Integrating Out New Fermions at One Loop

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Bridging Heavy BSM to Experiment

$$E = \Lambda_{\rm BSM} : \mathcal{L}_{\rm UV}[\varphi_H, \varphi_L]$$

↓ $\int$ out φ_H

$$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(\Lambda_{\rm BSM})\mathcal{O}_i$$

↓ RGE (done once and for all)

$$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(m_{\rm EW})\mathcal{O}_i \xrightarrow{\text{map (done once and for all)}} E = m_{\rm EW} : \text{observables}$$

Bridging Heavy BSM to Experiment

$$E = \Lambda_{\mathrm{BSM}} : \mathcal{L}_{\mathrm{UV}}[\varphi_H, \varphi_L] \xrightarrow{\text{Feynman diagrams}} \langle \varphi_L \varphi_L \cdots \varphi_L \rangle_{\mathrm{UV}}$$

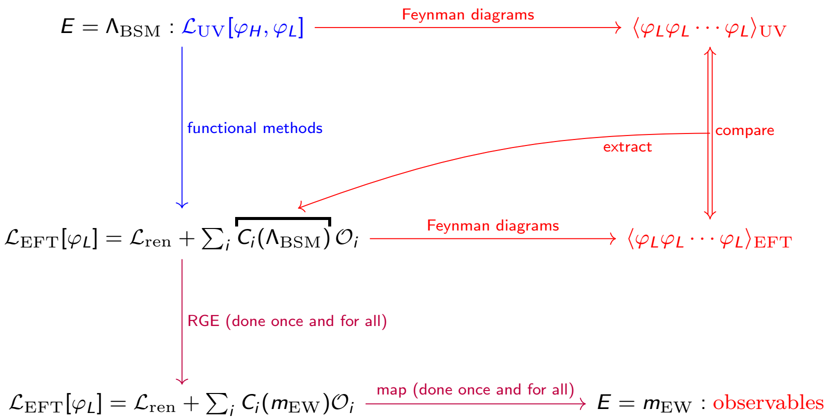
compare ($\Updownarrow$) — extract

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i \overline{C_i(\Lambda_{\mathrm{BSM}})}\, \mathcal{O}_i \xrightarrow{\text{Feynman diagrams}} \langle \varphi_L \varphi_L \cdots \varphi_L \rangle_{\mathrm{EFT}}$$

$\downarrow$ RGE (done once and for all)

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i C_i(m_{\mathrm{EW}})\mathcal{O}_i \xrightarrow{\text{map (done once and for all)}} E = m_{\mathrm{EW}} : \text{observables}$$

Bridging Heavy BSM to Experiment



Contents

Overview

The Effective Action Through Functional Methods

- Path integral definition of the effective action:

$$e^{i\,S_{eff}[\varphi_L]} = \int [\mathcal{D}\varphi_H] e^{i\,S_{UV}[\varphi_H,\varphi_L]}$$

- Stationary phase approx. $\Rightarrow$ classical solution:

$$\frac{\delta S_{UV}[\varphi_H,\varphi_L]}{\delta \varphi_H} = 0 \quad \Rightarrow \quad \varphi_H^{cl} = \varphi_H^{cl}(\varphi_L)$$

- Taylor expansion around the minimum (η = quantum fluctuations, assume no φ_L in loops):

$$S_{UV}[\varphi_H^{cl}+\eta,\varphi_L] = S_{UV}[\varphi_H^{cl}(\varphi_L),\varphi_L] + \frac{1}{2}\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H^2}\bigg|_{\varphi_H^{cl}} \eta^2 + \mathcal{O}(\eta^3)$$

- Integrate over $\eta \Rightarrow$ Gaussian integral ($c_s = \frac{1}{2}$ for real scalars, $-\frac{1}{2}$ for Weyl fermions):

$$e^{i\,S_{eff}[\varphi_L]} = e^{i\,S_{UV}[\varphi_H^{cl}(\varphi_L),\varphi_L]} \left[\det\left(-\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H^2}\bigg|_{\varphi_H^{cl}}\right)\right]^{-c_s} \xrightarrow{\det(A)=\exp(\mathrm{Tr}\log A)}$$

$$S_{eff}[\varphi_L] = \underbrace{S_{UV}[\varphi_H^{cl}(\varphi_L),\varphi_L]}_{=\text{ tree level}} + \underbrace{i c_s \mathrm{Tr}\log\left(-\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H^2}\bigg|_{\varphi_H^{cl}}\right)}_{=\text{ 1–loop}}$$

The Effective Action at One Loop

$$\mathcal{L}_{UV}[\varphi_H,\varphi_L] = \mathcal{L}_{IR}[\varphi_L] + \left(\varphi_H^\dagger T[\varphi_L] + \text{h.c.}\right) + \varphi_H^\dagger\Big(\underbrace{-D^2 - M^2}_{\text{diagonal}} - U[\varphi_L]\Big)\varphi_H$$

- Assume only heavy fields in the loop ⇒ "heavy–only" contribution:

$$S_{\text{eff}}^{\text{1-loop}} = ic_s \mathrm{Tr}\log\left(D^2 + M^2 + U[\varphi_L]\right) \equiv ic_s \mathrm{Tr}\log \mathcal{Q}_H$$

- Tr includes coordinate space trace, tr over internal indices (spin, gauge, flavour):

$$\begin{aligned} S_{\text{eff}}^{\text{1-loop}} &= ic_s\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d}\langle p|\log\mathcal{Q}_H|x\rangle\langle x|p\rangle \\ &= ic_s\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d} e^{-ipx}\log\left[\mathcal{Q}_H(x,\partial_x)\right]e^{ipx} \\ &= ic_s\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d}\log\left[\mathcal{Q}_H(x,\partial_x + ip)\right] \\ &= ic_s\,\mathrm{tr}\int d^dx \int \frac{d^dp}{(2\pi)^d}\log\left[-(iD_\mu - p_\mu)^2 + M^2 + U\right] \end{aligned}$$

- Fine print → assume no open covariant derivatives in U
- Never separate $D_\mu = \partial_\mu + igG_\mu^a T^a$
  ⇒ Covariant Derivative Expansion (CDE) (in contrast to Feynman diagrams)

The Universal One–Loop Effective Action (UOLEA) (Murayama+ '14, Ellis+ '15)

$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = i c_s \,\text{tr} \int \frac{d^d p}{(2\pi)^d} \log\left[-(iD_\mu - p_\mu)^2 + M^2 + U\right]$$

- Gaillard '86, Cheyette '88: momentum shift → insert $e^{\pm i D_\mu \partial/\partial p_\mu}$:

$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = i c_s \,\text{tr} \int \frac{d^d p}{(2\pi)^d} e^{i D_\mu \partial/\partial p_\mu} \log\left[-(iD_\mu - p_\mu)^2 + M^2 + U\right] e^{-i D_\mu \partial/\partial p_\mu}$$
$$= i c_s \,\text{tr} \int \frac{d^d p}{(2\pi)^d} \log\left[-\left(\tilde{G}_{\nu\mu} \frac{\partial}{\partial p_\nu} + p_\mu\right)^2 + M^2 + \tilde{U}\right]$$

- Gauge invariance preserved during all intermediate steps
⇒ D_μ's appearing only in commutators:

$$\tilde{G}_{\nu\mu} = \sum_{n=0}^{\infty} i^n \frac{n+1}{(n+2)!} [D_{\mu_1}, [\ldots [D_{\mu_n}, [D_\nu, D_\mu]]]] \frac{\partial^n}{\partial p_{\mu_1} \ldots \partial p_{\mu_n}},$$
$$\tilde{U} = \sum_{n=0}^{\infty} i^n \frac{1}{n!} [D_{\mu_1}, [\ldots [D_{\mu_n}, U]]] \frac{\partial^n}{\partial p_{\mu_1} \ldots \partial p_{\mu_n}}$$

- Momentum derivatives → cumbersome...

