

Forward jets - selected recent developments



The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences

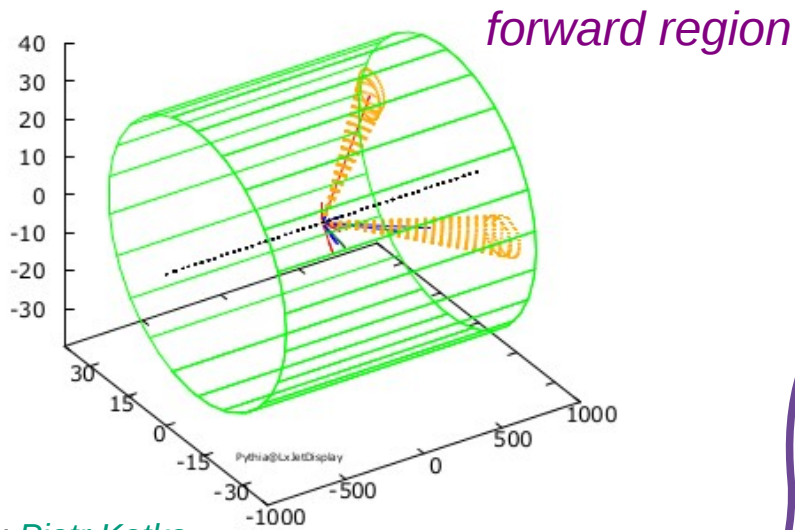
Krzysztof Kutak



NCN



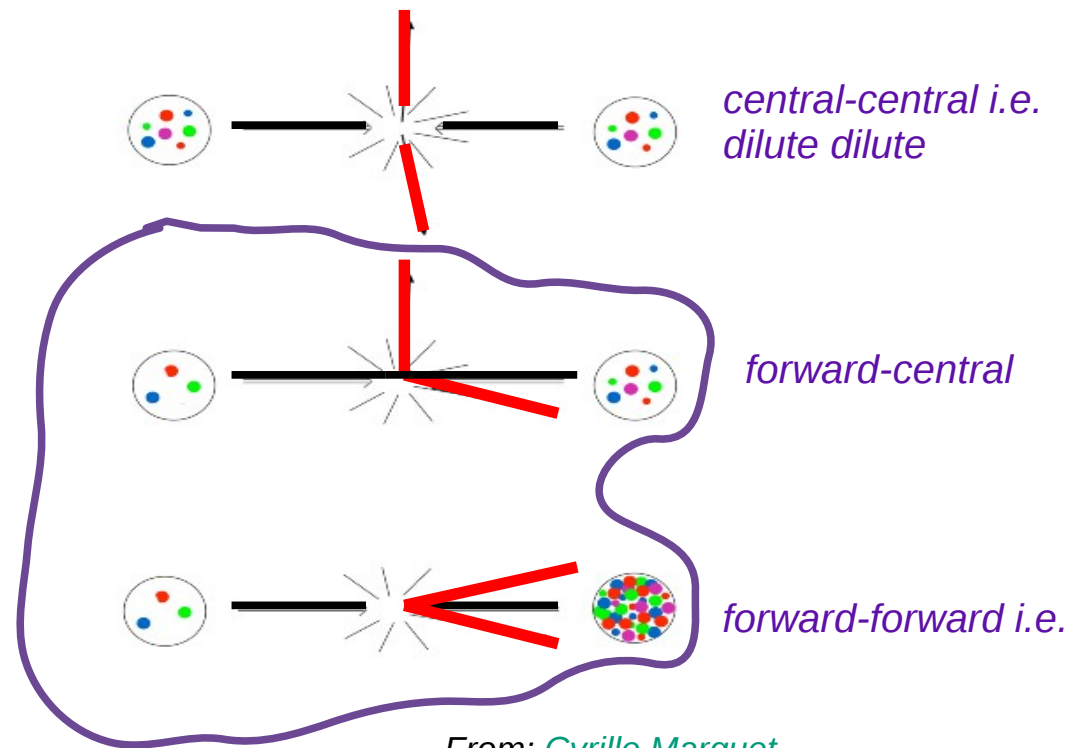
Forward jets



forward region

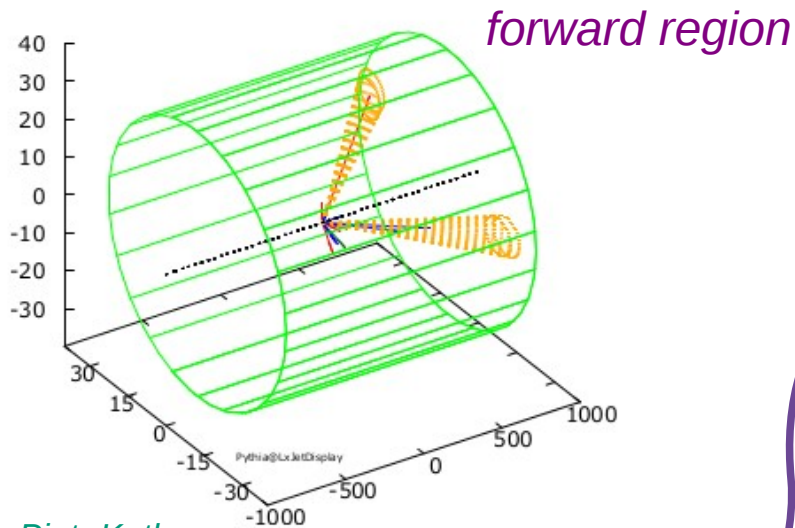
central region

From: Piotr Kotko



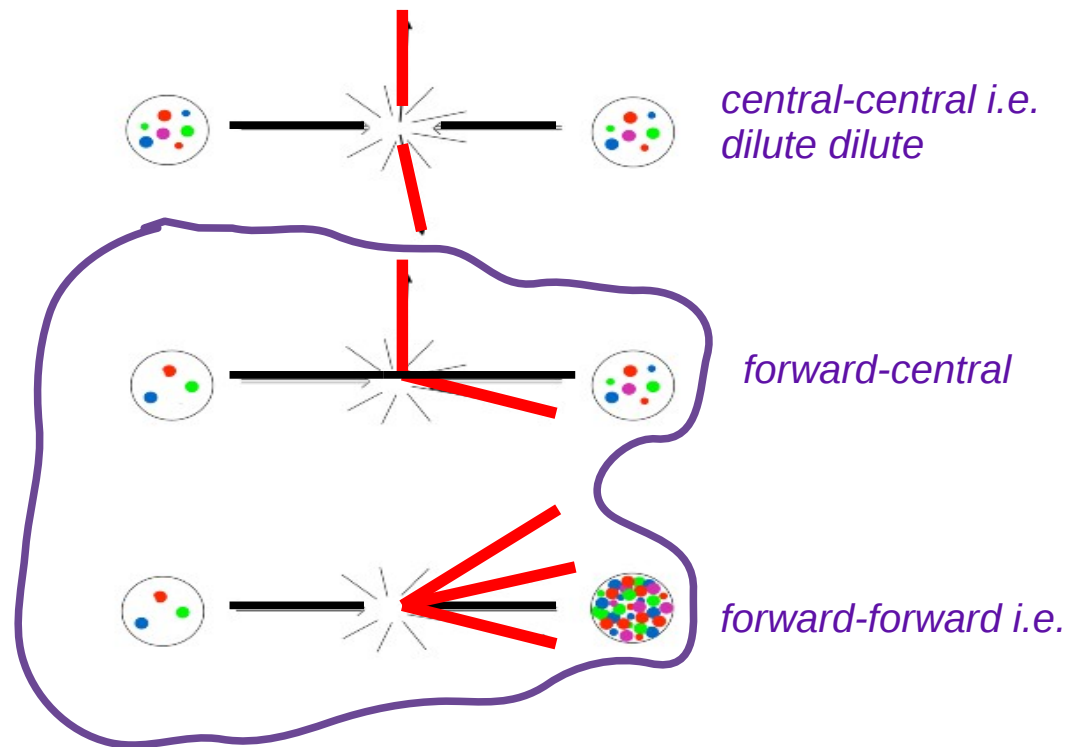
There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not

Forward jets



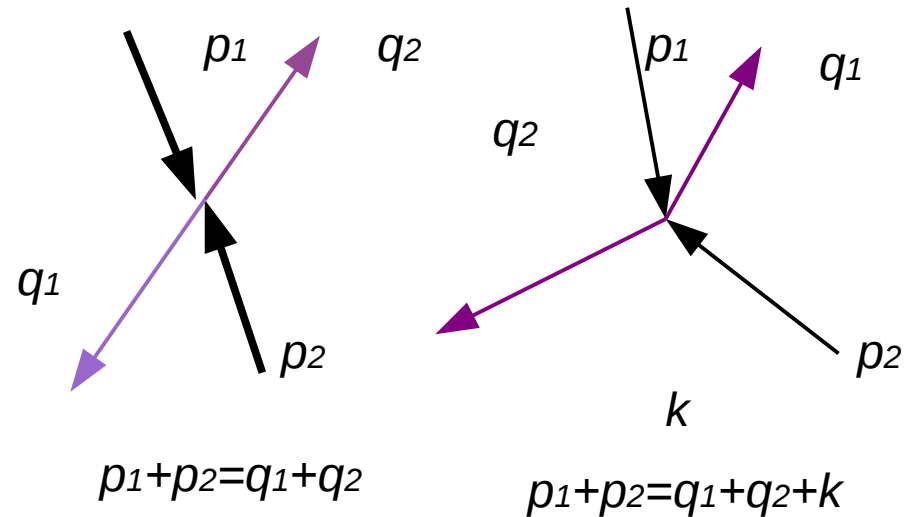
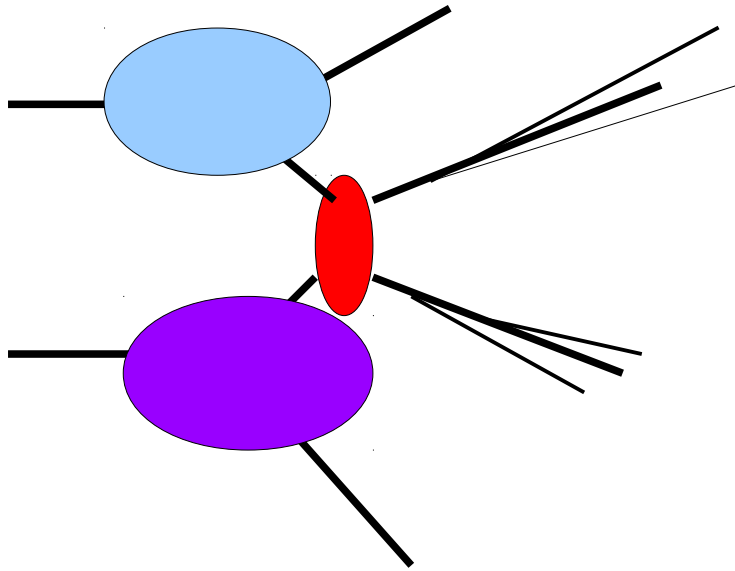
forward region

central region



There is certain class of processes where one can assume that partons in one of hadrons are just collinear with hadron and in other are not

