

# PDF Benchmarking

Robert Thorne

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University College London

I will discuss potential PDF benchmarking, outlining current exercises ongoing.

All discussions based on most recent sets, [NNPDF3.1](#), [CT18](#), [MSHT2020](#) (PDFs final - article imminent), along with [ABMP16](#) and [HERAPDF2.0](#).

Updates due to inclusion of new, largely [LHC](#) (different amounts and choices for different groups) but also some [HERA](#), [Tevatron](#), data sets.

First outline updates in [MSHT2020](#) since details not yet publically available.

## Theoretical Procedures

As before use a general mass variable flavour scheme based on the **Thorne-Roberts** scheme, using the “optimal” choice of parameters for smoothness near threshold.

Use deuteron and heavy nuclear corrections. Former fit using 4 parameter model, as in **MMHT14** and latter use same corrections ([arXiv:1112.6324](https://arxiv.org/abs/1112.6324)) as **MMHT14** with additional penalty-free freedom of order 1%.

Fit data with systematics uncertainties using either nuisance parameters if possible (preferred method) or with the correlation matrix provided. Some old data sets with domination of uncorrelated uncertainties and/or a limited understanding of correlations have errors added in quadrature. Now also use statistical correlations whenever provided.

Fit to absolute cross sections in preference to normalized if both available.

## Extension of parameterisation.

General parameterisation used  $A(1 - x)^\eta x^\delta (1 + \sum_{i=1}^n a_i T_i(1 - 2x^{\frac{1}{2}}))$ ,  
where  $T_i(1 - 2x^{\frac{1}{2}})$  are Chebyshev polynomials.

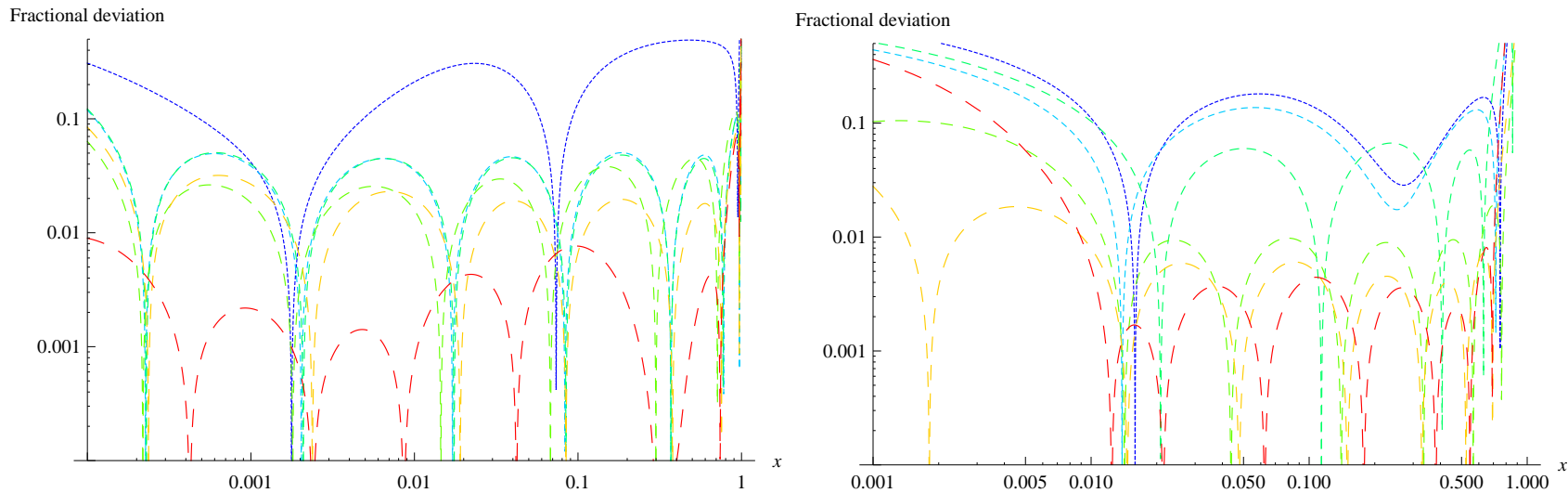


Illustration of precision possible with increasing  $n$ , sea-like (left) and valence-like (right) (where pseudo-data for  $x > 0.01$ ). Using  $n = 6$  would lead to much better than 1% precision.

For most PDFs  $n = 4$  is default for **MMHT2014** where there were **36** parameters in total.

Now extend parameters of PDFs using  $n = 6$ :

Parameterize  $(\bar{d}/\bar{u})$  instead of  $(\bar{d} - \bar{u})$ .  $(\bar{d}/\bar{u}) \rightarrow \text{constant}$  as  $x \rightarrow 0$ .

$$u_v(x, Q_0^2) = A_u(1-x)^{\eta_u} x^{\delta_u} (1 + \sum_{i=1}^6 a_{i,u} T_i(1-2x^{\frac{1}{2}})); A_u \text{ fixed by } \int_0^1 u_v dx = 2$$

$$d_v(x, Q_0^2) = A_d(1-x)^{\eta_d} x^{\delta_d} (1 + \sum_{i=1}^6 a_{i,d} T_i(1-2x^{\frac{1}{2}})); A_d \text{ fixed by } \int_0^1 d_v dx = 1$$

$$sea(x, Q_0^2) = A_S(1-x)^{\eta_S} x^{\delta_S} (1 + \sum_{i=1}^6 a_{i,S} T_i(1-2x^{\frac{1}{2}}));$$

$$s^+(x, Q_0^2) = A_s(1-x)^{\eta_s} x^{\delta_s} (1 + \sum_{i=1}^6 a_{i,s} T_i(1-2x^{\frac{1}{2}})); (a_{i,s} \neq a_{i,S}, i = 5, 6)$$

$$(\bar{d}/\bar{u})(x, Q_0^2) = A_{\text{rat}}(1-x)^{\eta_{\text{rat}}} (1 + \sum_{i=1}^6 a_{i,\text{rat}} T_i(1-2x^{\frac{1}{2}}));$$

$$g(x, Q_0^2) = A_g(1-x)^{\eta_g} x^{\delta_g} (1 + \sum_{i=1}^4 a_{i,g} T_i(1-2x^{\frac{1}{2}})) - A_{g-}(1-x)^{\eta_{g-}} x^{\delta_{g-}};$$

$$s^-(x, Q_0^2) = A_{s-}(1-x)^{\eta_{s-}} (1-x_o/x) x^{\delta_{s-}}. x_o \text{ fixed by } \int_0^1 s^- dx = 0, \delta_{s-} \text{ fixed.}$$

Change of to a maximum of **51** parton parameters.

When determining uncertainties go from **25** eigenvector pairs to **32** - one extra parameter for each PDF and two for  $s + \bar{s}$ .

## New LHC data fit.

Extremely high precision data on  $W, Z$  at 7 TeV from ATLAS, and high precision  $W^{+/-}$  data and double differential 8 TeV  $Z$  data at 8 TeV.

CMS 8 TeV precise data on the  $W^{+,-}$  rapidity distribution.

LHCb data at 7 and 8 TeV on  $W, Z$  rapidity distributions at higher rapidity.

$W + c$  jets data at 7 TeV from CMS.

ATLAS high mass Drell Yan data at 8 TeV.

ATLAS data on  $W^{+/-} + jets$  at 8 TeV.

$Z$   $p_T$  distributions at 8 TeV.

New data on  $\sigma_{t\bar{t}}$  at 8 TeV plus ATLAS single differential distributions in  $p_{T,t}, M_{t\bar{t}}, y_t, y_{t\bar{t}}$  and CMS double differential distributions in  $p_{T,t}, y_t$  both at 8 TeV.

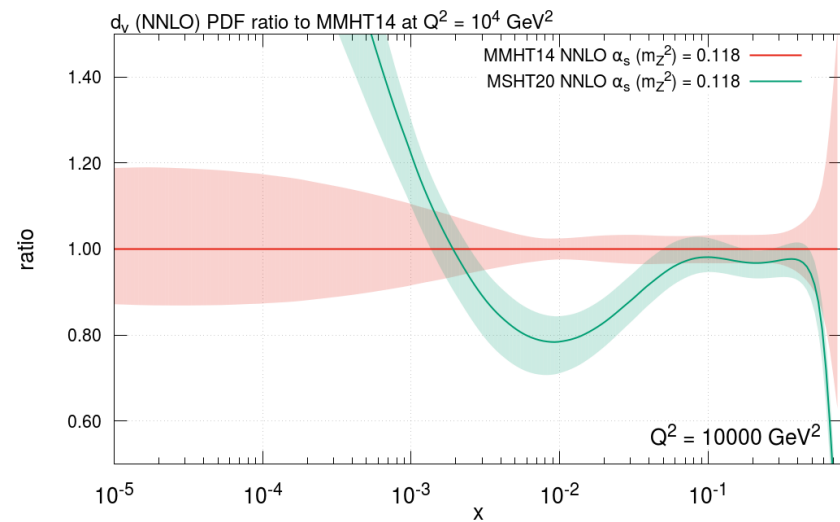
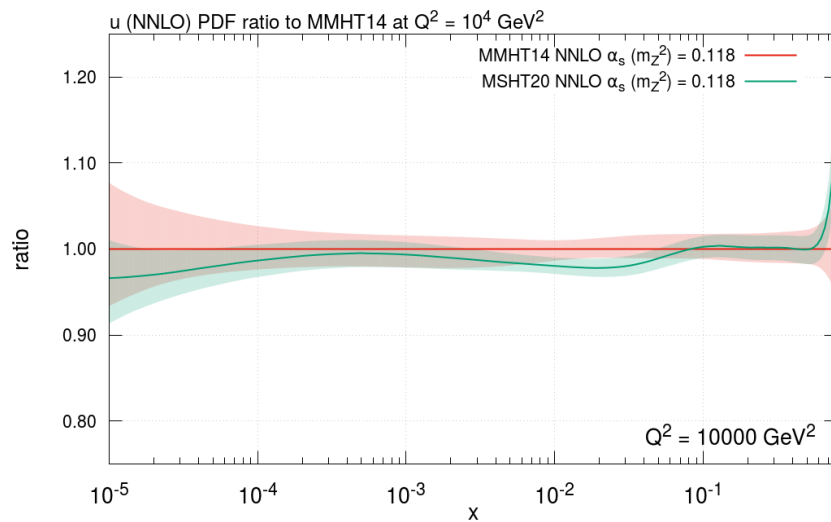
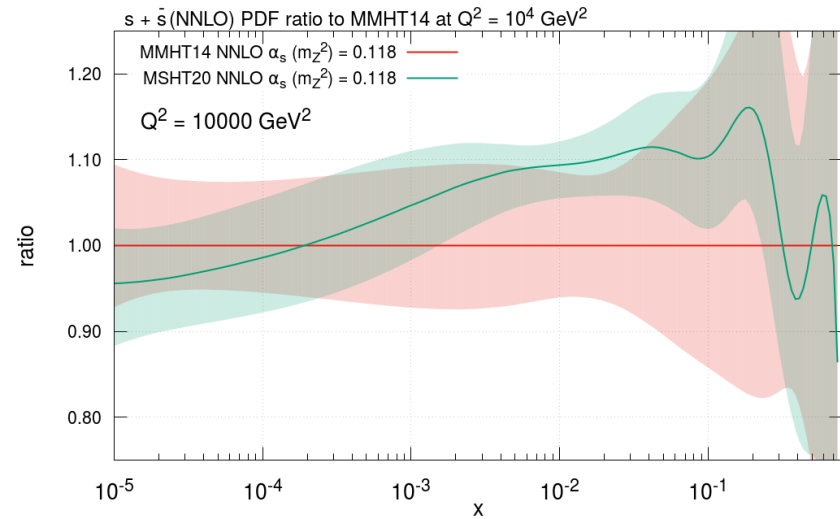
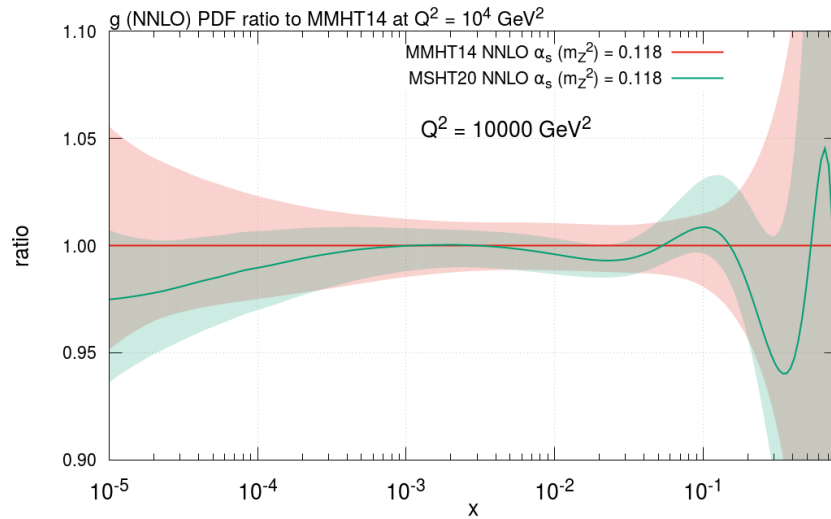
Inclusive jet data from ATLAS at 7 TeV and CMS at 2.76, 7 and 8 TeV.

