Spinning Cosmological Bootstrap

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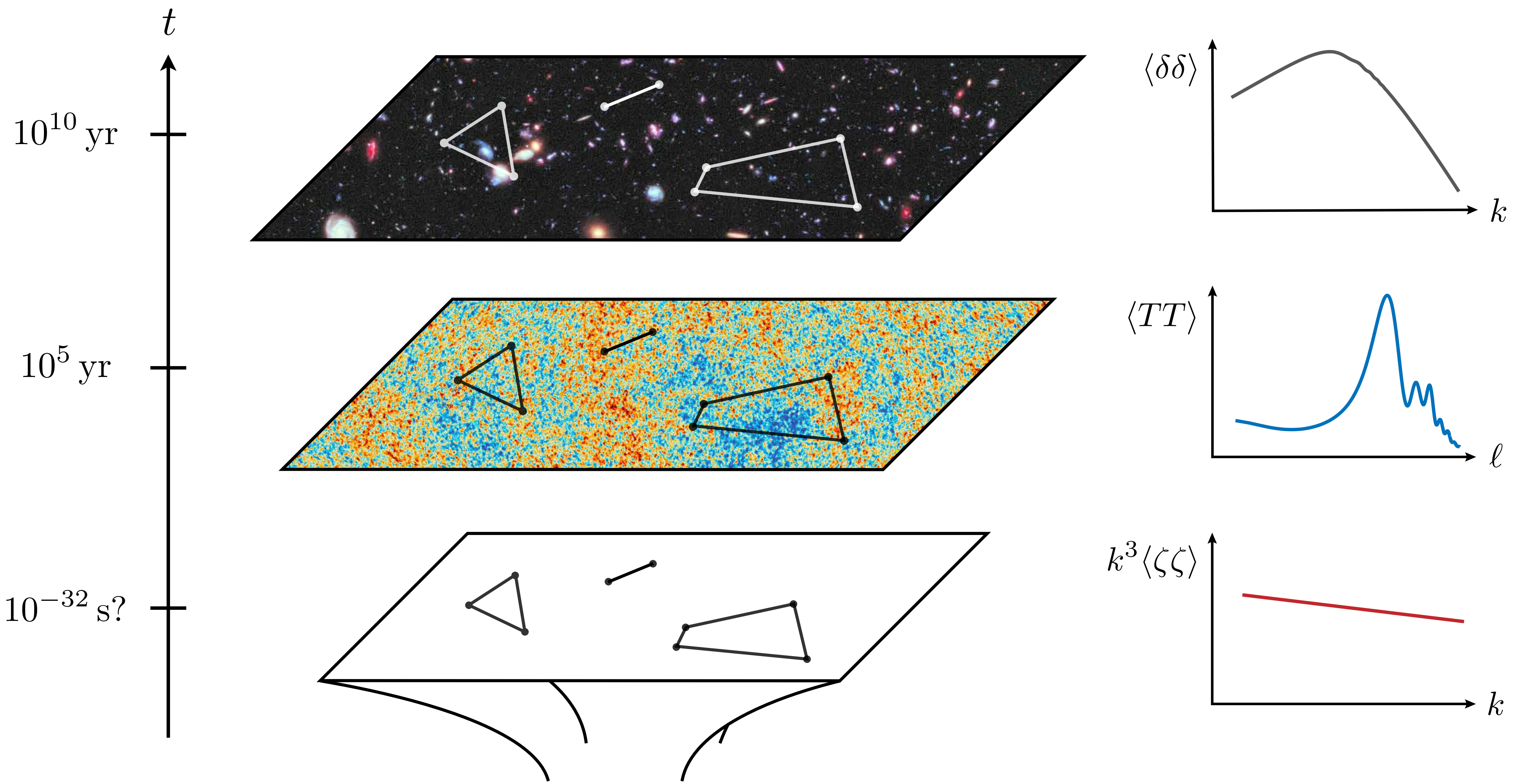
[1811.00024] w/ N. Arkani-Hamed, D. Baumann, G. L. Pimentel