QCD at high energies – k_t factorization



Schematic formula for cross section

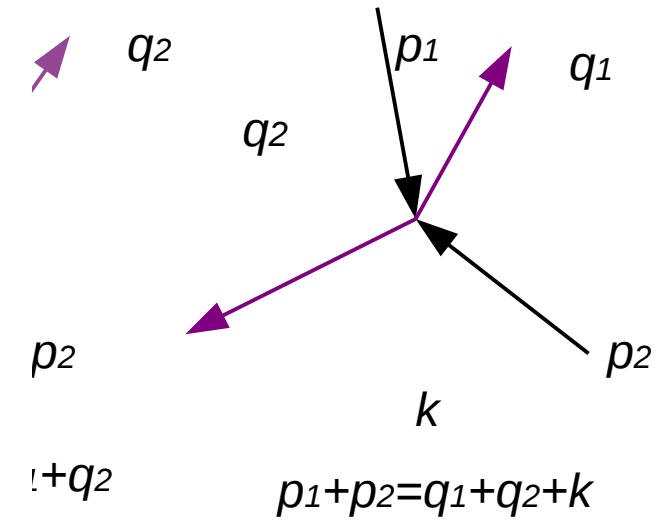
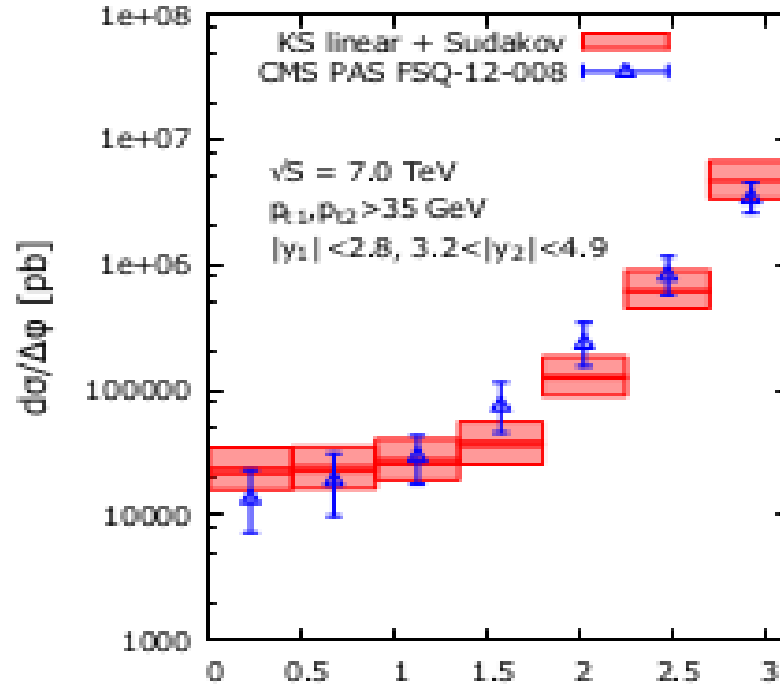
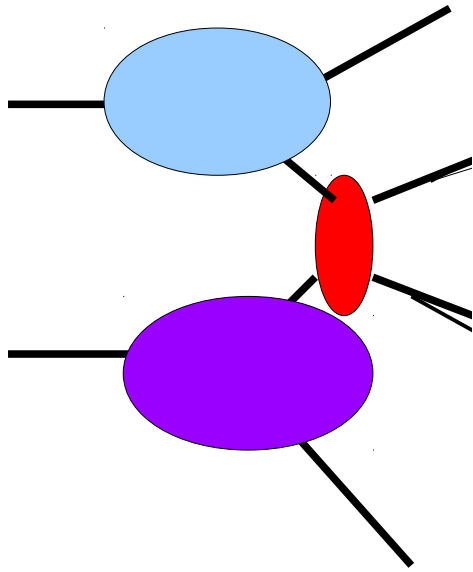
$$\frac{d\sigma}{dPS} \propto x f(x_1, \mu) \otimes \hat{\sigma}(x_1, x_2, k_T) \otimes \mathcal{F}(x_2, k_T)$$

Ciafaloni, Catani, Hautman '93
Collins, Ellis '93
Lipatov '95

New helicity based methods for ME

van Hameren, Kotko, K.Kutak, JHEP 1301 (2013) 078

QCD at high energies – k_t factorization



Schematic formula for

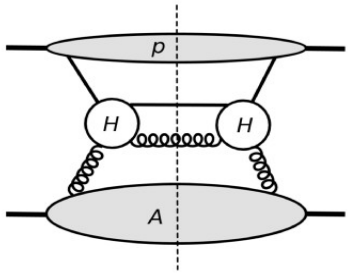
$$\frac{d\sigma}{dPS} \propto x f(x_1, \mu) \otimes v(x_1, x_2, n_T) \otimes v(x_2, n_T)$$

Ciafaloni, Catani, Hautman '93
 Collins, Ellis '93
 Lipatov '95

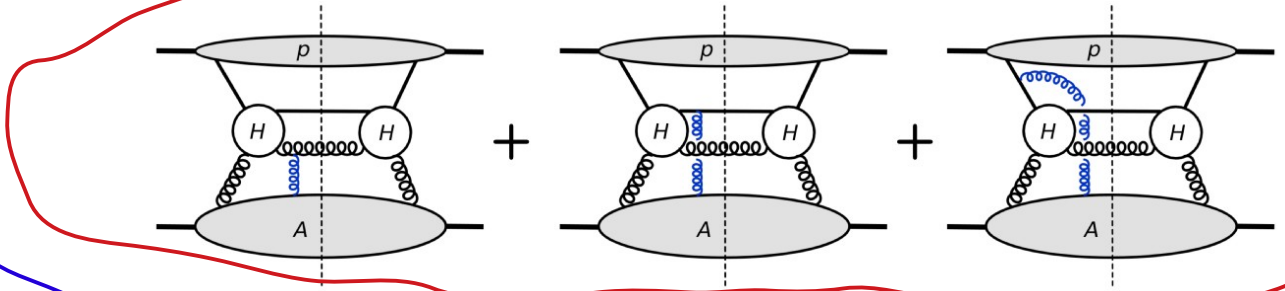
New helicity based methods for ME
 van Hameren, Kotko, K.K, JHEP 1301 (2013) 078

Formula for TMD gluon and gauge links

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \hat{F}^{i+}(\xi^+ = 0, \xi^-, \vec{\xi}_T) \right\} | P \rangle$$



Valid for large transversal momentum and was obtained in a specific gauge



similar diagrams with 2,3,...gluon exchanges. All this need to be resummed

From [S. Sapeta](#)

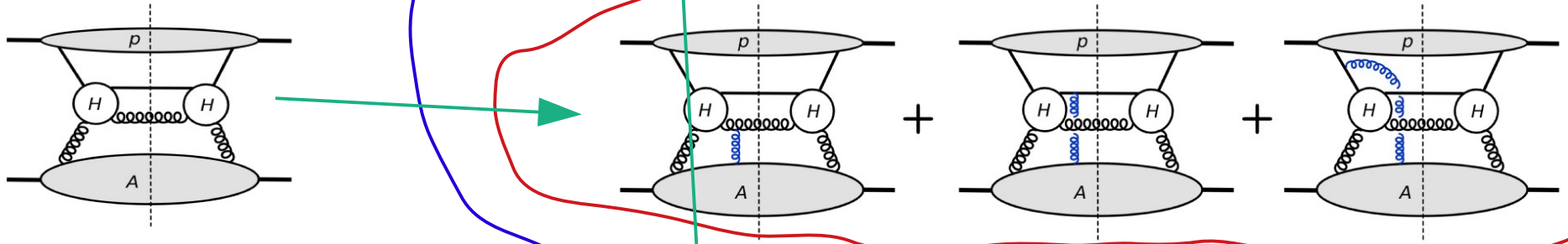
$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \mathcal{U}_{C_1} \hat{F}^{i+}(\xi) \mathcal{U}_{C_2} \right\} | P \rangle$$

Hard part defines the path of the gauge link $\mathcal{U}^{[C]}(\eta; \xi) = \mathcal{P} \exp \left[-ig \int_C dz \cdot A(z) \right]$

[C.J. Bomhof, P.J. Mulders, F. Pijlman](#)
[Eur.Phys.J. C47 \(2006\) 147-162](#)

Formula for TMD gluon and gauge links

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \hat{F}^{i+}(\xi^+ = 0, \xi^-, \vec{\xi}_T) \right\} | P \rangle$$



Valid for large transversal momentum and was obtained in a specific gauge

From [S. Sapeta](#)

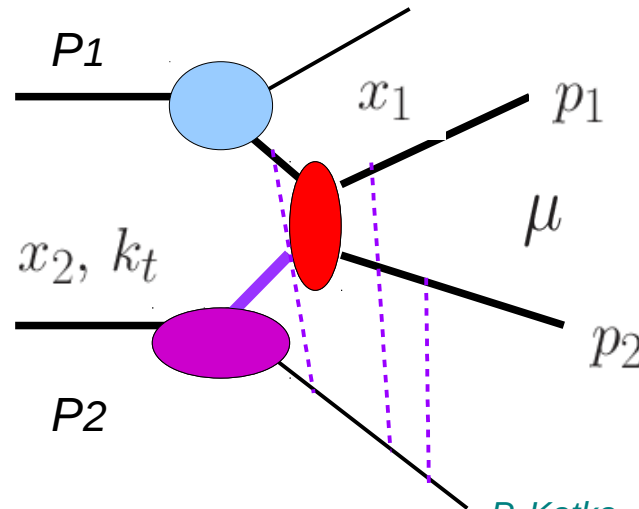
similar diagrams with 2,3,...gluon exchanges. All this need to be resummed

[C.J. Bomhof, P.J. Mulders, F. Pijlman](#)
[Eur.Phys.J. C47 \(2006\) 147-162](#)

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \mathcal{U}_{C_1} \hat{F}^{i+}(\xi) \mathcal{U}_{C_2} \right\} | P \rangle$$

Hard part defines the path of the gauge link $\mathcal{U}^{[C]}(\eta; \xi) = \mathcal{P} \exp \left[-ig \int_C dz \cdot A(z) \right]$

Improved Transverse Momentum Dependent Factorization



The same gauge link structure as in
 Fabio Dominguez, Bo-Wen Xiao, Feng Yuan
 Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan
 Phys.Rev. D83 (2011) 105005

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta,
 A. van Hameren,
 JHEP 1509 (2015) 106

Can be obtained from CGC
 T. Altinoluk, R. Boussarie, Piotr Kotko
 JHEP 1905 (2019) 156
 Comparison between CGC and ITMD
 Marquet, Fujii, Watanabe
 JHEP12(2020)181

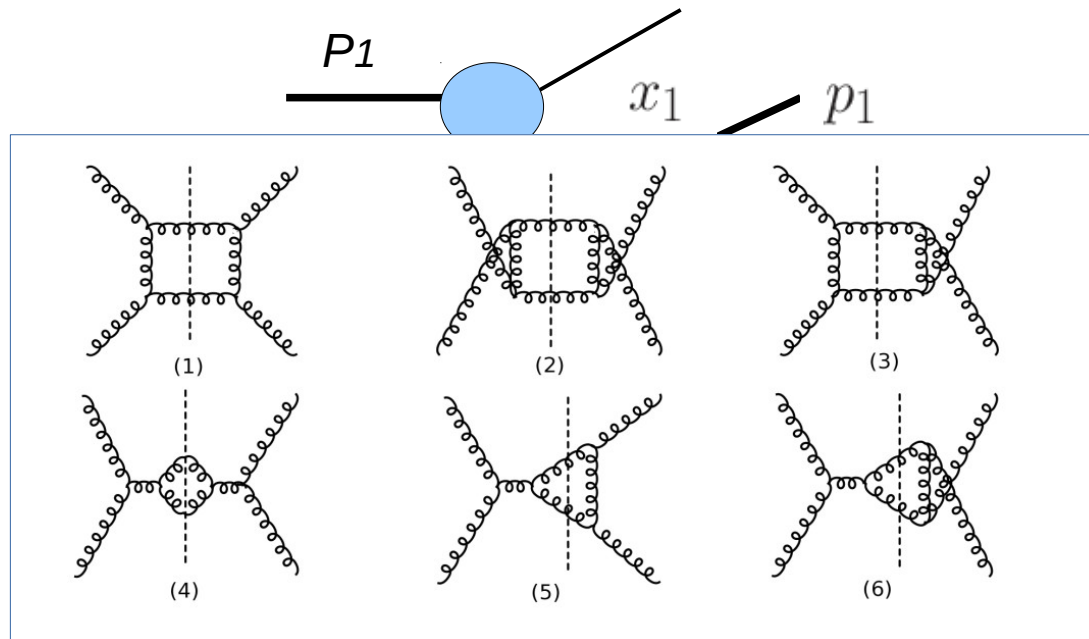
gauge invariant amplitudes with k_t and TMDs

Example for $g^* g \rightarrow g g$

$$\frac{d\sigma^{pA \rightarrow ggX}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} x_1 f_{g/p}(x_1, \mu^2) \sum_{i=1}^6 \mathcal{F}_{gg}^{(i)} H_{gg \rightarrow gg}^{(i)}$$