The Universal One–Loop Effective Action (UOLEA) (Murayama+ '14, Ellis+ '15)

- The (heavy–only) Universal One Loop Effective Action looks like ($F_{\mu\nu} \equiv -i[D_\mu, D_\nu]$):

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\text{1-loop}} = -ic_s \,\mathrm{tr}\Big\{ & f_2^i\, U_{ii} + f_3^i\, F_i^{\mu\nu} F_{\mu\nu,i} + f_4^{ij}\, U_{ij} U_{ji} \\ & + f_5^i\, [D^\mu, F_{\mu\nu,i}][D_\rho, F_i^{\rho\nu}] + f_6^i\, F^\mu{}_{\nu,i} F^\nu{}_{\rho,i} F^\rho{}_{\mu,i} \\ & + f_7^{ij}\, [D^\mu, U_{ij}][D_\mu, U_{ji}] + f_8^{ijk}\, U_{ij} U_{jk} U_{ki} + f_9^i\, U_{ii} F_i^{\mu\nu} F_{\mu\nu,i} \\ & + f_{10}^{ijkl}\, U_{ij} U_{jk} U_{kl} U_{li} + f_{11}^{ijk}\, U_{ij} [D^\mu, U_{jk}][D_\mu, U_{ki}] \\ & + f_{12}^{ij}\, \big[D^\mu, [D_\mu, U_{ij}]\big]\big[D^\nu, [D_\nu, U_{ji}]\big] + f_{13}^{ij}\, U_{ij} U_{ji} F_i^{\mu\nu} F_{\mu\nu,i} \\ & + f_{14}^{ij}\, [D^\mu, U_{ij}][D^\nu, U_{ji}] F_{\nu\mu,i} + f_{15}^{ij}\, \big(U_{ij}[D^\mu, U_{ji}] - [D^\mu, U_{ij}]U_{ji}\big)[D^\nu, F_{\nu\mu,i}] \\ & + f_{16}^{ijklm}\, U_{ij} U_{jk} U_{kl} U_{lm} U_{mi} + f_{17}^{ijkl}\, U_{ij} U_{jk} [D^\mu, U_{kl}][D_\mu, U_{li}] \\ & + f_{18}^{ijkl}\, U_{ij} [D^\mu, U_{jk}] U_{kl} [D_\mu, U_{li}] + f_{19}^{ijklmn}\, U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni} \Big\}\end{aligned}$$

- U_{ij}, $F_i^{\mu\nu}$ depend on the UV theory
- $f_N^{ijk\cdots} \equiv f_N(m_i, m_j, m_k, \ldots)$ are model independent $\Rightarrow$ universality!
- Assumptions: no open covariant derivatives in U + no light fields in the loops!

A Simpler Way? (Portoles+ '16)

- Expand the log $\Rightarrow$ the CDE is a power series in $\frac{1}{M}$ + momentum integrals factorize:

$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = \frac{c_s}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \, \mathrm{tr} \left\{ \left[\Delta \left(2ip_\mu D^\mu + D^2 + U \right) \right]^n \right\}, \quad \Delta = (p^2 - M^2)^{-1}$$

- Power counting:

$$p, M \sim \Lambda \quad \Rightarrow \quad \Delta \sim \frac{1}{\Lambda^2}, \quad D_\mu \sim \Lambda^0,$$
$$U \sim \Lambda^0 \quad \text{for} \quad \text{bosons},$$
$$U \sim \Lambda^1 \quad \text{for} \quad \text{fermions}$$

- Operators calculated order by order
- Dimension–6 operators $\Rightarrow$ truncate at $n = 6$!
- N.B.: For heavy bosons, $\mathcal{O}(U^{n>3})$ gives $\dim > 6$.

A Simpler Way? (Portoles+ '16)

- Key point → cyclic property of the trace
  ⇒ But does it apply for D_μ?
- Yes! Switching from tr back to Tr and using $\langle x|x\rangle = \delta^d(0) = V_d$:

$$\int d^d x\, \mathrm{tr}[f(x)] = \frac{1}{V_d}\int d^d x\, \mathrm{tr}[f(x)]\,\delta^d(0) = \frac{1}{V_d}\int d^d x\, \mathrm{tr}\left[\langle x|f(\hat{x})|x\rangle\right] = \frac{1}{V_d}\mathrm{Tr} f(\hat{x})$$

- For example, consider $\mathcal{O}(U^4 D^2)$ → only three independent terms:

$$\mathcal{L}_{U^4 D^2} \Rightarrow \mathrm{tr}\left(U^4 D^2\right),\ \mathrm{tr}\left(U^3 D_\mu U D^\mu\right),\ \mathrm{tr}\left(U^2 D_\mu U^2 D^\mu\right)$$

- Match to the most general gauge–invariant Lagrangian at $\mathcal{O}(U^4 D^2)$:

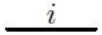
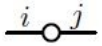
$$\mathcal{L}_{U^4 D^2} \Rightarrow \mathrm{tr}\left(U^2\,[D_\mu, U]\,[D^\mu, U]\right),\ \mathrm{tr}\left(U\,[D_\mu, U]\,U\,[D^\mu, U]\right)$$

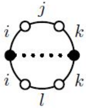
- Note redundancy: three independent terms vs. two gauge–invariant operators
- Bonus → can set up a diagrammatic computation!

