Building a Boostless Bootstrap

based on:

2004.09587 with Daniel Green
2007.00027 with David Stefanyszyn and Jakub Supeł
2009.xxxxx with Sadra Jazayeri and Harry Goodhew
2009.xxxxx to appear

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My brilliant collaborators: Thank you!

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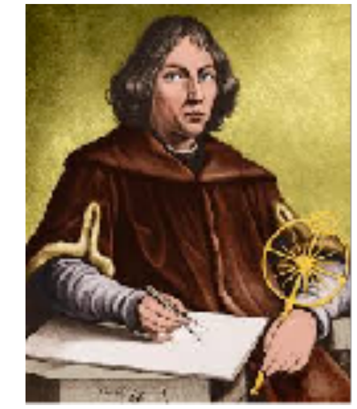
Summary

- Motivations: cosmology from the future
- The symmetries of cosmological perturbations
- A Boostless Bootstrap for the Bispectrum
- The Cosmological Optical Theorem
- Outlook and Open Questions

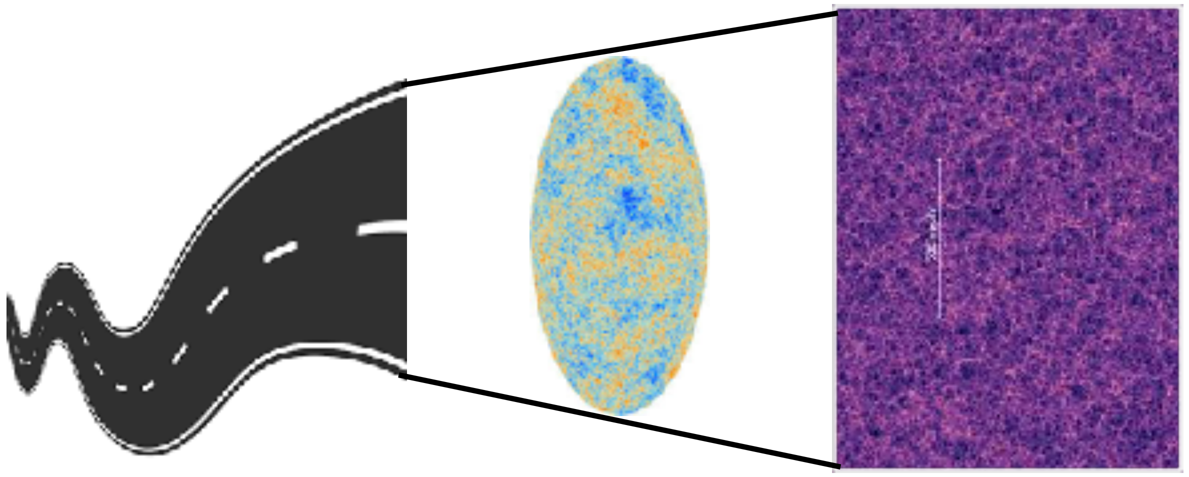
Motivations

Motivations



- Cosmological surveys are believed *measure* the future boundary of (perturbative) quantum correlators involving gravity around quasi de Sitter
- We hope this will teach us about the early universe and physics at the highest energies
- The space of models of the early universe is vast: (too) many spectra of particles, interactions and mechanisms
- We cannot observe the time evolution, but only the value of correlators in the far future, a.k.a. at the “boundary”

