Mixed QCDxEW corrections for Drell-Yan processes

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LHC EW Working Group General Meeting

Outline

Mixed QCDxEW corrections for Drell-Yan processes

- complexities and approximations
- first results for the off-shell case

Mixed QCDxEW corrections in the resonance region

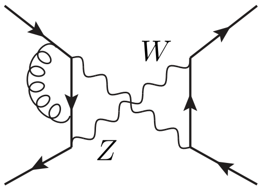
- motivations and simplifications
- recent progress: inclusive cross section and differential distributions.

Conclusions and outlook

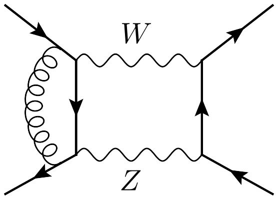
Mixed QCDxEW to Drell-Yan processes: why it is hard

The complete calculation of the mixed QCDxEW corrections for a fully off-shell dilepton system involve several technical complexities:

Amplitudes/Integrals



Two-loop integrals involving several energy scales

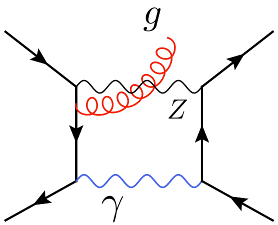


Scattering amplitudes not available yet

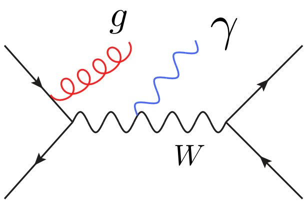
Recent developments:

[Bonciani, Di Vita, Mastrolia, Schubert 1604.08581] [Heller, von Manteuffel, Schabinger 1907.00491] [Mehedi Hasan, Schubert 2004.14908]

Subtraction of IR singularities



Complex infrared structure of the amplitude



Recent developments:

[Delto, Jaquier, Melnikov, Röntsch 1909.08428] [Cieri, De Florian, Der, Mazzitelli 2005.01315]

[Buonocore, Grazzini, Tramontano 1911.10166]

Fully off-shell case with charged leptons not available yet

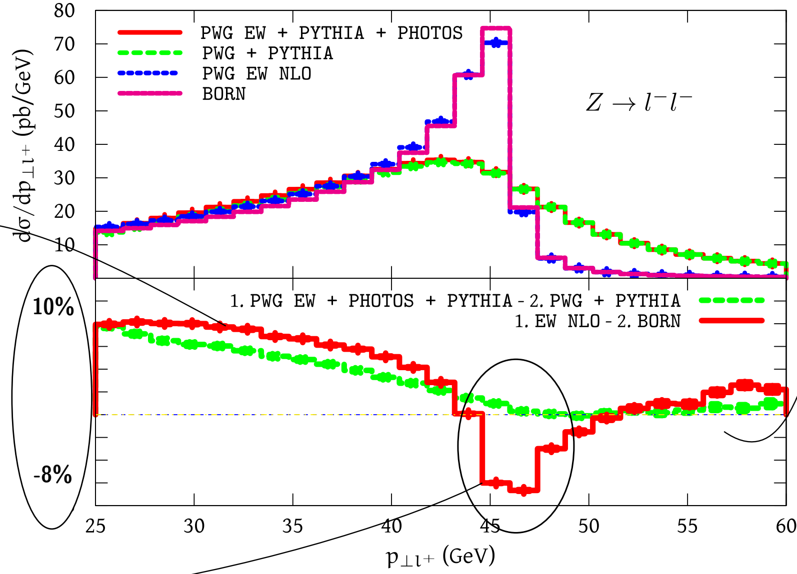
Available approximations to full NNLO QCDxEW: PS

NLOEW and QED multiple photon corrections within native NLO and QCD Parton Shower in POWHEG BOX [Balossini et al. 0907.0276, Barze' et al. 1302.4606]

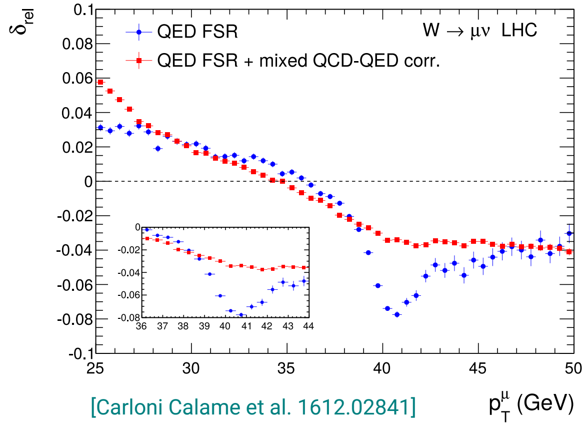
NLO QCD and EW + PS included into a single generator. Made available both for NCDY (Z) and CCDY (W)

Pure NLO EW effects

Red vs Green: impact of mixed corrections: up to several %



Combination of NLO QCD x NLO EW + PS (QCD & QED) vs NLO QCD+PS



[Carloni Calame et al. 1612.02841]

Detailed phenomenological study indicate that impact of mixed QCDxEW corrections on W mass extraction $\Delta M_W \sim -16$ MeV

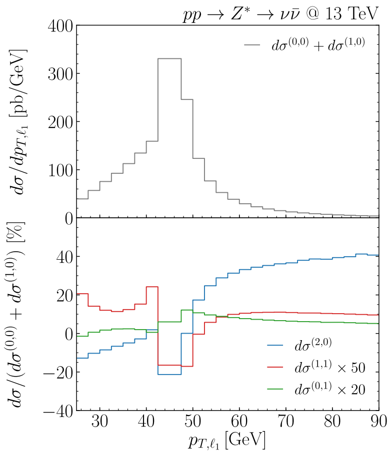
QCDxQED corrections to off-shell NCDY: $Z \to \nu\bar{\nu}$

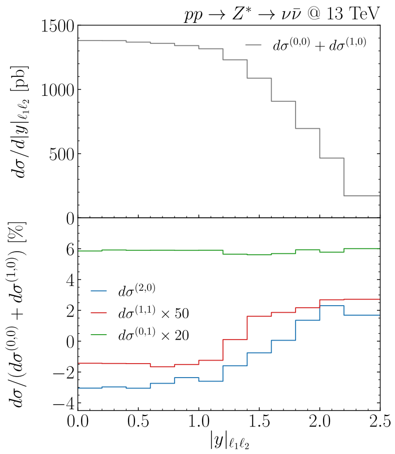
Computation of the **QCDxQED** corrections to fully off-shell NCDY using the qT subtraction formalism [Cieri, De Florian, Der, Mazzitelli 2005.01315]

QCDxQED effects present only in IS

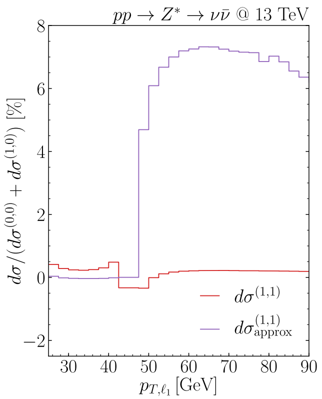
This takes care of IR divergencies of IS state type (regardless the Z decay)