[1910.14051] w/ D. Baumann, C. Duaso Pueyo, A. Joyce, G. L. Pimentel

[2005.04234] w/ D. Baumann, C. Duaso Pueyo, A. Joyce, G. L. Pimentel

Primordial fluctuations

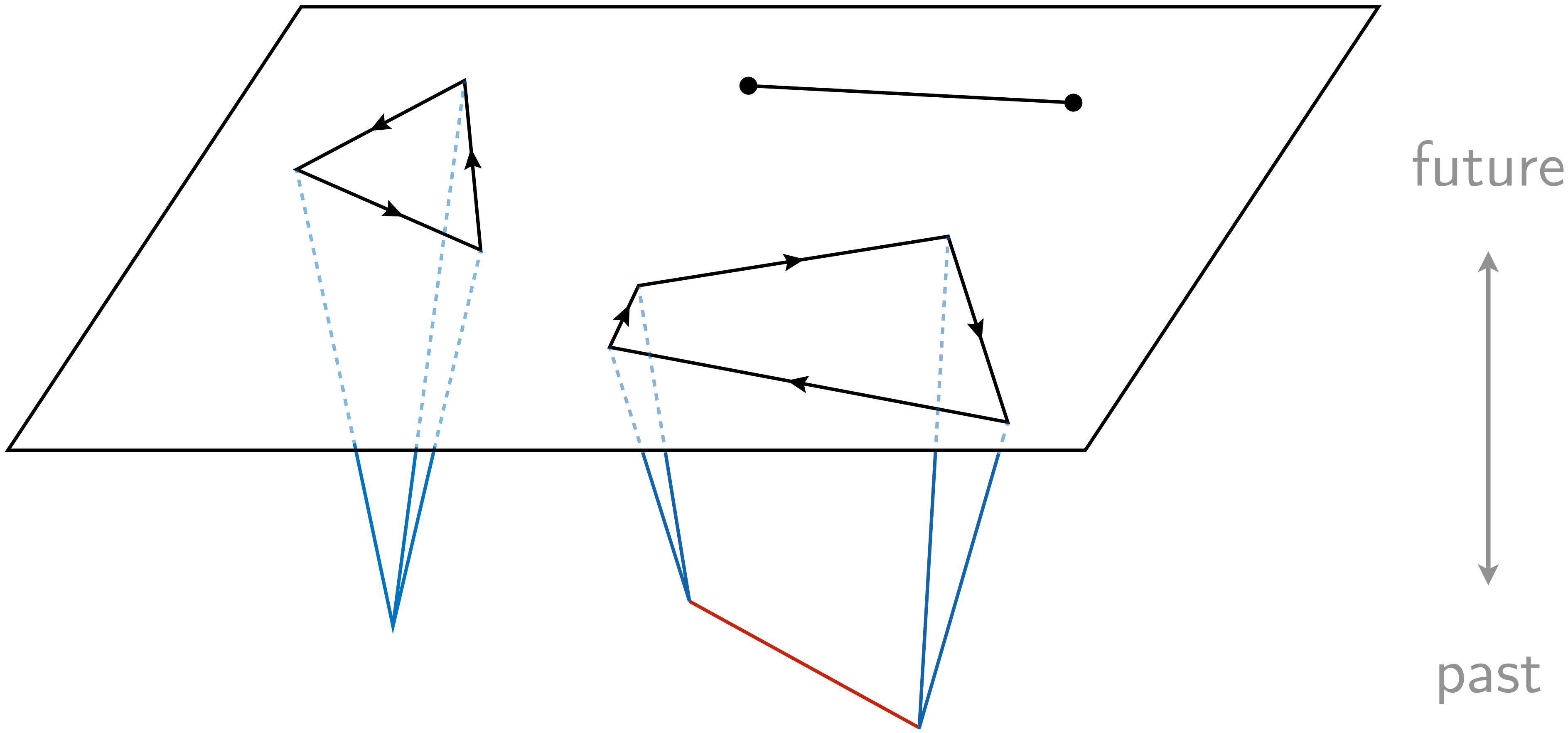


What cosmic history gave rise to primordial fluctuations?

Boundary correlators

In inflation, local physics is encoded on boundary correlators of dS.

Computations are complicated, yet the final answers are surprisingly simple.

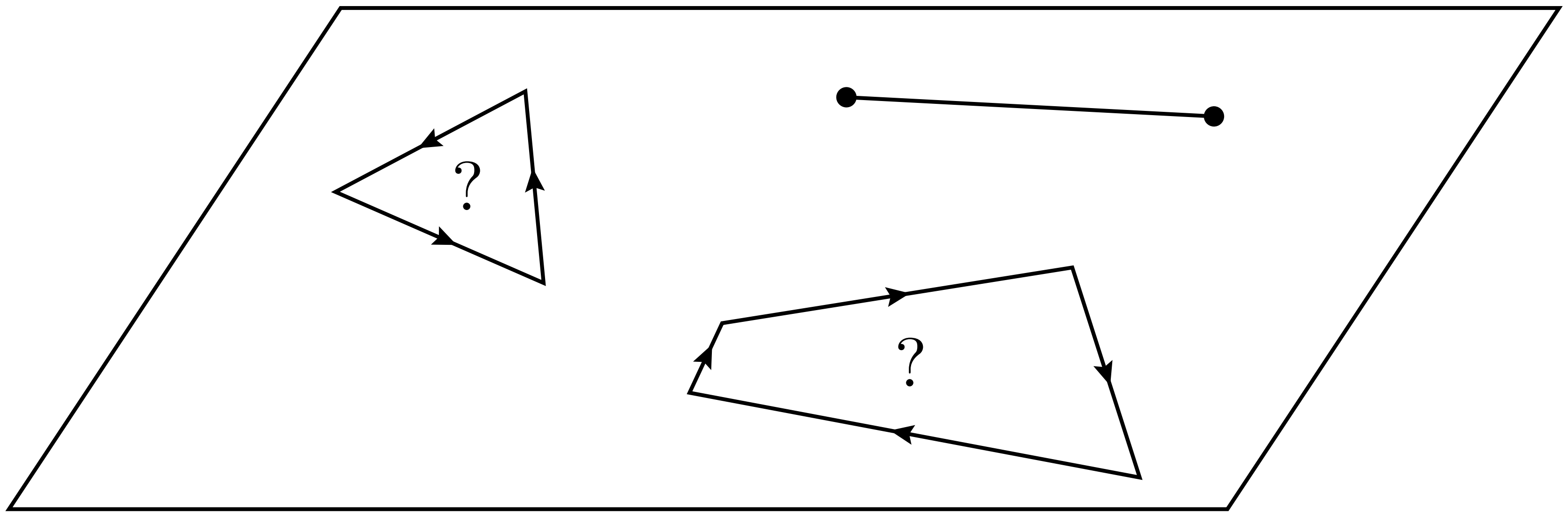


Can we “bootstrap” these correlators without bulk time evolution?

Cosmological bootstrap

What are the rules for consistent correlators?

What is the connection to the physics of scattering amplitudes?



New theoretical tools: extension of "cosmological collider physics"

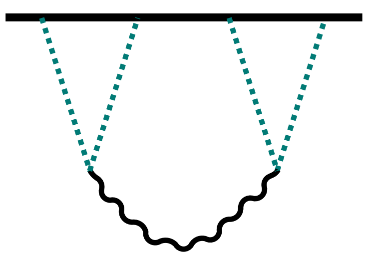
See e.g. Arkani-Hamed, Maldacena [2015], Arkani-Hamed, Benincasa, Postnikov [2017], Arkani-Hamed, Baumann, HL, Pimentel [2018], Benincasa [2018, 2019], Sleight, Taronna [2019], Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2019, 2020], Hillman [2020], Green, Pajer [2020], Goodhew, Jazayeri, Pajer [2020]

Cosmological bootstrap

The key idea of the cosmological bootstrap is to exploit

symmetries & singularities of cosmological correlators.

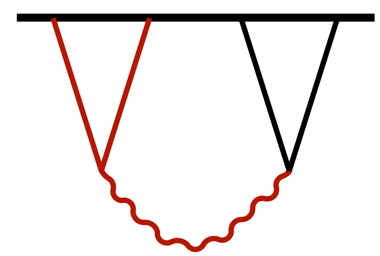
$E = \sum_i k_i$



$$\xrightarrow{E \to 0} \quad \frac{1}{E^{\#}} A_4$$

total energy singularity

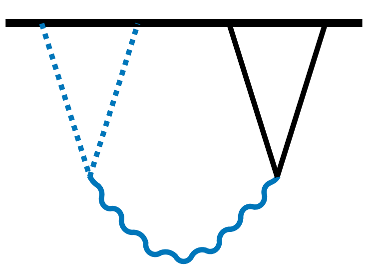
$s = |\vec{k}_1 + \vec{k}_2|$



$$\xrightarrow{k_1 + k_2 \to s} \quad \text{regular}$$

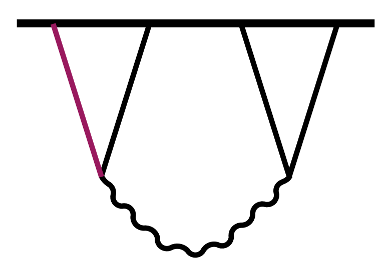
folded/collinear limit

$E_L = k_1 + k_2 + s$



$$\xrightarrow{E_L \to 0} \quad \frac{1}{E_L^{\#}} A_3 \otimes \widetilde{\Psi}_3$$

partial energy singularity



$$\xrightarrow{k_1 \to 0} \quad \text{regular}^*$$

soft limit

Outline

I. Scalar dS Correlators

II. Spinning dS Correlators

III. Cosmological Four-Particle Test

IV. Outlook

I. Scalar dS Correlators

Arkani-Hamed, Baumann, HL, Pimentel [2018]

