An analytical approach to the mechanics of superconducting magnets: the 11 T dipole collared coils

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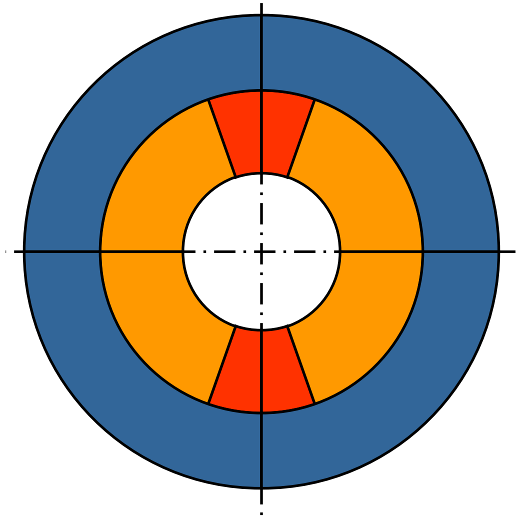
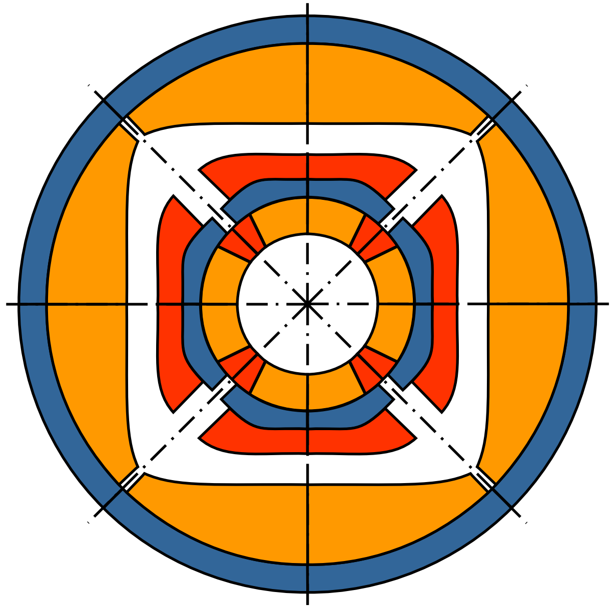
Contents

1. Motivation

The 2D cross section of superconducting magnets is modelled with great level of geometrical detail by the finite element method. However, due to the high degree of complexity of the models, sensitivity analysis and optimisations are long-lasting. Furthermore, potential relevant relationships within the system might be unnoticed.

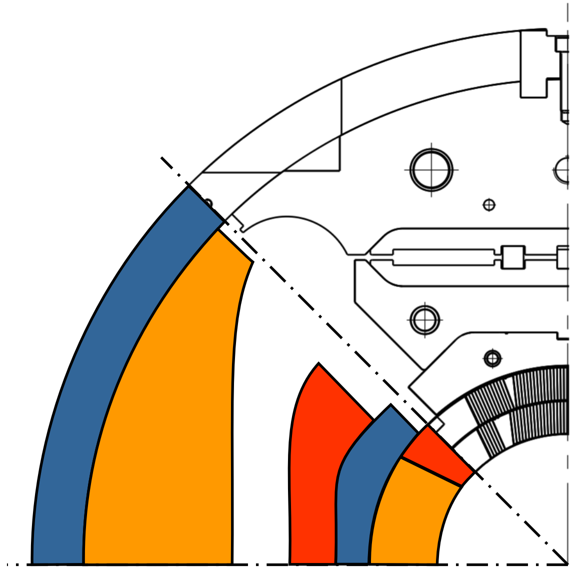


→ **Action item from the 11T Dipole Technical Meeting 2019:**
“Develop analytical formulae for the calculation of the deformed collared coils and resulting coil stress”



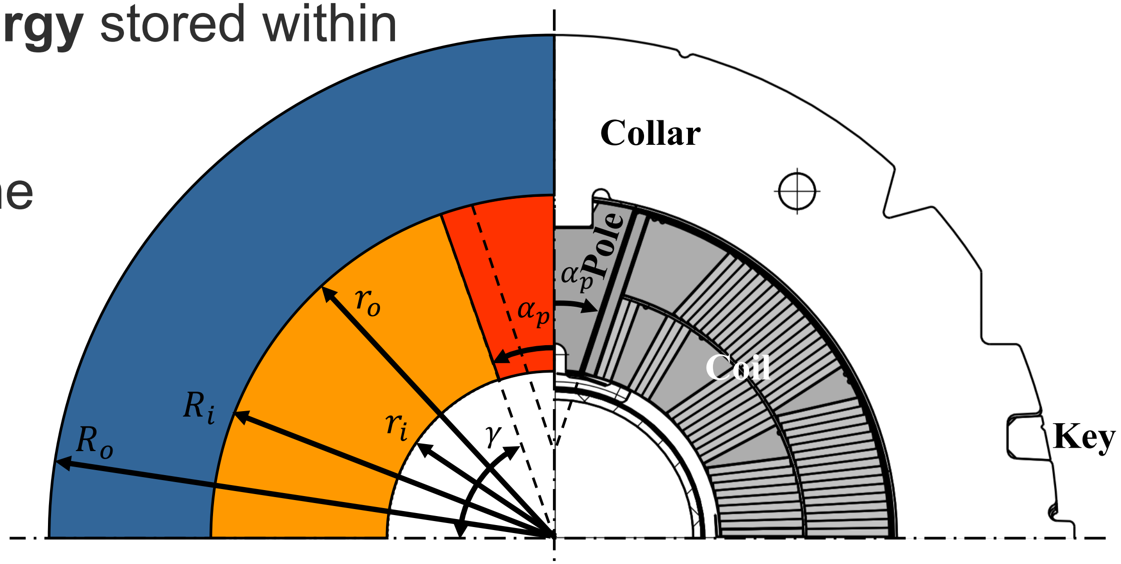
Development of the analytical formulae → *Generalisation of the method*:
The main objective of the method here presented is not to provide a tool to compute precise values of deformation or stress, but to establish an analytic framework to understand the main relationships between geometry and strain-stress developed within complex mechanical systems.

2. Introduction



The cross section of a superconducting magnet can be **conceptualised**, to different levels of complexity, by describing mathematically its components and the relationships between them.

If one considers all structural components as linearly elastic isotropic bodies, and all internal and external forces as conservative, the relation between material properties, stress, strain, and displacements can be expressed in terms of the **variation of elastic strain energy** stored within the structure.

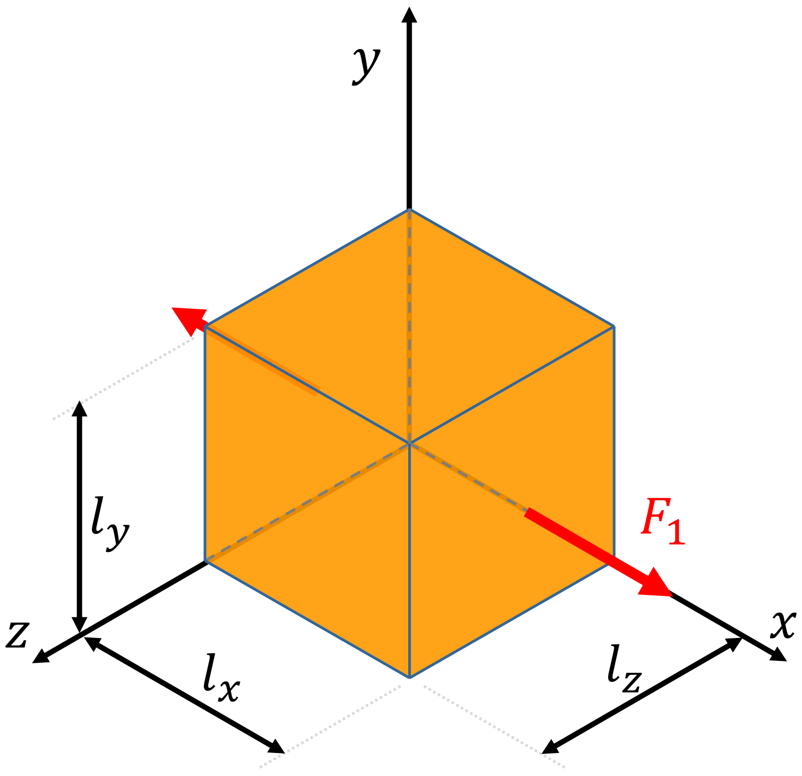


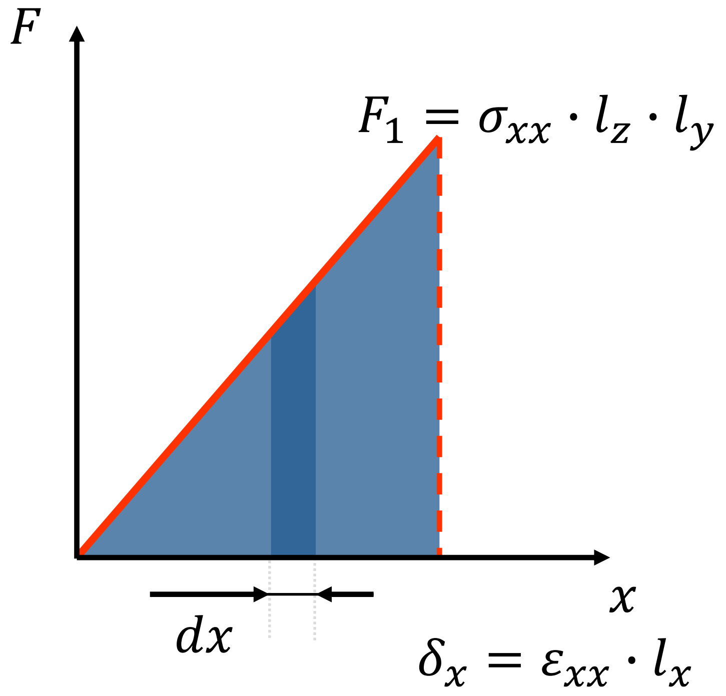
The proposed method formulates the structure through the concept of the **Λ function**, which is based on the idea of **geometrical interference** χ, and the partial derivative of the strain energy of each component, summarising, to a certain level of complication, the main geometrical interactions between components, and their temperature dependency.

3. Analytical formulation and method

A quick introduction to strain energy and deformation

First law of thermodynamics $dU = dQ - dW$





Elementary work

$$dW = Fdx$$

Total strain energy stored

$$U = \int_0^{\delta_x} Fdx$$

$$U_1 = \frac{1}{2} F_1 \delta_x$$

Considering $\delta_x(F_1)$

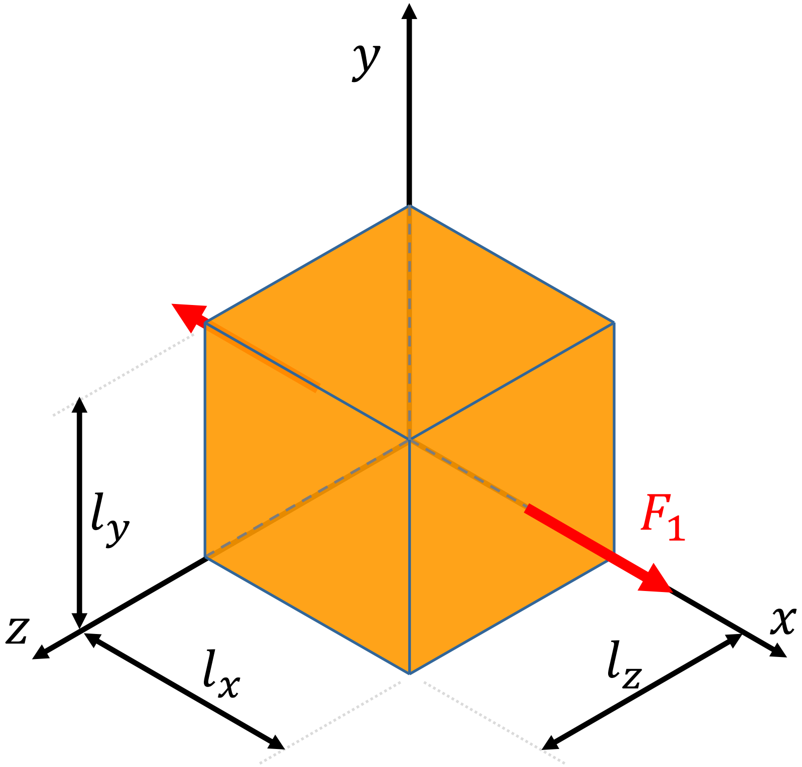
$$U_1 = \frac{l_x {F_1}^2}{2 l_y l_z E}$$

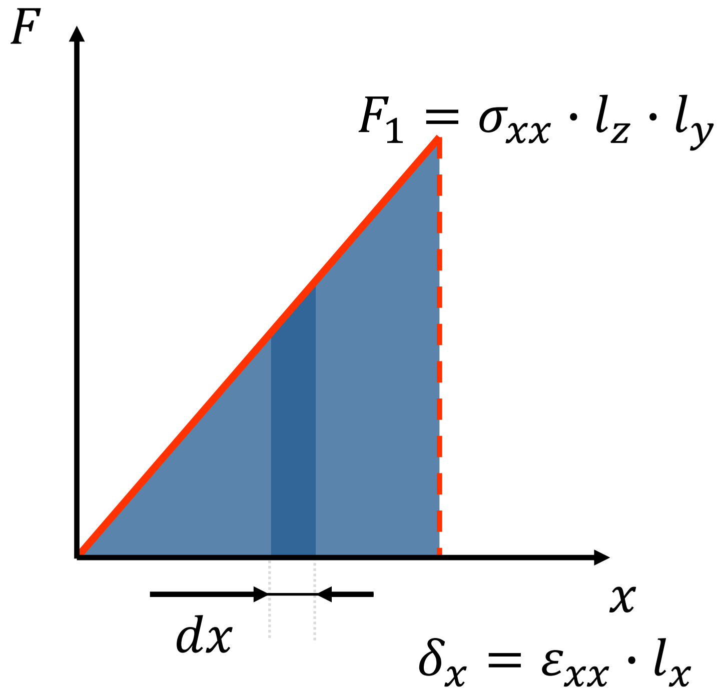
⋮

3. Analytical formulation and method

A quick introduction to strain energy and deformation

First law of thermodynamics $dU = dQ - dW$





Castigliano's second theorem

$U\colon \mathbb{R}^n \to \mathbb{R}$

$U(F_1, F_2, \dots, F_n)$

$$\frac{\partial U}{\partial F_1} = \delta_1, \dots, \frac{\partial U}{\partial F_n} = \delta_n$$

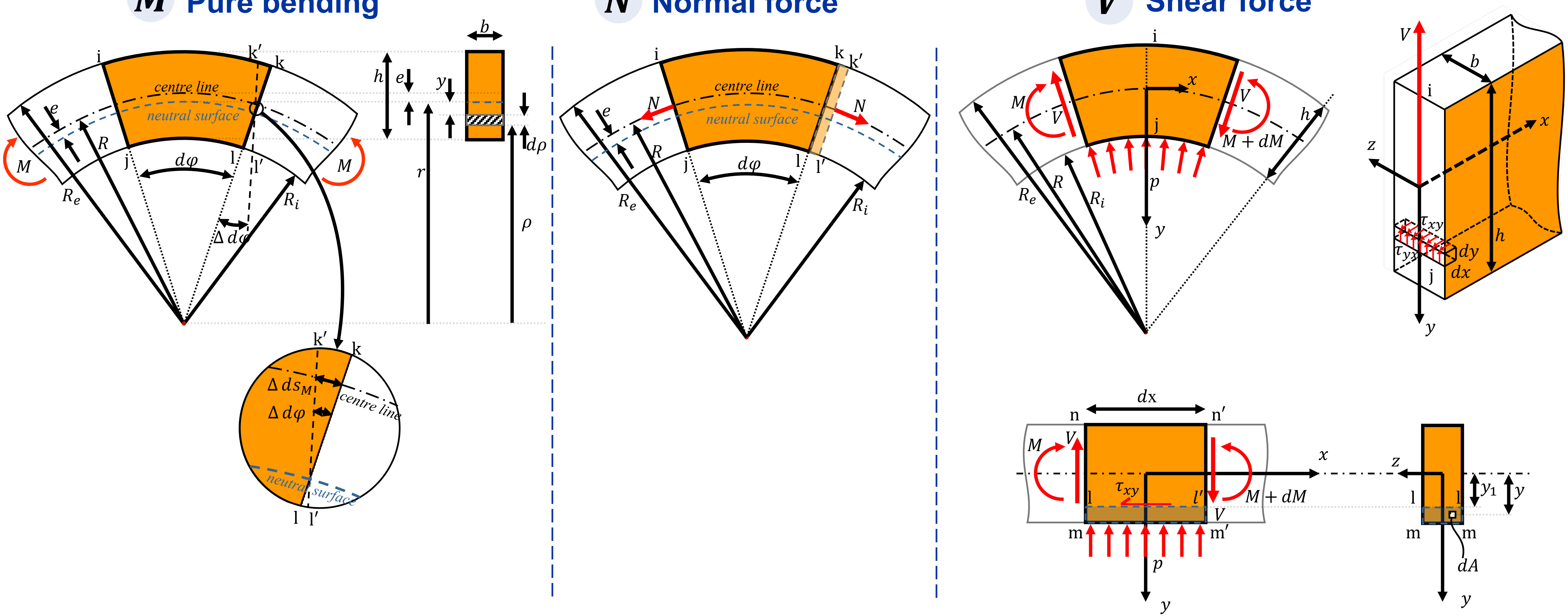
The displacement δ_x *is derived from the energy function as*

$$\frac{\partial U_1}{\partial F_1} = \frac{l_x F_1}{l_y l_z E}$$

3. Analytical formulation and method

A quick introduction to the variation of strain energy as a function of internal forces

M Pure bending | *N* Normal force | *V* Shear force



3. Analytical formulation and method

A quick introduction to the variation of strain energy as a function of internal forces

$\boldsymbol{M(\varphi)}$ **Pure bending *body function***

$\boldsymbol{N(\varphi)}$ **Normal force *body function***

$\boldsymbol{V(\varphi)}$ **Shear force *body function***

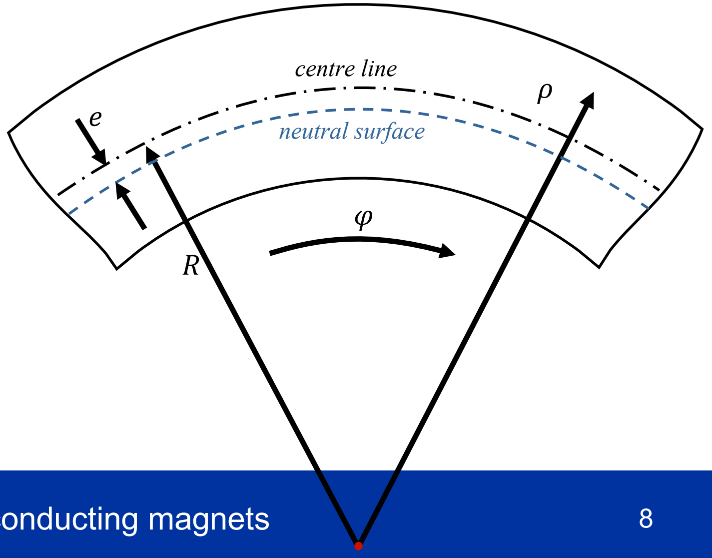
Global strain energy function

$$U_{body} = \int_0^s \left(\frac{M^2}{2AeER} + \frac{N^2}{2AE} - \frac{MN}{AER} + \psi \frac{V^2}{2AG} \right) ds$$

Transformation of internal forces into an azimuthal stress scalar field

$$\sigma_\theta = \frac{M(\varphi)y}{Ae(r-y)} + \frac{N(\varphi)}{A}$$

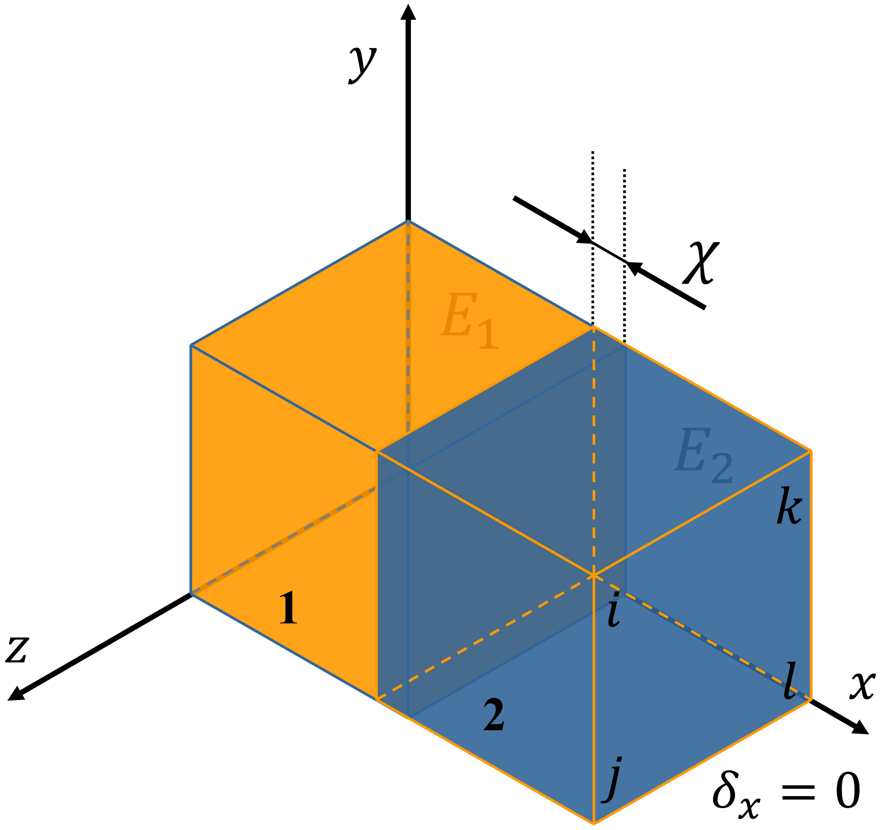
$$\sigma_\theta(\rho,\varphi) = \frac{M(\varphi)(R-e-\rho)}{Ae\rho} + \frac{N(\varphi)}{A}$$

