

Resummation Benchmarking

- Status Report

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On behalf of the Resummation Sub-group



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Introduction

- W mass measurement now possible to increasing precision at the LHC, **utilises Z p_T spectrum**.
- Necessitates increased accuracy in theory predictions - many development in this area.
- Sudakov double logarithms ($L = Q^2/q_T^2$) are left over from the cancellation of IR divergences:

$$\frac{d\sigma}{dq_T} \sim 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + 1) + \alpha_s^3(L^6 + L^5 + \dots) + \dots$$

$$\sim \sum_n \exp(\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1} + \dots)$$

- At low q_T $\alpha_s L^2 \sim 1$ and perturbative expansion breaks down \Rightarrow resummation.
- Resum these large logs up to given order by exponentiation and RGE evolution - **Possible up to N3LL**.
- Many different approaches - we wish to compare them to understand their differences, uncertainties and accuracy.

Different approaches to resummation

Schematic!

- CSS q_T resummation:

$$\frac{d\sigma_{res}}{dq_T} \sim e^S \times [(HC_1 C_2) \otimes f_1 \otimes f_2]$$

- TMD resummation:

$$\frac{d\sigma_{res}}{dq_T} \sim H \times F_1 \times F_2$$

- SCET resummation:

$$\frac{d\sigma_{res}}{dq_T} \sim H \times B_1 \times B_2 \times S$$

- Parton Shower-like (parton branching):

- ▶ Parton shower based with Sudakov factor S denoting probability of no resolvable branching emissions.
- ▶ Ordered emissions ensure control of sub-leading logs.

These different approaches are equivalent for the resummed piece at each order (up to power corrections $\mathcal{O}[(q_T/Q)^n]$).

Setup in CSS q_T resummation

Collins, Soper, Sterman, '85
 Catani, de Florian, Grazzini, '01
 Bozzi, Catani, de Florian, Grazzini, '05

$$\frac{d\sigma_{res}^F(p_1, p_2, Q^2, \mathbf{q}_T, y, \mathbf{\Omega})}{dQ^2 d^2\mathbf{q}_T dy d\mathbf{\Omega}} = \int \frac{d^2b}{(2\pi)^2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} W^F(\mathbf{b}, z_1, z_2, \dots)$$

$$\equiv \frac{Q^2}{s} \left[d\hat{\sigma}_{c\bar{c}}^{F, LO} \right] \int \frac{d^2b}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_T} \underbrace{S_c(Q^2, b_0^2/b^2)}_{\text{Sudakov}}$$

$$\times \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} \underbrace{H^F}_{\text{Hard Factor}} \underbrace{C_1 C_2}_{\text{Collinear Factors}} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2).$$

Perturbative Coefficients
- Universal

where

$$S_c(\mu_2^2, \mu_1^2) = \exp \left\{ - \int_{\mu_1^2}^{\mu_2^2} \frac{dq^2}{q^2} \left[A_c(\alpha_s(q^2)) \log \frac{\mu_2^2}{q^2} + B_c(\alpha_s(q^2)) \right] \right\},$$

$$H_q^F = \frac{|\tilde{M}_{q\bar{q} \Rightarrow F}|^2}{|M_{q\bar{q} \Rightarrow F^{(0)}}|^2}. \quad C_{qa}(z, \alpha_s) = \delta_{qa} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n C_{qa}^{(n)}(z).$$

Setup in CSS q_T resummationLow/High q_T High/Low b .

- b space - impact parameter space factorises kinematics:

$$\int d^2\mathbf{q}_T \exp(-i\mathbf{b}\cdot\mathbf{q}_T) \delta(\mathbf{q}_T - \sum_i \mathbf{q}_{iT}) = \prod_i \exp(-i\mathbf{b}\cdot\mathbf{q}_{iT}).$$

- Mellin space - Convolution of C factors and PDFs \Rightarrow product.
- Landau pole regularisation - b^* prescription ensures b freezes out at high b (low q_T) to avoid divergence (more later).
- Non-perturbative form factor - Can reintroduce non-perturbative effects via exponential S_{NP} term, form requires fitting to data.
- Modified Logs - “Nominal” Logs used $\log(Q^2 b^2) \Rightarrow \log(1 + Q^2 b^2)$ ensures resummed piece tends to 0 at low b (high q_T).
- Additive Matching resummed piece, low q_T finite piece, high q_T

$$\frac{d\hat{\sigma}}{dq_T^2} = \underbrace{\left[\frac{d\hat{\sigma}^{(\text{res})}}{dq_T^2} \right]_{\text{l.a.}}}_{\text{resummed piece, low } q_T} + \underbrace{\left[\frac{d\hat{\sigma}^{(\text{fin})}}{dq_T^2} \right]_{\text{f.o.}}}_{\text{finite piece, high } q_T}$$

- Many differences in the different formalisms.

Groups and Codes involved

- q_T resummation
 - ▶ DYRes/DYTURBO Camarda et al., '19
 - ▶ reSolve Coradeschi, T.C., '17
- TMD
 - ▶ NangaParbat Bacchetta et al., '19
 - ▶ arTeMiDe Scimemi, Vladimirov, '17
- SCET
 - ▶ SCETLib Ebert et al. '17
 - ▶ (CuTe) Becher et al. '11,'20
- Parton Shower-like/Branching
 - ▶ RadISH Monni et al. '16,'17
 - ▶ (PB-TMD) Martinez et al. '20

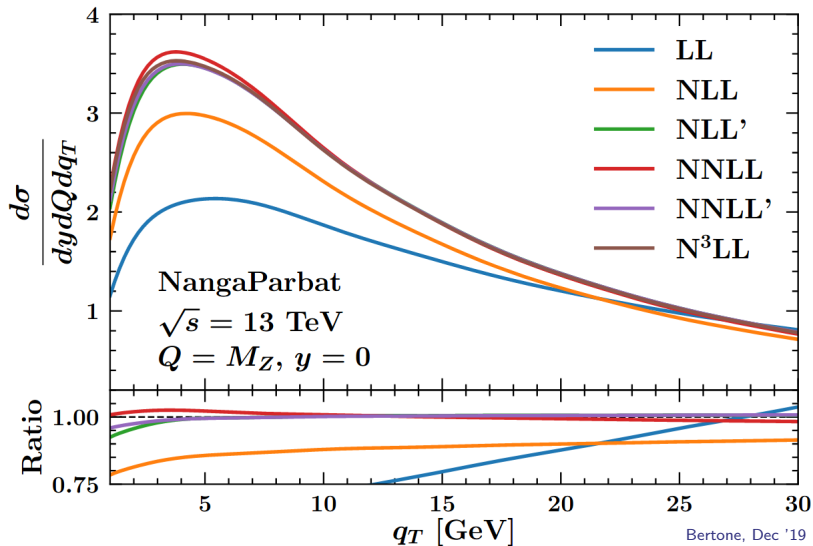
Many groups, well spread across the several different approaches.

