Iterative image reconstruction algorithm for computed tomography with very high energy electron beam

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Daiki Hayakawa, Hendrik Jansen, Paul Schütze

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Outline

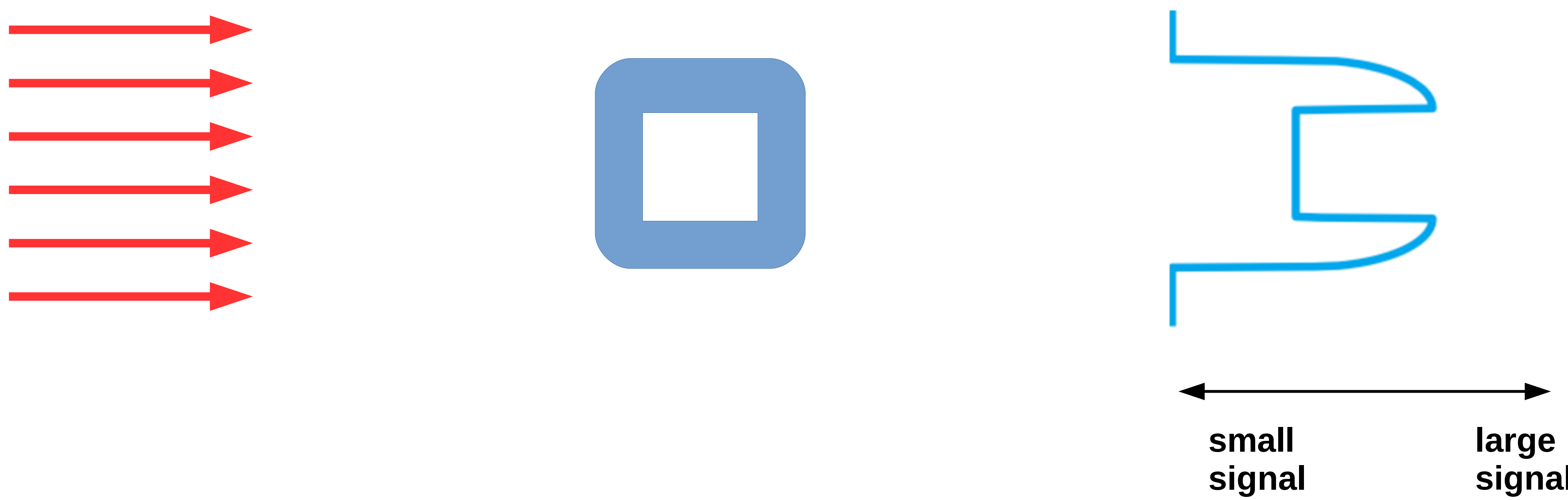
- Introduction of electron computed tomography
- Iterative image reconstruction algorithm
- Image analysis
- Future plans
- Conclusions

Imaging

Source
- x-ray, neutron
- charged particle

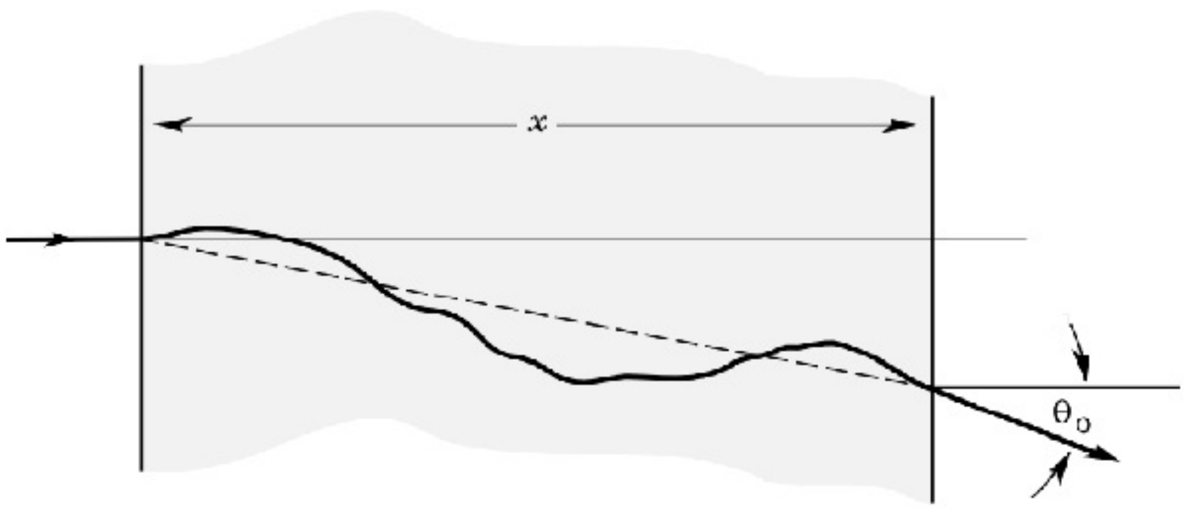
Interaction with the target
- absorption
- energy loss
- scattering

Measure physical quantity
- intensity
- remaining energy
- variance of the deflection angle



- Novel imaging modality using
  - High energy electron
  - Multiple Coulomb Scattering

Multiple Coulomb Scattering (MCS)



Highland's formula:

$$\Theta_0 = \left(\frac{13.6\ \mathrm{MeV}}{\beta c p} z\right) \sqrt{\varepsilon}(1 + 0.038 \ln \varepsilon)$$

Material budget

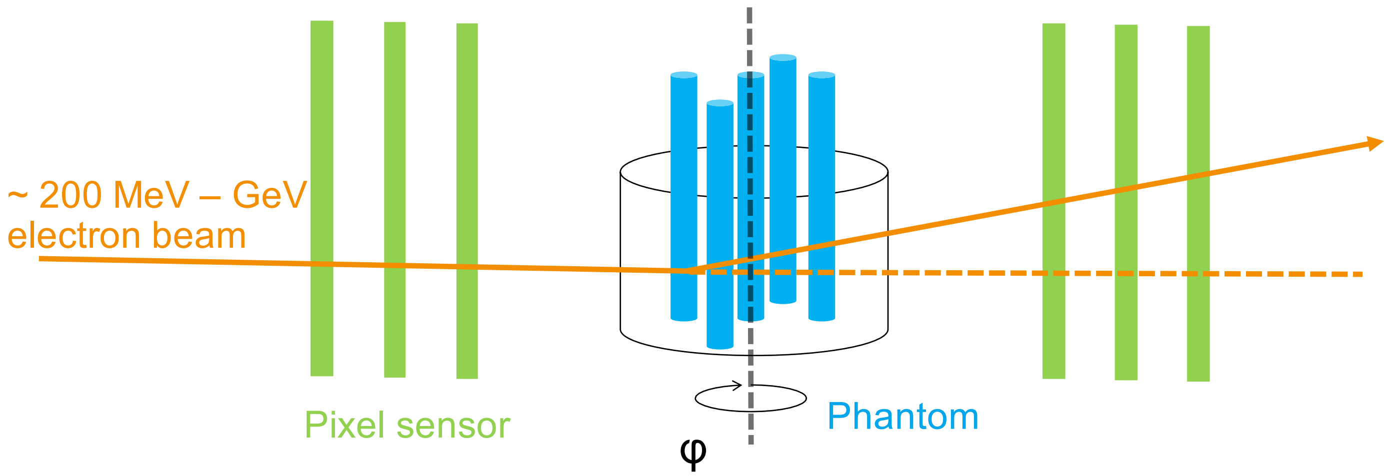
$$\varepsilon = \int \frac{1}{X_0(x)} \mathrm{d}x$$

