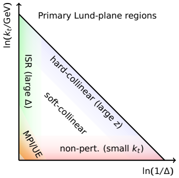
Calculating the QCD Primary Lund Jet Plane Density

Andrew Lifson

In collaboration with Gavin Salam and Gregory Soyez, hep-ph:2007.06578

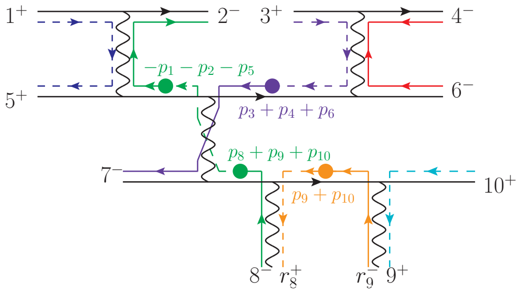
Work completed during master's thesis, 2018



Lund University

September 9^{th} 2020

Chirality-Flow: A Quick Interlude



In collaboration with Joakim Alnefjord, Christian Reuschle, and Malin Sjödahl.

Massless QED+QCD: hep-ph:2003.05877

Full Standard Model: Soon to be completed

$[ij]$ and $\langle ij\rangle$ are complex numbers

$$= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{s_{1\,2}\;s_{3\,4}\;s_{8\,9\,10}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{s_{1\,2\,5}\;s_{3\,4\,6}\;s_{8\,9\,10}\;s_{9\,10}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8r_8]\langle r_9 9\rangle}}_{\text{polarization vectors}} \quad [15]\langle 64\rangle[10\;9]$$

$$\times \left(\langle r_9 9\rangle[9 r_8] + \langle r_9 10\rangle[10 r_8]\right)\left(\underbrace{[33]}_{0}\langle 37\rangle + [34]\langle 47\rangle + [36]\langle 67\rangle\right)$$

$$\times \left(-\langle 89\rangle[91]\langle 12\rangle - \langle 89\rangle[95]\langle 52\rangle - \langle 8\,10\rangle[10\;1]\langle 12\rangle - \langle 8\,10\rangle[10\;5]\langle 52\rangle\right)$$

Overview

Why Look at Boosted Jets?

- Jets usually proxies for hard quark or gluon
- At (high) LHC energies, this correspondance breaks down
- Decay products of heavily-boosted objects ($p_t \gg m$) will be collimated
    - $\theta \sim m/p_t$
    - Clustered into same jet!
- We can miss interesting heavy physics
- Look inside a jet to determine its origin

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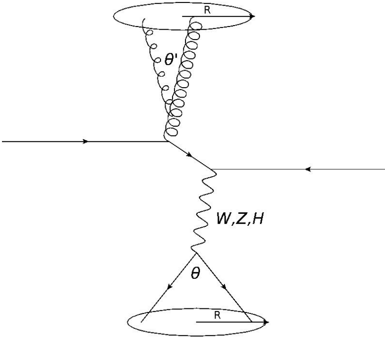

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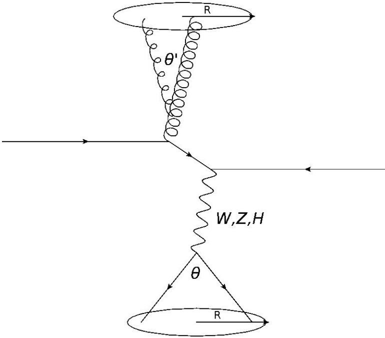

How to Look at Boosted Jets: Jet Substructure Predictions

Example: LL Integrated Cross Section of Jet Mass M_J

$$\Sigma_{\text{LO}}(m_J) \sim \frac{1}{\sigma_n} \int_{m_J/p_{TJ}}^{1} d\sigma_{n+g} \sim C_R \alpha_s \ln^2(p_{TJ}/m_J)$$

- $d\sigma_{n+g} \overset{\text{LL}}{=} \sigma_n C_R \frac{2\alpha_s}{\pi} \frac{dz}{z} \frac{d\theta}{\theta}$
- $\alpha_s \ln^2(1/m_{\text{jet}}) \gtrsim \mathcal{O}(1)$
  - Sudakov resummation
  - $\Sigma_{\text{res}} \sim e^{-\Sigma_{\text{LO}}}$
- Count logs, not α_s, have LL, NLL, etc.
- $\Sigma_{\text{LO}} = Lg_1(\alpha_s L) + g_2(\alpha_s L) + \ldots$

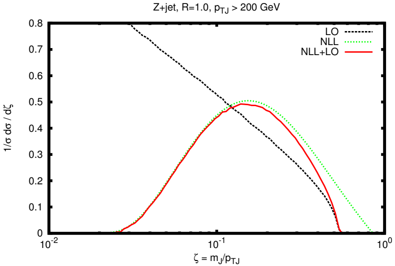

Image from G. Soyez

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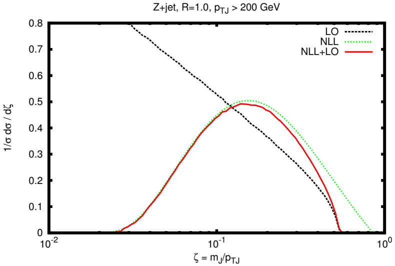

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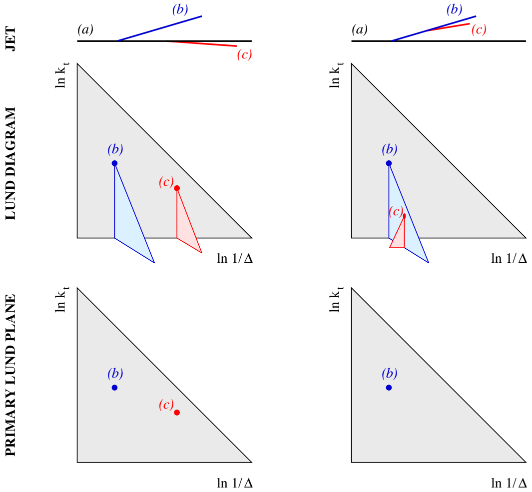
The Primary Lund Plane