Formalism implemented
 in Monte Carlo programs
 by A. van Hameren **KaTie**
 and P. Kotko **LxJet**

Improved Transverse Momentum Dependent Factorization



P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

Appropriate in any configuration

Can be obtained from CGC

T. Altinoluk, R. Boussarie, Piotr Kotko

JHEP 1905 (2019) 156

Comparison between CGC and ITMD

Marquet, Fujii, Watanabe

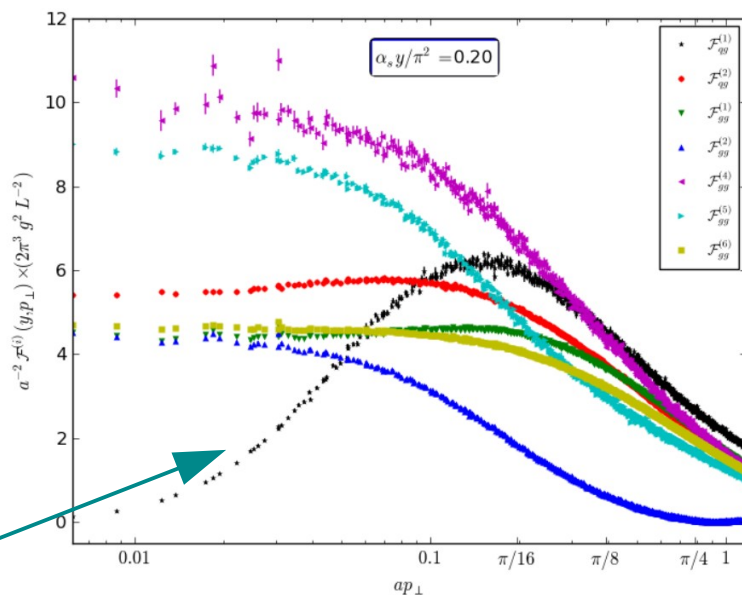
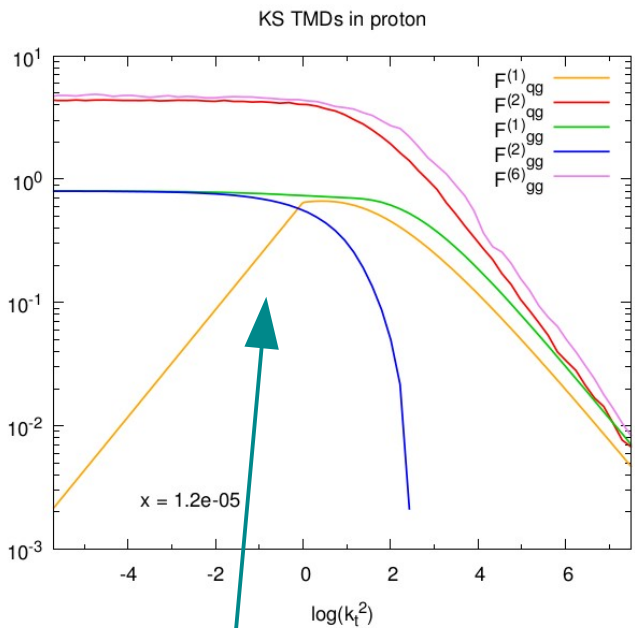
JHEP12(2020)181

gauge invariant amplitudes with k_t and TMDs

example for $g^ g \rightarrow g g$*

$$\frac{d\sigma^{pA \rightarrow ggX}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} x_1 f_{g/p}(x_1, \mu^2) \sum_{i=1}^6 \mathcal{F}_{gg}^{(i)} H_{gg \rightarrow gg}^{(i)}$$

Plots of ITMD gluons



Calculation – in large N_c approximation with analytic model for dipole gluon density – all gluons can be calculated from the dipole one

Kotko, .Kutak, Marquet, Petreska, Sapeta, van Hameren
 JHEP 1612 (2016) 034

Dipole gluon density (solution of BK)

The other densities are flat at low $k_t \rightarrow$ less saturation

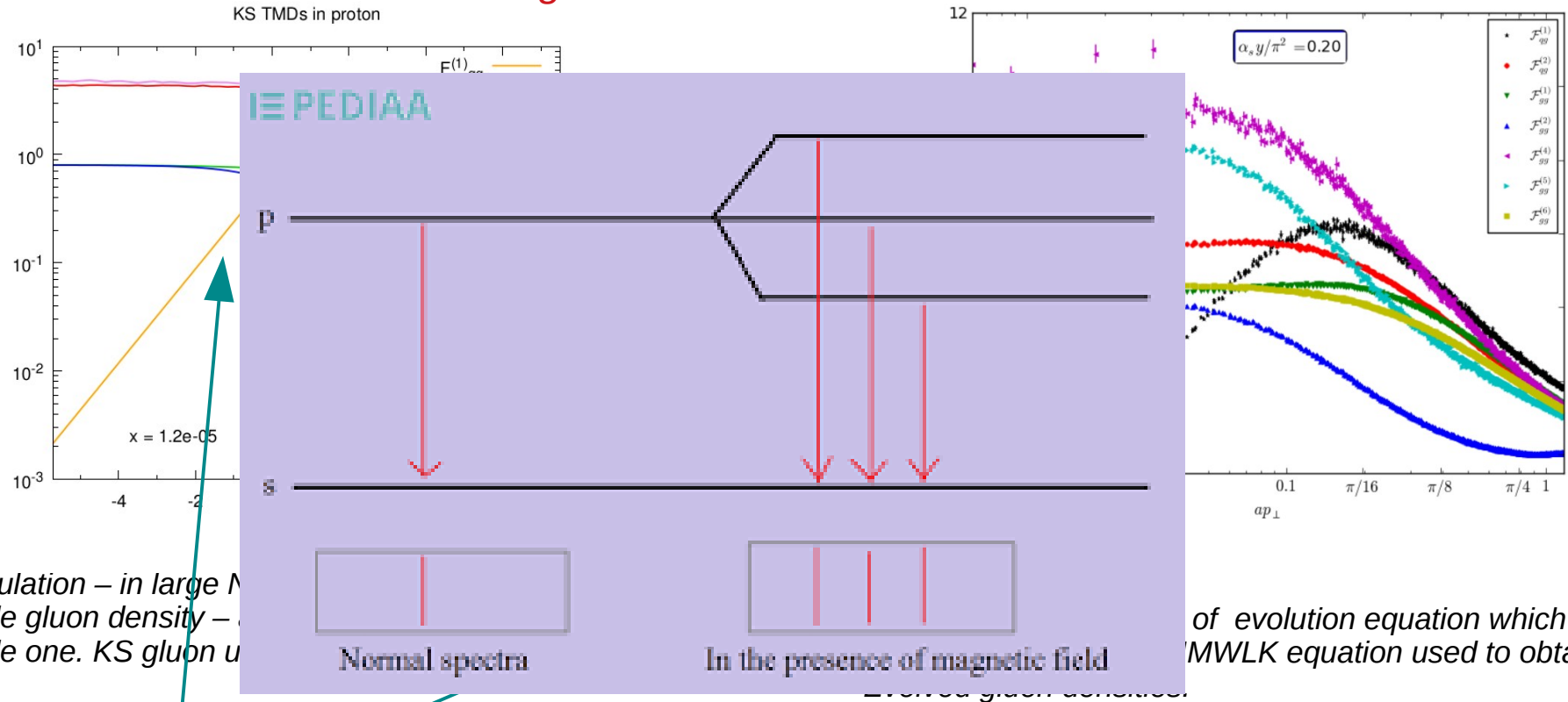
Not negligible differences at large $k_t \rightarrow$ differences at small angles

Obtained from solutions of evolution equation which accounts for finite N_c . JIMWLK equation used to obtain Evolved gluon densities.

The JIMWLK equation is a renormalization group equation for the Wilson lines, obtained by integrating out the quantum fluctuations at smaller and smaller Bjorken- x .
 C. Marquet, E. Petreska, C. Roiesnel
 JHEP 1610 (2016) 065

Plots of ITMD gluons

rough analogy to splitting of spectral lines in presence of a new scale – magnetic field



Calculation – in large N_c dipole gluon density – dipole one. KS gluon u

Kotko, K.K, Marquet, Petreska, Sapeta, van Hameren
JHEP 1612 (2016) 034

Dipole gluon density (solution of BK)

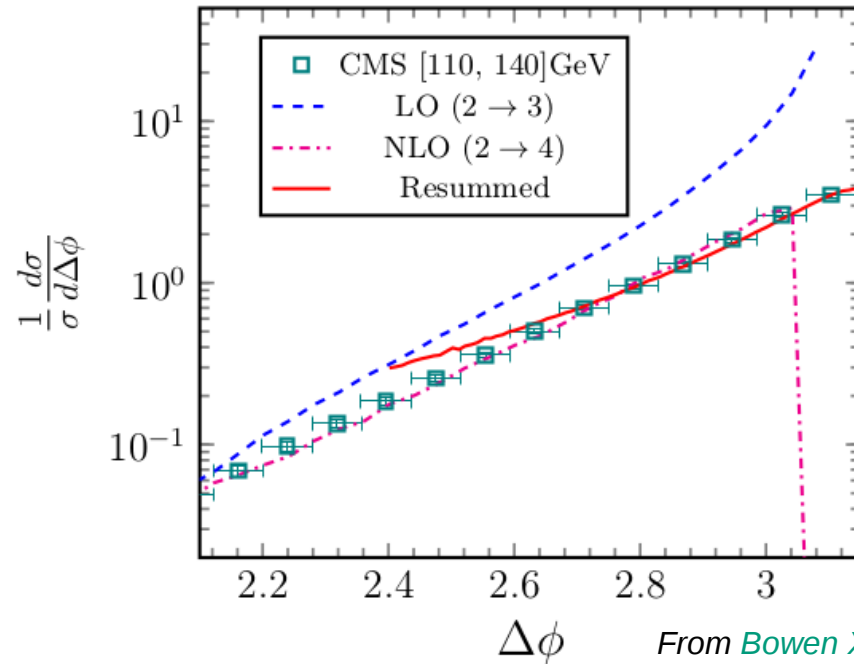
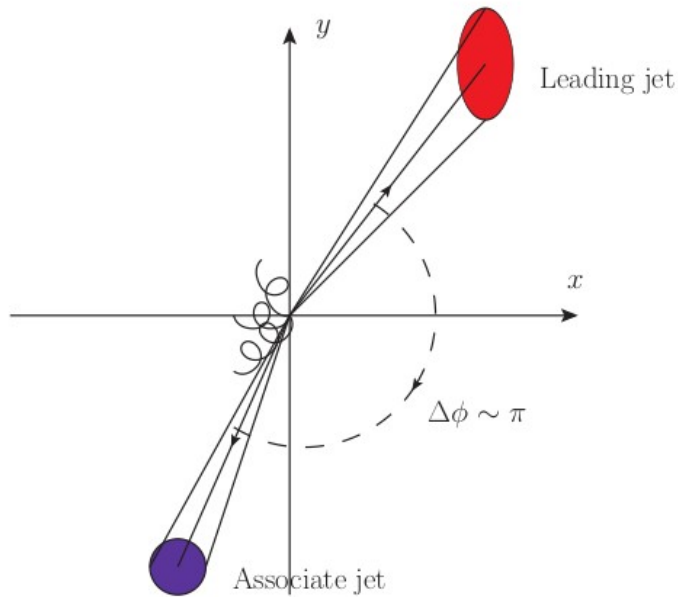
The other densities are flat at low $k_t \rightarrow$ less saturation

Not negligible differences at large $k_t \rightarrow$ differences at small angles

of evolution equation which MWLK equation used to obtain

The JIMWLK equation is a renormalization group equation for the Wilson lines, obtained by integrating out the quantum fluctuations at smaller and smaller Bjorken- x .
C. Marquet, E. Petreska, C. Roiesnel
JHEP 1610 (2016) 065