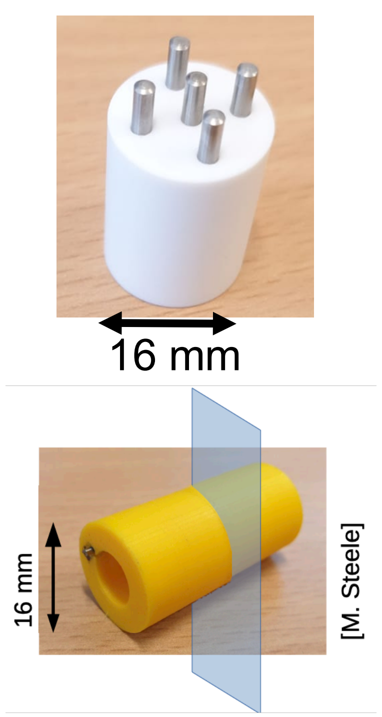
z: electron charge number
βc: electron velocity
p: electron momentum
X_0: material's radiation length

$$\Theta_0^2 \propto \varepsilon$$

- Electrons undergo MCS when transversing material
- Gaussian width of the deflection angle distribution correlates with material budget
- Material budget imaging by measuring the scattering angle distribution

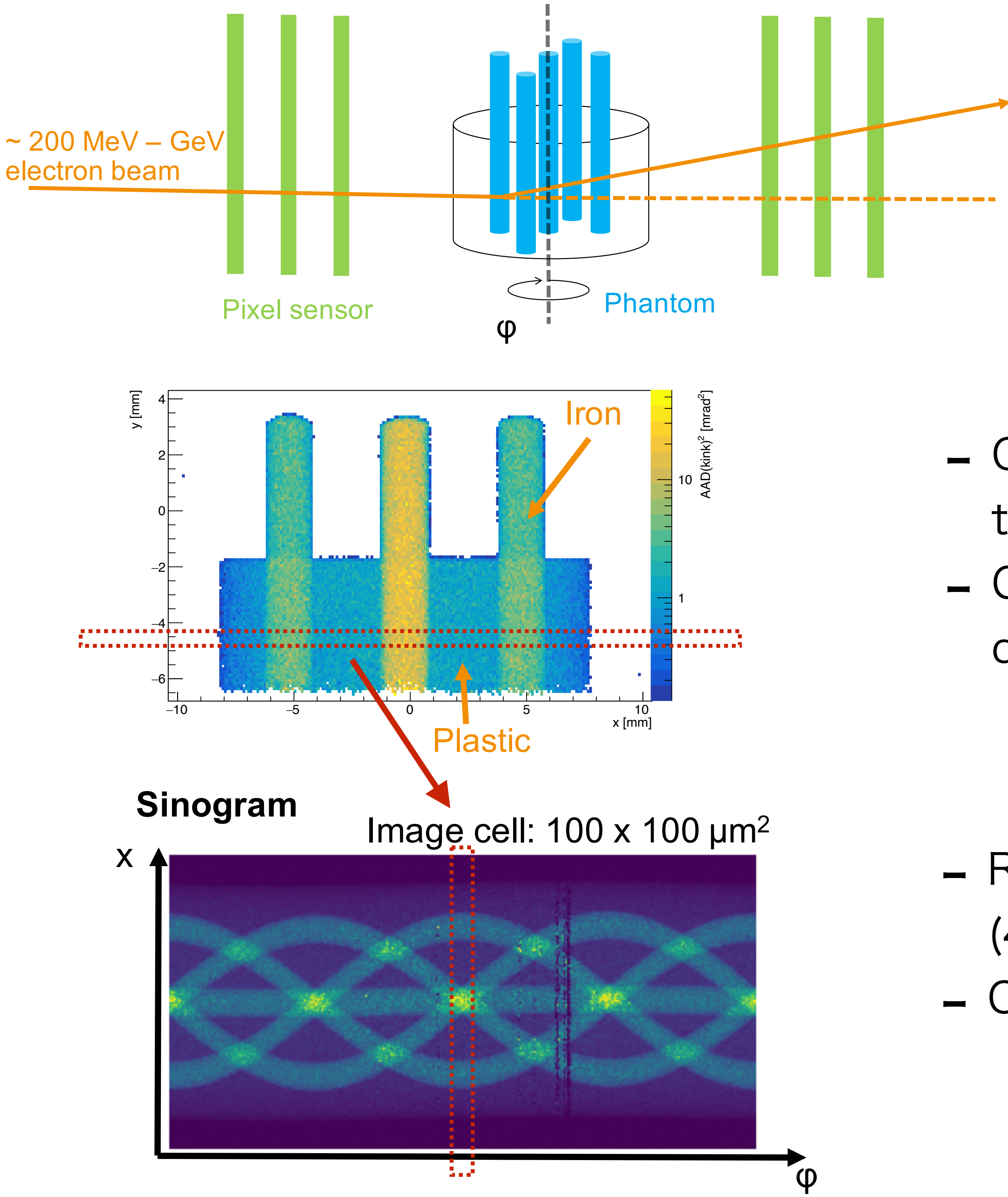
Testbeam Experiment at DESY





- Performed in June and August in 2020
- Electron energy: 2 GeV
- Test 2 phantoms
  - 5 iron rods + plastic
  - DESY logo

Electron Computed Tomography



- Measure trajectory of electrons
- Group electrons into pixels at a virtual plane though the phantom
- Calculate squared width of angular distribution per pixel (AAD_{90}^2)
- Repeat for various projections (400*0.9° = 360°)
- Create a sinogram

→ Tomographic image reconstruction

Tomographic Image Reconstruction Algorithm

1. Filtered Back Projection (FBP)
    - Analytical reconstruction using inverse Radon transform
    - Artefacts and noise due to statistical/systematic effects
2. Iterative reconstruction
    - Reconstruction based on a model with statistical noise using iterative methods
    - Artefacts and noise removal by adding a regularization method

Iterative Image Reconstruction

Data $\overrightarrow{y} = A\overrightarrow{x}$ Image (Truth)

Model

Suppose that:

$$\overrightarrow{y} = A\overrightarrow{x} + \overrightarrow{n}$$ n: noise

Reconstruct image x when noise n is minimized

Objective function:

$$J(\vec{x}) = \|A\vec{x} - \vec{y}\|^2$$

$$A^T A\vec{x} = A^T\vec{y} \;\; (\text{minimize } J(\vec{x}) \to \partial J(\vec{x})/\partial\vec{x} = 0)$$

Landweber method:

$$\overrightarrow{x}^{(k+1)} = \overrightarrow{x}^{(k)} + \alpha A^T(\overrightarrow{y} - A\overrightarrow{x}^{(k)})$$

k: number of iteration

α: step size

Simultaneous iterative reconstruction technique (SIRT)

TVS Regularization

β: regularization strength
U(x): energy function

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \alpha \left[A^T(\vec{y} - A\vec{x}^{(k)}) + \beta \frac{\partial U(\vec{x})}{\partial \vec{x}} \right]$$

Regularization

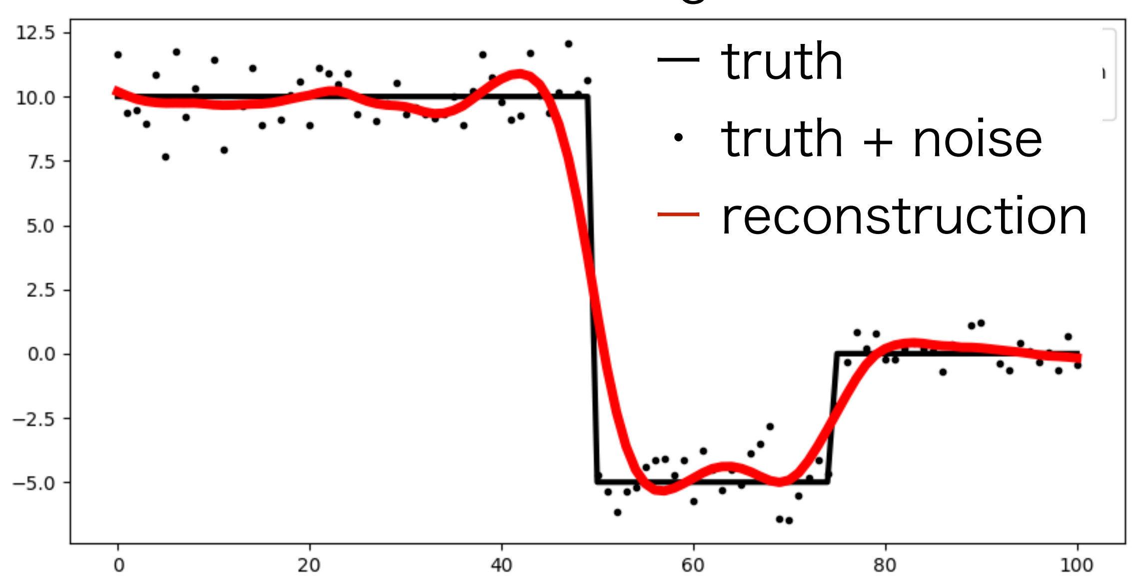
Total Variation Superiorization (TVS):

$$U(\vec{x}) = \sum_{i,j} \sqrt{(x_{i,j} - x_{i-1,j})^2 + (x_{i,j} - x_{i,j-1})^2}$$

