Efficient Multi-Jet Merging at High Multiplicities

Christian T Preuss

In collaboration with Helen Brooks and Peter Skands

Monash University

MCnet Meeting
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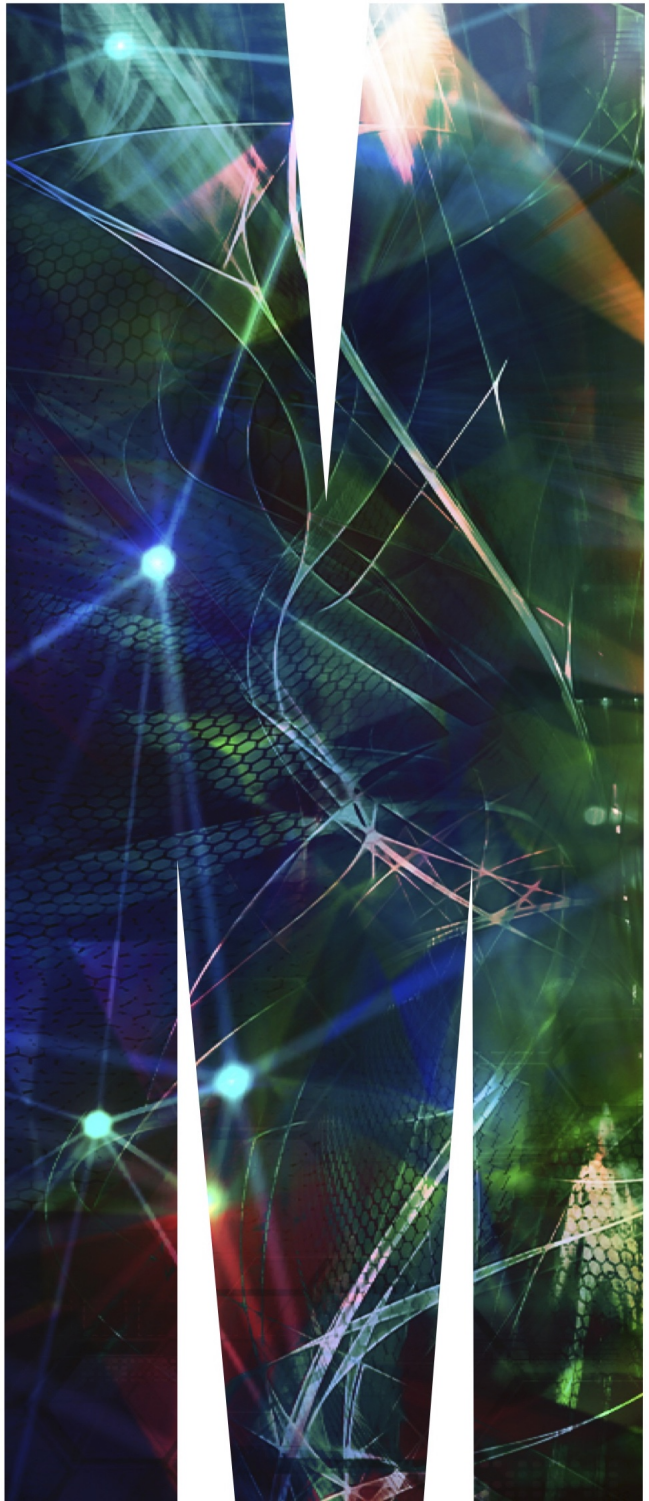
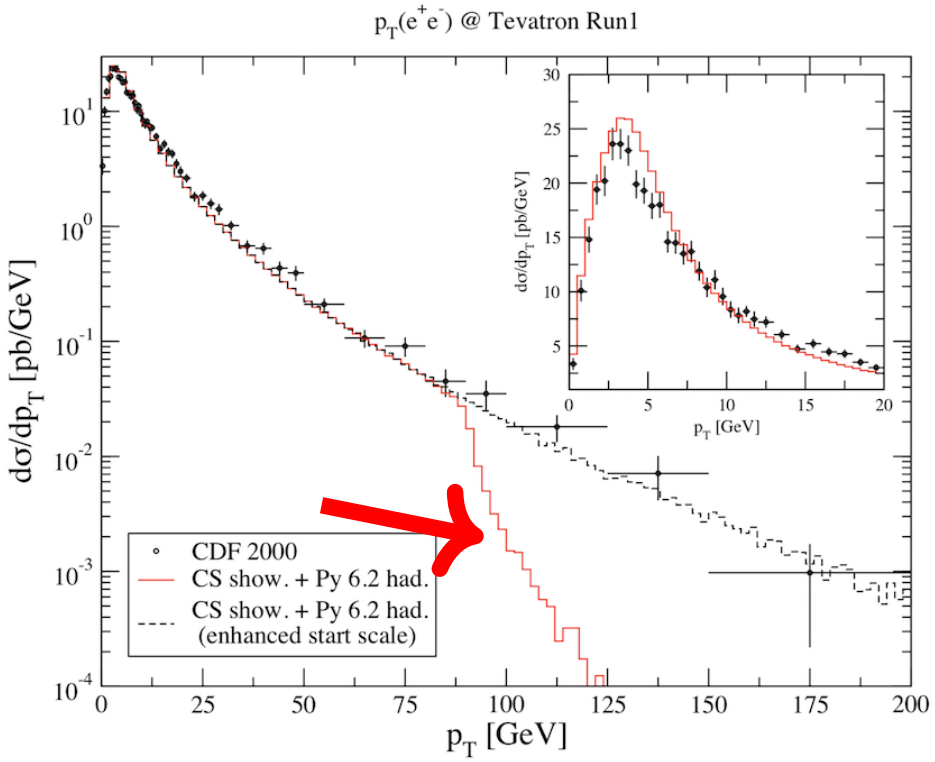


Parton Showers and Matrix Elements

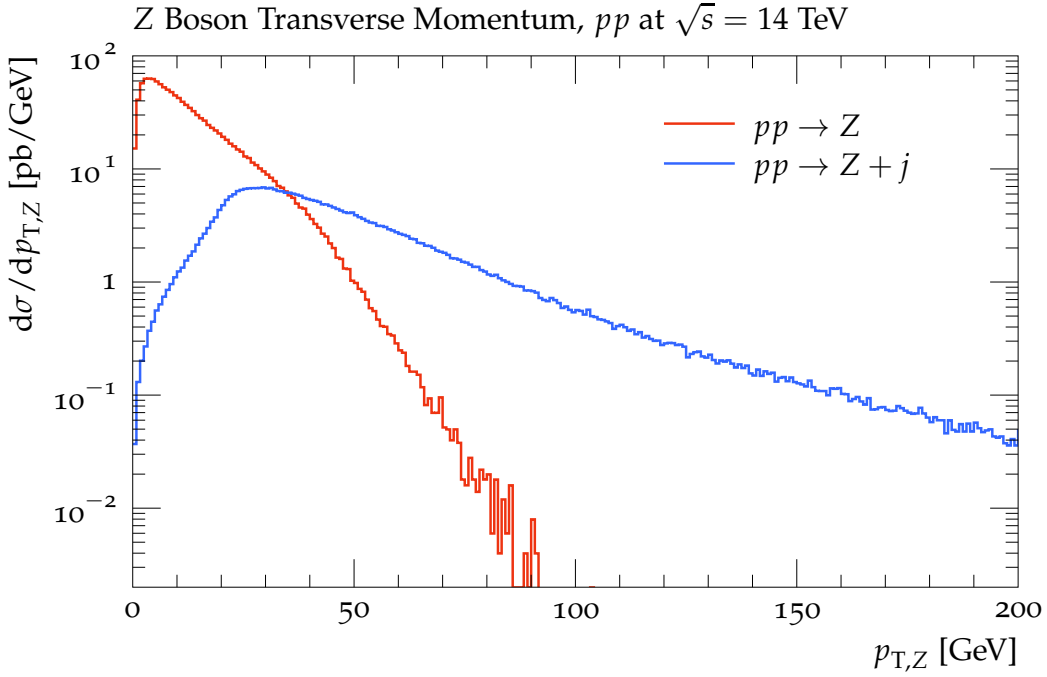
- parton showers work well in **soft** and **collinear** phase space regions
- **outside** these regions, higher-order perturbative corrections needed
- prime example: Drell-Yan processes, e.g. Z production at Tevatron:



[Höche, Krauss, Schumann, Siegert 0903.1219]

Combining ME+PS

- **Goal:** combine multiple configurations with different multiplicities, each dressed with parton shower:



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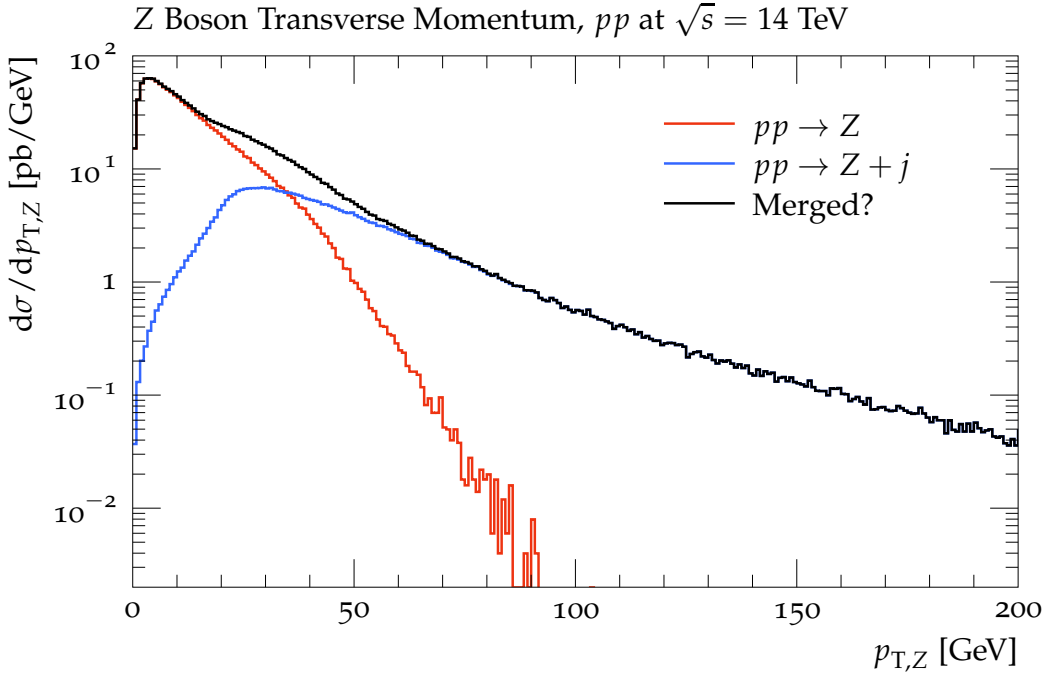

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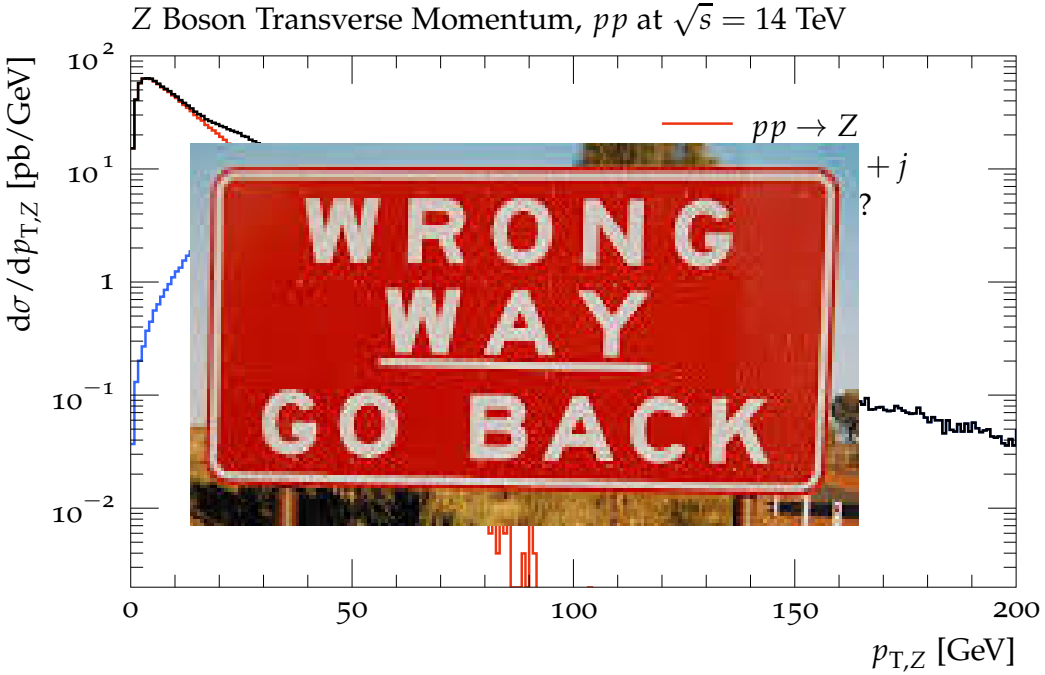
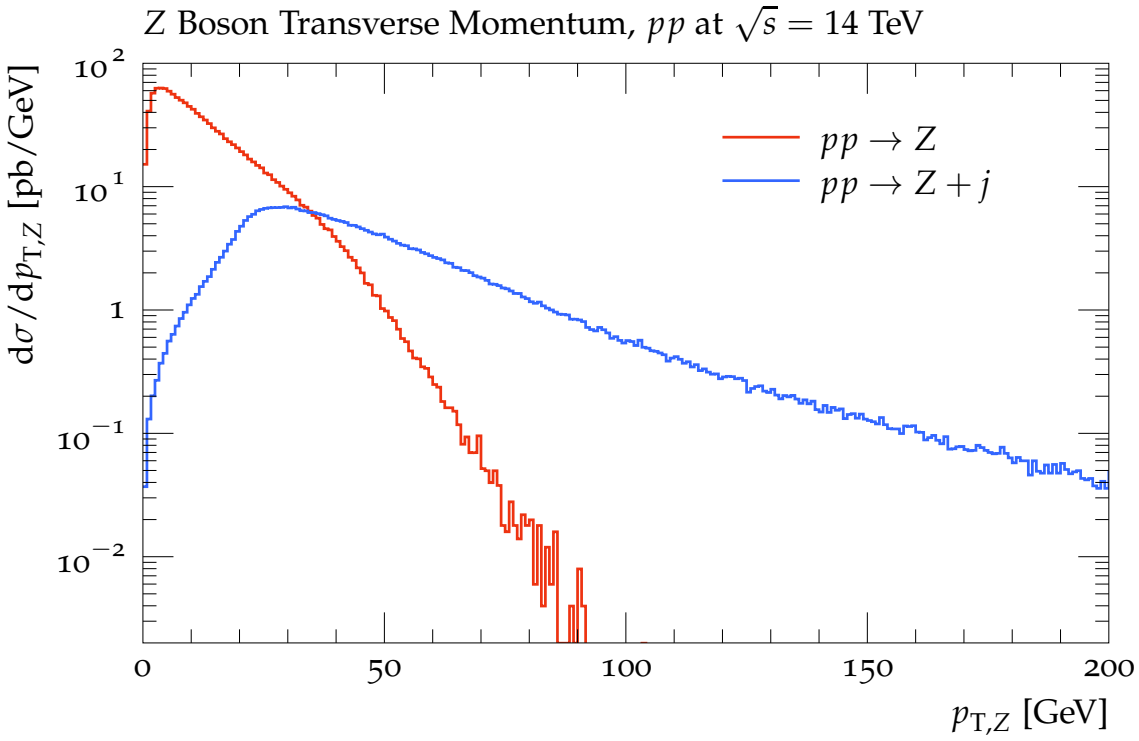


- simply adding the two **double-counts** emissions
  - both samples are **inclusive**, i.e. consider **at least** 0/1 jet(s)

How to do better: MEPS

- **Merging:** introduce (somewhat arbitrary) **merging scale** and let each calculation populate the phase space where it does best:

  Parton shower generates **soft/collinear** radiation
  Fixed-order calculation generates **hard** jet(s)



- now safe to combine samples, schematically:

$$\mathcal{B}_{\text{merged}} = \mathcal{B}_{\text{0-jet}}(< t_{\text{MS}}) + \Delta(0 \mapsto 1)\mathcal{B}_{\text{1-jet}}$$