Covariant Diagrams (Zhang '16)

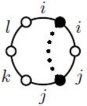
$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = \frac{c_s}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \, \text{tr}\left\{ \left[\Delta \left(2ip_\mu D^\mu + D^2 + U \right) \right]^n \right\}$$

- Three possible insertions: D^2, $2ip_\mu D^\mu$, U.
- Redundancy $\Rightarrow$ no need to consider D^2 or contracted adjacent $2ip_\mu D^\mu$ $\Rightarrow$ less diagrams!
  Notation: $p \leftrightarrow q$, $P_\mu \equiv iD_\mu$

Element of diagram	Symbol	Expression
heavy propagator (bosonic)		1
P insertion (bosonic, heavy)		$2P_\mu \delta_{ij}$
U insertion (heavy-heavy)		$U_{H\,ij}$



$$= -ic_s \frac{2^2}{2} \mathcal{I}[q^2]^{2121}_{ijkl} \, \text{tr}(P^\mu U_{ij} U_{jk} P_\mu U_{kl} U_{li})$$



$$= -ic_s 2^2 \, \mathcal{I}[q^2]^{2211}_{ijkl} \, \text{tr}(P^\mu U_{ij} P_\mu U_{jk} U_{kl} U_{li})$$

- Extract universal coefficients by matching onto:

$$f_{17}^{ijkl} \text{tr}\left(U_{ij} U_{jk} [P_\mu, U]_{kl} [P^\mu, U]_{li} \right) + f_{18}^{ijkl} \text{tr}\left(U_{ij} [P_\mu, U]_{jk} U_{kl} [P^\mu, U]_{li} \right)$$

$$\supset \left(-f_{17}^{ijkl} + f_{18}^{ijkl} + f_{18}^{jkli} \right) \text{tr}\left(P^\mu U_{ij} U_{jk} P_\mu U_{kl} U_{li} \right) + \left(f_{17}^{jkli} + f_{17}^{klij} - f_{18}^{ijkl} - f_{18}^{klij} \right) \text{tr}\left(P^\mu U_{ij} P_\mu U_{jk} U_{kl} U_{li} \right)$$

Heavy–Light Loops (Murayama+ '16, Ellis+ '16, Portoles+ '16, Zhang '16)

- What about both heavy and light fields running in the same loop?
  → consider quantum fluctuations for light fields too!

$$S_{eff}^{\text{1-loop}} \stackrel{?}{=} ic_s \log\det \begin{pmatrix} -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H^2}\right|_{\varphi_H^{cl}} & -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_H\delta\varphi_L}\right|_{\varphi_H^{cl}} \\ -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_L\delta\varphi_H}\right|_{\varphi_H^{cl}} & -\left.\frac{\delta^2 S_{UV}[\varphi_H,\varphi_L]}{\delta\varphi_L^2}\right|_{\varphi_H^{cl}} \end{pmatrix}$$

$$\equiv ic_s \log\det \begin{pmatrix} \mathcal{Q}_H & X_{HL} \\ X_{LH} & \mathcal{Q}_L \end{pmatrix}$$

- No, b/c it includes long–distance contributions, i.e. $p \sim m_{\varphi_L}$
  ⇒ 1PI effective action, not S_{eff}^{1-loop} !

- To get S_{eff}^{1-loop}, need to somehow isolate the short–distance $p \sim m_{\varphi_H}$ contributions...

Heavy–Light Loops (Portoles+ '16, Zhang '16)

- Method of "expansion by regions"
  $\rightarrow$ identity holds (in dim. reg.) order by order in $\frac{1}{M}$ $(M \gg m)$:

$$\int [d^d p] \frac{1}{(p^2 - M^2)(p^2 - m^2)^2} = \underbrace{\int [d^d p] \frac{1}{(p^2 - M^2)p^4}}_{\text{hard region: } p \sim M \gg m} + \underbrace{\int [d^d p] \frac{1}{(-M^2)(p^2 - m^2)^2}}_{\text{soft region: } p \sim m \ll M} + \mathcal{O}(M^{-4})$$

- Diagonalize the fluctuation matrix:

$$V \begin{pmatrix} \mathcal{Q}_H & X_{HL} \\ X_{LH} & \mathcal{Q}_L \end{pmatrix} V^\dagger = \begin{pmatrix} \mathcal{Q}_H - X_{HL}\mathcal{Q}_L^{-1}X_{LH} & 0 \\ 0 & \mathcal{Q}_L \end{pmatrix}$$

- The one–loop effective action is therefore:

$$S_{eff}^{\text{1-loop}} = i c_s \mathrm{Tr} \log \left(\mathcal{Q}_H - X_{HL}\mathcal{Q}_L^{-1}X_{LH} \right) \Big|_{\text{hard region}}$$

- The rest of the computation (e.g. momentum shift) proceeds as in the heavy–only case

Overview

The Fermionic Functional Determinant

- Start with a generic fermionic Lagrangian (heavy–only):

$$\mathcal{L}_{\rm VLF} = \overline{\Psi}\,(i\gamma_\mu D^\mu - M - W)\,\Psi$$

- D_μ → interactions with massless gauge fields (unbroken generators)
- W → interactions with scalars and/or massive gauge fields (broken generators)
- Can the previous master formula be used?
  → Yes! Square the quadratic operator, use $\mathrm{Tr}\log A + \mathrm{Tr}\log B = \mathrm{Tr}\log AB$:

$$\begin{aligned} S^{\text{1-loop}}_{\rm ferm} &= \frac{i}{2} c_f \left[\mathrm{Tr}\log\left(i\gamma_\mu D^\mu - M - W\right) + \mathrm{Tr}\log\left(-i\gamma_\mu D^\mu - M - W\right)\right] \\ &\equiv \frac{i}{2} c_f \,\mathrm{Tr}\log\left(D^2 + M^2 + U_{\rm ferm}\right), \\ U_{\rm ferm} &\equiv -\frac{i}{2}\sigma^{\mu\nu}[D_\mu, D_\nu] - i\gamma^\mu[D_\mu, W] - i[\gamma^\mu, W]D_\mu + \{M, W\} + W^2 \end{aligned}$$

- $[\gamma^\mu, W] \neq 0$ ⇒ open covariant derivative ⇒ UOLEA not applicable!
- Even if $[\gamma^\mu, W] = 0$, $U_{\rm ferm}$ complicated ⇒ applying UOLEA not straightforward!
- What if the fermionic quadratic operator is not "squared"?

The Fermionic Functional Determinant

- Same manipulations as before, but no "squaring":

$$\mathcal{L}^{\text{1-loop}}_{\text{ferm}} = \frac{-c_f}{16\pi^2} \operatorname{tr} \int [d^d p] \log \left[\not{p} - M - (-i\gamma_\mu D^\mu + W) \right]$$

$$= \frac{c_f}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \operatorname{tr} \left\{ \left[\not{\Delta} \left(-i\gamma_\mu D^\mu + W \right) \right]^n \right\}, \quad \not{\Delta} \equiv (\not{p} - M)^{-1}$$

- Power counting more transparent:

$$D_\mu,\ W \sim \Lambda^0, \quad \not{\Delta} \sim \Lambda^{-1}$$

$\Rightarrow$ all operators at dim–n given by n–th term in log expasion!

- The most general form of W (Quevillon '20):

$$W_{ij} = S_{ij} + i\gamma^5 P_{ij} + \gamma_\mu V^\mu_{ij} + \gamma_\mu \gamma^5 A^\mu_{ij} + \sigma_{\mu\nu} T^{\mu\nu}_{ij}$$

- Tensor term $\sigma_{\mu\nu} T^{\mu\nu}_{ij}$ non–renormalizable (dim–5 at least) $\to$ not considered here.
- N.B. $\to V_\mu \not\subset D_\mu$:
  - + massive gauge bosons (broken generators) $\subset V^\mu_{ij} \Rightarrow$ non–diagonal in flavour space!
  - + massless gauge bosons (unbroken generators) $\subset D^\mu \Rightarrow$ diagonal in flavour space!