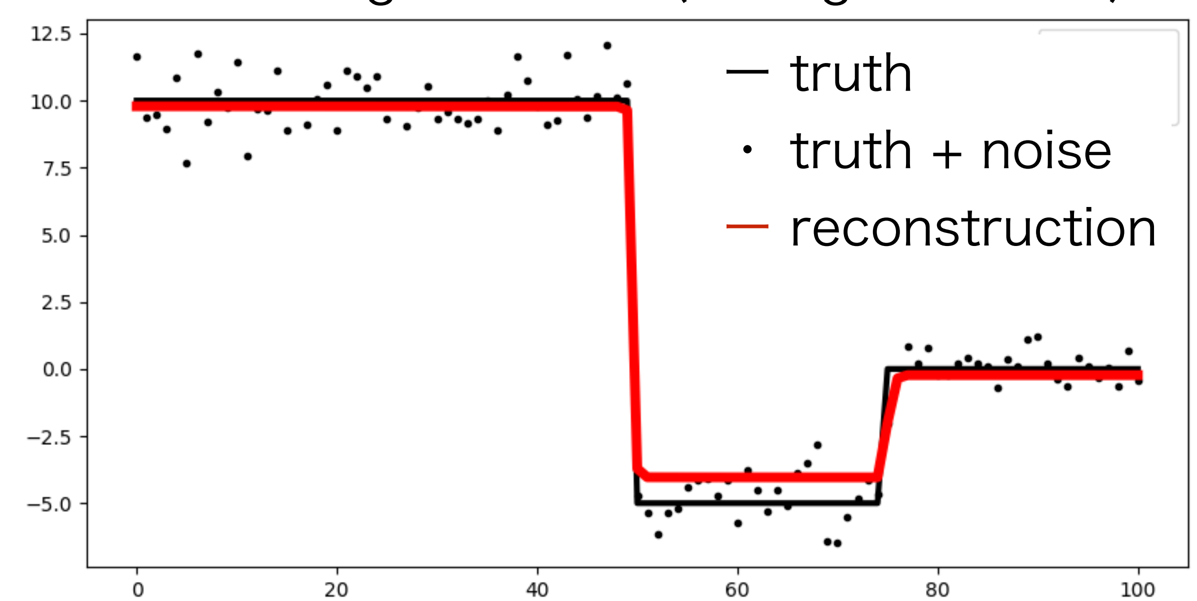
Magnitude of gradient

(Quote: https://pylops.readthedocs.io/en/latest/gallery/plot_tvreg.html)

Normal L2 regularization

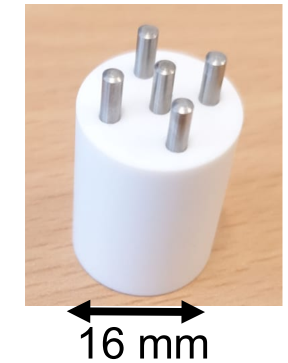


TV regularization (L1 regularization)



- Smoothing while preserving “edge”

Image analysis



β: regularization strength

FBP **SIRT + TVS (30 iterations)**

$\beta = 0$ $\beta = 3$ $\beta = 10$

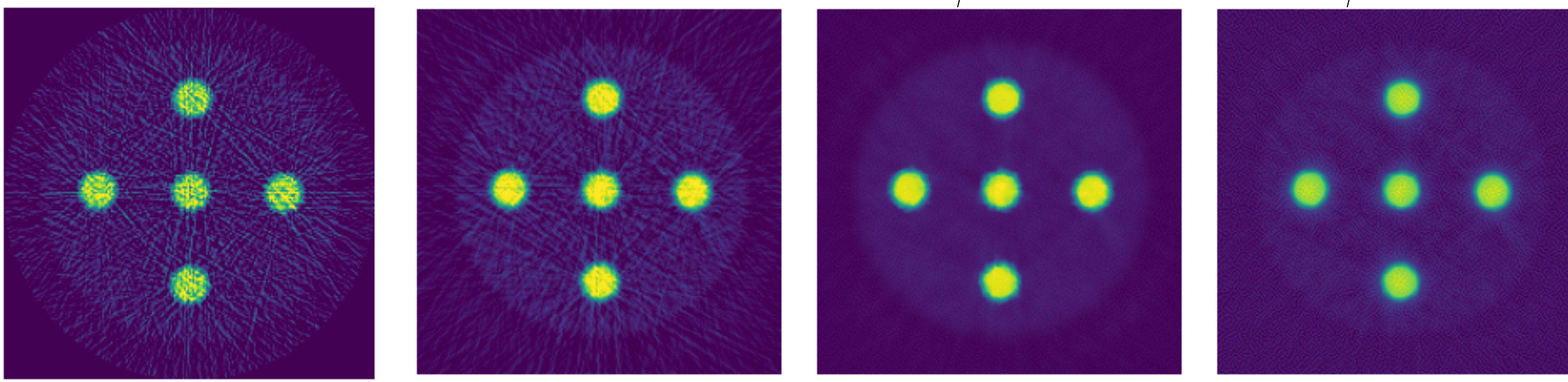
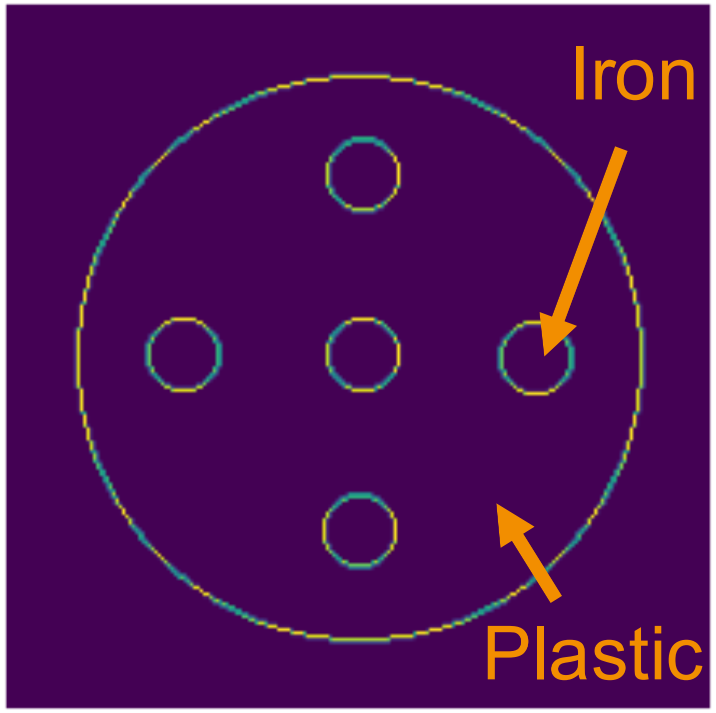