Include all our recent **LHC** data updates in the fit at **NNLO** (for default  $\alpha_S(M_Z^2) = 0.118$ ).

	no. points	NNLO $\chi^2$
D0 $W$ asymmetry	14	12.0
$\sigma_{t\bar{t}}$ Tevatron +CMS+ATLAS 7, 8TeV	17	14.5
LHCb 7+8 TeV $W + Z$	67	99.4
LHCb 8 TeV $e$	17	26.2
CMS 8 TeV $W$	22	12.7
ATLAS 7 TeV jets $R = 0.6$	140	221.6
CMS 7 TeV $W + c$	10	8.6
ATLAS 7 TeV $W, Z$	61	116.6
CMS 7 TeV jets $R = 0.7$	158	175.8
ATLAS 8 TeV $Z p_T$	104	188.5
CMS 8 TeV jets	174	261.3
ATLAS 8 TeV $t\bar{t} \rightarrow l + j$ single-diff	25	25.6
ATLAS 8 TeV $t\bar{t} \rightarrow l^+ l^-$ single-diff	5	3.5
ATLAS 8 TeV high-mass Drell-Yan	48	56.7
ATLAS 8 TeV $W^{+,-} + \text{jet}$	32	18.1
CMS 8 TeV $(d\sigma_{t\bar{t}}/dp_{T,t} dy_t)/\sigma_{t\bar{t}}$	15	22.5
ATLAS 8 TeV $W^+, W^-$	22	57.4
CMS 2.76 TeV jets	81	102.9
CMS 8 TeV $t\bar{t} y_t$ distribution	9	13.2
ATLAS 8 TeV double differential $Z$	59	85.6
total	4363	5122

Fit quality generally good. Relatively poor  $\chi^2$  values for some sets all observed by other groups.

# Changes in MSHT PDFs

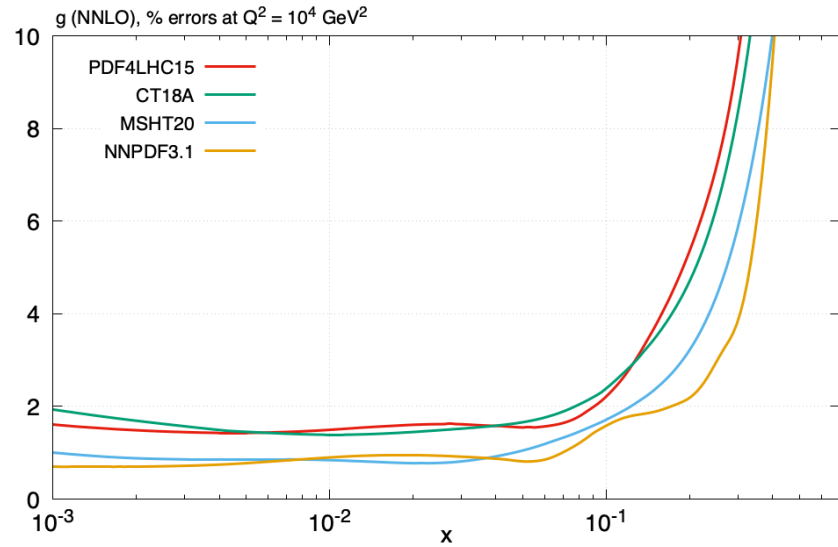
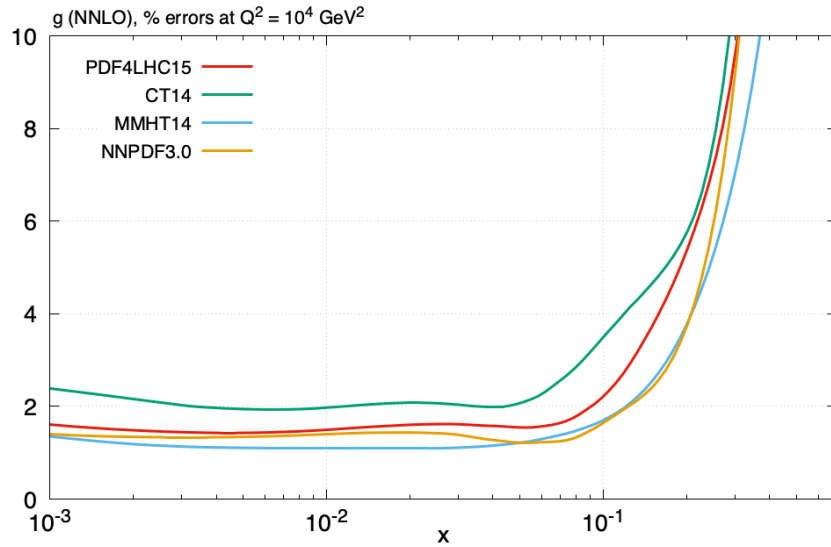


Most significant in  $d_V$  (parameterisation and new LHC data) and strange quark.

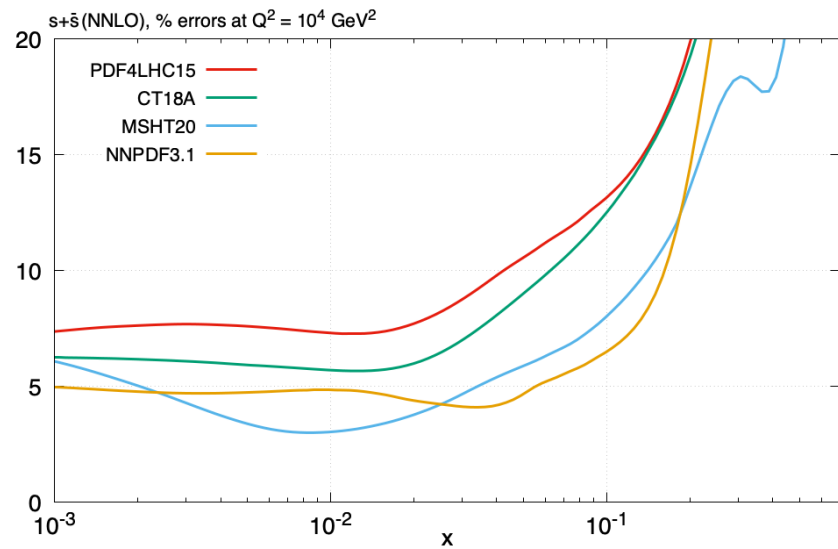
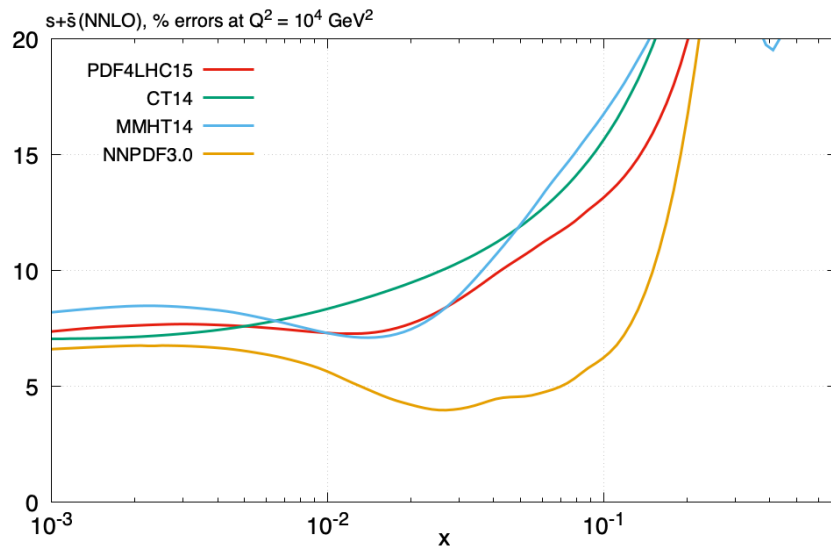


Changes in PDF uncertainties. Largely a decrease.

## Gluon

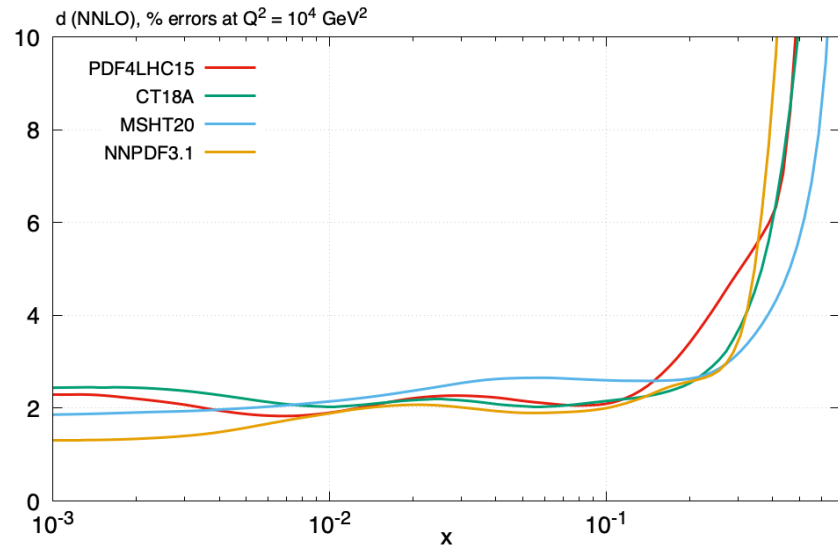
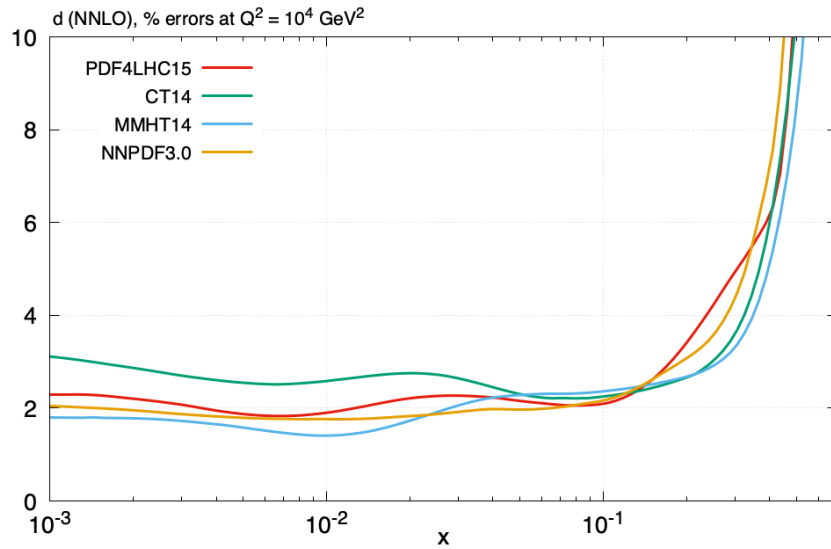


## Strange

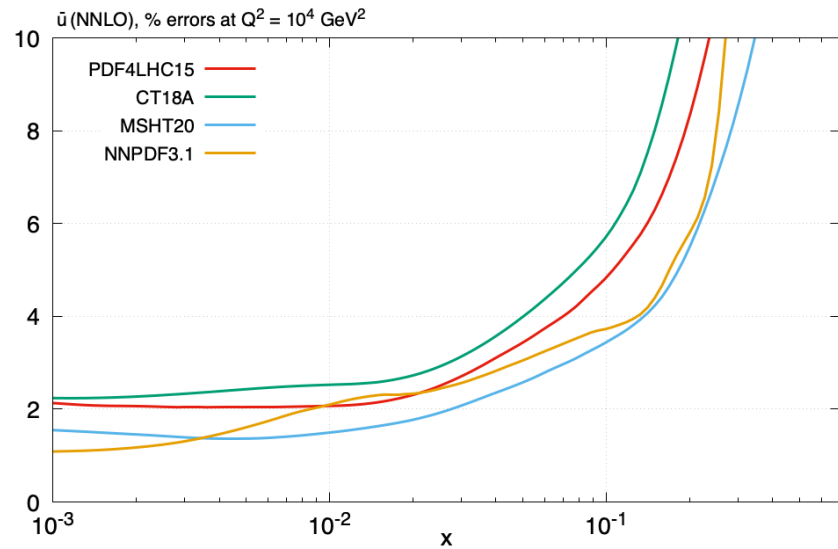
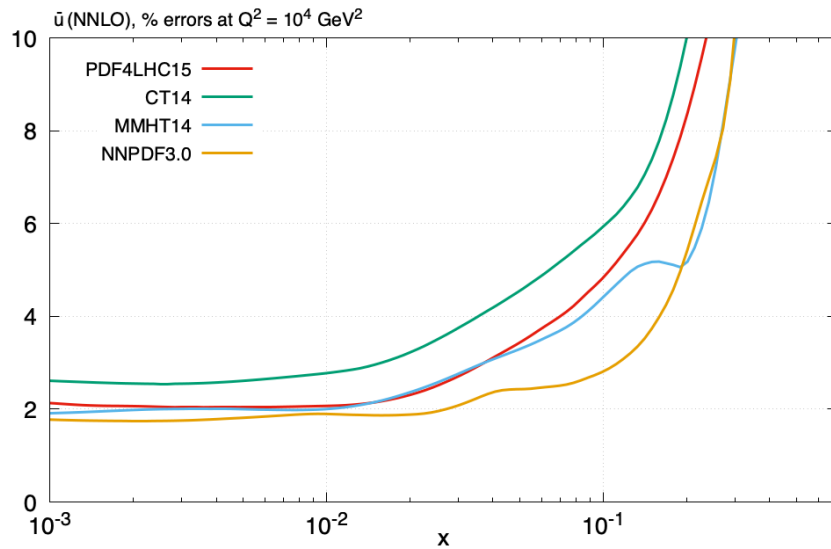


