Letter of Interest to be submitted to AF4: Multi-TeV Collider
Subject: Application of Vertical Excursion FFAs (vFFA) and Novel Optics in Muon Collider Accelerator Complex
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Acceleration and accumulation of the muon beams, whether for a neutrino physics facility or a multi-TeV muon collider, is a big challenge from machine design point of view. The life time of muons in the rest frame is only 2.2 micro second. Acceleration has to be as quick as possible to prevent muon decay and increase survival muons up to the final energy. As tertiary produced particles, muon beam emittance may not be as small as the conventional accelerator beams such as that of protons and electrons.

Use of Fixed Field Alternating Gradient Accelerators (FFAs) for muon acceleration had been proposed as the main accelerator for a neutrino factory since the 1990s [1-4]. When it was invented, one of big advantages of a FFA was a high repetition rate operation providing the high average beam current [5-7], that is limited in a synchrotron which requires ramping of the main lattice magnets. For the muon acceleration, short duration of an acceleration cycle rather than high repetition rate is essential.

R&Ds for the last 20 years advanced our knowledge on FFAs from beam optics to necessary hardware. New concepts of beam optics, such as a linear nonscaling FFA, a pumplet FFA, serpentine channel acceleration and a DF spiral FFA, have been proposed and some of them were demonstrated experimentally [8]. However, for the fastest possible acceleration of the muon beams, change of the revolution frequency through an acceleration cycle is a bottleneck in order to increase the momentum range from injection to extraction. In both serpentine channel acceleration and acceleration within a stationary RF bucket, the momentum ratio is about a few times at most.

A FFA with vertical orbit excursion (vFFA) was invented by Ohkawa in 1955 [9] and recently reinvented by Brooks in 2013 [10]. In a vFFA, the circular orbits moves up or down depending of the magnetic field shape when the beams are accelerated. As Ohkawa named it as ‘electron cyclotron’ in his paper, the momentum compaction factor of vFFA is zero for the entire momentum and the revolution frequency becomes independent of the particle momentum for ultra-relativistic particles. This opens up a new way of acceleration of unstable particles like muons.

For a muon collider of the final energy of 3 TeV, an order of 100 to 1000 times acceleration in terms of the momentum is required and we expect that would be done in circular ring accelerators. Although it might be inevitable to have multi-stage ring accelerators, the large momentum ratio between injection and extraction reduces the number of stages and the total cost. vFFA can use a fixed frequency RF system and this gives two major advantages. Firstly the energy gain per turn could be high because of the fixed frequency RF and the beams are always near the crest of the RF wave form. Secondly the momentum ratio is only determined by lattice magnet configuration, not by the longitudinal acceptance. If we could choose large field index and allow the reasonable orbit excursion, e.g. half a meter, the momentum ratio of 30 is achievable [11], which creates a scenario of the two stage muon accelerators from GeV to TeV.

A collider ring does not require the large momentum acceptance, but some of characteristics of vFFA is beneficial. For example, wriggling orbits both in horizontal and vertical planes help to reduce convergence of the neutrino beams to certain directions. Slight adjustments of optics will change the wriggling pattern of the orbits so that we can control the direction of the neutrino beams . The isochronous condition would keep the bunch shape fixed that is essential to maintain high luminosity.

A study has been initiated to design a collider ring extracting essential ingredients of vFFA. For example, the arc section optics comprising skew quadrupoles which is the lowest multipole of the vFFA magnets is investigated [12]. A skew quadrupole lattice does not provide isochronous condition for the wide range of momentum, but can make the zeroth order momentum compaction factor (alpha_c) zero. Unlike a common method employed in a normal quadrupole lattice to make zero alpha_c, the lattice with skew quadrupoles does not need negative bending magnets. Therefore the peak magnetic fields becomes lower or the circumference of a ring can be smaller.

Muon accumulator ring of the LEMMA scheme [13] has similar requirements as the muon collider ring in terms of optics design aiming at a compact ring and the isochronous condition for the momentum range of +/-10 to 20%. The arc with skew quadrupoles is looked at already showing a promising initial design. Both collider ring and accumulator ring have low beta insertions. It would be natural to consider the straight line optics with skew quadrupoles so that the matching between the arc and the low beta insertion could be done smoothly. Wriggling orbits in vertical as well as horizontal planes in the straight section is advantageous to enhance divergence of neutrino flux. The whole ring with skew quadrupoles could be converted to a normal quadrupole lattice when the reference coordinates are rotated by 45 degree. In that rotated frame, weak focusing at the bending of the arc becomes skew so that a correction skew quadrupole (it is a normal qudrupole in the normal horizontal and vertical frame) may be necessary.

Large acceptance of FFAs in general, not only of vFFA, could be ideal for the positron ring of the LEMMA scheme where the positron beams spread out when they goes through the target. A proton driver of the proton based muon collider scheme is another area where a FFA could be employed.

Despite its potential as an ideal accelerator for the muon collider complex, there are several issues to be further studies. As an accelerator, vFFA design as is has relatively strong reverse bending magnets. The circumference factor, which is defined roughly as a ratio of ring circumference to that of a ring with only bending magnets is large, at least 5 at the latest design. This could be further reduced by a proper choice of optics parameter, e.g. asymmetrical tune in two orthogonal planes. For the collider and accumulator ring, it is also important to minimise the ring size to reduce the cost. The skew quadrupole based lattice design for both arc and low beta insertion may require a new development of unconventional algorithms for orbit and optics corrections. Those are just a few example.

Finally, some of the team member were the major driver to design, construct and commission EMMA (Electron Model of Muon Accelerator) in 2012 [14], which is the first linear nonscaling FFA to demonstrate serpentine channel acceleration and resonance crossing with fast acceleration. Now some of them are also engaged in designing a vFFA as a prototype of future accelerator upgrade of ISIS spallation neutron and muon source, named ISIS-II [15]. If the prototype study is approved, the first vFFA will be constructed and commissioned within next 10 years. We expect much progress on the vFFA development beyond paper study in the coming years.

This works will be pursued within the new forming International Muon Collider collaboration.

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