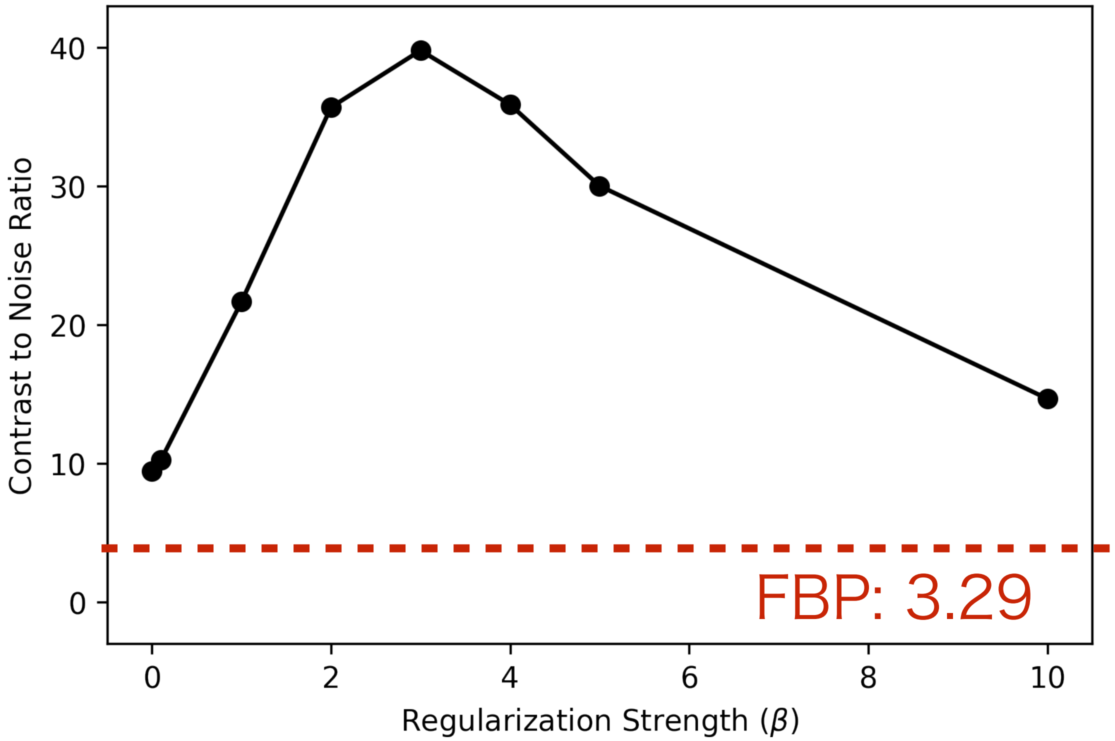
Image cell: 100 x 100 μm^2

- Quantitative image analysis
  - Contrast to noise ratio
  - Spatial resolution

Contrast to Noise Ratio (CNR)



- Circle detection to define the region of iron and plastic (https://scikit-image.org/docs/dev/auto_examples/edges/plot_circular_elliptical_hough_transform.html)



- Calculate the mean and the standard deviation within the area of iron and plastic
- Calculate CNR

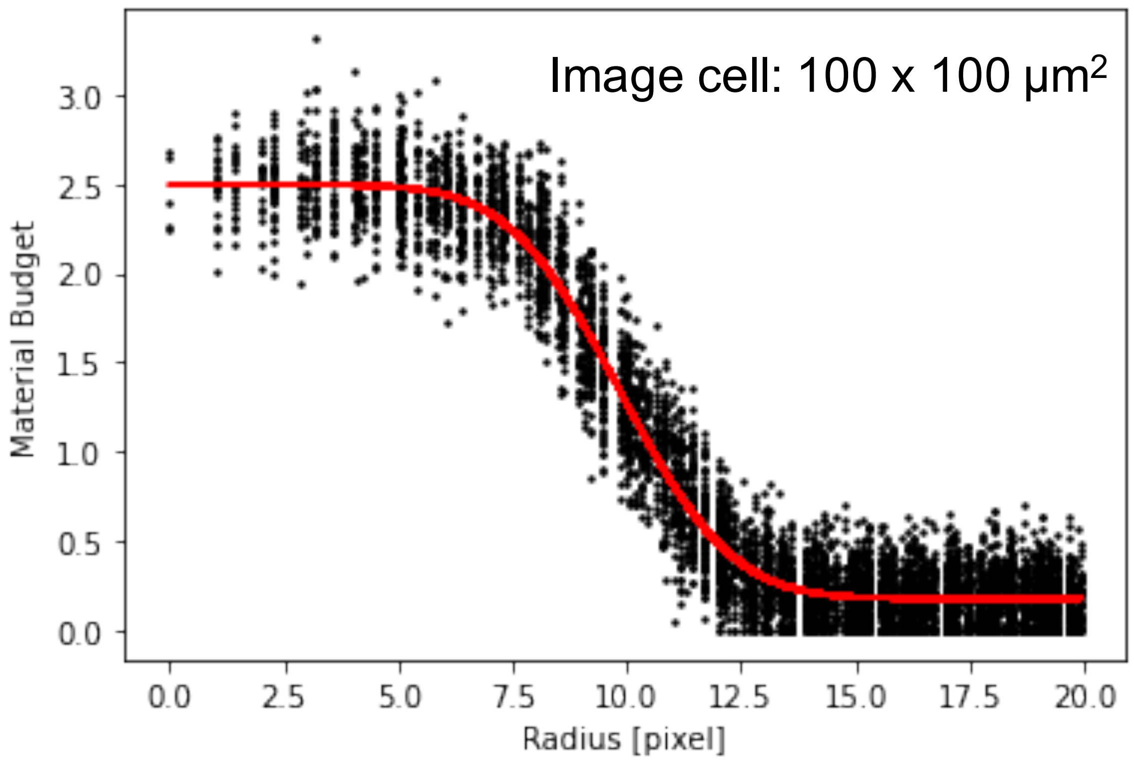
$$\mathrm{CNR} = \frac{\mu_{\mathrm{iron}} - \mu_{\mathrm{plastic}}}{\sqrt{\sigma_{\mathrm{iron}}^2 + \sigma_{\mathrm{plastic}}^2}}$$

μ: Mean

σ: Standard deviation

SIRT+TVS (β=3) shows the best CNR

Spatial Resolution

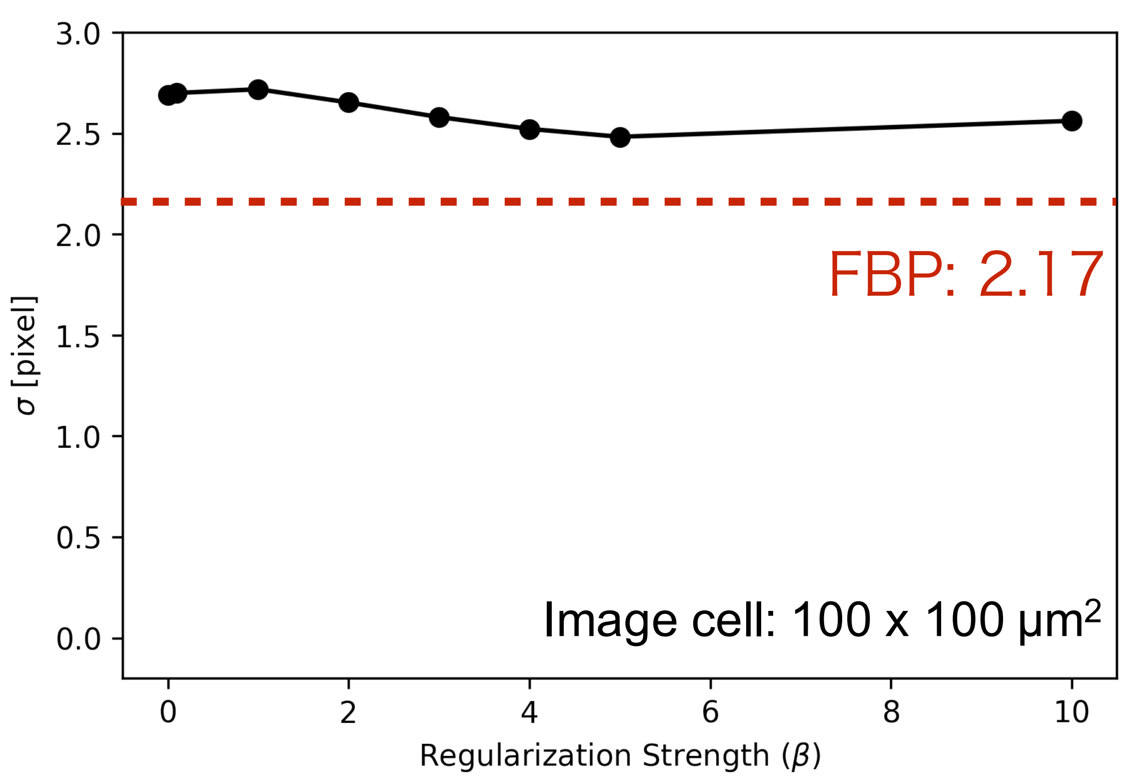


Edge-spread-function:

$$\mathrm{ESF}(x) \propto A + \mathrm{Erf}\left(\frac{x-\mu}{\sigma}\right)$$

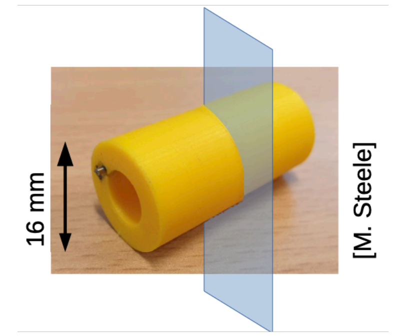
$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt$$

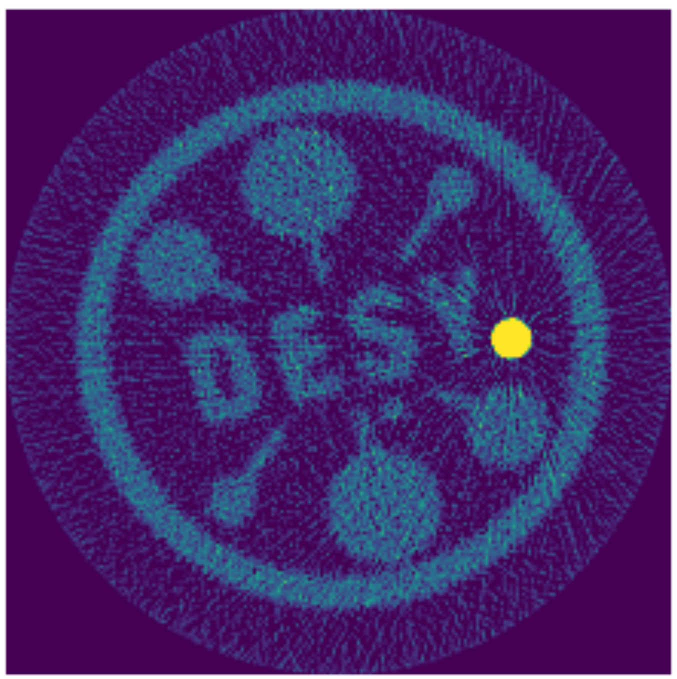
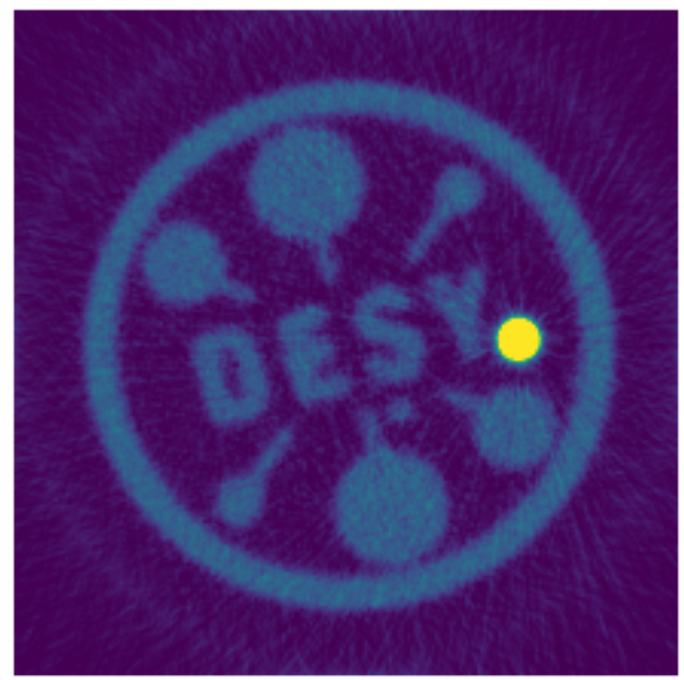
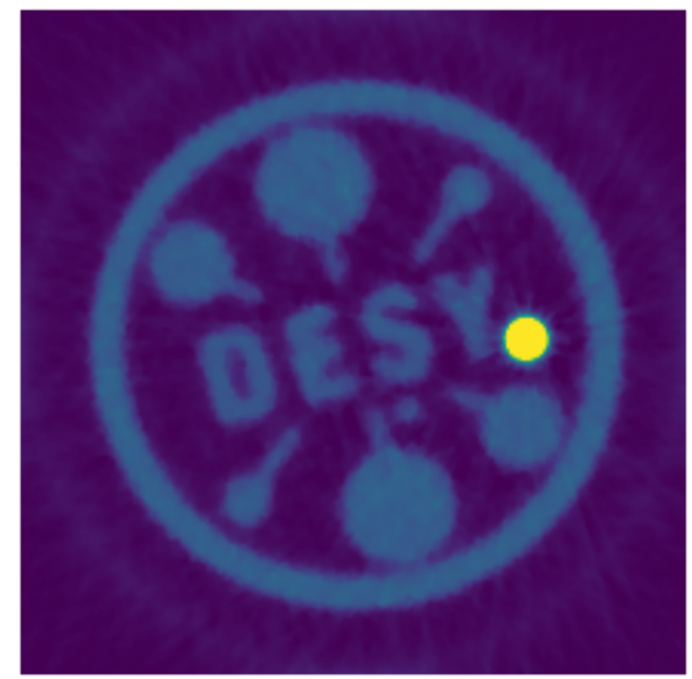
- Circle detection for 5 iron rods
- Plotting material budget as a function of radius from the center of the irons
- Fitting with ESF
- σ of the ESF corresponds to spatial resolution



TVS regularization only slightly worsen the spatial resolution

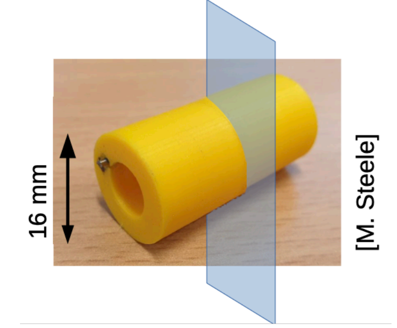
Image Analysis (DESY logo)



Number of projections	FBP	SIRT + TVS (30 iterations) $\beta = 0$	SIRT + TVS (30 iterations) $\beta = 0.4$
400			