Symmetries

Invariance under dilatations and SCTs imply

$$0 = \Big[1 + \sum_n \vec{k}_n \cdot \partial_{\vec{k}_n}\Big] F \qquad \Rightarrow \qquad F = s^{-1}\hat{F}$$

$$0 = \sum_n \Big[\vec{k}_n(\partial_{\vec{k}_n} \cdot \partial_{\vec{k}_n}) - 2(\vec{k}_n \cdot \partial_{\vec{k}_n})\partial_{\vec{k}_n} - 2\partial_{\vec{k}_n}\Big] F$$

$(m_\phi = \sqrt{2}H)$

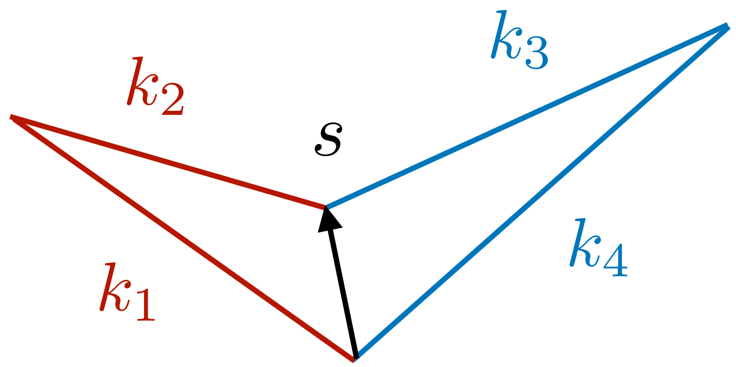
"conformally coupled"

For conformal scalars, these reduce to

$$(\Delta_u - \Delta_v)\hat{F} = 0$$

$(\Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u)$

"hypergeometric"

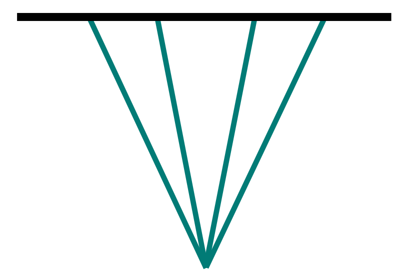


$$u \equiv \frac{s}{k_1 + k_2} \qquad v \equiv \frac{s}{k_3 + k_4}$$

Tree diagrams

Contact diagrams are controlled by the total energy singularity:

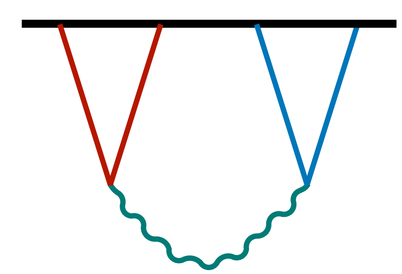
$$(\Delta_u - \Delta_v)\frac{1}{E} = 0 \qquad \Rightarrow$$



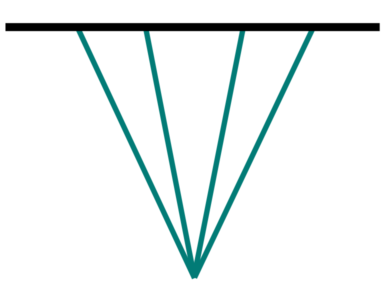
$$= \sum_n c_n \Delta_u^n \left(\frac{1}{E}\right)$$

Tree exchanges are additionally controlled by partial energy singularities:

$$\lim_{\mu \to \infty} \hat{F} = \frac{1}{\mu^2}\hat{C} \qquad \Rightarrow \qquad \begin{matrix} (\Delta_u + \mu^2) \\ (\Delta_v + \mu^2) \end{matrix}$$

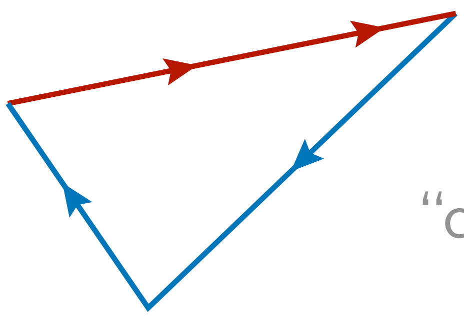


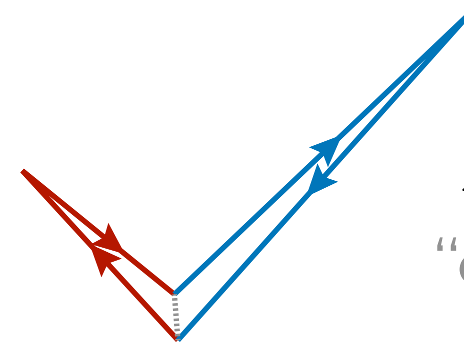
$$=$$



Tree exchange solution

Two physical boundary conditions are:

(I)  $\xrightarrow[\text{“collinear”}]{u \to 1}$ regular

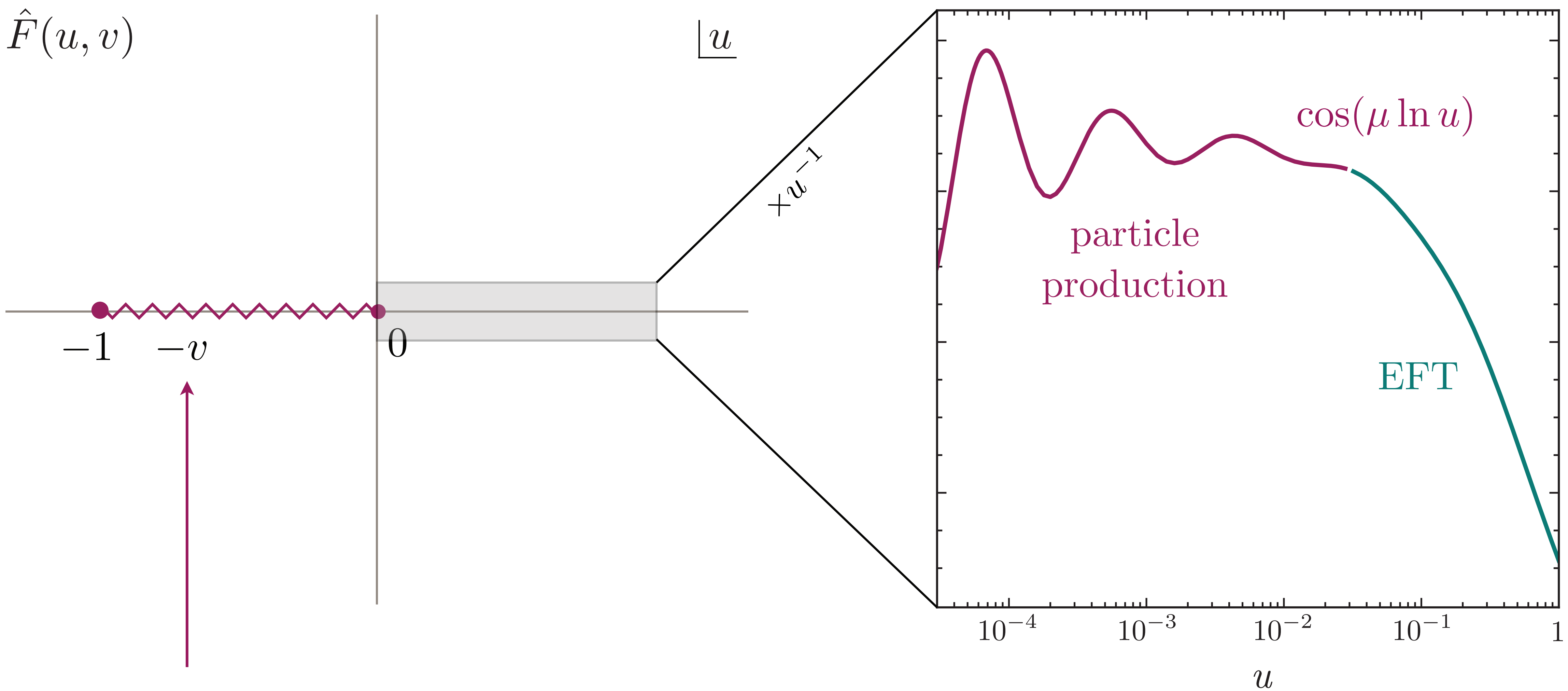
(II)  $\xrightarrow[\text{“collapsed”}]{u, v \to 0}$ $3\text{pt} \times 3\text{pt}$

This uniquely fixes the solution to be (e.g. for small u)

$$\hat{F} = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+\frac{1}{2})^2 + \mu^2} \left(\frac{u}{v}\right)^{n+1}}_{\text{EFT expansion}} + \underbrace{\frac{\pi}{\cosh \pi\mu} \frac{\left(\frac{u}{v}\right)^{\frac{1}{2}+i\mu} - c.c.}{2i\mu}}_{\text{particle production}}$$

Analytic structure

Analytic continuation provides a link between scattering in flat space and particle production in curved space.



$$\text{disc}\,\hat{F}(-v,v) = W[\hat{F}_+, \hat{F}_-] = A(2 \to 2)$$

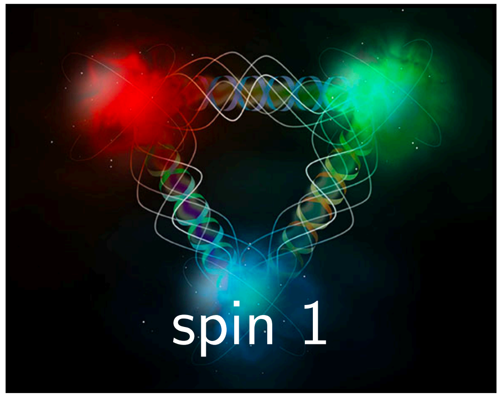
II. Spinning dS Correlators

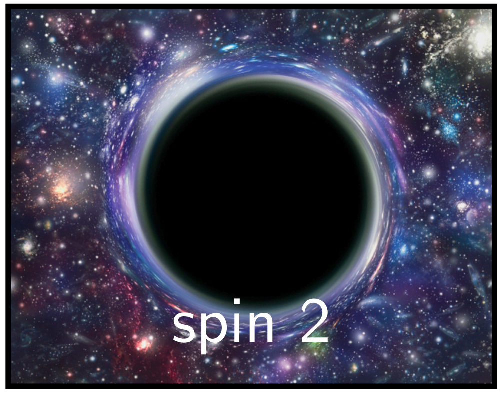
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]

Massless particles

Massless fields (ζ, γ) play an important role in cosmology, whose fluctuations get amplified during inflation and survive until late times.

Amplitudes for massless spinning particles are strikingly simple:





$$A_{\mathrm{YM}}(1^-2^-3^+4^+) = \frac{\langle 12\rangle^2[34]^2}{st} \qquad A_{\mathrm{GR}}(1^-2^-3^+4^+) = \frac{\langle 12\rangle^4[34]^4}{stu}$$

Is there a similar simplicity for spinning cosmological correlators?

Wavefunction of the universe

Cosmological fluctuations are captured by the wavefunction of the universe.

$$\Psi[\phi] = \exp\left[-\sum_{n=2}^{\infty}\frac{1}{n!}\int_{\vec{k}_1,\cdots,\vec{k}_n}\psi_n(\{\vec{k}_i\})\phi_{\vec{k}_1}\cdots\phi_{\vec{k}_n}\right]$$

$$\Updownarrow$$

$$\langle\phi_{\vec{k}_1}\cdots\phi_{\vec{k}_n}\rangle = \frac{1}{\mathcal{N}}\int[D\phi]\,\phi_{\vec{k}_1}\cdots\phi_{\vec{k}_n}|\Psi[\phi]|^2$$

The wavefunction coefficients can be interpreted as correlators of dual operators to bulk fields

$$\psi_n(\{\vec{k}_i\}) = \langle\mathcal{O}_{\vec{k}_1}\cdots\mathcal{O}_{\vec{k}_n}\rangle \times \delta^3(\textstyle\sum_a \vec{k}_a)$$

$$A_i \;\leftrightarrow\; J_i\,, \qquad \partial_i J_i = 0$$

$$\gamma_{ij} \;\leftrightarrow\; T_{ij}\,, \qquad \partial_i T_{ij} = 0$$

Ward-Takahashi identity

Correlators of conserved currents obey the Ward-Takahashi identity:

$$\langle (\vec{k}_1 \cdot J_{\vec{k}_1}) \mathcal{O}_{\vec{k}_2} \cdots \mathcal{O}_{\vec{k}_n} \rangle = \sum_{a=2}^{n} e_a \langle \mathcal{O}_{\vec{k}_2} \cdots \mathcal{O}_{\vec{k}_a + \vec{k}_1} \cdots \mathcal{O}_{\vec{k}_n} \rangle$$

$$\langle (\vec{k}_1 \cdot T_{\vec{k}_1} \cdot \vec{\xi}) \mathcal{O}_{\vec{k}_2} \cdots \mathcal{O}_{\vec{k}_n} \rangle = \sum_{a=2}^{n} \kappa_a (\vec{\xi} \cdot \vec{k}_1) \langle \mathcal{O}_{\vec{k}_2} \cdots \mathcal{O}_{\vec{k}_a + \vec{k}_1} \cdots \mathcal{O}_{\vec{k}_n} \rangle$$