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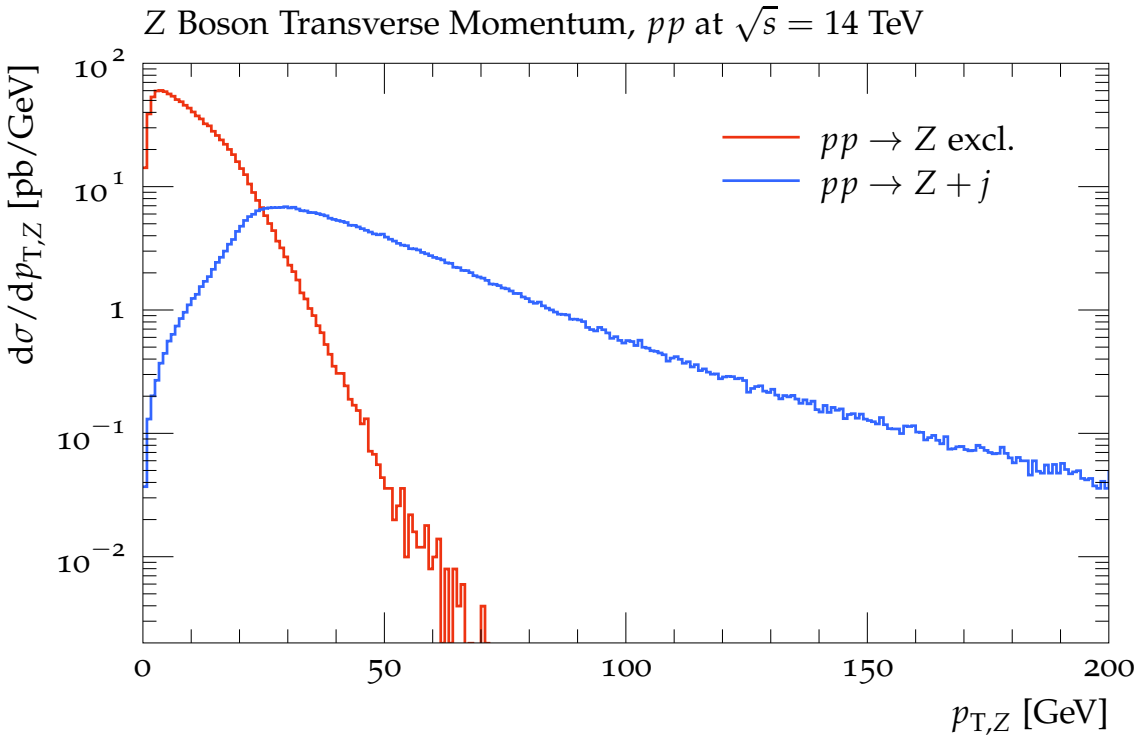

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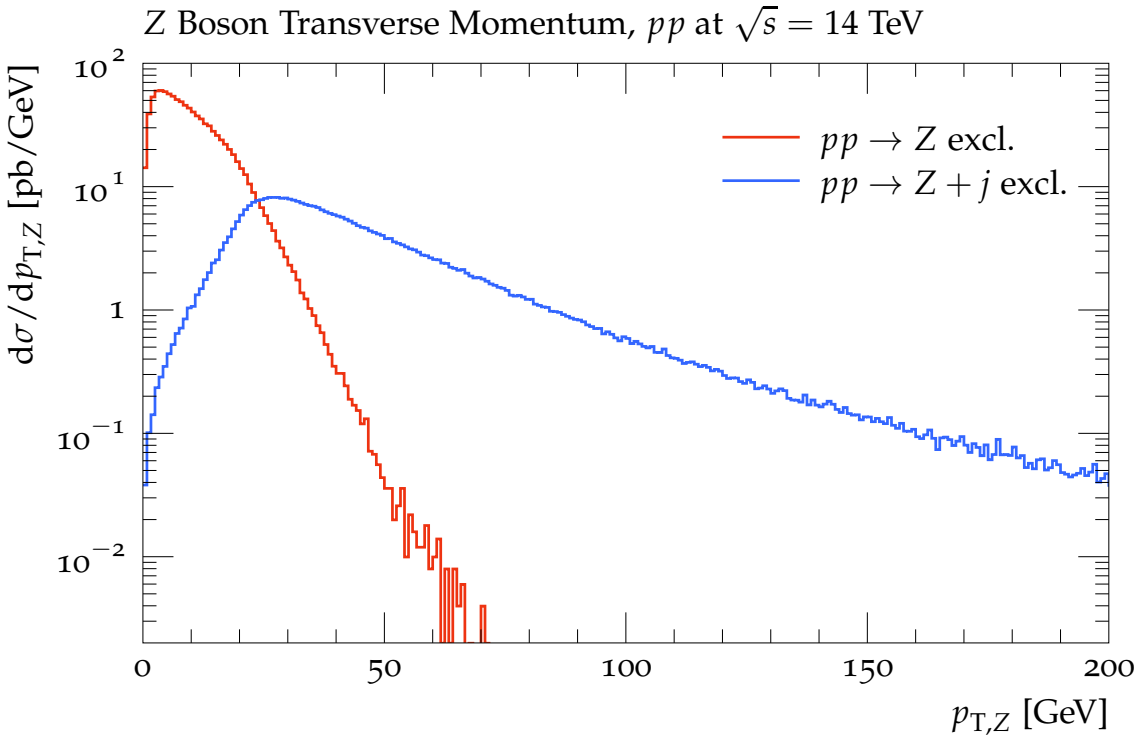

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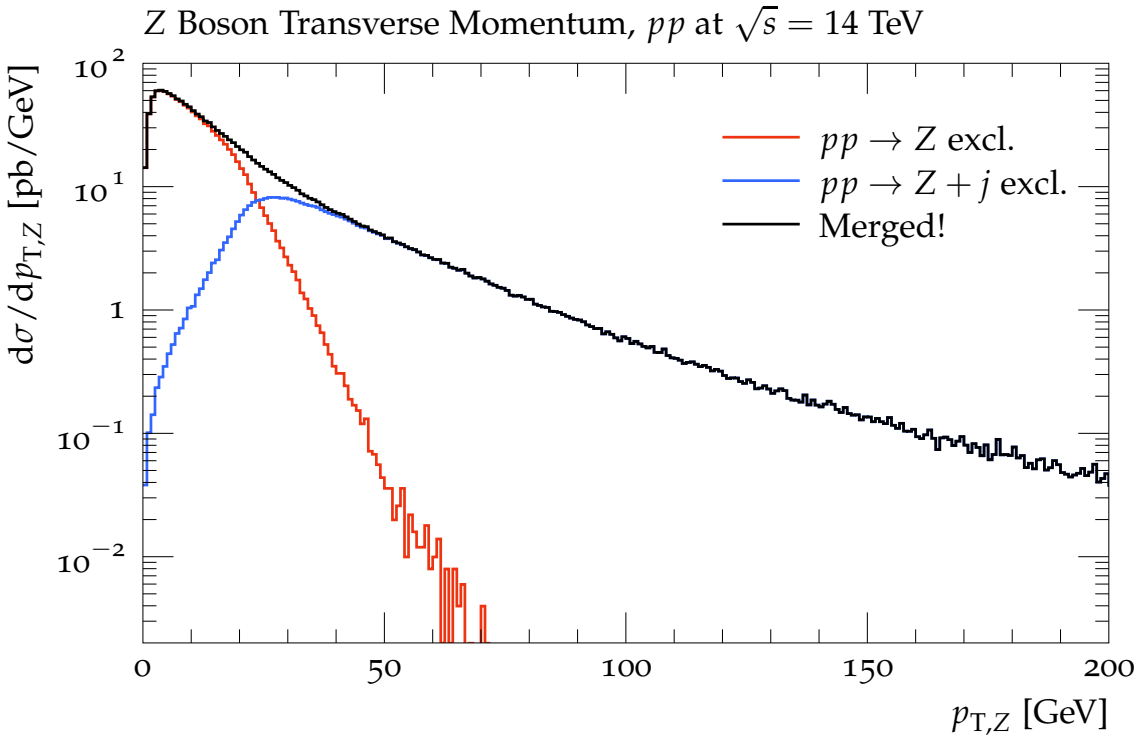