Level 1 settings

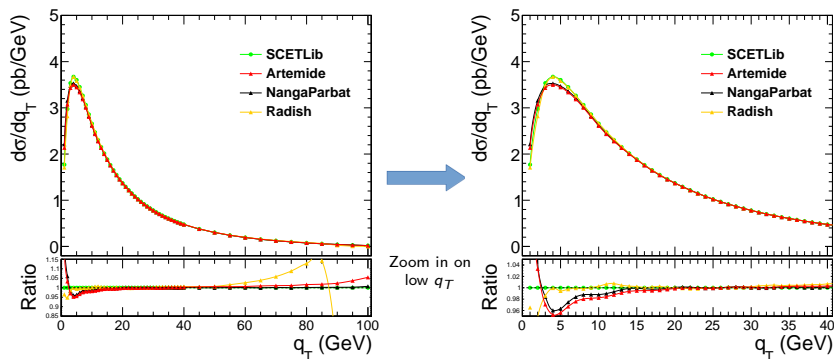
Essentially consistency in settings wherever possible!

- Consider triple differential cross-section $\frac{d\sigma}{dQdYdq_T}$.
- Z/γ^* production at 13TeV.
- Across values of $Q = m_Z, 1\text{TeV}$ and $y = 0, 2.4$; but focusing on $Q = m_Z, y = 0$.
- Consider all logarithmic orders up to NNLL'/N3LL.
- Only “Canonical” logs, i.e. unmodified $L = \log(Q^2 b^2)$.
- Resummed piece only.
- Across q_T range of 0 to 100GeV, focus though on low q_T where resummed piece relevant.
- No non-perturbative function S_{NP} , Landau pole regularisation treated differently but common value b_0/b_{max} used.
- Scales fixed to Q where naturally at hard scale.
- Same PDF choice, same $\alpha_s(m_Z) = 0.118$, same EW settings, no lepton cuts.

Perturbative Convergence



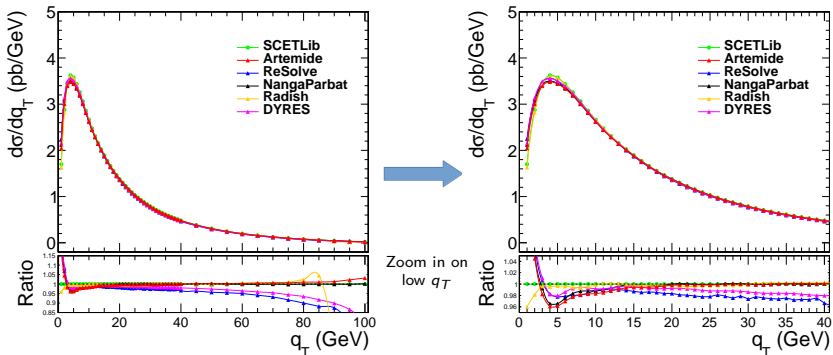
Level 1 - N3LL



Highest order computed in benchmarking.

Remember, spectrum not physical at Level 1 (or 2) stage outside low q_T .

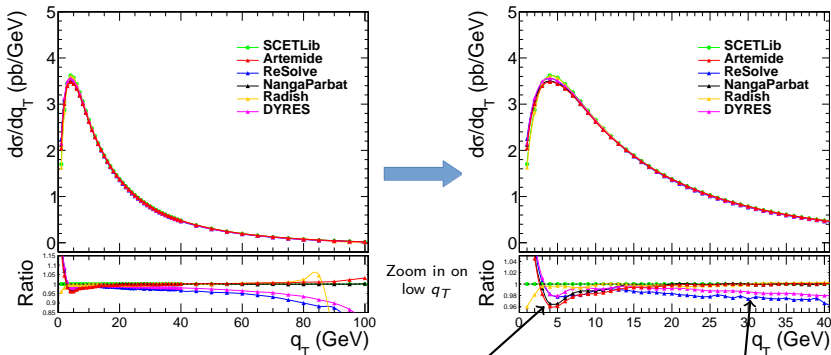
Level 1 - NNLL'



Some codes only go up to NNLL' (log counting differences).

Remember, spectrum not physical at Level 1 (or 2) stage outside low q_T .

Level 1 - NNLL' differences



Differences:

- Small differences at low q_T .
- q_T resummation codes show differences at intermediate - high q_T .

Landau Pole

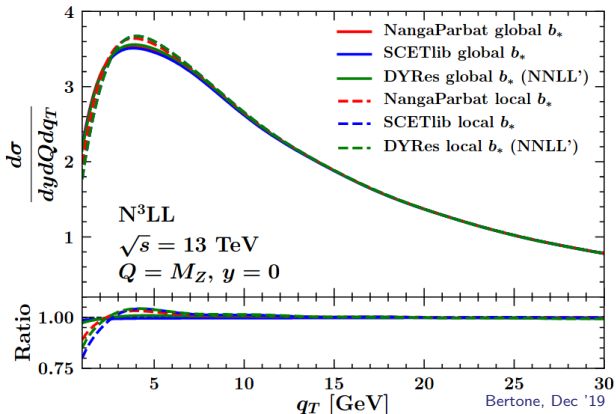
- One of main differences noted in Level 1 was in the small q_T region due to **Landau pole regularisation**.
- At low enough scales α_s will hit the Landau pole and so your result will diverge.
- In reality you are hitting a region of **non-perturbative effects**.
- Different formalisms avoid this in different manners, these would then be added back in by fitting to data via non-perturbative form factors.
- In b space the **b^* prescription** is often used:
 - ▶ Freezes b to b_{lim} at large b (low q_T).

$$b \Rightarrow b^* = \frac{b}{\sqrt{1 + b^2/b_{lim}^2}} = \begin{cases} b, & \text{for small } b/b_{lim} \ll 1, \text{ i.e. high } q_T. \\ b_{lim}, & \text{for large } b/b_{lim} \sim 1, \text{ i.e. low } q_T. \end{cases}$$

- Other methods, e.g. going around pole in complex plane (minimal prescription) available.

