Steinmann Relations, Causality and the Wavefunction of the Universe

Paolo Benincasa

Niels Bohr International Academy
&
Instituto de Física Teórica UAM/CSIC Madrid

8 September 2020 – Cosmological Correlators Workshop

based on:

P. B., A. J. McLeod, C. Vergu, 2009.03047

P.B., A. Di Tucci, C. Vergu, work in progress

N. Arkani-Hamed, **P.B.**, 1811.01125

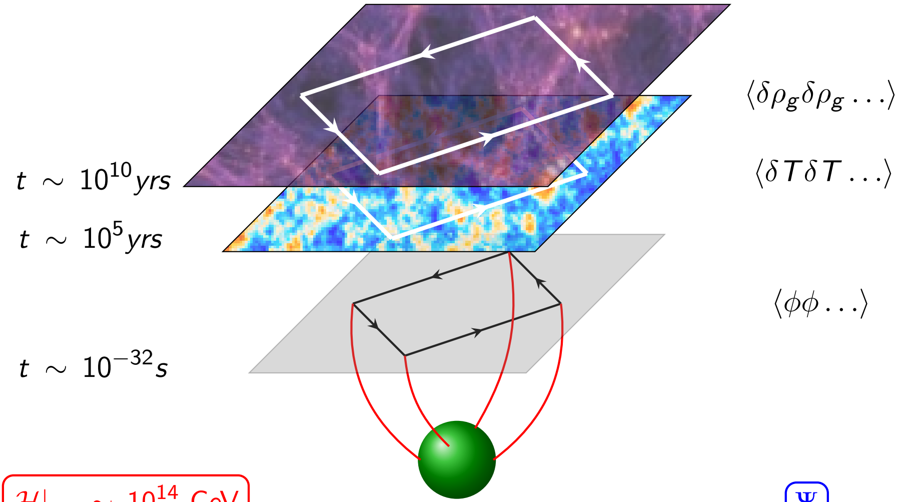
Other relevant works:

- N. Arkani-Hamed, **P. B.**, A. Postnikov, 1709.02813
- **P. B.**, 1811.02515, 1909.02517
- **P. B.**, M. Parisi, 2005.03612
- **P. B.**, A. Di Tucci, work in progress

Instituto de Física Teórica UAM-CSIC

The Niels Bohr International Academy

Observables and First Principles



$\mathcal{H}|_{\text{infl.}} \sim 10^{14}$ GeV

Ψ

Observables and First Principles

Flat space: *What are the rules governing physical processes?*

Lorentz Invariance

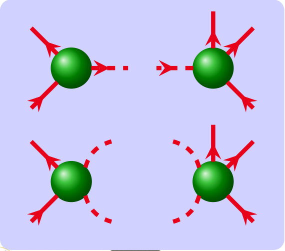
$$\mathcal{M}_n = \mathcal{M}_n(s_{i_1 \dots i_k})$$

Causality

Perturbative

Sufficient Analiticity: Poles, Branch cuts

Unitarity



- Tree-level & 1-loop fixed
- Higher loops: no complete set of rules

Instituto de Física Teórica UAM-CSIC

The Niels Bohr International Academy

Observables and First Principles

Flat space: *What are the rules governing physical processes?*

Lorentz Invariance | Causality | Unitarity

- 3-particle amplitudes fixed.
- Yang's theorem; Weinberg-Witten theorem.
- Charge conservation (spin 1); Equivalence principle (spin 2).
- Consistent interactions involve spins: 0, 1/2, 1, 3/2, 2.
- Spin-1 self-interactions just for different species.
- Graviton uniqueness theorem; $\mathcal{N} = 1$ Sugra
- Spin > 2: No self-interactions; no interactions with $s \leq 2$;
  No elementary massive particles

[Weinberg, 64; **P. B.**, Cachazo, 07; **P. B.**, Conde, 11; McGady, Rodina 13; Arkani-Hamed, Huang, Huang, 17]

Observables and First Principles

Cosmology: *What are the rules governing physical processes?*

Causality

Unitarity

What's their imprint and their consequences?

Observables and First Principles

Cosmology: *What are the rules governing physical processes?*

Causality

Unitarity

What's their imprint and their consequences? – First ε steps

- Cosmological Optical Theorem [Goodhew, Jazayeri, Pajer, 20]
- Steinmann-like relations [P. B., McLeod, Vergu, 20] [P. B., Di Tucci, Vergu, w.i.p.]
- Bootstrap programs [Arkani-Hamed, Baumann, Lee, Pimentel, 18] [Baumann, Duaso Pueyo, Joyce, Lee, Pimentel, 19-20] [Sleight, Taronna, 18-20]; [Pajer, Stefanyszyn, Supel, 20] [P. B., Di Tucci, w.i.p.]

