Mariana 12 August 2020

Lagrangian

$$\mathcal{L} = -\frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^A_{\alpha\beta} F^A_{\mu\nu} + F^B_{\alpha\beta} F^B_{\mu\nu}\right) +$$

$$+\frac{n^\alpha n^\mu}{4n^2} \varepsilon^{\mu\nu\gamma\delta}\left(F^B_{\alpha\nu} F^A_{\gamma\delta} - F^A_{\alpha\nu} F^B_{\gamma\delta}\right) - J_\mu A^\mu - K_\mu B^\mu$$

$$-\frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^{A_D}_{\mu\nu} + F^{B_D}_{\alpha\beta} F^{B_D}_{\mu\nu}\right)$$

$$+\frac{\epsilon\, e\, e_D}{n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^A_{\mu\nu} - F^{B_D}_{\alpha\beta} F^B_{\mu\nu}\right)$$

$$+\frac{n^\alpha n^\mu}{4n^2} \varepsilon^{\mu\nu\gamma\delta}\left(F^{B_D}_{\alpha\nu} F^{A_D}_{\gamma\delta} - F^{A_D}_{\alpha\nu} F^{B_D}_{\gamma\delta}\right) - J^D_\mu A^\mu_D - K^D_\mu B^\mu_D$$

$$F^X_{\mu\nu} = \partial_\mu X^\nu - \partial_\nu X^\mu$$

$$J_\mu = e\,\bar{\psi}_e \gamma^\mu \psi_e \qquad K_\mu = g\,\bar{\psi}_g \gamma^\mu \psi_g$$

$$J^D_\mu = e_D\,\bar{\psi}_{e_D} \gamma^\mu \psi_{e_D} \qquad K^D_\mu = g_D\,\bar{\psi}_{g_D} \gamma^\mu \psi_{g_D}$$

Note: normalization of Lagrangian is chosen for diagonalization of kinetic terms

g, g_D must match with Terning's or Verhaaren's convention

∴ It will amount to redifining $F^X_{\mu\nu}$ to include couplings.

Equations of motion

VISIBLE

$$(1)\quad \frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_A^{\alpha\nu} - n^\nu \partial_\nu F_A^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_B^{\gamma\beta}\right)$$

$$+ \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_{A_D}^{\alpha\nu} - n^\nu F_{A_D}^{\alpha\mu}\right) = J^\mu$$

$$(2)\quad \frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_B^{\alpha\nu} - n^\nu \partial_\nu F_B^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_A^{\gamma\beta}\right)$$

$$- \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_{B_D}^{\alpha\nu} - n^\nu F_{B_D}^{\alpha\mu}\right) = K^\mu$$

DARK

$$(3)\quad \frac{n_\alpha}{n^2}\left(n^\nu \partial_\nu F_{A_D}^{\alpha\nu} - n^\nu \partial_\nu F_{A_D}^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_{B_D}^{\gamma\beta}\right)$$

$$+ \frac{\epsilon e e_D}{n^2} n_\alpha \partial_\nu \left(n^\mu F_A^{\alpha\nu} - n^\nu F_A^{\alpha\mu}\right) = J_D^\mu + M_{A_D}^2 A_D^\mu A_{D\mu}$$

$$(4)\quad \frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_{B_D}^{\alpha\nu} - n^\nu \partial_\nu F_{B_D}^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_{A_D}^{\gamma\beta}\right)$$

$$- \frac{\epsilon e e_D}{n^2}\left(n_\alpha \partial_\nu n^\mu F_B^{\alpha\nu} - n_\nu^\nu F_B^{\alpha\mu}\right) = K_D^\mu + M_{B_D}^2 B_D^\mu B_{D\mu}$$

Rewrite equations, grouping terms

(1') $$\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu \left(F_A^{\alpha\nu} + \epsilon e e_D F_{A_D}^{\alpha\nu}\right) - n^\nu \partial_\nu \left(F_A^{\alpha\mu} + \epsilon e e_D F_{A_D}^{\alpha\mu}\right)$$
$$- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \, \partial_\nu F_B^{\gamma\beta} \Big] = J^\mu$$

(2') $$\frac{n_\alpha}{n^2} \, n^\mu \partial_\nu \left(F_B^{\alpha\nu} - \epsilon e e_D F_{B_D}^{\alpha\nu}\right) - n^\nu \partial_\nu \left(F_B^{\alpha\mu} - \epsilon e e_D F_{B_D}^{\alpha\mu}\right)$$
$$+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \, \partial_\nu F_A^{\gamma\beta} \Big] = K^\mu$$