Specializing to Vector–Like Fermions

$$\mathcal{L}^{\text{1-loop}}_{\text{ferm}} = \frac{c_f}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p] \, \mathrm{tr} \left\{ \left[\not{\Delta} \left(-i\gamma_\mu D^\mu + W \right) \right]^n \right\}$$

- Focus of this talk → Vector–Like Fermions!
- SMEFT in mind → work in the unbroken phase (no scalar VEVs)
  ⇒ The interaction term W has a simpler form:

$$W = S + i\gamma^5 P$$

- Why vector–like fermions (VLFs)?
  - + Composite Higgs / Xtra Dims / Twin Higgs etc. → Hierarchy problem
  - + No gauge anomalies (L and R chiralities have same quantum numbers)
  - + Appear in UV–complete solutions to the B-physics anomalies (Di Luzio+ '17, Isidori+ '17)
  - + Can induce a strong first-order electroweak phase transition (Egana-Ugrinovic '17, AA, P. Huang '18)

Example #1: Dimension–2 Terms

$$\mathcal{L}_{\rm ferm}^{\text{1-loop}} = \frac{c_f}{16\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \int [d^d p]\, \mathrm{tr} \left\{ \left[\Delta \!\!\!/ \left(-i\gamma_\mu D^\mu + S + i\gamma^5 P \right) \right]^n \right\}$$

- Work out the simple example of $n = 2 \rightarrow$ write explicitly the flavour indices $(i, j, k, \dots)$:

$$\mathcal{L}_{\rm ferm}^{n=2} = \frac{c_f}{16\pi^2} \frac{1}{2} \int [d^d p]\, \mathrm{tr} \left[\Delta \!\!\!/_i \left(S_{ij} + i\gamma^5 P_{ij} \right) \Delta \!\!\!/_j \left(S_{ji} + i\gamma^5 P_{ji} \right) \right] \qquad \left(\Delta \!\!\!/_i = \frac{p \!\!\!/ + m_i}{p^2 - m_i^2} \right)$$

$$= \frac{c_f\, n_D}{16\pi^2} \frac{1}{2} \left\{ \int [d^d p] \frac{\mathrm{tr_s}(\Delta \!\!\!/_i \Delta \!\!\!/_j)}{n_D}\, \mathrm{tr_g}(S_{ij} S_{ji}) + \int [d^d p] \frac{\mathrm{tr_s}\left[\Delta \!\!\!/_i (i\gamma^5) \Delta \!\!\!/_j (i\gamma^5) \right]}{n_D}\, \mathrm{tr_g}\left(P_{ij} P_{ji} \right) \right\}$$

$$\equiv \frac{c_f\, n_D}{16\pi^2} \left\{ \frac{1}{2} g_2^{ij}\, \mathrm{tr_g}(S_{ij} S_{ji}) + \frac{1}{2} \left(g_2^{i(j)} + \delta g_2^{ij} \right) \mathrm{tr_g}\left(P_{ij} P_{ji} \right) \right\}, \qquad n_D \equiv \mathrm{tr_s} \mathbb{1} = 4$$

- Cyclic property of the trace $\Rightarrow \mathbb{Z}_2$–symmetry for $\mathrm{tr_g}(S_{ij} S_{ji})$, $\mathrm{tr_g}(P_{ij} P_{ji})$ $\Rightarrow$ symmetry factor of $\frac{1}{2}$
- Universal coefficients:

$$g_2^{ij} \equiv g_2(m_i, m_j), \quad g_2^{i(j)} \equiv g_2(m_i, -m_j), \quad \delta g_2^{ij} \equiv \delta g_2(m_i, m_j)$$

- $\mathcal{O}(S^2)$ and $\mathcal{O}(P^2)$ universal coefficients related!

Systematic Treatment of γ^5

- Useful γ^5 identity ($d = 4 - 2\epsilon$ dimensions):

$$i\gamma^5(\not{p} + m)i\gamma^5 = (\not{p} - m) - 2\hat{g}_{\mu\nu}p^\mu\gamma^\nu, \quad g_{\mu\nu}\hat{g}^{\mu\nu} = \hat{g}_{\mu\nu}\hat{g}^{\mu\nu} = -2\epsilon$$

- "Breitenlohner–Maison–'t Hooft–Veltman" (BMHV) scheme for γ^5 in $d = 4 - 2\epsilon$ dimensions
$\Rightarrow$ evanescent part $\propto \hat{g}_{\mu\nu} \Rightarrow$ origin of δg_2^{ij}:

$$\frac{1}{n_D}\int [d^d p]\,\mathrm{tr_s}(\Delta_i \Delta_j) = \frac{1}{n_D}\int [d^d p]\frac{\mathrm{tr_s}\left[(\not{p} + m_i)(\not{p} + m_j)\right]}{(p^2 - m_i^2)(p^2 - m_j^2)} \qquad \Rightarrow g_2^{ij}$$

$$\frac{1}{n_D}\int [d^d p]\,\mathrm{tr_s}\left[\Delta_i(i\gamma^5)\Delta_j(i\gamma^5)\right] = \frac{1}{n_D}\int [d^d p]\frac{\mathrm{tr_s}\left[(\not{p} + m_i)(\not{p} - m_j)\right]}{(p^2 - m_i^2)(p^2 - m_j^2)} \qquad \Rightarrow g_2^{i(j)}$$

$$-\frac{2}{n_D}\int [d^d p]\frac{4\hat{g}_{\mu\nu}p^\mu p^\nu}{(p^2 - m_i^2)(p^2 - m_j^2)} \qquad \Rightarrow \delta g_2^{ij}$$

- Compute operators containing only $S \Rightarrow$ even powers of P follow immediately:
  - $+$ flip sign of certain masses
  - $+$ adjust symmetry factors
- Bonus $\rightarrow$ no evansesccent part for finite loops
$\Rightarrow$ simpler computation for dim–5 and dim–6 operators!