Flat-Space Causality and Steinmann Relations

1. General notion of causality: $[\phi(x_1), \phi(x_2)] = 0$ for $(x - x_1)^2 > 0$

2. Retarded products: [Lehmann, Symanzik, Zimmerman, 1957]

$$R(x;x_1) = -i\vartheta(x^0 - x_1^0)[\phi(x), \phi(x_1)]$$

$$R(x;x_1,\ldots,x_n) = (-i)^n \sum_{\pi\in\mathcal{S}_n} \vartheta(x^0 - x^0_{\pi(1)})\cdots\vartheta(x^0_{\pi(n-1)} - x^0_{\pi(n)})[[\cdots[\phi(x),\phi(x_{\sigma(1)})],\cdots],\phi(x_{\sigma(n)})]$$

3. Generalised Retarded Functions: [Steinmann, 1960; Ruelle, 1961; Araki, 1961]

$$r(x,\sigma;x_1,\sigma_1,\ldots,x_n,\sigma_n) = \langle 0|R(x,\sigma;x_1,\sigma_1,\ldots,x_n,\sigma_n)|0\rangle$$

4. (Generalised) Steinmann Relations: [Steinmann, 1960; Ruelle, 1961; Araki, 1961]

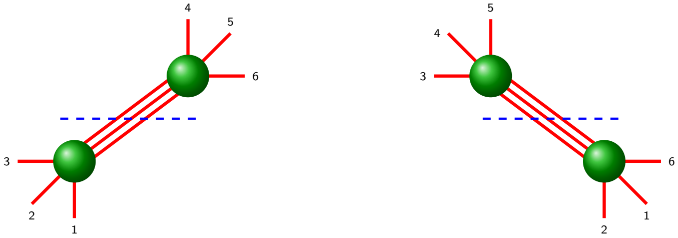
E.g.: $r_{jn} - a_{ik} = a_{kl} - r_{nj}$, $\left\{ \begin{array}{l} r_{jn} = r(x_j,+;,x_k.-;x_l,-;x_n,+) \\ a_{jn} = r(x_j,-;,x_k.+;x_l,+;x_n,-) \end{array} \right.$, $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ j & k & l & n \end{pmatrix}$

Flat-Space Causality and Steinmann Relations: S-matrix

5. Double discontinuities on partially overlapping channels vanish

[Stapp, 71; Cahill, Stapp, 73,75; Lassalle, 74]

$$\text{Disc}_{s_{\mathcal{I}}}\left(\text{Disc}_{s_{\mathcal{J}}}\mathcal{A}\right) = 0, \qquad \begin{cases} \mathcal{I} \not\subseteq \mathcal{J} \\ \mathcal{J} \not\subseteq \mathcal{I} \\ \mathcal{I} \cap \mathcal{J} \neq \varnothing \end{cases}$$



- Steinmann relations constrain the analytic structure of scattering amplitudes
- Used to bootstrap amplitudes and integrals in planar $\mathcal{N} = 4$ SYM

[Caron-Huot, Dixon, Drummond, Dulat, Foster, Grudogan, Harrington, von Hippel, McLeod, Papathanasiou, Spradlin, 16-20]

- Also valid for individual Feynmann graphs

[Bourjaily, Hannesdottir, McLeod, Schwartz, Vergu, 20]

Questions

Big question:

What are the invariant properties that Ψ ought to satisfy in order to come from a consistent causal evolution in cosmological space-times?

Smaller questions:

1. Can we see flat-space causality emerging from the wavefunction?

2. Do similar constraints hold for the wavefunction?

Questions

Big question:

What are the invariant properties that Ψ ought to satisfy in order to come from a consistent causal evolution in cosmological space-times?

Smaller questions:

1. Can we see flat-space causality emerging from the wavefunction?

$$\text{Res}_{\mathcal{W}_{\mathfrak{g}_1}}\left(\text{Res}_{\mathcal{W}_{\mathfrak{g}_2}}\Omega(\mathcal{S}_{\mathcal{G}})\right) = 0 \quad \Longrightarrow \quad \text{Disc}_{s_{\mathfrak{g}_1}}\left(\text{Disc}_{s_{\mathfrak{g}_2}}\mathcal{A}_{\mathcal{G}}\right) = 0$$

2. Do similar constraints hold for the wavefunction?

$$\text{Res}_{\mathcal{W}_{\mathfrak{g}_1}}\left(\text{Res}_{\mathcal{W}_{\mathfrak{g}_2}}\Omega(\mathcal{P}_{\mathcal{G}})\right) = 0$$

Questions

Big question:

What are the invariant properties that Ψ ought to satisfy in order to come from a consistent causal evolution in cosmological space-times?