# Changes in PDF uncertainties. Largely a decrease.

## Down

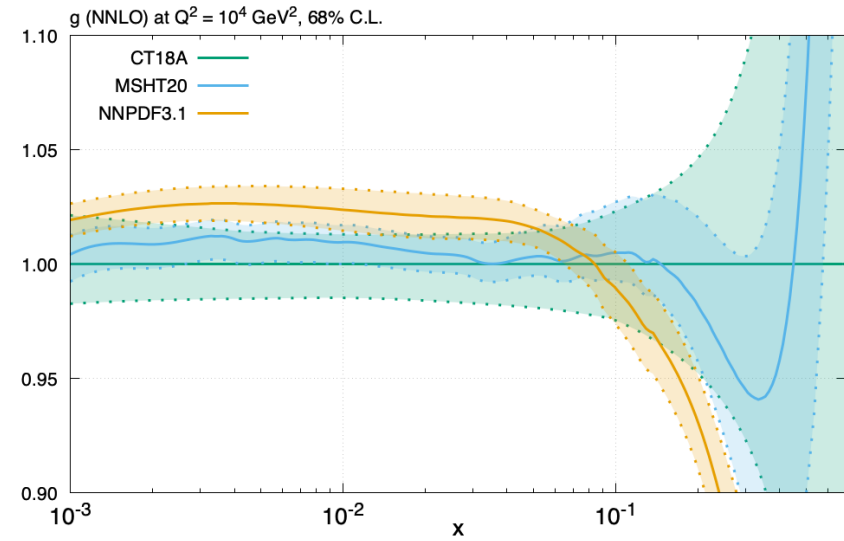
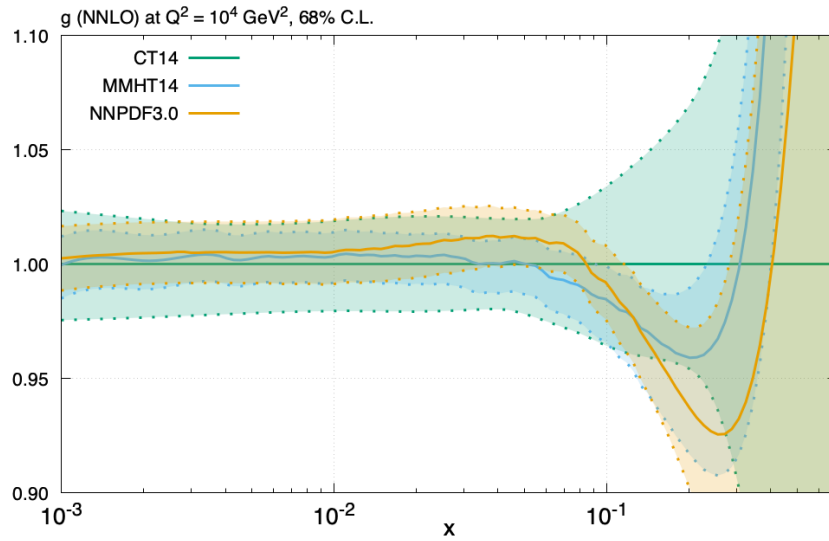


## Anti-up

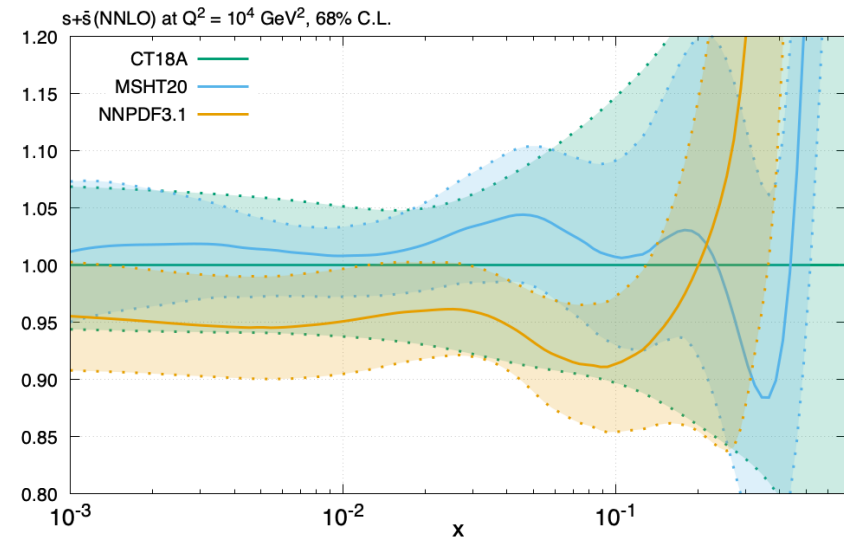
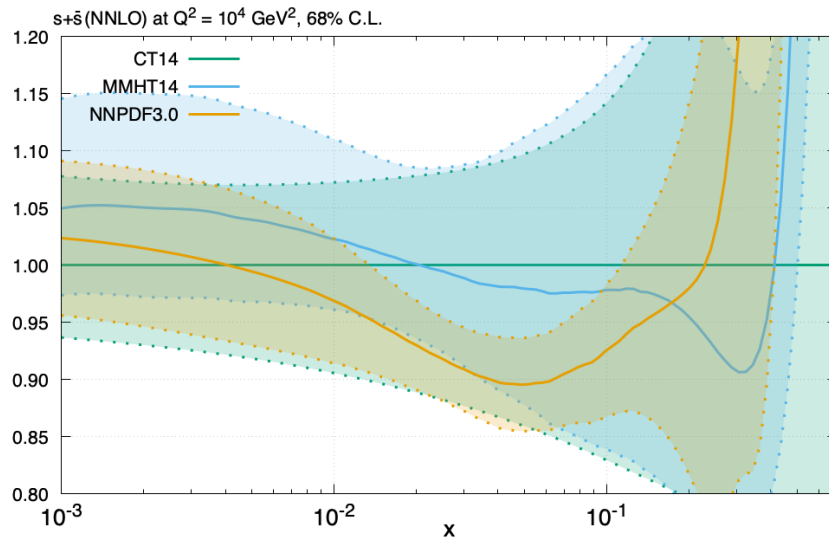


Changes in PDF central values not so good.

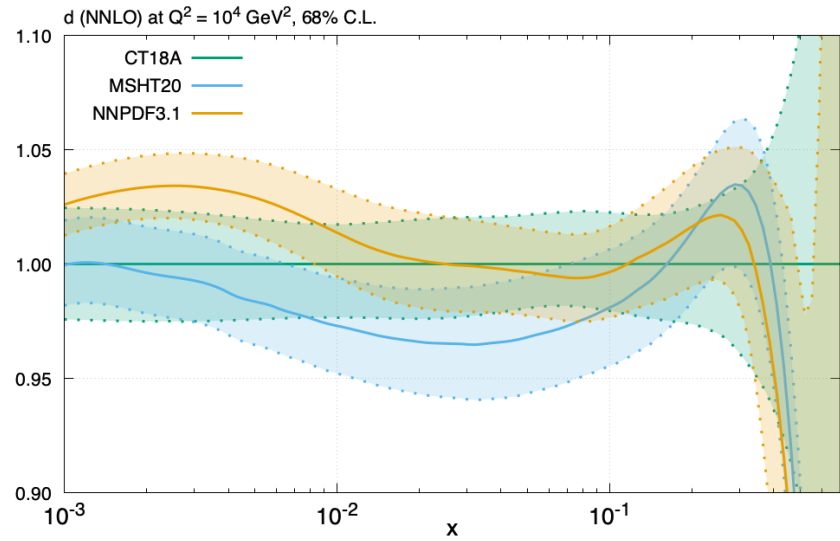
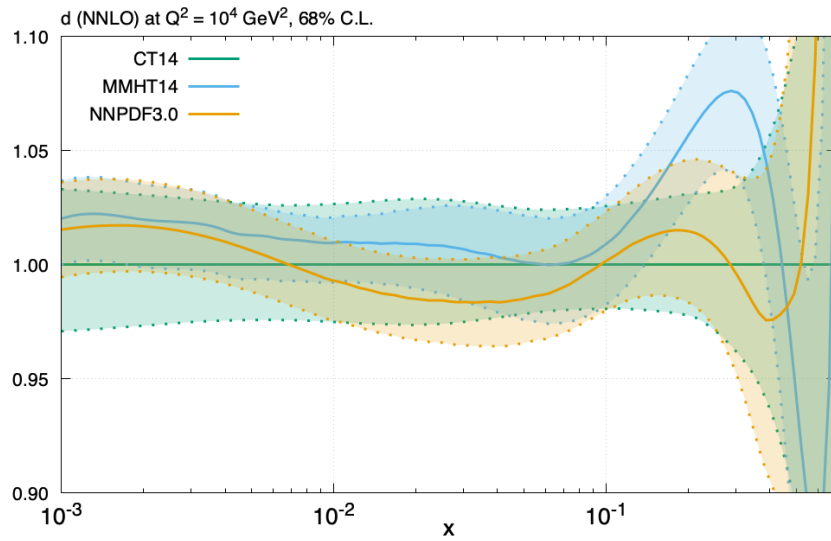
## Gluon



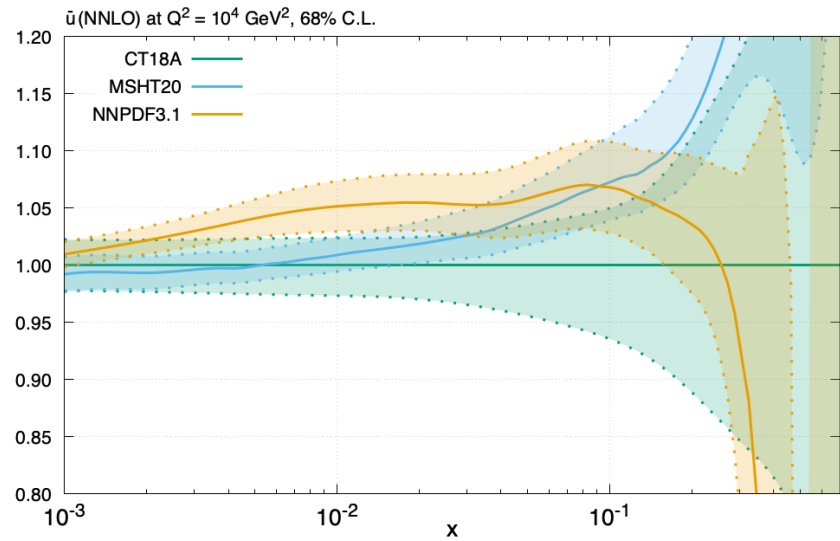
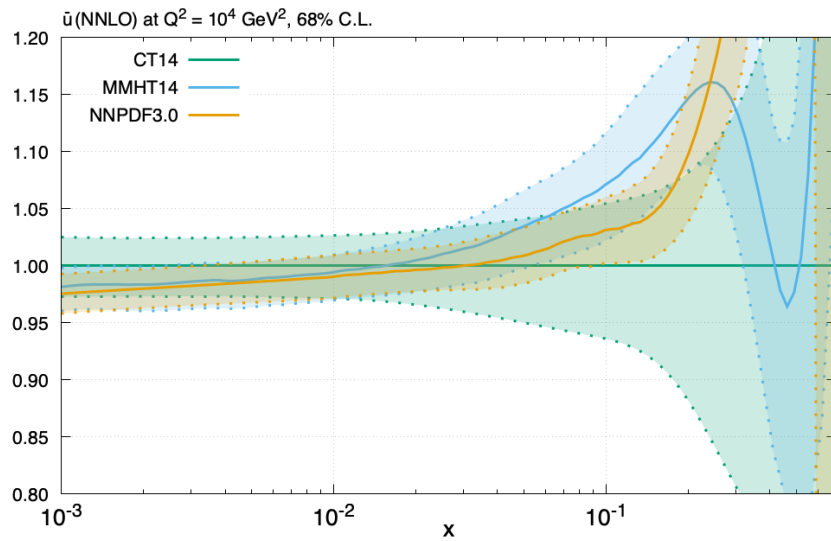
## Strange



# Down

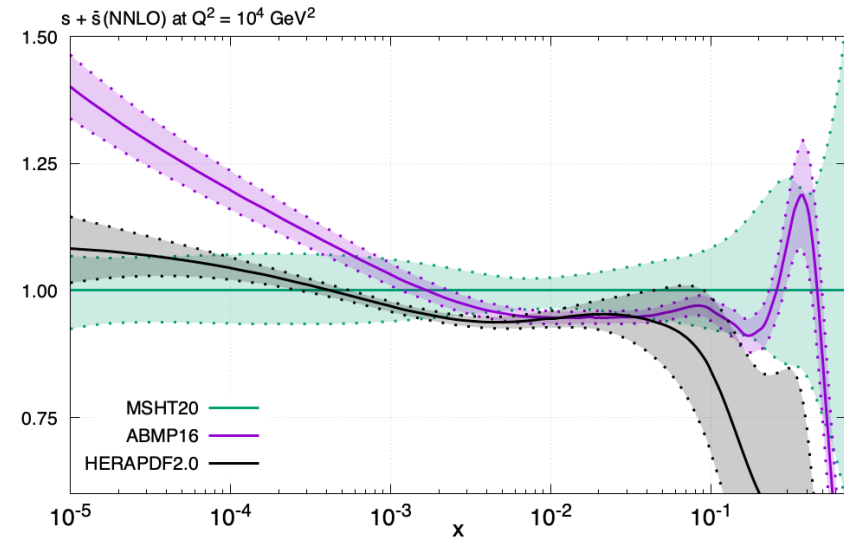
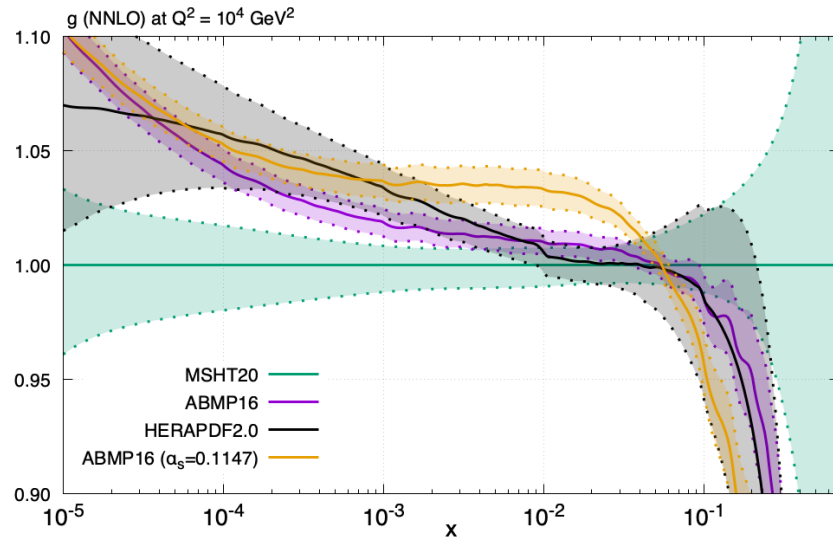