A roadmap

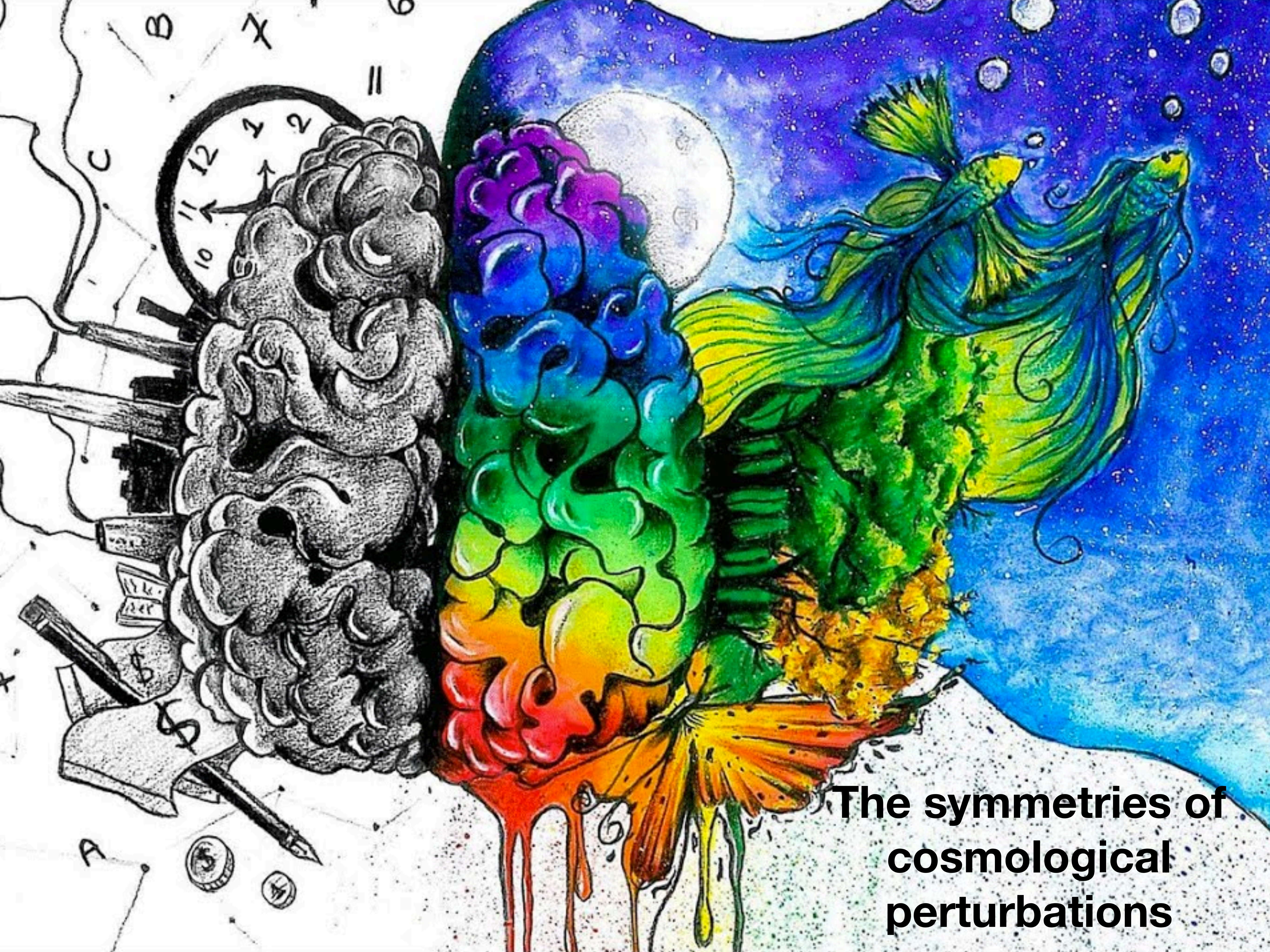


- The goal of this talk is to understand these observables from a **boundary** perspective, as opposed to the more standard bulk approach (in-in formalism).

- Boundary observables are both phenomenologically relevant and theoretically well-defined in a theory of quantum gravity

- For related work see contributions from Hayden Lee, Kostas Skenderis, Austin Joyce, Massimo Taronna, Carlos Duaso Pueyo, Harry Goodhew, Paul McFadden,

The symmetries of cosmological perturbations



Three theorems [Green & EP '20]

Assuming homogeneity, isotropy and scale invariance, I'll prove:

Theorem 1: The correlators of curvature perturbations are uniquely characterized by their soft limits (no field redefinition ambiguities)

Theorem 2: The only theory of curvature perturbations with full de Sitter symmetries is the free theory

Theorem 3: de Sitter symmetries are the largest possible set of symmetries for any single scalar field

Th. 1 & 2 are valid *only in single-clock cosmology* (while Th. 3 is general)

There are no further assumptions about the particle content (any mass and spin) or the interactions

Th. 1: What do we observe?

A crucial step is defining what we observe at the boundary:

- In particle physics, a scattering is defined by the quantum numbers (momentum, spin and charges) of the ingoing and outgoing states.

- In cosmo, we define curvature perturbations ζ, say on constant-energy slices, in the Lagrangian and compute.

- But if I hand you a correlator, how do you know what field it refers to? What are the “quantum numbers” of ζ?

ζ vs $\tilde{\zeta} = \zeta + \zeta^2 + \nabla^{-2}(\partial_i \zeta \partial^i \zeta) + \ldots$

Th. 1: the soft limits

- In single-clock inflation, ζ correlators must obey soft theorems that generalize Maldacena consistency relation

$$\langle \zeta(q)\zeta(k)\zeta(k')\rangle \to P(q)P(k)\left[(n_s-1)+0\times q+\mathcal{O}(q^2)\right]$$

- Soft theorems follow if the theory is invariant under

$$D_{\rm NL}:\delta\zeta=-1-\vec{x}\cdot\vec{\partial}\zeta\,,$$
$$K^i_{\rm NL}:\delta\zeta=-2x^i-2x^i\left(\vec{x}\cdot\vec{\partial}\zeta\right)+x^2\partial^i\zeta\,,$$
$$D:\delta\zeta=-\vec{x}\cdot\vec{\partial}\zeta\,.$$

- We prove that these transformations uniquely define ζ

Th.1: sketch of the proof

Each field redefinition is forbidden by some symmetry

$$\tilde{\zeta} = \zeta + F(\zeta) + G(\nabla_i, \zeta) + H(\nabla^{-2}, \nabla_i, \zeta)$$
$$= \zeta + [\zeta^2 + \dots] + [(\nabla\zeta)^2 + \dots] + [\nabla^{-2}(\nabla_i\zeta)^2 + \dots]$$

0 0 0

D_{NL} D K_{NL}

Th. 1: A given correlator is a correlators of ζ if and only if it obeys all single-clock soft theorems

Th. 2: Conformal = free

- Full de Sitter isometry is extremely powerful and many correlators can be computed directly via the cosmological bootstrap [Maldacena, Arkani-Hamed, Baumann, Pimentel, Joyce, Lee, Duaso Pueyo…]

- But in single field inflation we know we get large non-Gaussianity iff there is some condensate that breaks boosts, e.g. giving a speed of sound $c_s<1$ (EFT of Inflation, $P(X,\phi)$).

- I'll now prove that this is true to all orders and for all possible interactions with field of any mass and spin

Th. 2: symmetries

- De Sitter symmetries act on ζ as

Exact on in the decoupling slow-roll limit ε, η →0

$$P_i : \delta\zeta = -\partial_{x^i}\zeta\,,$$ translations

$$M_{ij} : \delta\zeta = 2x_{[i}\partial_{x^{j]}}\zeta\,,$$ rotations

$$D : \delta\zeta = -\vec{x}\cdot\vec{\partial}\zeta\,,$$ dilations

$$K^i : \delta\zeta = -2x^i\left(\vec{x}\cdot\vec{\partial}\zeta\right) + x^2\partial_{x^i}\zeta\,,$$ boosts