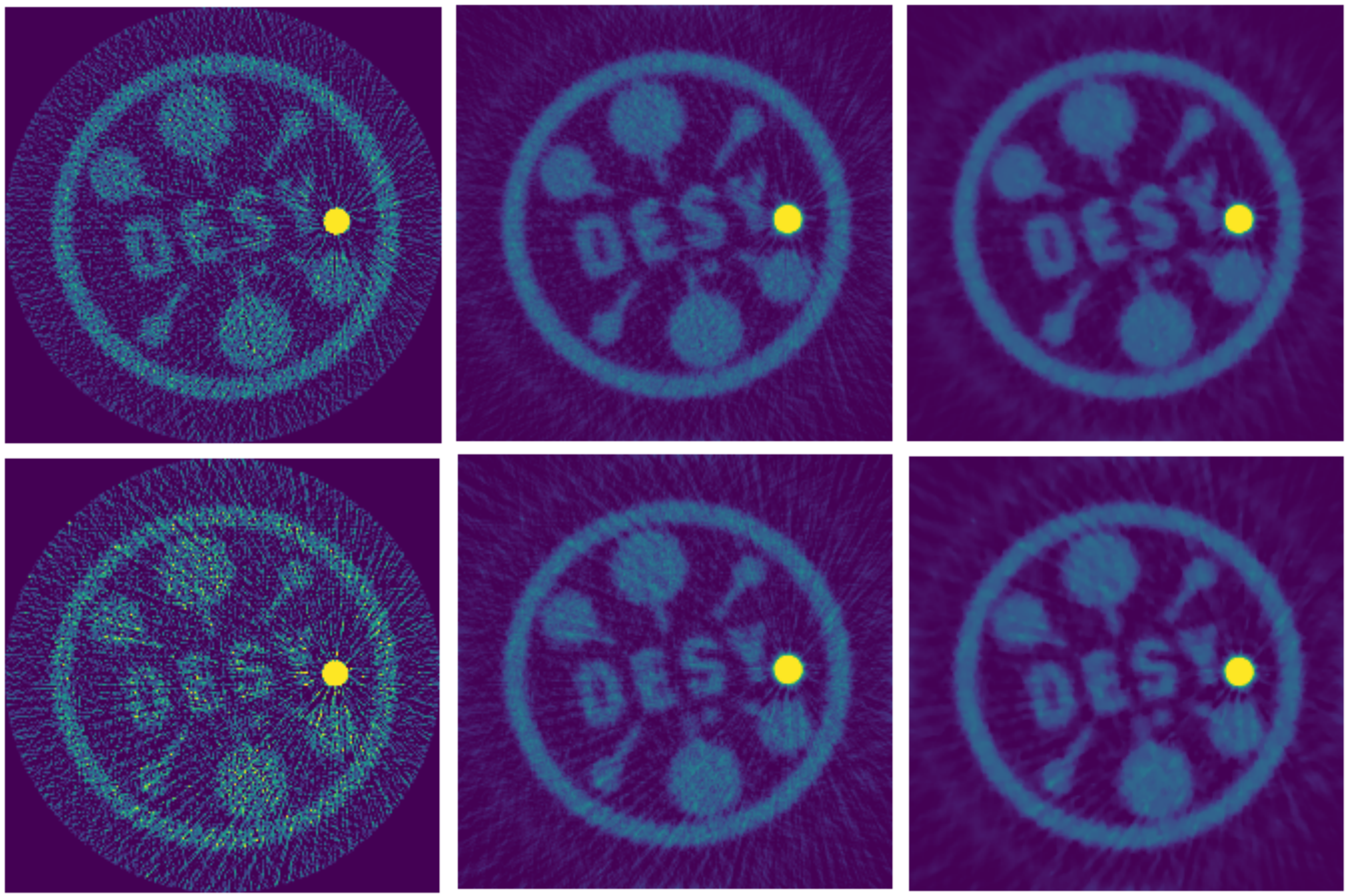
Iterative reconstruction yields noise reduction while maintaining contrast and spatial resolution

Image Analysis (DESY logo)



CAD

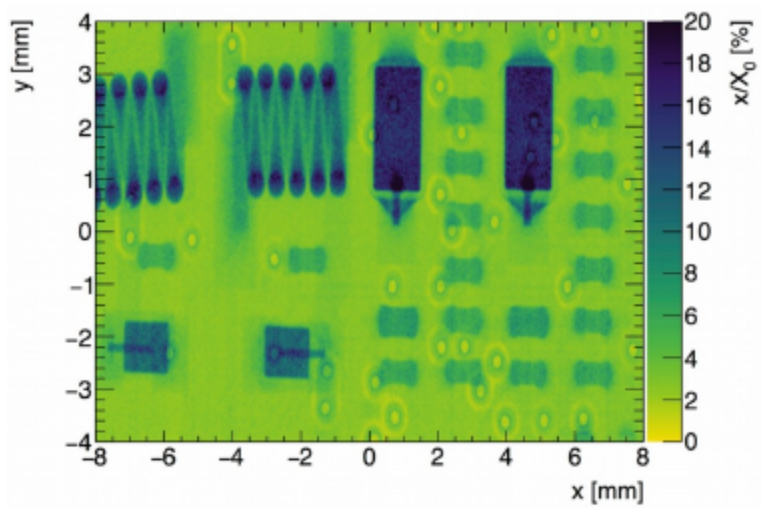
	FBP	SIRT + TVS (30 iterations) $\beta = 0$	SIRT + TVS (30 iterations) $\beta = 0.4$
Number of projections 200			
100			



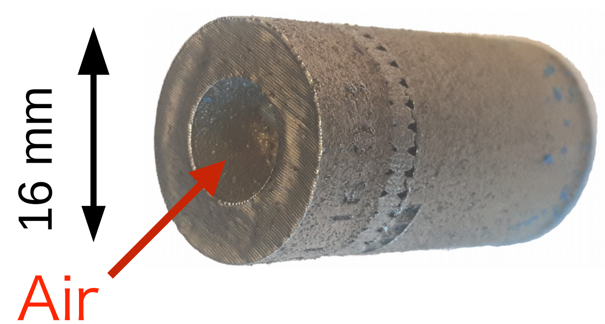
Noise increases as the number of projections decreased in FBP