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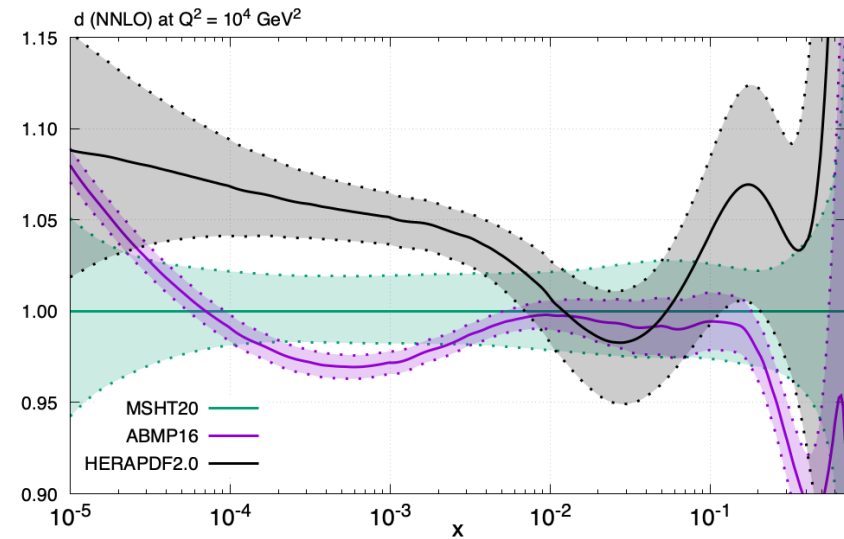
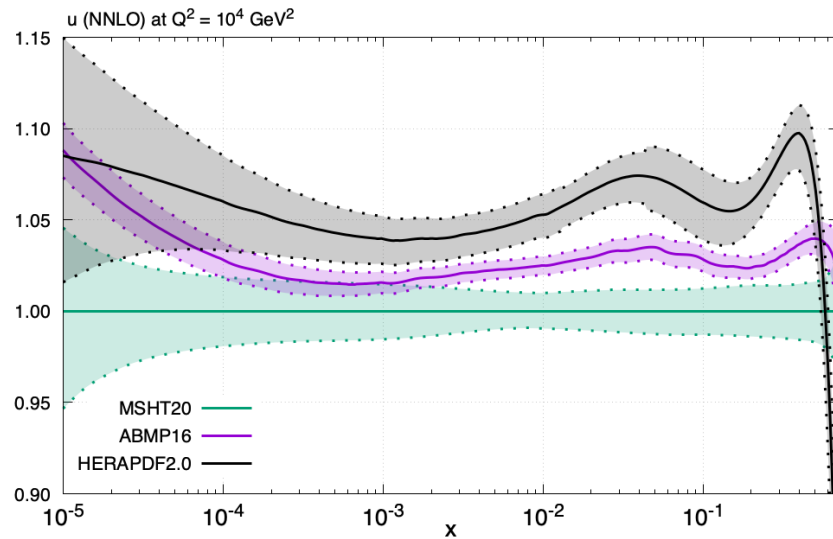


Comparisons to other PDFs. Some big differences.

## Gluon and Strange



## Up and Down



## New PDF4LHC Benchmarking exercise

Fit to a subset of data such that all groups (CT, MSHT, NNPDF) fit it (very largely) the same way.

Make definition flexible enough that a decent set of constraints on all PDF flavours and combinations is achieved.

Use most conservative cuts applied by any group – avoid most questionable kinematic regions.

Overall list is surprisingly small.

Many data sets fit in some non-trivially different manner by one or all groups.

Many data sets only fit by two groups or even one.

- NMC deuteron to proton ratio in DIS.
- NuTeV dimuon cross sections.
- HERA I + II inclusive cross sections from DIS.
- E866 fixed target Drell-Yan  $pd/pp$  data.
- D0  $Z$  rapidity distribution.
- ATLAS  $W, Z$  7 TeV rapidity distributions, only  $Z$  peak and not forward rapidity.
- CMS 7 TeV  $W$  asymmetry.
- CMS 8 TeV inclusive jet data.
- LHCb 7, 8 TeV  $W, Z$  rapidity distributions.
- BCDMS proton and deuteron DIS (MSHT use averaged data).

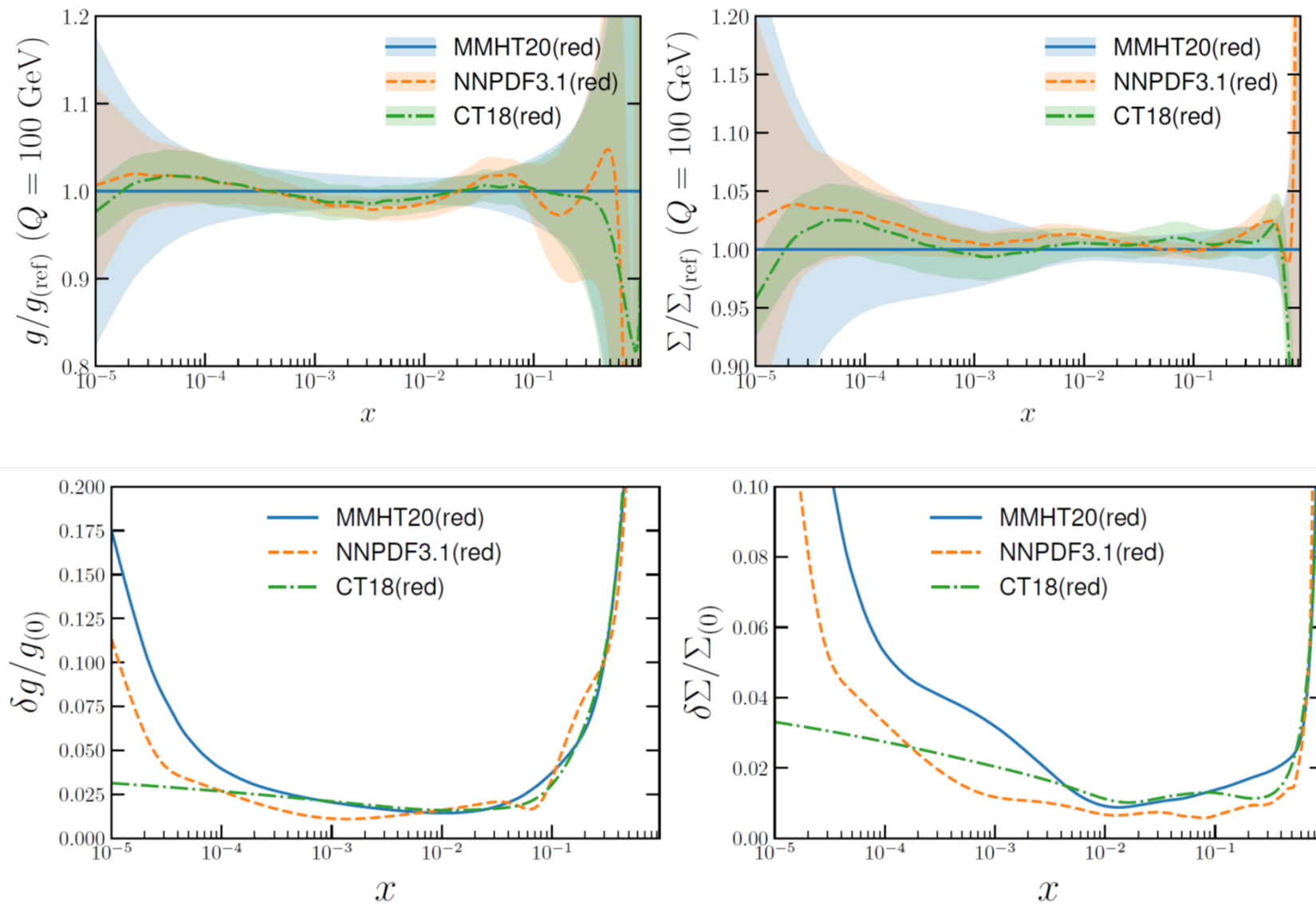
Set as many theoretical procedural choices the same as possible. Again, can differ between groups.

- $s - \bar{s} = 0$
- Perturbative charm only.
- Positive definite quark distributions (lack of constraints allow negative fluctuations).
- Common values of  $\alpha_S(M_Z^2)$  and  $m_{c,b}$ .
- No deuteron or nuclear corrections.
- Fixed branching ratio for charm hadrons  $\rightarrow$  muons.
- NNLO corrections for dimuon data.

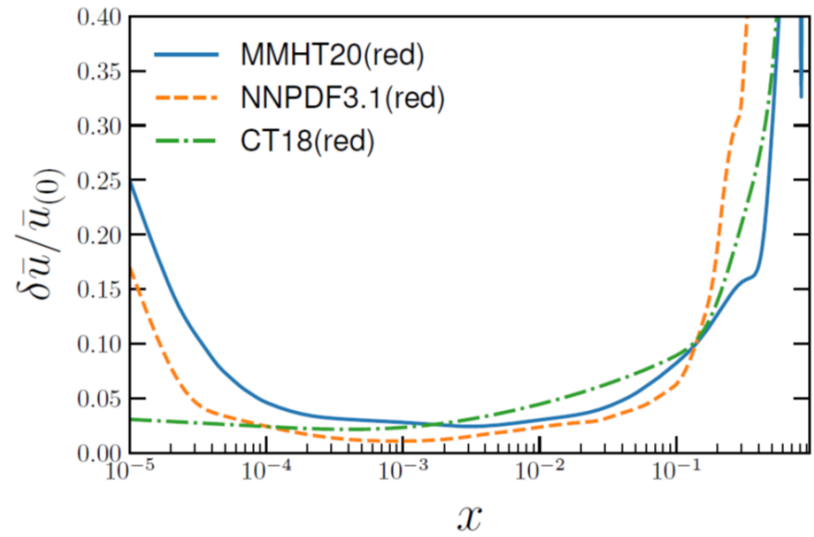
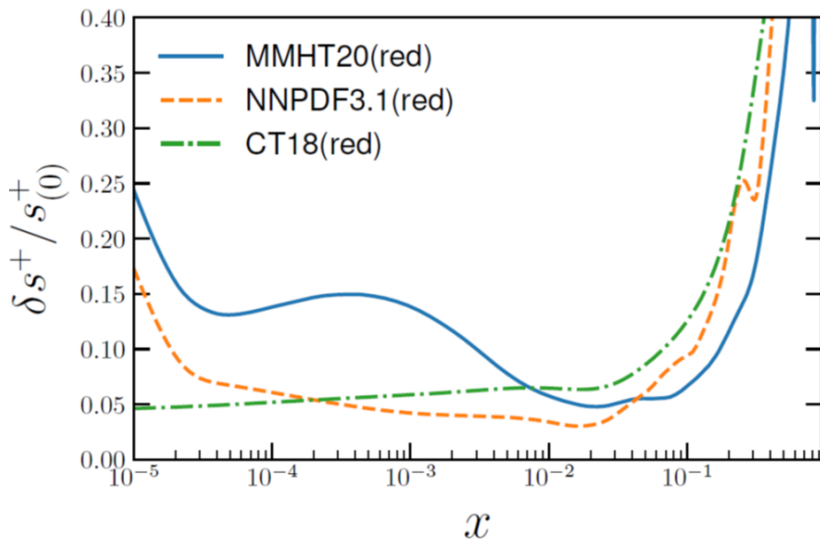
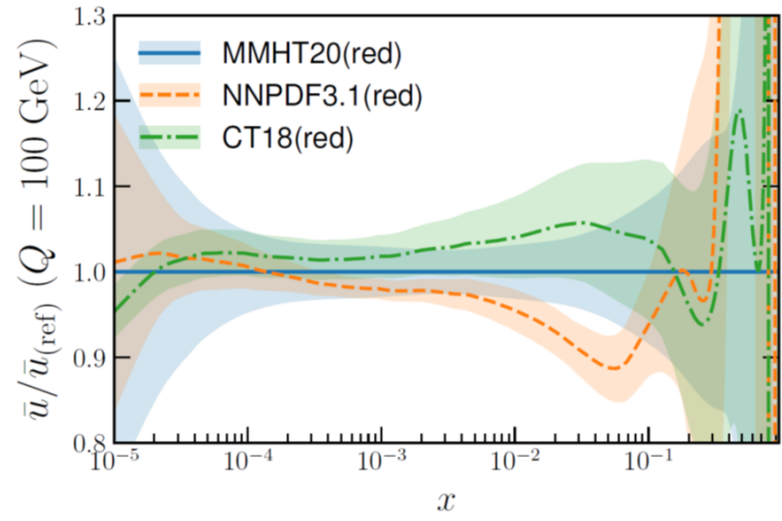
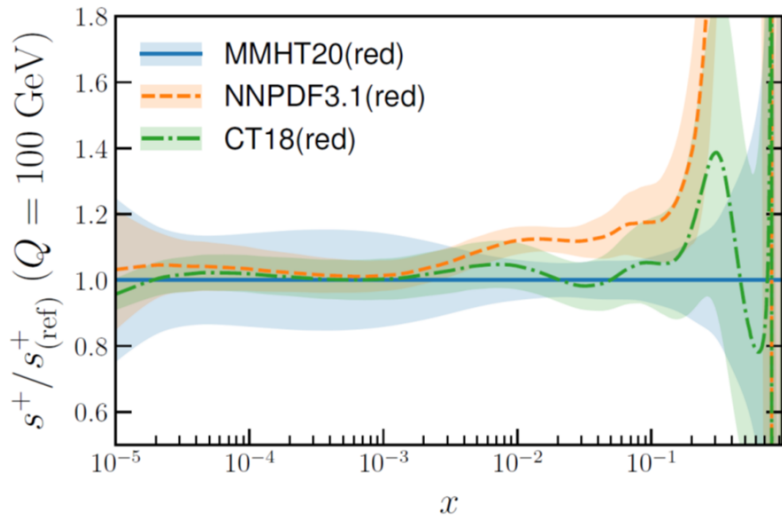
Note this is for simplicity and these are not fully the defaults of any group, or indeed, always recommended practice in a global fit.