To impose conformal symmetry, we consider the special conformal generator in spinor-helicity variables

$$\widetilde{K}^i = (\sigma^i)^\alpha{}_\beta \frac{\partial^2}{\partial \lambda^\alpha \partial \overline{\lambda}_\beta} \qquad \Longrightarrow \qquad \begin{aligned} \widehat{J}^{(\ell)} &= \xi^{i_1} \cdots \xi^{i_\ell} J^{(\ell)}_{i_1 \cdots i_\ell} \\ \widetilde{K}^i \widehat{J}^{(\ell)} &\sim \underbrace{K^i \widehat{J}^{(\ell)}}_{=0} + \frac{\xi^i}{k^2} (k \cdot \widehat{J}^{(\ell)}) \end{aligned}$$

Conformal Ward-Takahashi identity

Combining the WT identity with the special conformal generator, we obtain

$$\sum_{a=1}^{n}(\vec{b}\cdot\widetilde{K}_a)\langle\widehat{J}^{(\ell)}_{\vec{k}_1}\mathcal{O}_{\vec{k}_2}\cdots\mathcal{O}_{\vec{k}_n}\rangle = \sum_{a=2}^{n} e_a^{(\ell)}(\vec{b}\cdot\vec{\xi})^{\ell-1}\langle\mathcal{O}_{\vec{k}_2}\cdots\mathcal{O}_{\vec{k}_a+\vec{k}_1}\cdots\mathcal{O}_{\vec{k}_n}\rangle$$

The solution can be written as

$$\langle J^{(\ell)}\mathcal{O}\cdots\mathcal{O}\rangle = F^{\text{part.}}(\vec{k}_i,\vec{\xi}) + \Pi\cdot G^{\text{hom.}}(\vec{k}_i,\vec{\xi})$$

- particular solution $\leftrightarrow$ minimal bulk coupling
- homogeneous solution $\leftrightarrow$ non-minimal bulk coupling

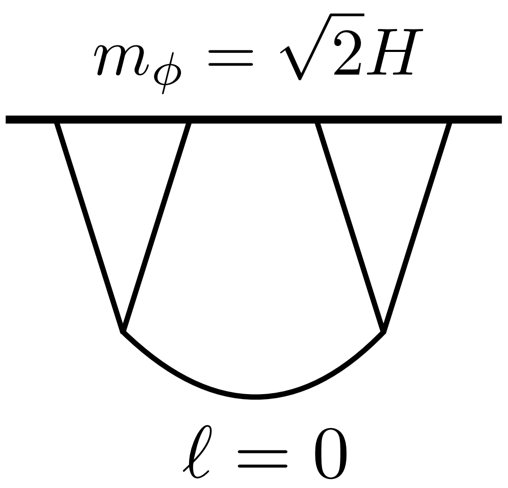
Maldacena, Pimentel [2011]
Bzowski, McFadden, Skenderis [2014]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]

Spinning cosmological correlators

We use two approaches to construct spinning correlators in dS:

1. Weight-shifting operators [Carlos & Austin's talks]

$$\langle \mathcal{O}_{\Delta_1,\ell_1} \cdots \mathcal{O}_{\Delta_4,\ell_4} \rangle = \sum_a c_a \mathcal{D}^{(a)}_{\Delta_i,\ell_i}(\vec{k}_i, \partial_{\vec{k}_i})$$



- Coefficients are fixed by the Ward-Takahashi identity.
- Algorithmic but computationally intensive

Costa, Penedones, Poland, Rychkov [2011]
Karateev, Kravchuk, Simmons-Duffin [2017]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2019, 2020]

Spinning cosmological correlators

We use two approaches to construct spinning correlators in dS:

2. Singularities [this talk]

$$\langle \mathcal{O}_{\Delta_1,\ell_1} \cdots \mathcal{O}_{\Delta_4,\ell_4} \rangle \;\to\; \begin{cases} \dfrac{1}{E^{\#}} A_4 & E \to 0 \\ \dfrac{1}{E_L^{\#}} A_{3,L} \otimes \widetilde{\Psi}_{3,R} & E_L \to 0 \\ \text{regular} & u, v \to 1 \end{cases}$$

- Advantage: much simpler building blocks.
- In many cases, these conditions fix the answer completely.

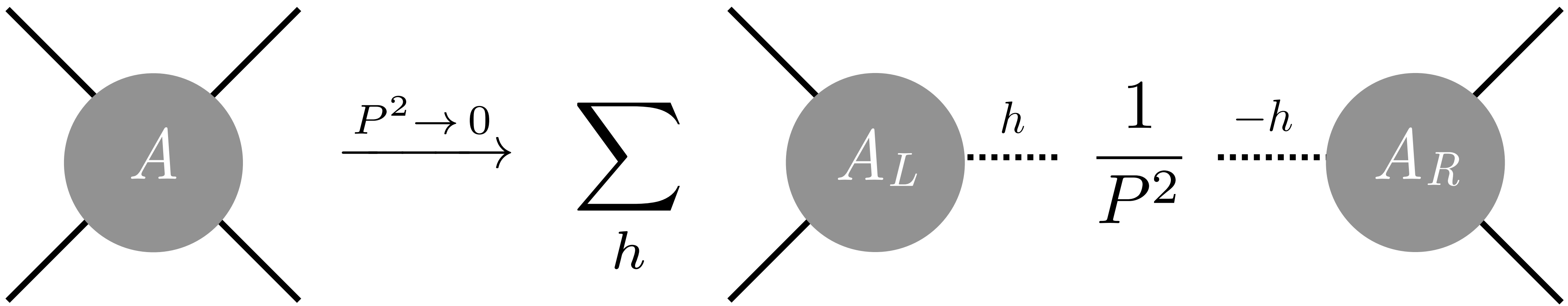
Arkani-Hamed, Benincasa, Postnikov [2017]
Benincasa [2018, 2019]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]

III. Cosmological Four-Particle Test

Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]

Factorization of amplitudes

Amplitudes at tree level have poles whose residues must factorize into a product of lower-point amplitudes.