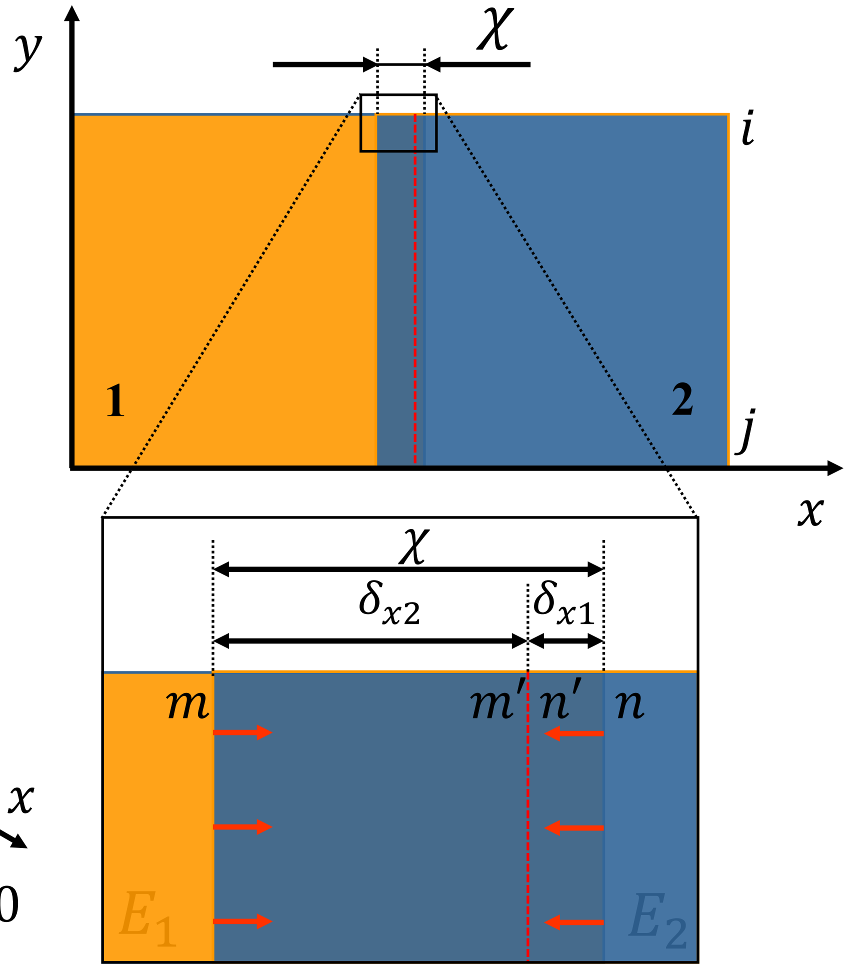


3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and Λ functions

General definition





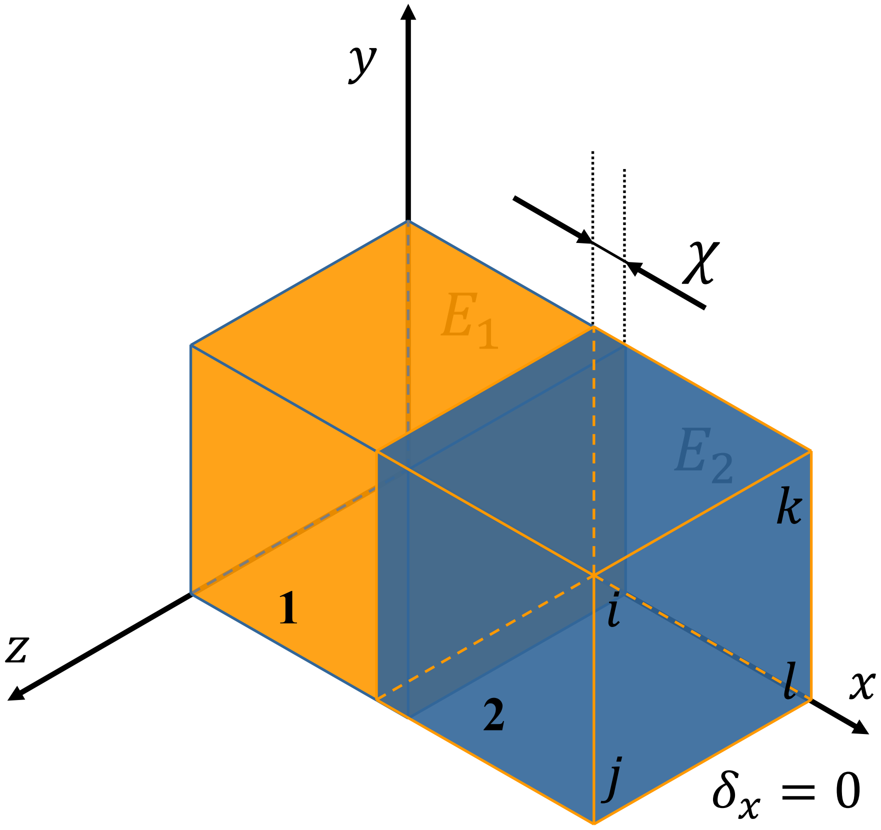
A **Λ function** of a system in non-equilibrium, is defined, in general terms, as a function that relates the partial derivative of the strain energy of the bodies of the system, with the **geometrical interference**, where the equilibrium solution is found at $\Lambda = 0$.

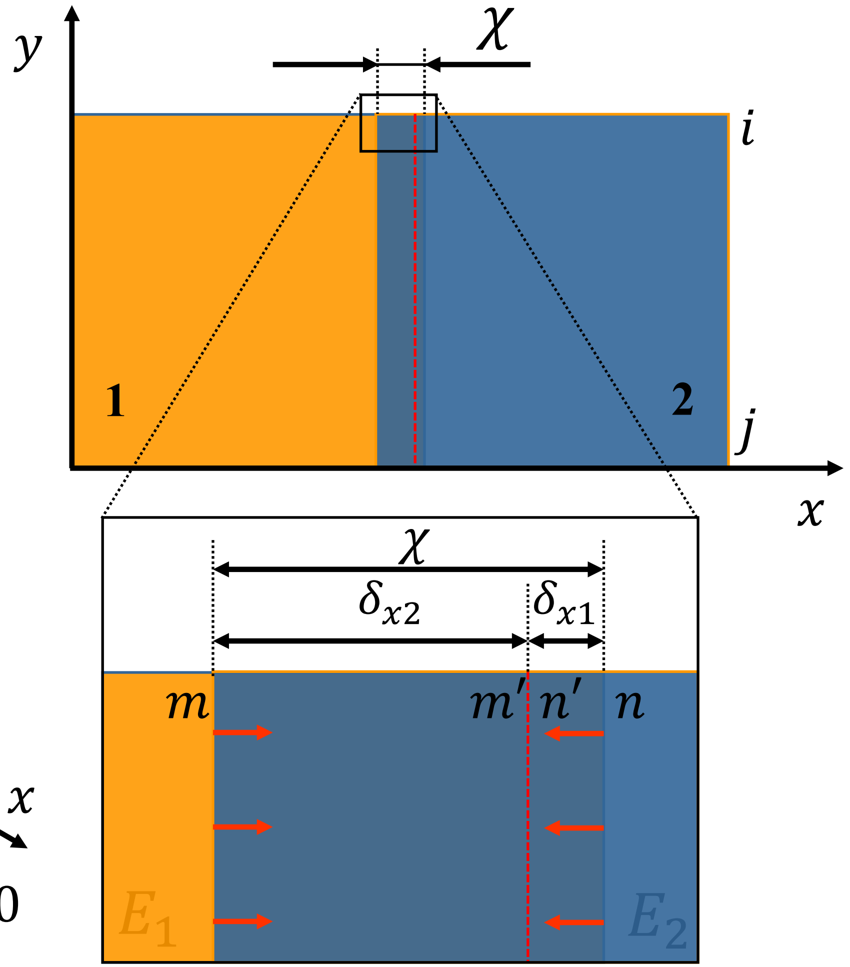
$$\Lambda_1 = 0$$
$$\Lambda_2 = 0$$
$$\vdots$$
$$\Lambda_n = 0$$

3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and Λ functions

General definition





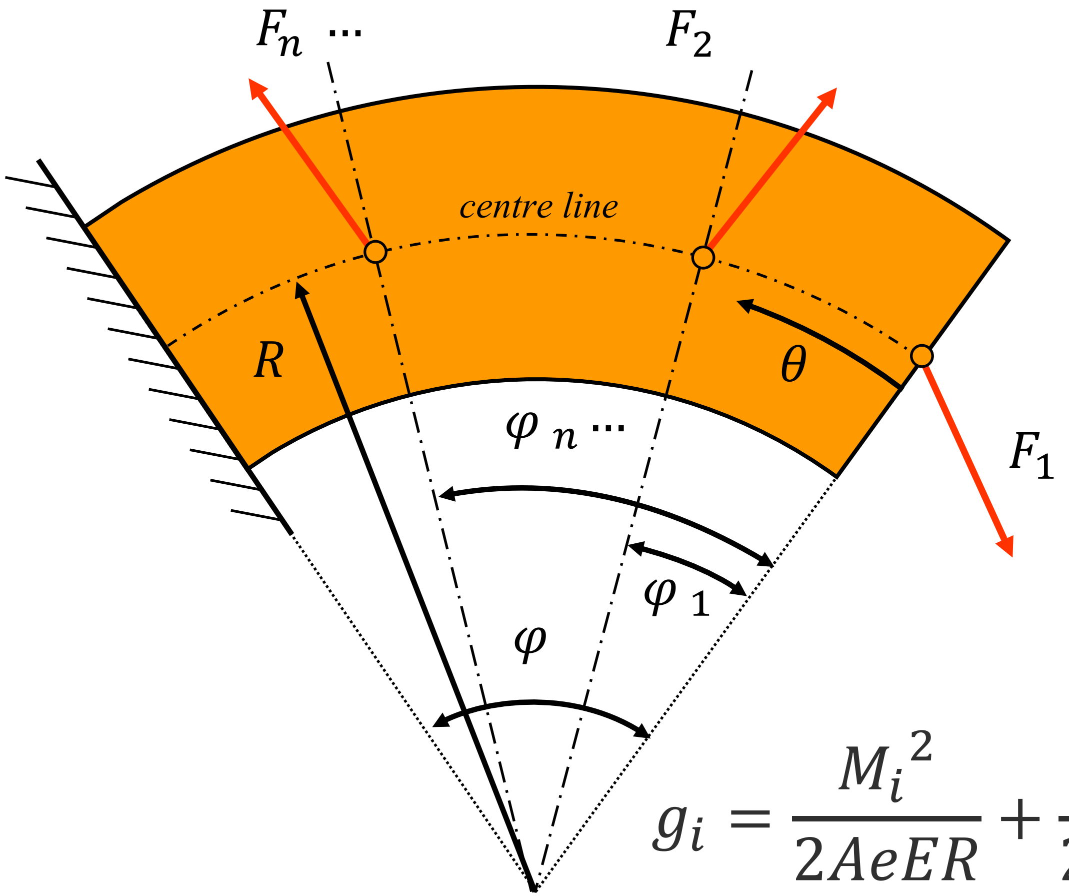
$$\Lambda = \frac{\partial U_1}{\partial F} + \frac{\partial U_2}{\partial F} - \chi$$

$$F = \left(\frac{1}{E_1} + \frac{1}{E_2}\right)^{-1} \frac{l_y l_z}{l_x} \chi$$

3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and Λ functions

Discontinuities



$$M,N,V = \begin{cases} M_1, N_1, V_1 & for\ \varphi_0 < \theta \le \varphi_1 \\ M_2, N_2, V_2 & for\ \varphi_1 < \theta \le \varphi_2 \\ \vdots & \\ M_{n+1}, N_{n+1}, V_{n+1} & for\ \varphi_n < \theta \le \varphi_{n+1} \end{cases}$$

Global strain energy function

$$U_{body} = \int_0^s \left(\frac{M^2}{2AeER} + \frac{N^2}{2AE} - \frac{MN}{AER} + \psi \frac{V^2}{2AG} \right) ds$$

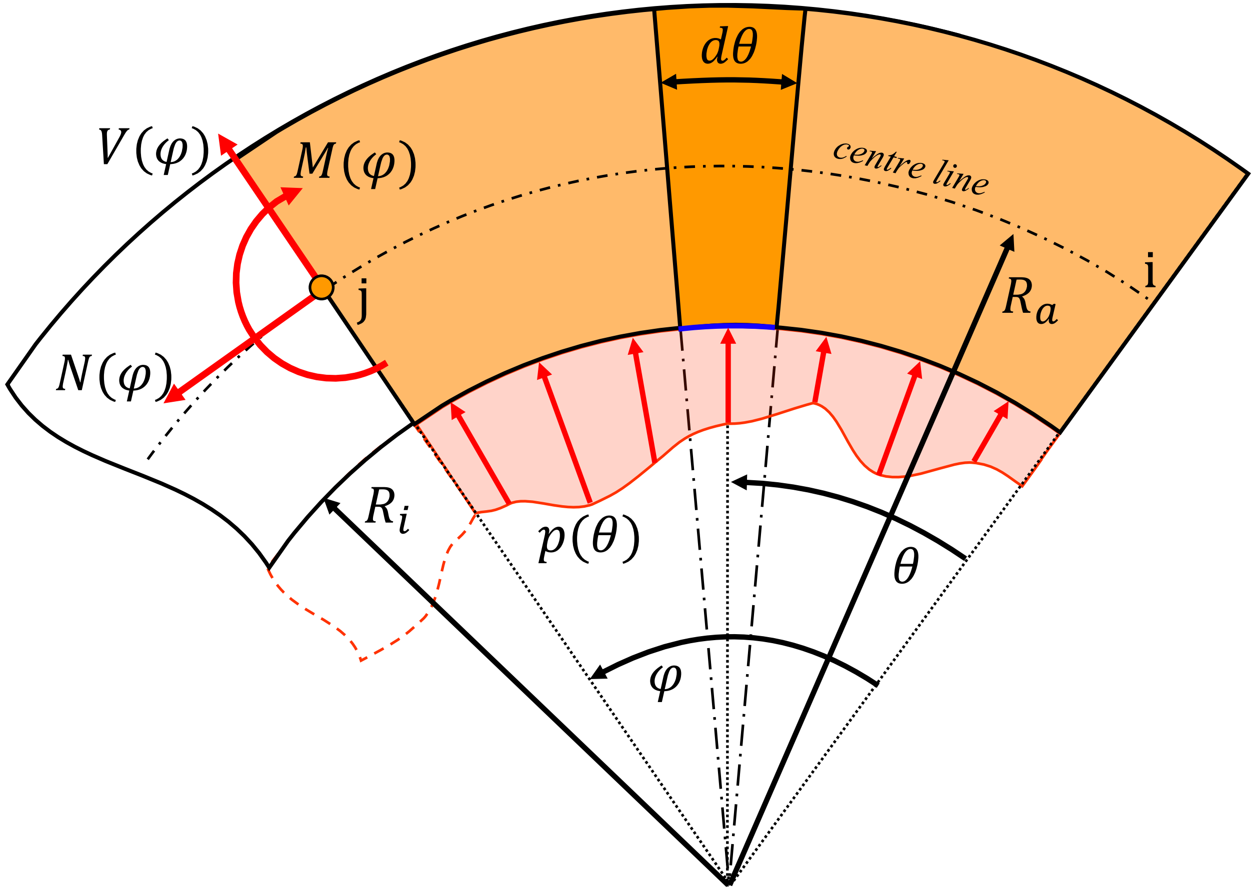
$$g_i = \frac{{M_i}^2}{2AeER} + \frac{{N_i}^2}{2AE} - \frac{M_i N_i}{AER} + \psi \frac{{V_i}^2}{2AG} \rightarrow$$

$$U_{body} = \sum_{i=1}^{n+1} \left(\int_{\varphi_{i-1}}^{\varphi_i} g_i R d\varphi \right)$$

3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and Λ functions

Contribution of external variable pressure to internal forces



Differential force and bending moment created by a differential angular element $d\theta$

$$dF = p(\theta) R_i d\theta$$

$$dM = p(\theta) R_i R_a \sin(\varphi - \theta)\, d\theta$$

Overall contribution to the internal forces

$$M(\varphi) = R_a R_i \int_0^{\varphi} p(\theta) \sin(\varphi - \theta)\, d\theta$$

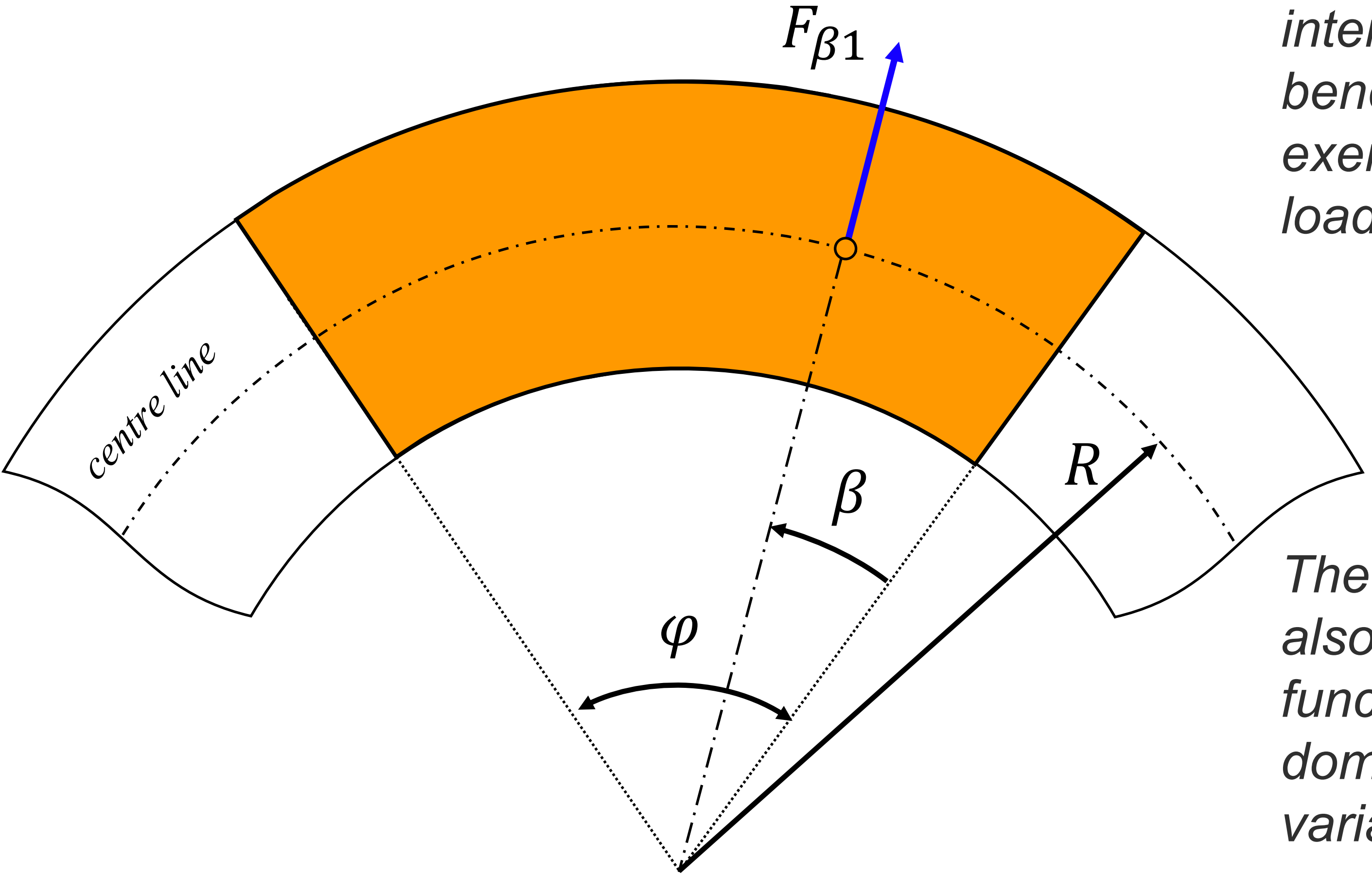
$$N(\varphi) = R_i \int_0^{\varphi} p(\theta) \sin(\varphi - \theta)\, d\theta$$

$$V(\varphi) = R_i \int_0^{\varphi} p(\theta) \cos(\varphi - \theta)\, d\theta$$

3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and Λ functions

Imaginary loads, displacements and rotations



In order to derive the displacement of the points of interest (or rotation in the case of an imaginary bending moment), in which the imaginary loads are exerted, it is necessary to include all imaginary loads within the energy formulation.

