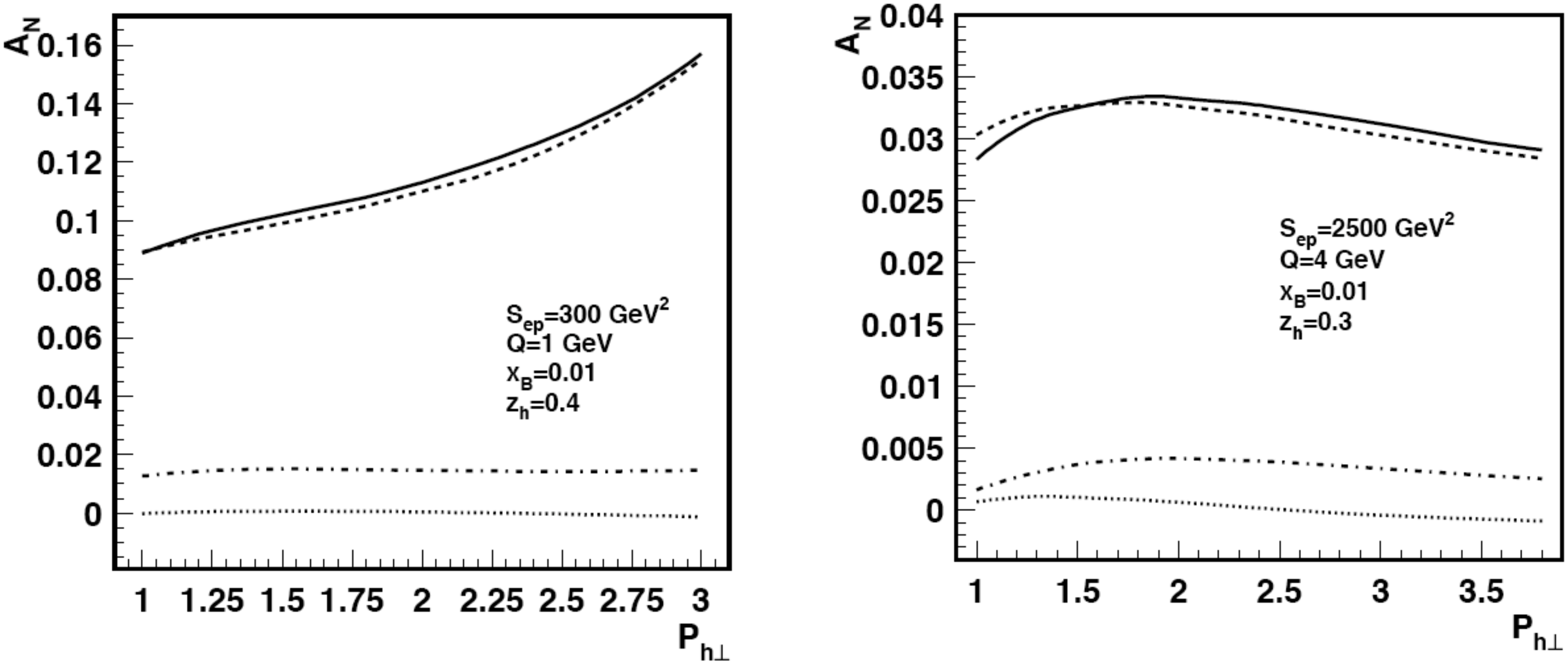
STAR PRL, 101, 222001, 2008

Kang, Qiu, Vogelsang, Yuan, 2010

Maximum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

- **SSA for D^0 production (λ_f only):**



- **The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T \gg m_c$**
- **For the region, $P_T \sim m_c$,**

$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

Interpretation of twist-3 distributions?

- Quark-gluon correlation as an example:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Normal twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

- Difference – the operator in Red:

$$\left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right]$$

How can we interpret the "expectation value" of this operator?