- Single-clock cosmology must also be invariant under

$$D_{\rm NL} : \delta\zeta = -1 - \vec{x}\cdot\vec{\partial}\zeta\,,$$

$$K^i_{\rm NL} : \delta\zeta = -2x^i - 2x^i\left(\vec{x}\cdot\vec{\partial}\zeta\right) + x^2\partial^{x^i}\zeta\,.$$

soft theorems

Th. 2: an infinite algebra

- The combination of two symmetries is a symmetry so these symmetries must close to form algebra, but this requires adding infinitely many other symmetries

$$\frac{1}{2}\left(K_i - K_i^{\mathrm{NL}}\right) = x_i \equiv V_i$$

$$[V_l, K_i] = \left(-2x_i x_l + x^2\delta_{il}\right) \equiv V_{li}$$

$$[V_{ij}, K_l] = 8x_i x_j x_l - 2x^2 x_{(l}\delta_{ij)} \equiv V_{ijl}$$

$$\dots$$

$$[V_{i_1 \dots i_n}, K_N L] \sim x^n \equiv V_{i_1 \dots i_n}$$

Th. 2: the OPE

- To study the consequence of these infinitely many symmetries we look at the OPE

$$\zeta(\vec{x})\zeta(\vec{y}) - \left[\sum_{a=0}^{\infty} \frac{\vec{x}^n}{n!}\vec{\partial}^n\zeta(0)\right]\zeta(0) = \sum_n c_n(\vec{x})\mathcal{O}_n(0)$$

Theory - Free theory = Interactions

- The action of the $V_{(i1..in)}$ on the LHS is always zero, so it must be zero on the RHS. This imposes that all c_n vanish.

Th. 2: implications

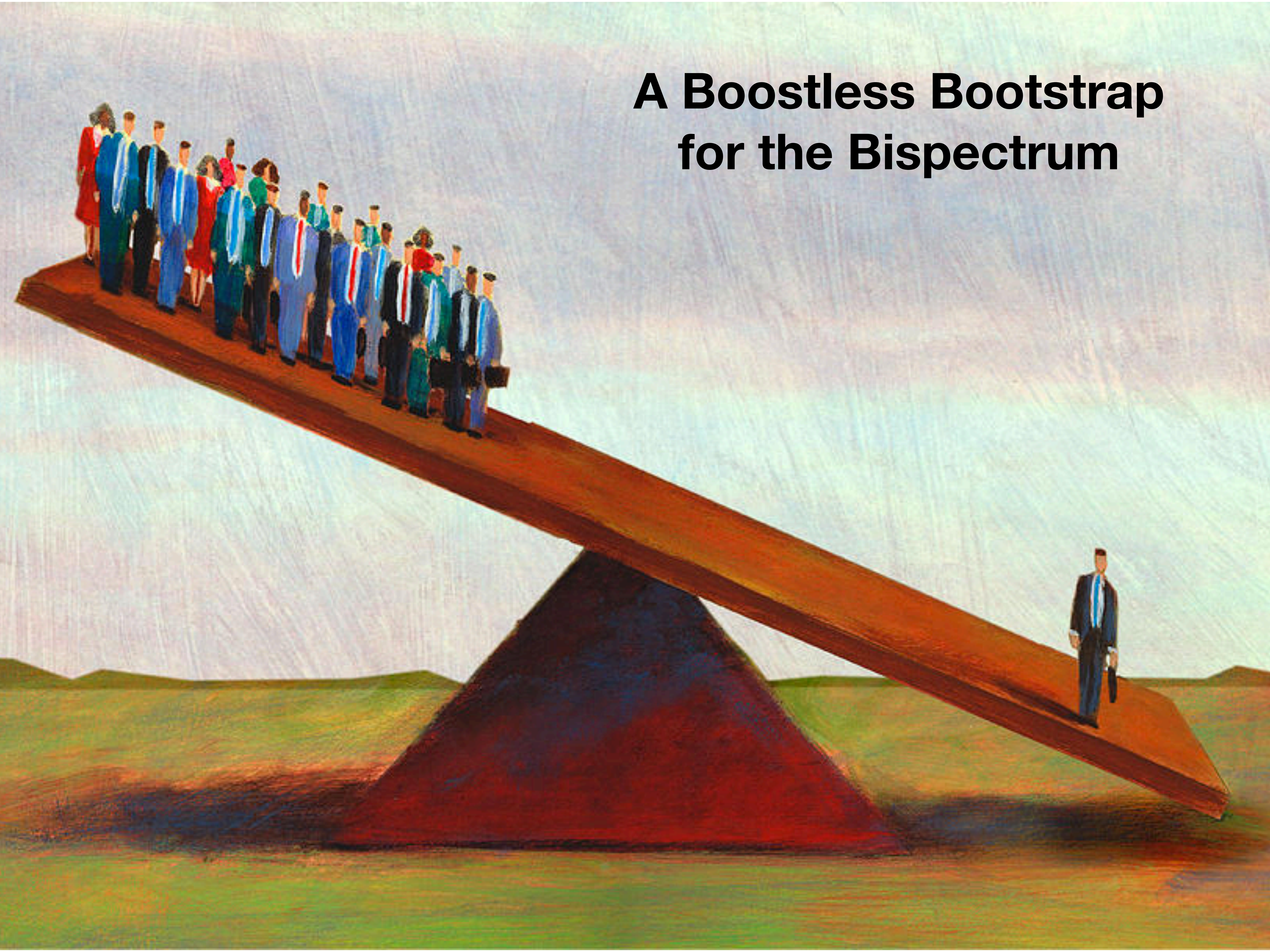
- Th. 2 tells us that in single-clock cosmology, ζ can be *exactly* invariant under all dS isometries only if theory is free, i.e. all connected correctors vanish.

- In realistic models, dilations and boosts are only approximate symmetries with corrections of order the slow-roll parameters.