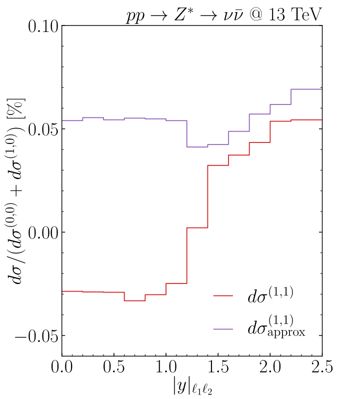
First step towards extension of qT subtraction to massive charged FS leptons





General comment: **corrections are small** (below 1%) however **strongly dependent on the kinematics** (phase-space and cuts)





Comparison of full QCDxQED vs factorised approximation

$$d\Delta^{(i,j)} = d\sigma^{(i,j)}/d\sigma^{(0,0)}$$

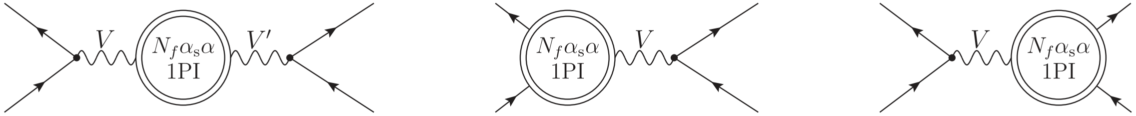
$$d\sigma^{(1,1)}_{\text{approx}} = d\sigma^{(0,0)} d\Delta^{(1,0)} d\Delta^{(0,1)}$$

Factorised approximation generally not a good approximation (at least in this case)

Discrepancies between full QCDxQED and factorisaion enhanced at differental level

QCDxEW corrections to off-shell DY: $O(N_f \alpha_s \alpha)$ contribution

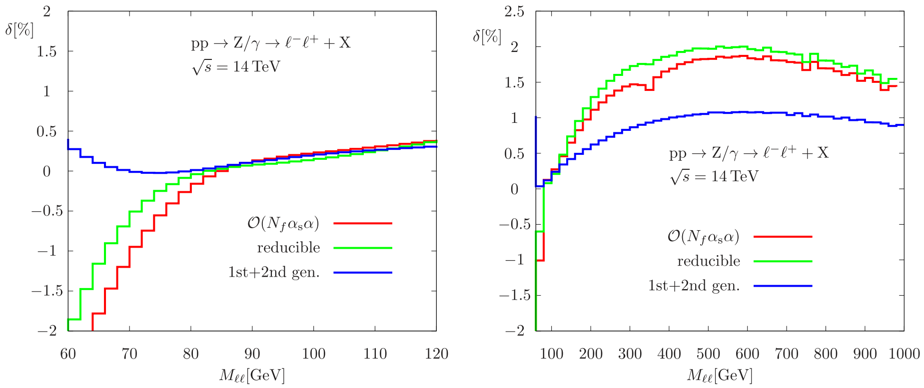
Mixed QCDxEW corrections to off-shell W and Z production coming from closed fermion loops [Dittmaier, Schmidt, Schwarz 2009.02229]



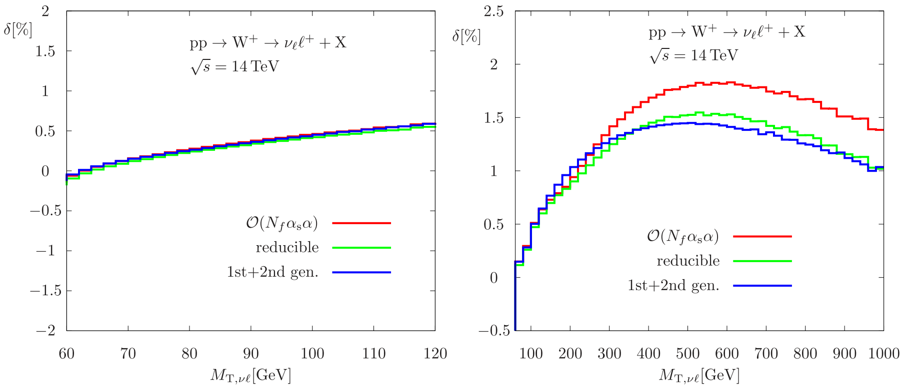
Gauge-invariant subset of mixed QCDxEW corrections. It can be investigated on its own

$O(N_f \alpha_s \alpha)$ contribution is sufficient for the generalisation of the complex mass scheme for the complete $O(\alpha_s \alpha)$ corrections

NCDY



CCDY



General comment: in regions dominated by the resonance corrections are small O(1‰). They increase to O(1-2%) in off-shell regions

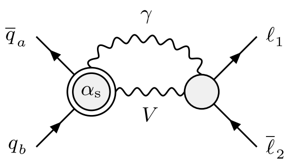
Drell-Yan at the resonace region: can we simplify it?

Resonance region relevant for EW precision studies at the LHC.

Moreover: remarkable technical simplification wrt fully off-shell case. One can work in the narrow width approximation [Fadin, Khoze, Martin hep-ph/9309234]

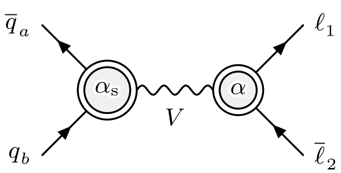
$$\sigma = \text{prod} \times \text{dec} + \mathcal{O}\left(\frac{\Gamma}{M}\right)$$

The pole approximation [Stuart '91] well suited for describing (mixed qcd-) electroweak effects near the resonance region



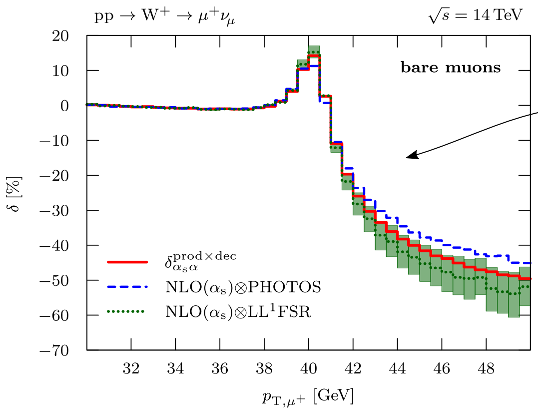
[Dittmaier, Huss, Schwinn. 1403.3216]

IS-FS non-factorizable contributions subdominant in the resonance region. Phenomenologically negligible



[Dittmaier, Huss, Schwinn. 1511.08016]

Dominant effects from IS-FS factorizable contributions. Authors consider initial-final and final-final.



Comparison of IS-FS correction with NLO QCD (IS) x QED PS (FS). Generally good agreement

(one of the) **Outcome** of the study: Mixed QCDxEW corrections can have an impact as **14 MeV in the extraction of the W mass**.