(3') $$\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu \left(F_{A_D}^{\alpha\nu} + \epsilon e e_D F_A^{\alpha\nu}\right) - n^\nu \partial_\nu \left(F_{A_D}^{\alpha\mu} + \epsilon e e_D F_A^{\alpha\mu}\right)$$
$$- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \, \partial_\nu F_{B_D}^{\gamma\beta} \Big] = J_D^\mu + M_{A_D}^2 A_D^\mu A_{D\mu}$$

(4') $$\frac{n_\alpha}{n^2} \, n^\mu \partial_\nu \left(F_{B_D}^{\alpha\nu} - \epsilon e e_D F_B^{\alpha\nu}\right) - n^\nu \partial_\nu \left(F_{B_D}^{\alpha\mu} - \epsilon e e_D F_B^{\alpha\mu}\right)$$
$$+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \, \partial_\nu F_{A_D}^{\gamma\beta} \Big] = K_D^\mu + M_{B_D}^2 B_D^\mu B_{D\mu}$$

Rotations

$$A_\mu = (\cos\phi - \epsilon e e_D \sin\phi)\bar{A}_\mu + (-\sin\phi - \epsilon e e_D \cos\phi)\bar{A}^D_\mu$$
$$A_{D\mu} = \sin\phi \, \bar{A}_\mu + \cos\phi \, \bar{A}_{D\mu}$$
$$B_\mu = \cos\phi \, \bar{B}_\mu - \sin\phi \, \bar{B}_{D\mu}$$
$$B_{D\mu} = (\sin\phi + \epsilon e e_D \cos\phi)\bar{B}_\mu + (\cos\phi - \epsilon e e_D \sin\phi)\bar{B}^D_\mu$$

$\to$ in (1') $F_A^{\alpha\nu} + \epsilon e e_D F_{A_D}^{\alpha\nu}$

$$= (\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_A^{\alpha\nu} + \epsilon e e_D (\sin\phi \bar{F}_A^{\alpha\nu} + \cos\phi \bar{F}_{A_D}^{\alpha\nu})$$

$$(-\sin\phi - \epsilon e e_D \cos\phi)\bar{F}_{A_D}^{\alpha\nu} = \cos\phi \bar{F}_A^{\alpha\nu} + \sin\phi \bar{F}_{A_D}^{\alpha\nu}$$

$$(1'') \quad \boxed{\begin{aligned} &\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu(\cos\phi \bar{F}_A^{\alpha\nu} + \sin\phi \bar{F}_{A_D}^{\alpha\nu}) - n^\nu \partial_\nu(\cos\phi \bar{F}_A^{\alpha\mu} + \sin\phi \bar{F}_{A_D}^{\alpha\mu}) \\ &- n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \partial_\nu(\cos\phi \bar{F}_B^{\gamma\beta} - \sin\phi \bar{F}_{B_D}^{\gamma\beta})\Big] = J_\mu \end{aligned}}$$

$$\Rightarrow \underline{J_\mu = \cos\phi \bar{J}_\mu + \sin\phi \bar{J}_\mu^D}$$

$\to$ In (2') $F_B^{\alpha\nu} - \epsilon e e_D F_{B_D}^{\alpha\nu} =$

$$= \cos\phi \bar{F}_B^{\alpha\nu} - \sin\phi \bar{F}_{B_D}^{\alpha\nu} - \epsilon e e_D (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_B^{\alpha\nu}$$

$$- \epsilon e e_D (\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_{B_D}^{\alpha\nu}$$

$$\cong (\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\nu} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{B_D}^{\alpha\nu}$$

$$(2'') \quad \boxed{\begin{aligned} &\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu\{(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\nu} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{B_D}^{\alpha\nu}\} \\ &- n^\nu \partial_\nu\{(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_B^{\alpha\mu} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{B_D}^{\alpha\mu}\} \\ &+ n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \partial_\nu\big[(\cos\phi - \epsilon e e_D \sin\phi)\bar{F}_A^{\gamma\beta} - (\sin\phi + \epsilon e e_D \cos\phi)\bar{F}_{A_D}^{\gamma\beta}\big] \\ &= K^\mu \end{aligned}}$$

$$\Rightarrow \underline{K_\mu = (\cos\phi - \epsilon e e_D \sin\phi)\bar{K}_\mu - (\sin\phi + \epsilon e e_D \cos\phi)\bar{K}_\mu^D}$$