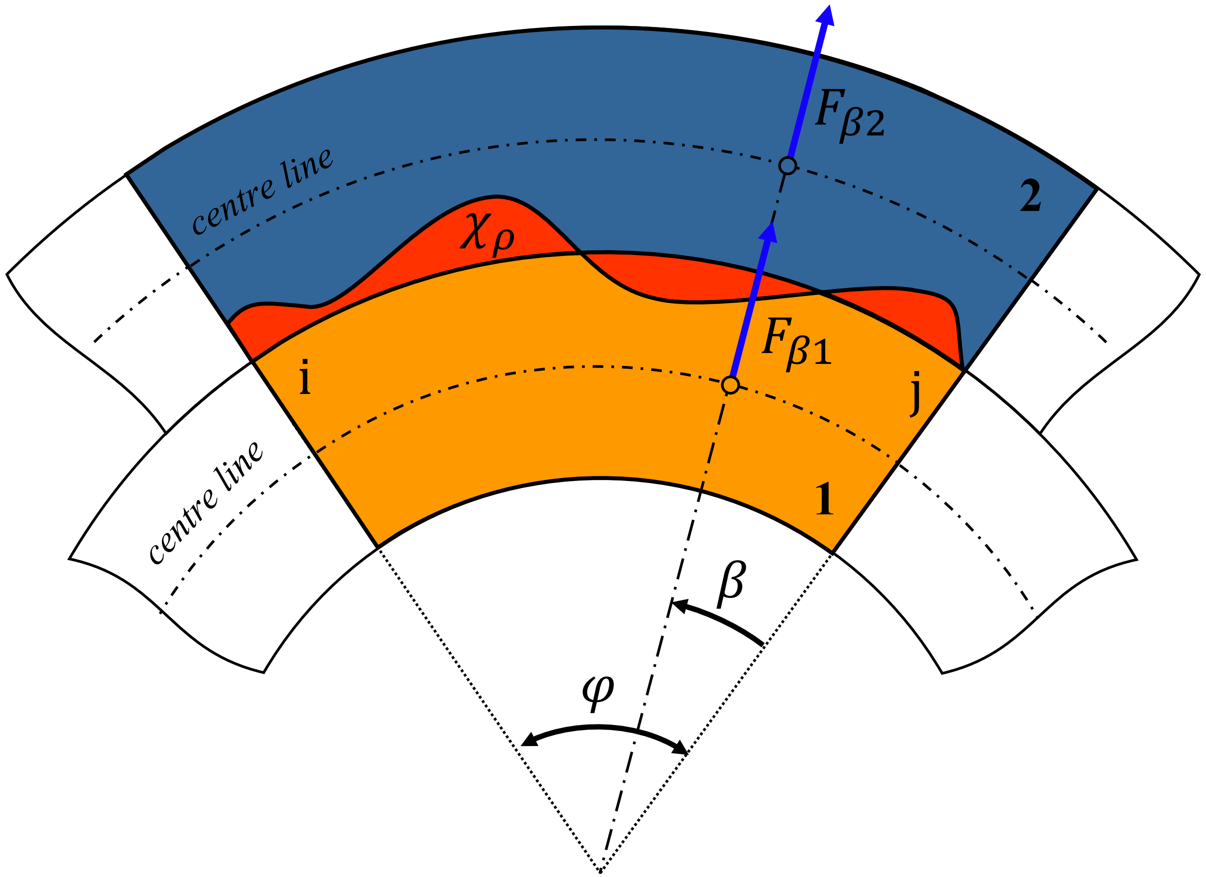
$$\delta_\beta = \left. \frac{\partial U_1}{\partial F_\beta} \right|_{F_\beta = 0}$$

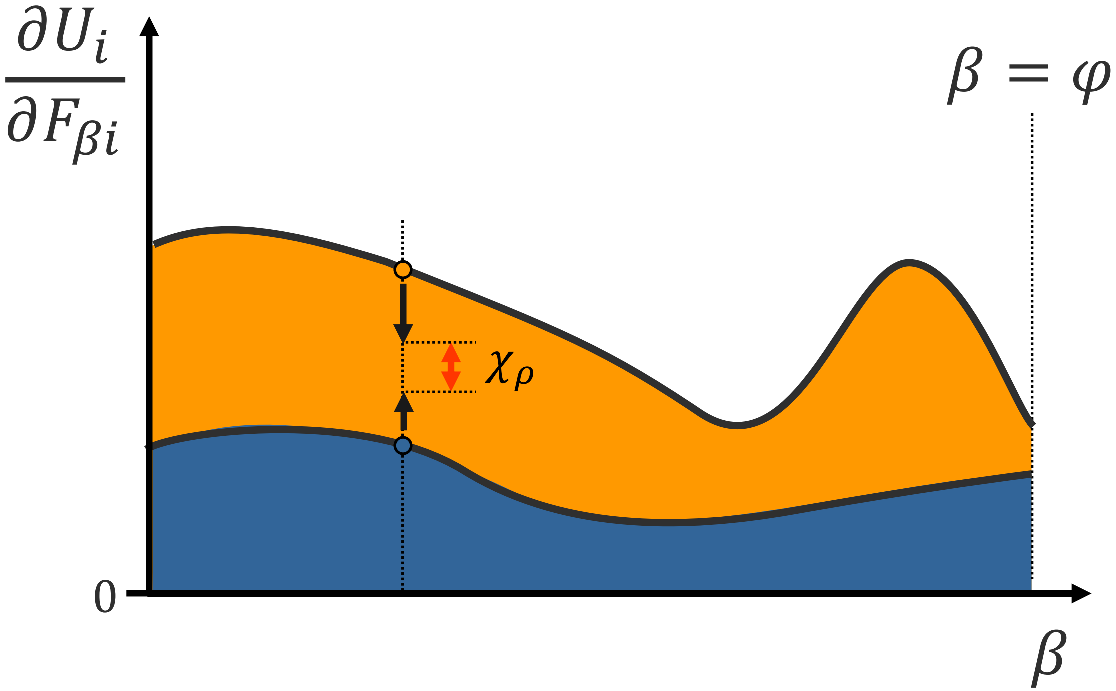
The position and direction of imaginary loads could also be parametrised. In this case, the internal functions will have to be defined over dynamic sub-domains, parametrised based on the same variables used for the imaginary load.

3. Analytical formulation and method

An analytical method based on the concepts of geometrical interference and **Λ** functions

Interaction between bodies

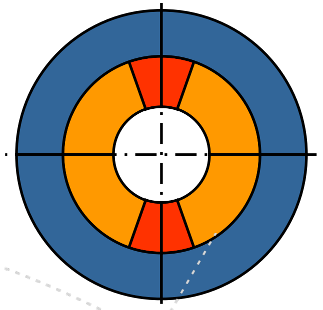


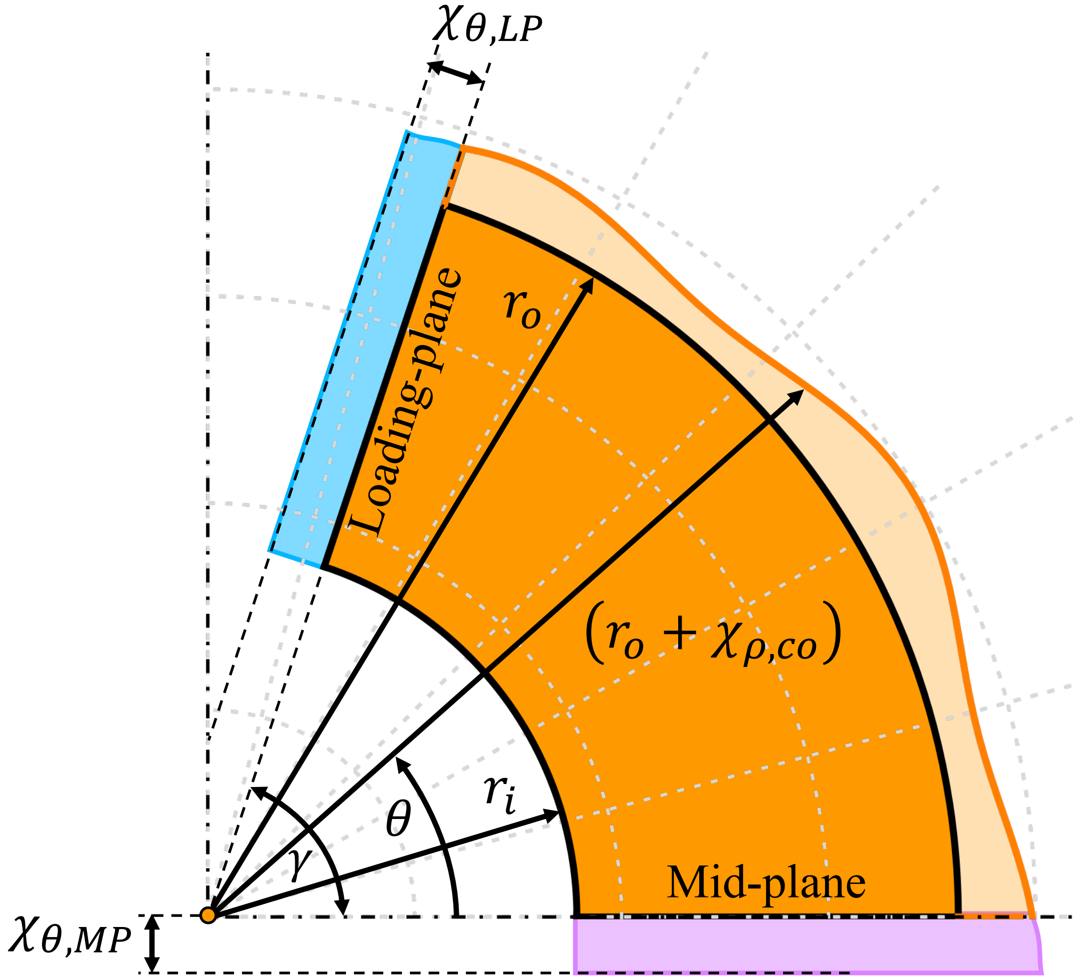


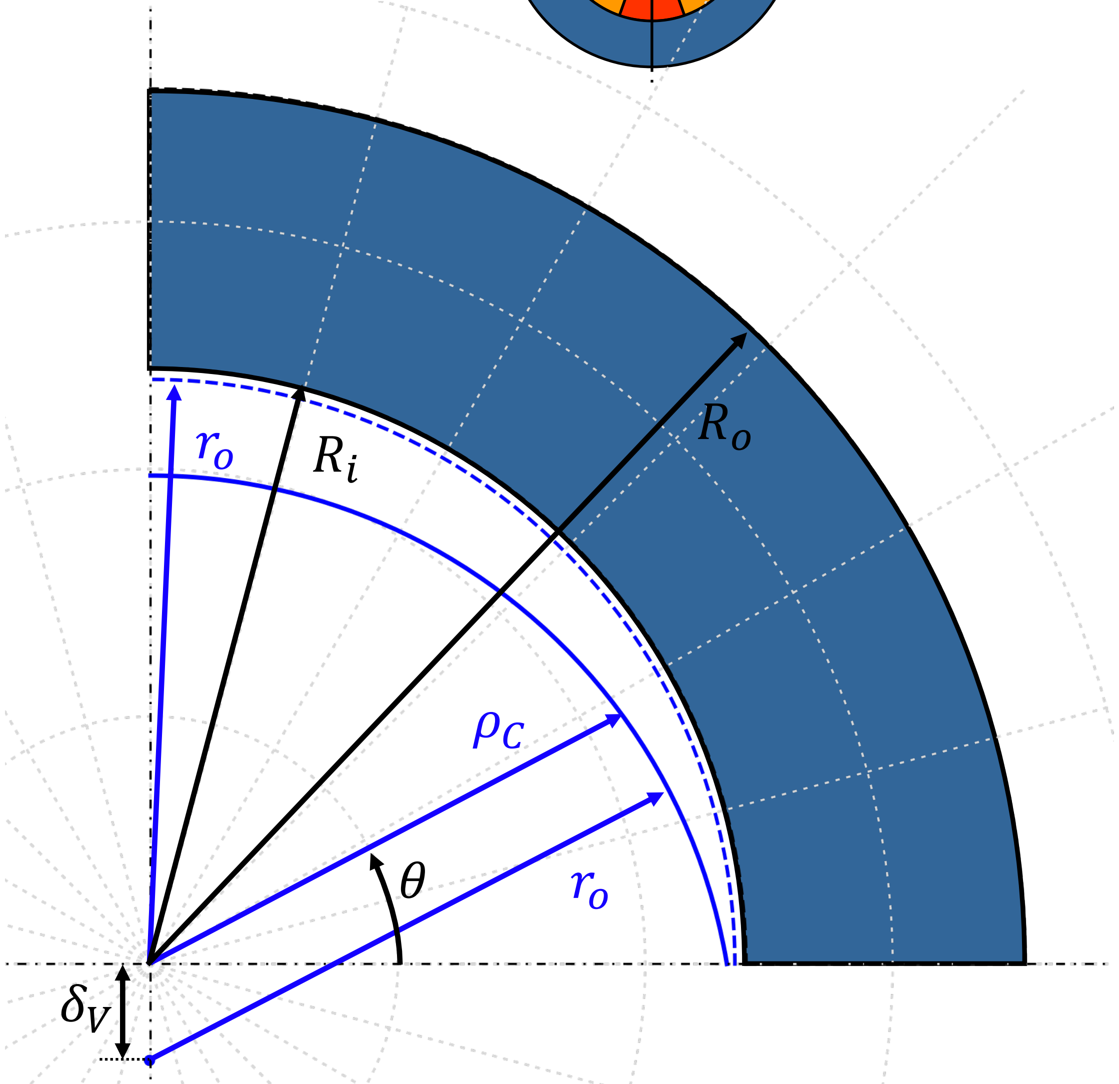
$$\Lambda = \int_0^{\varphi} \left(\frac{\partial U_1}{\partial F_{\beta 1}} - \frac{\partial U_2}{\partial F_{\beta 2}} + \chi_\rho \right)^2 d\beta$$

4. The 11 T collared coils structure

Geometry and temperature *General definitions*

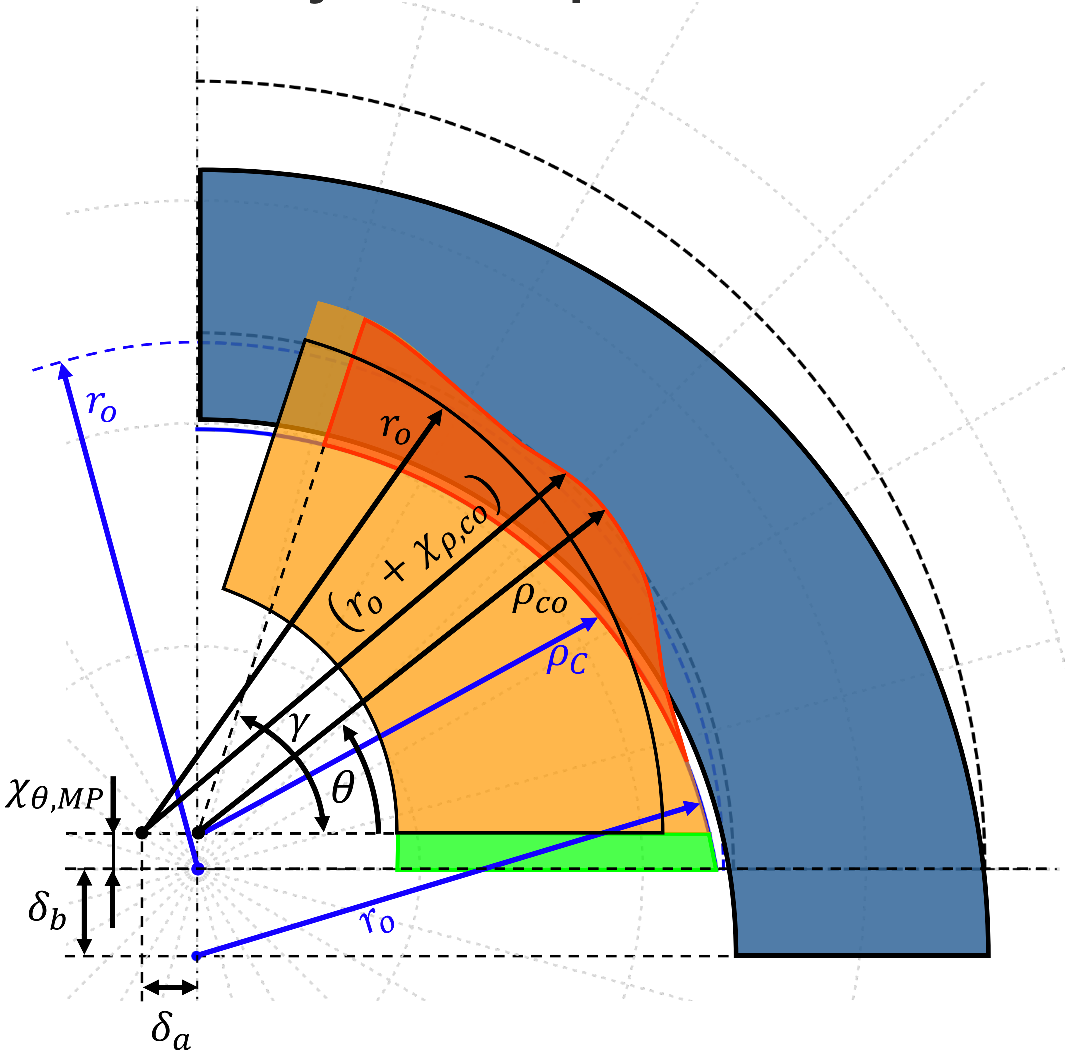






4. The 11 T collared coils structure

Geometry and temperature *General definitions*



Generalisation of the inner radial boundary of the ***collar***

$$\rho_C = (\delta_b - \chi_{\theta,MP}) \sin\theta + \sqrt{(r_o + R_i \alpha_C \Delta T_C)^2 - (\delta_b - \chi_{\theta,MP})^2 \cos^2\theta}$$

Generalisation of the outer radial boundary of the ***coil***

$$\rho_{co} = \delta_a \cos\theta + \sqrt{\left((r_o + \chi_{\rho,co})(1 + \alpha_{co}\Delta T_{co})\right)^2 - {\delta_a}^2 \sin^2\theta}$$

Generalisation of the radial geometrical interference, χ_ρ

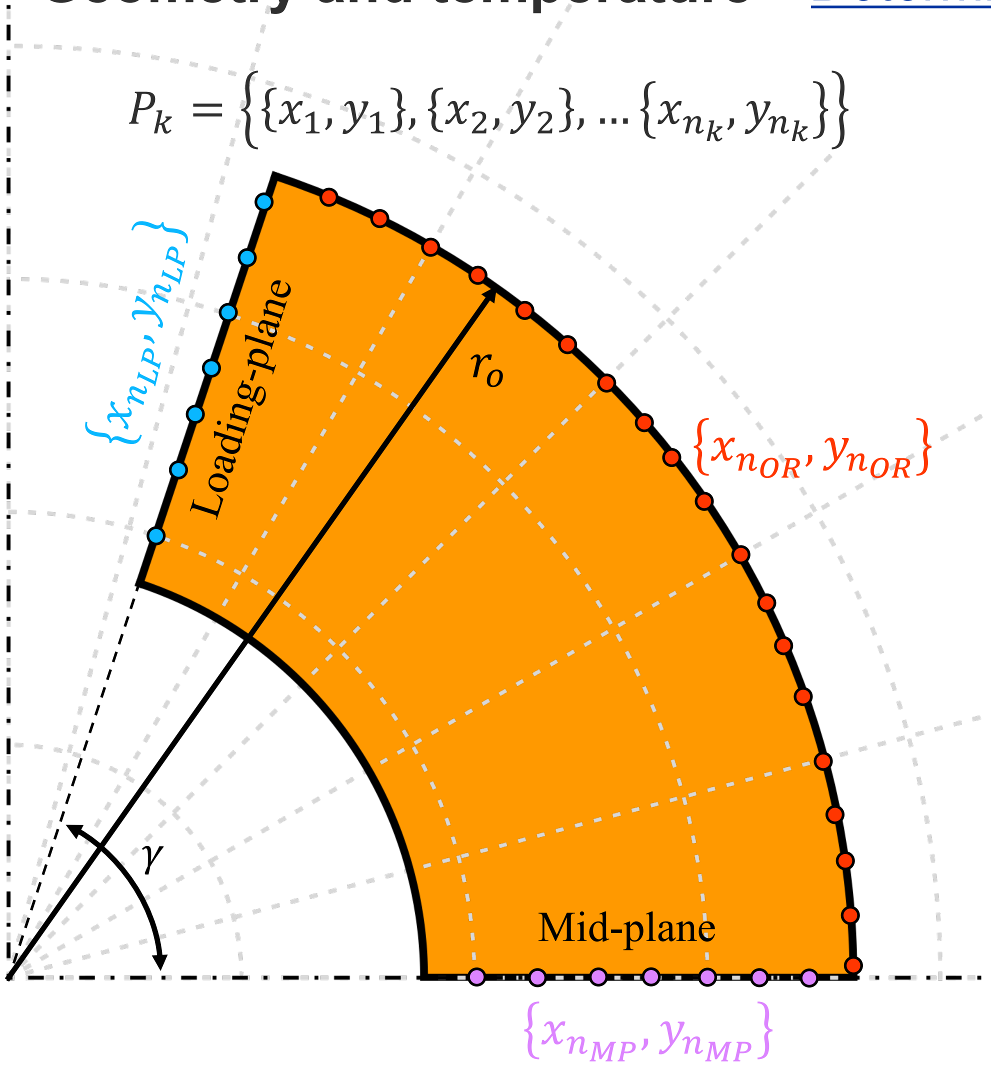
$$\chi_\rho = \rho_{co} - \rho_C$$

Geometrical interference created by the pole

$$\chi_{\theta,p} = \alpha_p \Delta T_p \left(\frac{\pi}{2} - \gamma\right) r_a - (R_i \alpha_C \Delta T_C + \delta_b - \chi_{\theta,MP}) \cos\gamma$$