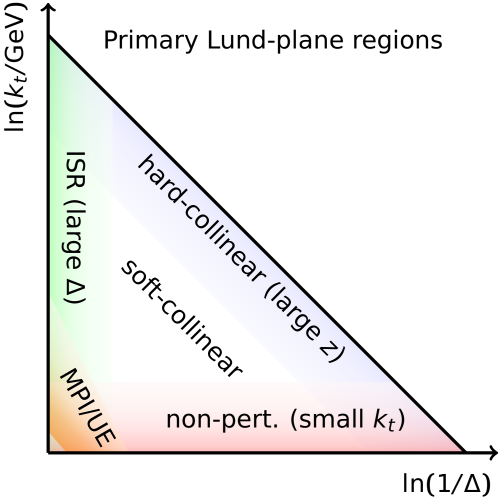
- $\Delta = \sqrt{(\Delta y)^2 + (\Delta\phi)^2}$
  $k_t = p_{t,b}\Delta$
- Used in MC, Resummation



Dreyer, Salam, Soyez hep-ph:1807.04758

The Primary Lund Plane

- $\Delta = \sqrt{(\Delta y)^2 + (\Delta\phi)^2}$
  $k_t = p_{t,b}\Delta$
- Used in MC, Resummation
- Useful to visualise emissions and logs within jet
- Nicely separates emission types



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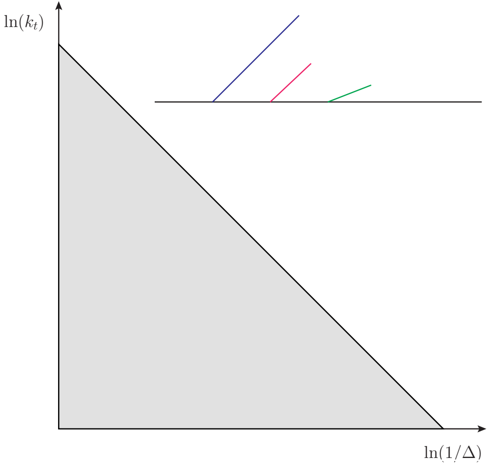
The Primary Lund Plane as an Observable

1. (Re-)Cluster all particles into a jet using C/A
2. De-cluster first particle, put softer in Lund plane
3. Following hardest branch, go back to step 2



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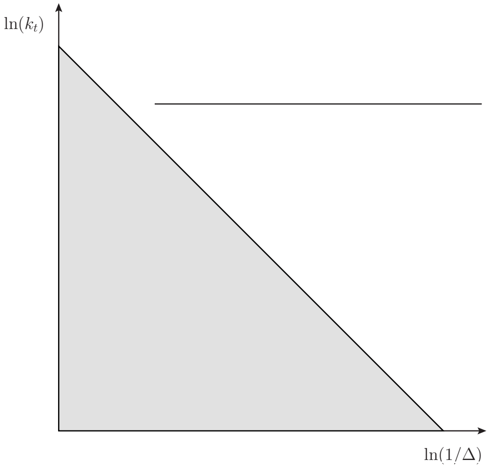

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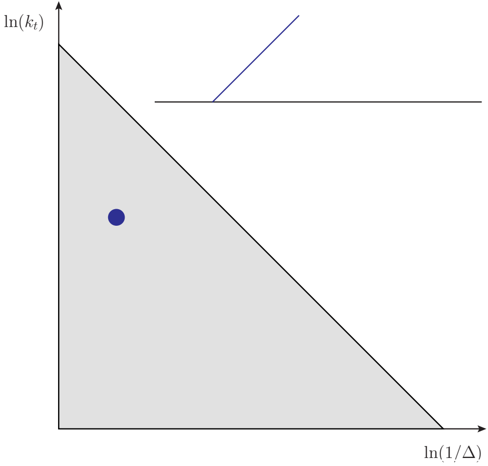

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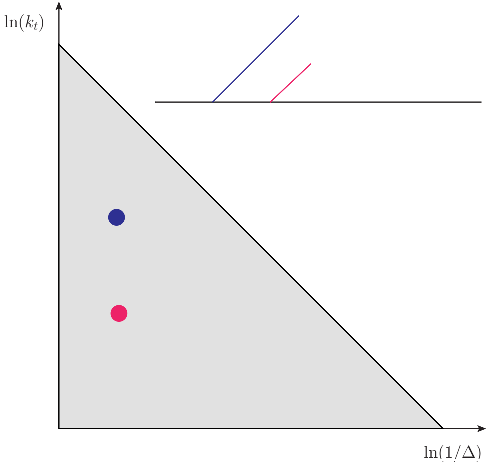

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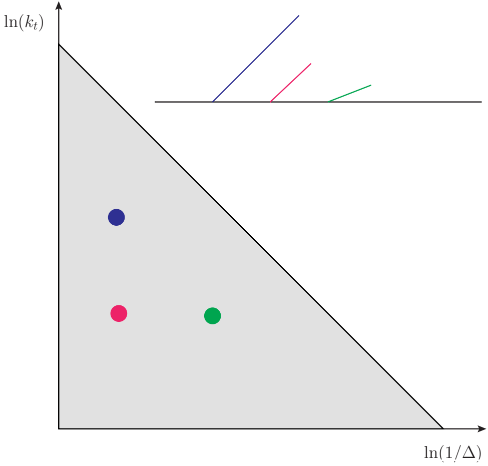