Sudakov, back-to-back jets and collinear physics



From Bowen Xiao lecture at QCD 2019 master class

In collinear physics at LO for $2 \rightarrow 2$ we get delta function since the colliding partons do not carry transverse momentum. Adding more jet we get some improvement $2 \rightarrow 3$, $2 \rightarrow 4$. The unobserved partons can be soft and can introduce large logs. **Note: k_t factorization also smears the delta function but it takes into account low x effects**

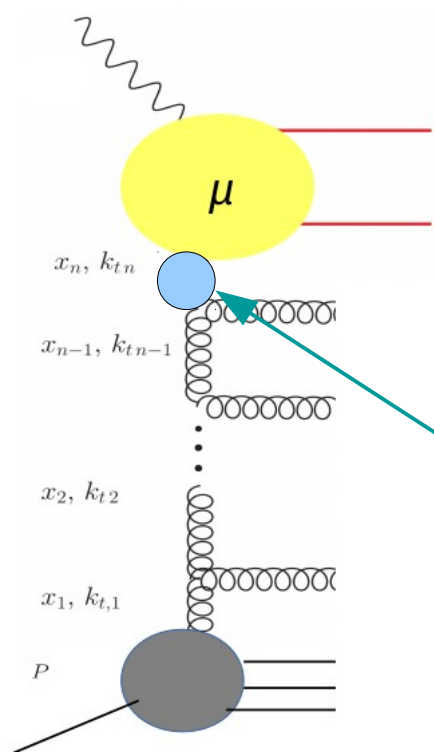
$$p_t \gg k_t$$

leading jet

imbalance between leading jet and associated jet – in forward jet scenario this can be linked to k_t of incoming parton

divergence $L \sim \ln^2 \frac{p_t^2}{k_t^2}$ needs to be resummed

Sudakov and KMRW



$$\frac{\partial a(x, \mu^2)}{\ln \mu^2} = \frac{\alpha_s}{2\pi} \sum_{a'=q,g} \int_x^1 dz P_{aa'}(z, \mu^2) a' \left(\frac{x}{z}, \mu^2 \right)$$

includes plus prescription

one introduces cutoff and rewrites the equation as

$$\frac{\partial a(x, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S}{2\pi} \left[\sum_{a'} \int_x^{1-\Delta} P_{aa'}(z) a' \left(\frac{x}{z}, \mu^2 \right) dz - a(x, \mu^2) V^a(\Delta) \right]$$

$$V^a(\Delta) = \sum_{a'} \int_0^{1-\Delta} P_{a'a}(z) dz$$

unregulated
virtual contribution

$$T^a(k_t, \mu) = \exp \left(- \int_{k_t^2}^{\mu^2} \frac{dk_t'^2}{k_t'^2} \frac{\alpha_S(k_t'^2)}{2\pi} V^a(\Delta) \right)$$

$$\Delta \equiv 1 - z_{\max} = \frac{k_t}{\mu + k_t}$$

$$f_a(x, k_t^2, \mu^2) \equiv \frac{\partial}{\partial \ln k_t^2} \left[a(x, k_t^2) T_a(k_t^2, \mu^2) \right]$$

$$= T^a(k_t, \mu) \left[\frac{\alpha_S(k_t^2)}{2\pi} \sum_{a'} \int_x^{1-\Delta} P_{aa'}(z) a' \left(\frac{x}{z}, k_t^2 \right) dz \right]$$

M. A. Kimber, A. D. Martin, M. G. Ryskin
 Phys.Rev.D63:114027,2001
 Phys.Rev.D64:094017,2001

A. Martin, M. Ryskin, G. Watt
 Phys.Rev.D70:014012,2004

B. Guiot
 Phys. Rev. D 101, 054006 (2020)

K. Golec-Biernat, A. Stasto
 Phys.Lett.B 781 (2018) 633-638

M. Nefedov, V. Saleev
 2009.13188

Eur.Phys.J.C 78 (2018) 2, 137
 M. Bury, A van Hameren, H. Jung, K. Kutak, S. Sapeta

Sudakov and TMD gluon density

- **Survival probability of the gap** * without emissions re-weighting of observable taking into account Sudakov and preserving total cross-section

$$\mathcal{O} = \frac{\sigma}{W} \sum_i w_i F_i^{\mathcal{O}}(X_i)$$

observable

function defining observable: cuts etc.

$$\mathcal{O} \sim \sum_i w_i F_i^{\mathcal{O}}(X_i) \Theta(\mu_i > k) + \sum_j w_j F_j^{\mathcal{O}}(X_j) \Theta(k_j > \mu_j)$$

total cross-section

$$\bar{\mathcal{O}} \sim \sum_i w_i F_i^{\mathcal{O}}(X_i) T(k_i, \mu_i) \Theta(\mu_i > k) + \sum_j w_j F_j^{\mathcal{O}}(X_j) \Theta(k_j > \mu_j)$$

total weight

van Hameren, Kotko, Kutak, Sapeta
PLB '14

- **KS hardscale unintegrated** ** gluon density with Sudakov preserving normalisation

$$\mathcal{F}(x, k^2, \mu^2) := \theta(\mu^2 - k^2) T_s(\mu^2, k^2) \frac{xg(x, \mu^2)}{xg_{hs}(x, \mu^2)} \mathcal{F}(x, k^2) + \theta(k^2 - \mu^2) \mathcal{F}(x, k^2)$$

$$xg_{hs}(x, \mu^2) = \int^{\mu^2} dk^2 T_s(\mu^2, k^2) \mathcal{F}(x, k^2) \quad xg(x, \mu^2) = \int^{\mu^2} dk^2 \mathcal{F}(x, k^2)$$

Kutak PRD '14

ITMD and HEF for dijets

$$\frac{d\sigma^{pA \rightarrow ggX}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} x_1 f_{g/p}(x_1, \mu^2) \sum_{i=1}^6 \mathcal{F}_{gg}^{(i)} H_{gg \rightarrow gg}^{(i)}$$

*moderate x almost linear
regime hybrid HEF*

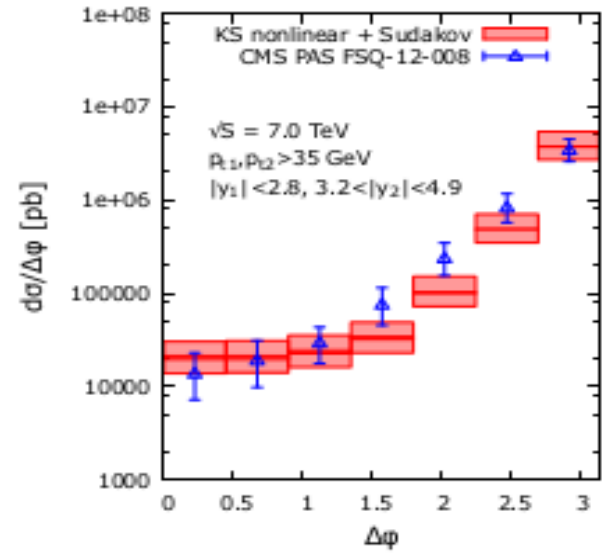
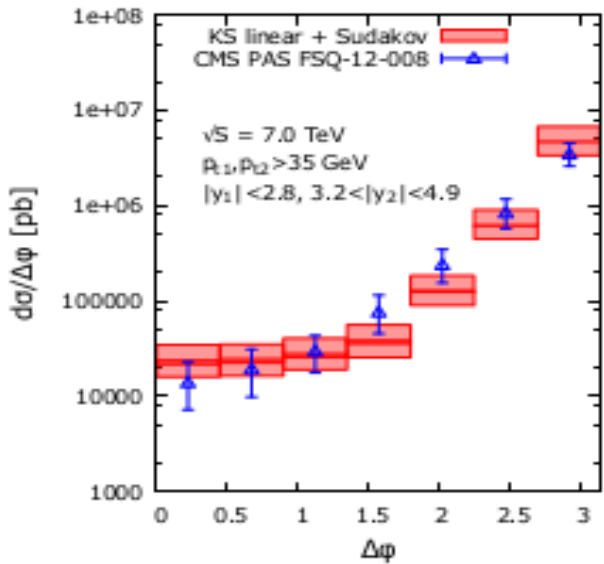
*A, Dumitru, A. Hayashigaki J. Jalilian-Marian
Nucl.Phys. A765 (2006) 464-482M.*

Marquet Nucl.Phys.A 796 (2007) 41-60

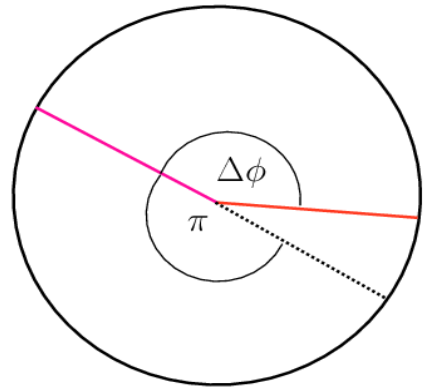
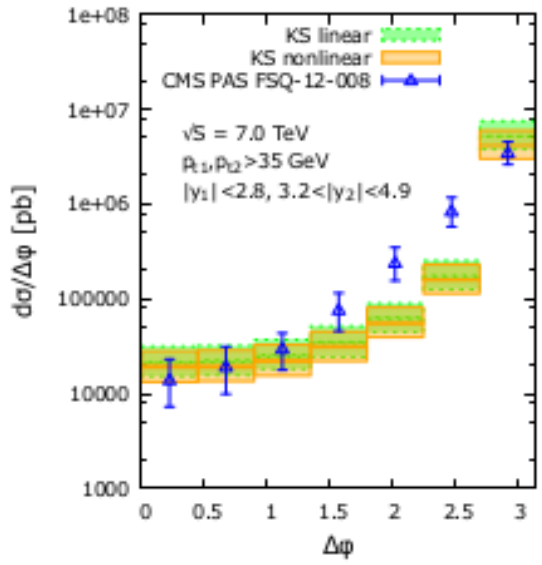
*Deak, F. Hautmann, H. Jung, K. Kutak
JHEP 0909 (2009) 121*

$$\frac{d\sigma_{\text{SPS}}^{P_1 P_2 \rightarrow \text{dijets} + X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1, \mu^2) |\overline{\mathcal{M}_{ag^* \rightarrow cd}}|^2 \mathcal{F}_{g/P_2}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}$$

Decorelations inclusive scenario

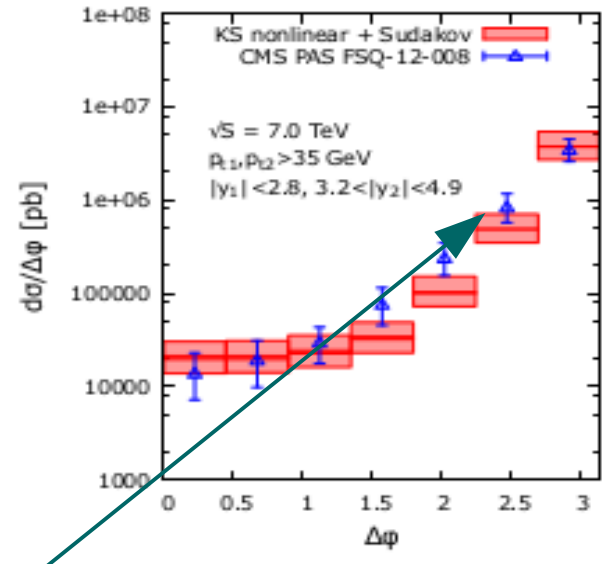
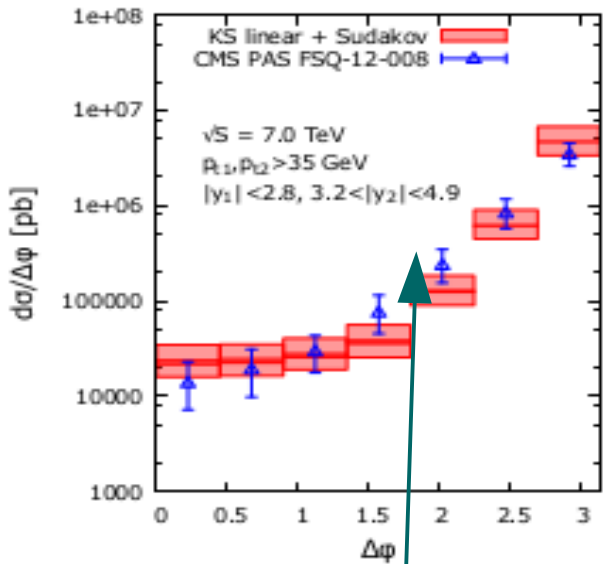