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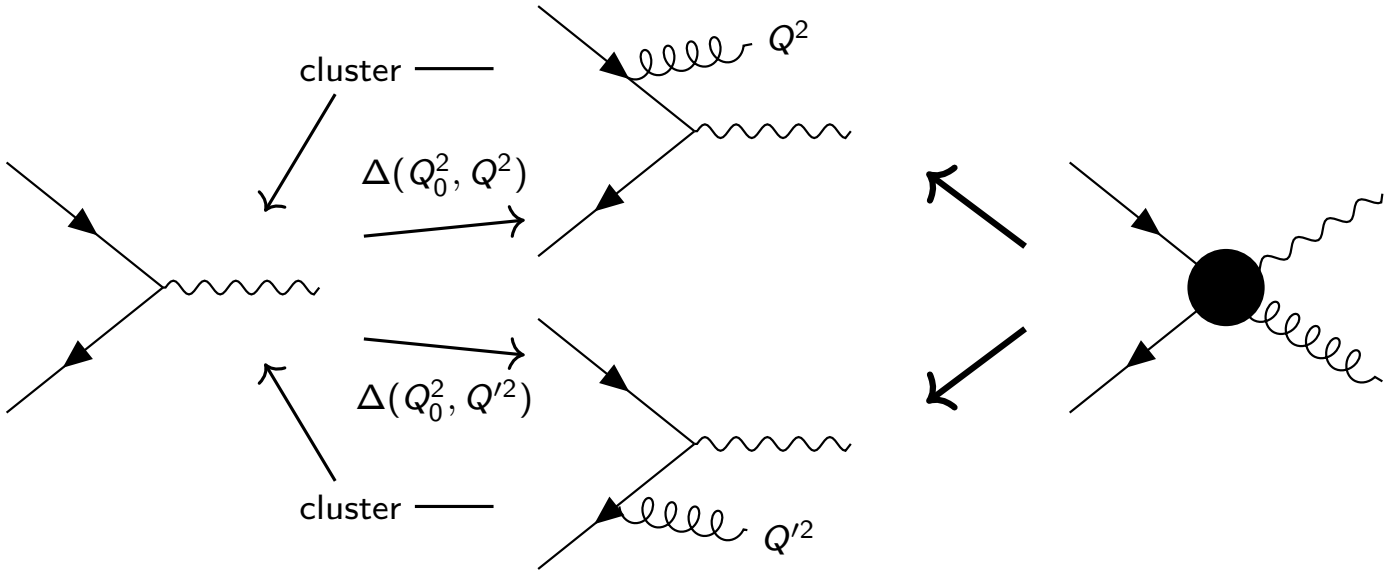
What we can learn from history

- basic CKKW idea [Catani, Krauss, Kuhn, Webber hep-ph/0109231], [Krauss hep-ph/0205283]:
  - select hard n-jet configuration
  - construct "shower history"
  - re-weight event by Sudakov factors between nodes

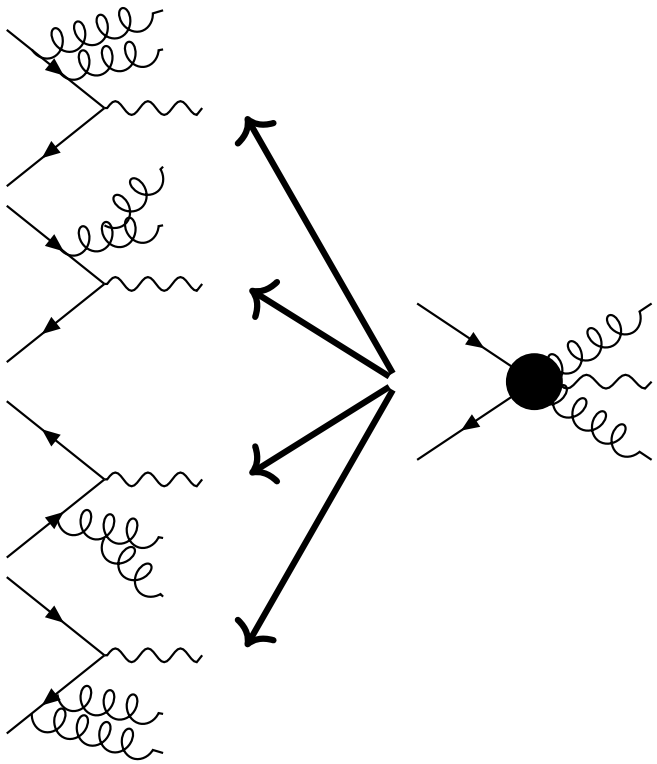


Merging schemes

- could use deterministic jet clustering and analytic Sudakovs, but want to do better:
  - use the same Sudakov factor in reweighting as in shower
  - account for shower evolution in history construction (or vice versa)
- solved in different ways, e.g.
  - CKKW-L [Lönnblad hep-ph/0112284], [Lönnblad, Prestel 1109.4829]
  - METS [Höche, Krauss, Schumann, Siegert 0903.1219], [Hamilton, Richardson, Tully 0905.3072]
- here consider CKKW-L:
  - construct **all possible** shower histories, choose **most likely**
  - let (truncated) **trial showers** generate Sudakov factors

The scaling of the method: complexity

- to include the $pp \to Z + 2$ jet matrix element:



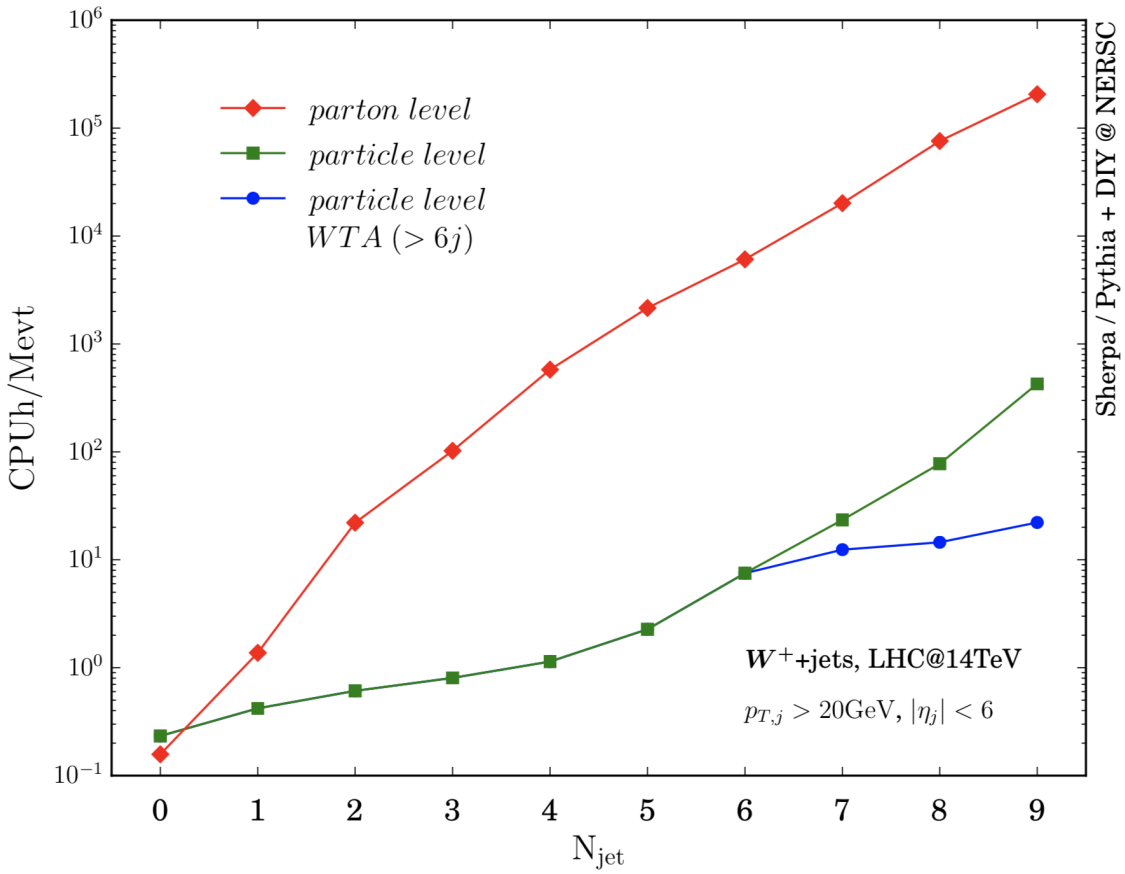
- **conventional** showers: $\mathcal{O}\left(2^n n!\right)$ histories contribute to n-particle final state