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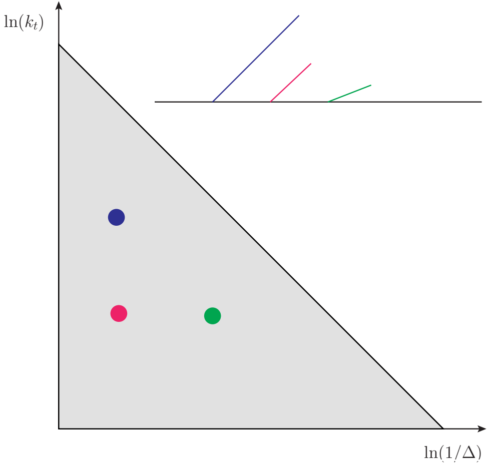
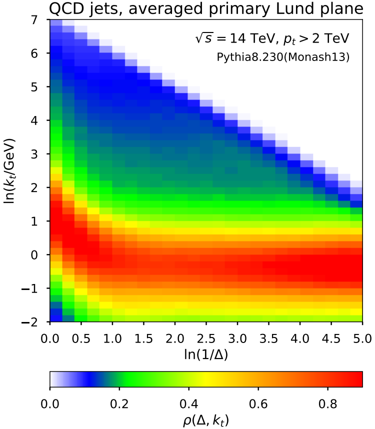
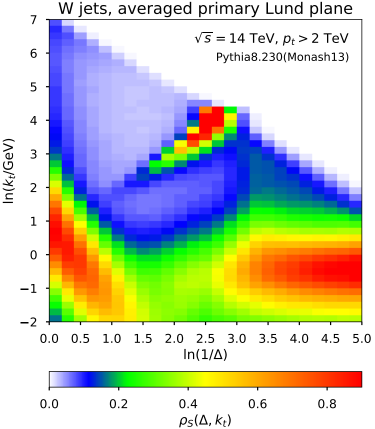


Observable Definition

$$\rho(\Delta, k_t) = \frac{1}{N_{\text{jets}}} \frac{dn_{\text{emissions}}}{d\ln(1/\Delta)d\ln(k_t)}$$

The Primary Lund Jet Plane Density: W vs QCD





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The QCD Primary Lund Jet Plane Density: Calculations

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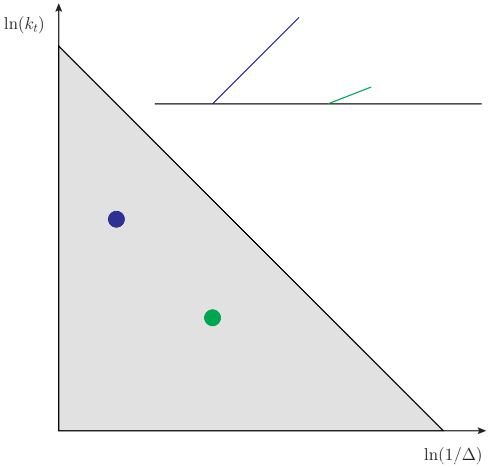

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$$\rho(\Delta, k_t) = \frac{1}{N_{\text{jets}}} \frac{dn_{\text{emissions}}}{d\ln(1/\Delta) d\ln(k_t)}$$

$$\rho_{\text{LL}} = \frac{2\alpha_s C_R}{\pi}$$

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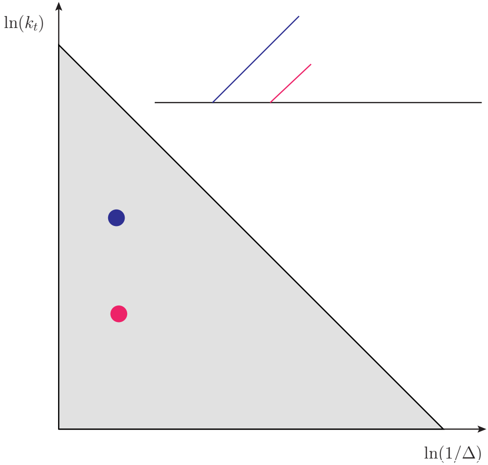

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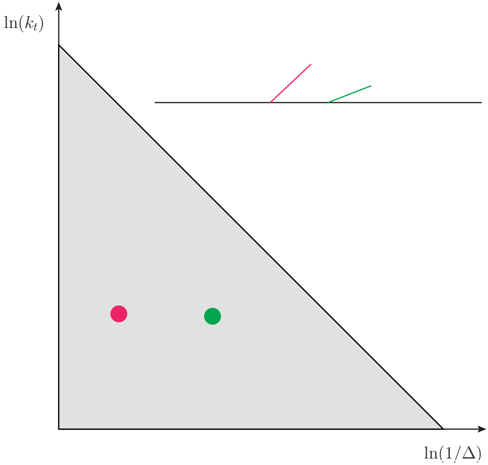

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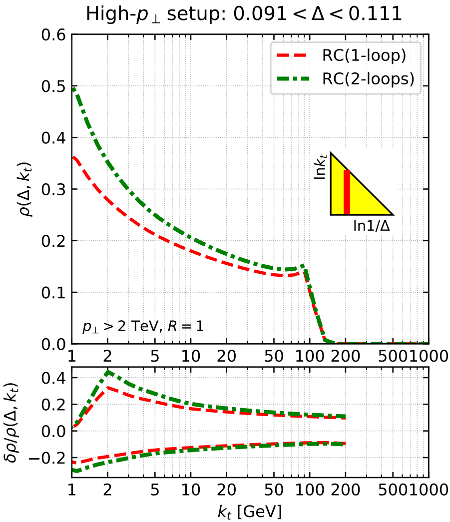
NLL Corrections to QCD Lund Jet Plane Density



- Running coupling ($\alpha_s(k_t)$)
  - Use 2 loop running
- Collinear effects ($\alpha_s^n \ln^{n-1}(\Delta)$)
- Soft effects ($\alpha_s^n \ln^{n-1}(k_t)$)
  - Soft & large angle emissions
  - C/A clustering effects