4. The 11 T collared coils structure

Geometry and temperature *Determination of the geometry of a coil*



Normal deviation function of the loading plate

$$N_{dev,LP} = \sqrt{x^2 + y^2}\sin\left(\arctan\left(\frac{y}{x}\right) - \gamma\right)$$

Normal deviation function of the outer radius

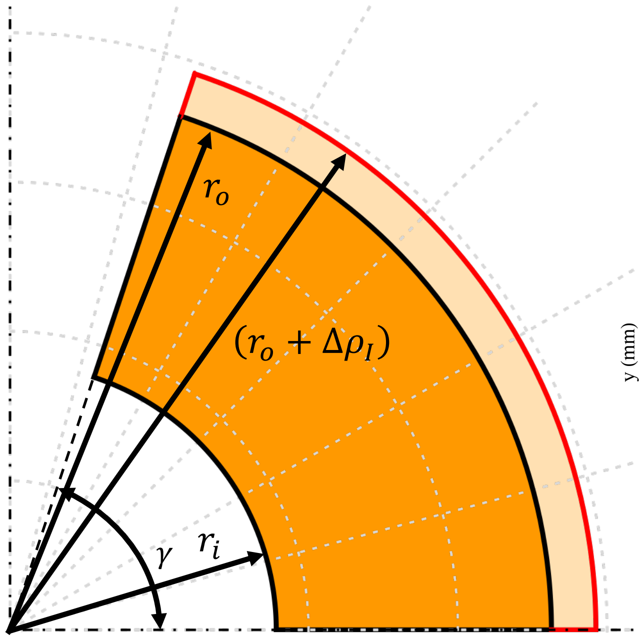
$$N_{dev,OR} = r_o - \sqrt{x^2 + y^2}$$

Global fitting minimisation function (i.e. alignment)

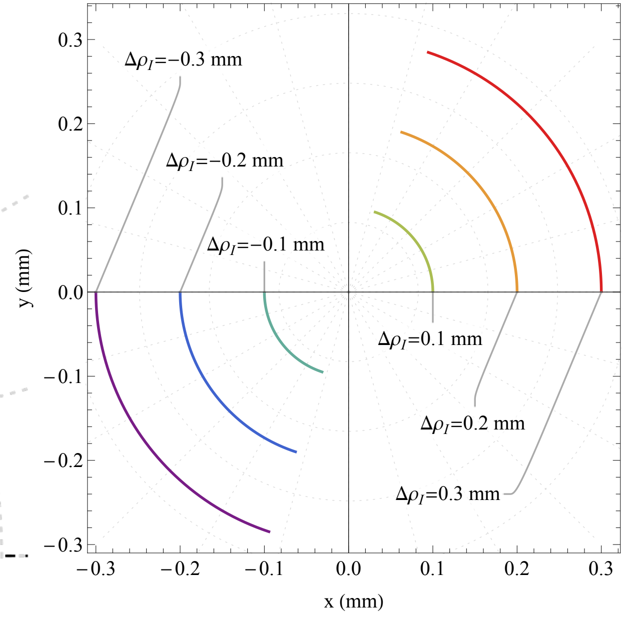
$$\min_{\{x_f, y_f, \theta_f\} \in \mathbb{R}} \left(\sum_{i=1}^{n_{LP}} \left(N_{dev,LP}\right)^2 + \sum_{i=1}^{n_{OR}} \left(N_{dev,OR}\right)^2 \right)$$

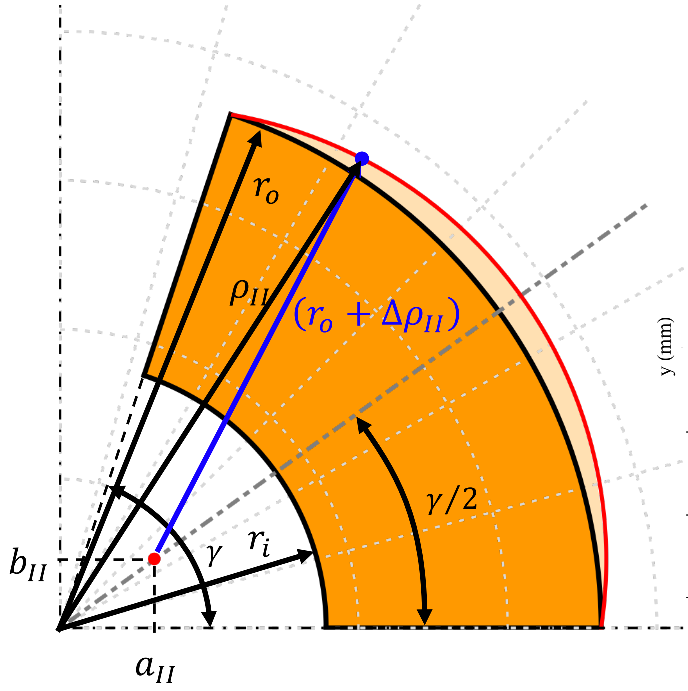
4. The 11 T collared coils structure

Geometry and temperature *Determination of the geometry of a coil*

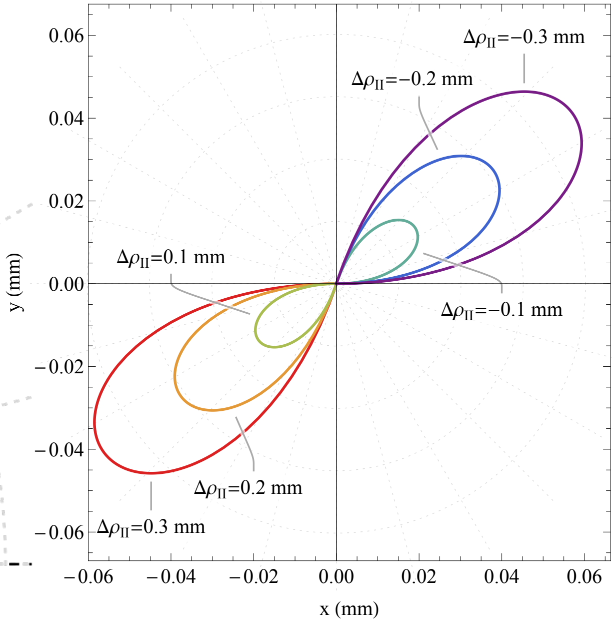


Type I χ_ρ





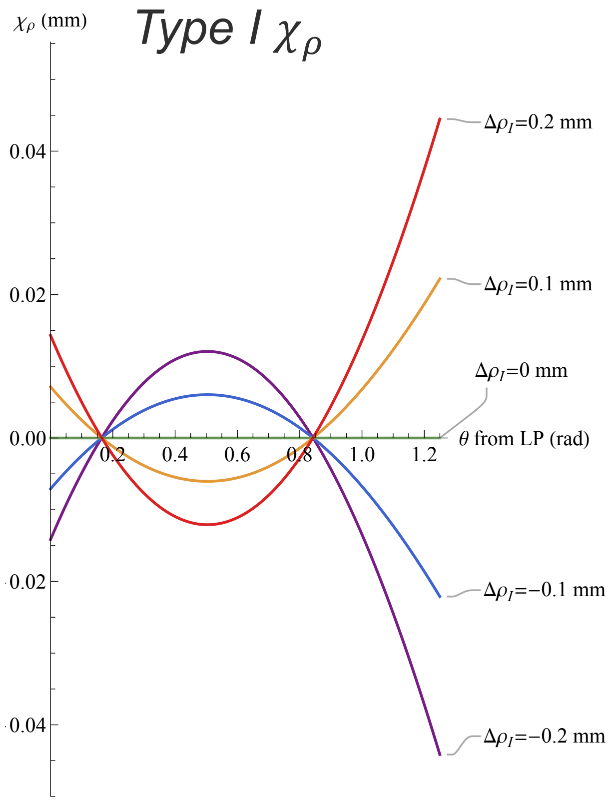
Type II χ_ρ

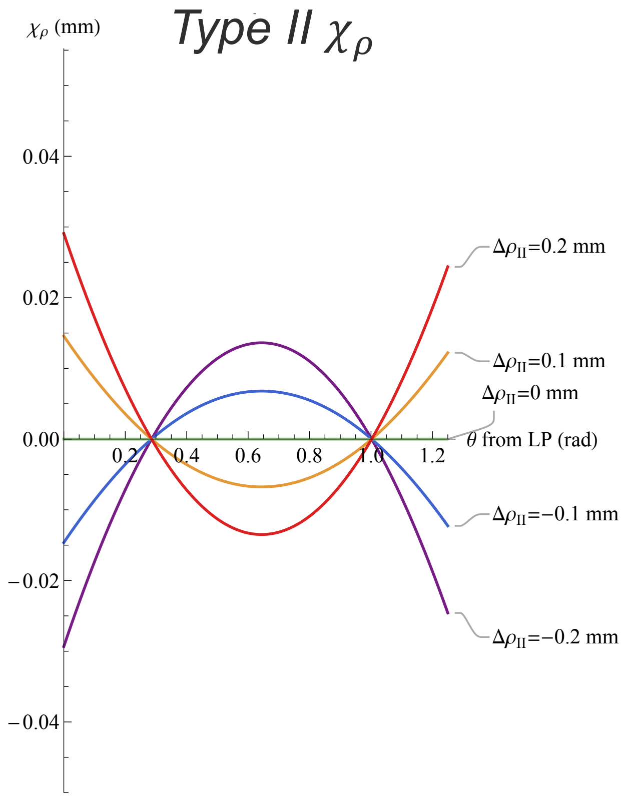


4. The 11 T collared coils structure

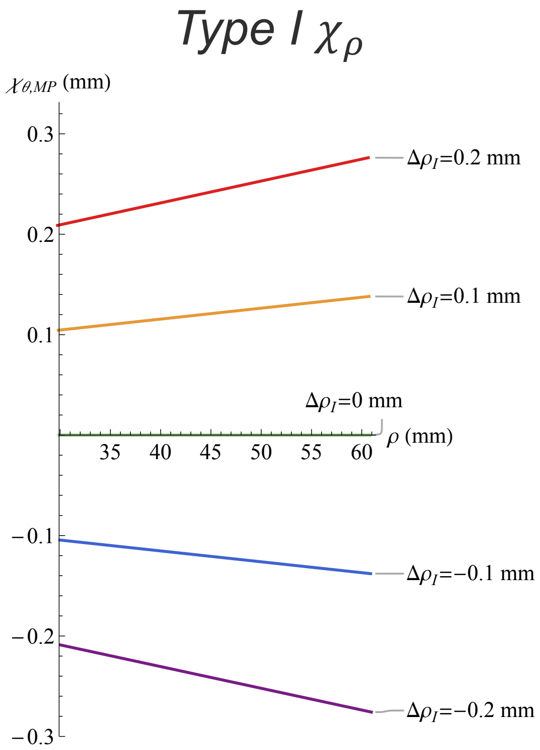
Geometry and temperature *Determination of the geometry of a coil*

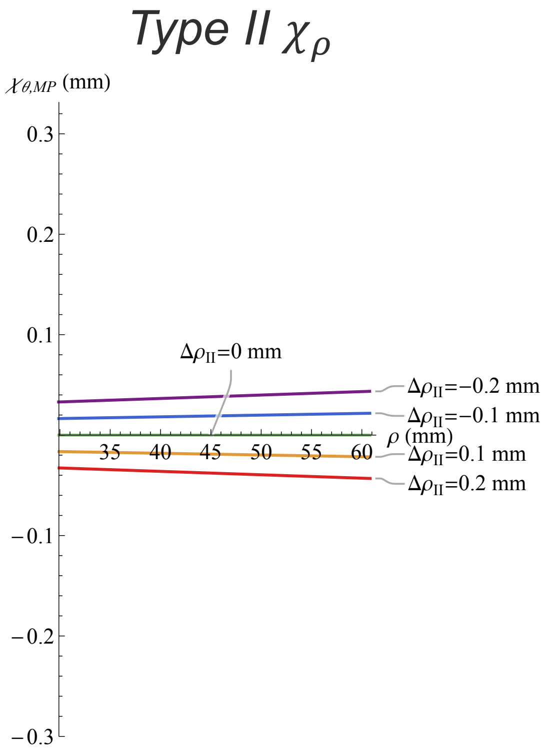
Apparent χ_ρ after alignment





Apparent azimuthal interference after alignment





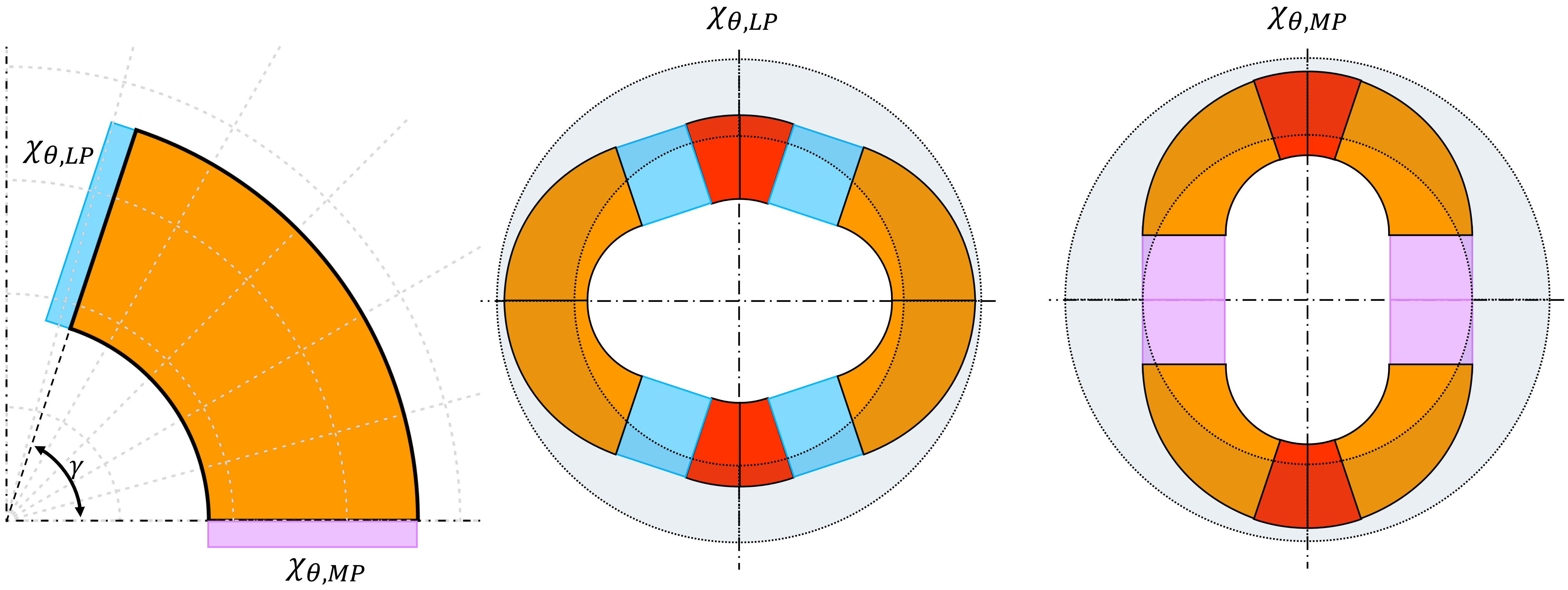
$$\bar{\chi}_{\theta,I} = \gamma\Delta\rho_I$$

$$\bar{\chi}_{\theta,II} = \left(2(r_o + \Delta\rho_{II}) \arcsin\left(\frac{r_o}{r_o + \Delta\rho_{II}} \sin\frac{\gamma}{2}\right)\right) - \gamma r_o$$

4. The 11 T collared coils structure

Geometry and temperature

Are $\chi_{\theta,LP}$ and $\chi_{\theta,MP}$ really not equivalent? A quick Gedankenexperiment

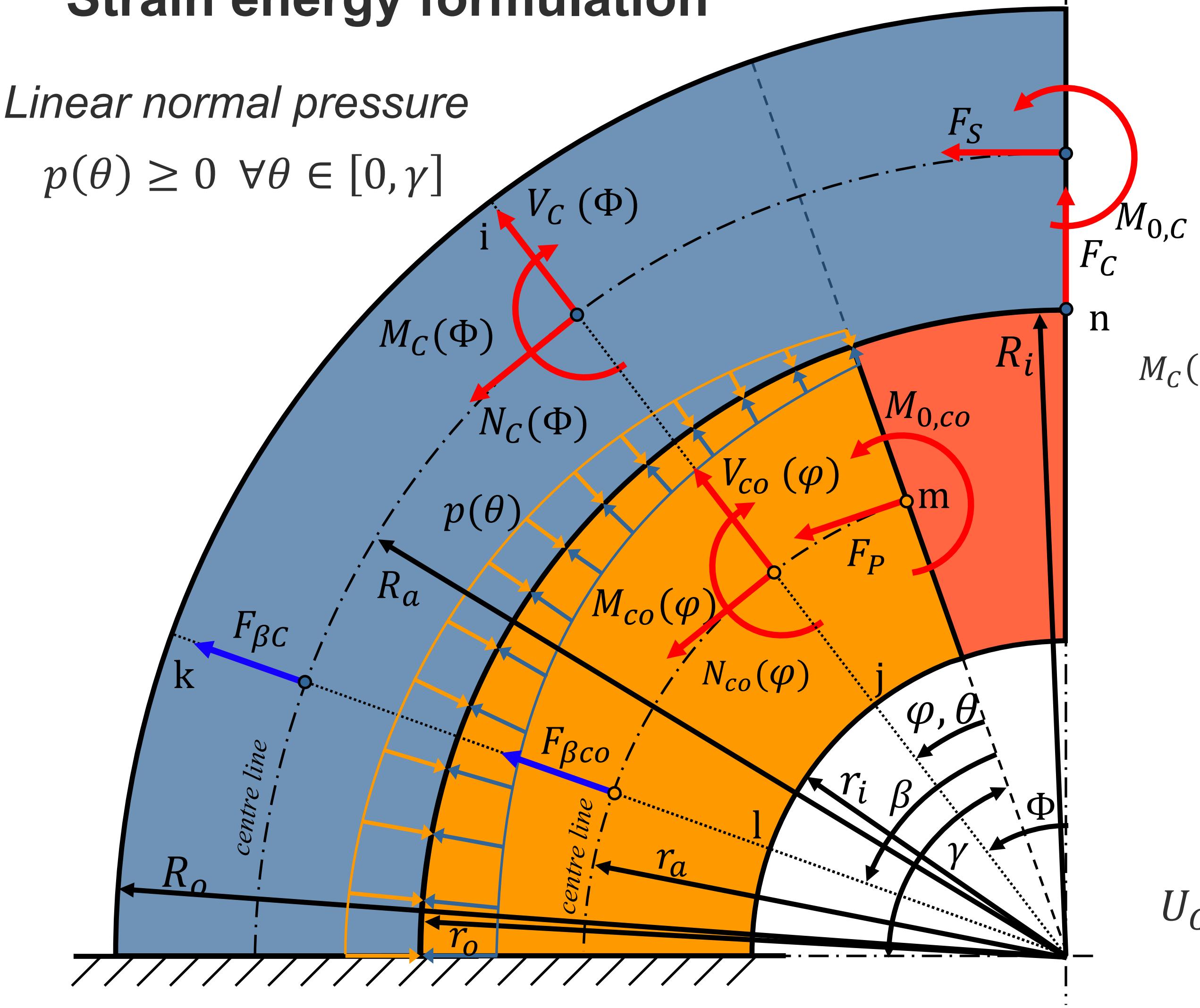


4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \;\; \forall \theta \in [0, \gamma]$



Energy formulation of the collar

$$F_S = F_{\beta C}\cos(\gamma - \beta) + r_o \int_0^{\gamma} p(\theta)\cos(\gamma - \theta)\, d\theta$$

$$M_C(\Phi) = \begin{cases} M_{1,C}(\Phi) = M_{0,C} + F_C R_a \sin\Phi - F_S R_a (1 - \cos\Phi) \quad for\ 0 < \Phi \leq \left(\frac{\pi}{2} - \gamma\right) \\ M_{2,C}(\Phi) = M_{1,C} + R_a r_o \int_0^{\Phi - \left(\frac{\pi}{2} - \gamma\right)} p(\theta) \sin\left(\Phi - \left(\frac{\pi}{2} - \gamma\right) - \theta\right) d\theta \\ \qquad for\ \left(\frac{\pi}{2} - \gamma\right) < \Phi \leq \left(\frac{\pi}{2} - \gamma + \beta\right) \\ M_{3,C}(\Phi) = M_{2,C} + F_{\beta C} R_a \sin\left(\Phi - \left(\frac{\pi}{2} - \gamma\right) - \beta\right) \\ \qquad for\ \left(\frac{\pi}{2} - \gamma + \beta\right) < \Phi \leq \frac{\pi}{2} \end{cases}$$

