

Impact of the longitudinal distribution on the transverse stability at flat top in the LHC X. Buffat, with many thanks to A. Oeftiger for his studies and his help!

- Summary of observations and past studies
- The sine-holed Gaussian distribution
  - Impact on the stability threshold and growth rate
- The q-Gaussian distribution
  - Benchmark
  - Impact on the stability threshold and growth rate

#### Summary of the LHC observations



 Schottky spectrum suggest that the longitudinal distributions at flat to in the LHC features a hole at a given synchrotron frequency, i.e. at a given longitudinal action [E. Shaposhnikova, et al., WP2 meeting 17 Jan. 2017, CERN-ACC-NOTE-2017-0016, CERN-ACC-NOTE-2019-0021]

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  - By comparing with the expected longitudinal tune spread, we can deduce that the hole is between 1.5 and 2σ
- Longitudinal profile measurement suggest that the tails are underpopulated [S. Papadopoulou @ IPAC'17]



#### 27.07.2020

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• We start with a Gaussian distribution in radial coordinates, that we multiply by a 'hole function':  $\prod_{n \in \mathcal{N}} \mathcal{U}(n)$ 

$$\Psi_{SHG}(r) = \frac{\Psi_G(r)\mathcal{H}(r)}{\int\limits_0^\infty \Psi_G(r)\mathcal{H}(r)dr}$$

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r [σ]

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, otherwise.

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, otherwise.  $\partial \Psi_C(r_2)$ 

$$\alpha_{\mathcal{H}} = 1 - 2w_{\mathcal{H}} \frac{\partial \Psi_G(r_{\mathcal{H}})}{\partial r}$$

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- The correction factor is such that the area removed by the first part of the hole function is approximatively added by the second, nevertheless a renormalisation remains needed
- Note that the center of the 'hole' is in fact  $r_{\mathcal{H}} w_{\mathcal{H}}/2$
- The advantage of the SHG is to maintain the core of the beam, it mimics a diffusion mechanism that would have moved the particles from first part of the hole function to the second part







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 $\rightarrow$  To be clarified with RF (Not the same beam, misinterpretation of the Schottky,... )



Parameter	Value
Energy [TeV]	6.5
Bunch intensity [10 <sup>11</sup> p]	1
Trans. emit. [µm]	2
r.m.s. bunch length [cm]	8
Q <sub>s</sub>	0.00184
Wake model	Flat top 2018
f <sub>RF</sub> [MHz]	400.8
ADT damping time [turns]	100
Nb. of slices	80
Nb. of rings	40



 BimBim with linear RF and dipolar wake only (LHC2018 flat top impedance model)



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- Holes that do not affect the core below 1o (right of the dashed curve) can impact the threshold at a fixed chromaticity
- With our current strategy to consider the maximum over the uncertainty on the chromaticity (Q'~10-20 units), the impact of the hole is marginal



#### **Non-linear RF**



 When including non-linear RF and/or quadrupolar, Landau damping cannot be addressed with the usual stability diagram

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- When including non-linear RF and/or quadrupolar, Landau damping cannot be addressed with the usual stability diagram
- The holes (affecting the tails beyond 1σ only) can have a significant impact on the growth rate of the most unstable mode at a fixed chromaticity. Again given the uncertainty on the chromaticity the maximum growth rate that can be expected is marginally impacted.

# Non-linear RF and quadrupolar wake fields





RF

#### Non-linear RF and quadrupolar wake fields





#### Non-linear RF and quadrupolar wake fields



In all configurations the quadrupolar wake has a weak impact on the growth rate

#### The q-Gaussian distribution in 2D

$$\Psi_{qG}(r,q) = \begin{cases} \Psi_G(r) &, \text{ if } q = 1\\ \frac{1}{r} \frac{1}{K_q} \left( 1 - \frac{1-q}{6-4q} r^2 \right)^{\frac{1}{1-q}} &, \text{ otherwise} \end{cases}$$

$$K_q = \begin{cases} \pi \frac{6-4q}{1-q} \frac{\Gamma\left(\frac{2-q}{1-q}\right)}{\Gamma\left(\frac{2-q}{1-q}+1\right)} &, \text{ if } q < 1\\ \pi \frac{6-4q}{q-1} \frac{\Gamma\left(\frac{1}{q-1}-1\right)}{\Gamma\left(\frac{1}{q-1}\right)} &, \text{ if } 1 < q < \frac{3}{2} \end{cases}$$

