Efficient Multi-Jet Merging at High Multiplicities

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Parton Showers and Matrix Elements

- parton showers work well in **soft** and **collinear** phase space regions
- **outside** these regions, higher-order perturbative corrections needed
- prime example: Drell-Yan processes, e.g. $Z$ production at Tevatron:

[Höche, Krauss, Schumann, Siegert 0903.1219]
Combining ME+PS

- **Goal:** combine multiple configurations with different multiplicities, each dressed with parton shower:

![Graph showing Z boson transverse momentum](image)

- $p_T,Z$ [GeV]
- $d\sigma/dp_T,Z$ [pb/GeV]
Combining ME+PS

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![Z Boson Transverse Momentum, pp at $\sqrt{s} = 14$ TeV]

- double-counts emissions
- both samples are inclusive, i.e. consider at least 0/1 jet(s)
Combining ME+PS

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  \[ pp \rightarrow Z \]
  \[ pp \rightarrow Z + j \]

  \[
  \frac{d\sigma}{dp_T,Z} \quad [\text{pb}/\text{GeV}]
  \]

  - simply adding the two **double-counts** emissions
    - both samples are **inclusive**, i.e. consider at least 0/1 jet(s)
How to do better: MEPS

- **Merging**: introduce (somewhat arbitrary) **merging scale** and let each calculation populate the phase space where it does best:
  - **Parton shower** generates **soft/collinear** radiation
  - **Fixed-order calculation** generates **hard** jet(s)

Now safe to combine samples, schematically:

$$B_{\text{merged}} = B_{0\text{-jet}}(< t_{\text{MS}}) + \Delta(0 \rightarrow 1) B_{1\text{-jet}}$$
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![Graph of Z Boson Transverse Momentum](image)

```
\begin{align*}
\left< Z \text{ Boson Transverse Momentum, } \sqrt{s} = 14 \text{ TeV} \right>
\end{align*}
```

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![Graph showing Z Boson Transverse Momentum for pp at $\sqrt{s} = 14$ TeV]

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What we can learn from history

- basic CKKW idea [Catani, Krauss, Kuhn, Webber hep-ph/0109231], [Krauss hep-ph/0205283]:
  - select hard $n$-jet configuration
  - construct “shower history”
  - re-weight event by Sudakov factors between nodes

\[ \Delta(Q_0^2, Q^2) \]

\[ \Delta(Q_0^2, Q'{}^2) \]
Merging schemes

- could use deterministic jet clustering and analytic Sudakovs, but want to do better:
  - use the same Sudakov factor in reweighting as in shower
  - account for shower evolution in history construction (or vice versa)
- solved in different ways, e.g.
  - CKKW-L [Lönnblad hep-ph/0112284], [Lönnblad, Prestel 1109.4829]
  - METS [Höche, Krauss, Schumann, Siegert 0903.1219], [Hamilton, Richardson, Tully 0905.3072]
- here consider CKKW-L:
  - construct all possible shower histories, choose most likely
  - let (truncated) trial showers generate Sudakov factors
The scaling of the method: complexity

- to include the $pp \rightarrow Z + 2 \text{ jet}$ matrix element:

- **conventional** showers: $\mathcal{O} (2^n n!)$ histories contribute to $n$-particle final state

<table>
<thead>
<tr>
<th>Number of Histories for $n$ Branchings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 1$</td>
</tr>
<tr>
<td>CS Dipole</td>
</tr>
<tr>
<td>Global Antenna</td>
</tr>
</tbody>
</table>

(starting from one colour-anticolour pair)
The scaling of the method: time

- hard process most expensive, but can be generated “once and for all”
- deterministic jet clustering ("WTA" – winner takes all) improves scaling, but does not represent shower history

[Höche, Prestel, Schulz 1905.05120]
**Sector showers**: combine antenna shower with deterministic jet-clustering algorithm

- let shower only generate emissions that would be clustered by a $(3 \rightarrow 2)$ jet algorithm ($\sim \text{ARCLUS}$ [Lönnblad Z.Phys.C 58 (1993)])

![Diagram of shower processes]

- only **one** (most singular) splitting kernel contributes per phase space point

\[
p^2_{\perp,ijk} = \frac{2(p_i \cdot p_j)(p_j \cdot p_k)}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}
\]
Phase space sectors

- branching phase space gets divided into **non-overlapping sectors**
- e.g. for FF $gg \rightarrow ggg$ branchings:

![Diagrams showing phase space sectors](image)

(Adapted from P. Skands)

- branchings in the shower are accepted **if and only if** they correspond to the **correct sector**
- sectors defined by minimal $p_\perp$ in event, but always contain:
  - soft endpoint
  - “full” collinear region for $qg$
  - “half” of the collinear region for $gg$ with boundary at $z = \frac{1}{2}$