# PDF comparison in reduced fits

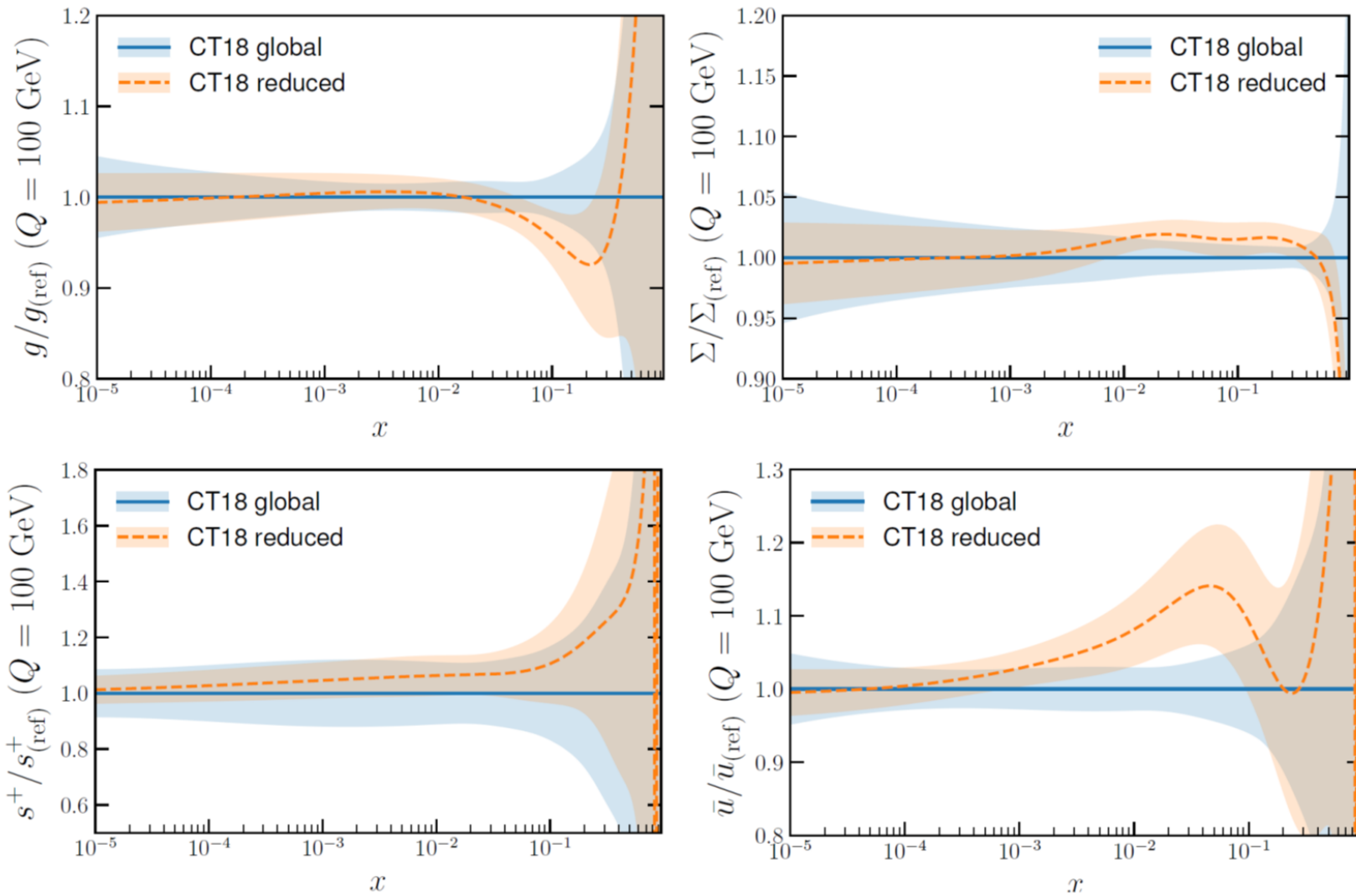


Plots from [J. Rojo](#).



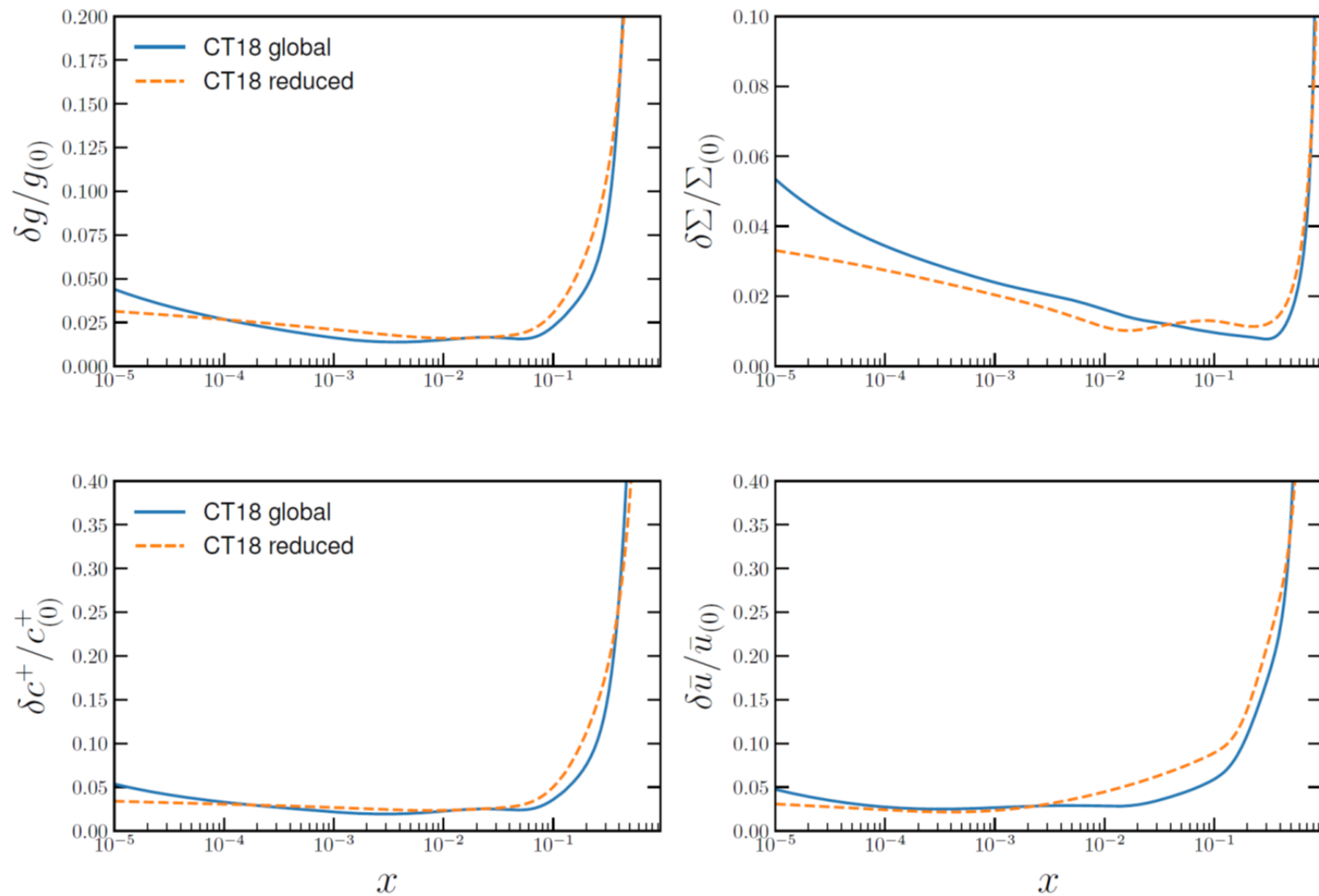
Plots from [J. Rojo](#).

# CT18 changes



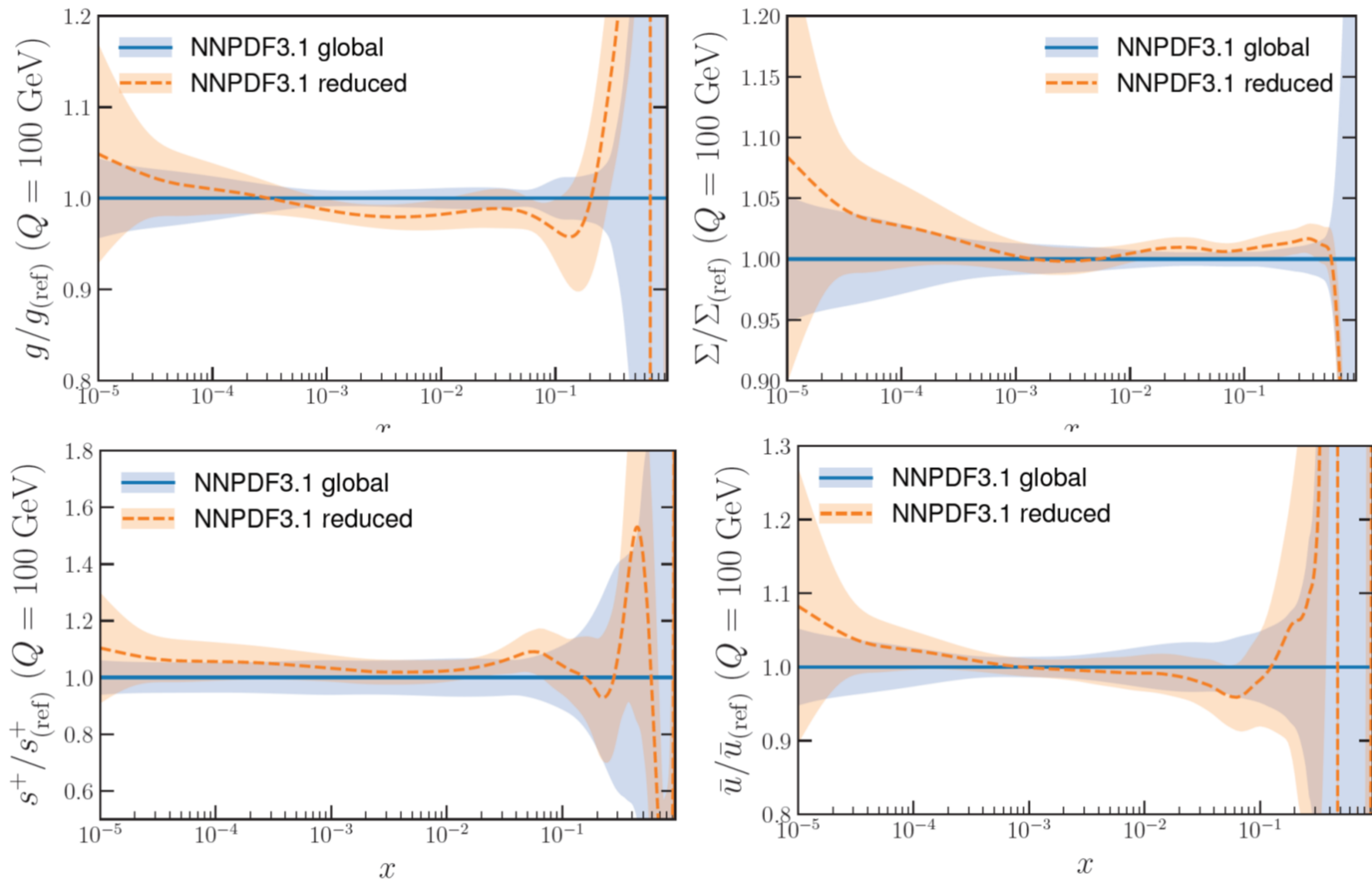
Plots from [J. Rojo](#).