Example #2: $\mathcal{O}(X^4 D^2)$ Terms

- Specify all gauge invariant independent operators at $\mathcal{O}(S^4 D^2)$, expand commutators, group terms:

$$16\pi^2 \mathcal{L}_{S^4 D^2} = c_f\, n_D \left[g_{12}^{ijkl} \mathrm{tr_g}\left(S_{ij} S_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, S\right]_{li} \right) + \frac{1}{2} g_{13}^{ijkl} \mathrm{tr_g}\left(S_{ij} \left[D_\mu, S\right]_{jk} S_{kl} \left[D^\mu, S\right]_{li} \right) \right]$$
$$= c_f\, n_D\, \mathrm{tr_g}\left[-g_{12}^{ijkl} \left(S_{ij} S_{jk} S_{kl} D^2 S_{li} \right) + \left(g_{12}^{ijkl} + g_{12}^{jkli} - g_{13}^{jkli} \right) \left(S_{ij} S_{jk} S_{kl} D^\mu S_{li} D_\mu \right) \right.$$
$$\left. + \frac{1}{2} \left(g_{13}^{ijkl} + g_{13}^{jkli} - g_{12}^{ijkl} - g_{12}^{klij} \right) \left(S_{ij} S_{jk} D^\mu S_{kl} S_{li} D_\mu \right) \right]$$

- Write down terms from CDE (log expansion):

$$16\pi^2 \mathcal{L}_{S^4 D^2} = -c_f \left[\int [d^d p]\, \mathrm{tr_s}\left(\Delta_i \Delta_j \Delta_k \Delta_l \gamma_\mu \Delta_l \gamma_\nu \Delta_l \right) \mathrm{tr_g}\left(S_{ij} S_{jk} S_{kl} D^\mu D^\nu S_{li} \right) \right.$$
$$+ \int [d^d p]\, \mathrm{tr_s}\left(\Delta_i \Delta_j \Delta_k \Delta_l \gamma_\mu \Delta_l \Delta_i \gamma_\nu \right) \mathrm{tr_g}\left(S_{ij} S_{jk} S_{kl} D^\mu S_{li} D^\nu \right)$$
$$\left. + \frac{1}{2} \int [d^d p]\, \mathrm{tr_s}\left(\Delta_i \Delta_j \Delta_k \gamma_\mu \Delta_k \Delta_l \Delta_i \gamma_\nu \right) \mathrm{tr_g}\left(S_{ij} S_{jk} D^\mu S_{kl} S_{li} D^\nu \right) \right]$$

- Equate the two expressions $\Rightarrow g_{12}^{ijkl}, g_{13}^{ijkl} = \ldots$ (note redundancy $\rightarrow$ serves as cross–check!)

Example #2: $\mathcal{O}(X^4 D^2)$ Terms

- $\mathcal{O}(X^4 D^2)$ universal coefficients for even powers of P? Use γ^5 identities (or variations):

$$i\gamma^5(\not{p} + m_i)i\gamma^5 = \not{p} - m_j$$
$$i\gamma^5(\not{p} + m_i)(\not{p} + m_j)i\gamma^5 = -(\not{p} - m_i)(\not{p} - m_j)$$

- $\Rightarrow$ flip mass signs, adjust symmetry factors:

$$\frac{1}{2} g_{13}^{ijkl} \mathrm{tr_g}\left(S_{ij}\left[D_\mu, S\right]_{jk} S_{kl}\left[D^\mu, S\right]_{li}\right) \to g_{13}^{ijk(l)} \mathrm{tr_g}\left(S_{ij}\left[D_\mu, S\right]_{jk} P_{kl}\left[D^\mu, P\right]_{li}\right)$$
$$\to \frac{1}{2} g_{13}^{i(j)k(l)} \mathrm{tr_g}\left(P_{ij}\left[D_\mu, P\right]_{jk} P_{kl}\left[D^\mu, P\right]_{li}\right)$$
$$g_{12}^{ijkl} \mathrm{tr_g}\left(S_{ij} S_{jk}\left[D_\mu, S\right]_{kl}\left[D^\mu, S\right]_{li}\right) \to g_{12}^{(i)jkl} \mathrm{tr_g}\left(P_{ij} S_{jk}\left[D_\mu, S\right]_{kl}\left[D^\mu, P\right]_{li}\right)$$
$$\to -g_{12}^{i(j)(k)l} \mathrm{tr_g}\left(P_{ij} S_{jk}\left[D_\mu, P\right]_{kl}\left[D^\mu, S\right]_{li}\right)$$

Example #2: $\mathcal{O}(X^4 D^2)$ Terms

- Full $\mathcal{O}(X^4 D^2)$ contribution (including P insertions):

$$
\begin{aligned}
16\pi^2 \mathcal{L}_{X^4 D^2} = c_f\, n_D \Big\{ \mathrm{tr_g} \Big[& g_{12}^{ijkl} \left(S_{ij} S_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, S\right]_{li}\right) + g_{12}^{ijk(l)} \left(S_{ij} S_{jk} \left[D_\mu, P\right]_{kl} \left[D^\mu, P\right]_{li}\right) \\
& + g_{12}^{ij(k)l} \left(S_{ij} P_{jk} \left[D_\mu, P\right]_{kl} \left[D^\mu, S\right]_{li}\right) + g_{12}^{i(j)kl} \left(P_{ij} P_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, S\right]_{li}\right) \\
& + g_{12}^{(i)jkl} \left(P_{ij} S_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, P\right]_{li}\right) - g_{12}^{ij(k)(l)} \left(S_{ij} P_{jk} \left[D_\mu, S\right]_{kl} \left[D^\mu, P\right]_{li}\right) \\
& - g_{12}^{i(j)(k)l} \left(P_{ij} S_{jk} \left[D_\mu, P\right]_{kl} \left[D^\mu, S\right]_{li}\right) + g_{12}^{i(j)k(l)} \left(P_{ij} P_{jk} \left[D_\mu, P\right]_{kl} \left[D^\mu, P\right]_{li}\right) \Big] \\
+ \mathrm{tr_g} \Big[& \frac{1}{2} g_{13}^{ijkl} \left(S_{ij} \left[D_\mu, S\right]_{jk} S_{kl} \left[D^\mu, S\right]_{li}\right) + g_{13}^{ijk(l)} \left(S_{ij} \left[D_\mu, S\right]_{jk} P_{kl} \left[D^\mu, P\right]_{li}\right) \\
& + g_{13}^{ij(k)l} \left(S_{ij} \left[D_\mu, P\right]_{jk} P_{kl} \left[D^\mu, S\right]_{li}\right) - \frac{1}{2} g_{13}^{ij(k)(l)} \left(S_{ij} \left[D_\mu, P\right]_{jk} S_{kl} \left[D^\mu, P\right]_{li}\right) \\
& - \frac{1}{2} g_{13}^{i(j)(k)l} \left(P_{ij} \left[D_\mu, S\right]_{jk} P_{kl} \left[D^\mu, S\right]_{li}\right) + \frac{1}{2} g_{13}^{i(j)k(l)} \left(P_{ij} \left[D_\mu, P\right]_{jk} P_{kl} \left[D^\mu, P\right]_{li}\right) \Big] \Big\}
\end{aligned}
$$

- γ^5 identities + symmetry factor adjustments $\Rightarrow$ only two terms to compute instead of 14!
- Streamlined computation $\Rightarrow$ automation!