Smaller questions:

1. Can we see flat-space causality emerging from the wavefunction?

$$\text{Res}_{\mathcal{W}_{\mathfrak{g}_1}}\left(\text{Res}_{\mathcal{W}_{\mathfrak{g}_2}}\Omega(\mathcal{S}_{\mathcal{G}})\right) = 0 \quad \Longrightarrow \quad \text{Disc}_{s_{\mathfrak{g}_1}}\left(\text{Disc}_{s_{\mathfrak{g}_2}}\mathcal{A}_{\mathcal{G}}\right) = 0$$

2. Do similar constraints hold for the wavefunction?

$$\text{Res}_{E_{\mathfrak{g}_1}}\left(\text{Res}_{E_{\mathfrak{g}_2}}\psi_{\mathcal{G}}\right) = 0 \quad \Longrightarrow \quad \text{Disc}_{s_{\mathfrak{g}_1}}\left(\text{Disc}_{s_{\mathfrak{g}_2}}\Psi_{\mathcal{G}}^{\text{tree}}\right) = 0$$

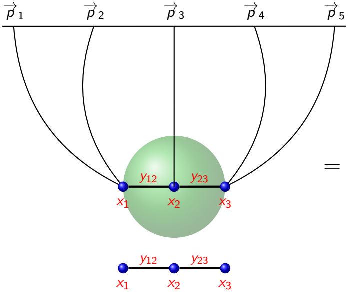
Combinatorics and the Wavefunction: the Cosmological Polytopes

- Mathematical objects with their own first-principles definition (no reference to Hilbert space and space-time)
- Any question about the (pertubative) wavefunction becomes a combinatoric-geometrical question

A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]; [P.B., 19]

$$S[\phi] = \int_{-\infty}^{0} d\eta \int d^d x \left[\frac{1}{2}(\partial\phi)^2 - \sum_{k\geq 3}\frac{\lambda(\eta)}{k!}\phi^k\right], \qquad \lambda(\eta) = \int_{-\infty}^{+\infty} d\varepsilon\, e^{i\varepsilon\eta}\tilde{\lambda}_k(\varepsilon)$$

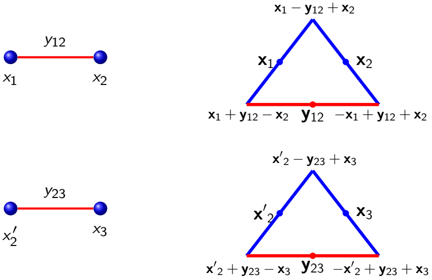


$$= \prod_{v\in\mathcal{V}}\left[\int_{X_v}^{+\infty} dx_v\,\tilde{\lambda}(x_v)\right]\underbrace{\int_{-\infty}^{0}\prod_{v\in\mathcal{V}}\left[d\eta_v\, e^{ix_v\eta_v}\right]\prod_{e\in\mathcal{E}} G(y_e,\eta_{v_e},\eta_{v'_e})}_{\psi(x_v,\,y_e)}$$

$$G(\eta_i,\eta_j) = \frac{1}{2y_e}\Big[\underbrace{e^{-iy_e(\eta_{v_e}-\eta_{v'_e})}\vartheta(\eta_{v_e}-\eta_{v'_e}) + e^{iy_e(\eta_{v_e}-\eta_{v'_e})}\vartheta(\eta_{v'_e}-\eta_{v_e})}_{\text{Feynman propagator}}\overbrace{-e^{iy_e(\eta_{v_e}+\eta_{v'_e})}}^{\text{b.c.}}\Big]$$

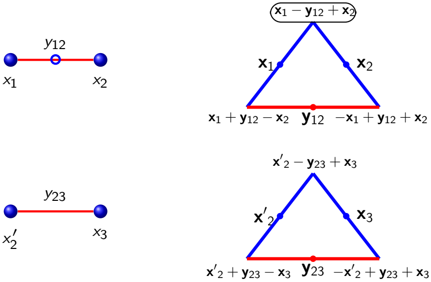
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]



$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v + n_e - 1}$$

A Combinatorial Description of the Wavefunction

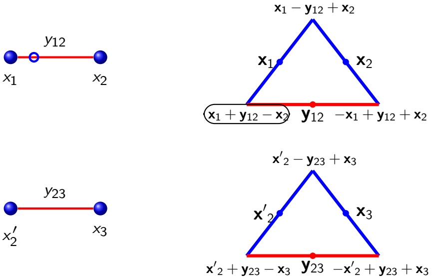


[N. Arkani-Hamed, P.B., A. Postnikov, 17]