Global vs local b^*

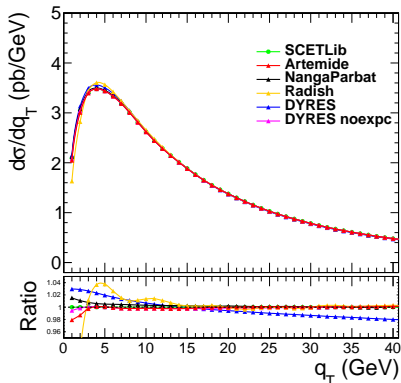
- Different implementations of b^* prescription - **global or local**, latter only $b \Rightarrow b^*$ replacement in α_s and PDFs but not logs.



- Affects **low q_T** end of spectrum - responsible for differences seen here in Level 1.

Resummation scheme - Intermediate and High q_T

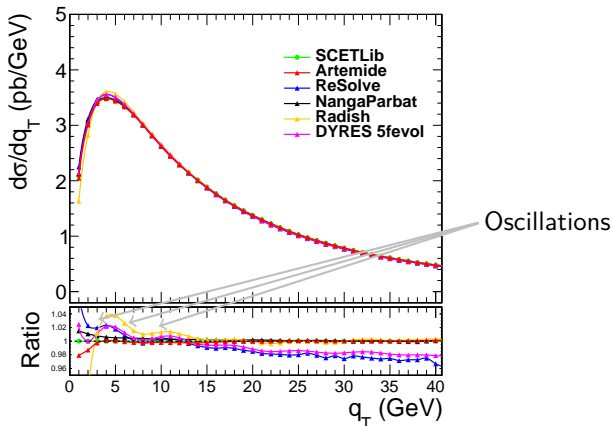
- Differences at intermediate and high q_T expected at level 2 due to transition functions, profile scales and matching in general.
- However at level 1 those effects are removed, therefore why the difference at intermediate - high q_T (where resummed piece not physical) - **Resummation scheme**



- Differences in the resummation schemes used, this **mainly affects the C and B coefficients** - presence in Sudakov and evolution or not - in different formalisms.
- An approximate scheme change can remove this and causes the difference in DYRES at intermediate-high q_T to disappear.

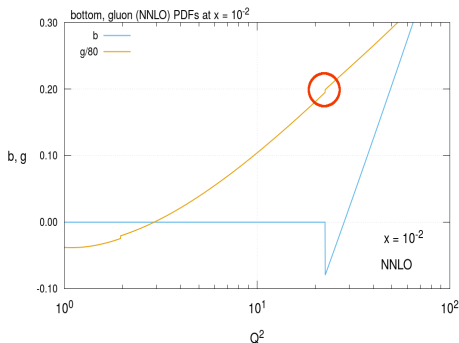
Small oscillations in ratios

- Looking closely at the ratio plots for Level 1 benchmarking, **coherent small oscillations** are seen in **ratios** between reSolve, DYRes, RadISH and SCETlib, NangaParbat, arTeMiDe - why?



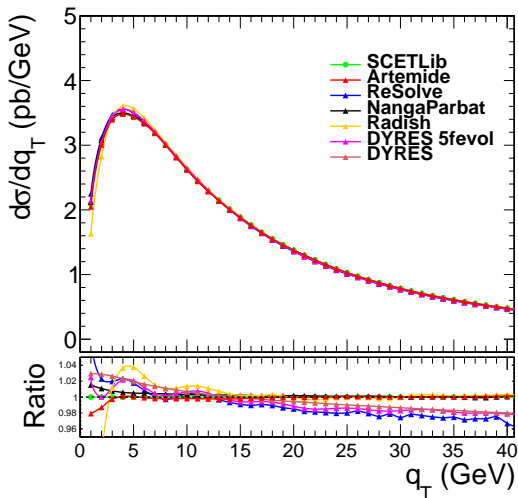
Small oscillations in ratios - PDF evolution

- reSolve and DYRes use PDFs in Mellin space, then **backward evolved internally** down to low scales, cross no thresholds.
- SCETlib, NangaParbat, arTeMiDe use LHAPDF evolution, this **crosses quark mass thresholds** \Rightarrow PDF discontinuities, done in b space \Rightarrow tiny oscillations in q_T space evident in ratios.
- RadISH will cross the **discontinuities** in q_T space \Rightarrow discontinuity in its spectrum rather than oscillation (oscillates *relative* to SCETlib, etc.).
- These are **tiny effects**, only clear in the ratios, but interesting to understand them nonetheless!



Small oscillations in ratios

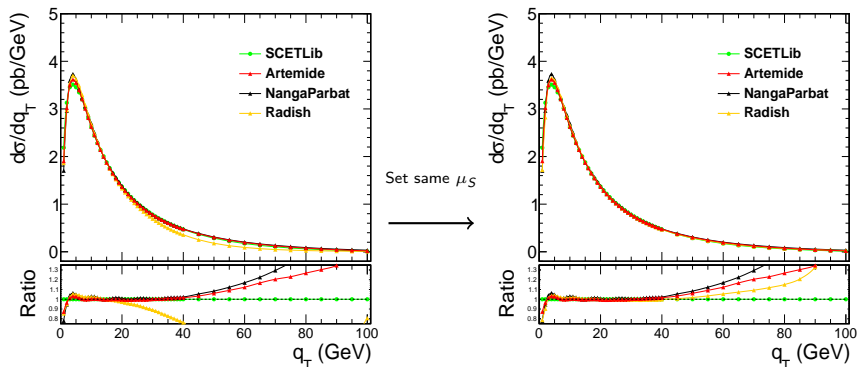
- DYRes can perform either LHAPDF evolution or 5-flavour backwards Mellin space internal evolution.
- When the LHAPDF evolution is performed it will cross the same thresholds and gain the same oscillations.
- Therefore oscillations removed from the ratio.