Example #3: $\mathcal{O}(X^6)$ terms

- Another example $\to \mathcal{O}(X^6)$ operators:

$$
\begin{aligned}
16\pi^2 \mathcal{L}_{X^6} = c_f\, n_D \Big[& \frac{1}{6}\, g_{11}^{ijklmn}\, \mathrm{tr_g}\left(S_{ij}S_{jk}S_{kl}S_{lm}S_{mn}S_{ni}\right) + g_{11}^{ijklm(n)}\, \mathrm{tr_g}\left(S_{ij}S_{jk}S_{kl}S_{lm}P_{mn}P_{ni}\right) \\
& - g_{11}^{ijkl(m)(n)}\, \mathrm{tr_g}\left(S_{ij}S_{jk}S_{kl}P_{lm}S_{mn}P_{ni}\right) + \frac{1}{2}\, g_{11}^{ijk(l)(m)(n)}\, \mathrm{tr_g}\left(S_{ij}S_{jk}P_{kl}S_{lm}S_{mn}P_{ni}\right) \\
& + g_{11}^{ijk(l)m(n)}\, \mathrm{tr_g}\left(S_{ij}S_{jk}P_{kl}P_{lm}P_{mn}P_{ni}\right) - g_{11}^{ij(k)(l)m(n)}\, \mathrm{tr_g}\left(S_{ij}P_{jk}S_{kl}P_{lm}P_{mn}P_{ni}\right) \\
& + \frac{1}{2}\, g_{11}^{ij(k)lm(n)}\, \mathrm{tr_g}\left(S_{ij}P_{jk}P_{kl}S_{lm}P_{mn}P_{ni}\right) + \frac{1}{6}\, g_{11}^{i(j)k(l)m(n)}\, \mathrm{tr_g}\left(P_{ij}P_{jk}P_{kl}P_{lm}P_{mn}P_{ni}\right) \Big],
\end{aligned}
$$

- γ^5 identities + symmetry factor adjustments $\Rightarrow$ only one term to compute instead of 8!

The Fermionic Universal Master Formula

- Building blocks: S, $[D_\mu, S]$, $[D_\mu,[D^\mu,S]]$, $F_{\mu\nu}$, $[D^\mu, F_{\mu\nu}]$, $\widetilde{F}_{\mu\nu}$ (and $[S \to P]$)

$$
\begin{aligned}
\mathcal{L}^{\text{1-loop}}_{\text{VLF}} \supset \frac{c_f\, n_D}{16\pi^2}\text{tr}_{\text{g}} \Big[& g_1^i\, S_{ii} + \frac{1}{2} g_2^{ij}\left(S_{ij}S_{ji}\right) + \frac{1}{3} g_3^{ijk}\left(S_{ij}S_{jk}S_{ki}\right) + \frac{1}{4} g_4^{ijkl}\left(S_{ij}S_{jk}S_{kl}S_{li}\right) \\
& + \frac{1}{2} g_5^{ij}\left([D_\mu,S]_{ij}[D^\mu,S]_{ji}\right) + \frac{1}{2} g_6^{i}\left(F_{\mu\nu,i}F_i^{\mu\nu}\right) + \frac{1}{5} g_7^{ijklm}\left(S_{ij}S_{jk}S_{kl}S_{lm}S_{mi}\right) \\
& + g_8^{ijk}\left(S_{ij}[D_\mu,S]_{jk}[D^\mu,S]_{ki}\right) + g_9^{i}\left(S_{ii}F_i^{\mu\nu}F_{\mu\nu,i}\right) + g_{10}^{i}\left(P_{ii}\widetilde{F}_i^{\mu\nu}F_{\mu\nu,i}\right) \\
& + \frac{1}{6} g_{11}^{ijklmn}\left(S_{ij}S_{jk}S_{kl}S_{lm}S_{mn}S_{ni}\right) + g_{12}^{ijkl}\left(S_{ij}S_{jk}[D_\mu,S]_{kl}[D^\mu,S]_{li}\right) \\
& + \frac{1}{2} g_{13}^{ijkl}\left(S_{ij}[D_\mu,S]_{jk}S_{kl}[D^\mu,S]_{li}\right) + \frac{1}{2} g_{14}^{ij}[D_\mu,[D^\mu,S]]_{ij}[D_\nu,[D^\nu,S]]_{ji} \\
& + g_{15}^{ij}\left(S_{ij}S_{ji}F_i^{\mu\nu}F_{\mu\nu,i}\right) + \frac{1}{2} g_{16}^{ij}\left(S_{ij}F_j^{\mu\nu}S_{ji}F_{\mu\nu,i}\right) + i\, g_{17}^{ij}\left(S_{ij}[D_\mu,S]_{ji}[D_\nu,F^{\nu\mu}]_i\right) \\
& + g_{18}^{ij}\left((S_{ij}P_{ji} + P_{ij}S_{ji})\widetilde{F}_i^{\mu\nu}F_{\mu\nu,i}\right) + g_{19}^{ij}\left(S_{ij}\widetilde{F}_j^{\mu\nu}P_{ji}F_{\mu\nu,i}\right) \\
& + g_{20}^{i}\,[D^\mu,F_{\mu\nu}]_i\,[D_\rho,F^{\rho\nu}]_i + \frac{i}{3} g_{21}^{i}\left(F^\mu{}_\nu F^\nu{}_\rho F^\rho{}_\mu\right) \Big]
\end{aligned}
$$

Overview

Model Description (AA, P. Huang '18)

- Consider a vector–like lepton model that can accommodate a strong EW phase transition:

$$L_{L,R} = \begin{pmatrix} N \\ E \end{pmatrix}_{L,R} \sim (1,2,Y), \quad N'_{L,R} \sim (1,1,Y+\frac{1}{2}), \quad E'_{L,R} \sim (1,1,Y-\frac{1}{2})$$

- VLL Lagrangian (assume negligible mixing with the SM):

$$\mathcal{L}_{VLL} = \overline{L}(i\gamma_\mu D^\mu_L - m_L)L + \overline{N}'(i\gamma_\mu D^\mu_N - m_N)N' + \overline{E}'(i\gamma_\mu D^\mu_E - m_E)E' \\ - \left(y_{N_R}\overline{L}_L\tilde{H}N'_R + y_{N_L}\overline{L}_R\tilde{H}N'_L + y_{E_R}\overline{L}_L H E'_R + y_{E_L}\overline{L}_R H E'_L + \mathrm{h.c.}\right)$$

- M matrix in the $(L,\ E',\ N')$ basis:

$$M = \begin{pmatrix} m_L\,\mathbb{1}_{2\times 2} & 0_{2\times 1} & 0_{2\times 1} \\ 0_{1\times 2} & m_E & 0 \\ 0_{1\times 2} & 0 & m_N \end{pmatrix}$$

Model Description (AA, P. Huang '18)

- Define Yukawa linear combinations:

$$y_A \equiv \frac{y_{A_L} + y_{A_R}}{2}, \quad z_A \equiv \frac{y_{A_L} - y_{A_R}}{2}, \quad A = N, E$$

- $\Rightarrow$ S and P matrices:

$$S = \begin{pmatrix} 0_{2\times 2} & y_E\, H_{2\times 1} & y_N\, \widetilde{H}_{2\times 1} \\ y_E^*\, H^\dagger_{1\times 2} & 0 & 0 \\ y_N^*\, \widetilde{H}^\dagger_{1\times 2} & 0 & 0 \end{pmatrix}, \quad P = i \begin{pmatrix} 0_{2\times 2} & z_E\, H_{2\times 1} & z_N\, \widetilde{H}_{2\times 1} \\ -z_E^*\, H^\dagger_{1\times 2} & 0 & 0 \\ -z_N^*\, \widetilde{H}^\dagger_{1\times 2} & 0 & 0 \end{pmatrix}$$