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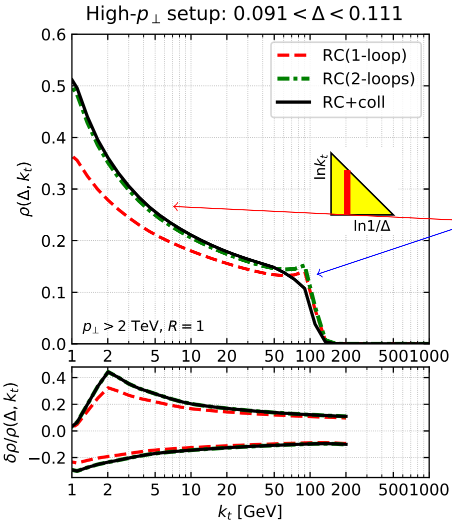

- Running coupling $(\alpha_s(k_t))$
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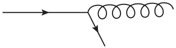
  - Loses energy $(z \sim \frac{1}{2})$
  - Changes flavour



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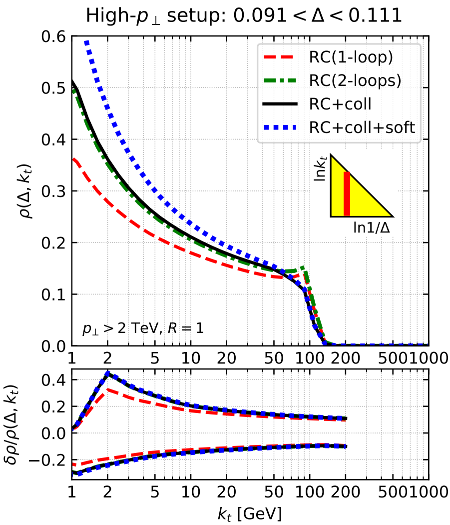

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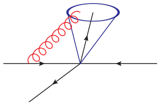

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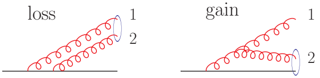
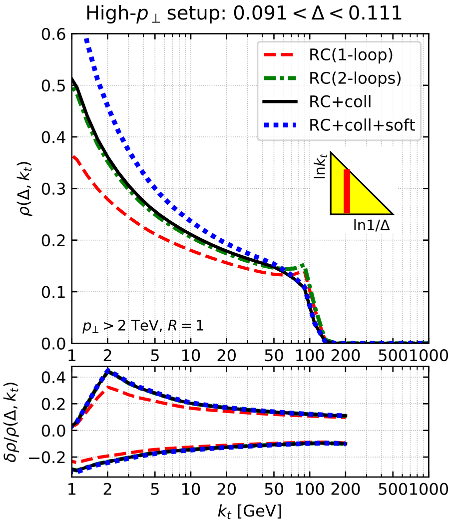


$z_1 >> z_2$, $\Delta_1 \sim \Delta_2$

NLL Corrections to QCD Lund Jet Plane Density



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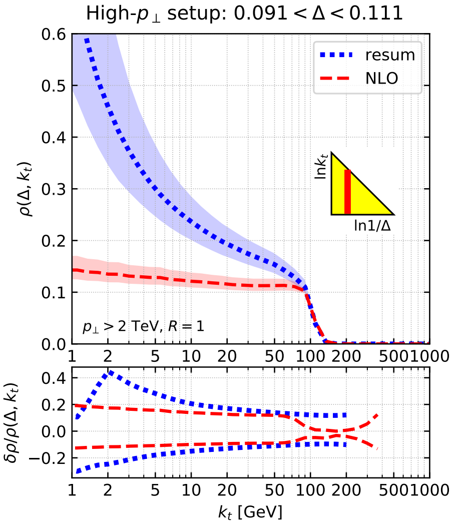
- Soft effects ($\alpha_s^n \ln^{n-1}(k_t)$)
  - Soft & large angle emissions
  - C/A clustering effects
- New logs found at jet edge
  - Not resummed
  - $\alpha_s^n \ln^{n-1}(R - \Delta)$
  - Interplay b/w anti-k_t, C/A

QCD Lund Jet Plane Density: Matching to NLO



- Exact $\mathcal{O}(\alpha_s^2)$ from NLOJet++
- Match NLO and resum
  - $\rho_{\text{NLO+resum}} = \frac{\rho_{\text{resum}}\rho_{\text{NLO}}}{\rho_{\text{resum,NLO}}}$

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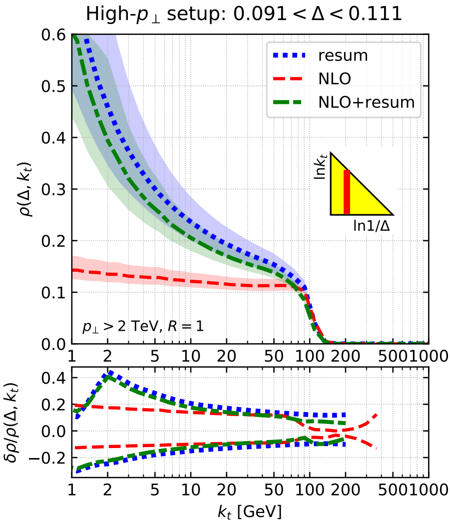

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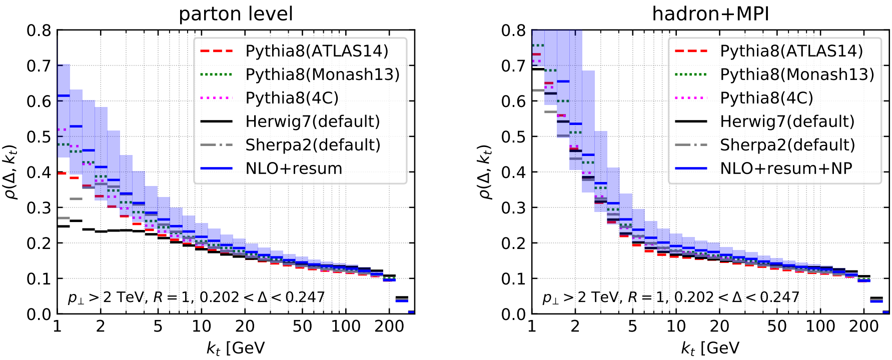
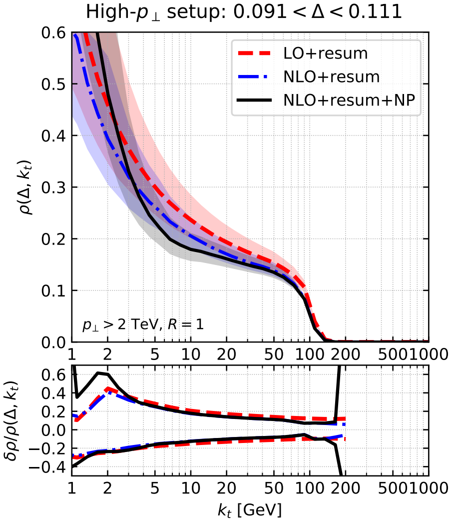
MC Comparisons