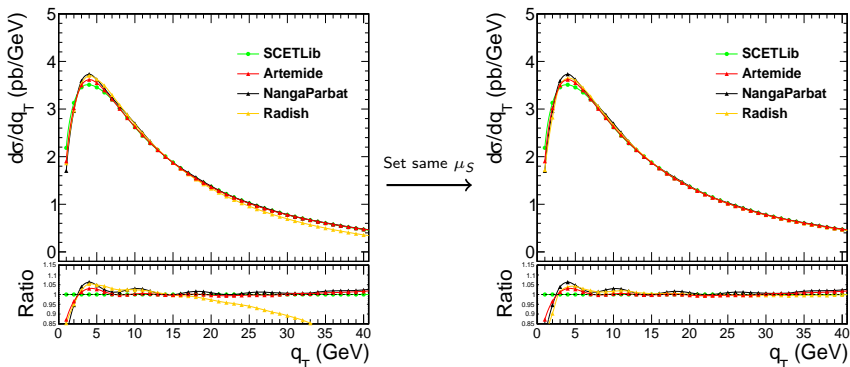
Level 2 Settings

- Still no non-perturbative S_{NP} factor included.
- Still no matching to finite piece - resummed piece only.
- Groups use their own *default settings* beyond this:
 - ▶ Different Landau pole regularisations, local vs global b^* , b_{lim} setting etc. \Rightarrow Will affect low q_T .
 - ▶ **Nominal Modified Logs** now used $\log(Q^2 b^2) \Rightarrow \log(1 + Q^2 b^2)$, different groups have their own settings \Rightarrow Will affect high q_T .
 - ▶ Choose own **scales** - e.g. resummation scale $\mu_S = Q/2, Q$, i.e. resumming $\log(1 + (m_Z)^2 b^2)$ or $\log(1 + (m_Z/2)^2 b^2)$ respectively. \Rightarrow Will affect intermediate q_T most.
 - ▶ Potential inclusion of damping functions, profile scales, different modified logs, etc.

Level 2 - N3LL

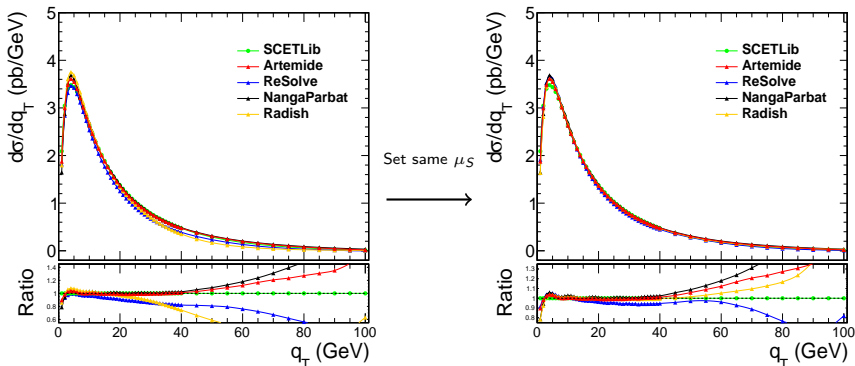


- Main difference is due to choice of central resummation scale $\mu_S = m_Z$ or $m_Z/2$, alters logs being resummed, should be absorbed in matching with the finite piece (level 3).
- Differences in b^* , resummation scheme, etc as in level 1.
- Additional differences at intermediate-high q_T due to different damping functions, transition functions, modified logs etc

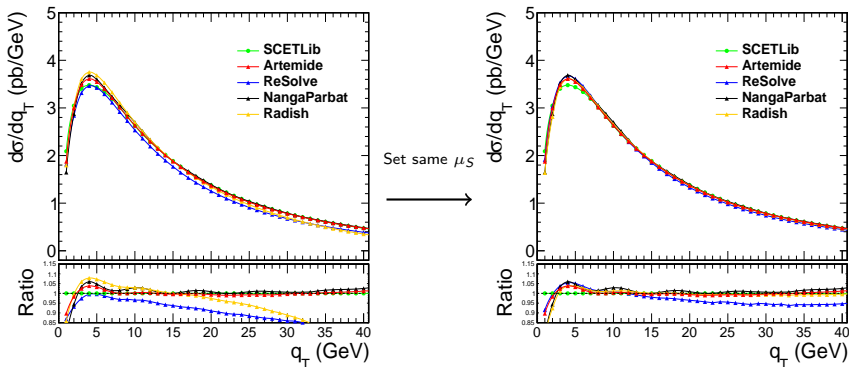
Level 2 - N3LL low q_T 

- Main difference is due to choice of central resummation scale $\mu_S = m_Z$ or $m_Z/2$, should be absorbed in matching with the finite piece (level 3).
- Differences in b^* , resummation scheme, etc as in level 1.

Level 2 - NNLL'



- Main difference is due to choice of central resummation scale $\mu_S = m_Z$ or $m_Z/2$, should be absorbed in matching with the finite piece (level 3).
- Differences in b^* , resummation scheme, etc as in level 1.
- Additional differences at intermediate-high q_T due to different damping functions, transition functions, modified logs etc

Level 2 - NNLL' low q_T 

- Main difference is due to choice of central resummation scale $\mu_S = m_Z$ or $m_Z/2$, should be absorbed in matching with the finite piece (level 3).
- Differences in b^* , resummation scheme, etc as in level 1.
- reSolve difference resummation scheme or power corrections between canonical and modified logs?