⋮

Collar's global strain energy function

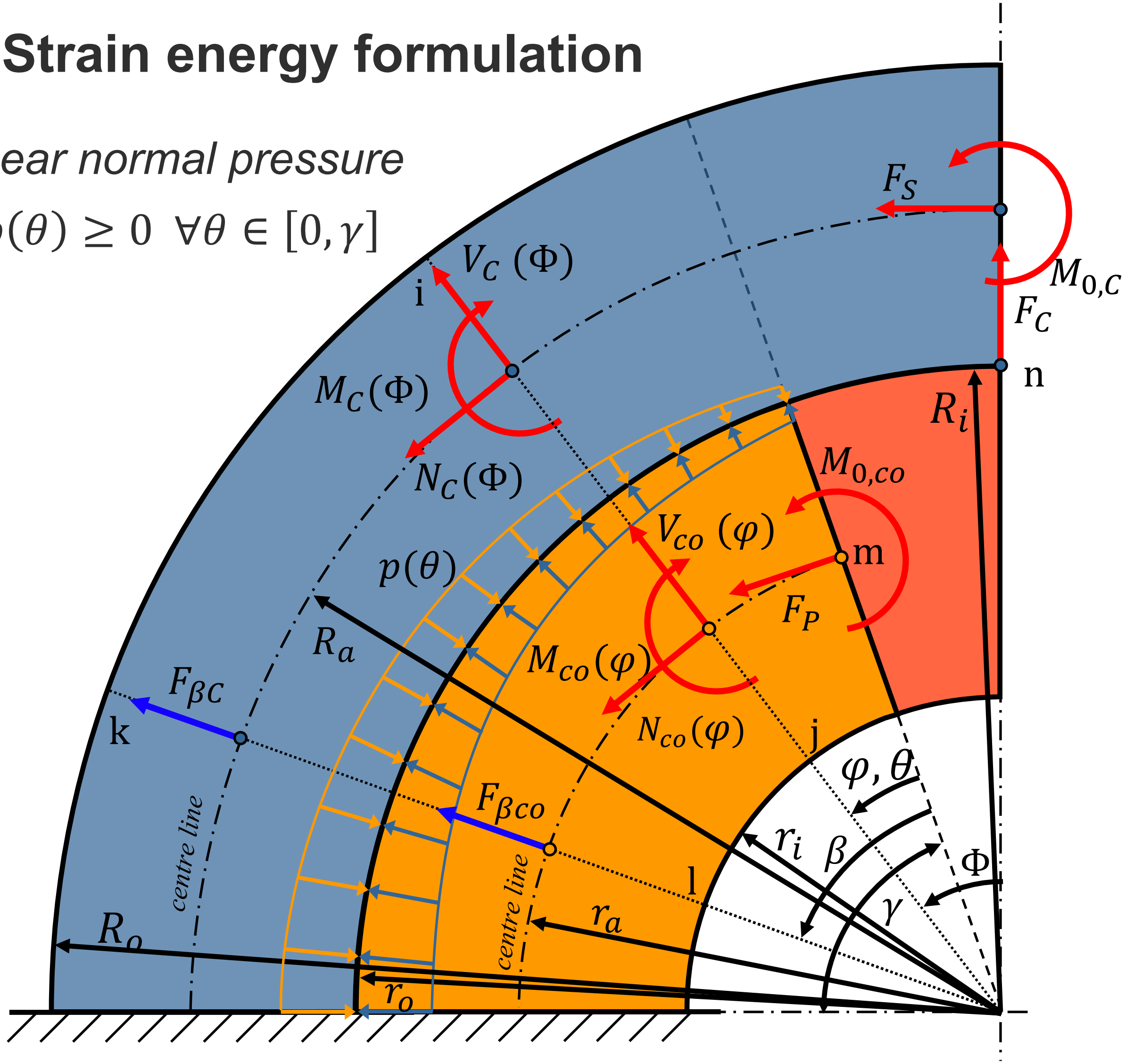
$$U_C = \int_0^{\left(\frac{\pi}{2} - \gamma\right)} g_{1,C}\, d\Phi + \int_{\left(\frac{\pi}{2} - \gamma\right)}^{\left(\frac{\pi}{2} - \gamma + \beta\right)} g_{2,C}\, d\Phi + \int_{\left(\frac{\pi}{2} - \gamma + \beta\right)}^{\frac{\pi}{2}} g_{3,C}\, d\Phi$$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \ \ \forall\theta \in [0, \gamma]$



Energy formulation of the coil

$$M_{co}(\varphi) = \begin{cases} M_{1,co}(\varphi) = M_{0,co} + F_P r_a (1 - \cos\varphi) - r_o r_a \int_0^{\varphi} p(\theta)\sin(\varphi - \theta)\, d\theta & \text{for } 0 < \varphi \leq \beta \\ M_{2,co}(\varphi) = M_{1,co}(\varphi) + F_{\beta co} r_a \sin(\varphi - \beta) & \text{for } \beta < \varphi \leq \gamma \end{cases}$$

$$N_{co}(\varphi) = \begin{cases} N_{1,co}(\varphi) = -F_P \cos\varphi - r_o \int_0^{\varphi} p(\theta)\sin(\varphi - \theta)\, d\theta & \text{for } 0 < \varphi \leq \beta \\ N_{2,co}(\varphi) = N_{1,co}(\varphi) + F_{\beta co} \sin(\varphi - \beta) & \text{for } \beta < \varphi \leq \gamma \end{cases}$$

$$V_{co}(\varphi) = \begin{cases} V_{1,co}(\varphi) = -F_P \sin\varphi + r_o \int_0^{\varphi} p(\theta)\cos(\varphi - \theta)\, d\theta & \text{for } 0 < \varphi \leq \beta \\ V_{2,co}(\varphi) = N_{1,co}(\varphi) - F_{\beta co} \cos(\varphi - \beta) & \text{for } \beta < \varphi \leq \gamma \end{cases}$$

Coil's global strain energy function

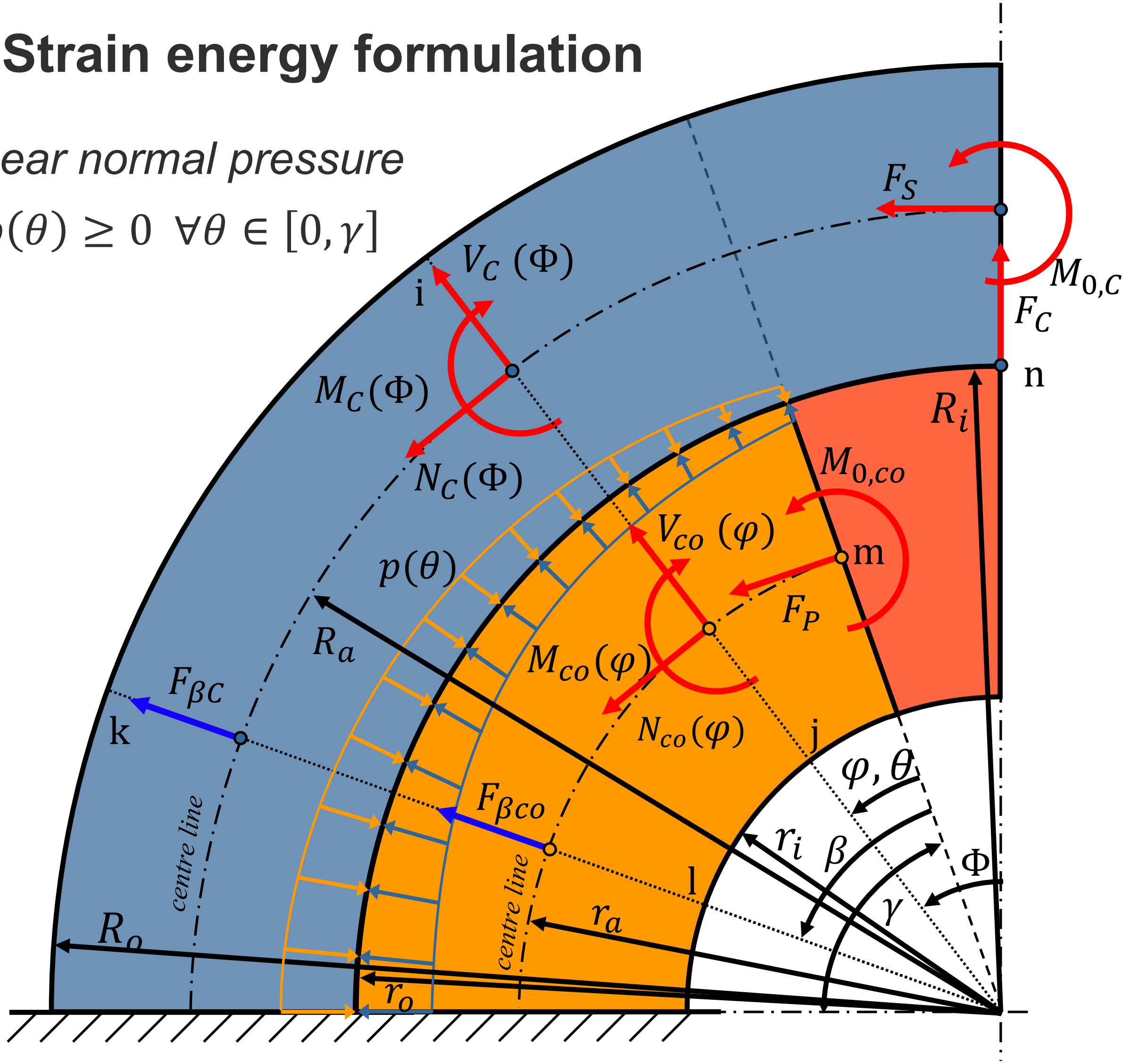
$$U_{co} = \int_0^{\beta} g_{1,co}\, d\varphi + \int_{\beta}^{\gamma} g_{2,co}\, d\varphi$$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \;\; \forall \theta \in [0, \gamma]$



Collar equilibrium

Equilibrium of vertical forces in the pole $F_C = F_P \cos\gamma$

Vertical stress in the collar nose $\sigma_{CN} = -\dfrac{2F_C}{b_C c_C}$

$$F_P = \frac{r_o}{\sin\gamma}\int_0^{\gamma} p(\theta)\cos(\gamma - \theta)\,d\theta$$

Collar internal equilibrium equation

$$F_C = \frac{r_o}{\tan\gamma}\int_0^{\gamma} p(\theta)\cos(\gamma - \theta)\,d\theta$$

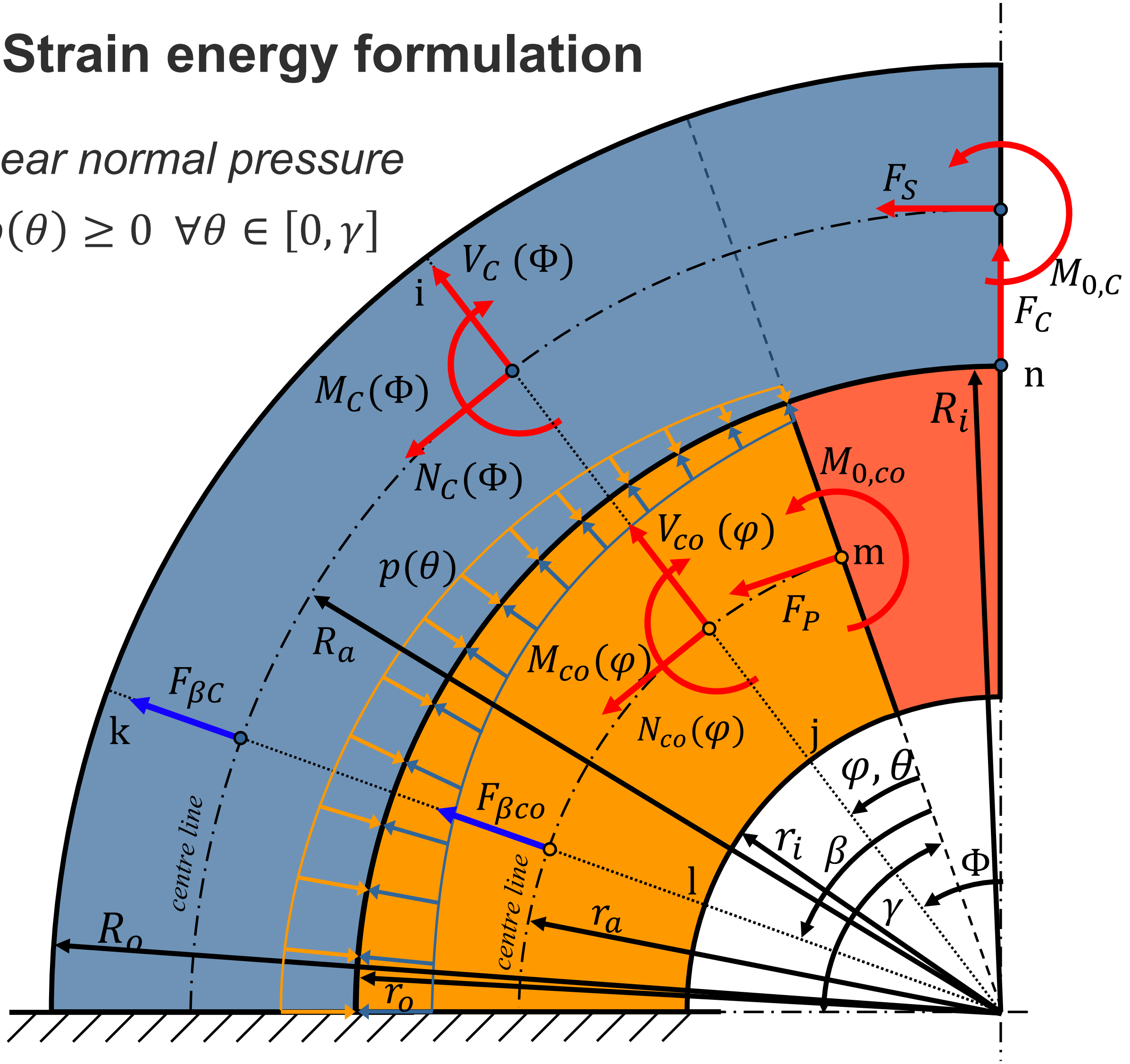
$$\sigma_{CN} = -\frac{r_o b_C c_C}{2\tan\gamma}\int_0^{\gamma} p(\theta)\cos(\gamma - \theta)\,d\theta$$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \ \ \forall \theta \in [0, \gamma]$



Collaring

Vertical integration of the pressure in the coil

$$F_{co,V} = r_o \int_0^{\gamma} p(\theta) \sin(\gamma - \theta)\, d\theta$$

Minimum collaring force

$$F_A = \frac{2\, r_o}{b} \int_0^{\gamma} p(\theta) \left(\sin(\gamma - \theta) + \frac{\cos(\gamma - \theta)}{\tan \gamma} \right) d\theta$$

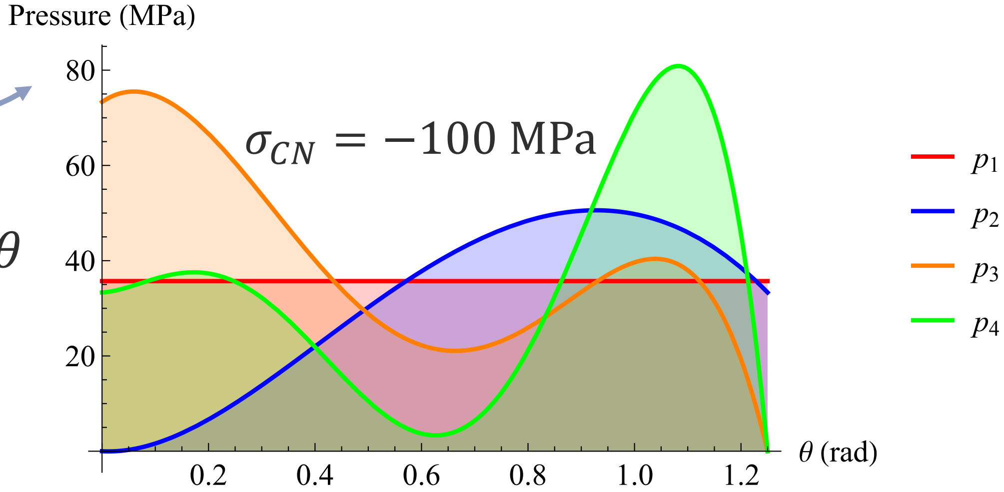
Dimensionless ratio between the total vertical force exerted by the press, F_A, and the force transferred through the collar nose

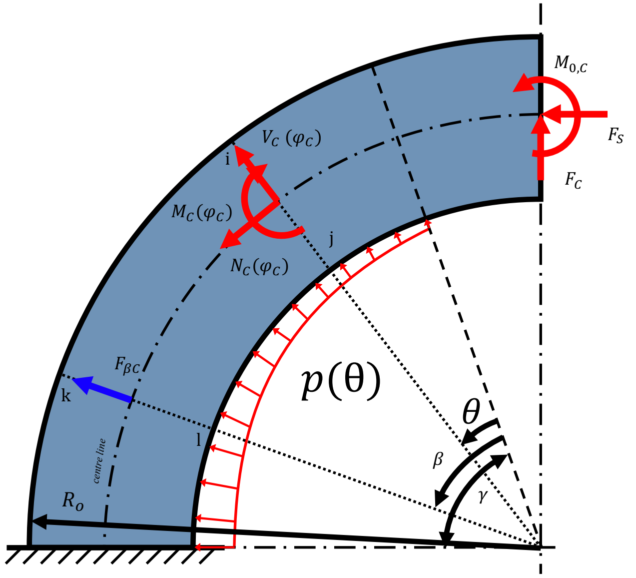
$$\zeta = b \frac{F_A}{2\, F_C}$$

4. The 11 T collared coils structure

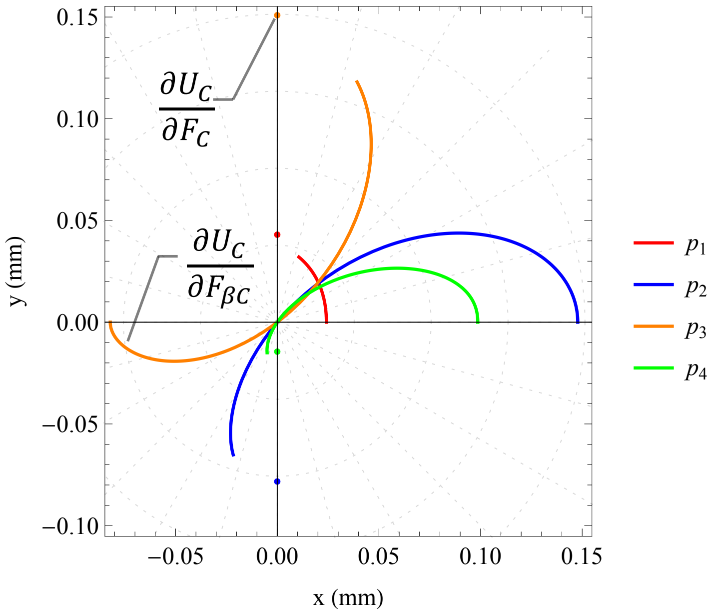
Strain energy formulation

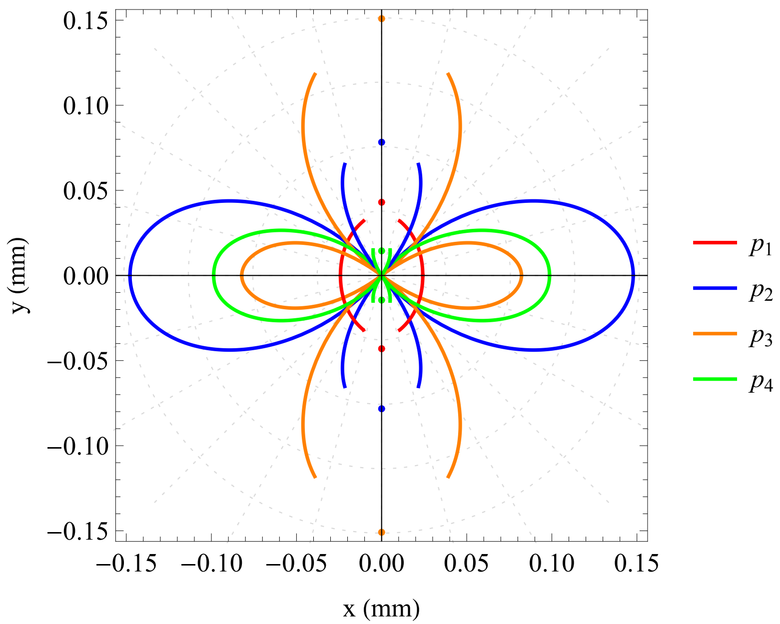
$$F_C = \frac{r_o}{\tan\gamma}\int_0^{\gamma} p(\theta)\cos(\gamma-\theta)\,d\theta$$