- Herwig 7.2.0, Pythia 8.2.30, Sherpa 2.2.8
- Good parton-level agreement in perturbative region
  - Disagreement Herwig $k_t \lesssim 5$ GeV, Sherpa $k_t \lesssim 1.5$ GeV
- Non-pert corrections made using average of Pythia & Sherpa
  - Herwig included in non-pert uncertainty

QCD Lund Jet Plane Density: Full Calculation



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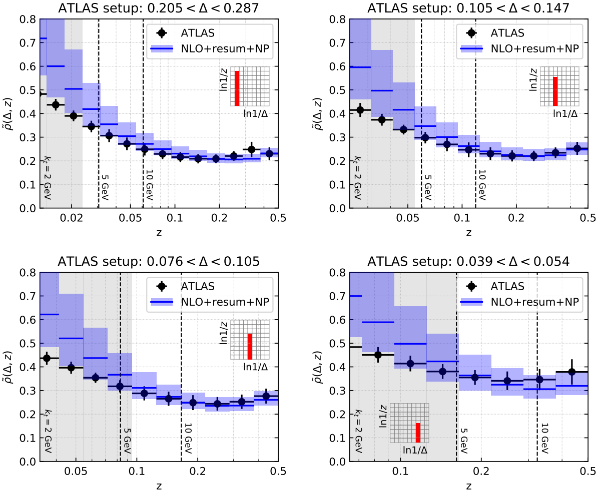
- Match NLO and resum
  - $\rho_{\mathrm{NLO+resum}} = \frac{\rho_{\mathrm{resum}}\rho_{\mathrm{NLO}}}{\rho_{\mathrm{resum,NLO}}}$
- Non-perturbative from MC
  - Pythia8 (3 tunes), Herwig7, Sherpa2
  - Averaged corrections
  - Required from $k_t \lesssim 20$ GeV

Comparison with ATLAS

Good agreement for $k_t \gtrsim 5$ GeV

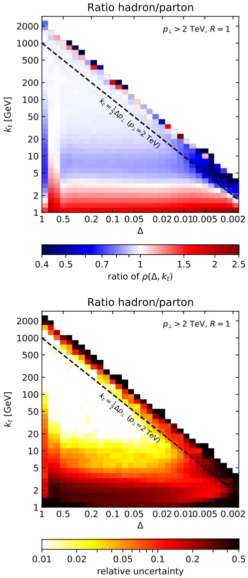


- Use $z \equiv$ mom frac
- Gray indicates large non-pert uncertainties

Summary

- High p_t jets can hide heavy particles
  - Required to look at their sub-structure to determine their origin
- Dreyer, Salam, Soyez recently proposed Lund-plane density as jet substructure observable
- Here we calculated observable at NLL for QCD jets
  - Resummed and matched to NLO
  - Hadronisation included
- Find good agreement between calculations and experiment above $k_t \gtrsim 5$ GeV
- Keep an eye out for massive chirality-flow paper

Non-perturbative MC corrections

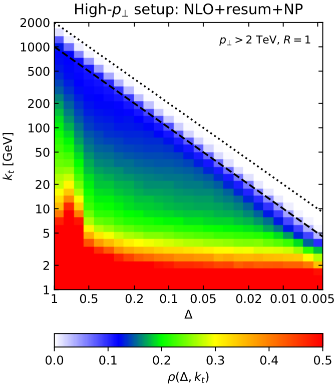


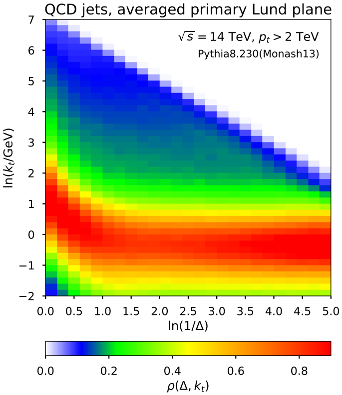
Origin of new $\ln(R - \Delta)$ Logs

- Occur at $\mathcal{O}(\alpha_s^2)$, with full form $\ln\left(\frac{R}{R-\Delta}\right)$
- Jet axis and distances different in anti-k_t, C/A
- Particles 1 and 2 collinear, both in anti-k_t jet
- After reclustering with C/A, particle 1 outside jet, particle 2 inside
- Can possibly remove with $R_{C/A}$ redef, e.g. $R_{C/A} \to \infty$
- NLL if $k_{t,1} \gg k_{t,2}$ (i.e. occurs with $\ln(k_t)$)
- If C/A for initial clustering, particle 2 clustered with 1
    - particle 2 on secondary Lund plane