- $[D_\mu, S]$ and $[D_\mu, P]$ matrices $\rightarrow$ replace $H, \widetilde{H}$ by $(D_\mu H), (D_\mu \widetilde{H})$
- Field strength matrix:

$$F_{\mu\nu} = \begin{pmatrix} g_1\, Y\, B_{\mu\nu}\, \mathbb{1}_{2\times 2} + \frac{g_2}{2}\, W^a_{\mu\nu}\sigma^a & 0_{2\times 1} & 0_{2\times 1} \\ 0_{1\times 2} & g_1 \left(Y - \frac{1}{2}\right) B_{\mu\nu} & 0 \\ 0_{1\times 2} & 0 & g_1 \left(Y + \frac{1}{2}\right) B_{\mu\nu} \end{pmatrix}$$

CP–Even Dimension–6 Operators (Huo '15)

- Set all masses equal to m and $y_{E_L} = y_{E_R} \equiv y_E$, $y_{N_L} = y_{N_R} \equiv y_N$:

$$
\begin{aligned}
16\pi^2 \mathcal{L}^{\rm CP}_{\rm dim-6} = {} & \frac{2\left(y_E^6 + y_N^6\right)}{15m^2}\, |H|^6 - \frac{2(y_E^2+y_N^2)^2}{5m^2}\, |H|^2 \Box |H|^2 - \frac{4(y_E^2-y_N^2)^2}{5m^2} \left| H^\dagger D_\mu H \right|^2 \\
& - \frac{y_E^4 - 4y_E^2 y_N^2 + y_N^4}{5m^2}\, |H|^2 \left(H^\dagger D^2 H + \text{h.c.} \right) - \frac{7\left(y_E^2+y_N^2\right)}{120m^2} g_2^2 |H|^2 W^i_{\mu\nu} W^i_{\mu\nu} \\
& - \frac{\left(7 - 40Y + 80Y^2\right) y_E^2 + \left(7 + 40Y + 80Y^2\right) y_N^2}{120m^2} g_1^2 |H|^2 B_{\mu\nu} B_{\mu\nu} \\
& + \frac{(3-20Y)y_E^2 + (3+20Y)y_N^2}{60m^2} g_1 g_2 \left(H^\dagger \sigma^i H \right) W^i_{\mu\nu} B_{\mu\nu} + \frac{y_E^2 + y_N^2}{5m^2} \left| D^2 H \right|^2 \\
& + \frac{2\left(y_E^2+y_N^2\right)}{15m^2} g_1 \left(H^\dagger i \overleftrightarrow{D}_\mu H \right) \partial_\nu B_{\nu\mu} + \frac{2\left(y_E^2+y_N^2\right)}{15m^2} g_2 \left(H^\dagger i \overleftrightarrow{D}^i_\mu H \right) D_\nu W^i_{\nu\mu} \\
& - \frac{1+8Y^2}{15m^2} g_1^2 (\partial_\mu B_{\mu\nu})^2 - \frac{1}{15m^2} g_2^2 (D_\mu W^i_{\mu\nu})^2 - \frac{1}{30m^2} \frac{g_2^3}{3!} \epsilon_{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}
\end{aligned}
$$

- Also, renormalization of EW gauge kinetic terms + Higgs kinetic, mass, and quartic terms.

CP–Odd Dimension–6 Operators

- Set all masses equal to m, but $y_{E_L} \neq y_{E_R}$, $y_{N_L} \neq y_{N_R}$ (otherwise no $\not{CP}$):

$$16\pi^2 \mathcal{L}^{\not{CP}}_{\dim-6} = -\frac{(|y_{N_L}|^2 + |y_{N_R}|^2)\mathrm{Im}(y_{N_L} y^*_{N_R}) - (|y_{E_L}|^2 + |y_{E_R}|^2)\mathrm{Im}(y_{E_L} y^*_{E_R})}{3m^2} \widetilde{\mathcal{O}}_f$$
$$- \frac{(1 + 6Y + 12Y^2)\,\mathrm{Im}(y_{N_L} y^*_{N_R}) + (1 - 6Y + 12Y^2)\,\mathrm{Im}(z_{E_L} y^*_{E_R})}{12m^2} g_1^2 |H|^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$$
$$+ \frac{(1 + 6Y)\,\mathrm{Im}(y_{N_L} y^*_{N_R}) + (1 - 6Y)\,\mathrm{Im}(y_{E_L} y^*_{E_R})}{12m^2} g_1 g_2 (H^\dagger \sigma^a H) \widetilde{W}^a_{\mu\nu} B^{\mu\nu}$$
$$- \frac{\mathrm{Im}(y_{N_L} y^*_{N_R} + y_{E_L} y^*_{E_R})}{12m^2} g_2^2 |H|^2 \widetilde{W}^a_{\mu\nu} W^{\mu\nu,a}, \quad \widetilde{\mathcal{O}}_f \equiv \frac{i}{2} |H|^2 \left[(D^2 H)^\dagger H - H^\dagger D^2 H \right]$$

- $\widetilde{\mathcal{O}}_f$ breaks custodial symmetry $\Rightarrow$ leading source of $\not{CP}$ at $\mathcal{O}(y^2_{E,N})$, not $\mathcal{O}(y^4_{E,N})$

Re–deriving Higgs Low Energy Theorems

- Goal → reproduce the Low Energy Theorem (LET) expression for $h \to \gamma\gamma$
- Broken phase → consider $E_{i=1,2}$ mass eigenstates, assume N_i's decoupled:

$$\mathcal{L}_{\mathrm{VLL}} \supset \overline{E}_i \left[i\gamma^\mu D_\mu \delta_{ij} - m_i \delta_{ij} - y_{ij} h - \tilde{y}_{ij} h (i\gamma^5) \right] E_j$$

$$D_\mu = \partial_\mu + i\, Q_E e\, A_\mu \;\Rightarrow\; F_{\mu\nu} = Q_E e\, A_{\mu\nu}, \quad S_{ij} = y_{ij} h, \quad P_{ij} = \tilde{y}_{ij} h$$

- Relevant universal terms for $h \to \gamma\gamma$ in the broken phase:

$$\mathcal{L}^{\text{1-loop}}_{\mathrm{VLF}} \supset \frac{c_f n_D}{16\pi^2} \mathrm{tr_g} \left[g_9^i \left(S_{ii} F_i^{\mu\nu} F_{\mu\nu,i} \right) + g_{10}^i \left(P_{ii} \widetilde{F}_i^{\mu\nu} F_{\mu\nu,i} \right) \right]$$

$$= \frac{Q_E^2 e^2}{24\pi^2} \left(\sum_i \frac{y_{ii}}{m_i} \right) h\, A_{\mu\nu} A^{\mu\nu} - \frac{Q_E^2 e^2}{16\pi^2} \left(\sum_i \frac{\tilde{y}_{ii}}{m_i} \right) h\, \widetilde{A}_{\mu\nu} A^{\mu\nu}$$

$$= \frac{Q_E^2 e^2}{24\pi^2} \left(\frac{\partial \log\left(|\det \mathcal{M}_E| \right)}{\partial v} \right) h\, A_{\mu\nu} A^{\mu\nu} - \frac{Q_E^2 e^2}{16\pi^2} \left(\frac{\partial \arg\left(\det \mathcal{M}_E \right)}{\partial v} \right) h\, \widetilde{A}_{\mu\nu} A^{\mu\nu}$$