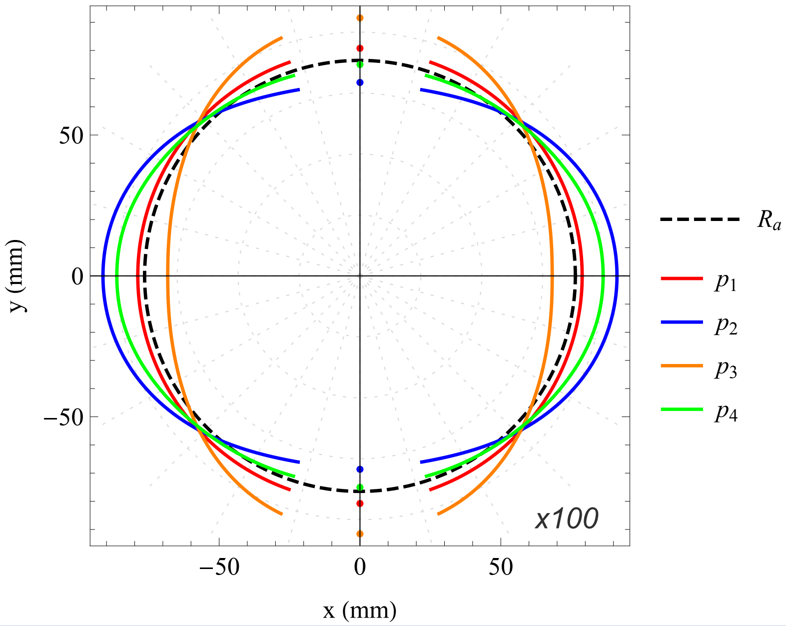


Radial deformation of the centre line of the collar structure for quadrant I





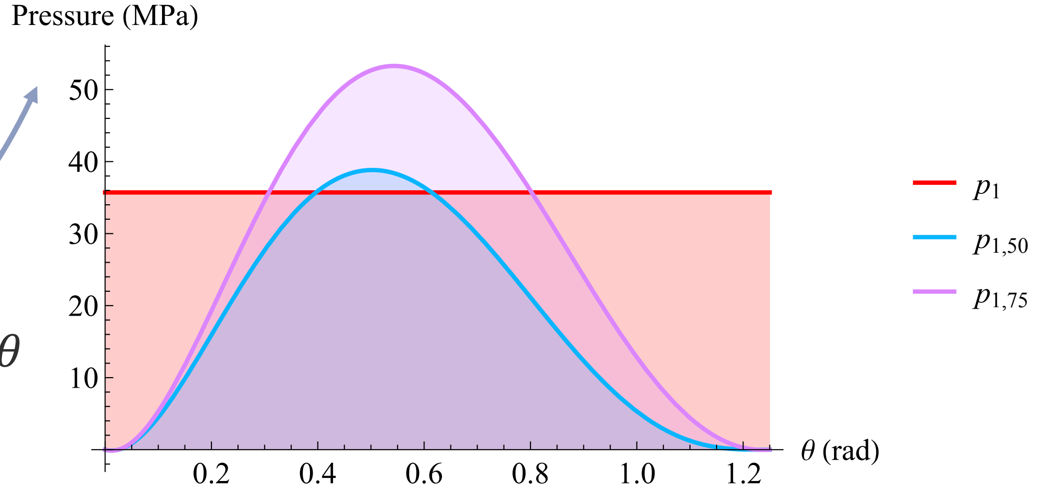
Visualisation of the centre line of the collar structure

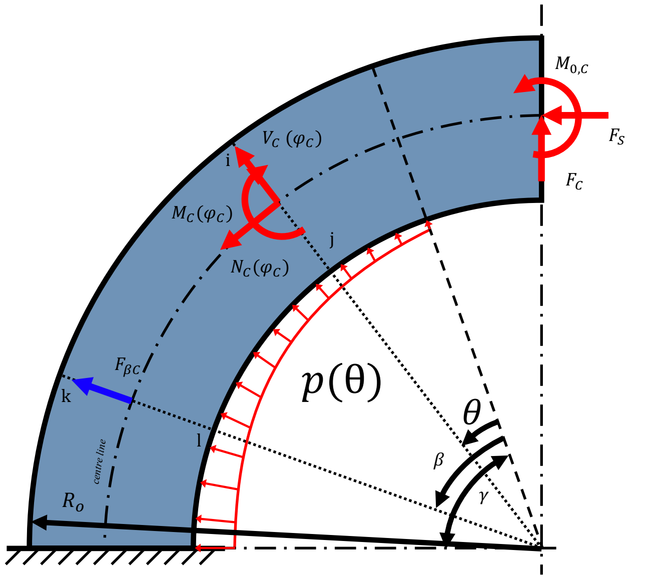


4. The 11 T collared coils structure

Strain energy formulation

$$\begin{cases} \left.\dfrac{\partial U_C}{\partial F_C}\right|_{F_C=F_{Ci},\, p(\theta)=p_{1,i}} = \delta_{C,V1} \\ F_{Ci} = \dfrac{r_o}{\tan\gamma}\displaystyle\int_0^{\gamma} p_{1,i}(\theta)\cos(\gamma-\theta)\, d\theta \end{cases}$$



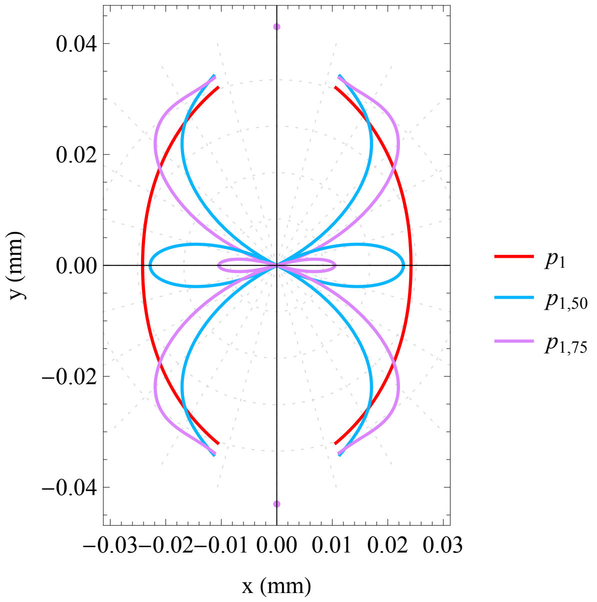


Solutions of $p(\theta)$ will be restricted to those in the form of polynomial expressions

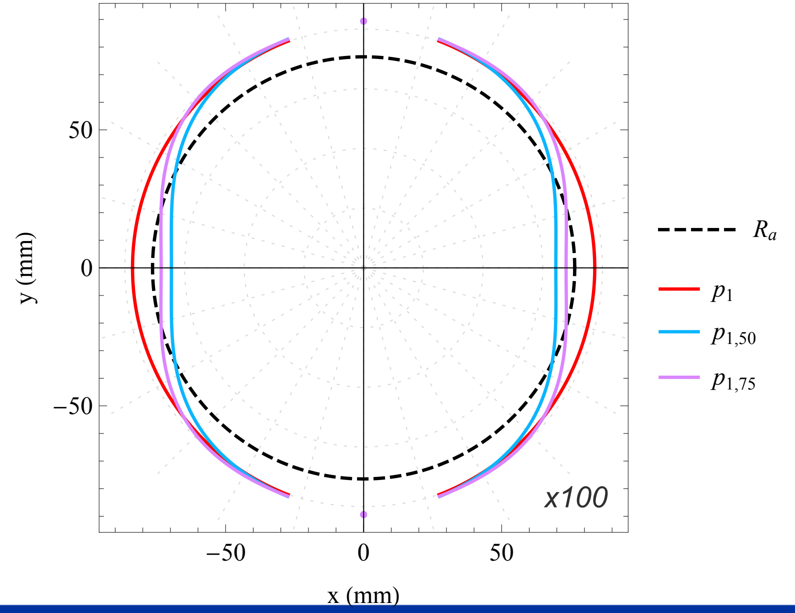
$$p(\theta) = \sum_{i=0}^{5} a_i \theta^i$$

Solutions for the polynomial coefficients to be found through a minimisation problem

$$\min_{\{a_i\}\in\mathbb{R}} \left| \delta_{C,V1} - \left.\frac{\partial U_C}{\partial F_C}\right|_{F_C=F_{Ci},\, p(\theta)=p_{1,i}} \right|$$

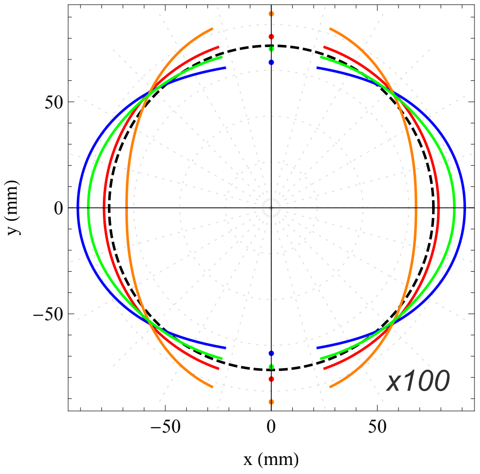


Visualisation of the centre line of the collar structure

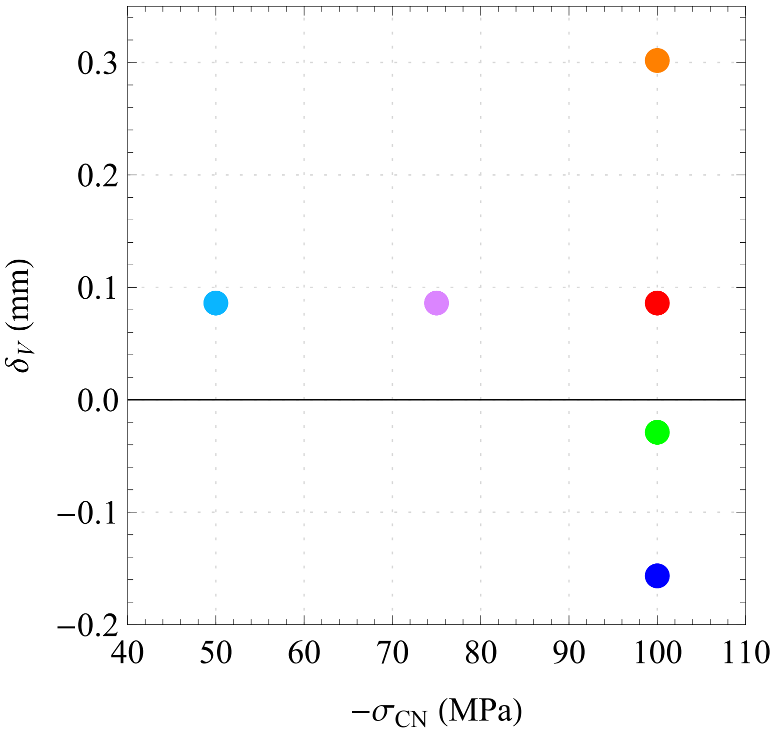


4. The 11 T collared coils structure

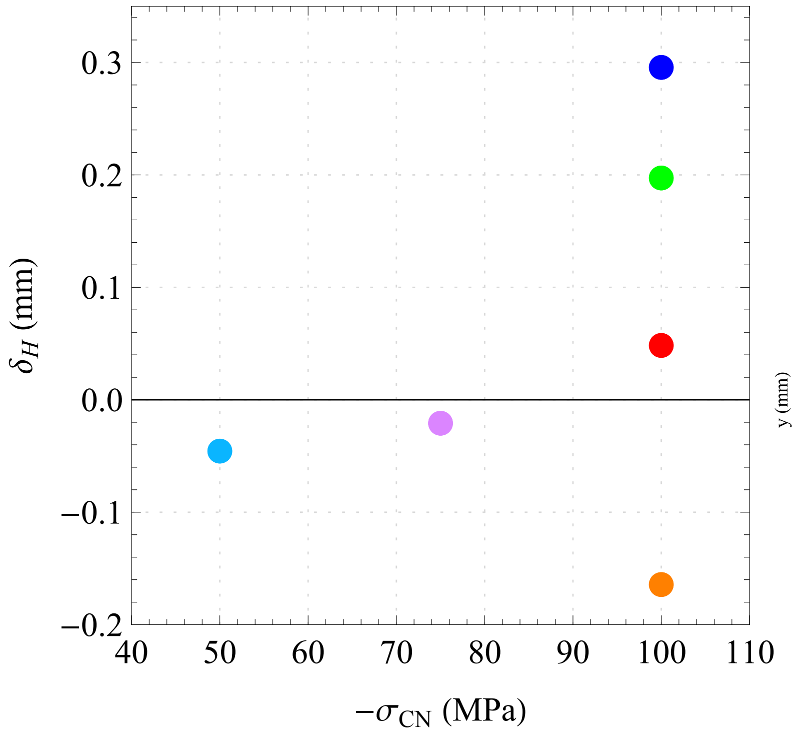
Strain energy formulation

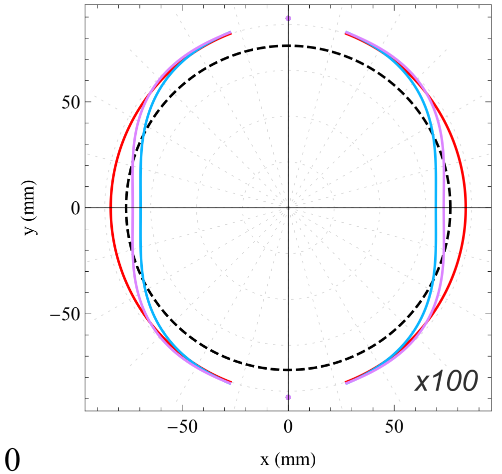


Total vertical deformation of the collars



Total horizontal deformation of the collars





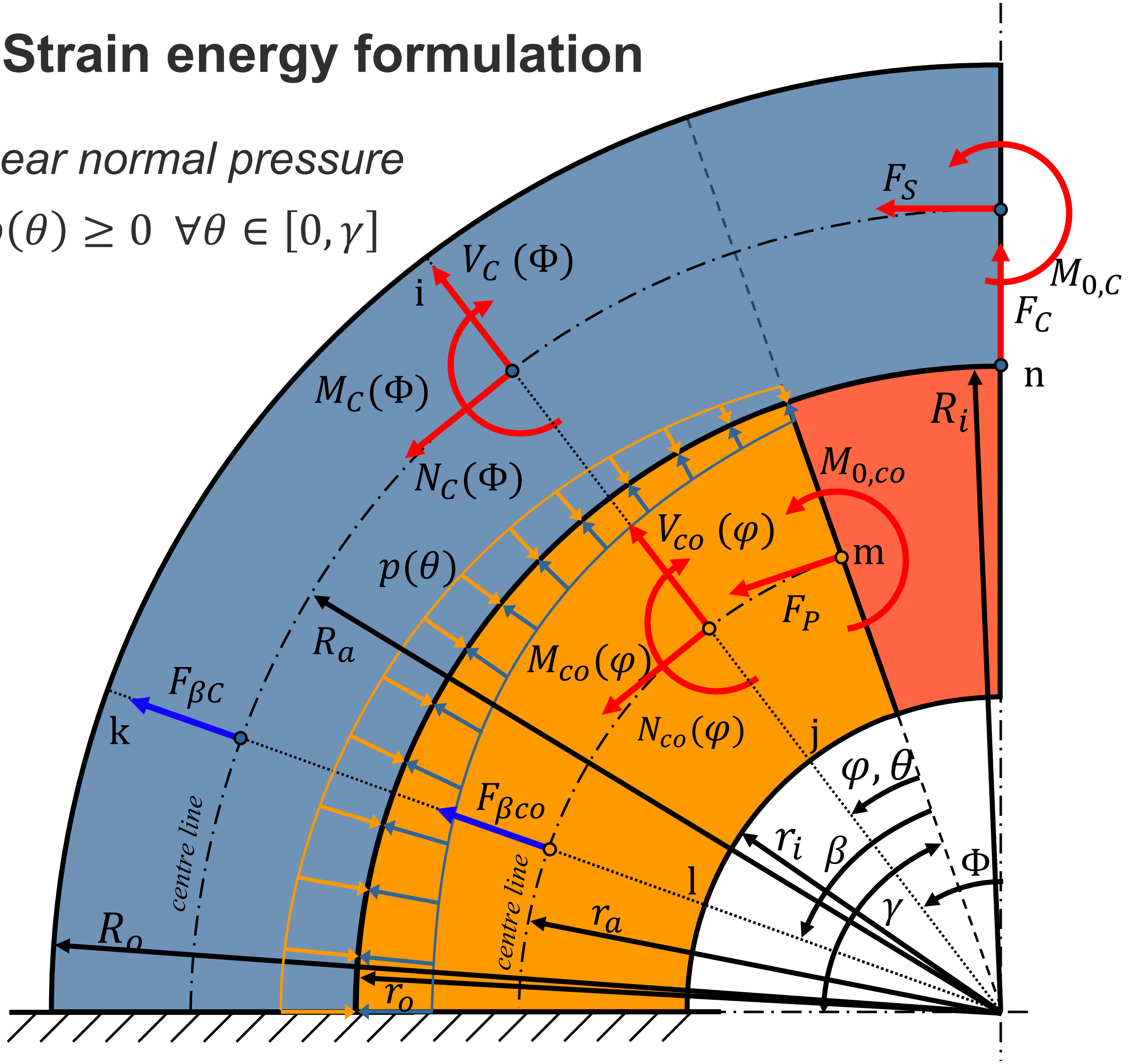
p_1 p_2 p_3 p_4 $p_{1,50}$ $p_{1,75}$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \;\; \forall \theta \in [0, \gamma]$



Global formulation of the collared coils

Pole and ground insulation deformation

$$\delta_{I\theta} = \frac{F_P}{b(r_o - r_i)}\left(\frac{\left(\frac{\pi}{2} - \gamma\right) r_a}{E_P} + \frac{t_{i\theta}}{E_i}\right)$$

Assembly geometrical interference

$$\chi_{\theta,a} = \chi_{shim} + \chi_{\theta,LP}$$

Overall azimuthal geometrical interference

$$\chi_\theta = \chi_{\theta,\Delta\rho} + \chi_{\theta T,co} + \chi_{\theta,p} + \chi_{\theta,a} - \delta_{I\theta}$$

Approximated radial deformation

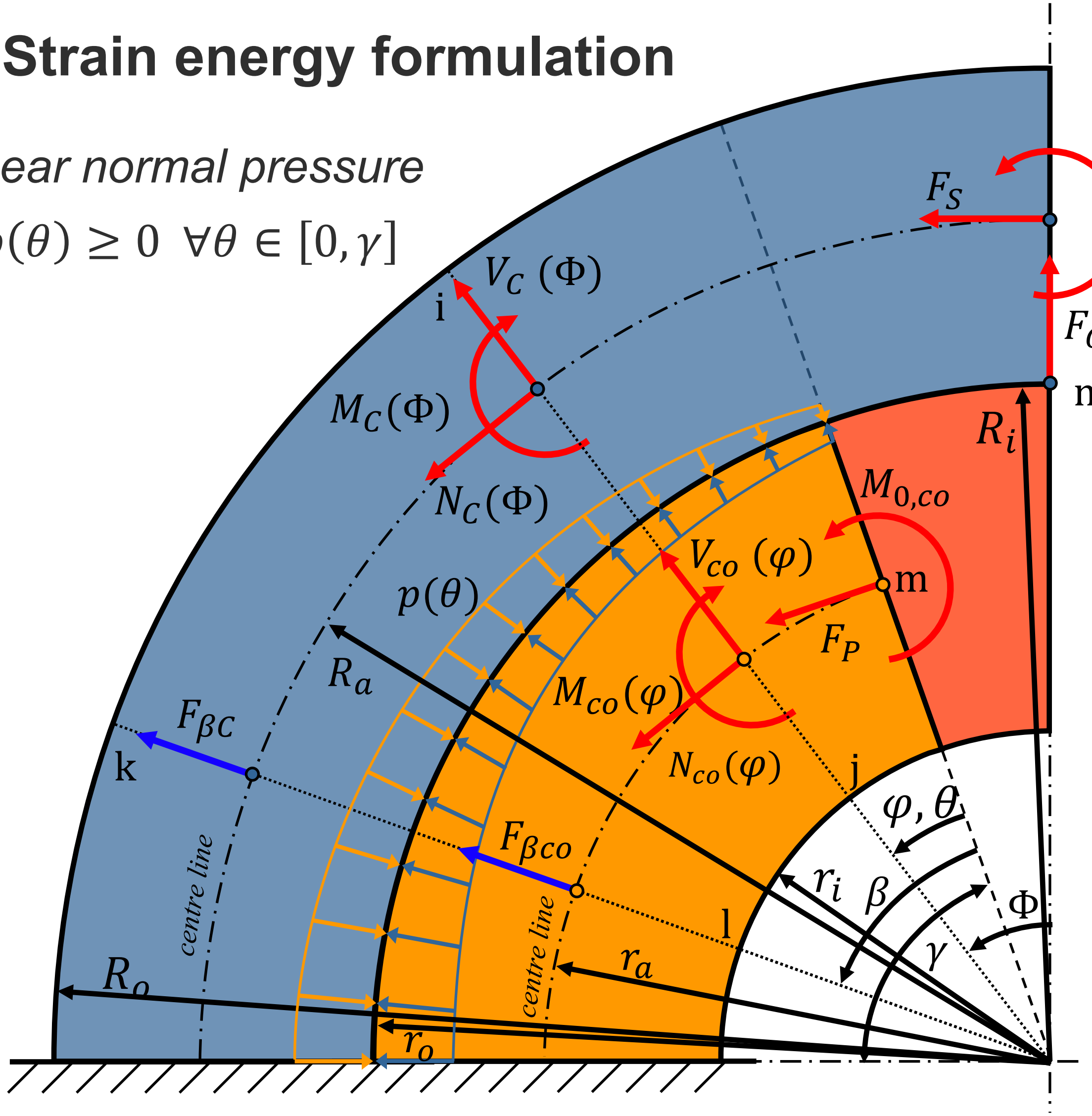
$$\delta_{I\rho} = \frac{1}{b_C}\left(\frac{t_{i\rho}}{E_i} + \frac{r_o - r_a}{E_{coil}} + \frac{r_o}{E_C}\left(\frac{R_a}{R_i} - 1\right)\right) p(\beta)$$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \;\; \forall\theta \in [0,\gamma]$