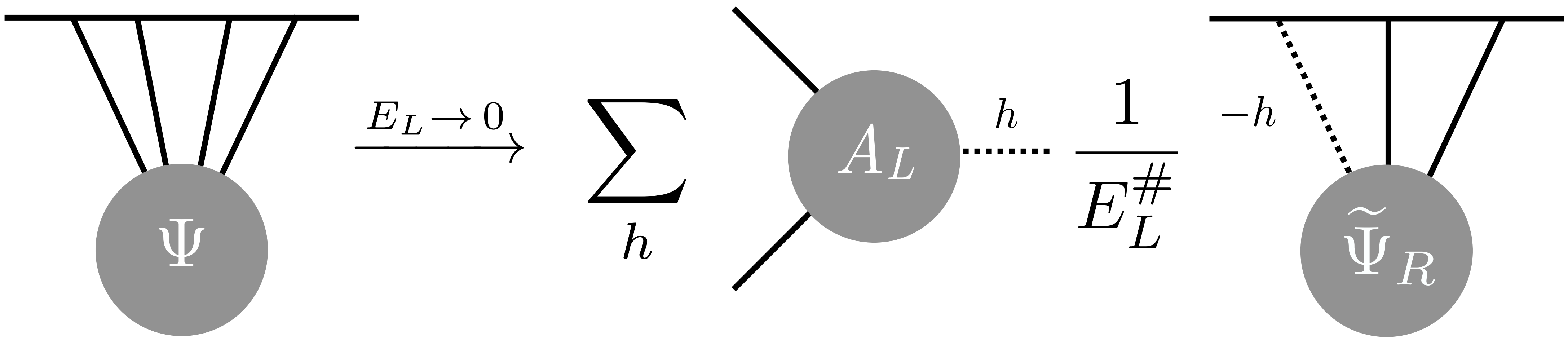


Consistent factorization does not only construct the full answer, but is also strong enough to rule out inconsistent theories.

Benincasa, Cachazo [2007]
Schuster, Toro [2008]
McGady, Rodina [2013]
Arkani-Hamed, Huang, Huang [2017]

Factorization of correlators

Correlators of conserved currents at tree level have poles whose residues must factorize into a product of lower-point amplitudes/correlators.



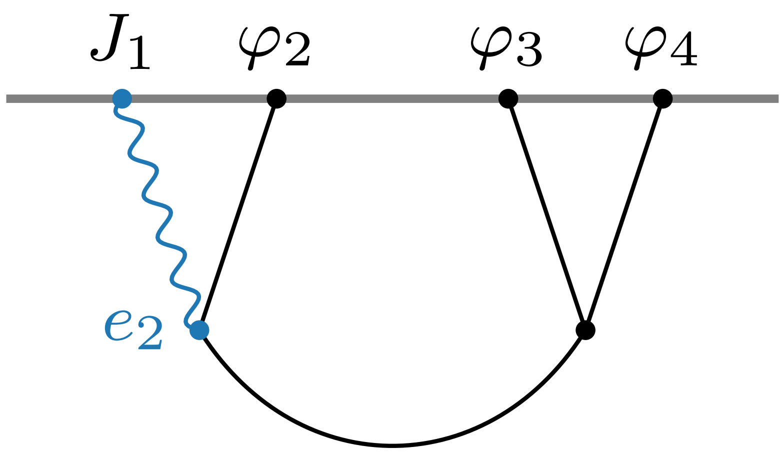
Consistent factorization, combined with other singularities, also provides a nontrivial constraint on cosmological correlators.

$$\widetilde{\Psi} = \frac{1}{2k_I}\left(\Psi|_{k_I \to -k_I} - \Psi\right)$$

Arkani-Hamed, Benincasa, Postnikov [2017]
Benincasa [2018, 2019]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]

JOOO: charge conservation

As a simple example, consider scattering of one photon with three scalars:



Let's construct this correlator from factorization. The building blocks are:

$$A_{J\varphi\varphi} = \vec{\xi}_1 \cdot \vec{k}_2 \qquad A_{\varphi\varphi\varphi} = 1$$

$$\widetilde{\Psi}_{J\varphi\varphi} = \frac{\vec{\xi}_1 \cdot \vec{k}_2}{(k_{12}+s)(k_{12}-s)} \qquad \widetilde{\Psi}_{\varphi\varphi\varphi} = \frac{1}{2s} \log\left(\frac{k_{34}+s}{k_{34}-s}\right)$$

JOOO: charge conservation

We require the following s-channel factorizations:

$$\langle J\varphi\varphi\varphi\rangle \xrightarrow{E_L\to 0} \frac{1}{E_L} A_{J\varphi\varphi}\cdot \widetilde{\Psi}_{\varphi\varphi\varphi} = \frac{1}{E_L}\frac{\vec{\xi}_1\cdot\vec{k}_2 \log(\frac{E_R}{k_{34}-s})}{-2s}$$

$$\langle J\varphi\varphi\varphi\rangle \xrightarrow{E_R\to 0} \widetilde{\Psi}_{J\varphi\varphi}\cdot A_{\varphi\varphi\varphi}\log(\frac{E_R}{\mu}) = \frac{1}{E_L}\frac{\vec{\xi}_1\cdot\vec{k}_2 \log(\frac{E_R}{\mu})}{(k_{12}-s)}$$

An object that factorizes correctly with no spurious singularity is

$$\langle J\varphi\varphi\varphi\rangle = \frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L(k_{12}-s)}\log\left(\frac{E_R}{E}\right)$$

Is this the final answer?

JOOO: charge conservation

To answer this, let's check the conformal WT identity.

The action of the conformal generator on the s-channel correlator gives

$$\sum_{a=1}^{4}(\vec{b}\cdot\widetilde{K}_a)\langle J\varphi\varphi\varphi\rangle_s = \frac{\vec{b}\cdot\vec{\xi}_1}{k_1^2}\left[e_2\log\left(\frac{k_{34}+s}{E}\right)+\frac{k_1}{E}\right]$$

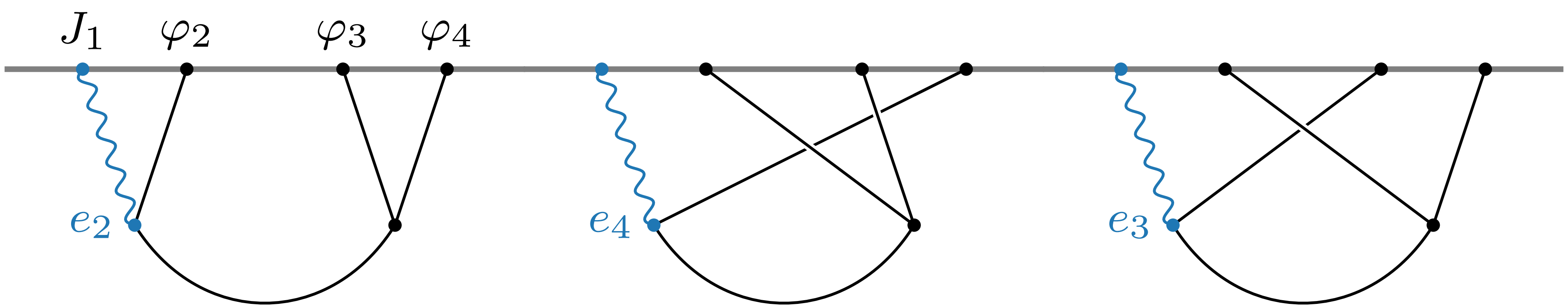
The conformal WT identity asserts that this should be equal to