Comparison to LL: Full Primary Lund Plane

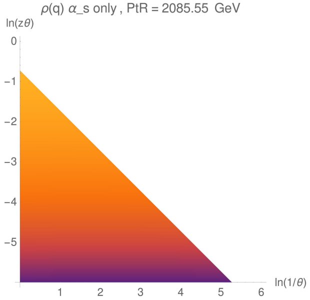
Different colour scales, axes

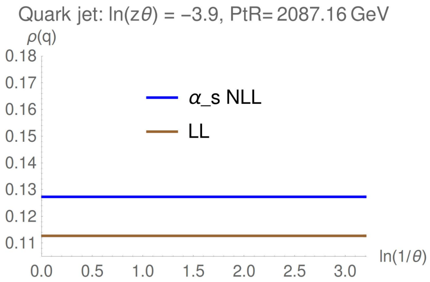




α_s Corrections

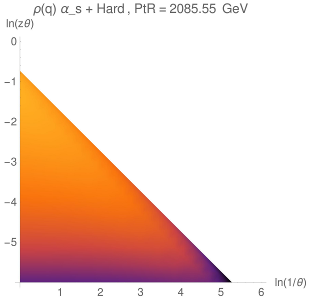
$$\alpha_s(k_t) = \frac{\alpha_s}{1 + 2\alpha_s\beta_0 \ln(z\theta)} - \alpha_s^2 \left\{ \frac{\beta_1}{\beta_0} \frac{\ln\left[1 + 2\alpha_s\beta_0 \ln(z\theta)\right]}{\left[1 + 2\alpha_s\beta_0 \ln(z\theta)\right]^2} - \frac{K}{2\pi} \frac{1}{\left[1 + 2\alpha_s\beta_0 \ln(z\theta)\right]^2} \right\}$$

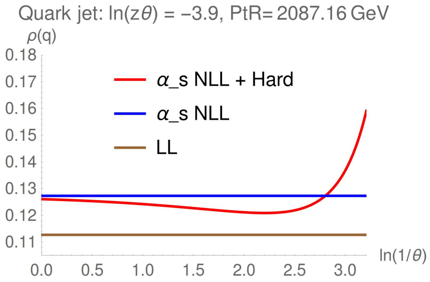




Hard collinear Corrections

- Use full splitting function
- $\frac{dz}{z} \to zP_{ij}(z)\frac{dz}{z}$



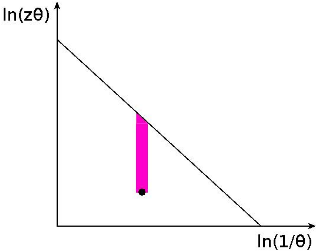


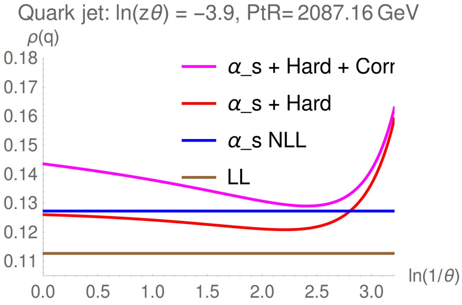
Correlated Emissions

$$M_{\mathrm{Corr}}(C_{R,\mathrm{jet}}) = \frac{2\alpha_s^{(0)}(k_t)}{\pi} C_{R,\mathrm{jet}} \frac{1}{\pi\beta_0} \ln\left[\frac{1-\lambda(\theta)}{1-\lambda(z\theta)}\right] K_A\left[C_A - C_{R,\mathrm{jet}}\right]$$

$$\lambda(x) = 2\alpha_s(p_t R)\beta_0 \ln(1/x)$$

- 2 emissions at similar angle
- Clustering ambiguous

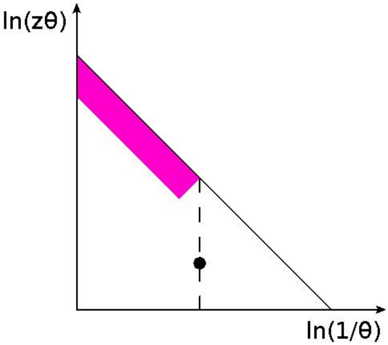




Flavour Changing Corrections

- Hard branch changes if $z > 1/2$ in $P_{gq}(z)$, or $g \to qq$

$$\begin{pmatrix} f_q(\theta) \\ f_g(\theta) \end{pmatrix} = \frac{1}{C_q + C_g} \begin{pmatrix} C_q - e^{-(C_g+C_q)I_\theta}[C_q(1 - f_q(\theta_0)) - C_g f_q(\theta_0)] \\ C_g + e^{-(C_g+C_q)I_\theta}[C_q(1 - f_q(\theta_0)) - C_g f_q(\theta_0)] \end{pmatrix}$$

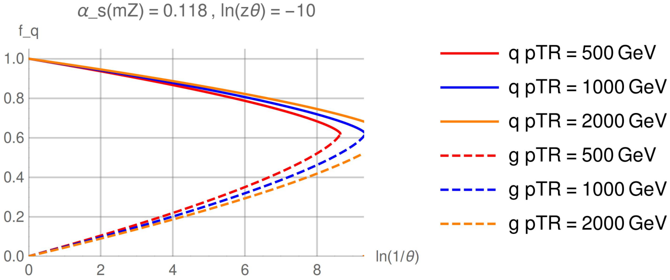


Flavour Changing Corrections

- Hard branch changes if $z > 1/2$ or $g \to qq$

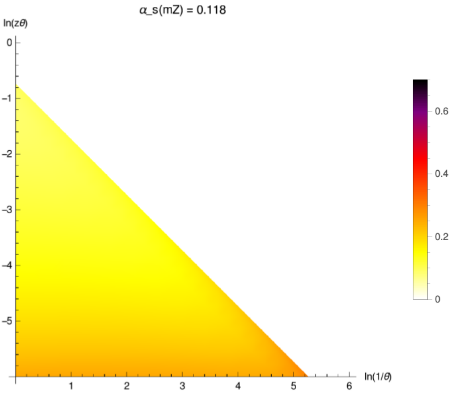
$$\begin{pmatrix} f_q(\theta) \\ f_g(\theta) \end{pmatrix} = \frac{1}{C_q + C_g} \begin{pmatrix} C_q - e^{-(C_q+C_g)I_\theta}[C_q(1 - f_q(\theta_0)) - C_g f_q(\theta_0)] \\ C_g + e^{-(C_q+C_g)I_\theta}[C_q(1 - f_q(\theta_0)) - C_g f_q(\theta_0)] \end{pmatrix}$$