$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v + n_e - 1}$$

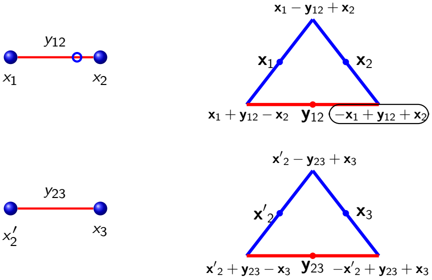
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]



$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

A Combinatorial Description of the Wavefunction



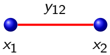
[N. Arkani-Hamed, P.B., A. Postnikov, 17]

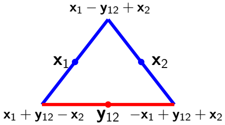
$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

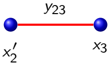
A Combinatorial Description of the Wavefunction

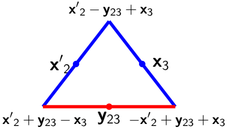
[N. Arkani-Hamed, P.B., A. Postnikov, 17]

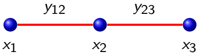
$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

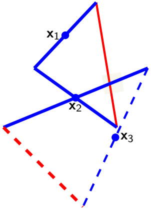




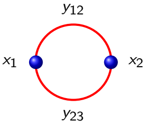




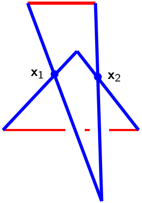




$$\mathbf{x}_2 = \mathbf{x}_2'$$



$$\mathbf{x}_2 = \mathbf{x}_2'$$
$$\mathbf{x}_1 = \mathbf{x}_3$$

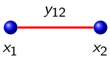


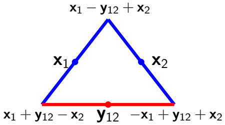
Instituto de Física Teórica UAM-CSIC

The Niels Bohr International Academy

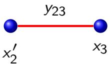
A Combinatorial Description of the Wavefunction

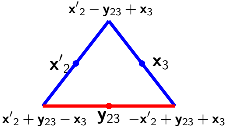
[N. Arkani-Hamed, P.B., A. Postnikov, 17]





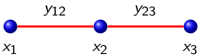
$$\mathcal{Y} = \sum_v x_v \mathbf{X}_v + \sum_e y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

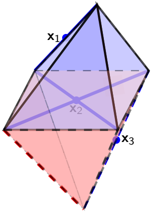


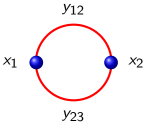


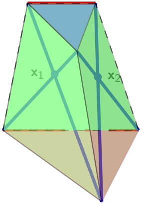
$$\psi \equiv \Omega = \frac{\langle 123\rangle^2}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}23\rangle} = \frac{1}{(x_1+x_2)(x_1+y)(y+x_2)}$$

meromorphic function with single poles only,
with residues corresponding to codimension-1 boundaries









$$\mathbf{x}_2 = \mathbf{x}'_2$$

$$\mathbf{x}_2 = \mathbf{x}'_2$$
$$\mathbf{x}_1 = \mathbf{x}_3$$

Instituto de Física Teórica UAM-CSIC

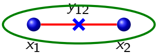
The Niels Bohr International Academy

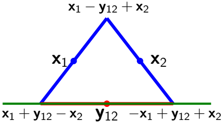
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]

$$\mathcal{Y} = \sum_{v} x_v \mathbf{X}_v + \sum_{e} y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

$$x_1 + x_2 = 0$$





$$\psi \equiv \Omega = \frac{\langle 123\rangle^2}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}23\rangle} = \frac{1}{(x_1+x_2)(x_1+y)(y+x_2)}$$

meromorphic function with single poles only,
with residues corresponding to codimension-1 boundaries

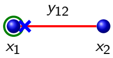
$$\Omega(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}}) \equiv \text{Res}_{\mathcal{W}_{\mathcal{G}}} \Omega(\mathcal{P}_{\mathcal{G}}) = \left.\frac{1}{(x_1+y)(y+x_2)}\right|_{x_1+x_2=0}$$

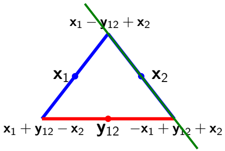
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]

$$\mathcal{Y} \,=\, \sum_{v} x_v \mathbf{X}_v + \sum_{e} y_e \mathbf{Y}_e \,\in\, \mathbb{P}^{n_v+n_e-1}$$

$$x_1 + y \,=\, 0$$





$$\psi \,\equiv\, \Omega \,=\, \frac{\langle 123\rangle^2}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}23\rangle} \,=\, \frac{1}{(x_1+x_2)(x_1+y)(y+x_2)}$$

meromorphic function with single poles only,
with residues corresponding to codimension-1 boundaries

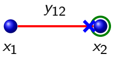
$$\Omega(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1}) \,\equiv\, \text{Res}_{\mathcal{W}_{\mathfrak{g}_1}} \Omega(\mathcal{P}_{\mathcal{G}}) \,=\, \left.\frac{1}{(x_1+x_2)(y+x_2)}\right|_{x_1+y=0}$$