- Hence curvature correlators computed using dS isometries must be slow-roll suppressed and therefore unobservable in the conceivable future

- Th. 2 does *not* apply to multifield models, where dS isometries can be very useful for practical calculations

A Boostless Bootstrap for the Bispectrum



Drop the boosts

De Sitter bootstrap:
lots of symmetry so very powerful and constraining but at odds with large non-Gaussianity in single field [Maldacena, Arkani-Hamed, Baumann, Pimentel; see Lee, Duaso Pueyo & Joyce talks]

A cosmological bootstrap

A "Boostless" Bootstrap
Less symmetry, but describes non-Gaussianity with small speed of sound (EFT of inflation - P(X) theories)

A Boostless Bootstrap

- Goal: Find a set of “Bootstrap Rules” on the boundary that determine all possible bispectra of relevance for cosmology/inflation

- How: use symmetries and general principles such as unitarity and locality

- Trick: start with the (better understood) amplitude and build the correlator out of it

- What: All bispectra (3-point correlators) of *massless* scalars and gravitons

Bootstrap Rules

All Bootstrap Rules are formulated at the boundary:

1. Rotations, Translations and Scale invariance
2. Tree-level bispectrum in de Sitter
3. Amplitude limit
4. Bose symmetry
5. Locality + Bunch-Davies vacuum
6. Soft limits

Rule 1: rotations, translations and scale invariance

- Bispectra must be constructed from SO(3) invariant contractions and must be proportional to an overall momentum-conserving delta function
- We can separate the bispectrum B into a polarization factor and a trimmed bispectrum with a fixed k scaling

$$
\begin{aligned}
B &= \sum_{\text{contractions}} \left[\epsilon^{h_1}(\mathbf{k}_1)\epsilon^{h_2}(\mathbf{k}_2)\epsilon^{h_1}(\mathbf{k}_3)\mathbf{k}_1^{\alpha_1}\mathbf{k}_2^{\alpha_2}\mathbf{k}_3^{\alpha_3}\right] \times \mathcal{B} \\
&= \sum_{\text{contractions}} (\text{polarization factor}) \times (\text{trimmed bispectrum}) \,,
\end{aligned}
$$

$$
\mathcal{B}(\lambda k_1, \lambda k_2, \lambda k_3) = \lambda^{-(6+\alpha_{\text{tot}})}\mathcal{B}(k_1, k_2, k_3)\,,
$$

Rule 2: tree-level in de Sitter

- The trimmed bispectrum must be a rational function of the k's

$$\mathcal{B} = \frac{\mathrm{Poly}_{\beta}(k_1, k_2, k_3)}{\mathrm{Poly}_{6+\alpha_{\mathrm{tot}}+\beta}(k_1, k_2, k_3)} \,,$$

- This is equivalent to tree-level (=contact) with de Sitter mode function (but not assumption of dS boosts!)

- The exception are logs, which will be discussed later on

Rule 3: Amplitude limit

- The residue of the total-energy pole (kT=k1+…+kn=0) of (tree-level) correlators is fixed by the (UV-limit of the) amplitude [Maldacena & Pimentel ’11; Raju ’12; Arkani-Hamed et al ’17-‘18; Benincasa ’18]

- The precise relation is (see Goodhew talk) [Goodhew, Jazayeri & EP ’20]

$$\lim_{k_T \to 0} B_n = \frac{(-1)^n H^{p+n-1}(p-1)!}{2^{n-1}} \times \frac{\mathrm{Re}\left(i^{1+n+p} A_n\right)}{\left(\prod_{a=1}^n k_a\right)^2 k_T^p},$$

$$\lim_{k_T \to 0} B_3 = -\frac{H^{p+2}(p-1)!}{4} \times \frac{\mathrm{Re}\left(i^p A_3\right)}{\left(k_1 k_2 k_3\right)^2 k_T^p},$$

- where p is fixed by dimensional analysis and scale invariance. For the bispectrum it’s simply the number of derivatives. More generally [EP ’20 to appear]

$$p = 1 + \sum_\alpha (D_\alpha - 4)$$

Rule 4: Bose symmetry

- For correlators of identical fields the trimmed bispectrum must be symmetric when the polarization factor is

- A theorem says that any symmetric polynomials can be written *uniquely* using Elementary Symmetric Polynomials (ESP)

$$k_T = e_1 = k_1 + k_2 + k_3\,, \quad e_2 = k_1 k_2 + k_2 k_3 + k_1 k_3\,, \quad e_3 = k_1 k_2 k_3$$

$$\mathcal{B}_{XXX} = \frac{\text{Poly}_\beta(k_T, e_2, e_3)}{\text{Poly}_{6+\alpha_{\text{tot}}+\beta}(k_T, e_2, e_3)}\,.$$

- These variables are extremely useful in practice!

Rule 5: Locality and the Bunch-Davies vacuum

- The denominator is fixed to be $(k_T)^p$ $(e_3)^3$

$$\mathcal{B}_{XYZ} = \frac{\text{Poly}_{3+p-\alpha_{\text{tot}}}(k_1, k_2, k_3)}{k_T^p e_3^3} \, .$$

- This fixed the order of the Poly at the numerator!

- Any deviation from implies a violation of locality, e.g. general interactions with inverse laplacians,

- OR a deviation from the Bunch-Davies vacuum, e.g. having both positive and negative frequencies next to creation or annihilation operators

Rule 6: Soft theorems

- Correlators of curvature perturbations ζ and gravitons ɣ must obey generalizations of Maldacena's consistency relation (Theorem 1!). This fixes the leading and next-to-leading order

$$\lim_{k_l \to 0} \langle \zeta(\mathbf{k}_l) X(\mathbf{k}_s - \mathbf{k}_l/2) Y(-\mathbf{k}_s - \mathbf{k}_l/2) \rangle' = P_\zeta(k_l) \frac{\partial}{\partial \log k_s} \left(k^3 \langle X(\mathbf{k}_s) Y(-\mathbf{k}_s) \rangle' \right) + \dots ,$$

$$\langle \gamma^h(\mathbf{k}_l) X(\mathbf{k}_s - \mathbf{k}_l/2) X(-\mathbf{k}_s - \mathbf{k}_l/2) \rangle' = \frac{3}{2} P_\gamma(k_l) \frac{\epsilon^h_{ij}(\mathbf{k}_l) k_s^i k_s^j}{k_s^2} P_X(k_s) + \dots ,$$