No saturation but visible Sudakov effects

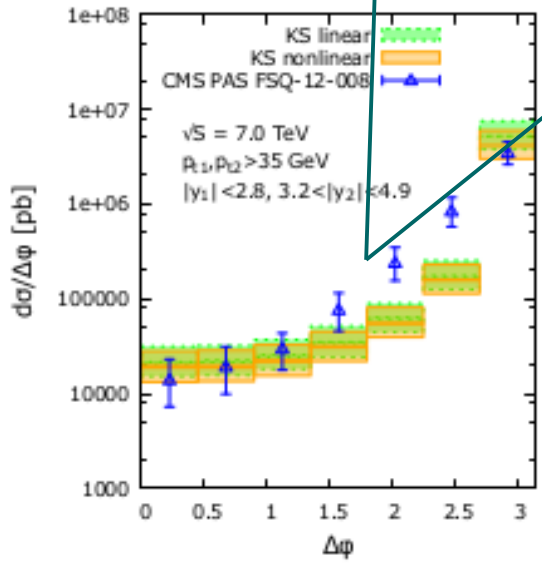


Phys.Lett. B737 (2014) 335-340
A. van Hameren, P. Kotko, K. Kutak, S. Sapeta

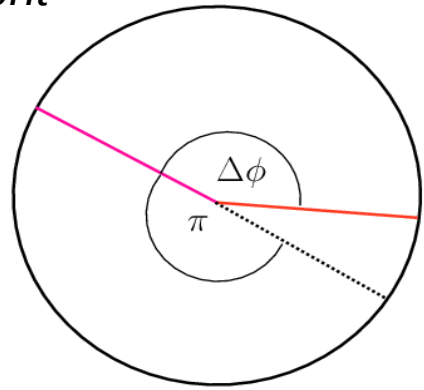
Decorelations inclusive scenario



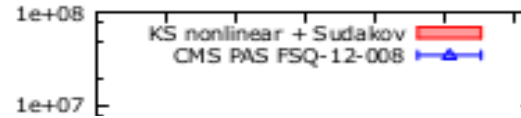
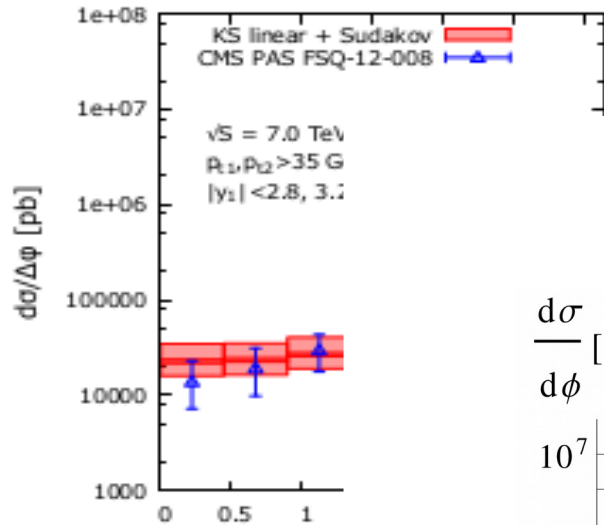
No saturation but visible Sudakov effects



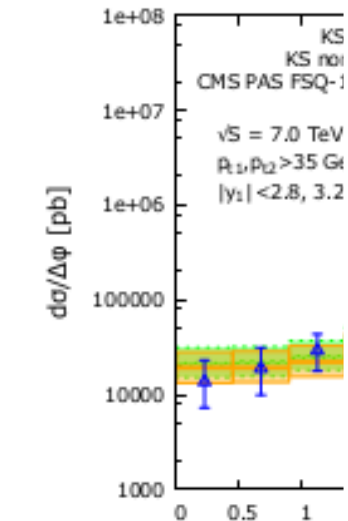
enhancement



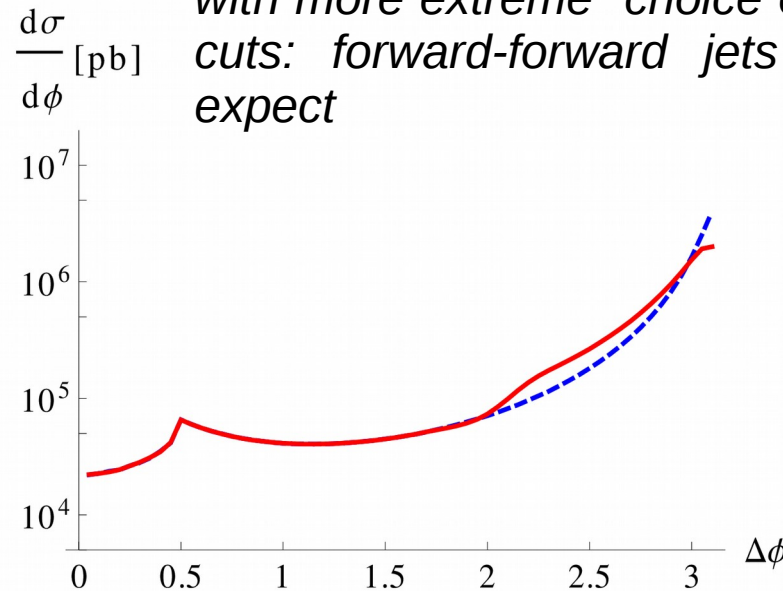
Decorelations inclusive scenario



No saturation...
Visible Sudakov effects



with more extreme choice of rapidity cuts: forward-forward jets we can expect

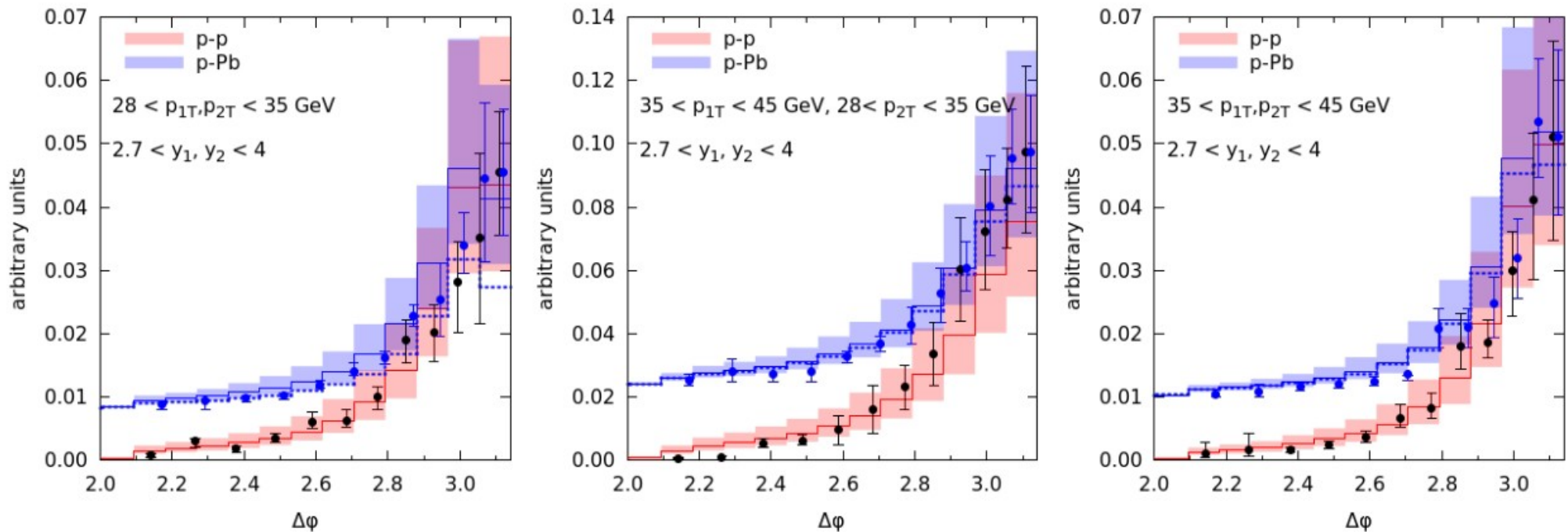


K. Kutak
Phys.Rev. D91 (2015) no.3, 034021

Signature of broadening in forward-forward dijets

ATLAS Phys.Rev. C100 (2019) no.3, 034903

A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515



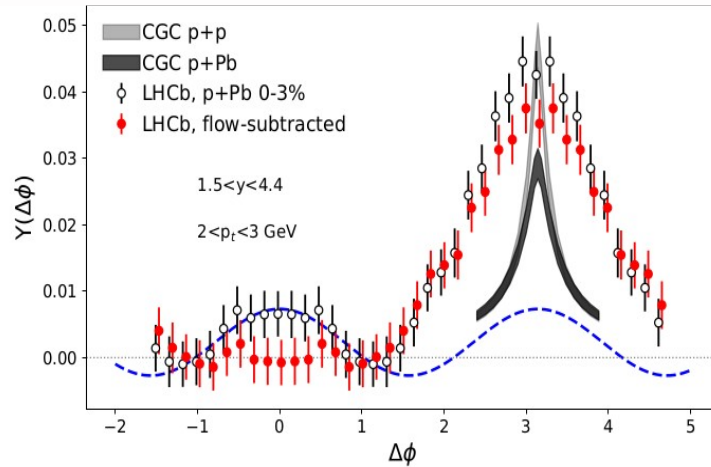
Data: number of dijets normalized to number of single inclusive jets. We can not calculate that. We can compare shapes.

Procedure: fit normalization to p-p data.

Use that both for p-p and p-Pb. Shift p-Pb data

The procedure allows for visualization of broadening

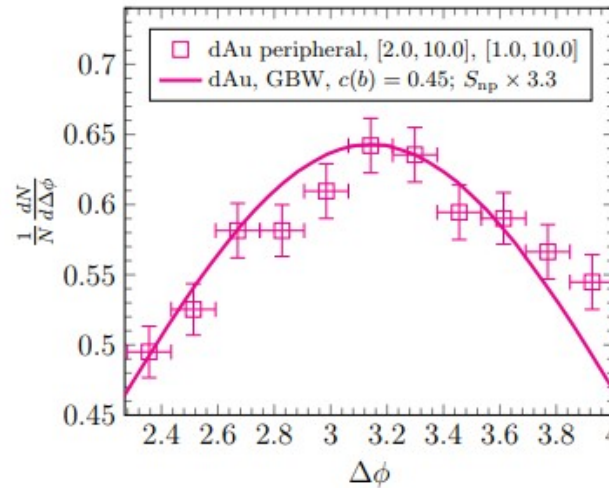
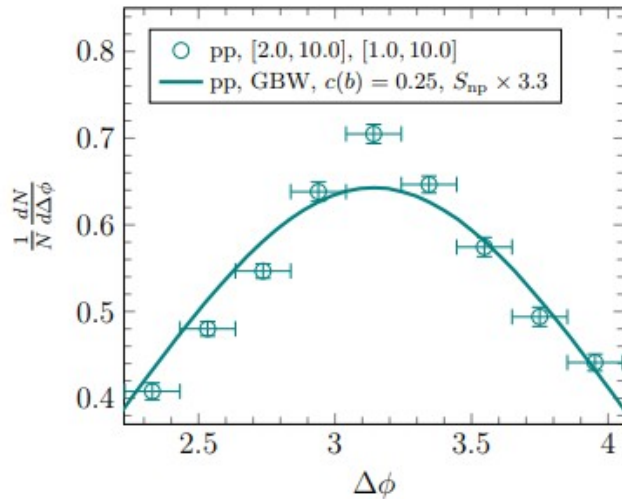
Di-hadrons production