# Uncertainties



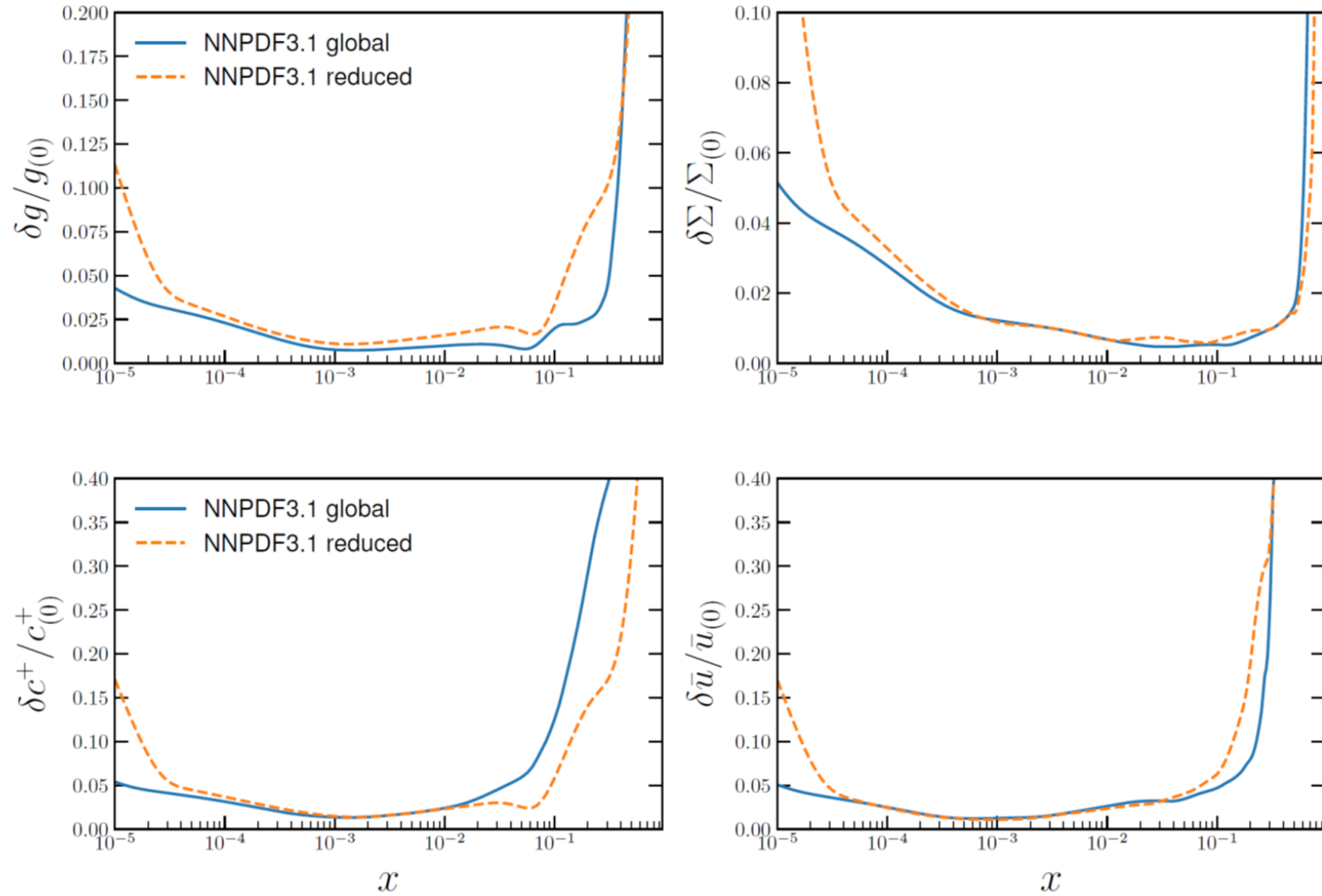
Plots from [J. Rojo](#).

# NNPDF3.1 changes



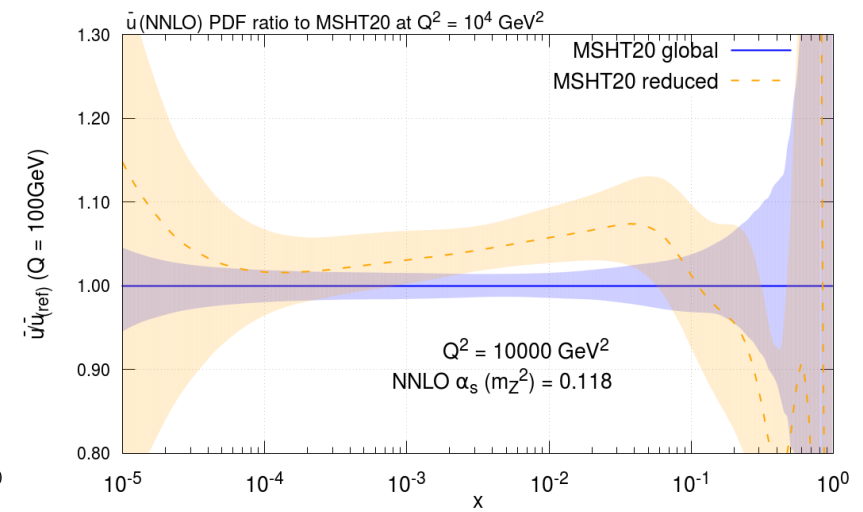
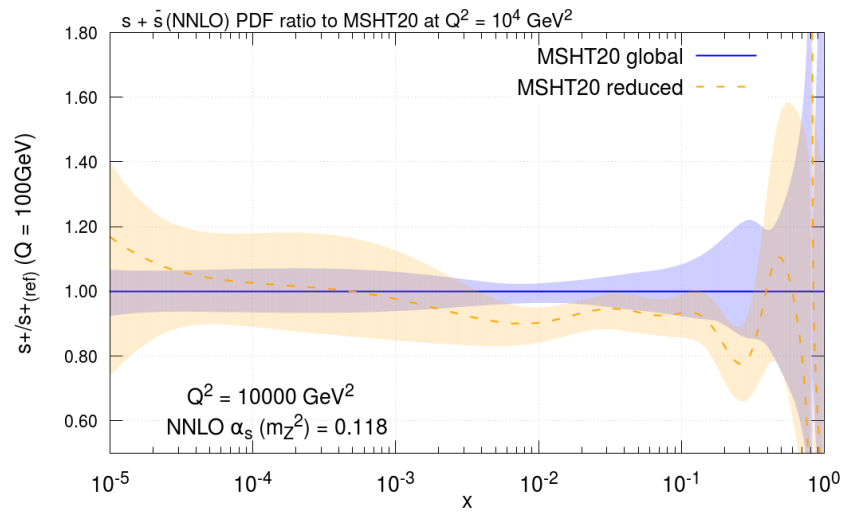
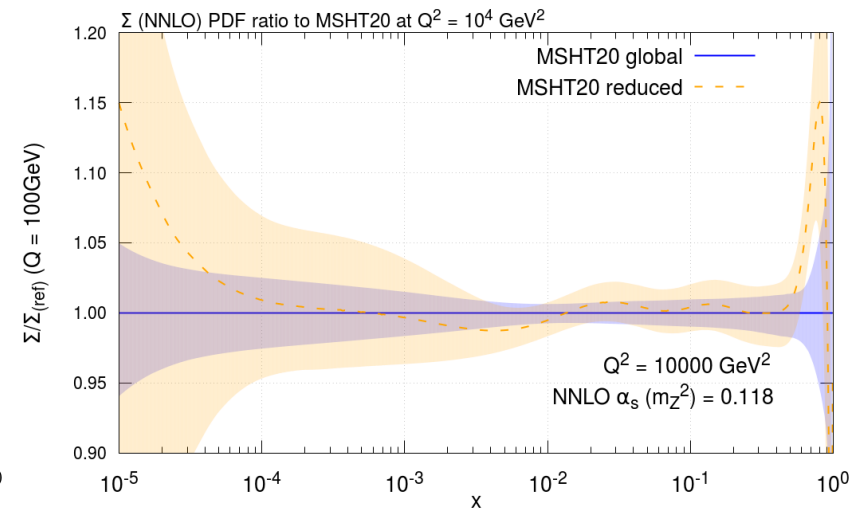
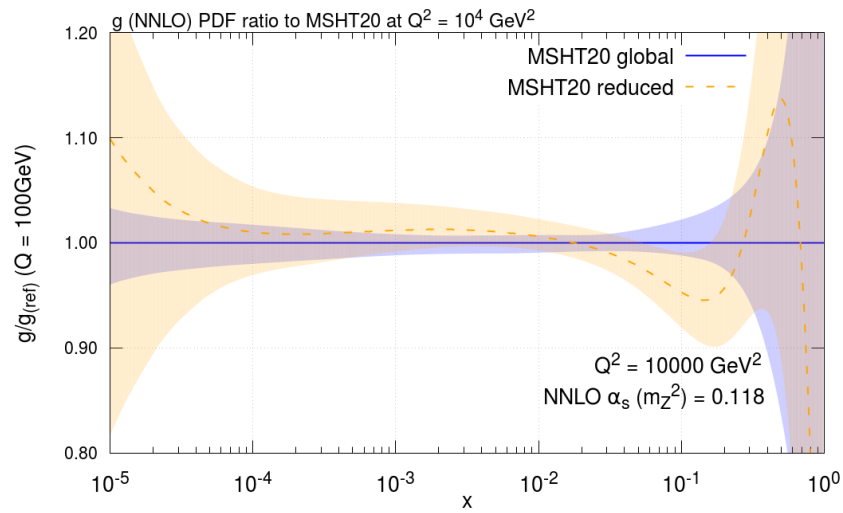
Plots from [J. Rojo](#).

# Uncertainties

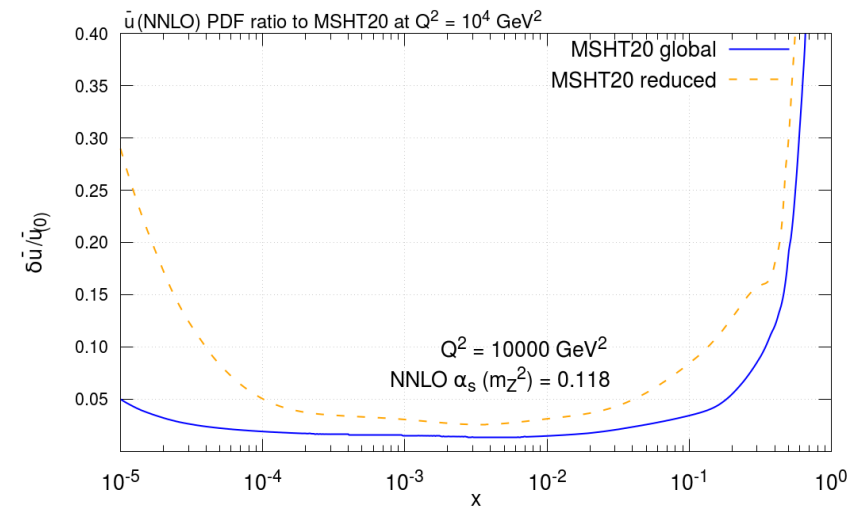
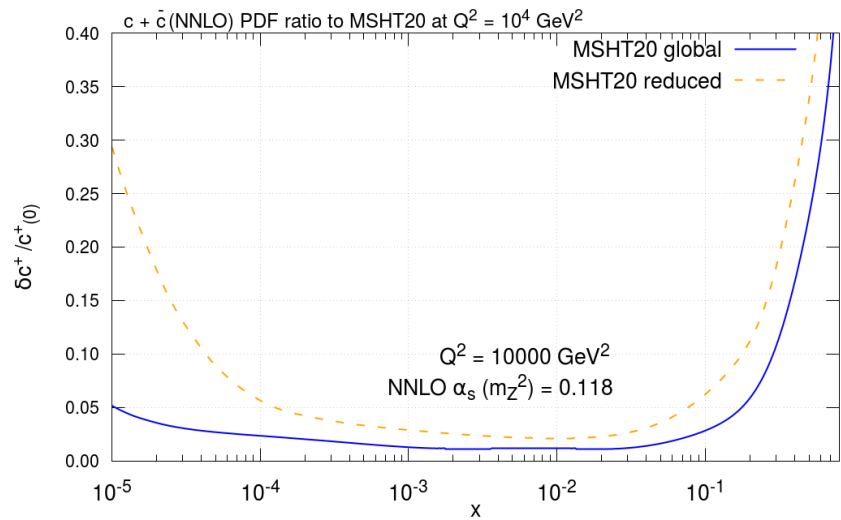
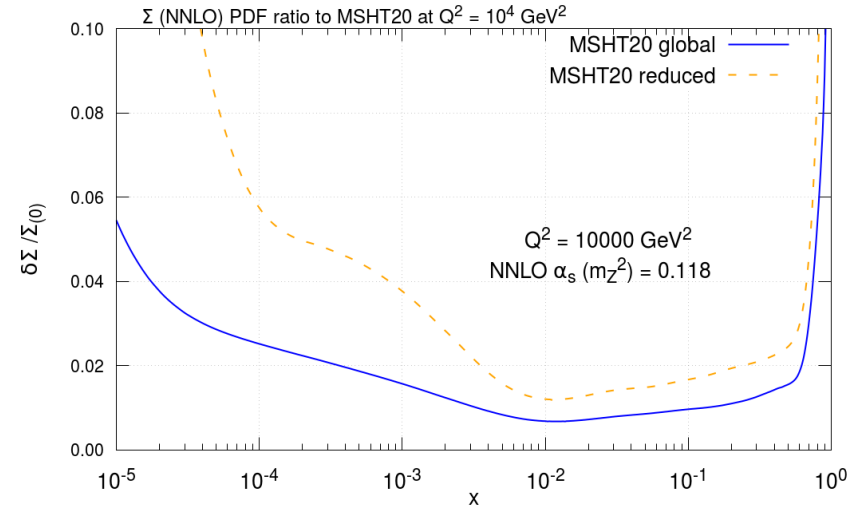
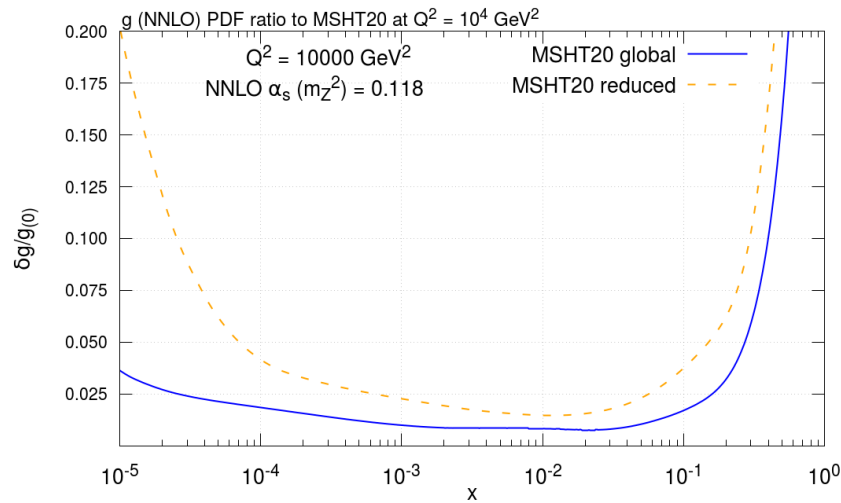


Plots from [J. Rojo](#).