Scales

- As expected, more differences at Level 2 as codes using (largely with some exceptions) default settings.
- Several sources of **theoretical uncertainties**, many of which can be probed by **scale variations**.
- However there are different scales in different formalisms and implementations:
 - reSolve, DYRes, RadISH have **factorisation** (μ_F), **renormalisation** (μ_R) and **resummation scale** (μ_S) uncertainties all set around the hard scale:

$$\log(Q^2 b^2) = \log(\mu_S^2 b^2) + \log(Q^2 / \mu_S^2)$$

- Usually do a **9-point variation** where μ_F and μ_R are varied together and μ_S separately, *envelope then taken as scale variation band*:

$$(\mu_R/Q, \mu_F/Q, \mu_S/Q) = (0.5, 0.5, 1), (0.5, 1, 1), (1, 0.5, 1), (1, 1, 1), \\ (1, 2, 1), (2, 1, 1), (2, 2, 1), (1, 1, 0.5), (1, 1, 2).$$

Scales

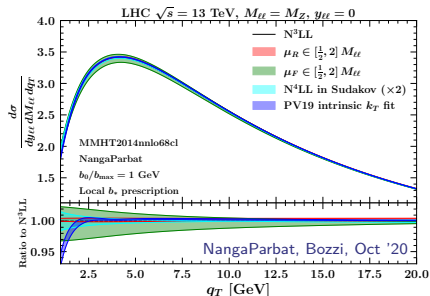
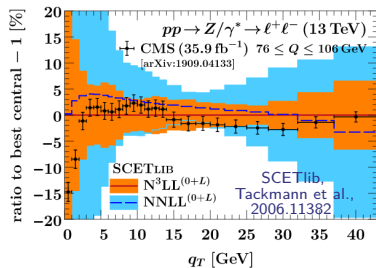
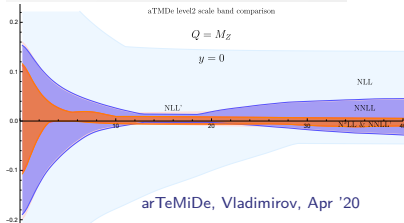
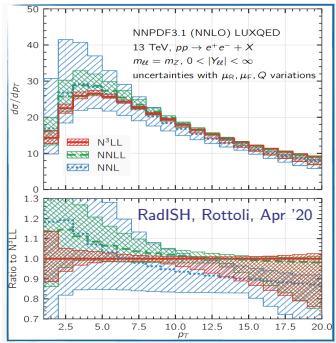
- In TMD factorisation you have **2 pairs scales** $(\mu, \zeta) \rightarrow (\mu_0, \zeta_0)$ - initial and final scales for 2D evolution:
 - ▶ Ultimately 2 of these are varied:
 - ⇒ 1 at high scale - related to **renormalisation scale** in CSS language.
 - ⇒ 1 at low scale - related to **PDFs** which in TMDs are computed exactly at **low scales** here.
 - ⇒ No resummation scale.
 - ▶ Differences in exact setup between NangaParbat and arTeMiDe.
- In SCETlib - again **2 starting scales** (μ_i, ν_i) and **2 ending scales** (μ, ν) , can in theory be set separately for the **Hard, Beam and Soft functions** (although not ν for H), central choice:

$$\mu_H = Q, \quad \mu_B = b_0/b, \quad \nu_B = Q, \quad \mu_S = \nu_S = b_0/b.$$

- **“Profile scales”** are used to switch μ_B, μ_S, ν_S between the resummed (b_0/b) at low q_T and fixed order (Q) at high q_T .
- **36 profile scale variations** in relevant log ratios by a factor of 2 (not 4) + fixed order scale Q varied by factor of 2.

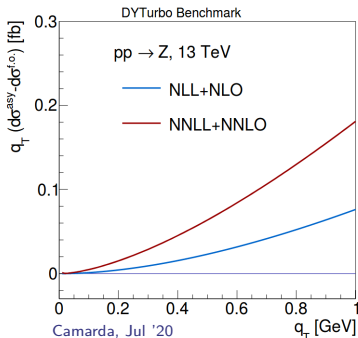
Level 2 with uncertainties

Not yet complete, an idea can be gained from separate results:



Matching - Level 3

- Matched finite piece to be calculated at **LO and NLO for $V+jet$** by DYTurbo to be used by the groups with their own resummed pieces and matching implementation.
- Enables Level 3 predictions where possible.
- Done for $Q = m_Z, y = 0$ point focused on.
- Renormalisation and factorisation scale variations up to factor of 2 provided.



Level 3.5? - Non-perturbative factors:

- For description at low q_T need to include a **non-perturbative contribution** - S_{NP} , this is an **exponential in b, x, Q_0** typically.
- Have both intrinsic transverse momentum dependence of initial states (boundary condition for TMDs) and non-perturbative contributions to evolution.
- This is modelled by **fits to relevant data for low values of q_T/Q** - e.g. Older Fermilab data, Tevatron data and new LHC DY data.
- NangaParbat - 9 parameter fit ($\lambda, N_1, \sigma, \alpha, N_{1B}, \sigma_B, \alpha_B, g_2, g_{2B}$).

- Can also fit along with SIDIS data, but need also high Q^2 , arTeMiDe have used HERMES and COMPASS data as well as DY data.

Scimemi et al., '19, '20

- Perhaps do a pseudodata fit?

$$f_{NP}(x, b_T, \zeta) = \left[\frac{1 - \lambda}{1 + g_1(x) \frac{b_T^2}{4}} + \lambda \exp\left(-g_{1B}(x) \frac{b_T^2}{4}\right) \right] \times \exp\left[-(g_2 + g_{2B} b_T^2) \ln\left(\frac{\zeta}{Q_0^2}\right) \frac{b_T^2}{4}\right]$$

Bacchetta et al., '19

"intrinsic" NP contribution
(x - and b_T -dependent)