The global Λ functions

The pole displacement contribution

$$\frac{\partial U_C}{\partial F_C}\cos\gamma$$

Compression on the loading-plane

$$\frac{\partial U_{co}}{\partial F_P}$$

Displacement of the coil due to deformation of the collar

$$\left(\left(\frac{\partial U_C}{\partial F_{\beta C}} + \delta_{I\rho}\right)\bigg|_{\beta=\gamma} + \delta_a\right)\sin\gamma$$

Λ function for the azimuthal direction

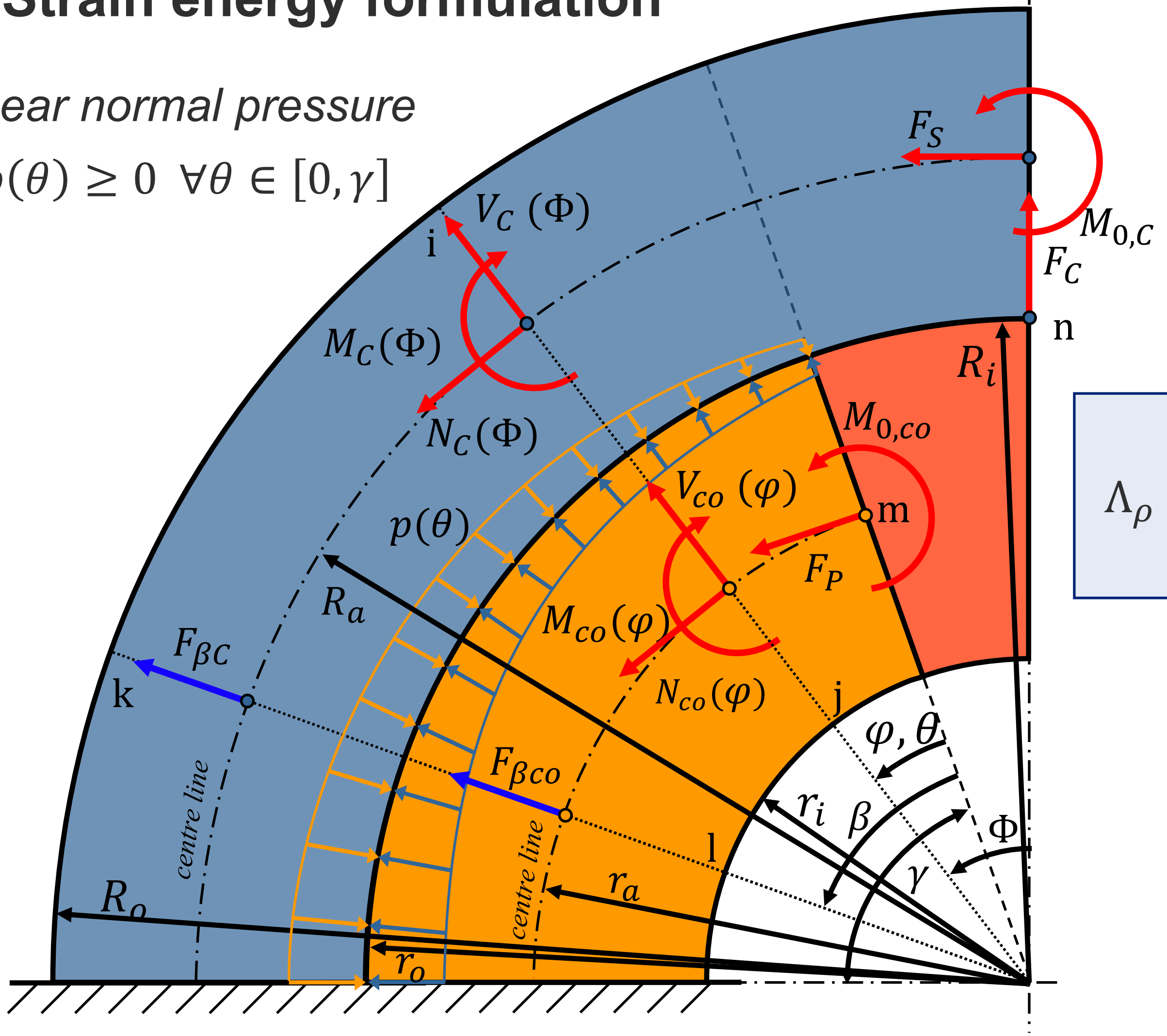
$$\Lambda_\theta = \frac{\partial U_C}{\partial F_C}\cos\gamma + \frac{\partial U_{co}}{\partial F_P} + \left(\left(\frac{\partial U_C}{\partial F_{\beta C}} + \delta_{I\rho}\right)\bigg|_{\beta=\gamma} + \delta_a\right)\sin\gamma - \chi_\theta$$

4. The 11 T collared coils structure

Strain energy formulation

Linear normal pressure

$p(\theta) \geq 0 \ \ \forall \theta \in [0, \gamma]$



The global $\boldsymbol{\Lambda}$ *functions*

$\boldsymbol{\Lambda}$ function for the rotation imposed in the loading-plane

$$\Lambda_{rot} = \frac{\partial U_{co}}{\partial M_P} - \chi_{rot}$$

$\boldsymbol{\Lambda}$ function for the radial direction

$$\Lambda_\rho = \int_0^\gamma \left(\frac{\partial U_C}{\partial F_{\beta C}} - \left(\frac{\partial U_{co}}{\partial F_{\beta co}} + \left(\frac{\partial U_C}{\partial F_{\beta C}} + \delta_{I\rho} \right) \Bigg|_{\beta=\gamma} \right) - \chi_\rho + \delta_{I\rho} \right)^2 d\beta$$

Finding solutions

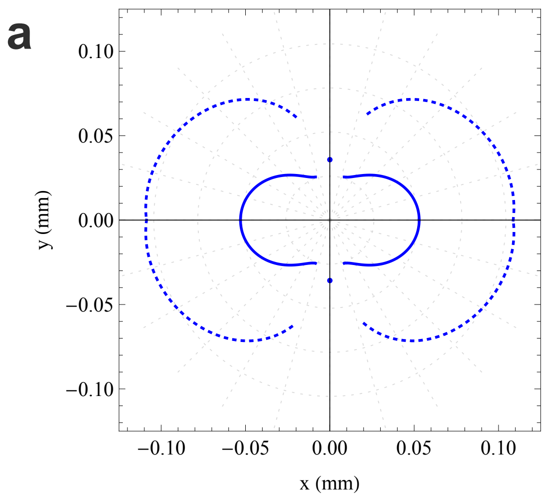
$$\begin{cases} \Lambda_\theta = 0 \\ \Lambda_{rot} = 0 \\ \Lambda_\rho = 0 \end{cases} \rightarrow \quad \min_{\{a_i\} \in \mathbb{R}} |\Lambda_\rho| \quad \text{subject to} \quad \begin{cases} \Lambda_\theta = 0 \\ \Lambda_{rot} = 0 \\ p(\theta) \geq 0 \ \ \forall \theta \in [0, \gamma] \end{cases}$$

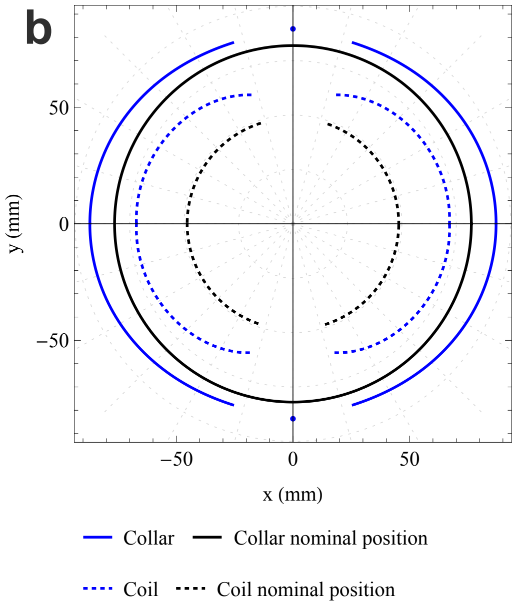
4. The 11 T collared coils structure

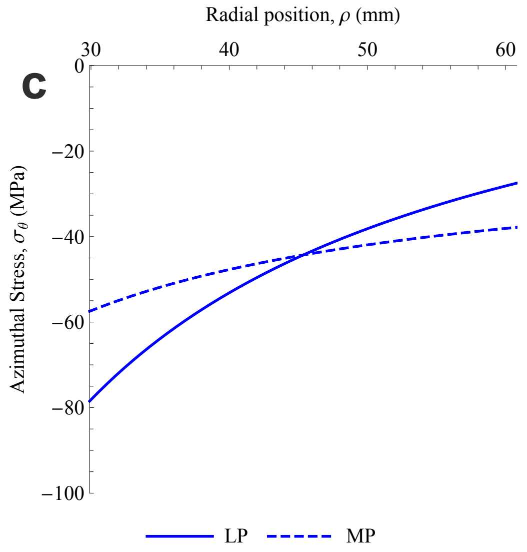
Solutions and discussion

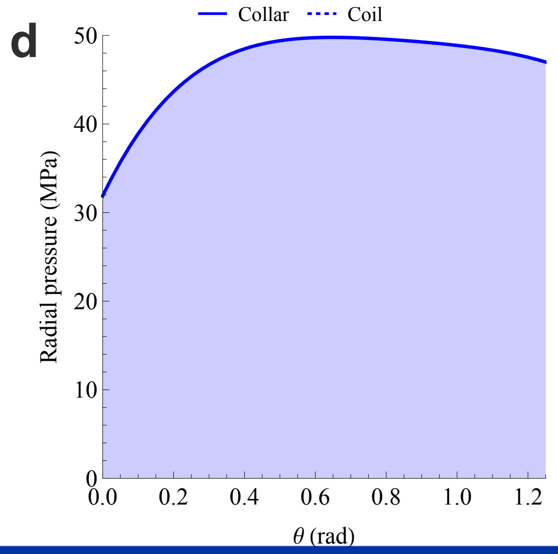
A nominal collared case

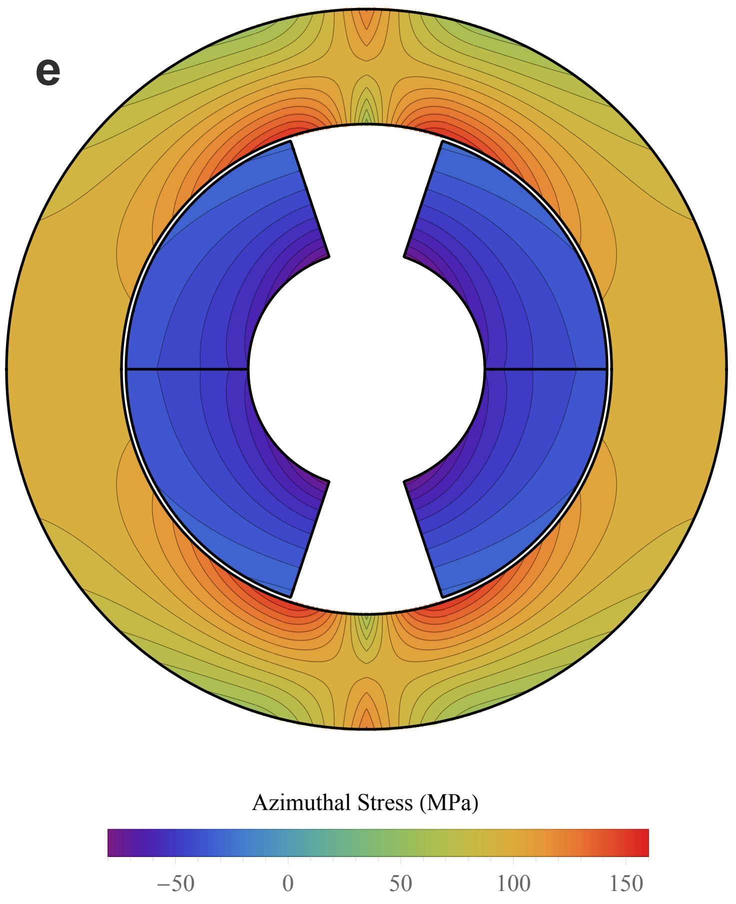
$$\chi_{\theta,aN} = \chi_{\theta,aN}^{*}\left(1+\frac{1}{\gamma}\arcsin\left(\frac{v_f\cos\gamma}{r_a}\right)\right)^{-1} \rightarrow 0.278\,mm$$







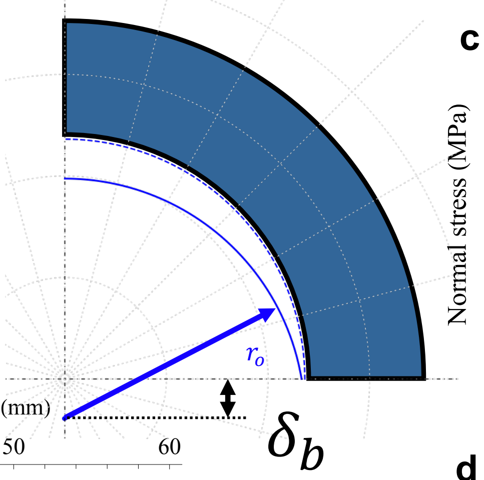


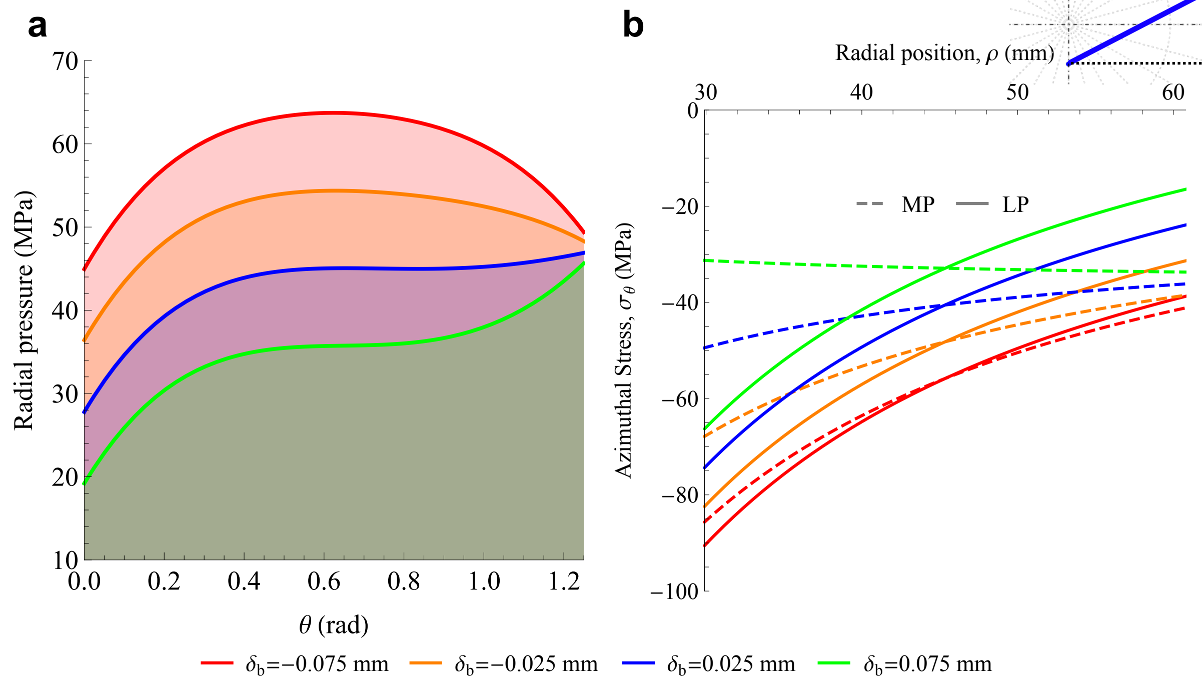


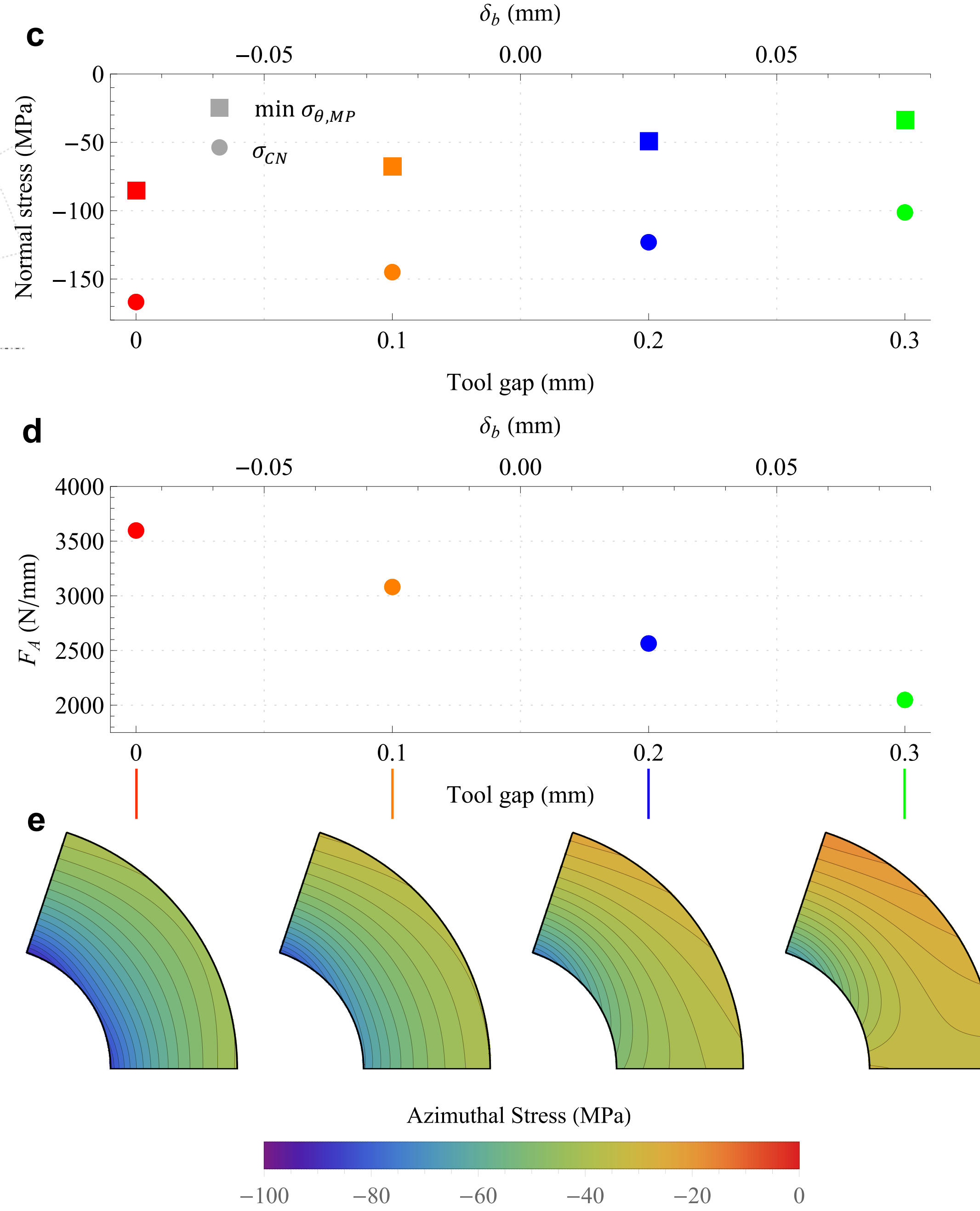
a, Polar representation of the magnitude of the radial displacement of the centre line of the coil and collar for all quadrants. **b,** Polar representation of the radial displacement of the centre line of the collar and coil in relation to the un-deformed nominal position. The deformation is amplified in the graph by a factor x200 for better visualisation. **c,** Azimuthal stress distribution in the coil's loading-plane (LP) and mid-plane (MP). **d,** Radial pressure distribution on the outer surface of the coil, where θ=0 represents the angular position of the loading-plane of the coil. **e,** Azimuthal stress scalar field for the collar and coil.

4. The 11 T collared coils structure

Solutions and discussion [Collaring]







a, Radial pressure distribution on the outer surface of the coil, where θ=0 represents the angular position of the loading-plane of the coil, as a function of δ_b. **b,** Azimuthal stress distribution in the coil's loading-plane (LP) and mid-plane (MP). **c,** Collar nose stress, and minimum azimuthal stress in the mid-plane of the coil. **d,** Force exerted by the press by length. **e,** Azimuthal stress scalar field for the coil at the different stages considered.