- The symmetric limit makes the next-to-leading order vanish automatically

- In terms of ESP this is just e3 -> 0

Example: <γγγ>

- The graviton bispectrum was derived both by explicit calculation [Maldacena ’02] and from de Sitter isometries [Maldacena & Pimentel ’11]. Here we re-derive it from the bootstrap rules

- From the two-derivative amplitude (e.g. from spinor helicity)

$$A_{\gamma\gamma\gamma} = -\frac{1}{\sqrt{2}M_{Pl}}\epsilon^{h_1}_{ii'}(\mathbf{k}_1)\epsilon^{h_2}_{jj'}(\mathbf{k}_2)\epsilon^{h_3}_{ll'}(\mathbf{k}_3)t_{ijl}t_{i'j'l'} \; ,$$

$$t_{ijl} = k_2^i\delta_{jl} + k_3^j\delta_{il} + k_1^l\delta_{ij}$$

- we make the p=2 Ansatz with 2 numerical unknowns C_1, C_3

$$B_{\gamma\gamma\gamma} = -\frac{1}{2}\left(\frac{H}{M_{Pl}}\right)^4 \frac{\epsilon^{h_1}_{ii'}\epsilon^{h_2}_{jj'}\epsilon^{h_3}_{ll'}t_{ijl}t_{i'j'l'}}{k_T^2 e_3^3}\left[e_3 + C_1 k_T e_2 + C_3 k_T^3\right] \; .$$

Example: <γγγ>

- The (symmetric) soft limit imposes two constraints

$$C_1 + 4C_3 = -3, \qquad 1 + 3C_1 + 4C_3 = 0\,,$$

- whose solution is C_1=1 and C_3=-1, which gives the correct result

$$B_{\gamma\gamma\gamma} = -\frac{H^4}{2M_{Pl}^4}\frac{\epsilon^{h_1}_{ii'}\epsilon^{h_2}_{jj'}\epsilon^{h_3}_{ll'}t_{ijl}t_{i'j'l'}}{k_1^3k_2^3k_3^3}\left[\frac{k_1k_2k_3}{k_T^2} + \frac{\sum_{a>b}k_ak_b}{k_T} - k_T\right]\,,$$

Example: <γζζ>

- The γζζ bispectrum was derived explicitly in [Maldacena ’02] and from approx de Sitter isometries [Mata, Raju, Trivedi ‘12]. It’s pretty much the same as γγγ

- From the two-derivative amplitude (e.g. from spinor helicity)

$$A_{\gamma\zeta\zeta} = \frac{\sqrt{2}}{M_{Pl}} \epsilon^h_{ij}(\mathbf{k}_1) k_2^i k_3^j \, .$$

- we make the p=2 Ansatz with 2 numerical unknowns C1, C3

$$B_{\gamma\zeta\zeta} = \frac{1}{4\epsilon} \frac{H^4}{M_{Pl}^4} \frac{\epsilon^h_{ij}(\mathbf{k}_1) k_2^i k_3^j}{e_3^3 k_T^2} \left[e_3 + C_1 e_2 k_T + C_3 k_T^3 \right] \, ,$$

Example: <γζζ>

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$$B_{\gamma\zeta\zeta} = \frac{1}{4\epsilon}\frac{H^4}{M_{Pl}^4}\frac{\epsilon^h_{ij}(\mathbf{k}_1)k_2^i k_3^j}{e_3^3 k_T^2}\left[e_3 + e_2 k_T - k_T^3\right]\,,$$

- In both examples the soft theorem provide the same constraining power as de Sitter boosts, but they are much more general and valid also when boosts are broken.

Example: <ζζζ>

- The scalar bispectrum is the most phenomenologically relevant of all.
- So far we considered only Lorentz Invariant amplitudes. But (Lorentz and de Sitter) boosts in cosmology are spontaneously broken by the coupling with the inflaton background that picks out a preferred frame (EFT of inflation)
- Boost-breaking amplitudes were recently studied [EP, Stefanyszyn & Supel '20]. For three identical scalars one finds

$$A(1^0, 2^0, 3^0) = F(E_1 E_2 + E_1 E_3 + E_3 E_2,\ E_1 E_2 E_3)$$

- In this talk, I discuss the boost-breaking case (see the paper for the Lorentz Invariant case, which gives the ε -> 0 limit in canonical single field [Maldacena '02; EP, Pimentel, van Wijck '16])

Example: Lorentz-invariant <ζζζ>

- Let's start with Lorentz invariant. This corresponds to the conformal limit of inflation ε -> 0 [EP, Pimentel, v.Wijck '16]

- The amplitude is constant (no derivatives) -> p=0

$$A^{LI}_{\phi\phi\phi} = \lim_{\epsilon \to 0} V''' = -\frac{3}{2}\,\frac{H^2}{\sqrt{2\epsilon}M_{Pl}}\,\dot{\eta} + \dots\,,$$

- This is a special case of the kT pole relation, where the amplitude fixes the logarithmic divergence

$$\lim_{k_T \to 0} B_3 = -\frac{H^2}{(k_1 k_2 k_3)^2}\log(-\tau_0 k_T)\,\mathrm{Re}\,A_3'\,.$$

- So we start with the p=0 Ansatz and 5 free parameters

$$B_{\zeta\zeta\zeta} = \left(\frac{H^2}{4\epsilon M_{Pl}^2}\right)^2 \frac{1}{e_3^3}\left[\frac{\dot{\eta}}{2H}\log\left(-k_T\tau_*\right)\left(3e_3 + C_1 k_T e_2 + C_3 k_T^3\right) + D_0 e_3 + D_1 k_T e_2 + D_3 k_T^3\right],$$

Example: Lorentz-invariant <ζζζ>

- Four parameters (C1, C3, D1, D3) are fixed by the squeezed limit, which has to vanish separately for the Log and polynomial part

$$C_1 = -3\,, \qquad D_0 = \frac{1}{2}\left[-4D_1 + \frac{\dot\eta}{H}(5-3\gamma_E) - 3\eta_*\right]\,,$$

$$C_3 = 1\,, \qquad D_3 = \frac{1}{8}\left[-2D_1 + \frac{\dot\eta}{H}(-2+\gamma_E) + \eta_*\right]\,.$$