Number of Histories for n Branchings

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040

(starting from one colour-anticolour pair)

The scaling of the method: time



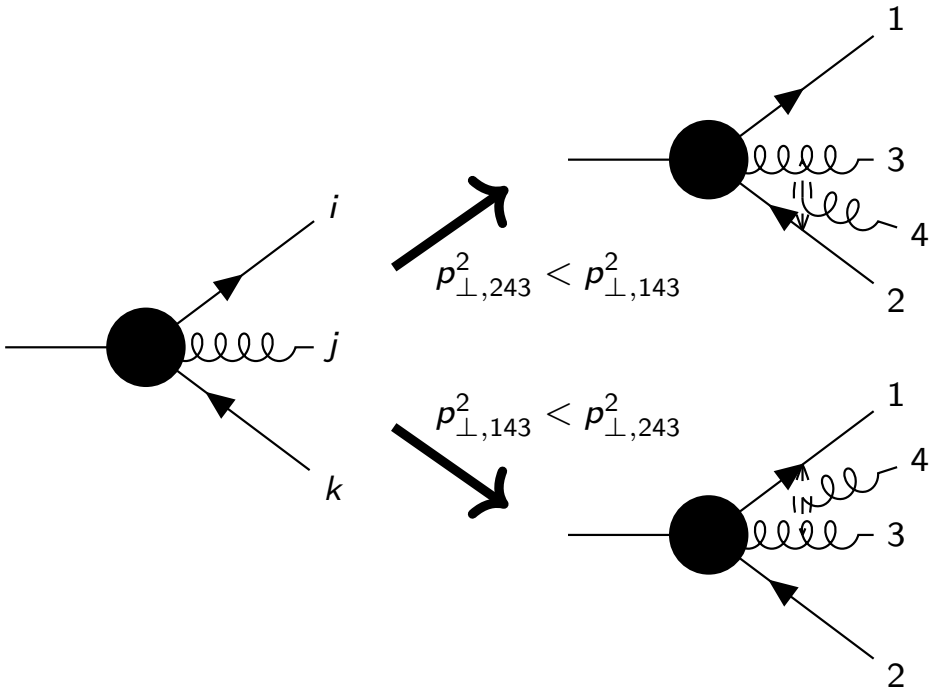
[Höche, Prestel, Schulz 1905.05120]

- hard process most expensive, but can be generated "once and for all"
- deterministic **jet clustering** ("WTA" – winner takes all) improves scaling, but does not represent **shower history**

Unconventional showers? Sector showers

[Brooks, CTP, Skands 2003.00702; Lopez-Villarejo, Skands 1109.3608]

- **Sector showers**: combine antenna shower with deterministic jet-clustering algorithm
- let shower only generate emissions that would be clustered by a $(3 \mapsto 2)$ jet algorithm ($\sim$ ARCLUS [Lönnblad Z.Phys.C 58 (1993)])

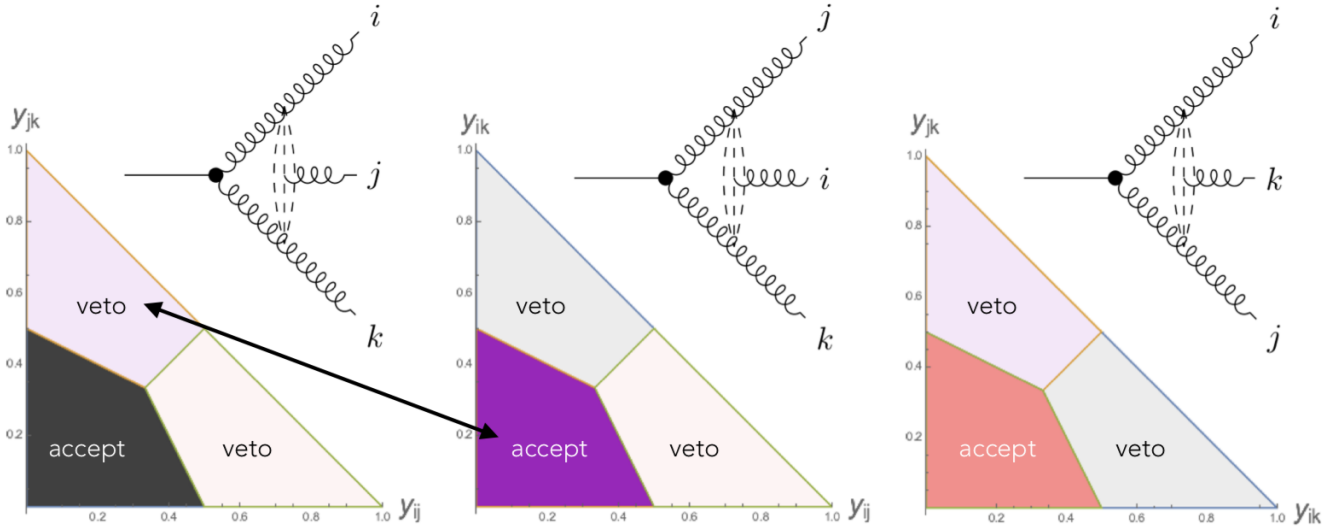


$$p^2_{\perp,ijk} = \frac{2(p_i \cdot p_j)(p_j \cdot p_k)}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}$$

- only **one** (most singular) splitting kernel contributes per phase space point

Phase space sectors

- branching phase space gets divided into **non-overlapping sectors**
- e.g. for FF $gg \mapsto ggg$ branchings:



(adapted from P. Skands)

- branchings in the shower are accepted **if and only if** they correspond to the **correct sector**
- sectors defined by minimal $p_\perp$ in event, but always contain:
  - soft endpoint
  - "full" collinear region for qg
  - "half" of the collinear region for gg with boundary at $z = \frac{1}{2}$

Note: in general, highly non-trivial sector boundaries away from the singular limits!

Sector antenna functions

- splitting kernels have to incorporate **full** single-unresolved limits for given PS point (KOSOWER subtraction terms [Kosower PRD 57 (1998) 5410, PRD 71 (2005) 045016])
- e.g. $qg \mapsto qgg$ (FF):

$$A_{q_i g_j g_k}(s_{ij}, s_{jk}, s_{ik}) \to \begin{cases} \frac{2 s_{ik}}{s_{ij} s_{jk}} & \text{if } j \text{ soft} \\ \frac{1}{s_{ij}} P_{q \mapsto qg}(z) & \text{if } i \parallel j \\ \frac{1}{s_{jk}} P_{g \mapsto gg}(z) & \text{if } j \parallel k \end{cases} \quad \text{where } s_{ij} = 2 p_i \cdot p_j$$

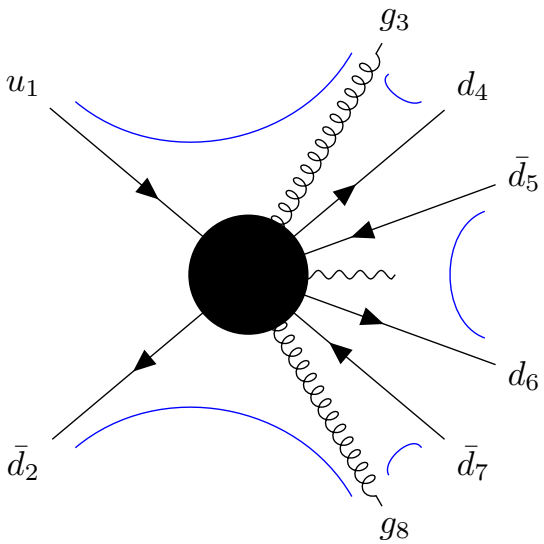
- VINCIA choice: GGG antenna functions [Gehrmann-de Ridder, Gehrmann, Glover JHEP 09 (2005) 056] (+ non-singular modifications)