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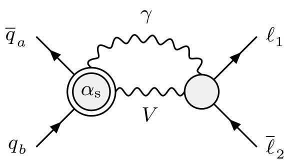

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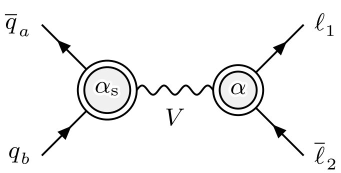

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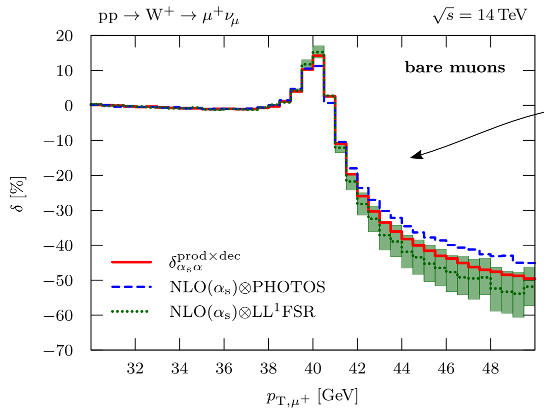

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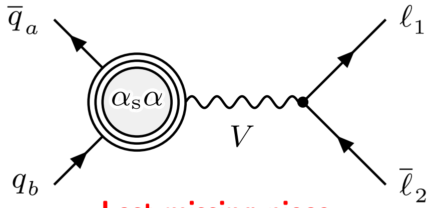


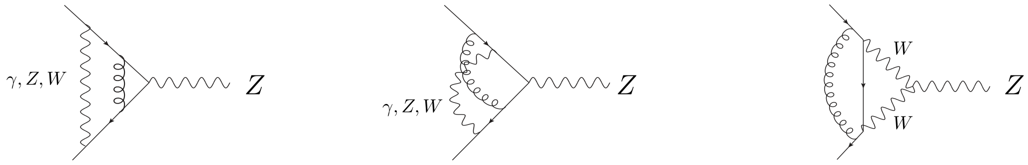
Last missing piece

Complete QCDxEW corrections to on-shell Z: fully inclusive XS

$$\sigma_{\text{tot}} = \int \mathrm{d}\sigma \qquad\qquad \mathrm{d}\sigma = \mathrm{d}\sigma^{\text{LO}} + \sum_{i,j} \frac{\alpha_s^i}{2\pi}\frac{\alpha^j}{2\pi}\delta\sigma^{i,j} = \mathrm{d}\sigma^{\text{LO}} + \sum_{i,j} \mathrm{d}\sigma^{(i,j)}$$

NNLO QCDxQED corrections to inclusive Z obtained through "abelianisation" [De Florian, Der, Fabre 1805.12214] of NNLO QCD [Hamberg, Matsuura, van Neerven '90]

NNLO QCDxEW corrections to inclusive Z production: $q\bar{q} \to Z$ channel [Bonciani, F.B, Rana, Triscari, Vicini 1911.06200]



Fully inclusive cross section for the production of an on-shell Z boson [Bonciani, F.B, Rana, Vicini 2007.06518]

Fully analytic computation of the amplitudes and of the required loop and phase-space integrals. Important benchmark for Monte Carlo calculations

$$\sigma^{(1,1)} = \sigma^{(1,1)}_{\text{VV}} + \sigma^{(1,1)}_{\text{RV}} + \sigma^{(1,1)}_{\text{VR}} + \sigma^{(1,1)}_{\text{RR}}$$

Two-loop integrals with mass effects

Virtual-QCD, Real-emission QED: same as QCDxQED

Virtual-EW, Real-emission QCD: most complicated bit

In the qq → qq QCDxEW interference effects which do not show up in QCDxQED

Complete QCDxEW corrections to on-shell Z: fully inclusive XS

Results presented here have been computed in the 4FS (massive b-quarks, no b pdf)

$$A_1 = \sigma^{\rm LO} + \sigma^{(1,0)} + \sigma^{(2,0)}$$

← Computed using NNPDF31_nnlo_as_0118_nf_4 → Pure NNLO QCD result

$$B_1 = \sigma^{\rm LO} + \sigma^{(1,0)} + \sigma^{(2,0)}$$

$$B_2 = \sigma^{\rm LO} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)}$$

$$B_3 = \sigma^{\rm LO} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)} + \sigma^{(1,1)}$$

$$B_{3,\gamma} = \sigma^{\rm LO} + \sigma^{(1,0)} + \sigma^{(2,0)} + \sigma^{(0,1)} + \sigma_\gamma^{(1,1)}$$

Computed using NNPDF_31_nnlo_as_0118_luxqed_nf_4 → It contains photon PDF (and corresponding evolution of PDFs)

$\mu_R = \mu_F = M_Z$ (results expressed in pb)

order	G_μ	$\alpha(0)$	$\delta_{G_\mu-\alpha(0)}$ (%)
A_1	55787	53884	3.53
B_1	55651	53753	3.53
B_2	55501	55015	0.88
B_3^γ	55516	55029	0.88
B_3	55469	55340	0.23

B_3 vs A_1 ~ -0.57 %

α_0 ($\alpha(0)$,M_Z,M_W) and G_μ (G_μ,M_Z,M_W) input schemes

Conservative estimate EW input scheme uncertainty

Reduction of uncertainty related to EW input scheme

7-pt μ_R-μ_F scale variation: $B_3 = 55469^{+0.65\%}_{-1.01\%}\,\text{pb}$

For a thourough assessment of theory uncertainty at this precision level, N3LO QCD corrections need to be considered as well

Mixed QCDxEW to on-shell Z/W production: going differential

Complete calculation of mixed QCDxEW corrections at the differential level to on-shell V-boson production at the LHC.
Z [F.B., Caola, Delto, Jaquier, Melnikov, Röntsch 2005.10221] and W [Behring, F.B., Caola, Delto, Jaquier, Melnikov, Röntsch 2009.10386]

$$\mathrm{d}\sigma_{pp\to\ell_1\ell_2} = \mathrm{Br}(V\to\ell_1\ell_2)\mathrm{d}\sigma_{pp\to V}\frac{\mathrm{d}\Gamma_{V\to\ell_1\ell_2}}{\Gamma_{V\to\ell_1\ell_2}}$$

Main (technical) aspects of the calculations:

- Calculation of 2-loop form factors: agreement with results available from the literature for the Z [Kotikov, Kuhn, Veretin hep-ph/0703013] and new results for the W. Computation performed for arbitrary EW gauge bosons masses.

- One-loop real-virtual integrals from OpenLoops2 [F.B., Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller 1907.13071] It guarantees numerical stability in unresolved IR regions.