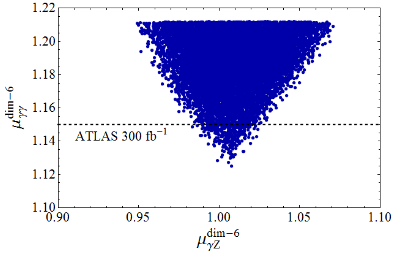
- Agreement with previous results in the literature!
- Agreement with symmetric phase calculation of $|H|^2 A_{\mu\nu} A_{\mu\nu}$ and $|H|^2 \widetilde{A}_{\mu\nu} A_{\mu\nu}$!

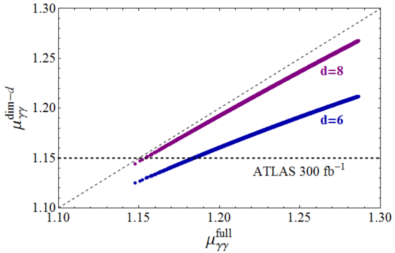
Loop–Induced Higgs Couplings: $h \to \gamma\gamma, \gamma Z$

- Where to expect better sensitivities?
  $\to$ Higgs couplings induced at 1 loop in the SM: $h \to \gamma\gamma$, $h \to \gamma Z$!
- Scan approach:

$$m_L, m_N, m_E \in [500, 1500]\ \mathrm{GeV},$$
$$y_{N_{L,R}}, y_{E_{L,R}} \in \left[2, \sqrt{4\pi}\right];$$

- Constraints $\to$ strong 1st order phase transition, $h \to \gamma\gamma$, EW precision tests
- $(h \to \gamma Z)_{\mathrm{NP}} \sim (-0.25 + s_W^2) \Rightarrow$ small impact for $h \to Z\gamma$:





Strong First Order Electroweak Phase Transition (SFOPT)

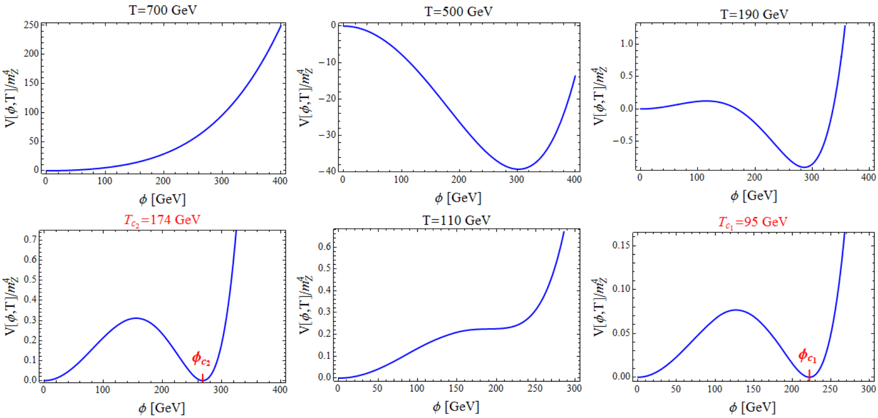


Figure: Typical thermal evolution of the potential → 1 crossover + 2 SFOPTs.

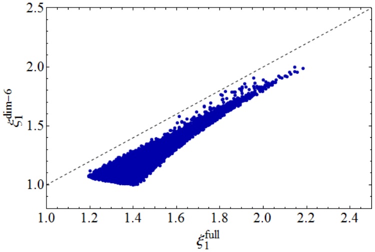
N.B.: Only the last SFOPT (responsible for BAU) can be captured at dim–6.

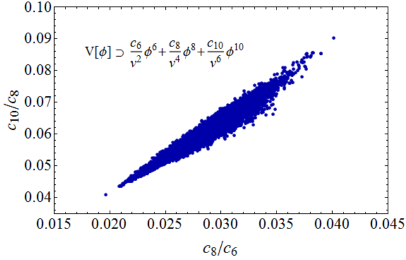
Strong First Order Electroweak Phase Transition (SFOPT)

- How much of the PT dynamics can be captured by $|H|^6$?

  → compare PT strength from full 1–loop potential vs $|H|^6$–only ⇒ good agreement!

  → check EFT validity ⇒ convergent expansion!





- Advantage?

  ⇒ simpler numerics (PT strength, GW calculation) with polynomial potential!

Overview

Summary and Conclusions

- Reviewed integration of heavy NP at 1–loop through path integral (functional) methods
- Functional methods for EFT matching → solid alternative to Feynman diagrams
  - + universal coefficients done once and for all ⇒ perhaps more model–independent?
  - + no tricky minus signs or symmetry factors!
- Applied functional methods to integrate out VLFs at 1–loop
  → Fermionic universal coefficients for nondegenerate masses computed for the 1st time!
- Argued how terms involving even powers of γ_5 follow immediately from terms with no γ_5's ⇒ a more streamlined calculation!
- Applied the results to a fermionic model for strong phase transitions

Bridging Heavy BSM to Experiment

$$E = \Lambda_{\mathrm{BSM}} : \mathcal{L}_{\mathrm{UV}}[\varphi_H, \varphi_L]$$

$\int$ out φ_H → done once and for all?

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i C_i(\Lambda_{\mathrm{BSM}})\mathcal{O}_i$$

RGE (done once and for all)

$$\mathcal{L}_{\mathrm{EFT}}[\varphi_L] = \mathcal{L}_{\mathrm{ren}} + \sum_i C_i(m_{\mathrm{EW}})\mathcal{O}_i \xrightarrow{\text{map (done once and for all)}} E = m_{\mathrm{EW}} : \text{observables}$$

THANK YOU FOR YOUR ATTENTION!

Bridging Heavy BSM to Experiment

$E = \Lambda_{\rm BSM} : \mathcal{L}_{\rm UV}[\varphi_H, \varphi_L]$

$\downarrow$ $\int$ out φ_H $\rightarrow$ done once and for all?

$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(\Lambda_{\rm BSM})\mathcal{O}_i$

$\downarrow$ RGE (done once and for all)

$\mathcal{L}_{\rm EFT}[\varphi_L] = \mathcal{L}_{\rm ren} + \sum_i C_i(m_{\rm EW})\mathcal{O}_i \xrightarrow{\text{map (done once and for all)}} E = m_{\rm EW}$: observables

THANK YOU FOR YOUR ATTENTION!