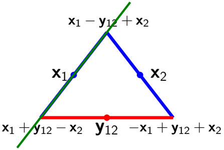
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]

$$\mathcal{Y} = \sum_{v} x_v \mathbf{X}_v + \sum_{e} y_e \mathbf{Y}_e \in \mathbb{P}^{n_v+n_e-1}$$

$$y + x_2 = 0$$





$$\psi \equiv \Omega = \frac{\langle 123\rangle^2}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}23\rangle} = \frac{1}{(x_1+x_2)(x_1+y)(y+x_2)}$$

meromorphic function with single poles only,
with residues corresponding to codimension-1 boundaries

$$\Omega(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_2}) \equiv \text{Res}_{\mathcal{W}_{\mathfrak{g}_2}} \Omega(\mathcal{P}_{\mathcal{G}}) = \left.\frac{1}{(x_1+x_2)(x_1+y)}\right|_{y+x_2=0}$$

Instituto de Física Teórica UAM-CSIC

The Niels Bohr International Academy

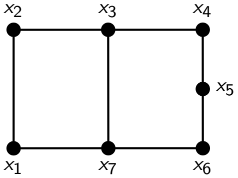
A Combinatorial Description of the Wavefunction

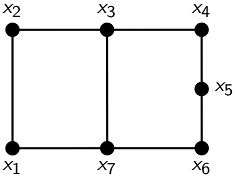
[N. Arkani-Hamed, P.B., A. Postnikov, 17]

Residues of $\psi \Longleftrightarrow$ Facets $\Longleftrightarrow$ Connected subgraphs $\mathfrak{g} \subseteq \mathcal{G}$

Facet: $\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-2}$

1. Marking all the internal edges of $\mathfrak{g}$ in the middle
2. Marking all the edge departing from $\mathfrak{g}$ close to $\mathfrak{g}$





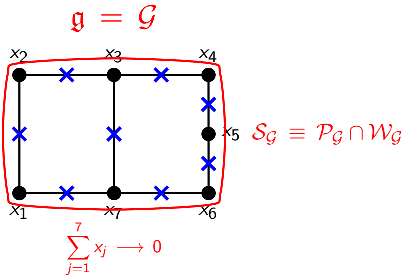
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]

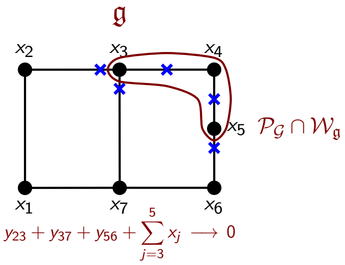
Residues of $\psi \Longleftrightarrow$ Facets $\Longleftrightarrow$ Connected subgraphs $\mathfrak{g} \subseteq \mathcal{G}$

Facet: $\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-2}$

1. Marking all the internal edges of $\mathfrak{g}$ in the middle
2. Marking all the edge departing from $\mathfrak{g}$ close to $\mathfrak{g}$



Scattering Facet



Instituto de Física Teórica UAM-CSIC

The Niels Bohr International Academy

A Combinatorial Description of the Wavefunction

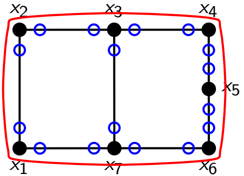
[N. Arkani-Hamed, P.B., A. Postnikov, 17]

Residues of $\psi \iff$ Facets $\iff$ Connected subgraphs $\mathfrak{g} \subseteq \mathcal{G}$

Facet: $\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-2}$

1. Marking all the internal edges of $\mathfrak{g}$ in the middle
2. Marking all the edge departing from $\mathfrak{g}$ close to $\mathfrak{g}$

$\mathfrak{g} = \mathcal{G}$

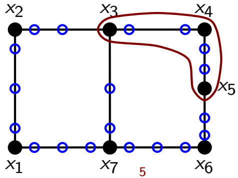


$\mathcal{S}_{\mathcal{G}} \equiv \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}}$

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

Scattering Facet

$\mathfrak{g}$



$\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}}$

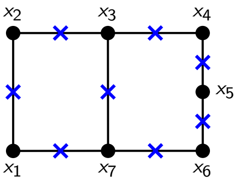
$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$

Emergence of Flat-Space Unitarity and Causality

Emergence of Flat-Space Unitarity

Scattering Facet: $\mathcal{S}_{\mathcal{G}} \equiv \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}} \subset \mathbb{P}^{n_v+n_e-2}$

[N. Arkani-Hamed, **P.B.**, 18]