# MSHT2020 changes



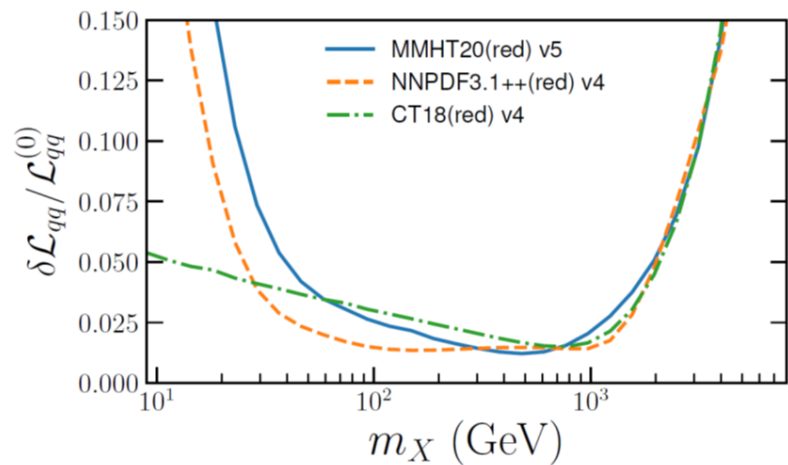
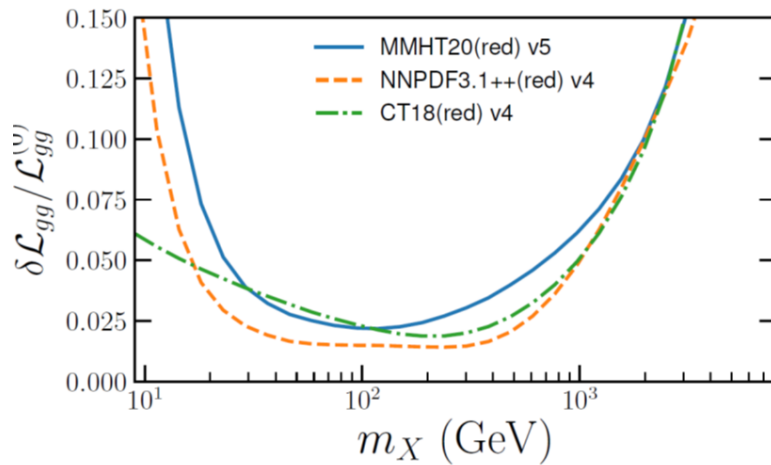
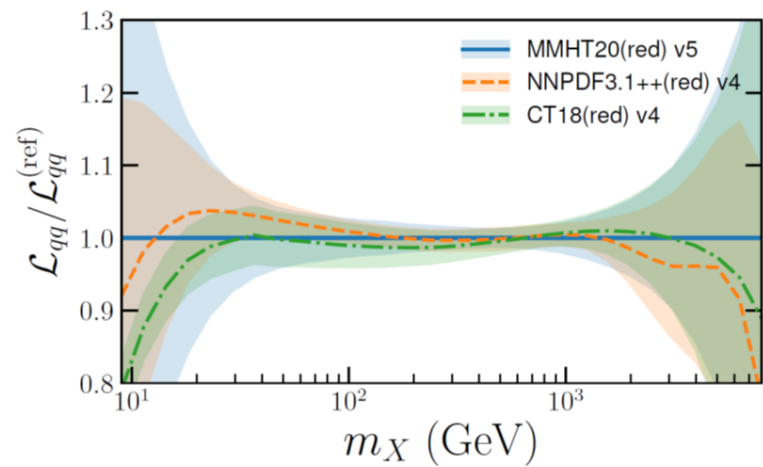
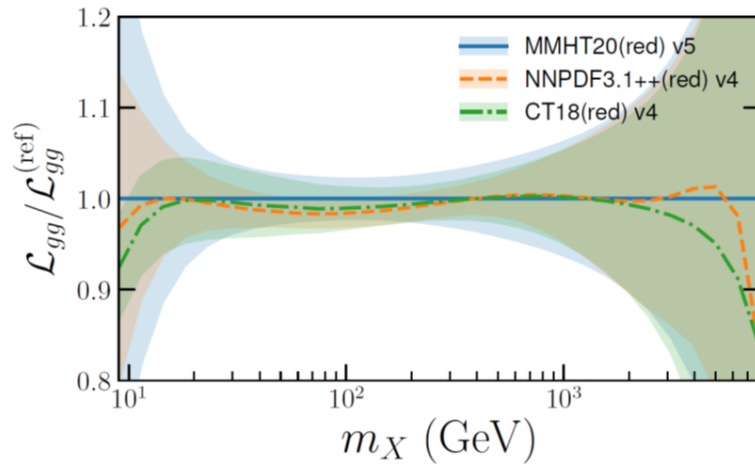
# Uncertainties



Noticable that only **MSHT20** see very definite overall increase in uncertainties when using reduced set.



# PDF luminosity comparison

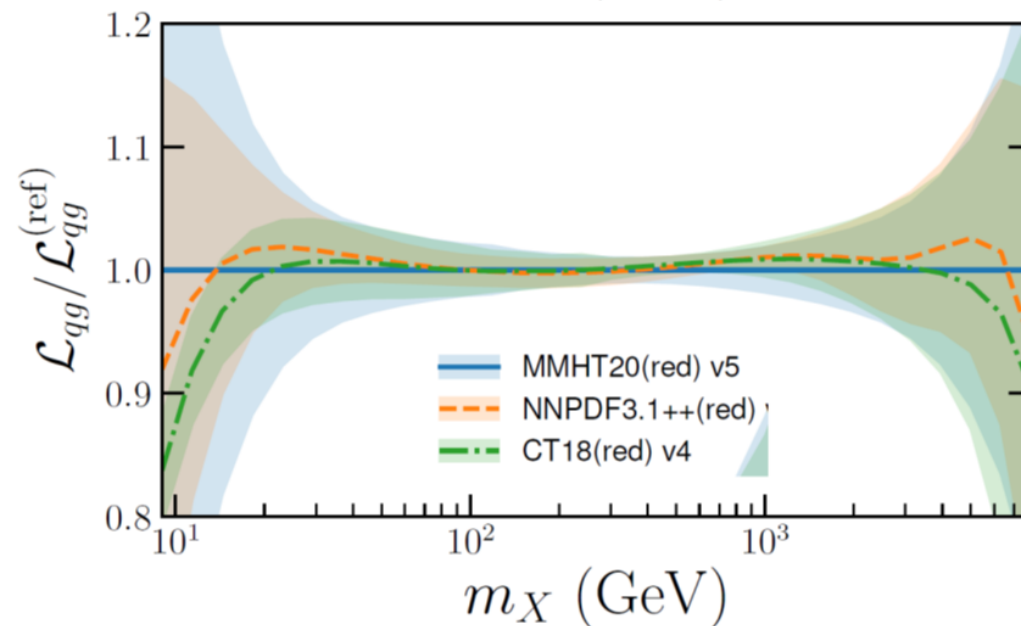
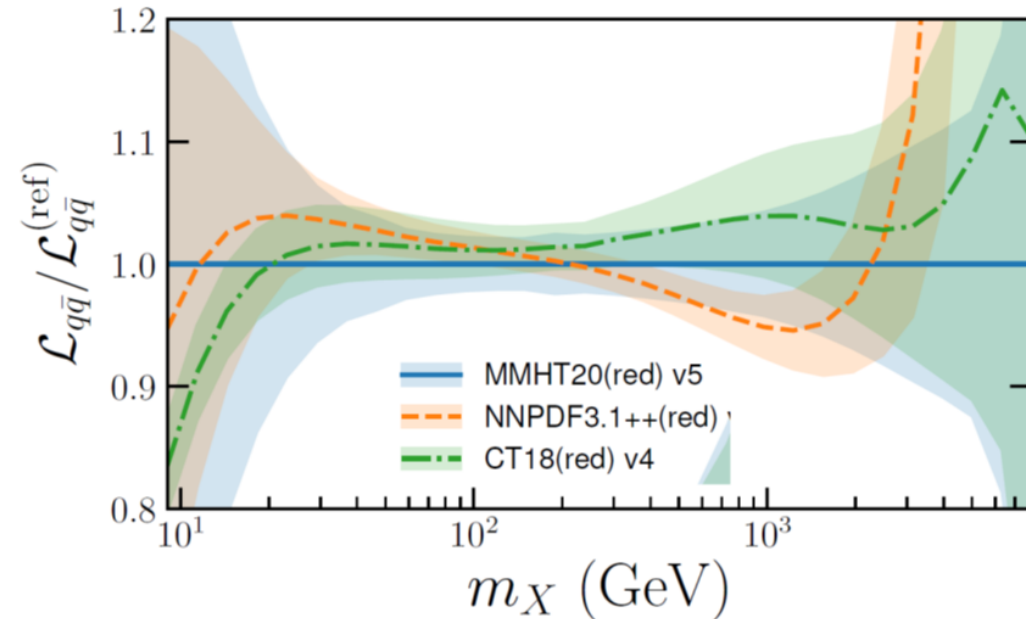


Good agreement in “main” luminosities. Plots from [J. Rojo](#).

Not so good in those that depend on antiquarks.

Again differences seen when flavour or quark-antiquark separation vital.

Plots from [J. Rojo](#).



## Theory comparisons

Also compare quality of fit obtained to “identical” data by each group.

comparison using the **resulting PDFs from the benchmark fits** from each group

ID	Expt.	$N_{pt}$	$\chi^2$ (CT)	$\chi^2$ (MSHT)	$\chi^2$ (NNPDF)
101	BCDMS $F_2^p$	329/163 <sup>††</sup> /325 <sup>†</sup>	348.96	163.86	393.06
102	BCDMS $F_2^d$	246/151 <sup>††</sup> /244 <sup>†</sup>	260.50	133.68	265.35
104	NMC $F_2^d/F_2^p$	118/117 <sup>†</sup>	111.54	109.87	103.38
124+125	NuTeV $\nu\mu\mu + \bar{\nu}\mu\mu$	38+33	62.03	48.46	92.53
160	HERAI+II	1120	1378.87	1344.87	1358.04
203	E866 $\sigma_{pd}/(2\sigma_{pp})$	15	21.35	12.52	5.49
245+250	LHCb 7TeV& 8TeV $W, Z$	29+30	64.65	70.17	82.57
246	LHCb 8TeV $Z \rightarrow ee$	17	23.42	24.91	26.24
248	ATLAS 7TeV $W, Z$ (2016)	34	65.65	66.28	75.54
260	D0 Z rapidity	28	17.51	16.28	17.39
267	CMS 7TeV electron $A_{ch}$	11	6.93	17.63	8.22
269	ATLAS 7TeV $W, Z$ (2011)	30	31.35	28.04	29.07
545	CMS 8TeV incl. jet	185/174 <sup>††</sup>	185.40	241.93	232.55
Total	$N_{pt}$	—	2263	1991	2256
Total	$\chi^2$	—	2584	2278.51	2689.42

Table from [T. Hobbs](#).

Do the same when using the same PDFs (PDF4LHC15) as input.

comparison using PDF4LHC15 as input PDF for the calculations of each group

ID	Expt.	$N_{pt}$	$\chi^2$ (CT)	$\chi^2$ (MSHT)	$\chi^2$ (NNPDF)
101	BCDMS $F_2^p$	329/163 <sup>††</sup> /325 <sup>†</sup>	442.07	195.90	479.67
102	BCDMS $F_2^d$	246/151 <sup>††</sup> /244 <sup>†</sup>	239.36	191.57	298.47
104	NMC $F_2^d/F_2^p$	118/117 <sup>†</sup>	109.11	110.31	108.22
124+125	NuTeV $\nu\mu\mu + \bar{\nu}\mu\mu$	38+33	60.76	35.94	34.04
160	HERAI+II	1120	1421.59	1394.38	1782.61
203	E866 $\sigma_{pd}/(2\sigma_{pp})$	15	6.68	8.15	7.70
245+250	LHCb 7TeV& 8TeV $W, Z$	29+30	103.04	79.21	154.97
246	LHCb 8TeV $Z \rightarrow ee$	17	22.93	28.10	45.63
248	ATLAS 7TeV $W, Z$ (2016)	34	228.16	253.80	241.70
260	D0 $Z$ rapidity	28	17.10	16.12	16.78
267	CMS 7TeV electron $A_{ch}$	11	33.18	5.54	7.99
269	ATLAS 7TeV $W, Z$ (2011)	30	35.92	36.91	40.73
545	CMS 8TeV incl. jet	185/174 <sup>††</sup>	282.23	329.00	326.81
Total	$N_{pt}$	—	2263	1991	2256
Total	$\chi^2$	—	3002.13	2684.92	3545.34

Table from T. Hobbs.