ITMD no Sudakov

Expectation: Sudakov will broaden the distribution

G. Giacalone, C. Marquet, M. Matas
 Phys.Rev. D99 (2019) no.1, 014002

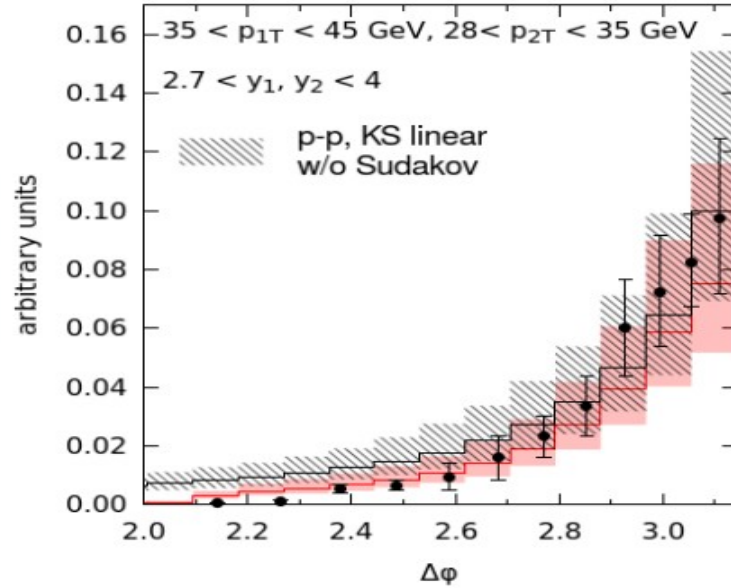
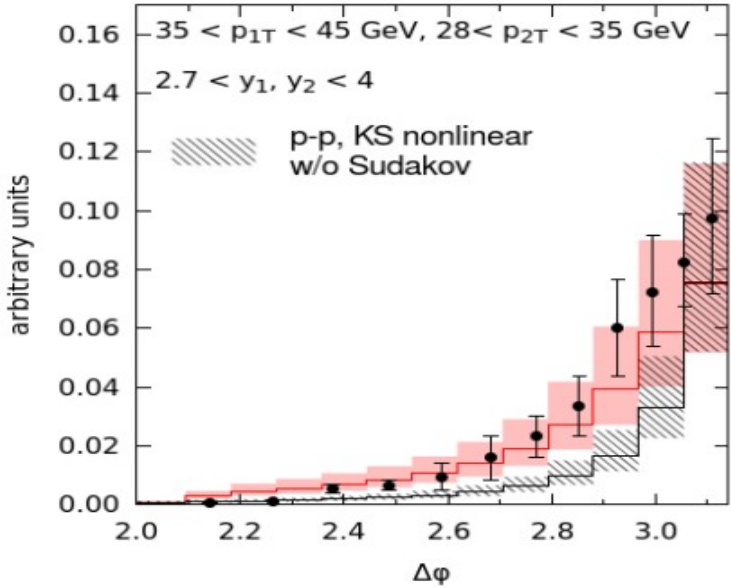


Correlation limit of CGC + Sudakov

A. Stasto, S. Wei, B. Xiao, F. Yuan
 Phys.Lett. B784 (2018) 301-306

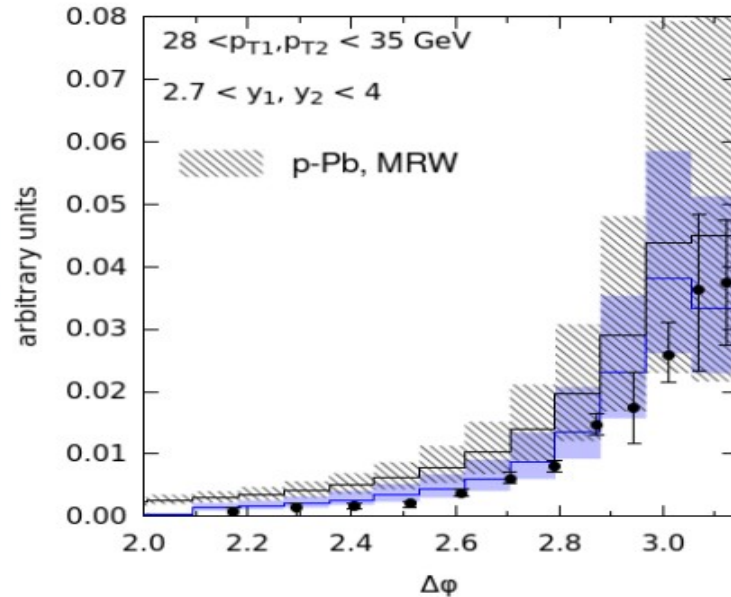
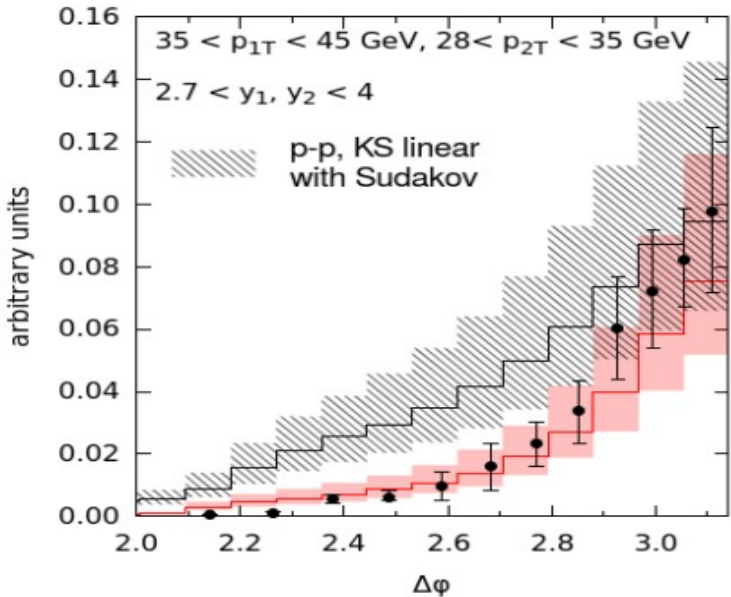
Other approaches

*nonlinearity
no Sudakov
too narrow
distribution*



*linear
no Sudakov
not too bad
but
different
shape*

*linear
too wide
Sudakov
acts too much*



*Linear +
Sudakov
Ordering in
kt
not too bad
but different
shape*

Drell-Yan and Sudakov in coordinate space

zero transverse momentum

$$\frac{d\sigma}{dQ^2 d^2 p_T} = \sum_q \int \frac{dx_1 dx_2}{x_1 x_2 S} q(x_1) q(x_2) \hat{\sigma}(q\bar{q} \rightarrow l^+ l^-) \int d^2 b_\perp e^{-p_T \cdot b_\perp}$$

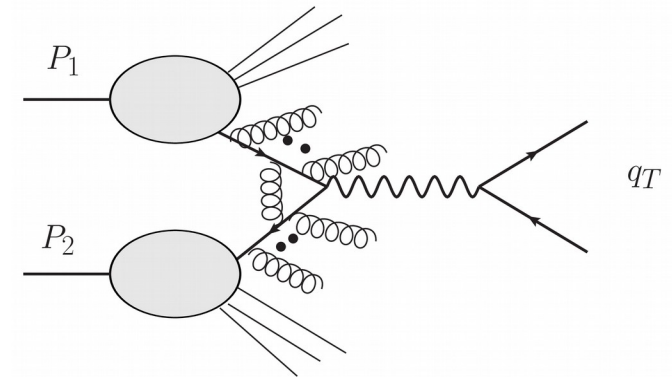
$$Y = \frac{1}{2} \ln \frac{x_1}{x_2} \quad x_1 = \frac{Q}{\sqrt{S}} e^Y \quad x_2 = \frac{Q}{\sqrt{S}} e^{-Y}$$

invariant mass

$$\frac{dP}{d^2 q_T} = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{q_T} \int_0^{1 - \frac{q_T^2}{Q^2}} dz \frac{1+z^2}{1-z} \frac{\alpha_s C_F}{\pi^2} \frac{1}{q_T} \ln \frac{Q^2}{q_T^2}$$

resummation of all soft emissions leads to

$$\frac{d\sigma}{dQ^2 d^2 p_T} = \sum_q \int d^2 b_\perp e^{-p_T \cdot b_\perp} e^{-S(b_T)} \int \frac{dx_1 dx_2}{x_1 x_2 S} q(x_1, \mu) q(x_2, \mu) \hat{\sigma}(q\bar{q} \rightarrow l^+ l^- + X)$$



process studied a lot in TMD factorization framework see also recent low x papers by
 Marquet, Wei, Xiao
 Phys.Lett.B 802 (2020) 135253
 Golec-Biernat, Stebel
 Eur. Phys. J. C (2020) 80:455

It is usually referred as the Sudakov form factor for which can be interpreted as the probability for emitting no gluons with transverse momentum greater than p_T . The momenta of emitted gluons should add up to p_T the transverse momentum conservation for arbitrary number of gluon radiations.

KS gluon with b space Sudakov

Before addressing the di-jets in forward-forward which are sensitive to both Sudakov and saturation we want to see what different Sudakov give.

$$\mathcal{F}_{g^*/B}^{ag \rightarrow cd}(x, k_T, \mu) = \frac{-N_c}{2\pi\alpha_s} \int \frac{d^2 b_T}{2\pi} e^{-ik_T \cdot b_T} e^{-S(\mu, b_T)_{Sud}} \nabla_{b_T}^2 (1 - N(x, b_T))$$

Nucl.Phys. B921 (2017) 104-126
B. Xiao, F. Yuan, J. Zhou.

Phys. Rev. D 88, 114010 (2013)
A. H. Mueller, Bo-Wen Xiao, Feng Yuan

$$S_p^{gg \rightarrow gg}(Q, b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[4C_A \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) - 3C_A \beta_0 \frac{\alpha_s}{\pi} \right]$$

A. Stasto, S. Wei, B. Xiao, F. Yuan,
Phys.Lett.B 784 (2018) 301-306

hard scale

$$S_p^{qg \rightarrow qg}(Q, b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[2(C_F + C_A) \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) - \left(\frac{3}{2}C_F + C_A \beta_0\right) \frac{\alpha_s}{\pi} \right]$$