$\rightarrow$ In (3') $F_{A_D}^{\alpha\nu} + \epsilon e e_D F_A^{\alpha\nu} =$

$\sin\phi\, \bar{F}_A^{\alpha\nu} + \cos\phi\, \bar{F}_{A_D}^{\alpha\nu} + \epsilon e e_D (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_A^{\alpha\nu}$

$- \epsilon e e_D (\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_{A_D}^{\alpha\nu}$

$\cong (\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_A^{\alpha\nu} + (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_{A_D}^{\alpha\nu}$

$$(3'')\quad \boxed{\begin{aligned} &\frac{n_\alpha}{n^2}\Big\{ n^\mu \partial_\nu \Big[(\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_A^{\alpha\nu} + (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_{A_D}^{\alpha\nu} \Big] \\ &\quad - n^\nu \partial_\nu \Big[(\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_A^{\alpha\mu} + (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_{A_D}^{\alpha\mu} \Big] \\ &\quad - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu \Big[(\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_B^{\gamma\beta} + (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_{A_D}^{\gamma\beta} \Big] \\ &\quad = J_D^\mu + M_{A_D}^2 (\sin^2\phi\, \bar{A}_\mu^2 + \cos^2\phi\, A_{\mu D}^2 + 2\sin\phi\cos\phi\, A_\mu A_{\mu D}) \end{aligned}}$$

$$\underline{J_{D_\mu} = (\sin\phi + \epsilon e e_D \cos\phi) J_\mu + (\cos\phi - \epsilon e e_D \sin\phi) \bar{J}_\mu^D}$$

$\rightarrow$ In (4') $F_{B_D}^{\alpha\nu} - \epsilon e e_D F_B^{\alpha\nu} =$

$(\sin\phi + \epsilon e e_D \cos\phi) \bar{F}_B^{\alpha\nu} + (\cos\phi - \epsilon e e_D \sin\phi) \bar{F}_{B_D}^{\alpha\nu}$

$- \epsilon e e_D \cos\phi\, \bar{F}_B^{\alpha\nu} + \epsilon e e_D \sin\phi\, \bar{F}_{B_D}^{\alpha\nu}$

$= \sin\phi\, \bar{F}_B^{\alpha\nu} + \cos\phi\, \bar{F}_{B_D}^{\alpha\nu}$

$$(4'')\quad \boxed{\begin{aligned} &\frac{n_\alpha}{n^2}\Big[n^\mu \partial_\nu (\cos\phi\, \bar{F}_{B_D}^{\alpha\nu} + \sin\phi\, \bar{F}_B^{\alpha\nu}) - n^\nu \partial_\nu (\cos\phi\, \bar{F}_{B_D}^{\alpha\mu} + \sin\phi\, \bar{F}_B^{\alpha\mu}) \\ &\quad + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta\, \partial_\nu (\cos\phi\, \bar{F}_{A_D}^{\gamma\beta} + \sin\phi\, \bar{F}_A^{\gamma\beta}) \Big] = \\ &= K_D^\mu + M_{B_D}^2 (\sin^2\phi\, \bar{B}_\mu^2 + \cos^2\phi\, \bar{B}_{D\mu}^2 + 2\cos\phi\sin\phi\, B_{D\mu} B^\mu) \end{aligned}}$$

$$\underline{K_{D\mu} = \cos\phi\, \bar{K}_{D\mu} + \sin\phi\, \bar{K}_\mu}$$

1 Normalization of Lagrangian terms with visible and dark sectors - 12 August 2020

Our current Lagrangian. Until now, we have worked with Lagrangian terms collected from Eqs. (2.4), (4.1) and (4.2) (with $\theta = 0$) in Ref. [1] (or Eq. (2.1) in Ref. [2])