Bertone, Mar '20

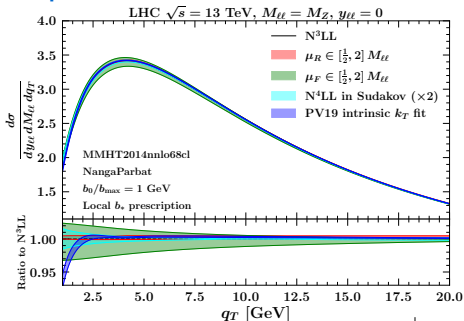
NP correction to pert. evolution
(b_T -dependent)

$$g_1(x) = \frac{N_1}{x\sigma} \exp\left[-\frac{1}{2\sigma^2} \ln^2\left(\frac{x}{\alpha}\right)\right],$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B} \exp\left[-\frac{1}{2\sigma_B^2} \ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

x-dep. width of TMDs

Non-perturbative fits



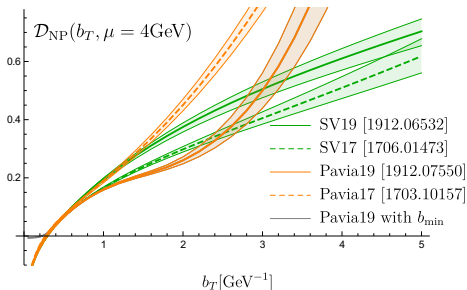
- SV19 fit by arTeMiDe results in non-perturbative factor \mathcal{D}_{NP} .
- Shows effect of inclusion of SIDIS data on top of Drell-Yan.

arTeMiDe, Vladimirov, 2003.02288

- Non-perturbative effects low q_T end of the spectrum.
- PV19 fit by NangaParbat shows potential sensitivity to these effects.

NangaParbat, Bozzi Oct '20

Bacchetta et al., 1912.07550



Conclusions

Overall **good progress** in the resummation sub-group and many **useful discussions!** Thanks to all participants and codes involved and also to Daniel and Aram for organising and coordinating.

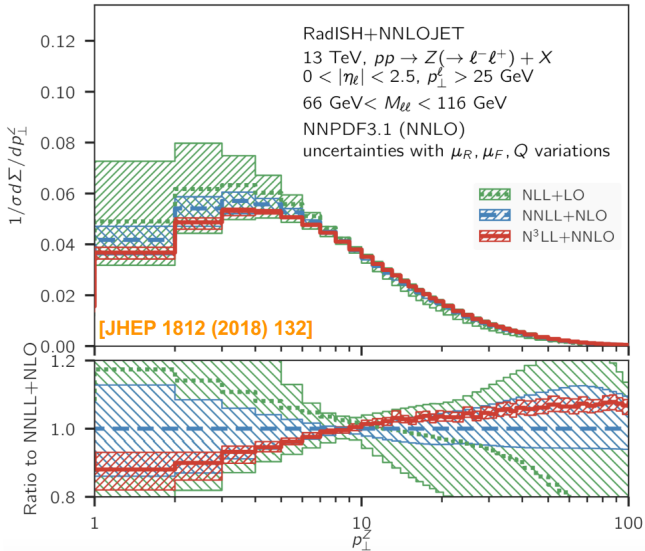
- ✓ Level 1 benchmarking complete
 - ✓ *Very good agreement* between codes, especially at low q_T .
 - ✓ Many new effects understood - e.g. Landau pole regularisations (b^* prescription), and PDF thresholds (small oscillations in ratios)
- ✓ Level 2 benchmarking well on its way -
 - ✓ Good agreement generally, some differences in intermediate and high q_T regions as expected.
 - ✓ Uncertainties provided via scale variations.
 - ✓ Many useful discussions about different scales and scale variations.
- Level 2 benchmarking with uncertainties to be finished soon.
- Level 3 benchmarking with matching has begun.
- Potential Level 3.5 including non-perturbative contributions.
- Beginning process of documenting this for *Yellow Report* and *separate publication*.

Differences in methods

- b (impact parameter) space or direct k_T space
- Treatment of non-perturbative effects and Landau pole - cut-off, freezing, etc
- Matching to fixed order
 - ▶ Multiplicative vs Additive.
 - ▶ Any damping function (profile scales, etc), transition functions, modified logs.
- Scales present (renormalisation, factorisation, resummation, rapidity, etc)

Many differences in methods, understanding of their impacts is part of the benchmarking exercise.

Perturbative convergence



Global vs local b^*

🍏 In b_T space the **unregularised** (diverging) cross section looks like this:

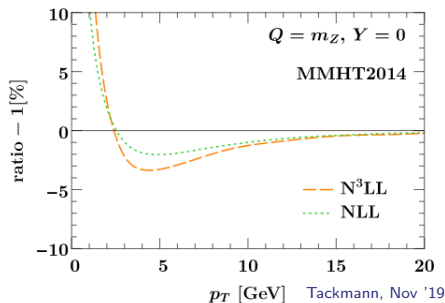
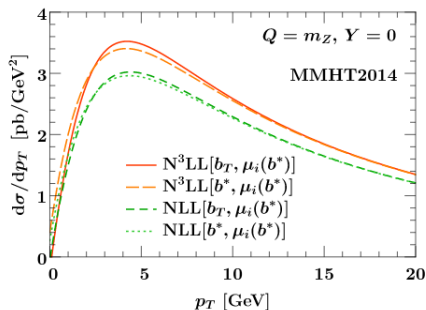
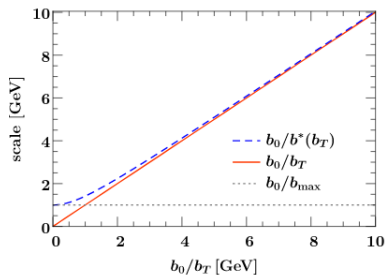
$$\frac{d\sigma}{dq_T} = \int_0^\infty db_T b_T J_0(b_T q_T) \left[\sum_{n=0}^\infty \alpha_s^n \left(\frac{1}{b_T} \right) \sum_{k=0}^{2n} \ln^k(Q^2 b_T^2) \frac{d\bar{\sigma}^{[n,k]}}{dq_T} \right] \otimes \mathcal{L} \left(\frac{1}{b_T} \right)$$