**Note:** in general, highly non-trivial sector boundaries away from the singular limits!
Sector antenna functions

- splitting kernels have to incorporate **full** single-unresolved limits for given PS point (Kosower subtraction terms [Kosower PRD 57 (1998) 5410, PRD 71 (2005) 045016])

  e.g. \( qg \mapsto qgg \) (FF):

\[
A_{q_iq_jq_k}(s_{ij}, s_{jk}, s_{ik}) \rightarrow \begin{cases} 
\frac{2s_{ik}}{s_{ij} s_{jk}} & \text{if } j \text{ soft} \\
\frac{1}{s_{ij}} p_{q_i} \mapsto qg(z) & \text{if } i \parallel j \quad \text{where } s_{ij} = 2p_i \cdot p_j \\
\frac{1}{s_{jk}} p_{g_j} \mapsto gg(z) & \text{if } j \parallel k
\end{cases}
\]

- **VINCIA** choice: GGG antenna functions
  [Gehrmann-de Ridder, Gehrmann, Glover JHEP 09 (2005) 056] (+ non-singular modifications)

\[
A(i, j, k) = \frac{|\mathcal{M}_3^0(i, j, k)|^2}{|\mathcal{M}_2^0(l, K)|^2} \quad \text{for } IK \mapsto ijk
\]
merging with sector shower relatively straight-forward:
start from CKKW-L and modify **history construction**

modulo quark permutations, sector showers have a **single history**, i.e. a **unique** inverse

to account for all possible \(q\bar{q}\)-clusterings, find all viable colour orderings

for each colour-ordering, shower history again **uniquely** defined by sectors

if multiple colour-orderings possible, choose one that **maximises** branching probability
Scaling revisited: time

- CPU time in $pp \rightarrow W^- + 9\ jet$ merging:
  
  **CPU time / history**

  ![Graph showing CPU time per history for $pp \rightarrow W^- + 9\ jet$.]

  **CPU time / event**

  ![Graph showing CPU time per event for $pp \rightarrow W^- + 9\ jet$.]

**Note:** kink for 9-jet sample on the rhs due to inclusive treatment of last node in CKKW-L
Scaling revisited: resources

- **memory** allocation/deallocation per 1k events in $pp \rightarrow Z + 10$ jet merging:

Note: the 10th jet only hypothetical, so that every multiplicity is treated as “intermediate” node
First results

- Parton-level results for merging in $pp \to Z$ with up to 9 additional jets (using event files from [Höche, Prestel, Schulz 1905.05120])

$k_\perp 1$-Jet Resolution ($D = 0.4$), $pp \to Z \to e^+e^-, \sqrt{s} = 14$ TeV

$k_\perp 9$-Jet Resolution ($D = 0.4$), $pp \to Z \to e^+e^-, \sqrt{s} = 14$ TeV
First results

- comparison with **Pythia** for $pp \rightarrow W^- + 9$ jets @parton level (using event files from [Höche, Prestel, Schulz 1905.05120])

$k_1$ 1-Jet Resolution ($D = 0.4$), $pp \rightarrow W^- \rightarrow e^- \bar{\nu}_e$, $\sqrt{s} = 14$ TeV

$k_9$ 9-Jet Resolution ($D = 0.4$), $pp \rightarrow W^- \rightarrow e^- \bar{\nu}_e$, $\sqrt{s} = 14$ TeV
Conclusions

- sector showers combine shower evolution with jet clustering to become uniquely invertible
- ME+SS combines simplicity of CKKW with accuracy of CKKW-L
  - linear scaling for time to construct histories
  - constant overall run time and memory usage
- stand-alone sector showers slower than conventional ones, but smarter sampling possible
- sector shower + merging implemented in VINCIA shower in Pythia

This is just the beginning...

- sector showers well validated against “global” VINCIA and Pythia
  (discontinuities? still searching...)
- sector-based merging so far rather academic study, stress test yet to follow
  (extension to NLO desirable)
- VINCIA sector showers + merging getting ready for Pythia release (8.304?)
  - VINCIA’s QCD showers fully “sectorised” (II, IF, FF, RF)
  - idea is to replace existing “global” showers
Backup: sector resolution criterion

For final-final clusterings, sector resolution defined by:

$$Q_{\text{res}, j}^2 = \begin{cases} \frac{s_{ij}s_{jk}}{s_{ijk}} & \text{if } j \text{ is a } g \\ s_{ij} \sqrt{\frac{s_{jk}}{s_{ijk}}} & \text{if } (i, j) \text{ is a } q\bar{q} \text{ pair} \end{cases}$$

Sectors defined by:

$$\Theta_{\text{sct}, j} = \theta(\min\{Q_{\text{res}, i}^2 - Q_{\text{res}, j}^2\})$$  \(1\)
Backup: some sector shower validations

Note: no dedicated “sector tune”, no MECs
Backup: memory profiles

Heap Profile Z + 0 Jets

Heap Profile Z + 3 Jets

Heap Profile Z + 6 Jets

Heap Profile Z + 9 Jets