- To find D0 I need to invoke dS boosts

$$(K_a - K_b)\, B_{\zeta\zeta\zeta}(k_1,k_2,k_3) = 0 \quad \Rightarrow \quad D_1 = \frac{\eta}{H}\left(1 - \frac{3}{2}\gamma_E\right) - \frac{3}{2}\eta_*$$

- This recovers the bispectrum in the “conformal limit of inflation, where ε -> 0 [EP, Pimentel, Wijck ’16]

$$B_{\zeta\zeta\zeta} = \left(\frac{H^2}{4\epsilon M_{Pl}^2}\right)^2 \left\{ \frac{\eta_*}{2}\frac{k_1^3+k_2^3+k_3^3}{k_1^3k_2^3k_3^3} + \frac{\dot\eta}{2H}\frac{1}{k_1^3k_2^3k_3^3} \times \left[(-1+\gamma_E+\log(-k_T\tau_*))\sum_{i=1}^{3}k_i^3 - \sum_{i\neq j}k_i^2k_j + k_1k_2k_3\right]\right\}$$

Example: Boost-breaking <ζζζ>

- Assuming one del per ϕ, we have p=3. The amplitude at three derivatives is

$$A(1^0, 2^0, 3^0) \propto E_1 E_2 E_3 \to k_1 k_2 k_3 = e_3$$

- The Ansatz has 7 parameters

$$B = \frac{C_0 e_3^2 + C_0' e_2^3 + C_1 k_T e_2 e_3 + C_2 k_T^2 e_2^2 + C_3 k_T^3 e_3 + C_4 k_T^4 e_2 + C_6 k_T^6}{e_3^3 k_T^3}.$$

so $C_0 = g$.

- The C'_0 term corresponds to a *non-local amplitude* so $C'_0 = 0$, leaving 6

- The soft theorem removes 1 parameter leaving 5

Example: Boost-breaking <ζζζ>

$$B = \frac{C_0 e_3^2 + C_1 k_T e_2 e_3 + C_2 k_T^2 e_2^2 + C_3 k_T^3 e_3 + C_4 k_T^4 e_2 + C_6(C_a) k_T^6}{e_3^3 k_T^3} \, .$$

- This recovers the bispectrum of a P(X,ϕ) theory [Chen et al 06], which has 5 parameters (linearly related to C's)
  - 2 parameters are f_{NL}^{ort} and f_{NL}^{equil}
  - 3 parameter are slow-roll terms ε, η, and s.

Boostless outlook

- We can write down the tree-level bispectrum *to all orders in derivatives*

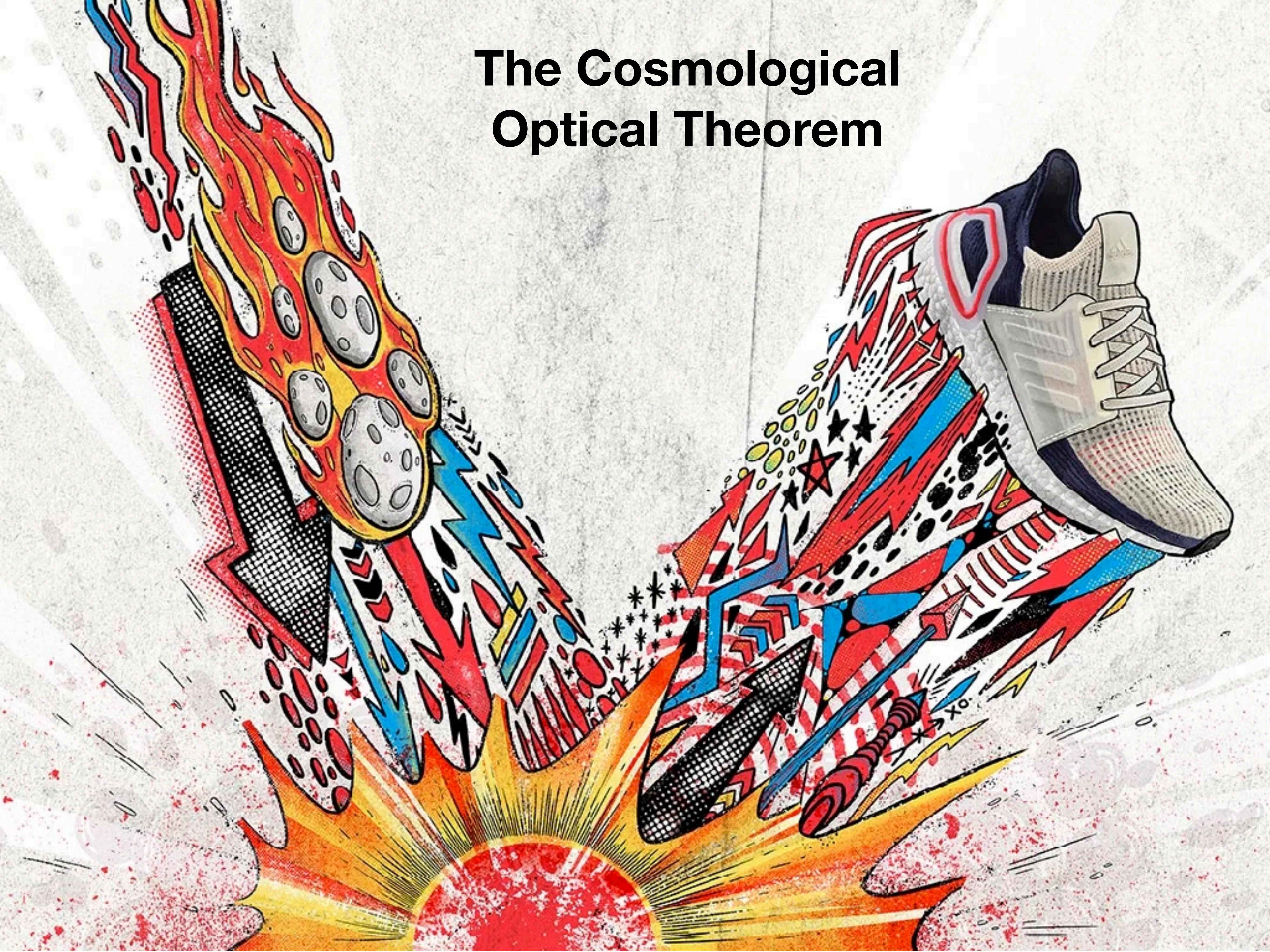
$$B_{\zeta\zeta\zeta} = \frac{\mathrm{Poly}_{3+p_{\max}}(k_T, e_2, e_3)}{e_3^3 k_T^{p_{\max}}}$$