- QCDxEW renormalization: independent calculation, agreement with [Djoaudi, Gambino hep-ph/9309298] [Dittmaier, Huss, Schwinn 1511.08016]

- Subtraction of IR singularities performed within the nested soft collinear subtraction formalism [Caola, Melnikov, Röntsch 1702.01352]
  Z: abelianisation of NNLO QCD for QCDxQED corrections [Delto, Jaquier, Melnikov, Röntsch 1909.08428]
  W: QED radiation off the W, so an extension of the subtraction scheme is needed (main goal of the paper with 2-loop FF)

Corrections to the integrated cross sections: Z case

For NCDY electroweak corrections can be further split as $\mathrm{d}\sigma^{(i,1)} = \mathrm{d}\sigma^{(i,1)}_{\mathrm{QED}} + \mathrm{d}\sigma^{(i,1)}_{\mathrm{weak}}$

bin-by-bin in differential distributions

Ratio to NLO QCD

$$\mathrm{d}\Delta^{(i,j)} = \frac{\mathrm{d}\sigma^{(i,j)}}{\mathrm{d}\sigma^{(0,0)} + \mathrm{d}\sigma^{(1,0)}}$$

the $Z \to e^+e^-$ branching ratio drops in the ratio

We present results for:

- 13 TeV LHC
- G_μ scheme: (G_μ, M_W, M_Z) as input
- $\mu_F = \mu_R = M_Z/2$

Standard selection criteria (cuts):

- p_{T,l_1} > 24 GeV (harder lepton)
- p_{T,l_2} > 16 GeV (softer lepton)
- $-2.4 < y_l < 2.4$
- m_{ll} > 50 GeV
- e-γ recombination for $R_{e\gamma} < 0.1$

$$R_{e\gamma} = \sqrt{(y_e - y_\gamma)^2 + (\varphi_e - \varphi_\gamma)^2}$$

	Inclusive	Cuts	Cuts (production)
$\Delta^{(0,1)}_{\mathrm{QED}}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta^{(0,1)}_{\mathrm{weak}}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
$\Delta^{(0,1)}$	-3.2×10^{-3}	-1.0×10^{-2}	-2.8×10^{-3}
$\Delta^{(2,0)}$	$+1.3 \times 10^{-2}$	$+5.8 \times 10^{-3}$	$+5.8 \times 10^{-3}$
$\Delta^{(1,1)}_{\mathrm{QED}}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
$\Delta^{(1,1)}_{\mathrm{weak}}$	-1.6×10^{-3}	-2.1×10^{-3}	-2.1×10^{-3}
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Tiny EW corrections due to G_μ scheme

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(QCDx)QED corrections strongly sensitive to selection cuts

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NNLO QCD unnaturally (and accidentally) small. QCDxEW compete (eventually larger)

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$$R_{e\gamma} = \sqrt{(y_e - y_\gamma)^2 + (\varphi_e - \varphi_\gamma)^2}$$

	Inclusive	Cuts	Cuts (production)
$\Delta^{(0,1)}_{\text{QED}}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta^{(0,1)}_{\text{weak}}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
$\Delta^{(0,1)}$	-3.2×10^{-3}	-1.0×10^{-2}	-2.8×10^{-3}
$\Delta^{(2,0)}$	$+1.3 \times 10^{-2}$	$+5.8 \times 10^{-3}$	$+5.8 \times 10^{-3}$
$\Delta^{(1,1)}_{\text{QED}}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
$\Delta^{(1,1)}_{\text{weak}}$	-1.6×10^{-3}	-2.1×10^{-3}	-2.1×10^{-3}
$\Delta^{(1,1)}$	-1.1×10^{-3}	-8.0×10^{-3}	-2.0×10^{-3}

Production process.
It removes (strong) dependence on lepton cuts:
weak corrections dominant over QED

Corrections to the integrated cross sections. $\mu_F = \mu_R = M_Z$

$$d\Delta^{(i,j)} = \frac{d\sigma^{(i,j)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

Ratio to NLO QCD

13 TeV LHC
G_μ scheme: (G_μ, M_W, M_Z) as input
$\mu_F = \mu_R = M_Z$

p_{T,l_1} > 24 GeV (harder lepton)
p_{T,l_2} > 16 GeV (softer lepton)
$-2.4 < y_l < 2.4$
m_{ll} > 50 GeV
e-γ recombination for $R_{e\gamma} < 0.1$

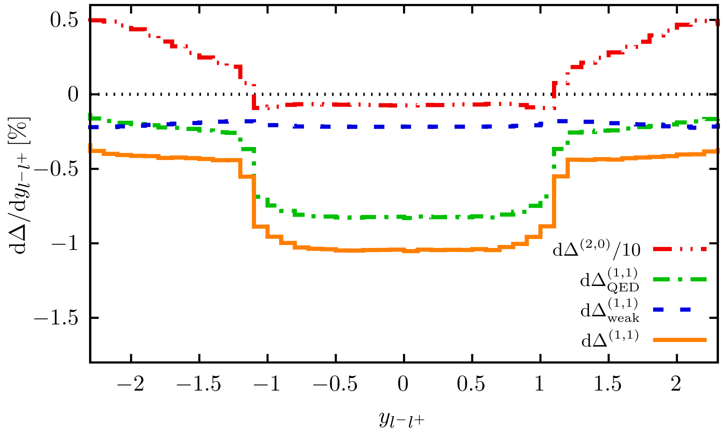
$\mu_F = \mu_R = M_Z/2$

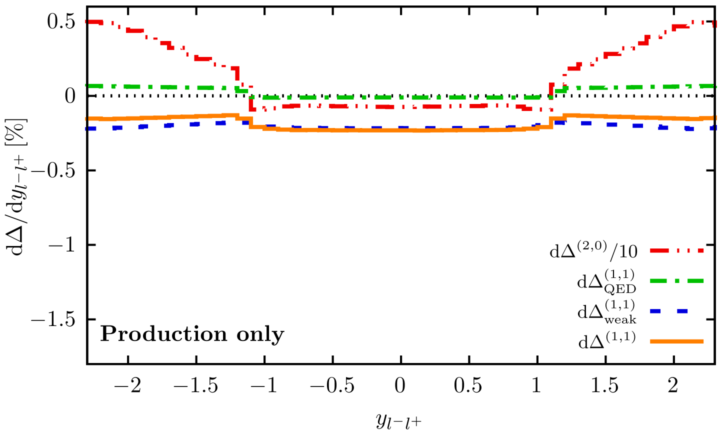
	Inclusive	Cuts	Cuts (production)
$\Delta^{(0,1)}_{\text{QED}}$	+2.3 x 10^{-3}	-5.3 x 10^{-3}	+2.2 x 10^{-3}
$\Delta^{(0,1)}_{\text{weak}}$	-5.5 x 10^{-3}	-5.0 x 10^{-3}	-5.0 x 10^{-3}
$\Delta^{(0,1)}$	-3.2 x 10^{-3}	-1.0 x 10^{-2}	-2.8 x 10^{-3}
$\Delta^{(2,0)}$	+1.3 x 10^{-2}	+5.8 x 10^{-3}	+5.8 x 10^{-3}
$\Delta^{(1,1)}_{\text{QED}}$	+5.5 x 10^{-4}	-5.9 x 10^{-3}	+1.4 x 10^{-4}
$\Delta^{(1,1)}_{\text{weak}}$	-1.6 x 10^{-3}	-2.1 x 10^{-3}	-2.1 x 10^{-3}
$\Delta^{(1,1)}$	-1.1 x 10^{-3}	-8.0 x 10^{-3}	-2.0 x 10^{-3}