The degradation is suppressed in iterative reconstruction

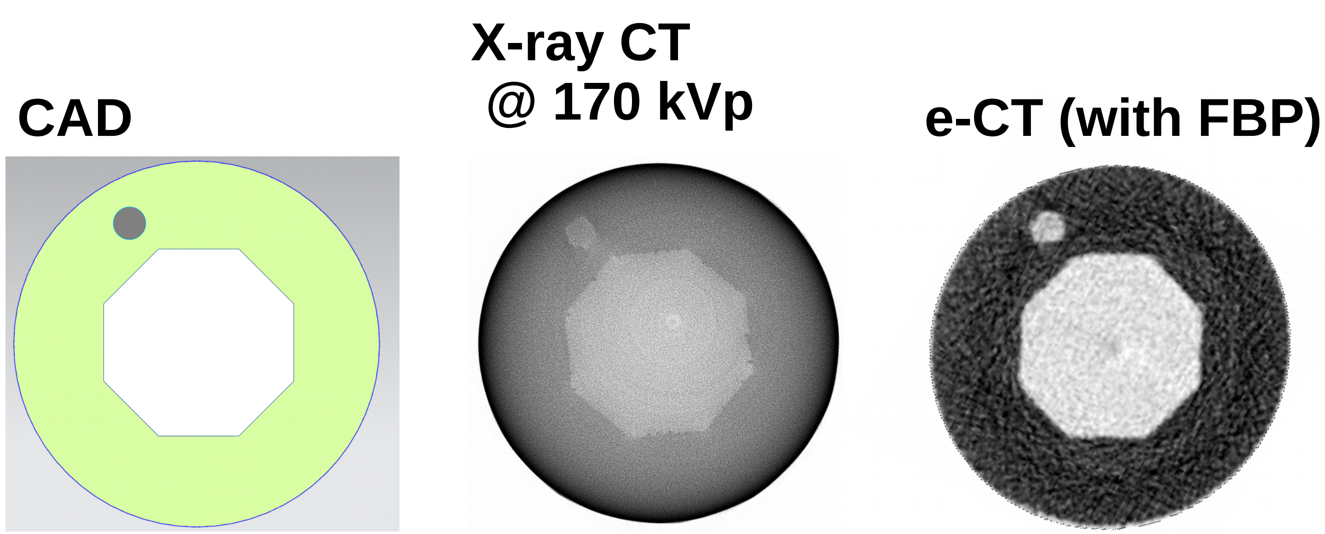
Applications



- Material Budget imaging for HEP detector upgrade



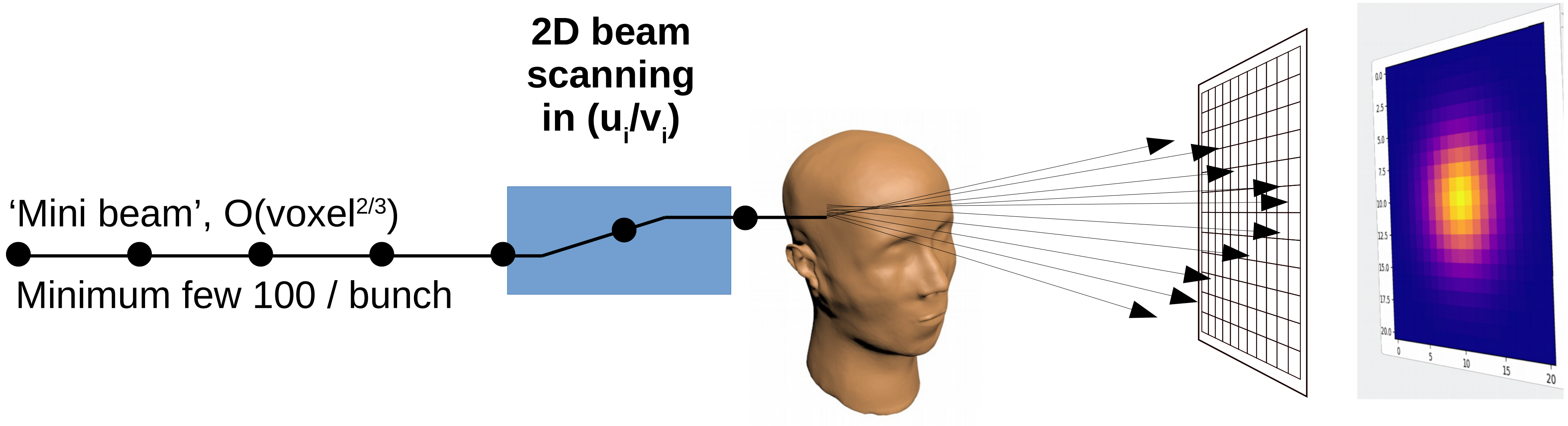
- Non-destructive imaging for high-Z materials



Future Plans

- Imaging for use in combination with Very High Energy Electron Radiotherapy (VHEE-RT)
  - Energy: 50-250 MeV
  - Better dose conformity than conventional photon radiotherapy
  - Reasonable cost and size compared to hadron therapy
  - Possibility of treatment with ultra-high dose rate (FLASH-RT)
- Portal Imaging with 200 MeV
  - Example: Detecting metal markers implanted in the body to determine the location of the target

Pulse Beam Imaging



This is not the final image!

Merely the response per bunch.

- Place a sensor only downstream of target
- 2D beam scanning with pulsed beam
- Read data from each bunch and calculate material budget from the beam spread without track reconstruction
- First tests planned at PITZ/DESY-Zeuthen

Conclusions

- Iterative image reconstruction algorithm was developed for novel electron computed tomography
- The proposed algorithm yields noise reduction and better contrast
- Small negative effect for spatial resolution
- The improvement of the algorithm is significant with lower statistics
- Under development for medical imaging with 200 MeV

Backups

Iterative Reconstruction for e-CT

- Statistical noise
  - Unknown statistical noise of the width (AAD_{90}) of the scattering angle distribution

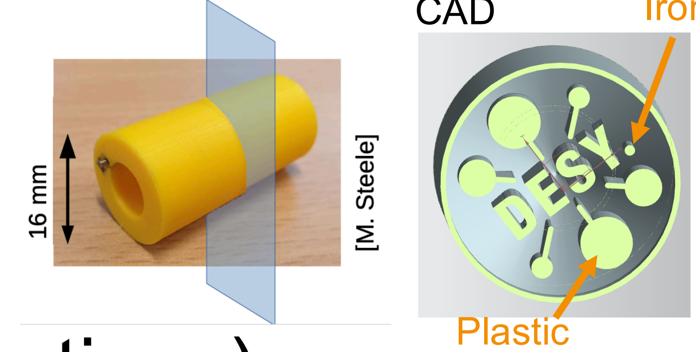
- Regularization
  - A large variety of regularization terms can be implemented in the iterative algorithm

U(x): energy function

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \alpha \left[A^T(\vec{y} - A\vec{x}^{(k)}) + \beta \frac{\partial U(\vec{x})}{\partial \vec{x}} \right]$$

Function: quadratic, gaussian, total variation, ⋯

Image Analysis (DESY logo)



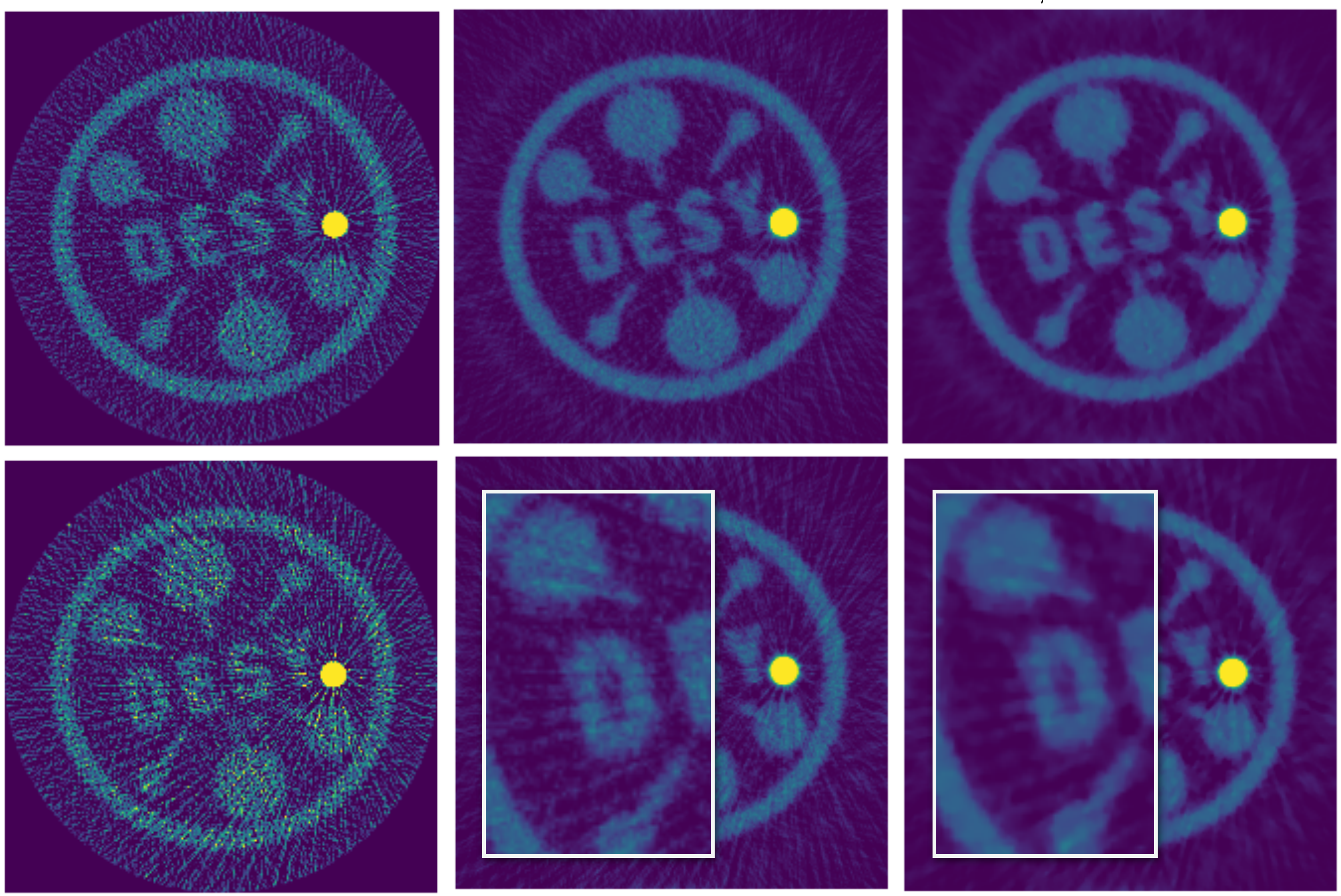
FBP | **SART + TVS** (30 iterations)