$$= \frac{\vec{b}\cdot\vec{\xi}_1}{k_1^2}\left[e_2\underbrace{\log\left(\frac{k_{34}+s}{\mu}\right)}_{\langle\varphi_{2+1}\varphi_3\varphi_4\rangle}+e_3\underbrace{\log\left(\frac{k_{24}+u}{\mu}\right)}_{\langle\varphi_2\varphi_{3+1}\varphi_4\rangle}+e_4\underbrace{\log\left(\frac{k_{23}+t}{\mu}\right)}_{\langle\varphi_2\varphi_3\varphi_{4+1}\rangle}\right]$$

[see Austin's talk]

JOOO: charge conservation

We thus find that the s-channel correlator itself is inconsistent.



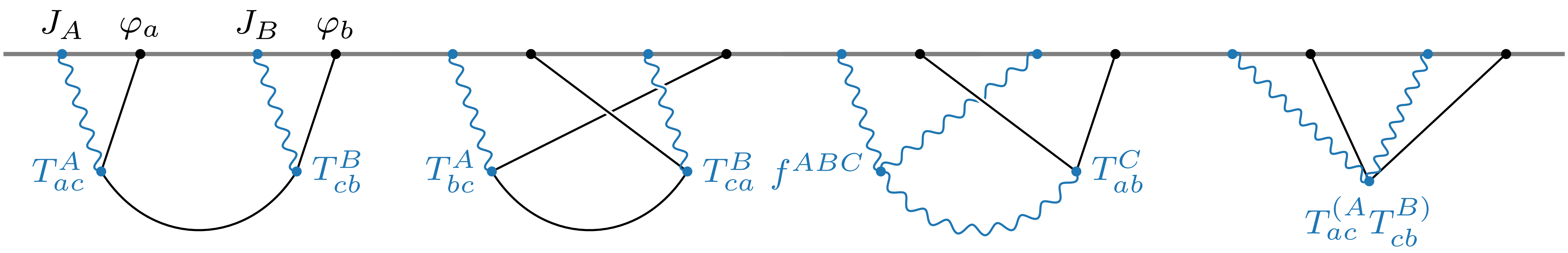
To conform with the conformal WT identity, we must add multiple channels with charges satisfying

$$k \cdot \langle J\varphi\varphi\varphi\rangle \sim \sum \langle \varphi\varphi\varphi\rangle \qquad \Rightarrow \qquad e_2 + e_3 + e_4 = 0$$

"charge conservation"

JOJO: Yang-Mills

Now, consider the non-Abelian Compton correlator:



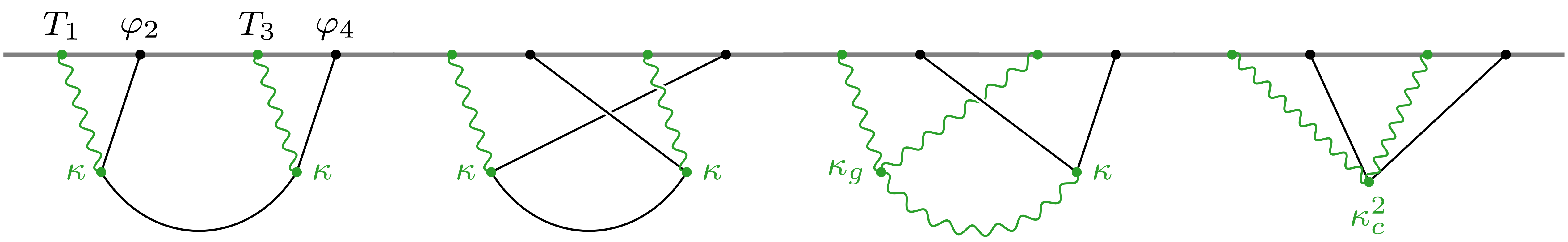
To conform with the conformal WT identity, we must add multiple channels and a contact diagram with couplings satisfying

$$k \cdot \langle J\varphi J\varphi \rangle \sim \sum \langle \varphi J \varphi \rangle \qquad \Rightarrow \qquad [T^a, T^b]_{ab} = f^{ABC} T^C_{ab}$$

"Lie algebra"

TOTO: equivalence principle

Similar result holds for the graviton Compton correlator:



To conform with the conformal WT identity, we must add multiple channels and a contact diagram with gravitational charges satisfying

$$k \cdot \langle T\varphi T\varphi \rangle \sim \sum \langle \varphi T \varphi \rangle + \langle \varphi\varphi \rangle \qquad \Rightarrow \qquad \kappa = \kappa_g = \kappa_c$$

"equivalence principle"

TOTO: more on-shell way

Consider taking the limits $E, k_1, k_3 \to 0$ simultaneously:

$$E^3\langle T^-\varphi T^+\varphi\rangle_s \xrightarrow{E,k_1,k_3\to 0} -\kappa^2\frac{S^3}{k_1k_3}$$

$$E^3\langle T^-\varphi T^+\varphi\rangle_t \xrightarrow{\qquad\qquad} -\kappa^2\frac{T^3}{k_1k_3}$$

$$E^3\langle T^-\varphi T^+\varphi\rangle_u \xrightarrow{\qquad\qquad} -\kappa\kappa_g\frac{U(6S^2+6SU+U^2)}{6k_1k_3}$$

$$E^3\langle T^-\varphi T^+\varphi\rangle_c \xrightarrow{\qquad\qquad} \kappa_c^2\frac{U(12ST-5U^2)}{6k_1k_3}$$

The Lorentz-violating poles vanish if the couplings are taken to be the same:

$$E^3\langle T^-\varphi T^+\varphi\rangle_{s+t+u+c} \xrightarrow[\kappa=\kappa_g=\kappa_c]{E,k_1,k_3\to 0} -\kappa^2\frac{S(T+U)(S+T+U)}{k_1k_3}=0$$

A no-go example

Consider coupling a massless spin-ℓ particle to three scalars in flat space:

$$\langle J^{(\ell)}\varphi\varphi\varphi\rangle = \frac{\langle 12\rangle^{\ell}\langle 1\bar{2}\rangle^{\ell}}{k_1^{\ell}}\frac{1}{E E_L E_R}$$

Adding other channels and taking the $E, k_1 \to 0$ limit, we find

$$\langle J^{(\ell)}\varphi\varphi\varphi\rangle_{s+t+u} \xrightarrow{E,k_1\to 0} \frac{1}{k_1^{\ell}E}\left(g_2 S^{\ell-1} + g_4 T^{\ell-1} + g_3 U^{\ell-1}\right)$$

- $\ell = 1$: charge conservation
- $\ell = 2$: equivalence principle
- $\ell > 2$: no matter couplings with massless higher-spins

IV. Outlook

Solving on-shell constraints

Can we derive conformally-invariant cosmological correlators with a minimal assumption on their singularity structure?