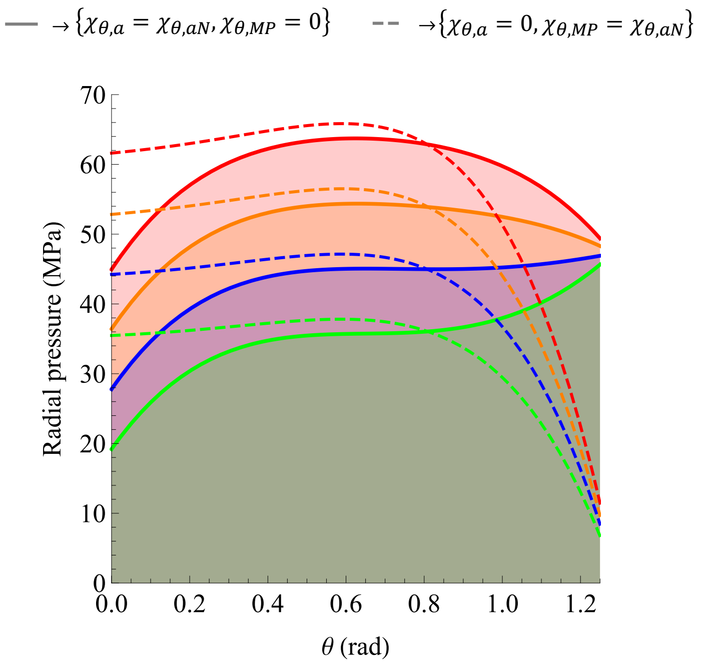
4. The 11 T collared coils structure

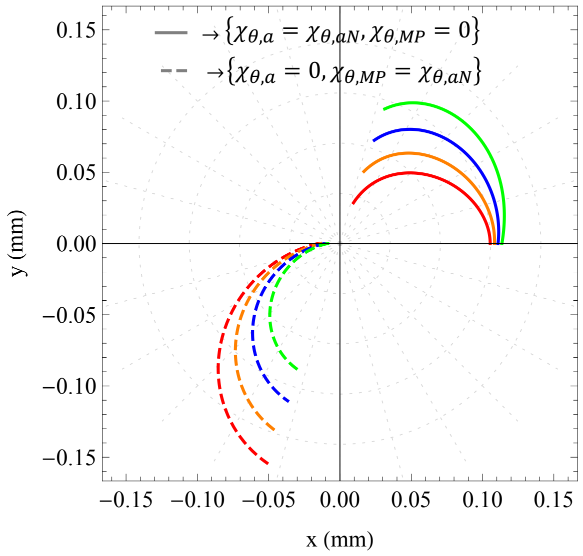
Solutions and discussion

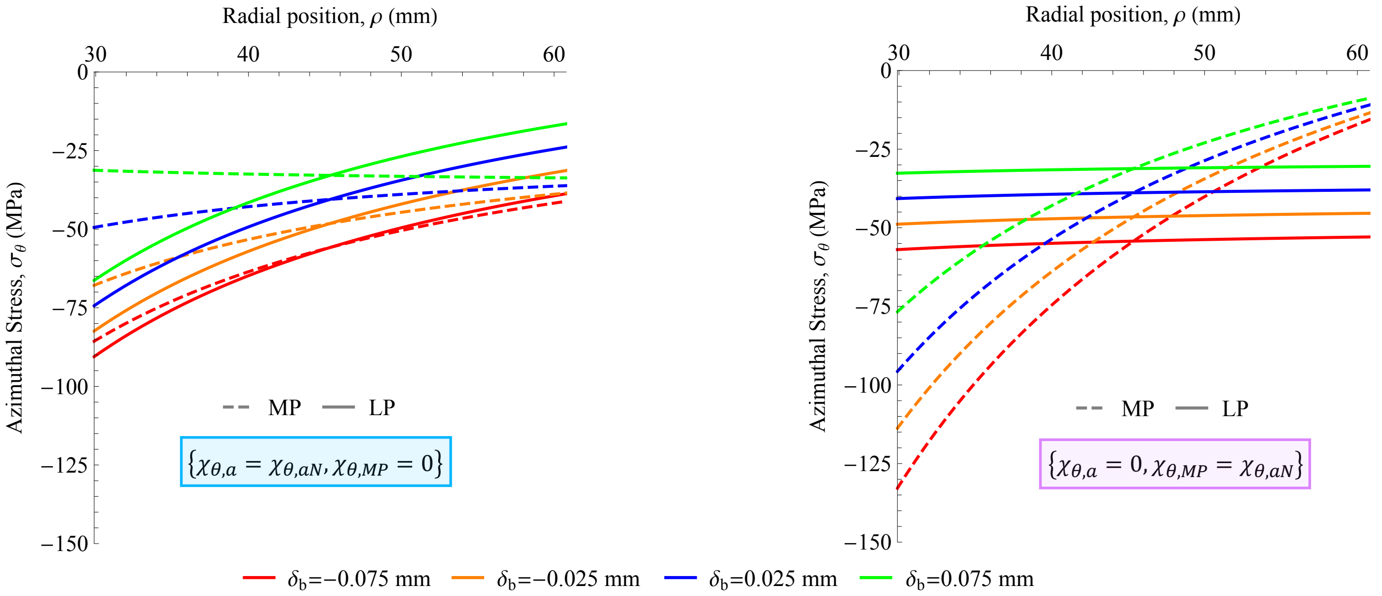
Geometrical non-equivalence principle

$$\chi_{\theta,aN} = \chi_{\theta,aN}^{*}\left(1+\frac{1}{\gamma}\arcsin\left(\frac{v_f\cos\gamma}{r_a}\right)\right)^{-1} \rightarrow 0.278\ mm$$

$$\chi_{\theta,t} = \chi_{\theta,a} + \chi_{\theta,MP} \quad \rightarrow \quad \begin{cases} \text{case I} & \{\chi_{\theta,a} = \chi_{\theta,aN}, \chi_{\theta,MP} = 0\} \\ \text{case II} & \{\chi_{\theta,a} = 0, \chi_{\theta,MP} = \chi_{\theta,aN}\} \end{cases}$$







4. The 11 T collared coils structure

Solutions and discussion

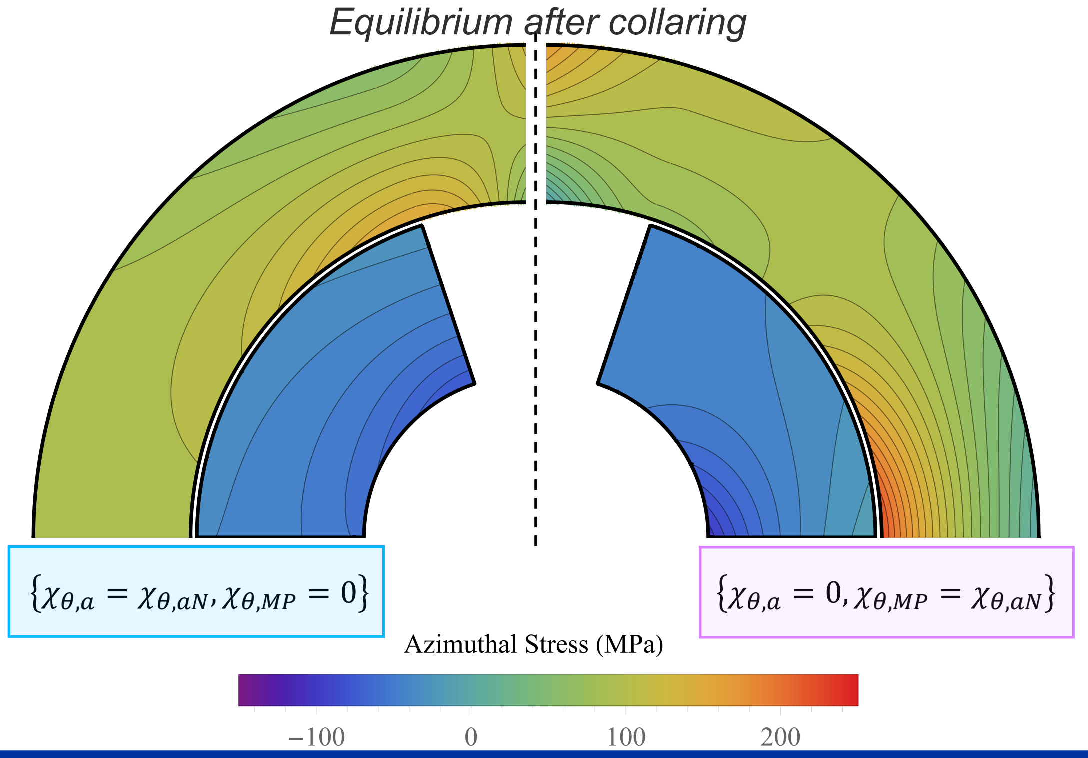
case I $\{\chi_{\theta,a} = \chi_{\theta,aN}, \chi_{\theta,MP} = 0\}$

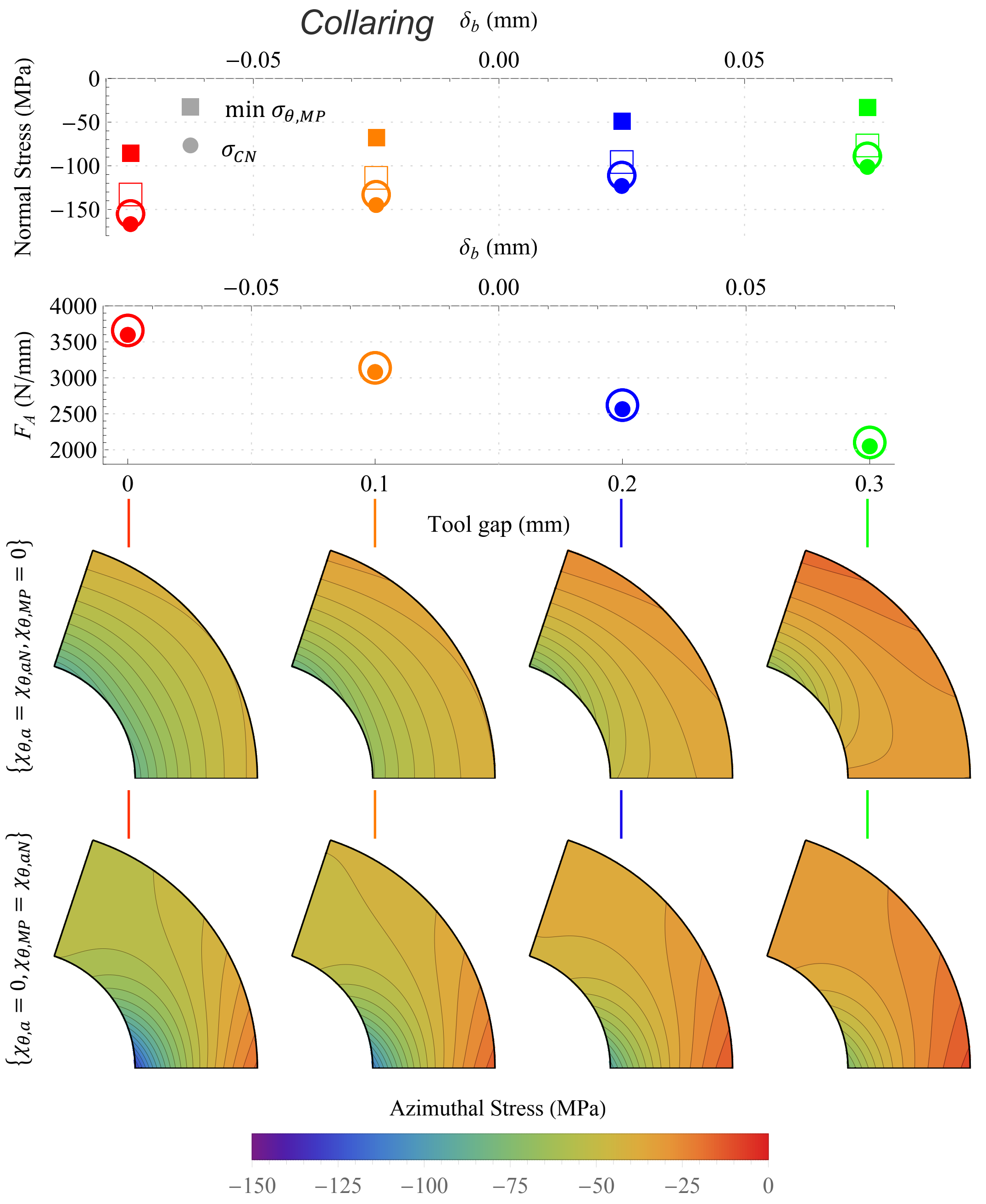
case II $\{\chi_{\theta,a} = 0, \chi_{\theta,MP} = \chi_{\theta,aN}\}$

Geometrical non-equivalence principle

$\{\chi_{\theta,a} = \chi_{\theta,aN}, \chi_{\theta,MP} = 0\}$

$\{\chi_{\theta,a} = 0, \chi_{\theta,MP} = \chi_{\theta,aN}\}$

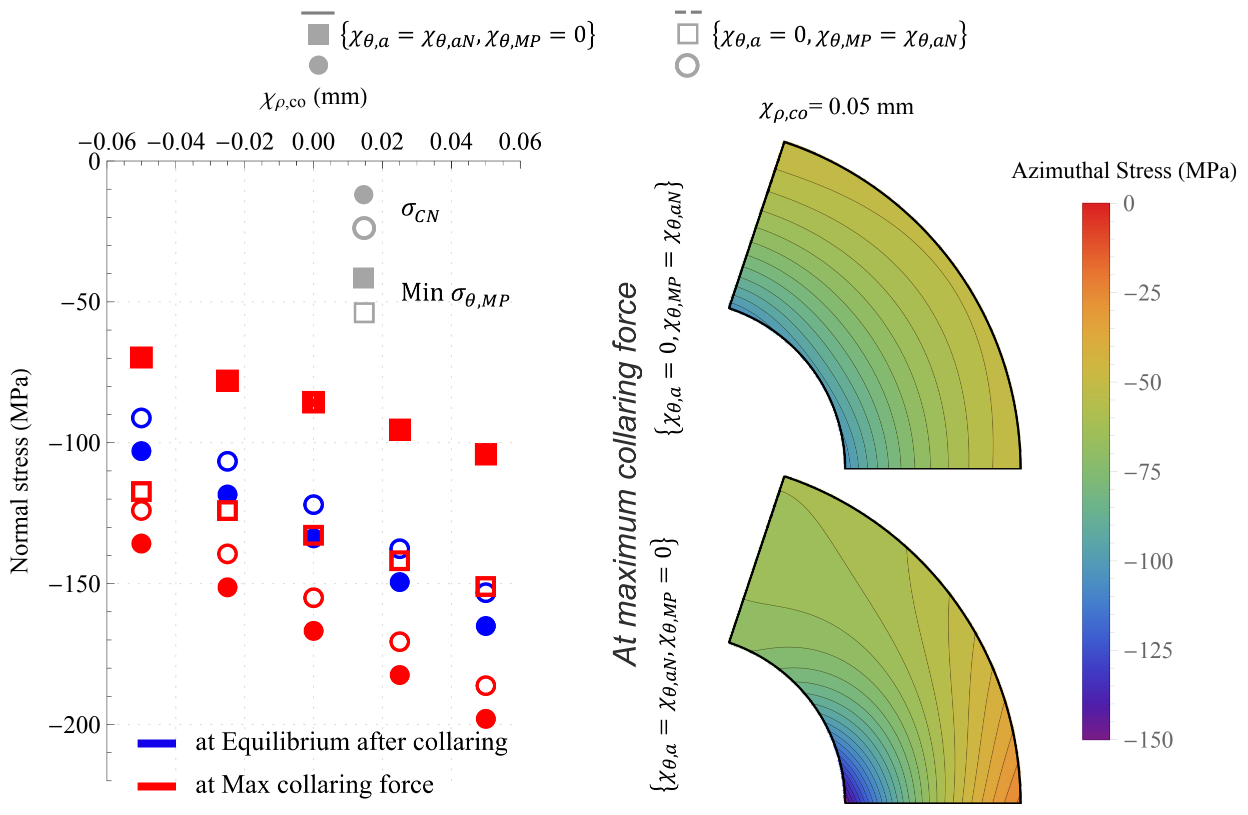




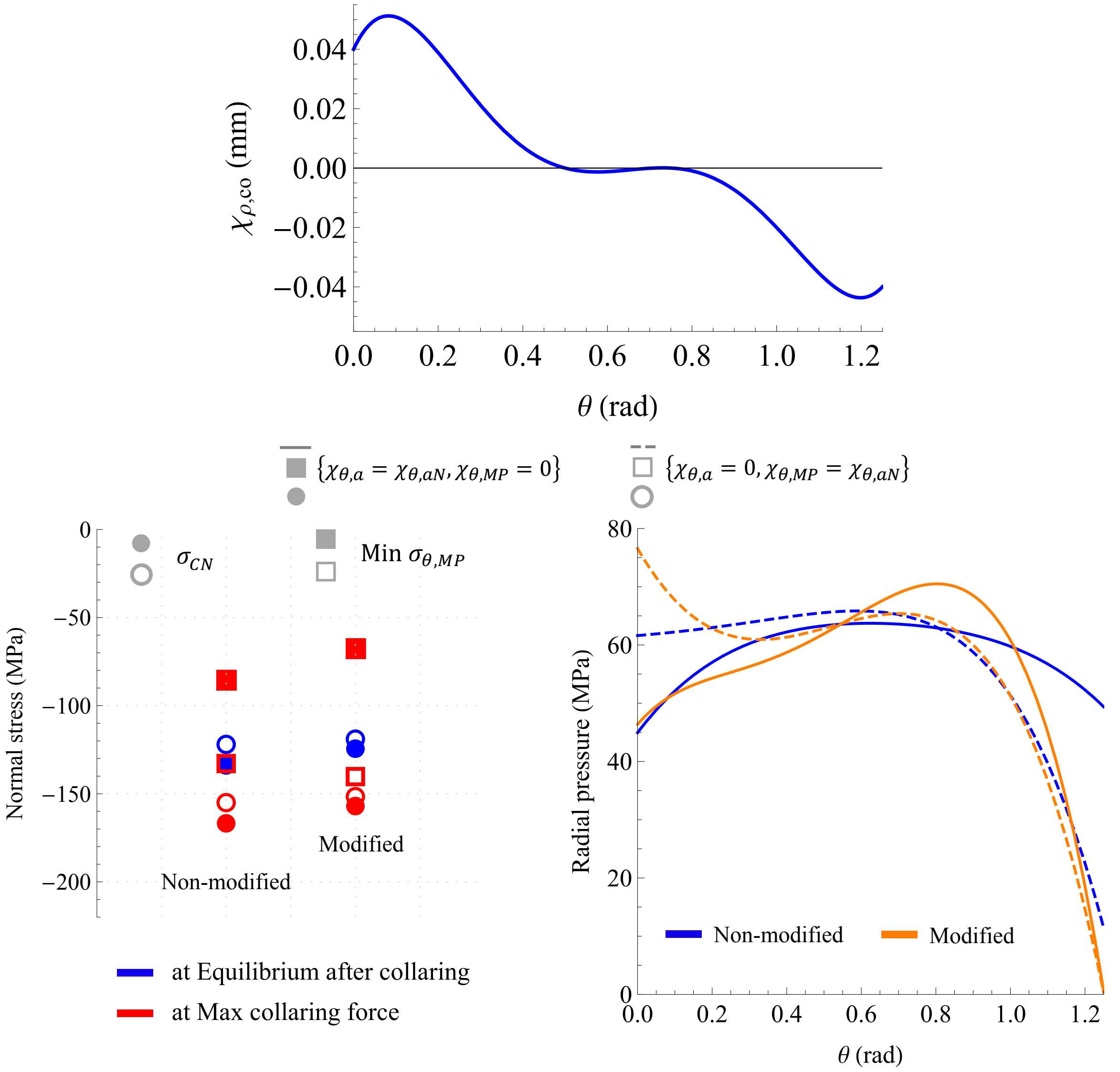
4. The 11 T collared coils structure

Solutions and discussion

Uniform radial interference



Non-uniform radial interference



5. Conclusions I

- A method created around the idea of the Λ **functions**, based on the concepts of geometrical interference and the variation of elastic strain energy, has been developed for the structural analysis of complex structures.

- The potential of the method has been explored through the development of a mathematical framework in relation to the conceptualised collared coils of the $\cos\theta$ 11 T dipole.

- The importance and main challenges of characterising the geometry of the coil has also been discussed in depth.

- It has been shown that there are **infinite** number of solutions to the collar internal equilibrium equation, and therefore it is incorrect to establish *transfer functions* between the value of the collar nose stress (or pole stress) to the peak stress in the coil, without taking geometrical interferences into consideration. This concept can also be extended to establishing higher degree of relationships within the magnet hierarchical structure, e.g. inferring the peak stress in the coil from the strain within the shell of the magnet.

5. Conclusions II

- It has been also demonstrated through the geometrical non-equivalence principle, that the geometrical interference created by an excessive size at the level of the mid-plane of the coil, cannot be balanced by creating azimuthal interference at the level of the pole, as different geometrical interference patterns are created. However, a compensation through variable radial interference should still be possible.

- Only the *simultaneous* consideration of all geometrical deviations of the regions of interests of the coil would allow to accurately estimate the stress-strain in the coil.

- Geometrical considerations are especially relevant in the context of Nb_3Sn coils as the stress management is critical. Further, there is potential for optimisations based on geometrical parameters, e.g. $\chi_{\rho,co}$ to minimise the maximum stress during collaring, or to enhance radial support of the coil during powering at cold.

Thanks!

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