$$\begin{aligned}\mathcal{L} = &- \frac{n^\alpha n^\mu}{8\pi n^2} g^{\beta\nu} g \left(F^A_{\alpha\beta} F^A_{\mu\nu} + F^B_{\alpha\beta} F^B_{\mu\nu}\right) + \frac{n^\alpha n_\mu}{16\pi n^2} \epsilon^{\mu\nu\gamma\delta} g \left(F^B_{\alpha\nu} F^A_{\gamma\delta} - F^A_{\alpha\nu} F^B_{\gamma\delta}\right) - J_\mu A^\mu - g K_\mu B^\mu \\ &- \frac{n^\alpha n^\mu}{8\pi n^2} g^{\beta\nu} g_D \left(F^{A_D}_{\alpha\beta} F^{A_D}_{\mu\nu} + F^{B_D}_{\alpha\beta} F^{B_D}_{\mu\nu}\right) + \frac{n^\alpha n_\mu}{16\pi n^2} \epsilon^{\mu\nu\gamma\delta} g_D \left(F^{B_D}_{\alpha\nu} F^{A_D}_{\gamma\delta} - F^{A_D}_{\alpha\nu} F^{B_D}_{\gamma\delta}\right) - e_D J_\mu {A_D}^\mu - e_D g_D K_\mu {B_D}^\mu \\ &- \frac{1}{2} M^2_{A_D} A^2_D - \frac{1}{2} M^2_{B_D} B^2_D + \epsilon\, e\, e_D \frac{n^\alpha n^\mu}{n^2} g^{\beta\nu} \left(F^A_{D\alpha\beta} F^A_{\mu\nu} - F^B_{D\alpha\beta} F^B_{\mu\nu}\right) \end{aligned} \tag{1}$$

where $g = \frac{4\pi}{e^2}$, $g_D = \frac{4\pi}{e_D^2}$ and $F^X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ ($X = A$, B, A_D, B_D). Note that I added an extra e_D here in the last term of line 2: $-e_D g_D K_\mu {B_D}^\mu$, to agree with Eq. (4.1) of Ref. [1]. The mass terms are from Eqs. (5.1) and (5.3) in Ref. [1] and Ref. [3].

We did not use them yet but it is not clear how the currents would be defined; we tentatively suggested

$$\begin{aligned} J_\mu &= e \overline{\Psi}_e \gamma_\mu \Psi_e, \qquad & K_\mu &= g \overline{\Psi}_g \gamma_\mu \Psi_g, \\ J_{D\mu} &= e_D \overline{\Psi}_{De} \gamma_\mu \Psi_{De}, \qquad & K_{D\mu} &= g_D \overline{\Psi}_{Dg} \gamma_\mu \Psi_{Dg}, \end{aligned} \tag{2}$$

where Ψ_e and Ψ_g are some sort of projections on the electric and magnetic components.

Would this be a better normalization? The lack of symmetry with the charges in the interaction terms of Eq. (1) seems to have been remedied in p.6 of Ref. [4]. Also, in p. 4 (as well, see Eq. (3.9) or Ref. [1], they use the magnetic coupling $b = \frac{4\pi}{e}$ and likewise for b_D, rather than g. But I did not check the kinetic terms of Verhaaren's p. 6. It looks like Eq. (1) would be replaced with

$$\begin{aligned}\mathcal{L} = &- \frac{n^\alpha n^\mu}{8\pi n^2} g^{\beta\nu} b \left(F^A_{\alpha\beta} F^A_{\mu\nu} + F^B_{\alpha\beta} F^B_{\mu\nu}\right) + \frac{n^\alpha n_\mu}{16\pi n^2} \epsilon^{\mu\nu\gamma\delta} b \left(F^B_{\alpha\nu} F^A_{\gamma\delta} - F^A_{\alpha\nu} F^B_{\gamma\delta}\right) - e J_\mu A^\mu - b K_\mu B^\mu \\ &- \frac{n^\alpha n^\mu}{8\pi n^2} g^{\beta\nu} b_D \left(F^{A_D}_{\alpha\beta} F^{A_D}_{\mu\nu} + F^{B_D}_{\alpha\beta} F^{B_D}_{\mu\nu}\right) + \frac{n^\alpha n_\mu}{16\pi n^2} \epsilon^{\mu\nu\gamma\delta} b_D \left(F^{B_D}_{\alpha\nu} F^{A_D}_{\gamma\delta} - F^{A_D}_{\alpha\nu} F^{B_D}_{\gamma\delta}\right) - e_D J_\mu {A_D}^\mu - b_D K_\mu {B_D}^\mu \\ &- \frac{1}{2} M^2_{A_D} A^2_D - \frac{1}{2} M^2_{B_D} B^2_D + \epsilon\, e\, e_D \frac{n^\alpha n^\mu}{n^2} g^{\beta\nu} \left(F^A_{D\alpha\beta} F^A_{\mu\nu} - F^B_{D\alpha\beta} F^B_{\mu\nu}\right) \end{aligned} \tag{3}$$

where $b = \frac{4\pi}{e}$, $b_D = \frac{4\pi}{e_D}$. It seems some terms in Eq. (3) are obtained by multiplying terms in Eq. (1) by e (but not all?!?) Now with the charges included explicitly in the interaction terms, should we use We did not use them yet but it is not clear how the currents would be defined; we tentatively suggested

$$\begin{aligned} J_\mu &= \overline{\Psi}_e \gamma_\mu \Psi_e, \qquad & K_\mu &= \overline{\Psi}_g \gamma_\mu \Psi_g, \\ J_{D\mu} &= \overline{\Psi}_{De} \gamma_\mu \Psi_{De}, \qquad & K_{D\mu} &= \overline{\Psi}_{Dg} \gamma_\mu \Psi_{Dg}, \end{aligned} \tag{4}$$

rather than Eq. (2)?