• This definition is such that the r.m.s. remains equal to 1 independently of q



C. Vignat and A. Plastino. Central limit theorem and deformed exponentials. Journal of Physics A: Mathematical and Theoretical, 20(45), 2007

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- This definition is such that the r.m.s. remains equal to 1 independently of q
  - To compare with Adrian's result and experimental data, we can adjust the 'bunch length' to rather maintain the FWHM
- As opposed to the Gaussian, the projection of the 2D q-Gaussian in 1D is not a 1D q-Gaussian with the same parameters

 $\rightarrow$  The values of *q* are not exactly identical to the one used in S. Papadopoulou @ IPAC17, yet they are comparable









HL-LHC configuration with March 2017 wake model at 15cm. The FWHM of the distributions is fixed to the one of the Gaussian

parameter	value
intensity	$N = 2.3 \times 10^{11}$
chromaticity	$-10 \le Q'_{x,y} \le 40$
damping rate	50 turns
RF voltage	$V_{RF} = 16 \mathrm{MV}$
flat-top energy	7 TeV
momentum compaction	$\alpha_c = 53.86^{-2}$
transverse tunes	$(Q_x, Q_y) = (62.31, 60.32)$
synchrotron tune	$Q_s \approx 2.12 \times 10^{-3}$
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- The agreement between the two codes is rather good in particular concerning the maximum growth rate
- The lack of instabilities with Q'>0 is possibly linked to the limited number of turns in PyHT (6·10<sup>5</sup>), as the rise time is rather long (>2·10<sup>5</sup> turns)







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- The agreement between the two codes is again rather good in particular concerning the maximum growth rate
- It is unclear why the transition between the modes does not occur at the same chromaticity in the two approaches

#### Impact on the stability threshold (LHC 2018) Fixed RMS



 As for the hole, the impact is marginal on the maximum threshold over a wide range of frequency, but the impact at a fixed chromaticity can be significant (~50%)

#### Impact on the stability threshold <u>Fixed</u> FWHM





- The impact on the threshold at low chromaticity is visible
  - It is comparable to an impact of the bunch length for a Gaussian distribution

 $\rightarrow$  As opposed to Adrian with pyHT, we seem to find that the RMS is more relevant for the transverse instability threshold than the FWHM

#### Maximum growth rate



- For low q, the impact of the non-linearity of the RF, the quadrupolar wake and the ADT demodulation on the maximum growth rate with positive chromaticities is marginal (details in backup)
- There are some 'interesting' behaviour for high q in the presence of the ADT demodulation. Since they are not that relevant experimentally, I do not investigate today...

#### **Digression : Impact of the ADT demodulation**



 The longitudinal tails have a strong impact on the growth rate when taking into account the ADT demodulation

# **Digression : Impact of the ADT demodulation**



0 -20

-10

20

10

0 Chromaticity



 The holes affecting the longitudinal tails beyond 1σ do not seem to significantly affect our stability threshold predictions because they are based on a maximum over a wide range of frequency

 $\rightarrow$  At a fixed chromaticity, both the instability threshold and the growth rate can be significantly affected. This effect could explain the lack of reproducibility often observed between threshold measurement and with operational beams.

 $\rightarrow$  At Q'~0 the effect remains marginal and is consequently not the cause for the *puzzling* discrepancy at low chromaticity



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• The agreement between BimBim and Adrian's PyHT simulation on HL-LHC configuration with q-Gaussian longitudinal distributions is reasonably good

#### Impact of bunch length for a Gaussian distribution



#### Impact on the growth rate q-Gaussian with fixed RMS

#### No quadrupolar wake

#### Quadrupolar wake





**Non-linear RF** 

 $\boldsymbol{\alpha}$ 

Linear

#### Impact on the growth rate q-Gaussian with fixed FWHM

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Linear Rl

# Non-linear **RF**

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