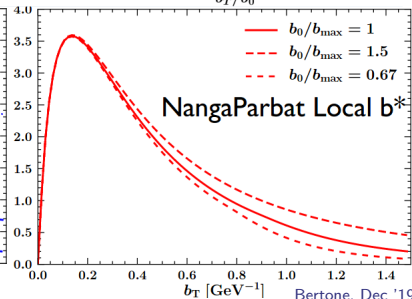
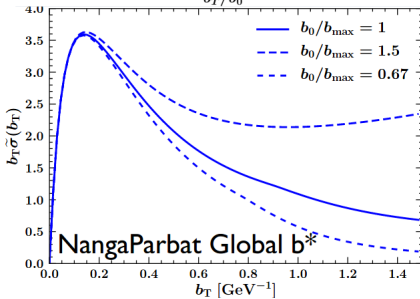
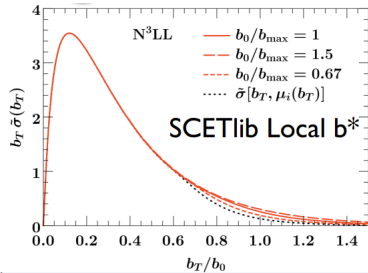
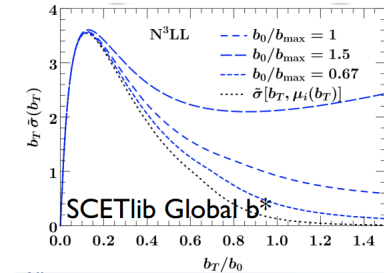
🍏 The **global** b^* prescription is:

$$\frac{d\sigma}{dq_T} = \int_0^\infty db_T b_T J_0(b_T q_T) \left[\sum_{n=0}^\infty \alpha_s^n \left(\frac{1}{b_*(b_T)} \right) \sum_{k=0}^{2n} \ln^k(Q^2 b_*^2(b_T)) \frac{d\bar{\sigma}^{[n,k]}}{dq_T} \right] \otimes \mathcal{L} \left(\frac{1}{b_*(b_T)} \right)$$

🍏 The **local** b^* prescription is:

$$\frac{d\sigma}{dq_T} = \int_0^\infty db_T b_T J_0(b_T q_T) \left[\sum_{n=0}^\infty \alpha_s^n \left(\frac{1}{b_*(b_T)} \right) \sum_{k=0}^{2n} \ln^k(Q^2 b_T^2) \frac{d\bar{\sigma}^{[n,k]}}{dq_T} \right] \otimes \mathcal{L} \left(\frac{1}{b_*(b_T)} \right)$$

Global vs Local b^* in q_T space

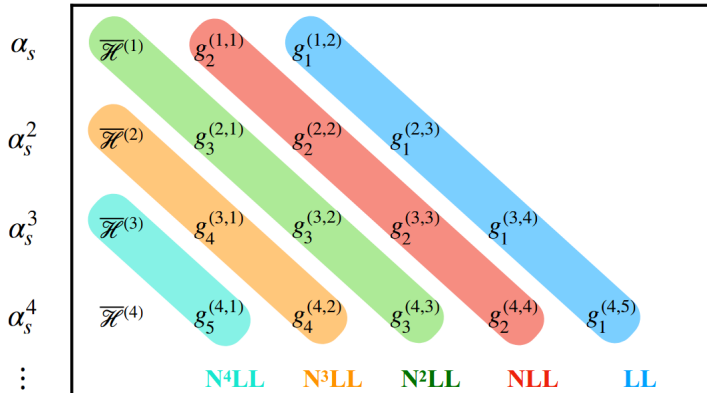
Global vs Local b^* in b space

Bertone, Dec '19

Logarithmic Counting - unprimed N(n)LL

$$\ln\left(\frac{d\sigma}{dq_T}\right) \propto \ln\left(1 + \sum_{m=1}^{\infty} \alpha_s^m \mathcal{H}^{(m)}\right) + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{n+1} g^{(n,k)} L^k \quad \alpha_s L \sim 1$$

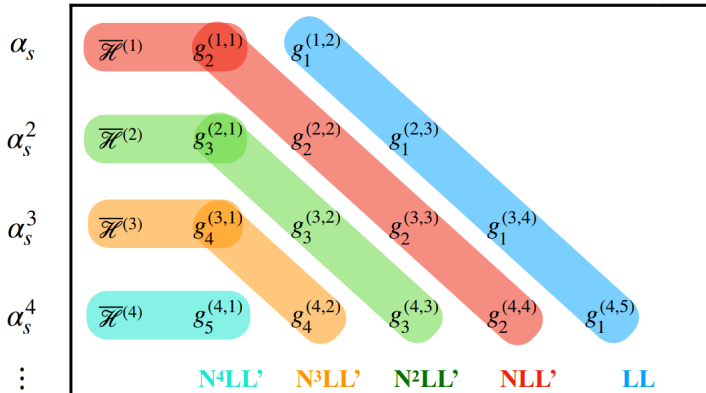
1 L L^2 L^3 L^4 L^5 ...



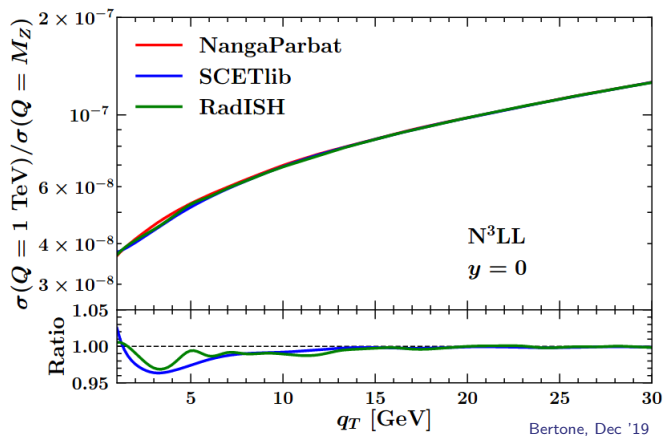
Logarithmic Counting - primed $N(n)LL'$

$$\ln\left(\frac{d\sigma}{dq_T}\right) \propto \ln\left(1 + \sum_{m=1}^{\infty} \alpha_s^m \mathcal{H}^{(m)}\right) + \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=1}^{n+1} g^{(n,k)} L^k \quad \alpha_s L \sim 1$$

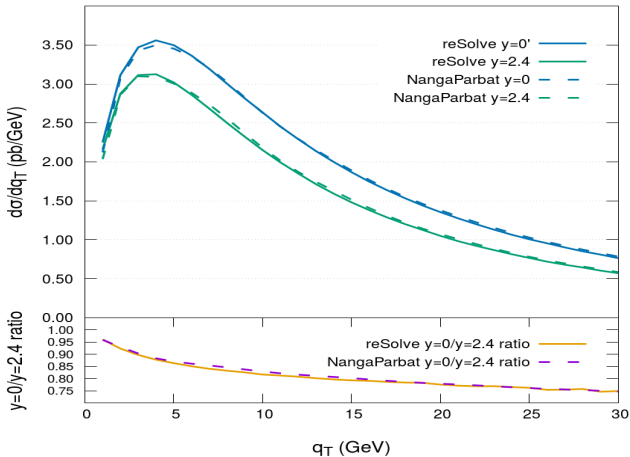
1 L L^2 L^3 L^4 L^5 ...