Task: Solve the conformal WT identity with the following *ansatze*:

$$\langle J_\ell^- J_\ell^- J_\ell^+ \rangle = \langle 12\rangle^\ell \langle 2\overline{3}\rangle^\ell \langle \overline{3}1\rangle^\ell \sum_{\boldsymbol{m},n}^{N_3(\ell)} c_{\boldsymbol{m}n} \frac{k_1^{m_1} k_2^{m_2} k_3^{m_4}}{E^n}$$

$$\langle J_\ell^- J_\ell^- J_\ell^+ J_\ell^+ \rangle = \langle 12\rangle^{2\ell} \langle \overline{34}\rangle^{2\ell} \sum_{\boldsymbol{m},\boldsymbol{n}}^{N_4(\ell)} d_{\boldsymbol{m}\boldsymbol{n}} \frac{k_1^{m_1} \cdots k_4^{m_4} s^{m_s} t^{m_t} u^{m_u}}{E^{n_1} \prod_{a=\{s,t,u\}} E_{a,L}^{n_{a,2}} E_{a,R}^{n_{a,3}}} + \cdots$$

Solvable at 3 points, more nontrivial at 4 points.

Boels, Medina [2016]
Arkani-Hamed, Rodina, Trnka [2016]
[work in progress]

Codifying time evolution

What boundary equation governs time evolution when there is no conformal symmetry?

Flat-space correlators have simpler structures than their dS counterparts, with the two related by integro-differential equations; e.g.

$$\langle \varphi\varphi\varphi\varphi \rangle_{\mathrm{dS}} = \int \mathrm{d}k_{12}\, \mathrm{d}k_{34}\, \langle \varphi\varphi\varphi\varphi \rangle_{\mathrm{flat}}$$

$$\langle J_\ell \varphi\varphi\varphi \rangle_{\mathrm{dS}} = \prod_{j=1}^{\ell} \Big[(2j-1) - k_1 \partial_{k_1} \Big] \int \mathrm{d}k_{34}\, \langle J_\ell \varphi\varphi\varphi \rangle_{\mathrm{flat}}$$

Can we “boost-strap” correlators in flat space?

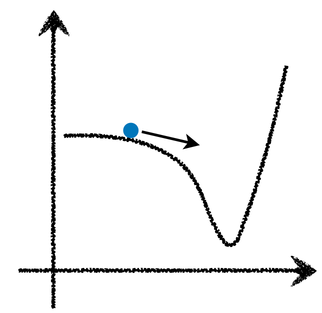
[work in progress]

Open problems

- What is the "Parke-Taylor correlator"?
- On-shell recursion for spinning correlators
- More four-particle tests in dS
- Conformally-invariant building blocks [Paul, Kostas's talks]
- Analytic structure beyond trees [Aaron, Dan, Matthew, Victor's talks]
- Implications of bulk unitarity [Harry, Enrico's talks]
- Bulk toolkits (Mellin/frequency space) [Massimo's talk]
- Geometric/combinatoric structure [Paolo's talk]

Thank you for listening!

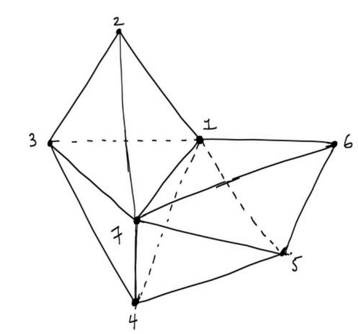
Inflation



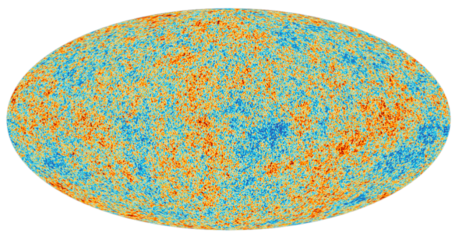
(A)dS/CFT

Cosmological bootstrap

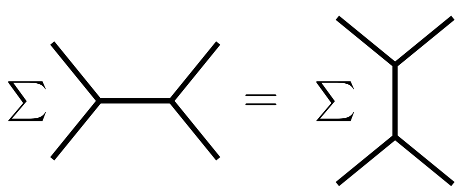
Scattering amplitudes



Observational cosmology



Conformal bootstrap



Back-up

Cosmological spinor-helicity formalism

As in flat space, we can complexity momenta and use spinor representations.

$$k^\mu = (k, \vec{k}) \qquad \Rightarrow \qquad \lambda^\alpha \bar{\lambda}_\beta = k_i (\sigma^i)^\alpha{}_\beta + k \mathbb{1}^\alpha{}_\beta$$

Novelty in dS: can contract between barred and un-barred spinors.

$$\text{e.g.} \quad \langle \bar{\lambda} \lambda \rangle = 2k$$

In spinor-helicity variables, the conformal generator is

$$\boxed{\widetilde{K}^i = 2(\sigma^i)^\alpha{}_\beta \frac{\partial^2}{\partial \lambda^\alpha \partial \bar{\lambda}_\beta}}$$

Maldacena, Pimentel [2011]
Witten [2003]

Cosmological spinor-helicity formalism

The conformal generator acts nicely on operators with $\Delta = 2$:

$$\widetilde{K}^i \varphi = -K^i \varphi$$

$$\widetilde{K}^i J^\pm = -K^i J^\pm + \frac{2}{k^2}\xi^i_\pm (k \cdot J) \qquad (J^\pm = \xi^\pm \cdot J)$$

$$\widetilde{K}^i \widehat{T}^\pm = \underbrace{-K^i \widehat{T}^\pm}_{=0} + \frac{12}{k^2}\xi^i_\pm \underbrace{(k \cdot \widehat{T} \cdot \xi_\pm)}_{\text{divergence}} \qquad (k\widehat{T}^\pm = \xi^\pm \cdot T \cdot \xi^\pm)$$

Thanks to this very useful property, it is possible to check the WT identity

with just keeping the transverse part of correlators.

Maldacena, Pimentel [2011]
Mata, Raju, Trivedi [2012]
Baumann, Duaso Pueyo, Joyce, HL, Pimentel [2020]