where (i) one parameter is fixed by the soft theorem and (ii) the highest pole must have a factor of e3 to be local.

- The recently derived **boostless amplitudes** for all massless particles of any spin [Pajer, Stefanyszyn, Supel '20] can be used to bootstrap all the graviton & scalar bispectra to all order in derivatives!

- This can reproduce many results, e.g. that <ɣɣɣ> is unchanged to leading order in derivatives [Creminelli et al '14] & ɣ non-Gaussianity in the EFT of I (see Cabass contribution) [Cabass & Bordin '20]

The Cosmological Optical Theorem



Beyond the bispectrum

- The Boostless Bootstrap Rules fix the tree-level scalar and graviton bispectra partially because *contact interactions are particularly simple*

- But de Sitter isometries are also extremely constraining for 4-point functions, which are indeed the foundation of the cosmological bootstrap (see Lee, Joyce & Duaso Pueyo's talks)

- If we want a more comprehensive Boostless Bootstrap we need more constraining power.

- Here I discuss the **Cosmological Optical Theorem**, a set of conditions on boundary wavefunction and correlators implied by unitarity (see Harry Goodhew's talk for details on the derivation)

Unitarity

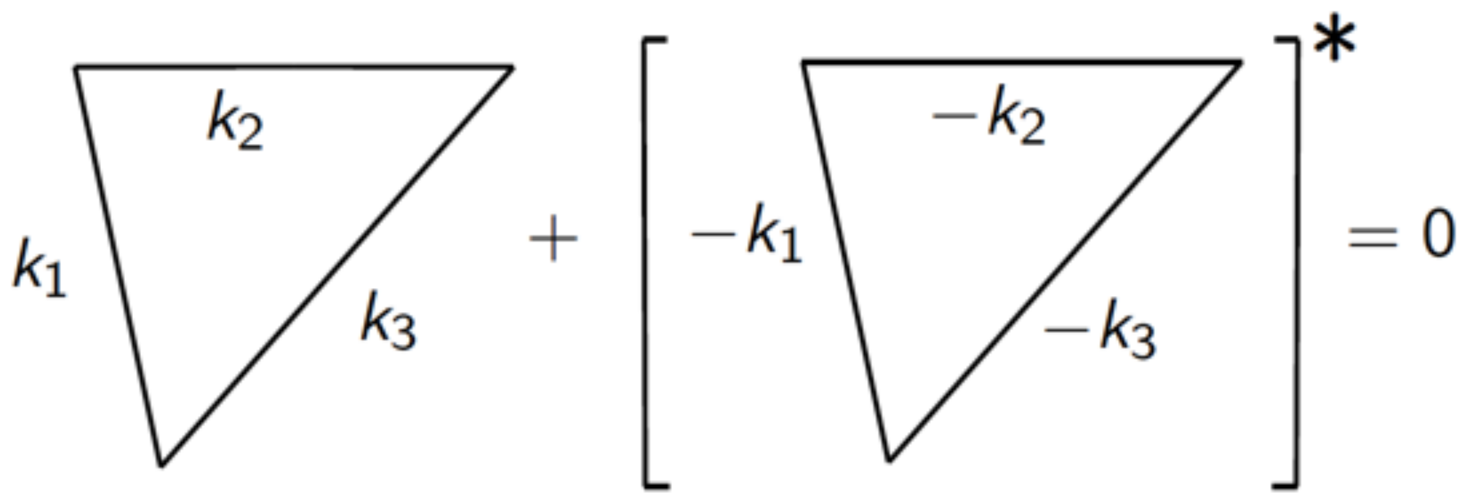
- The *unitarity* of time evolution is a pillar of our understand of physics through quantum field theory
- The consequences of unitarity are well-understood for amplitudes, e.g. in the Optical Theorem and in the Cutting Rules.
- But how is unitary *time evolution* encode in the future value of *spacelike* correlators? [see e.g. many Arkani-Hamed talks]
- Here we provide (necessary) conditions on the coefficients ψ_n of the wavefunction of scalar fields to arise from unitary evolution

$$\Psi[\phi] = \exp\left[-\sum_{n=2}^{\infty} \int \psi_n \phi(\mathbf{k}_1) \ldots \phi(\mathbf{k}_n) \right],$$

Cosmological Optical Theorem: contact diagrams

- Staring from UU†=1, and using the *analytic structure* of de Sitter mode functions, we find for any contact diagram

$$\psi'_n(k_a, \hat{k}_a \cdot \hat{k}_b) + \left[\psi'_n(-k_a^*, \hat{k}_a \cdot \hat{k}_b)\right]^* = 0 \,, \qquad k_a \in \mathbb{C}^{n-} \,.$$



Cosmological Optical Theorem: contact diagrams

$$\psi_n'(k_a, \hat{k}_a \cdot \hat{k}_b) + \left[\psi_n'(-k_a^*, \hat{k}_a \cdot \hat{k}_b)\right]^* = 0\,, \qquad k_a \in \mathbb{C}^{n-}\,.$$