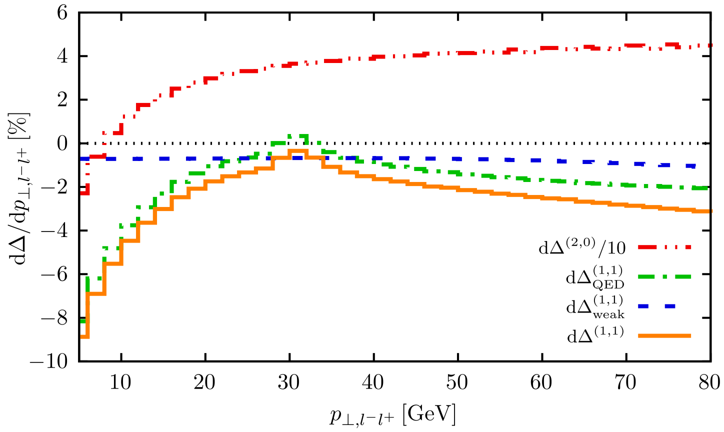
$\mu_F = \mu_R = M_Z$

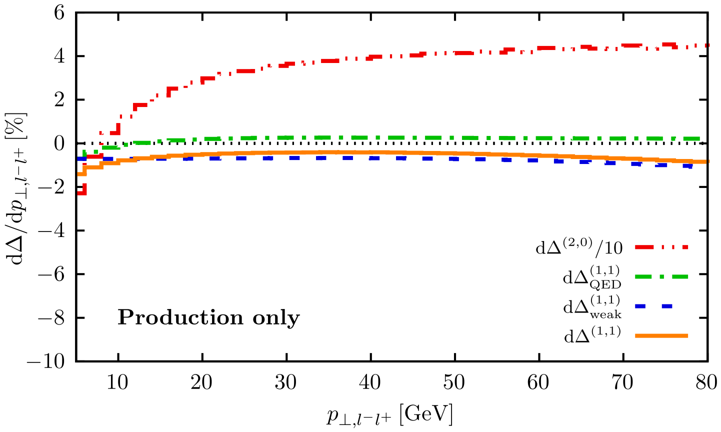
	Inclusive	Cuts	Cuts (production)
$\Delta^{(0,1)}_{\text{QED}}$	+3.1 x 10^{-3}	-5.5 x 10^{-3}	+3.0 x 10^{-3}
$\Delta^{(0,1)}_{\text{weak}}$	-6.2 x 10^{-3}	-5.8 x 10^{-3}	-5.8 x 10^{-3}
$\Delta^{(0,1)}$	-3.1 x 10^{-3}	-1.1 x 10^{-2}	-2.9 x 10^{-3}
$\Delta^{(2,0)}$	-6.3 x 10^{-3}	-1.2 x 10^{-2}	-1.2 x 10^{-2}
$\Delta^{(1,1)}_{\text{QED}}$	+2.9 x 10^{-4}	-5.2 x 10^{-3}	-1.5 x 10^{-4}
$\Delta^{(1,1)}_{\text{weak}}$	-9.2 x 10^{-4}	-1.3 x 10^{-3}	-1.3 x 10^{-3}
$\Delta^{(1,1)}$	-6.4 x 10^{-4}	-6.5 x 10^{-3}	-1.5 x 10^{-3}

Differential distributions: Z case





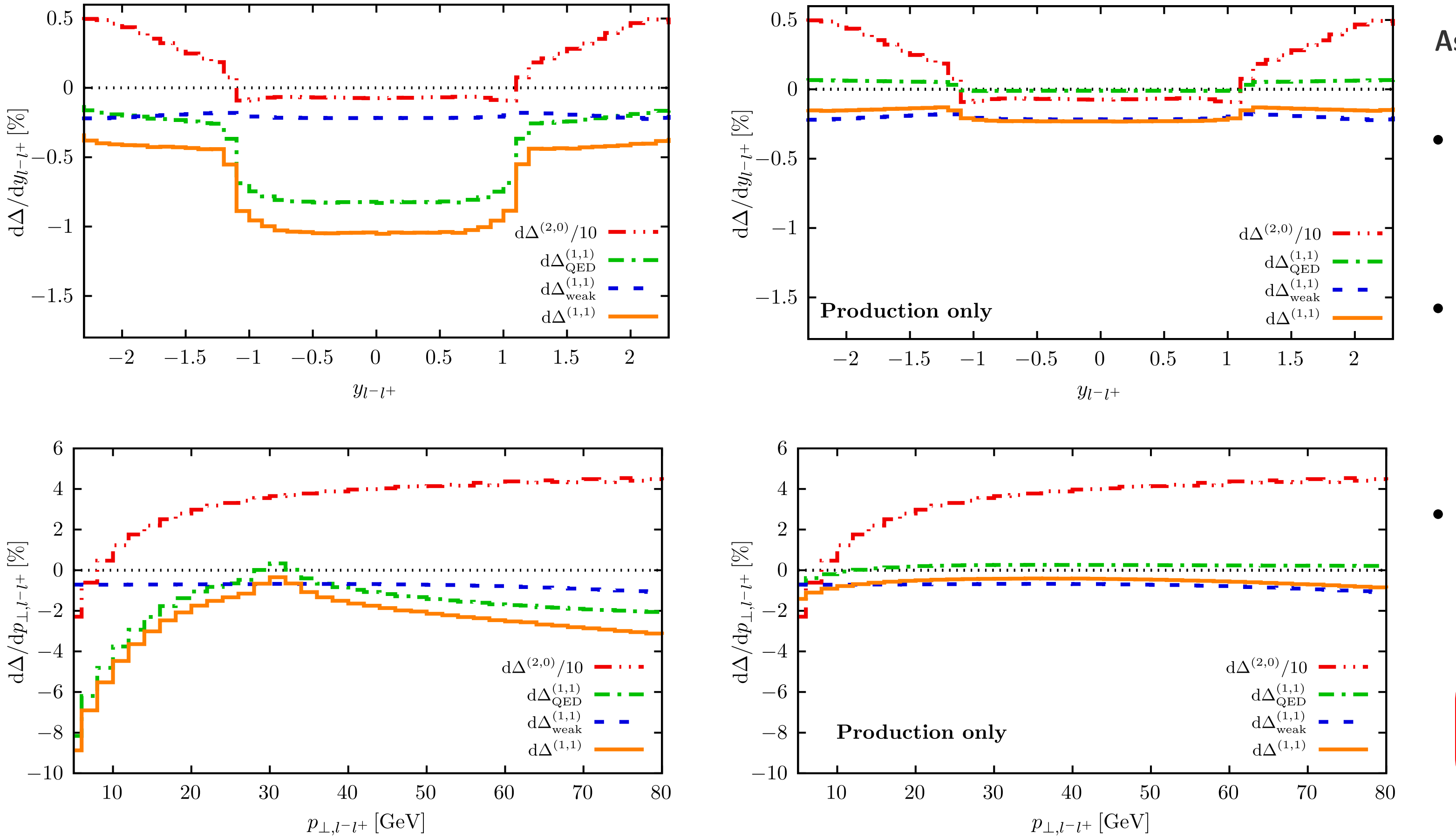




As for the integrated XS:

- QCDxWeak corrections dominate over QCDxQED in the production mechanism
- In certain kinematic regions QCDxEW effects comparable to NNLO QCD ones, e.g. central y_{ll} and low $p_{T,ll}$
- The impact of mixed QCDxEW, as well as the QED-Weak interplay is observable dependent

Differential distributions: Z case



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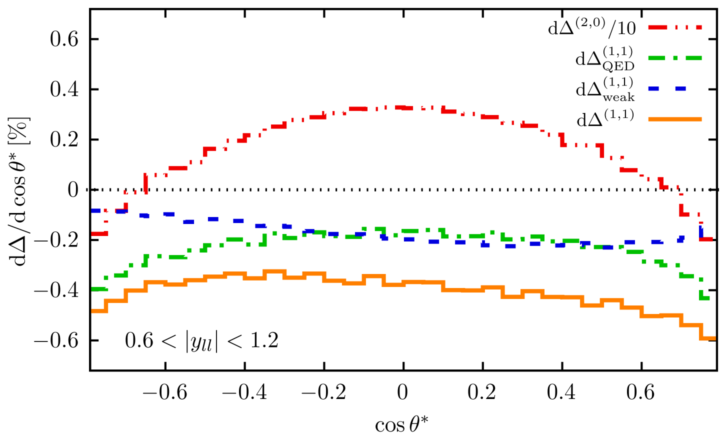
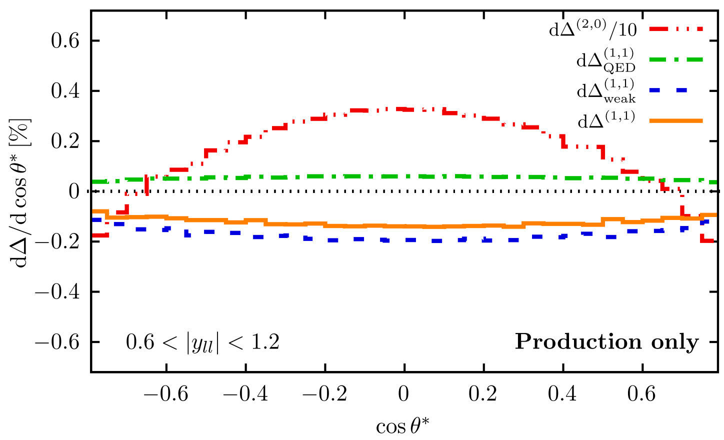
- The **impact of mixed QCDxEW**, as well as the **QED-Weak interplay** is **observable dependent**

both EW and QED effects must be considered for the required $O(\alpha\alpha_s)$ accuracy

Differential distributions: Z case

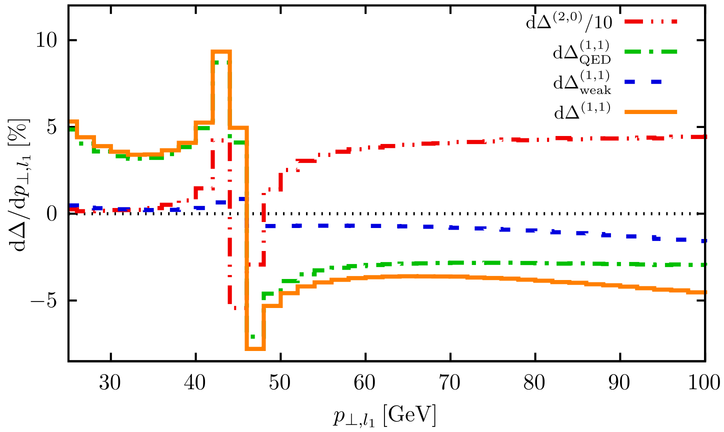


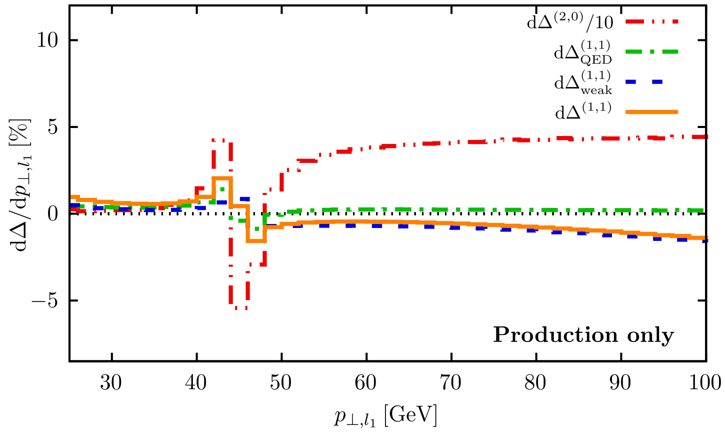


Collins-Soper angle ϑ*

$$\cos\theta^* = \mathrm{sgn}(p_{z,ll}) \frac{P^+_{l^-}P^-_{l^+} - P^-_{l^-}P^+_{l^+}}{\sqrt{m^2_{ll}\left(m^2_{ll} + p^2_{\perp,ll}\right)}}$$

cosϑ* allows for a precise determination of the weak mixing angle at the LHC





- for cosϑ* QED and weak effects have a similar impact even when fs corrections are included
- LO kinematic boundary $p_{T,l1} < M_Z/2$.
  Effects more pronounced when FS QED corrections are included.
  Sensitivity to selection cuts

Comparison against factorised approximation: Z case

Preliminary

Exact: $d\sigma = d\sigma^{LO}\left(1 + \delta_{QCD} + \delta_{EW} + \delta_{QCD\times EW}\right)$ $\delta_{QCD} \sim$ NLO+NNLO QCD

Multiplicative approximation:

$$d\sigma = d\sigma^{LO}\left(1 + \delta_{QCD}\right)\left(1 + \delta_{EW}\right)$$