$\beta = 0$ | $\beta = 0.4$

Number of projection

200

100



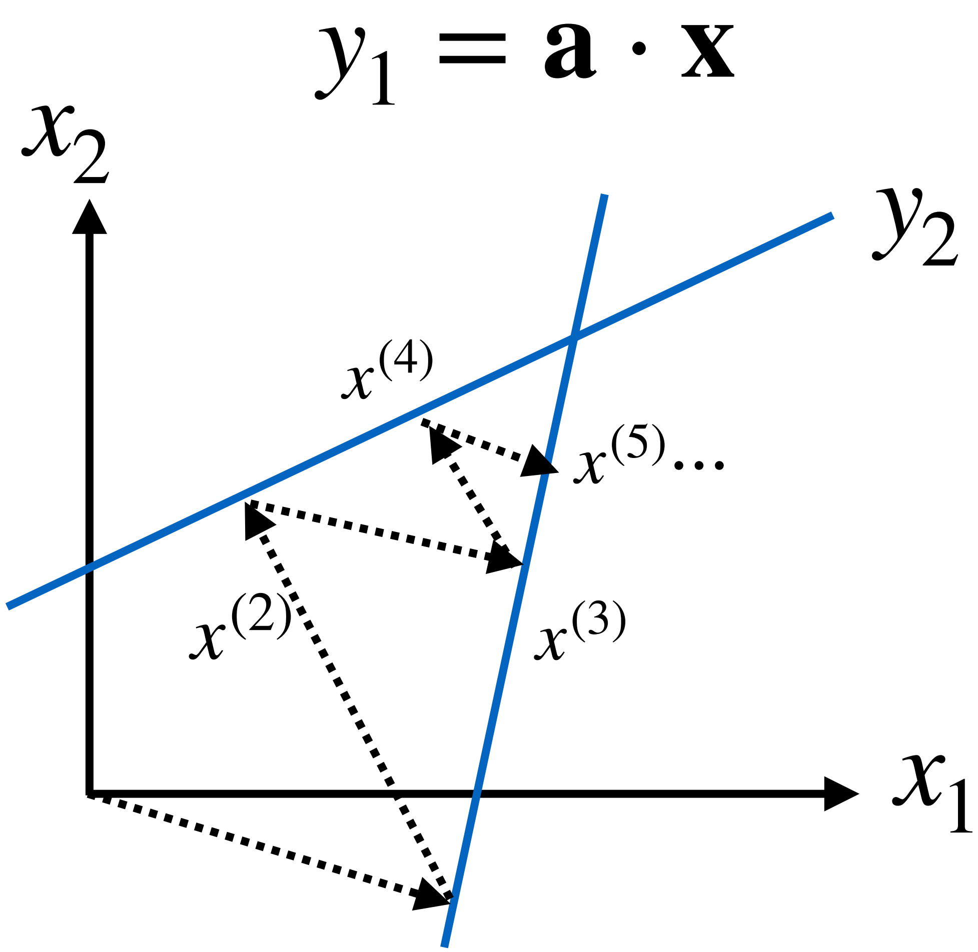
Not significant improvement like 5 iron rods?

→ TVS works (large uniform + clear edge) structure

Projection method

$$\overrightarrow{x}^{(k+1)} = \overrightarrow{x}^{(k)} + \sum_{i=1}^{I} \alpha \|\overrightarrow{a_i}\|^2 \frac{y_i - \overrightarrow{a_i} \cdot x_i^{(k)}}{\|\overrightarrow{a_i}\|^2} \overrightarrow{a_i}$$

= 1 → Algebraic reconstruction technique



$\mathbf{y} = (y_1, y_2)^T$

$\mathbf{x} = (x_1, x_2)^T$

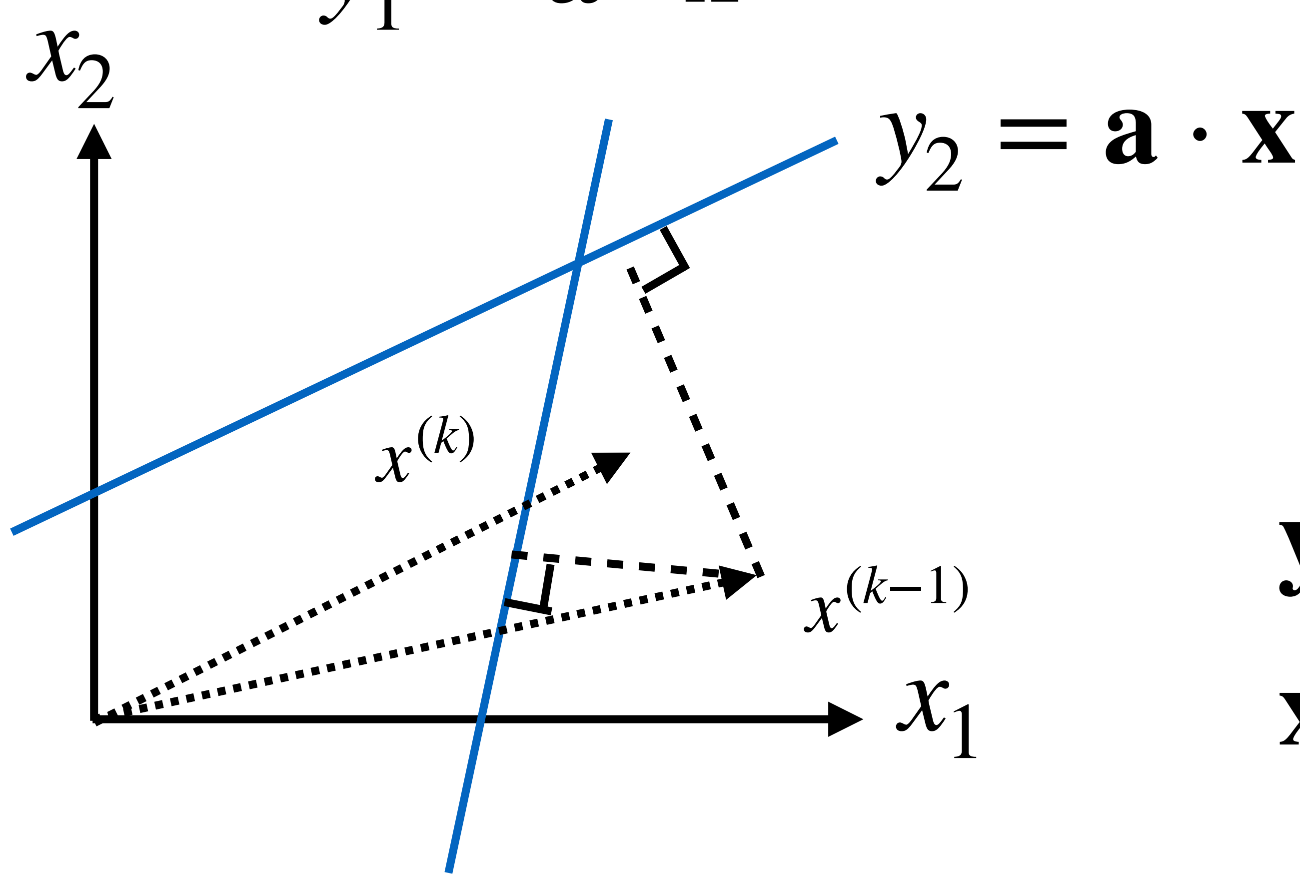
SIRT Projection

$$\overrightarrow{x}^{(k+1)} = \overrightarrow{x}^{(k)} + \sum_{i=1}^{I} \alpha \|\overrightarrow{a_i}\|^2 \frac{y_i - \overrightarrow{a_i} \cdot x_i^{(k)}}{\|\overrightarrow{a_i}\|^2} \overrightarrow{a_i}$$

Simultaneous projection with weighting $\sum_{i=1}^{I} \alpha \|\overrightarrow{a_i}\|^2$

α: step size (free parameter)

$y_1 = \mathbf{a} \cdot \mathbf{x}$



$$\mathbf{y} = (y_1, y_2)^T$$

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