2 Diagonalization of the kinetic term

The other equations that are not clear to me are the transformation that alledgedly lead to the diagonal basis $\overline{V}$ (with $V = A$, B, A_D, B_D) of the kinetic term:

$$\begin{aligned} A_\mu &= (\cos\phi + \epsilon e e_D \sin\phi)\, \overline{A}_\mu + (-\sin\phi + \epsilon e e_D \cos\phi)\, \overline{A}_{D\mu}, \\ A_{D\mu} &= \sin\phi\, \overline{A}_\mu + \cos\phi\, \overline{A}_{D\mu}, \\ B_\mu &= \cos\phi\, \overline{B}_\mu - \sin\phi\, \overline{B}_{D\mu}, \\ B_{D\mu} &= (\sin\phi - \epsilon e e_D \cos\phi)\, \overline{B}_\mu + (\cos\phi + \epsilon e e_D \sin\phi)\, \overline{B}_{D\mu}, \end{aligned} \tag{5}$$

along with the diagonal currents

$$
\begin{aligned}
e\overline{J}_\mu &= (\cos\phi + \epsilon e e_D \sin\phi)\, eJ_\mu + \sin\phi\; e_D J_{D\mu}, \\
e_D \overline{J}_{D\mu} &= (-\sin\phi + \epsilon e e_D \cos\phi)\, eJ_\mu + \cos\phi\; e_D J_{D\mu}, \\
\frac{1}{e}\overline{K}_\mu &= \cos\phi\; \frac{1}{e} K_\mu + (\sin\phi - \epsilon e e_D \cos\phi)\, \frac{1}{e_D} K_{D\mu}, \\
\frac{1}{e_D}\overline{K}_{D\mu} &= -\sin\phi\; \frac{1}{e} K_\mu + (\cos\phi + \epsilon e e_D \sin\phi)\, \frac{1}{e_D} \overline{K}_{D\mu}.
\end{aligned} \tag{6}
$$

By solving for the currents in the non-diagonal basis, I obtained (to be verified)

$$
\begin{aligned}
eJ_\mu &= \cos\phi\; e\overline{J}_\mu - \sin\phi\; e_D \overline{J}_{D\mu}, \\
e_D J_{D\mu} &= (\sin\phi - \epsilon e e_D \cos\phi)\, e\overline{J}_\mu + (\cos\phi + \epsilon e e_D \sin\phi)\, e_D \overline{J}_{D\mu}, \\
\frac{1}{e} K_\mu &= (\cos\phi + \epsilon e e_D \sin\phi)\, \frac{1}{e}\overline{K}_\mu + (-\sin\phi + \epsilon e e_D \cos\phi)\, \frac{1}{e_D}\overline{K}_{D\mu}, \\
\frac{1}{e_D} K_{D\mu} &= \sin\phi\; \frac{1}{e}\overline{K}_\mu + \cos\phi\; \frac{1}{e_D}\overline{K}_{D\mu}.
\end{aligned} \tag{7}
$$

Now should Eqs. (5) and (6) diagonalize the mixing of the kinetic term in Eq. (1)? I tried –probably the wrong way– and it did not work. And how does one obtain Eqs. (5) and (6) anyways?

References

[1] J. Terning and C. B. Verhaaren, JHEP **12**, 123 (2018) doi:10.1007/JHEP12(2018)123 [arXiv:1808.09459 [hep-th]].

[2] J. Terning and C. B. Verhaaren, JHEP **12**, 152 (2019) doi: 10.1007/JHEP12(2019)152 [arXiv: arXiv:1906.00014v2 [hep-ph]].

[3] A. Hook and J. Huang, Phys. Rev. D **96**, no.5, 055010 (2017) doi:10.1103/PhysRevD.96.055010 [arXiv:1705.01107 [hep-ph]].

[4] C. Verhaaren, Magnetic Monopole Dark Matter, Talk given at the International Conference on Neutrinos and Dark Matter (NDM-2020), Hurghada, Egypt, 13 January 2020.