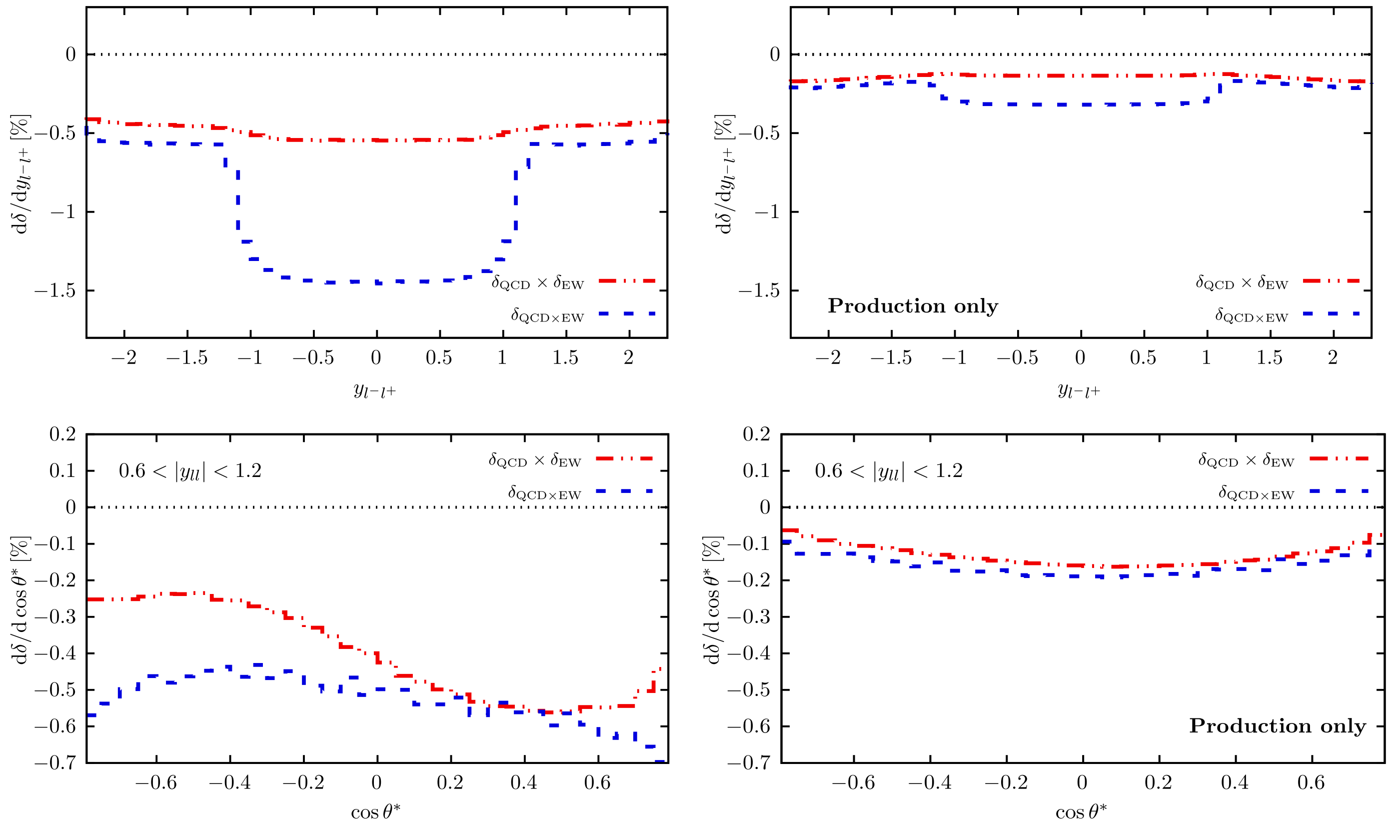
$\delta_{QCD} \times \delta_{EW}$

widely adopted method to estimate missing mixed QCDxEW effects.

NB: δ defined wrt LO

At the integrated cross section level:

	$\delta_{QCD} \times \delta_{EW}$	$\delta_{QCD\times EW}$
Inclusive	-1.17 x 10^{-3}	-1.40 x 10^{-3}
Cuts	-0.51 x 10^{-2}	-1.09 x 10^{-2}
Cuts (production)	-1.38 x 10^{-3}	-2.65 x 10^{-3}



Similar studies in [Dittmaier, Huss, Schwinn. 1511.08016]

Corrections to the integrated cross sections: W case

DISCLAIMER: we have not performed extensive phenomenological studies for the W yet. Work in progress for a future publication

To present results for the fiducial cross section we write: $\sigma_{pp\to W^+} = \sigma_{\mathrm{LO}} + \Delta\sigma_{\mathrm{NLO},\alpha_s} + \Delta\sigma_{\mathrm{NLO},\alpha} + \Delta\sigma_{\mathrm{NLO},\alpha\alpha_s}$

We present results for:

- 13 TeV LHC
- G_μ scheme: (G_μ, M_W, M_Z) as input
- $\mu_F = \mu_R = M_W, M_W/2, M_W/4$

Selection criteria (cuts):

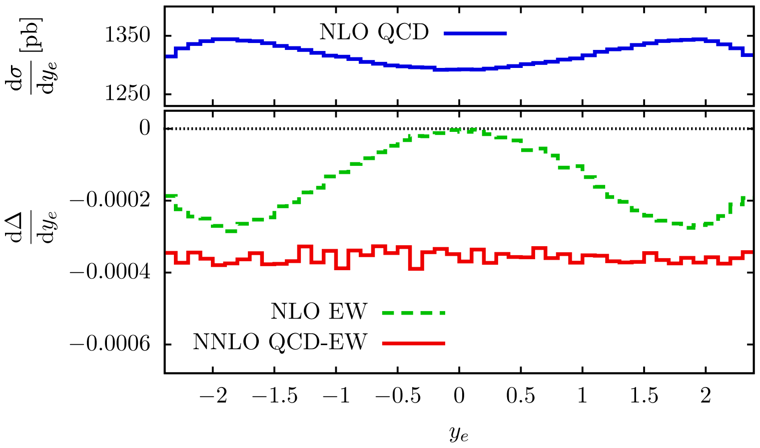
- $p_{T,e+} > 15$ GeV
- $p_{T,miss} > 15$ GeV
- $-2.4 < y_{e+} < 2.4$

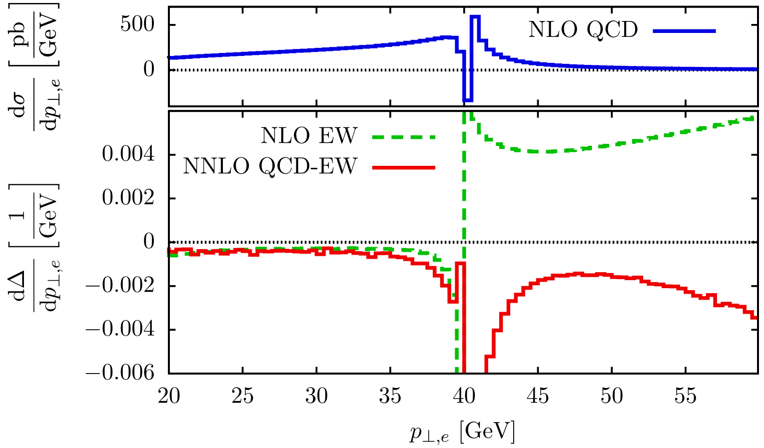
NLO EW corrections are tiny: O(0.02%): mostly due to the G_μ scheme

Mixed QCDxEW corrections also very small (below permille) though larger than NLO EW (at least for this setup)

σ[pb]	channel	$\mu = M_W$	$\mu = M_W/2$	$\mu = M_W/4$
σ_{LO}		6007.6	5195.0	4325.9
$\Delta\sigma_{\mathrm{NLO},\alpha_s}$	all ch.	508.8	1137.0	1782.2
	$q\bar{q}'$	1455.2	1126.7	839.2
	qg/gq	-946.4	10.3	943.0
$\Delta\sigma_{\mathrm{NLO},\alpha}$	all ch.	2.1	-1.0	-2.6
	$q\bar{q}'$	-2.2	-5.2	-6.7
	$q\gamma/\gamma q$	4.2	4.2	4.04
$\Delta\sigma_{\mathrm{NNLO},\alpha_s\alpha}$	all ch.	-2.4	-2.3	-2.8
	$q\bar{q}'/qq'$	-1.0	-1.2	-1.0
	qg/gq	-1.4	-1.2	-2.1
	$q\gamma/\gamma q$	0.06	0.03	-0.04
	$g\gamma/\gamma g$	-0.12	0.04	0.30