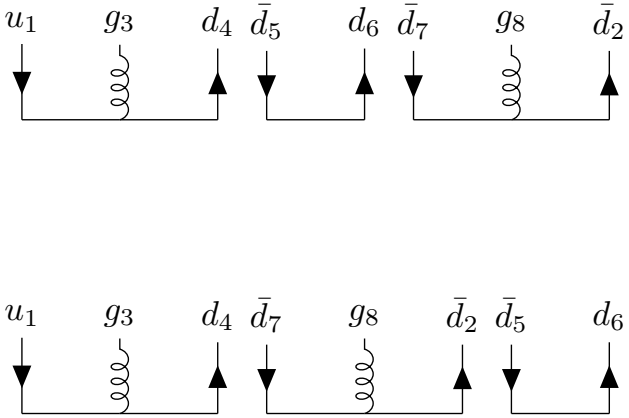
$$A(i,j,k) = \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2} \quad \text{for } IK \mapsto ijk$$

Taming the MESS [Brooks, CTP 2008.09468]

- merging with sector shower relatively straight-forward:
  start from CKKW-L and modify **history construction**

- modulo quark permutations, sector showers have a **single history**, i.e. a **unique** inverse
- to account for all possible $q\bar{q}$-clusterings, find all viable colour orderings



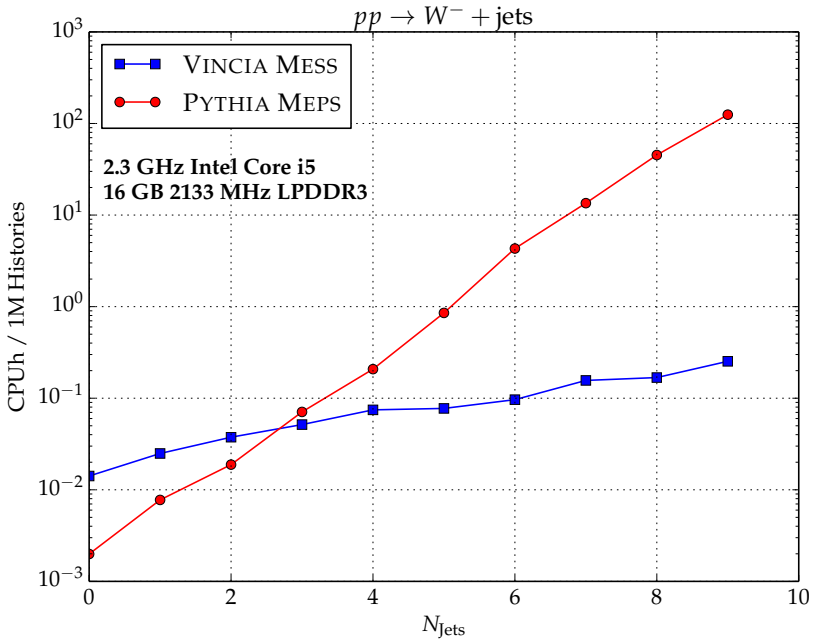


- for each colour-ordering, shower history again **uniquely** defined by sectors
- if multiple colour-orderings possible, choose one that **maximises** branching probability

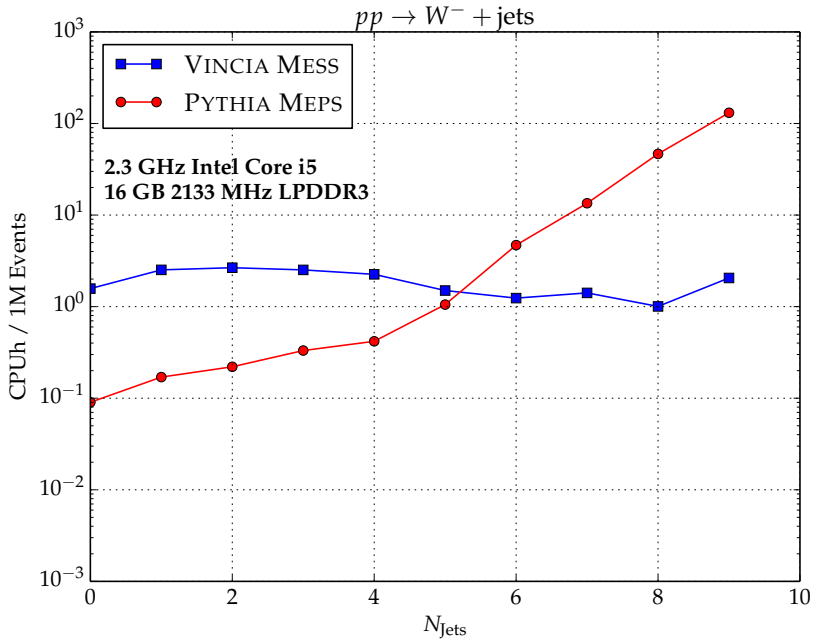
Scaling revisited: time

- CPU time in $pp \to W^- + 9$ jet merging:

CPU time / **history**



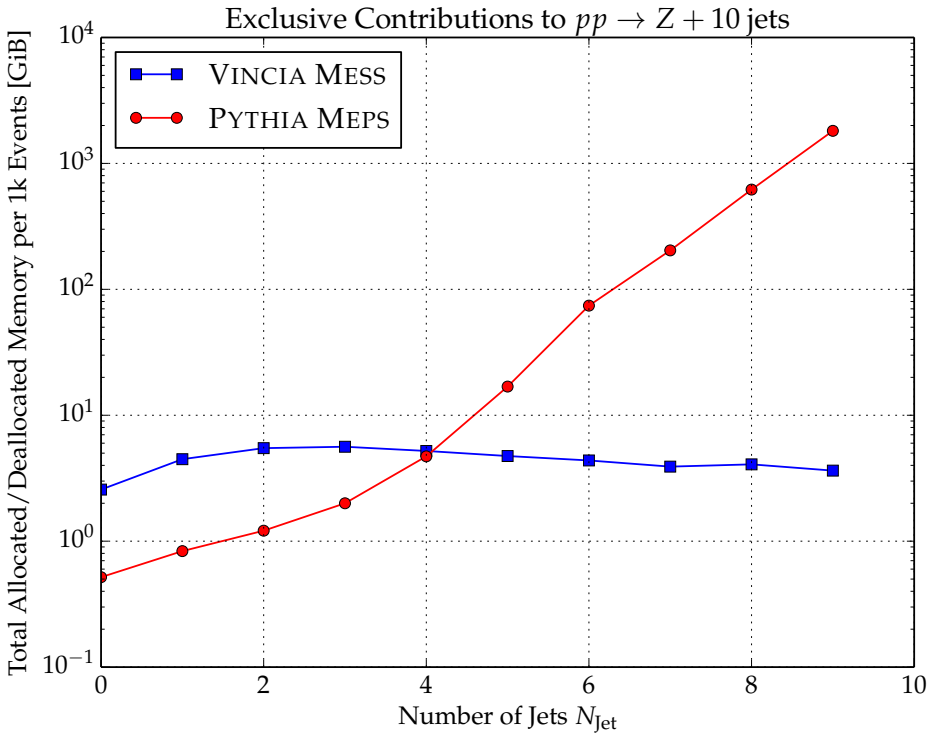
CPU time / **event**



Note: kink for 9-jet sample on the rhs due to inclusive treatment of last node in CKKW-L