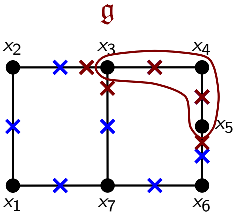
Emergence of Flat-Space Unitarity

Scattering Facet: $\mathcal{S}_{\mathcal{G}} \equiv \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}} \subset \mathbb{P}^{n_v+n_e-2}$

[N. Arkani-Hamed, P.B., 18]

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$



Lower-dimensional Face:
$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-3}$

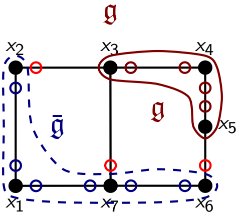
Emergence of Flat-Space Unitarity

Scattering Facet: $\mathcal{S}_{\mathcal{G}} \equiv \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}} \subset \mathbb{P}^{n_v+n_e-2}$

[N. Arkani-Hamed, P.B., 18]

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$



Lower-dimensional Face:
$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-3}$

$$\Omega(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}}) = \Omega(\Sigma_{\not{\mathcal{E}}}) \times \Omega(\mathcal{S}_{\mathfrak{g}}) \times \Omega(\mathcal{S}_{\bar{\mathfrak{g}}}) = \left(\prod_{e \in \not{\mathcal{E}}} \frac{1}{2y_e}\right) \times \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}]$$

cutting rules!

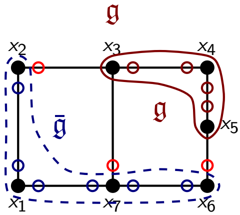
Emergence of Flat-Space Unitarity

Scattering Facet: $\mathcal{S}_{\mathcal{G}} \equiv \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathcal{G}} \subset \mathbb{P}^{n_v+n_e-2}$

[N. Arkani-Hamed, P.B., 18]

$$\sum_{j=1}^{7} x_j \longrightarrow 0$$

$$y_{23} + y_{37} + y_{56} + \sum_{j=3}^{5} x_j \longrightarrow 0$$



Lower-dimensional Face:
$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}} \subset \mathbb{P}^{n_v+n_e-3}$

$$\Omega(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}}) = \Omega(\Sigma_{\not{\mathcal{E}}}) \times \Omega(\mathcal{S}_{\mathfrak{g}}) \times \Omega(\mathcal{S}_{\bar{\mathfrak{g}}}) = \left(\prod_{e\in\not{\mathcal{E}}} \frac{1}{2y_e}\right) \times \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}]$$

cutting rules!

Measure for the Lorentz Invariant Phase Space

Emergence of Flat-Space Causality

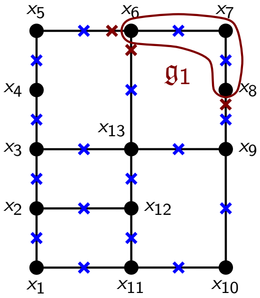
Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

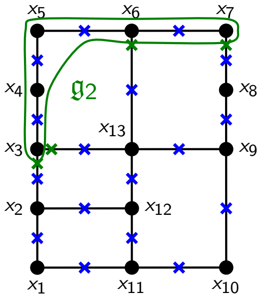
Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

Subgraphs $\mathfrak{g}_1$, $\mathfrak{g}_2$	$\mathfrak{g}_1 \cap \mathfrak{g}_2 \neq \varnothing$	$\mathfrak{g}_1 \cap \bar{\mathfrak{g}}_2 \neq \varnothing$
	$\bar{\mathfrak{g}}_1 \cap \mathfrak{g}_2 \neq \varnothing$	$\bar{\mathfrak{g}}_1 \cap \bar{\mathfrak{g}}_2 \neq \varnothing$



$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1}$

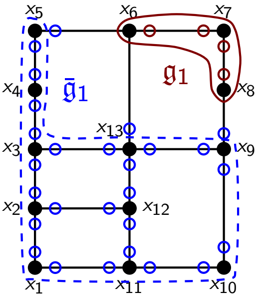


$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_2}$

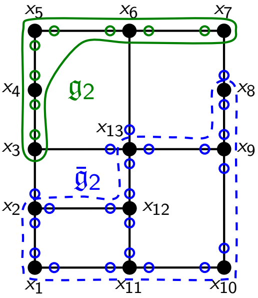
Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

Subgraphs $\mathfrak{g}_1,\ \mathfrak{g}_2$ | $\mathfrak{g}_1 \cap \mathfrak{g}_2 \neq \varnothing \quad \mathfrak{g}_1 \cap \bar{\mathfrak{g}}_2 \neq \varnothing$; $\bar{\mathfrak{g}}_1 \cap \mathfrak{g}_2 \neq \varnothing \quad \bar{\mathfrak{g}}_1 \cap \bar{\mathfrak{g}}_2 \neq \varnothing$