- This is the cosmological equivalent of Hermitian analyticity for amplitudes. It comes with a prescription for the analytic continuation of ψ_n and implies

$$2\,\mathrm{Re}\,\psi_n'(-i\lambda_a, \hat{k}_a \cdot \hat{k}_b) = \mathrm{Disc}\left[\psi_n'(-i\lambda_a, \hat{k}_a \cdot \hat{k}_b)\right] \quad (\lambda_a > 0)$$

- Consequences: IR-finite ψ_n must be real for massless scalars

- Consequences: IR-finite correlators with odd n of conformally coupled fields vanish

$$\mathrm{Im}(\psi_n') = 0 \quad (n = \mathrm{even})\,, \qquad \mathrm{Re}(\psi_n') = 0 \quad (n = \mathrm{odd})$$

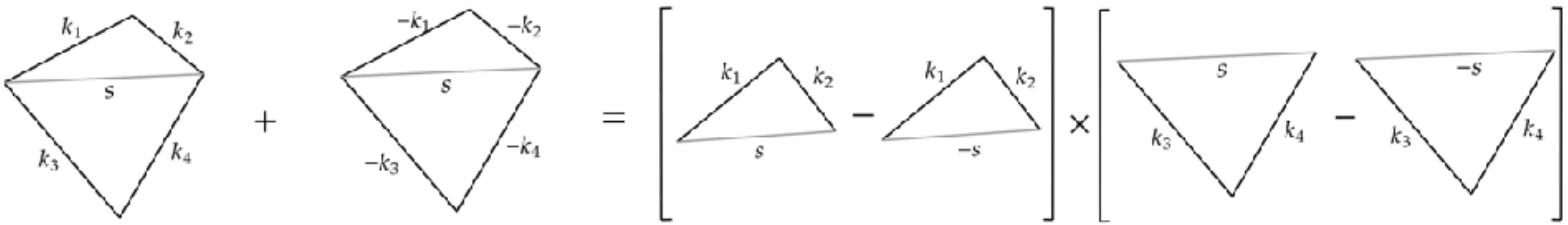
Cosmological Optical Theorem: contact diagrams

- For IR-divergent ψ_n, log divergences introduce branch points/cuts and must be accompanied by an imaginary part
- For example, ϕ^3 for massless scalar gives

$$\psi_3' = \frac{g}{H^4}\Big[(-2+2\gamma+i\pi)e_1^3 + (4-6\gamma-3i\pi)e_2e_1 + (2+6\gamma+3i\pi)e_3 + 2\left(e_1^3-3e_2e_1+3e_3\right)\log\left(-e_1\eta_0\right) + \frac{2i}{\eta_0}(e_1^2-2e_2) + \frac{2i}{\eta_0^3}\Big],$$

- So the Im part is tied to the Log

Cosmological Optical Theorem: exchange diagrams



- Unitarity implies a relation between ψ_4 and $(\psi_3)^2$:

$$s = |\mathbf{k}_1 + \mathbf{k}_2|$$

$$\psi_4'^{s}(k_1, k_2, k_3, k_4, s) + [\psi_4'^{s}(-k_1, -k_2, -k_3, -k_4, s)]^* =$$
$$P_\sigma(s)\left[\psi_3'^{\phi\phi\sigma}(k_1, k_2, s) - \psi_3'^{\phi\phi\sigma}(k_1, k_2, -s)\right]\left[\psi_3'^{\phi\phi\sigma}(k_3, k_4, s) - \psi_3'^{\phi\phi\sigma}(k_3, k_4, -s)\right].$$

- It can be easily summed over all 3 channels

- This **determines the singularities** of ψ_4! they can be seen in the total energy poles of ψ_3

Cosmological Optical Theorem: exchange diagrams

- Consequences: the trispectrum is related to the bispectrum

$$B_4^s(k_1,k_2,k_3,k_4,s)+B_4^s(-k_1,-k_2,-k_3,-k_4,s)=$$
$$\frac{1}{2P_\sigma(s)}\left[B_3(k_1,k_2,s)-B_3(k_1,k_2,-s)\right]\left[B_3(k_3,k_4,s)-B_3(k_3,k_4,-s)\right]\,.$$

- This fixed ψ_4 and B_4 up to contact diagrams

- We checked this for the trispectrum in the EFT of single-clock inflation (P(X,ϕ)) [Chen et al '06; Smith et al. 15] and for graviton-exchanged trispectrum in canonical inflation [Seery et al'08]

Summary

- The goal is understand observables, i.e. correlators/wavefunction from the boundary of quasi dS: *Cosmological Bootstrap*
- General and phenomenologically interesting predictions break (Lorentz/de Sitter) boosts: *Boostless Bootstrap*
- Scalar and tensor bispectra can be derived from transparent *Bootstrap Rules*. This reproduces Maldacena's results but extends to boost-breaking correlators in the EFT of inflation to all orders in derivatives
- Unitarity implies a *Cosmological Optical Theorem* from which we can bootstrap higher point (e.g. 4-point) from lower point (e.g. 3-point)

Open questions for discussion

- What are the consequences of locality for correlators/wavefunction? Maldacena's bispectrum has a non-local amplitude…
- Bootstrap all boost-breaking scalar-tensor correlators?
- Cosmological Optical Theorem to all/higher order? Non-perturbative formulation? General-mass exchange? Deviation from de Sitter mode functions?
- How well can you measure observationally the order of the kT pole?

Locality in curved spacetime

- The scalar bispectrum for canonical single-field inflation [Maldacena '02] has p=1!? The residue of kT pole then must be (and indeed is) the (boost-breaking) **non-local** amplitude

$$A_{\text{Non-local}} \sim \frac{e_2^2}{e_3} \sim \frac{E_1 E_2 + E_1 E_3 + E_2 E_3}{E_1 E_2 E_3}$$

- This amplitude is not allowed by locality in Minkowski, but arises in the flat-space limit of curved spacetime!

- What are the consequences of locality for correlators?

- What amplitudes can appear on kT poles of local theories?