Different Q



Different calculations seeing same behaviour as the hard scale is increased, modulo small q_T differences already present.

Different y 

Different calculations seeing same behaviour at larger rapidities.
 Therefore we can focus on $Q = m_Z, y = 0$.

Resummation Scheme

- The master formula for resummation in this b -space Mellin space formalism is **invariant** under the transformation:

$$\begin{aligned}
 H_c^F(\alpha_s) &\Rightarrow H_c^F(\alpha_s)[h_c(\alpha_s)]^{-1}, \\
 B_c(\alpha_s) &\Rightarrow B_c(\alpha_s) - \beta(\alpha_s) \frac{d \log h_c(\alpha_s)}{d \log \alpha_s}, \\
 C_{ab}(\alpha_s, z) &\Rightarrow C_{ab}(\alpha_s, z) \sqrt{h_c(\alpha_s)}.
 \end{aligned}$$

where $h_c(\alpha_s)$ is a perturbative function $h_c(\alpha_s) = 1 + \mathcal{O}(\alpha_s)$.

- Can use this to make scheme choice - “**resummation scheme**”, can set H_c , all choices formally theoretically equivalent.
- In fact this is how the *universality of the formalism for different processes* arises.

Catani, de Florian, Grazzini, Nucl. Phys. B596: 299-312, 2001 arXiv:hep-ph/0008184 .

Resummation Scheme

- Many applications choose “*hard scheme*”, where factors in the flavour off-diagonal parts of the collinear functions $C_{ab}(z)$ proportional to $\delta(1-z)$ are removed \Rightarrow “*physical choice*”.
- In fact many choices are possible \Rightarrow in general collinear factors differ depending on the initiating partons at LO (gg or $q\bar{q}$).
- One can define an arbitrary scheme in which 1 $q\bar{q}$ initiated process F has $H_q^{F(n)} = 0$ and 1 gg initiated process F' has $H_g^{F'(n)} = 0$.
- In reSolve the “**Drell-Yan - Higgs**” scheme is chosen:

$$H_q^{DY(n)} = 0, \quad H_g^{h(n)} = 0 \quad \text{for all orders } n$$

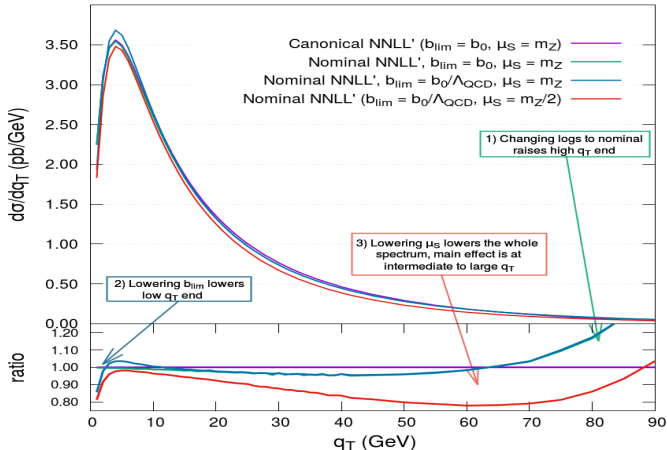
Could these resummation scheme differences be responsible for some of the differences at intermediate to high q_T in the comparison, where the spectrum is no longer physical?

reSolve Level 1 \rightarrow Level 2 changes

reSolve changes level 1 \Rightarrow level 2:

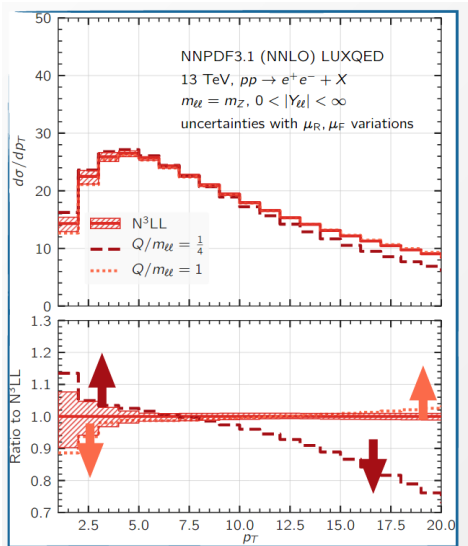
- 1 Canonical \Rightarrow nominal logs, affects high q_T tail.
- 2 b^* prescription - b_{lim} raised so reach higher b , default $b_{lim} = b_0 \exp(1/2\alpha_s\beta_0)/\mu_S \sim \Lambda_{QCD}^{-1} \Rightarrow$ affects low q_T .
- 3 Resummation scale $\mu_S = m_Z/2$ taken, affects whole spectrum, largely intermediate q_T .
- 4 No further suppression of resummed piece at intermediate/large q_T at this stage

Level 1 to Level 2 changes

reSolve NNLL' $Q=mZ$ $y=0$ 

- Resummation scale change is large at intermediate to large $q_T \Rightarrow$ need to match with fixed order finite piece \Rightarrow counter term would cancel out much of this variation after matching.

Resummation Scale



Rottoli, Apr '20

- **Unitarity constraint** - Total cross-section fixed to finite order total in both CSS and RadISH \Rightarrow effect in tails is absorbed when the matching to fixed order is performed.
- As a result downwards scale variation on μ_S when the resummed piece alone is considered is too large.