$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1}$



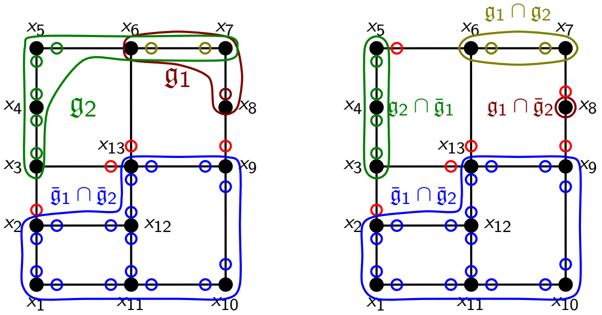
$\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_2}$

Emergence of Flat-Space Causality

Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$



Emergence of Flat-Space Causality

Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

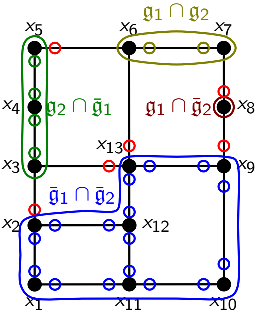
[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

If $\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} \neq \varnothing$ then:

1. $\dim(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 4,$
2. Factorisation: $\Omega = \Omega(\Sigma_{\not{g}}) \times \prod_{\mathfrak{g}} \Omega(\mathcal{S}_{\mathfrak{g}})$

$$\dim(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 1 - \sum_{\mathcal{S}_{\mathfrak{g}}} 1 = n_v + n_e - 5$$



Emergence of Flat-Space Causality

Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

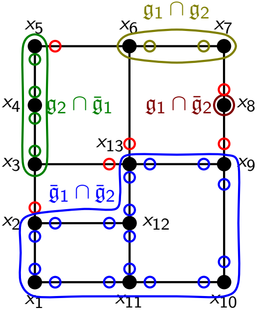
[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

If $\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} \neq \varnothing$ then:

1. $\dim(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 4,$
2. Factorisation: $\Omega = \Omega(\Sigma_{\not{\mathfrak{g}}}) \times \prod_{\mathfrak{g}} \Omega(\mathcal{S}_{\mathfrak{g}})$

$$\dim(\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 1 - \sum_{\mathcal{S}_{\mathfrak{g}}} 1$$
$$= n_v + n_e - 5$$



Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

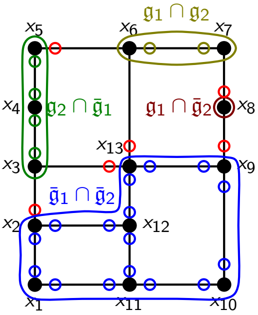
[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

Why don't contribution come from higher codimension intersections?

1. - Zeros of Ω
   - Locus of the intersections of the facets outside $\mathcal{P}_{\mathcal{G}}$

2. These higher codim intersections lay on this locus



Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

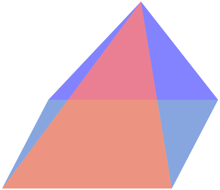
[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

Why don't contribution come from higher codimension intersections?

1. $\begin{cases} \bullet \text{ Zeros of } \Omega \\ \bullet \text{ Locus of the intersections of the facets outside } \mathcal{P}_{\mathcal{G}} \end{cases}$

2. These higher codim intersections lay on this locus



Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{\mathcal{W}_{\mathfrak{g}_1}} \text{Res}_{\mathcal{W}_{\mathfrak{g}_2}} \Omega(\mathcal{S}_{\mathcal{G}}) = 0$$

Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{s_{\mathfrak{g}_1}} \text{Res}_{s_{\mathfrak{g}_2}} \mathcal{A}(\mathcal{G}) = 0$$

Emergence of Flat-Space Causality

Vanishing of the double discontinuities along partially-overlapping channels (Steinmann Relations)

[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-4} \supset \mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{s_{\mathfrak{g}_1}} \text{Res}_{s_{\mathfrak{g}_2}} \mathcal{A}(\mathcal{G}) = 0$$

$$\Downarrow$$

$$\text{Disc}_{s_{\mathfrak{g}_1}} \text{Disc}_{s_{\mathfrak{g}_2}} \mathcal{A}(\mathcal{G}) = 0$$

What about the Wavefunction?

[P.B., A. J. McLeod, C. Vergu, 20]

What about the wavefunction?
Let's ask the same question!

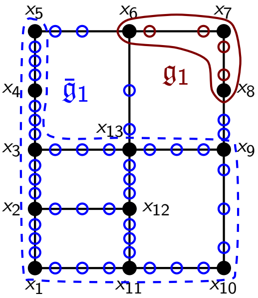
$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

Steinmann-like Relations and the Wavefunctions

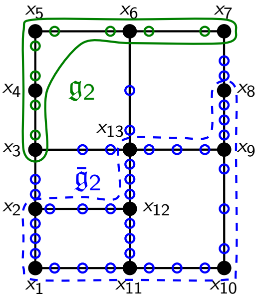
[P.B., A. J. McLeod, C. Vergu, 20]

What about the wavefunction?
Let's ask the same question!