- At small angles, **Always** $\sim 63\%$ quarks

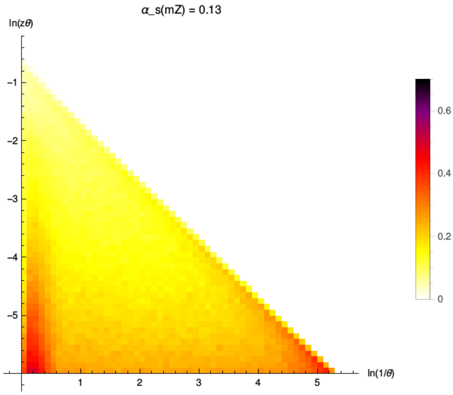


Monte Carlo Comparison: Quark Densities

Compared to Pythia 8.235 at parton level



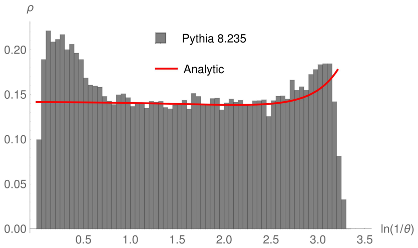
(a) Analytical



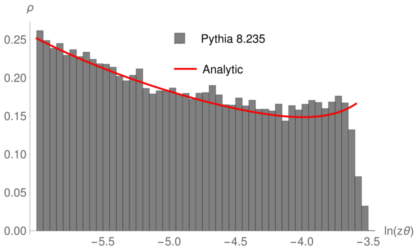
(b) Pythia

Monte Carlo Comparison: Quark Density Slices

Compared to Pythia 8.235 at parton level



(c) Quark: slice $\ln(z\theta) \in (-3.8, -4.0)$



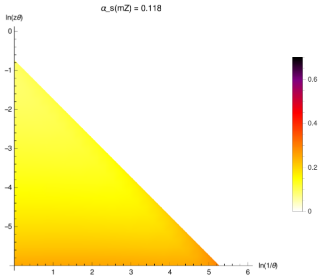
(d) Quark: slice $\ln(1/\theta) \in (2.8, 3.0)$

Quark-Gluon Differences

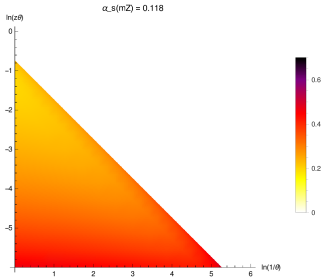
- Distribution in Lund plane Poisson-distributed:

$$P_j(n_i) = \frac{1}{n_i!} e^{-\rho_j} \rho_j^{n_i}$$

- ρ_j given by my NLL density prediction for $j = q, g$



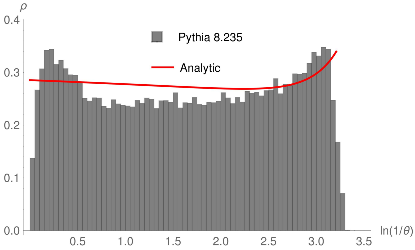
(e) Quark



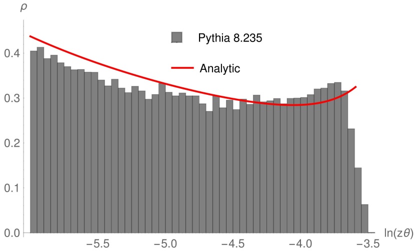
(f) Gluon

Gluon Slices

Compared to Pythia 8.235 at parton level



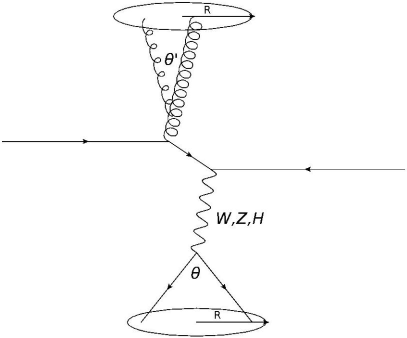
(g) Gluon: slice $\ln(z\theta) \in (-3.8, -4.0)$



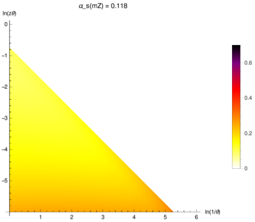
(h) Gluon: slice $\ln(1/\theta) \in (2.8, 3.0)$

Jet substructure

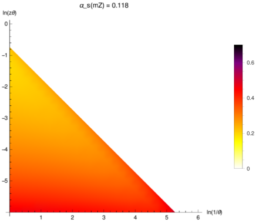
- Look inside a jet to determine its substructure
- Can count hard prongs
  - top $= 3$, $W, Z, H = 2$, $q, g = 1$, BSM $=$?
- Look at differences in radiation pattern
  - g or q? Use e.g. $C_A \approx 2C_F$