Arises due to incomplete cancelation
of collinear singularities

Arises due to incomplete cancelation
of soft divergencies

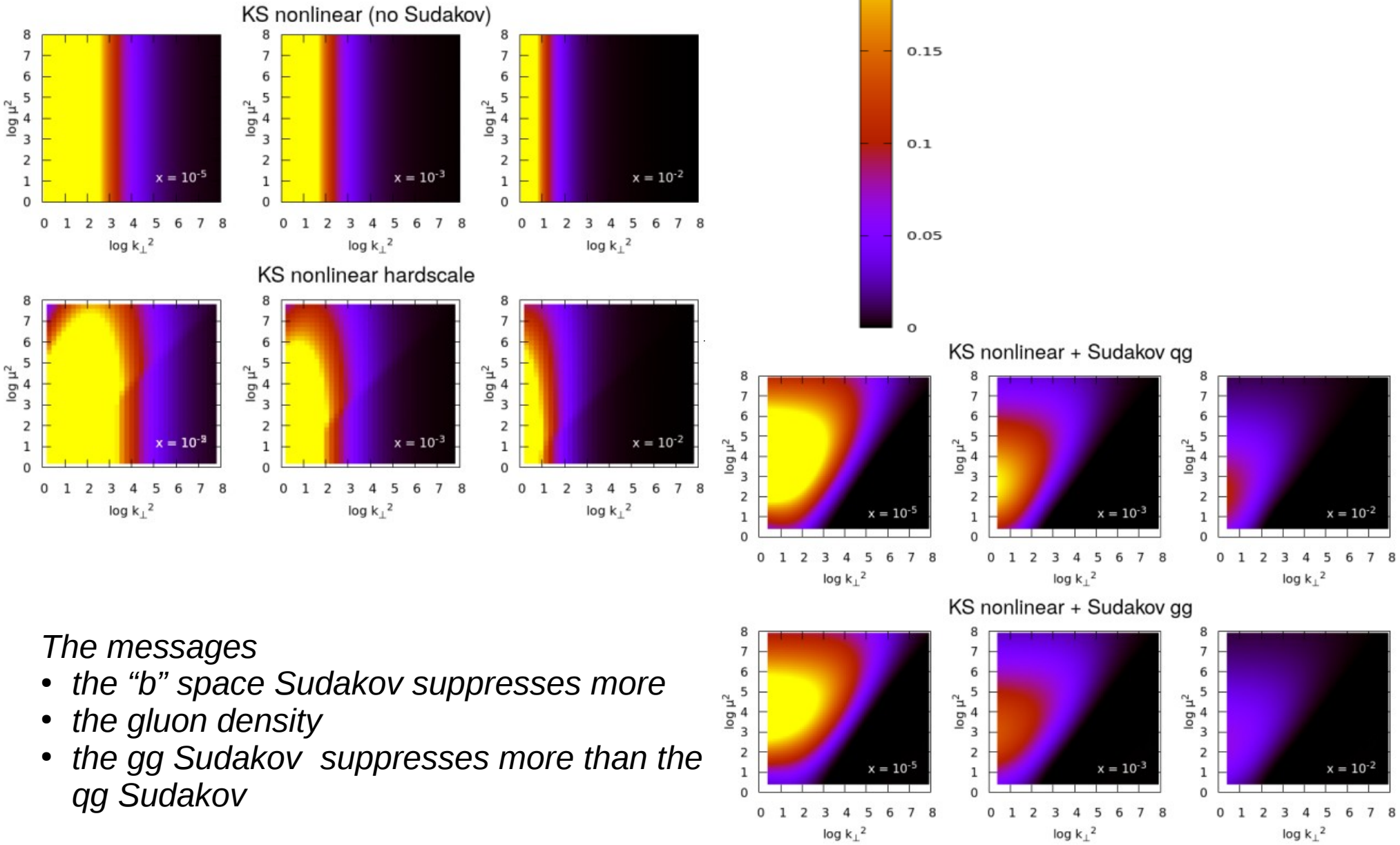
$$\mathcal{F}_{g^*/B}^{ab \rightarrow cd}(x, k_\perp, \mu) = \int db_\perp \int dk'_\perp b_\perp k'_\perp J_0(b_\perp k'_\perp) J_0(b_\perp k_\perp) \mathcal{F}_{g^*/B}(x, k'_\perp) e^{-S_{Sud}^{ab \rightarrow cd}(\mu, b_\perp)}$$

van Hameren, Kotko, Kutak, Sapeta
to appear soon

Solution of BK with kinematical constraint and nonsingular pieces of DGLAP splitting function

KS gluon with Sudakov

van Hameren, Kotko, Kutak, Sapeta
2010.13066

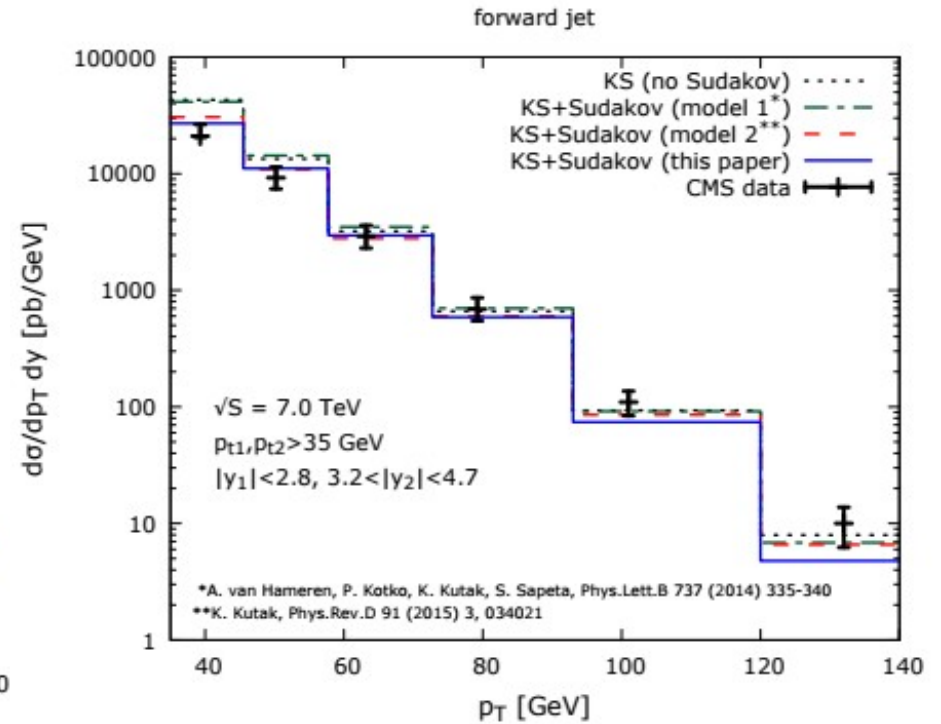
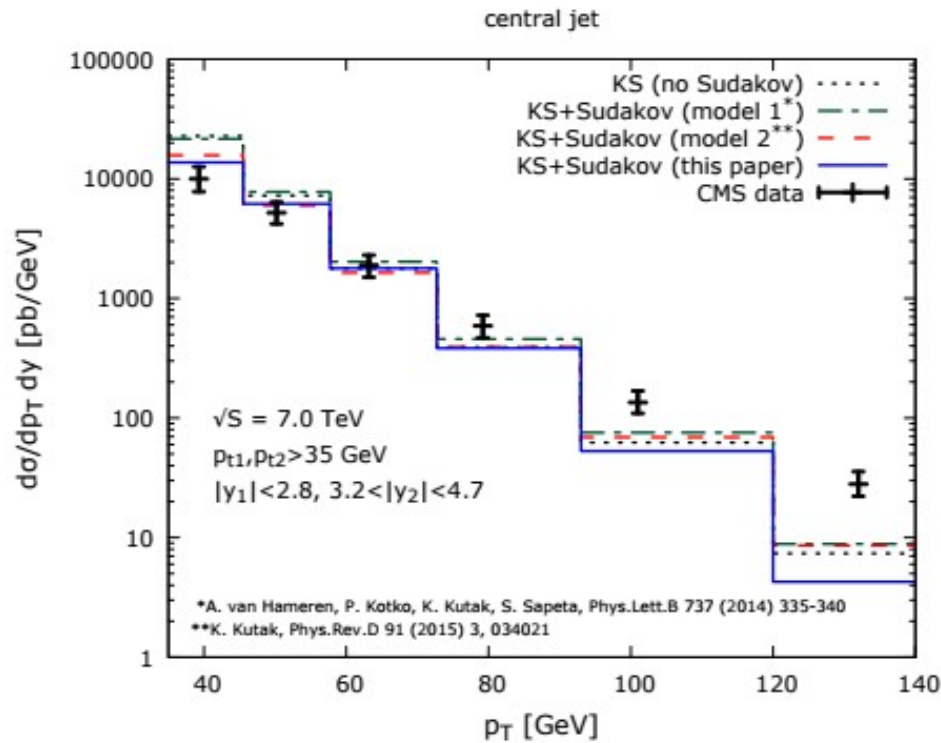


The messages

- the “b” space Sudakov suppresses more
- the gluon density
- the gg Sudakov suppresses more than the qq Sudakov

P_T spectra – no Sudakov vs. “b” space Sudakov vs. * model vs. ** model

van Hameren, Kotko, Kutak, Sapeta
2010.13066

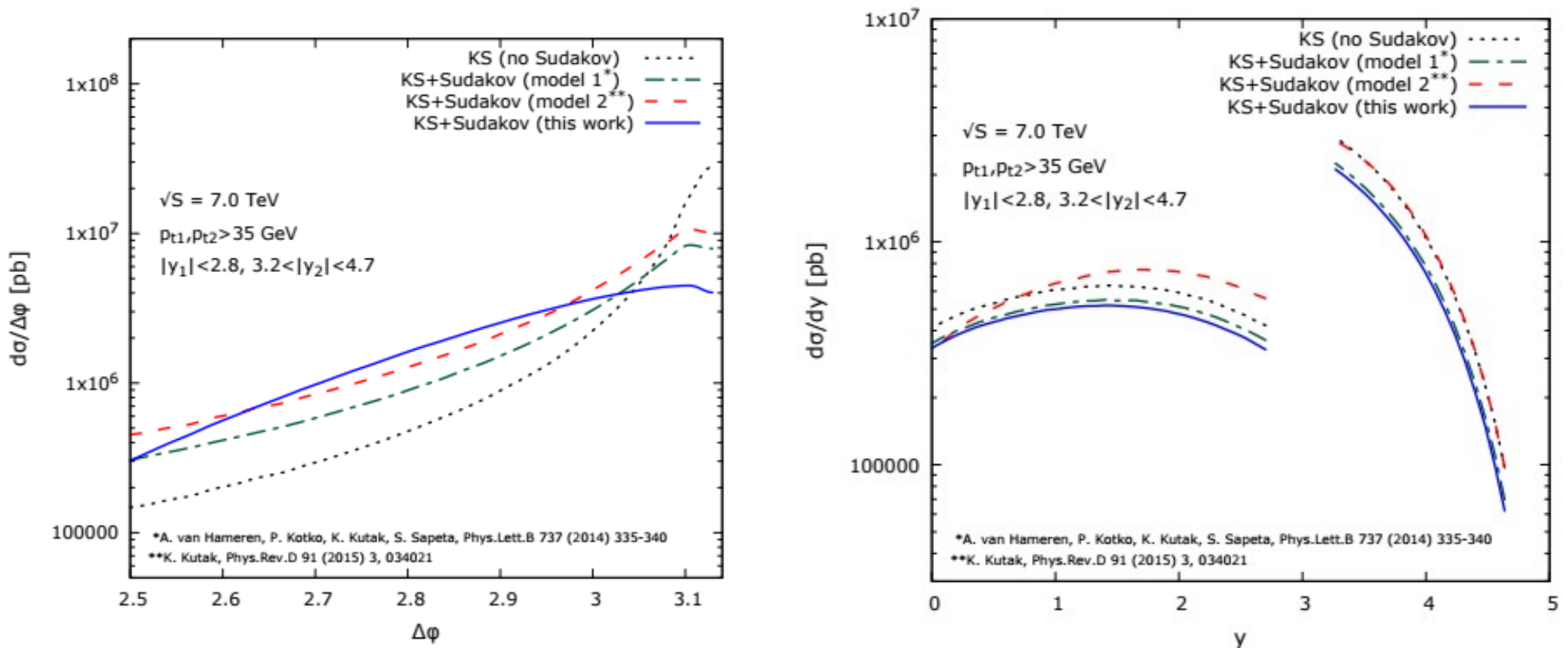


P_T spectra give similar description. Not very useful but confirm that the framework is in principle fine.

At larger p_T we expect the parton shower to play significant role.

Azimuthal angle decorrelations and rapidity distributions

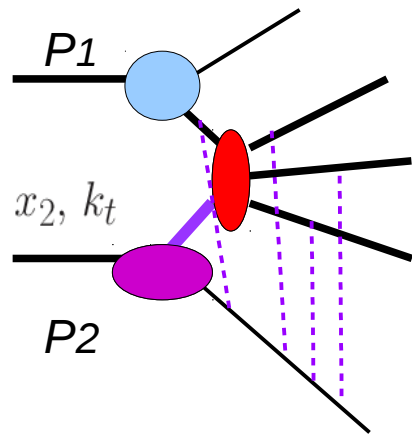
van Hameren, Kotko, Kutak, Sapeta
arxiv: 2010.13066



All approaches lead to suppression of the distribution at the back-to-back region and broadening of the cross-section. Clearly the "b" space Sudakov suppresses cross section more than the DGLAP based one. The shape of distribution away from back-to-back is very different: concave vs. convex.