In order to investigate differences will look in detail at theoretical predictions from each group with same input PDFs.

Some differences expected from e.g. choice of heavy flavour scheme.

## Benchmarking with Toy data sets for purposes of Correlations

Suggested a shorter subset of data which should provide reasonable constraints.

- HERA I + II inclusive cross sections from DIS.
- E866 fixed target Drell-Yan  $pd/pp$  data.
- CDF  $Z$  rapidity distribution.
- ATLAS  $W, Z$  7 TeV rapidity distributions.
- D0 1.96 TeV  $W$  asymmetry.
- CMS 7 TeV inclusive jet data.

Significant, but not complete overlap with base set considered above.

Have been generated in **xFitter** input format.

- To check correlations between different datasets we generated toys for all datasets together.
- List of datasets:

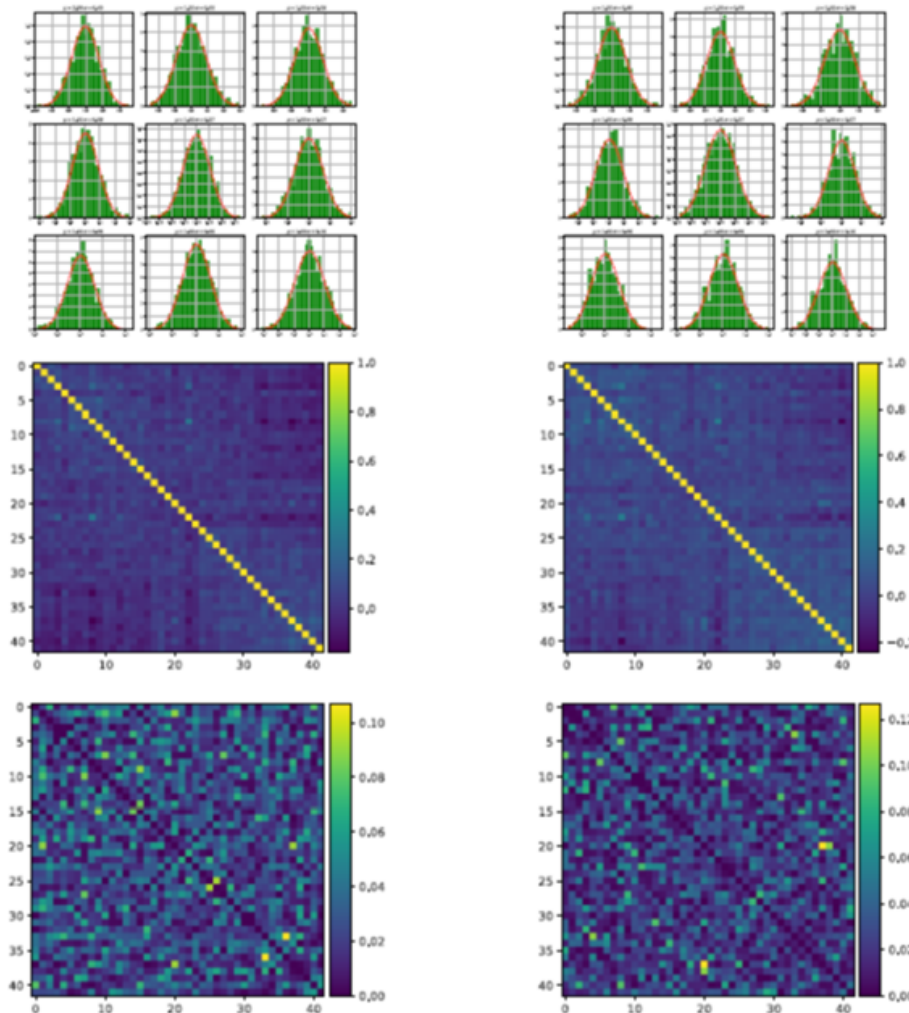
Experiment	DataSet Name
ATLAS	W-,W+, Z peak CC, Z peak CF, Z high mass CF, Z high mass CC, Z low mass
CMS	inclusive jets
HERA1+2	CCem, CCep, NCem, NCep 460, NCep 575, NCep 820, NCep 920
Tevatron	Tab11 (instead of E866), CDF Z Boson Rapidity, D0 Wel pt25

- For each dataset Valiantsin generated 999 toys.

“checked that the covariance matrices computed from the systematic uncertainties reported in the paper are well reproduced if they are recomputed from the toys”

# Example for the HERA inclusive data.

## HERA CEm

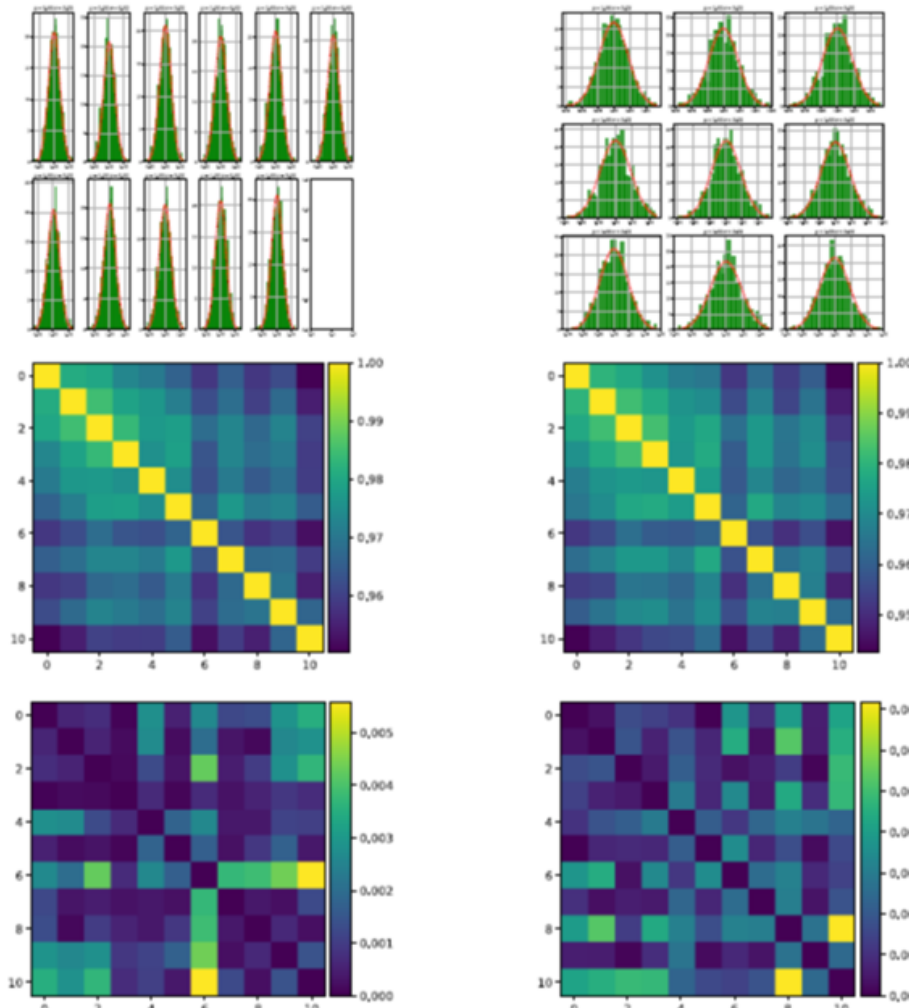


- Left plots – separate generated toys, right – together;
- Top plots – Gaussian with mean 1.00;
- Mid plots – There are some bins with negative coeff in right plot;
- Bot plots – Abs difference corr matrices from xfitter and toys matrix – As a result of negative bins – larger difference;



# Example for the ATLAS $W$ data.

## ATLAS $W_+$

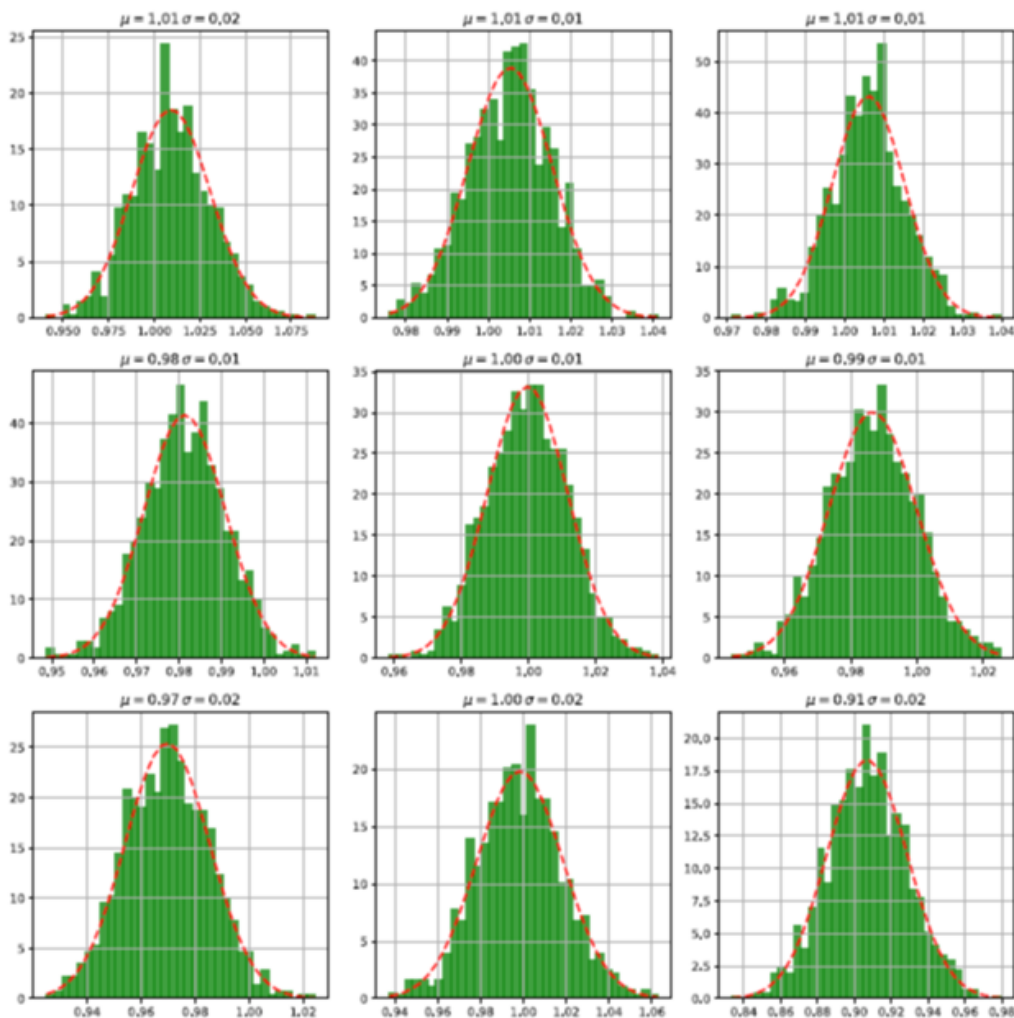


- Left plots – separate generated toys, right – together;
- Top plots –  $\frac{\sigma_{toys}}{\sigma_{Orig}}$  – Gaussian with mean 1.00;
- Mid plots – Correlation toys matrices – left and right looks very similar;
- Bot plots – Abs difference corr matrices from xfitter and toys matrix – difference < 0.04;
- Other ATLAS datasets on git.



Example for the E866 Drell Yan ratio data.

## Tevatron Tab11 bins



- Bin number 9 (bot right) have mean = 0.91.
- It is not a statistical problem.

Slight differences to data sets within PDF4LHC exercise.

Discover some data sets treated a little differently by different groups, e.g. different versions of data (CDF rapidity, CMS jets).

Not necessarily a problem - PDF4LHC study initially concentrating on data common to all groups – not necessarily limited selection giving best constraints. However, overlap clearly not perfect.

MSHT2020-like PDF fit gives stable results with (smaller than) base Toy set. May well not be true for all other groups (partially luck). Achieving stability can introduce bias not present in global fit (tendency of any fit to restricted data sets).

## Conclusions

Many updates from PDF groups in the recent past. Include large but varying amounts of LHC data – starting to have a very significant impact on PDF extractions.

Theory catching up for precision data, e.g. NNLO jets, differential top,  $Z, W p_T$  ... More data leads to possible improvements in parameterisation.

Uncertainties generally come down, but not always – in some regions just more realistic.

Agreement between groups generally better, more so in details of flavour separation (e.g. ATLAS fits with a number of extra sets beyond HERA data can get some reasonable  $\bar{d} - \bar{u}$  constraint). Agreement not always better in more general PDF, e.g. gluon.

Benchmarking exercise gives good agreement between groups on most generally constrained quantities, i.e. gluon, singlet quarks. Differences surprisingly large in PDFs depending on flavour separation.

Still to be understood, but certainly partially due to biases from fitting PDFs with lack of full constraint. Needs to be understood before really pushing further.

Underlines (clarifies) importance of fitting maximal range of data types to get true constraints. Some PDFs, in particular details of flavour separation, constrained partially, but largely compatibly, by a wide variety of data types.

In these cases more data improves agreement even when different choices of those data, assumptions, theory choices etc. used.

When a small number of data sets constrain a PDF then more sensitivity to procedures/choices, i.e. biases.