Scaling revisited: resources

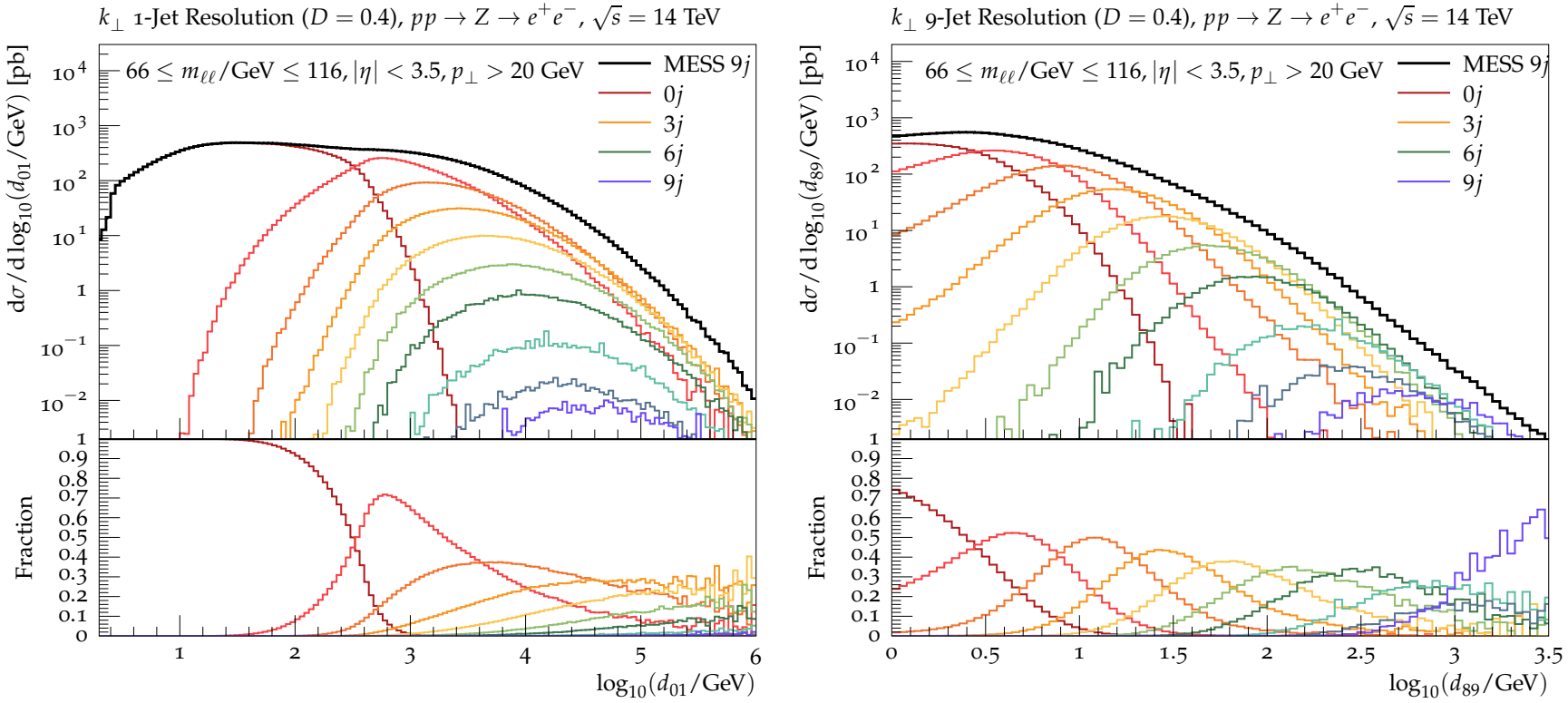
- **memory** allocation/deallocation per 1k **events** in $pp \to Z + 10$ jet merging:



Note: the 10th jet only hypothetical, so that every multiplicity is treated as "intermediate" node

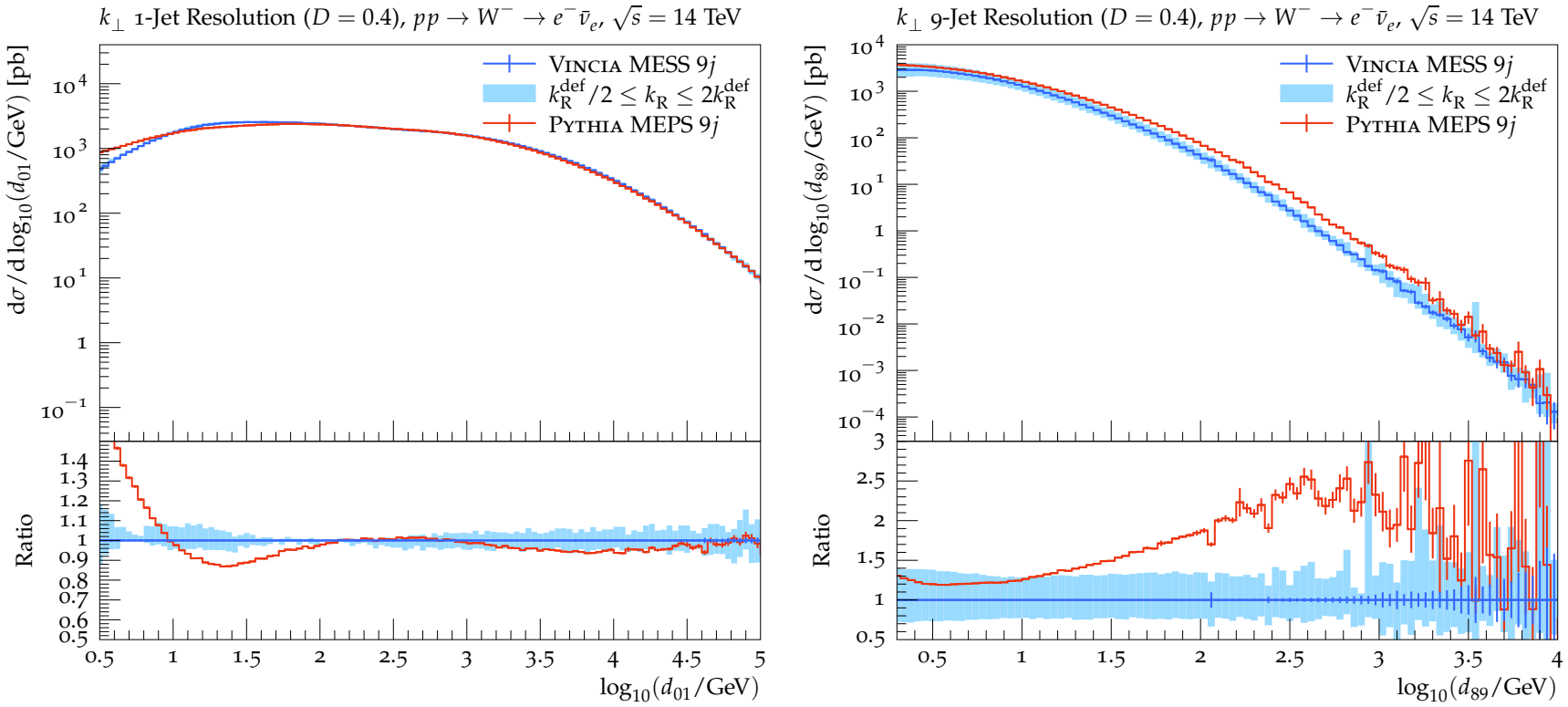
First results

- parton-level results for merging in $pp \to Z$ with up to 9 additional jets (using event files from [Höche, Prestel, Schulz 1905.05120])



First results

- comparison with PYTHIA for $pp \to W^- + 9$ jets @parton level (using event files from [Höche, Prestel, Schulz 1905.05120])