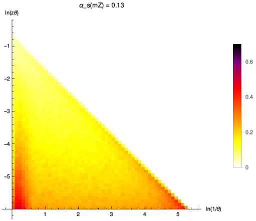
Pythia Densities



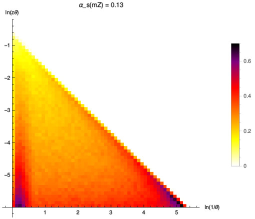
(i) Analytical: Quark



(j) Analytical: Gluon



(k) Pythia: Quark



(l) Pythia: Gluon

Full Results

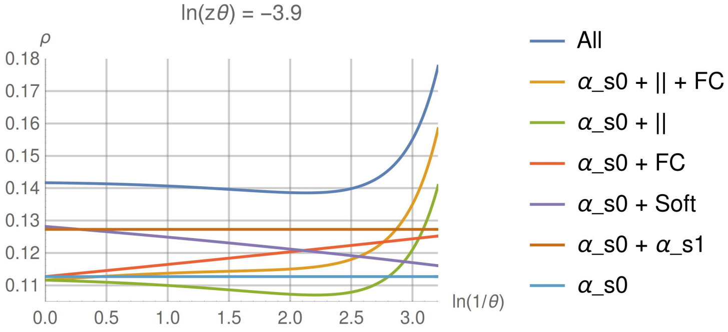
$$\rho = M^{(0)} \left(C_F \left[1 + M^{(1)}_{\alpha_s} + M^{(1)}_{\parallel,q} + M^{(1)}_{\text{soft}}(C_F) \right] \quad C_A \left[1 + M^{(1)}_{\alpha_s} + M^{(1)}_{\parallel,g} \right] \right) \cdot \begin{pmatrix} f_q(\theta) \\ f_g(\theta) \end{pmatrix}$$

$M^{(0)}$	$\frac{2\alpha_s^{(0)}(k_t)}{\pi} \Theta(z < 1/2)$
$M^{(1)}_{\alpha_s}$	$\alpha_s^{(1)}(k_t)/\alpha_s^{(0)}(k_t)$
$M^{(1)}_{\parallel,q}$	$\left(-z + \frac{z^2}{2} + \frac{z}{2} \frac{1+z^2}{1-z} \right) \Theta(z < 1/2)$
$M^{(1)}_{\parallel,g}$	$\left[-z + \frac{z^2}{1-z} + z^2(1-z)^2 + n_f z \frac{T_R}{2C_A} \left(z^2 + (1-z)^2 \right) \right] \Theta(z < 1/2)$
$M^{(1)}_{\text{soft}}(C_F)$	$\frac{1}{\pi\beta_0} \ln \left[\frac{1-\lambda(\theta/2)}{1-\lambda(z\theta)} \right] K_A \left[C_A - C_F \right]$
$f_q(\theta)$	$\frac{1}{C_g+C_q} \left(C_q - (1-\lambda(\theta))^{\frac{C_g+C_q}{2\pi\beta_0}} \left[C_q(1-f_q(\theta_0)) - C_g f_q(\theta_0) \right] \right)$
$f_g(\theta)$	$\frac{1}{C_g+C_q} \left(C_g + (1-\lambda(\theta))^{\frac{C_g+C_q}{2\pi\beta_0}} \left[C_q(1-f_q(\theta_0)) - C_g f_q(\theta_0) \right] \right)$

Table: Summary of NLL functions which appear above. We remind that $z = e^{\ln(z\theta)+\ln(1/\theta)}$, $\lambda(x) = 2\alpha_s\beta_0 \ln(1/x)$, $C_g = C_F\,(2\ln(2) - 5/8)$, $C_q = n_f T_R 2/3$, and $K_A = 0.3231$.

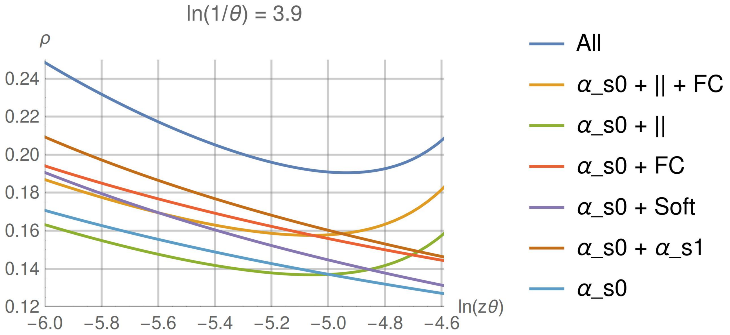
Effects of Different Contributions

Quark density with different effects



Effects of Different Contributions

Quark density with different effects



DGLAP Splitting Functions

$$P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z}$$
$$P_{gg}(z) = 2C_A \left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right)$$
$$P_{qg}(z) = T_R \left(z^2 + (1-z)^2 \right)$$
$$P_{xg}(z) = \frac{1}{2} P_{gg}(z) + n_f P_{qg}(z)$$

QCD Running Coupling

$$\alpha_s(Q^2) = \frac{\alpha_s}{1 + \alpha_s \beta_0 \ln\left(\frac{Q^2}{\mu_R^2}\right)}$$

$$- \alpha_s^2 \left\{ \frac{\beta_1}{\beta_0} \frac{\ln\left[1 + \alpha_s \beta_0 \ln\left(Q^2/\mu_R^2\right)\right]}{\left[1 + \alpha_s \beta_0 \ln\left(Q^2/\mu_R^2\right)\right]^2} - \frac{K}{2\pi} \frac{1}{\left[1 + \alpha_s \beta_0 \ln\left(Q^2/\mu_R^2\right)\right]^2} \right\}$$

$$\beta_0 = \frac{11 C_A - 4 n_f T_R}{12\pi} = \frac{33 - 2n_f}{12\pi}$$

$$\beta_1 = \frac{17 C_A^2 - n_f T_R(10 C_A + 6 C_F)}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_f = \frac{67 - 3\pi^2}{6} - \frac{5}{9} n_f$$

Jet Algorithm: The Generalised k_t algorithm [?]

- Iteratively cluster together "closest" particles in list
- Two distance types:
  - **Interparticle** $d_{ij} = \min\left(p_{t,i}^{2p}, p_{t,j}^{2p}\right)\theta_{ij}^2$
  - **Beam** $d_{iB} = p_{t,i}^{2p} R^2$
- Build all distances, find smallest
  - If smallest is d_{ij}, combine $i + j \to k$, put k in list
  - Else if d_{iB} is smallest, i is called a jet, removed from list
- p is free parameter
  - $p = 1$: k_t algorithm, soft first
  - $p = 0$: C/A algorithm, angular ordered
  - $p = -1$: anti-k_t algorithm, most circular