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$



$\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1}$



$\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_2}$

What about the wavefunction?
Let's ask the same question!

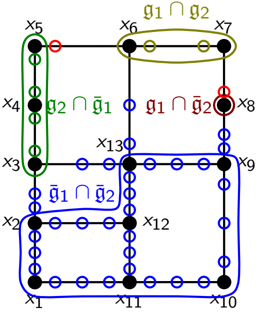
[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

If $\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} \neq \varnothing$ then:

1. $\dim(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 3$,
2. Factorisation: $\Omega = \Omega(\Sigma_{\mathcal{G}}) \times \prod_{\mathfrak{g}} \Omega(\mathcal{S}_{\mathfrak{g}})$

$$\dim(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 1 - \sum_{\mathcal{S}_{\mathfrak{g}}} 1 = n_v + n_e - 4$$



$\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}$

Steinmann-like Relations and the Wavefunctions

What about the wavefunction?
Let's ask the same question!

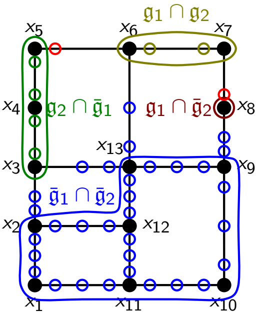
[**P.B.**, A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

If $\mathcal{S}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} \neq \varnothing$ then:

1. $\dim(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 3$,
2. Factorisation: $\Omega = \Omega(\Sigma_{\mathscr{G}}) \times \prod_{\mathfrak{g}} \Omega(\mathcal{S}_{\mathfrak{g}})$

$$\dim(\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}) = n_v + n_e - 1 - \sum_{\mathcal{S}_{\mathfrak{g}}} 1$$
$$= n_v + n_e - 4$$



$\mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2}$

Steinmann-like Relations and the Wavefunctions

What about the wavefunction?
Let's ask the same question!

[P.B., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{\mathcal{W}_{\mathfrak{g}_1}} \text{Res}_{\mathcal{W}_{\mathfrak{g}_2}} \Omega(\mathcal{P}_{\mathcal{G}}) = 0$$

Steinmann-like Relations and the Wavefunctions

[P.B., A. J. McLeod, C. Vergu, 20]

What about the wavefunction?
Let's ask the same question!

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{E_{\mathfrak{g}_1}} \text{Res}_{E_{\mathfrak{g}_2}} \psi_{\mathcal{G}} = 0$$

What about the wavefunction?
Let's ask the same question!

[𝔓.𝔅., A. J. McLeod, C. Vergu, 20]

$$\mathbb{P}^{n_v+n_e-3} \supset \mathcal{P}_{\mathcal{G}} \cap \mathcal{W}_{\mathfrak{g}_1} \cap \mathcal{W}_{\mathfrak{g}_2} = \varnothing$$

$$\text{Res}_{E_{\mathfrak{g}_1}} \text{Res}_{E_{\mathfrak{g}_2}} \psi_{\mathcal{G}} = 0$$

$$E_{\mathfrak{g}} = \sum_{v\in\mathfrak{g}} x_v + \sum_{e\in\mathcal{E}^{\text{ext}}_{\mathfrak{g}}} y_e$$

$$\text{Disc}_{E_{\mathfrak{g}_1}} \text{Disc}_{E_{\mathfrak{g}_2}} \Psi^{\text{tree}}_{\mathcal{G}} = 0$$

1. The double residues of the universal integrand $\psi_{\mathcal{G}}$ vanish $\forall\, \mathcal{G}$

2. The double discontinuities of the tree-level wavefunction $\Psi^{\text{tree}}_{\mathcal{G}}$ vanish ($\Psi^{\text{tree}}_{\mathcal{G}}$ is given in terms of polylogs)

[Arkani-Hamed, 𝔓.𝔅., Postnikov, 17; Hillman, 19]

Conclusion

- Steinmann relations for amplitudes are implied by causality.
- They arise in a simple way from the combinatorics of the scattering facet of the cosmological polytope.

- Steinmann-like constraints for the wavefunction.
- They arise via the same mechanism as the flat-space ones.

- The analysis holds for a large class of models in FRW cosmologies.

- A final proof of the relation between these new constraints on the wavefunction and causality yet to be established [P. B., A. Di Tucci, C. Vergu; work-in-progress]

  Obs: Perturbative integrands of in-in correlators as R-products