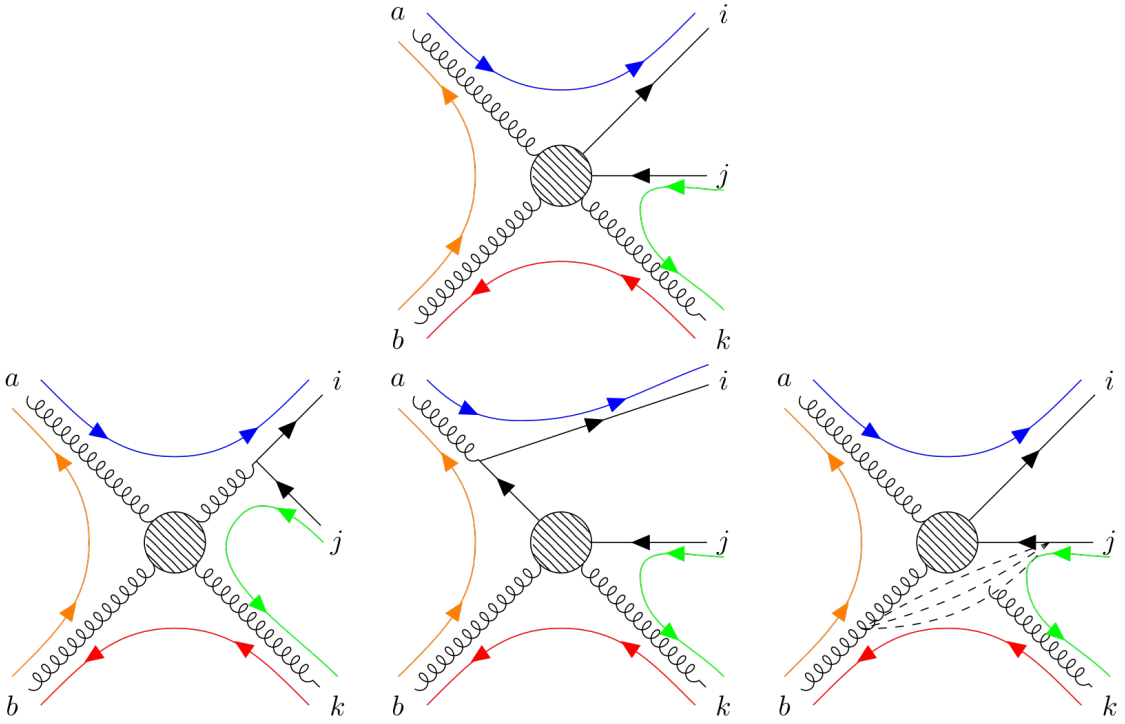
Conclusions

- sector showers combine shower **evolution** with jet **clustering** to become **uniquely** invertible
- ME+SS combines simplicity of CKKW with accuracy of CKKW-L
  - **linear** scaling for time to construct histories
  - **constant** overall run time and memory usage
- stand-alone sector showers **slower** than conventional ones, but smarter sampling possible
- sector shower + merging implemented in VINCIA shower in PYTHIA

This is just the beginning...

- sector showers well validated against "global" VINCIA and PYTHIA (discontinuities? still searching...)
- sector-based merging so far rather academic study, stress test yet to follow (extension to NLO desirable)
- VINCIA sector showers + merging getting ready for PYTHIA release (8.304?)
  - VINCIA's QCD showers fully "sectorised" (II, IF, FF, RF)
  - idea is to **replace** existing "global" showers

Backup: sector resolution criterion



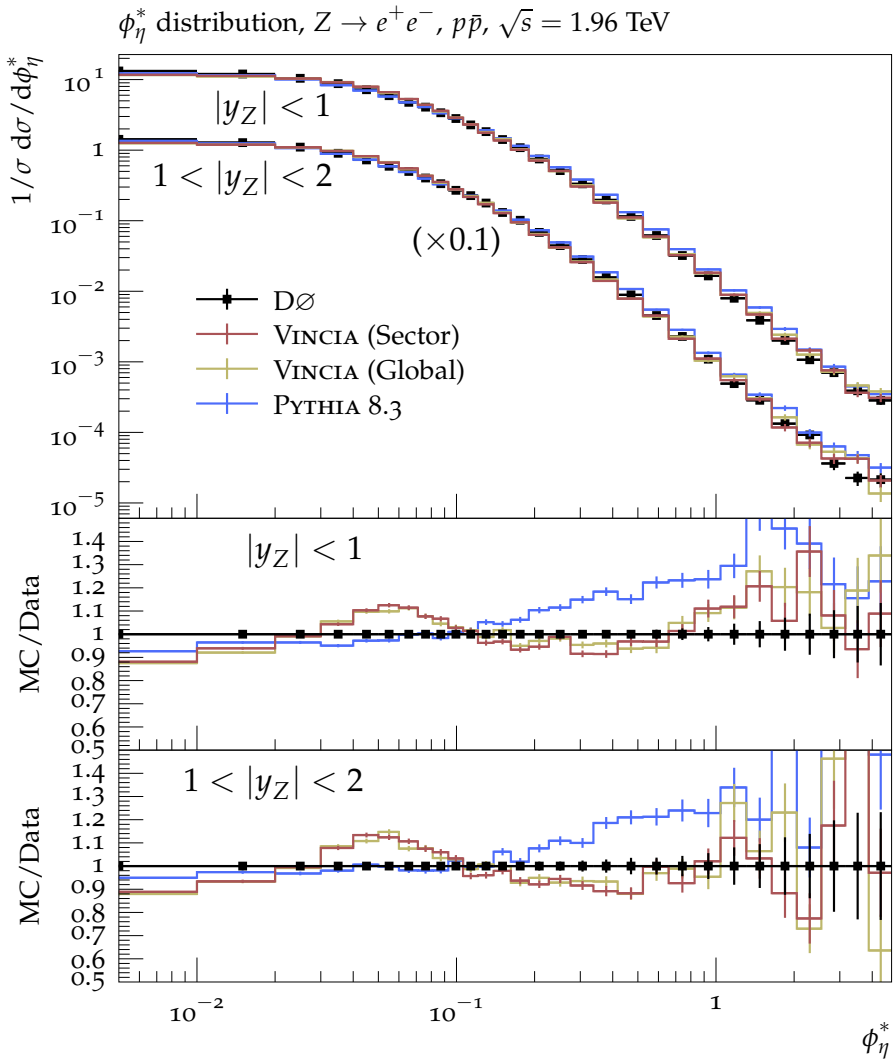
For final-final clusterings, sector resolution defined by:

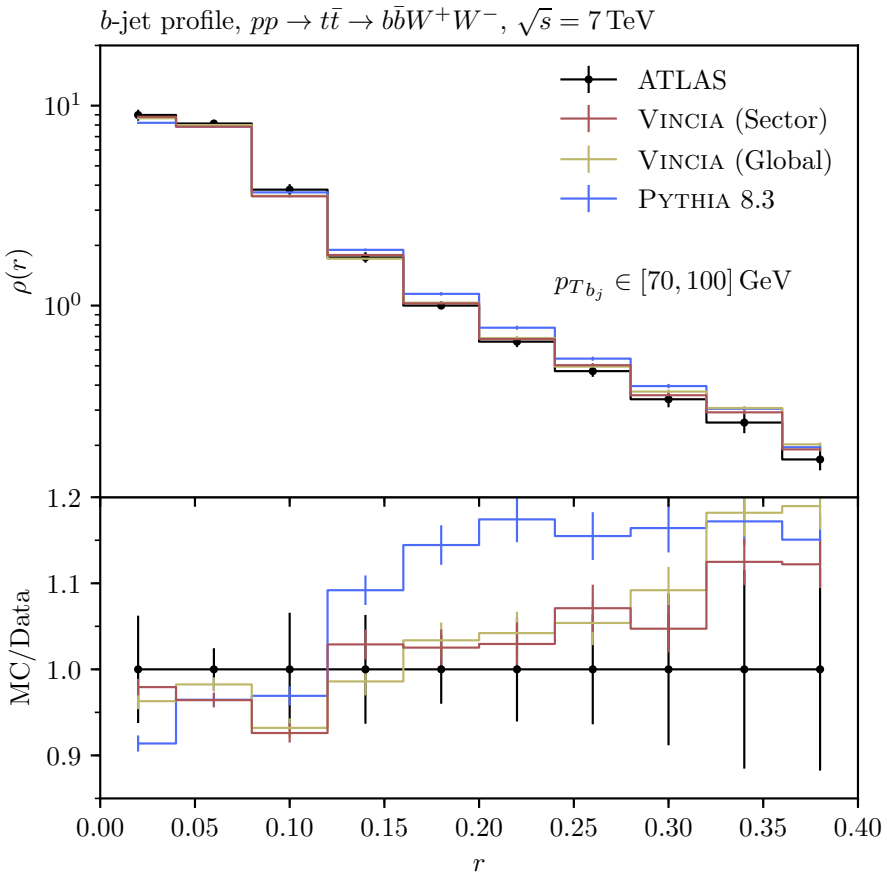
$$Q^2_{\text{res},j} = \begin{cases} \frac{s_{ij}s_{jk}}{s_{ijk}} & \text{if } j \text{ is a } g \\ s_{ij}\sqrt{\frac{s_{jk}}{s_{ijk}}} & \text{if } (i,j) \text{ is a } q\bar{q} \text{ pair} \end{cases}$$

Sectors defined by:

$$\Theta_{\text{sct},j} = \theta(\min\{Q^2_{\text{res},i}\} - Q^2_{\text{res},j}) \tag{1}$$

Backup: some sector shower validations





Note: no dedicated "sector tune", no MECs

Backup: memory profiles