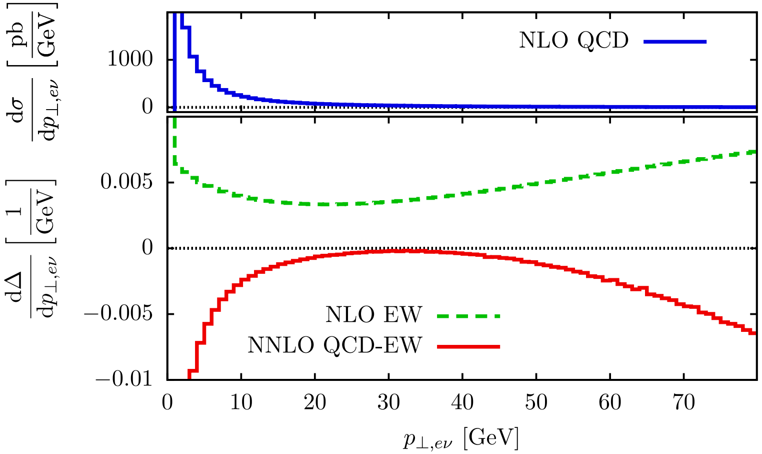
Differential distributions: W case

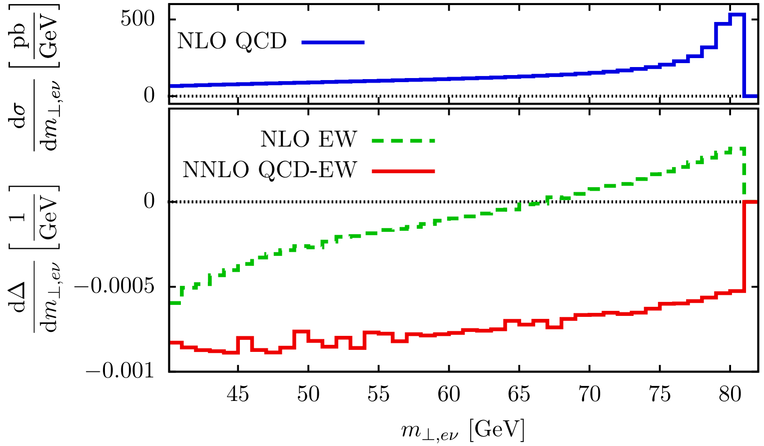




$$d\Delta^{(i,j)} = \frac{d\sigma^{(i,j)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}}$$

Bin-by-bin ratios of NLO EW and NNLO mixed QCDxEW corrections





As for the integrated cross section, NLO EW and QCDxEW are tiny, with the latter often larger

However, shapes are significantly different

Conclusions

For the required level of accuracy and for the upcoming HL-LHC phase, NNLO QCDxEW corrections to DY need to be taken into account.

We are witnessing fast and constant progress from the side of theoretical calculations.

First results towards a complete fully off-shell calculation

- QCDxQED corrections to off-shell Z-boson decaying to neutrinos
- $O(N_f \alpha \alpha_s)$ corrections to off-shell W/Z-boson

Mixed QCDxEW corrections to Z production at the resonance:

- mixed QCDxEW corrections to inclusive XS. Reduction of EW theory uncertainty
- mixed QCDxEW corrections are generally small: $O(10^{-3})$, however…
- interplay between QCDxQED and QCDxWeak
- effects vary with the observable and depend strongly on the selection cuts

complete mixed QCDxEW corrections needed (where possible) if accuracy target is < 1%

Mixed QCDxEW corrections to W production at the resonance:

- mixed QCDxEW corrections are tiny: $O(10^{-3})$, however…
- They must be included if we aim at O(10 MeV) accuracy on the W-mass
- Mixed QCDxEW corrections to Z and W both crucial for pheno studies

Outlook

Detailed phenomenological studies at the resonance

- Combined mixed QCDxEW corrections to both Z and W are crucial for pheno studies of precision observables
- Proper assessment of impact on W mass extraction is now possible

Mixed QCDxEW corrections away from the resonance

- Crucial for studies in high-invariant mass regions
- Assessment of validity of most used approximations
- Many interesting formal aspects of perturbative calculations

Inclusion of N3LO QCD corrections in the analysis of uncertainties

- N3LO QCD corrections to the production of a virtual photon [Duhr, Dulat, Mistlberger 2001.07717]
- N3LO QCD corrections to CCDY [Duhr, Dulat, Mistlberger 2007.13313]

Backup

On-shell Z production in the narrow-width approximation

The production of a Z boson is computed in the narrow width approximation

$$\mathrm{d}\sigma_{pp\to e^+e^-} = \mathrm{Br}\left(Z \to e^+e^-\right) \mathrm{d}\sigma_{pp\to Z} \frac{\mathrm{d}\Gamma_{Z\to e^+e^-}}{\Gamma_{Z\to e^+e^-}}$$

The cross section for the process pp → Z → e^+e^- can be expanded in power of α_s and α

$$\mathrm{d}\sigma = \sum_{i,j} \frac{\alpha_s^i}{2\pi}\frac{\alpha^j}{2\pi}\delta\sigma^{(i,j)} = \sum_{i,j}\mathrm{d}\sigma^{(i,j)}$$

The partial decay width is expanded perturbatively, so is the ratio dΓ/Γ:

$$\Gamma_{Z\to e^+e^-} = \Gamma^0 \times \left(1 + \alpha\delta_{\mathrm{dec}}^{(0,1)} + \alpha\alpha_s\delta_{\mathrm{dec}}^{(1,1)}\right) + \mathcal{O}(\alpha^2, \alpha_s^2) \qquad \mathrm{d}\Gamma_{Z\to e^+e^-} = \mathrm{d}\Gamma^{(0,0)} + \mathrm{d}\Gamma^{(0,1)} + \mathrm{d}\Gamma^{(1,1)} + \mathcal{O}(\alpha^2, \alpha_s^2)$$

The mixed QCDxEW corrections to the cross sections thus read

$$\mathrm{d}\sigma^{(1,1)} = \mathrm{Br}(Z\to e^+e^-) \times \Bigg[\mathrm{d}\sigma_{pp\to Z}^{(1,1)} \times \frac{\mathrm{d}\Gamma^{(0,0)}}{\Gamma^0} +$$

"production only"

$$\mathrm{d}\sigma_{pp\to Z}^{(1,0)} \times \left(\frac{\mathrm{d}\Gamma^{(0,1)}}{\Gamma^0} - \alpha\frac{\mathrm{d}\Gamma^{(0,0)}}{\Gamma^0}\delta_{\mathrm{dec}}^{(0,1)}\right) + \mathrm{d}\sigma_{pp\to Z}^{(0,0)} \times \left(\frac{\mathrm{d}\Gamma^{(1,1)}}{\Gamma^0} - \alpha\alpha_s\frac{\mathrm{d}\Gamma^{(0,0)}}{\Gamma^0}\delta_{\mathrm{dec}}^{(1,1)}\right)\Bigg]$$

final-state effects