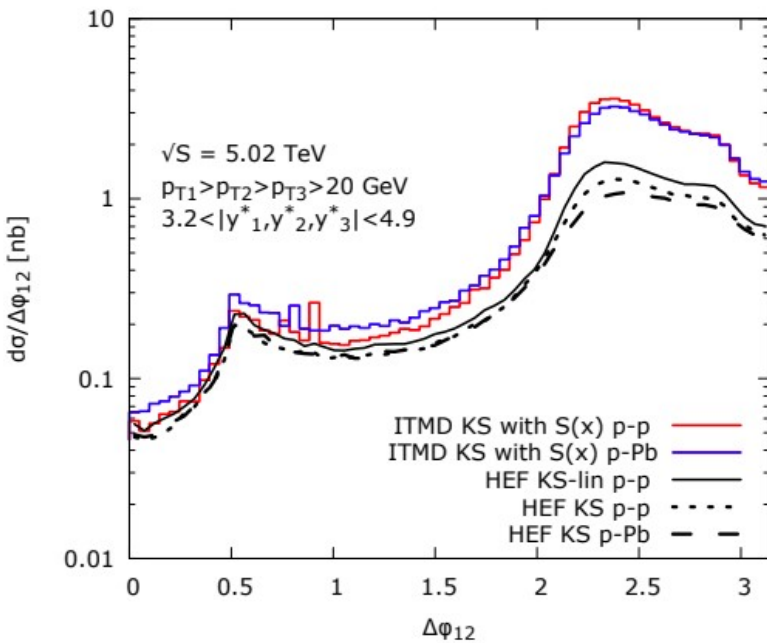
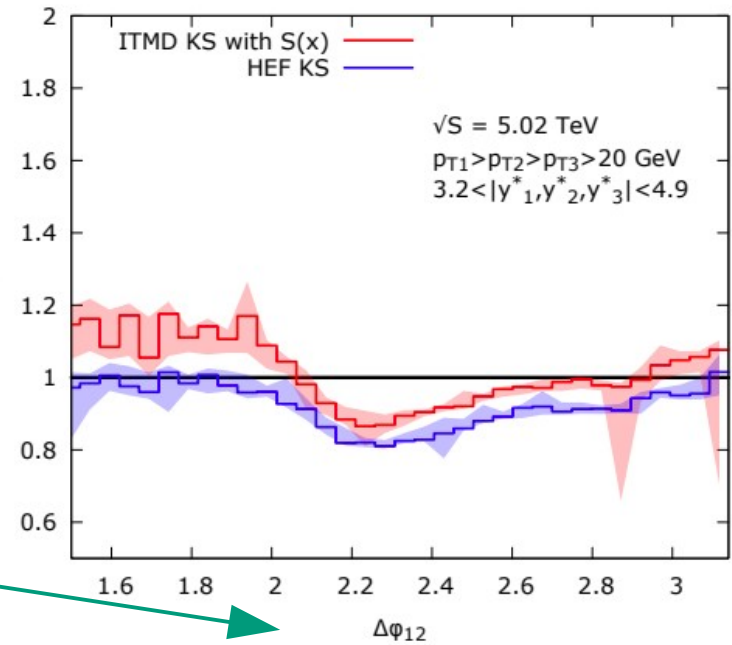
ITMD for three jets in $p+p, p+A$

ratio $p-A/p-p$

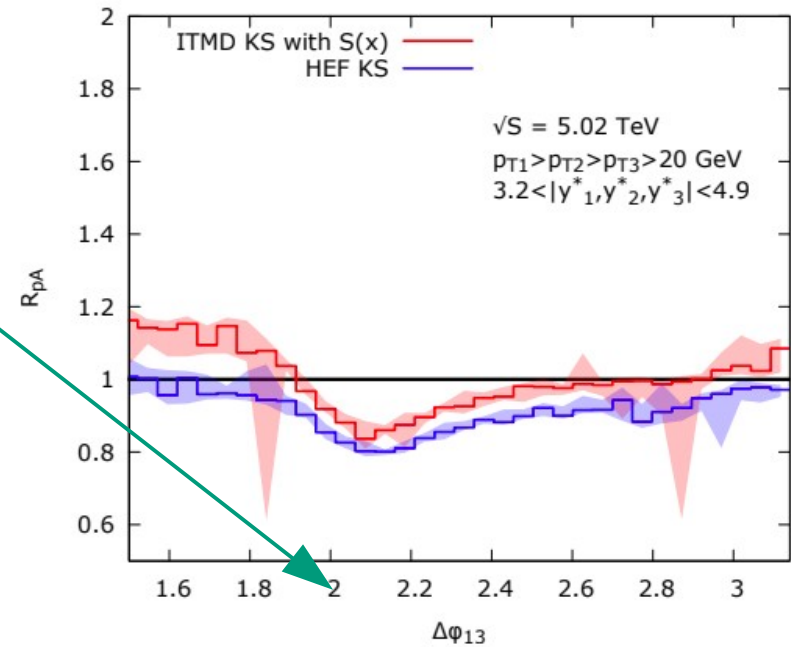


There are regions where $R_{pA} > 1$

angle between leading and subleading jet



angle between leading and softest jet

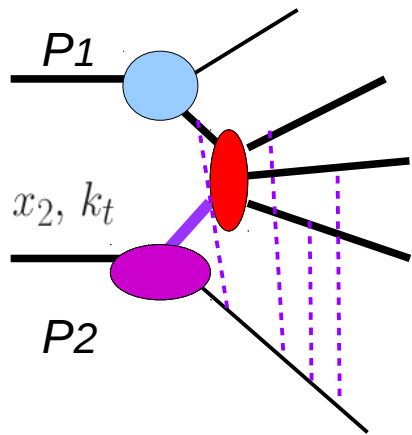


Bury, van Hameren, Kotko, Kutak
JHEP09(2020)175

See also for results in correlation limit (no k_t in ME)
Altinoluk, Boussarie, Marquet, Taels JHEP 07 (2020) 143

Iancu, Mulian Nucl.Phys.A 985 (2019) 66-127

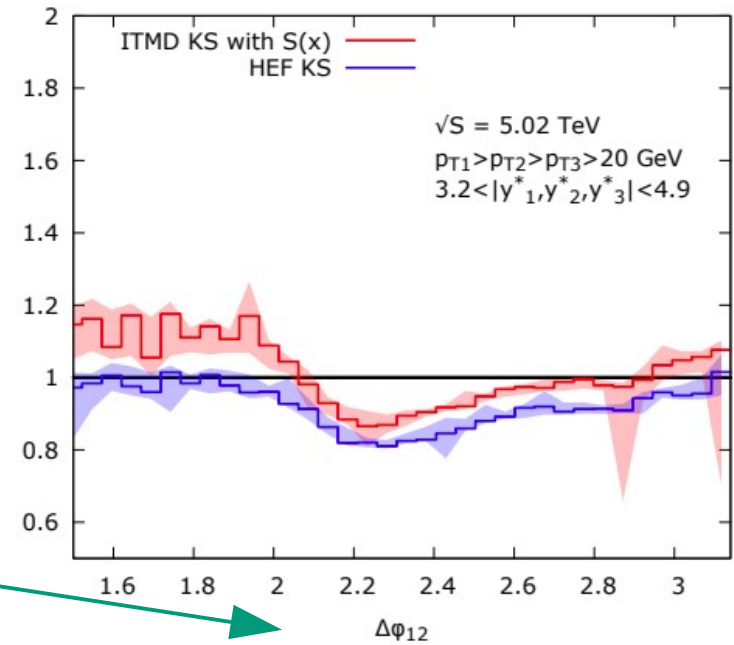
ITMD for three jets in $p+p$, $p+A$



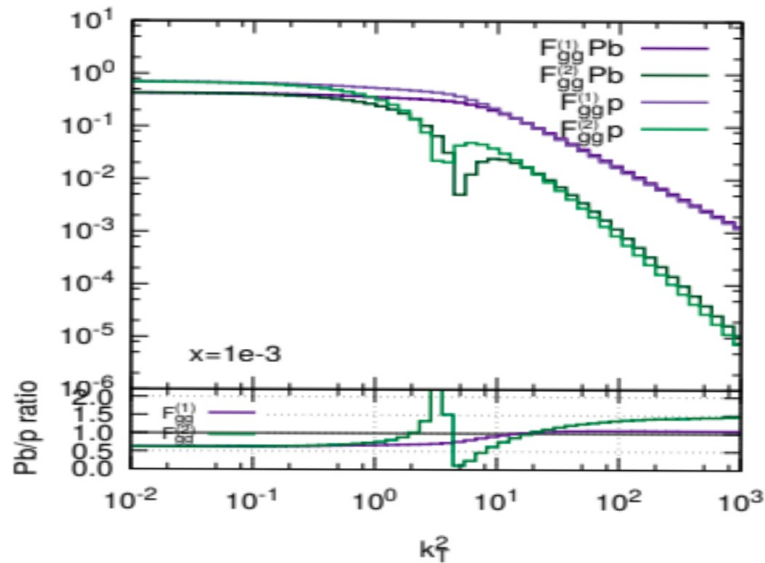
ratio $p-A/p-p$

There are regions where $R_{pA} > 1$

angle between leading and subleading jet

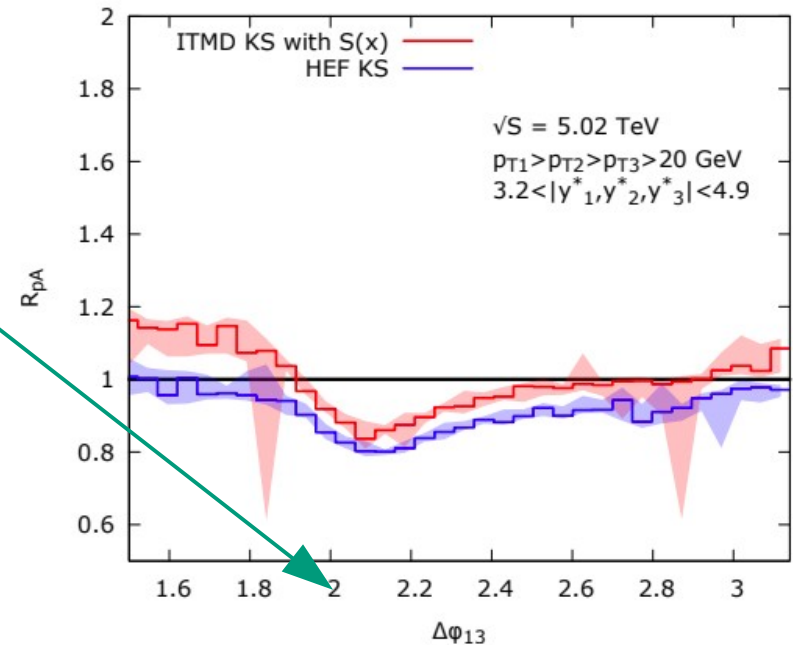


KS gluon TMDs in proton and lead



Bury, van Hameren, Kotko, Kutak
JHEP09(2020)175

angle between leading and softest jet

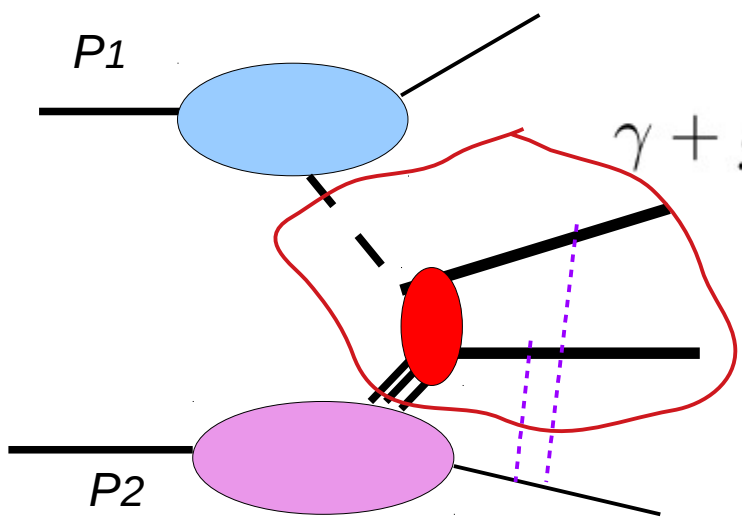


See also for results in correlation limit (no k_t in ME)
Altinoluk, Boussarie, Marquet, Taels JHEP 07 (2020) 143

Iancu, Mulian Nucl.Phys.A 985 (2019) 66-127

UPC collision of Pb-Pb

Direct relation relation to DIS

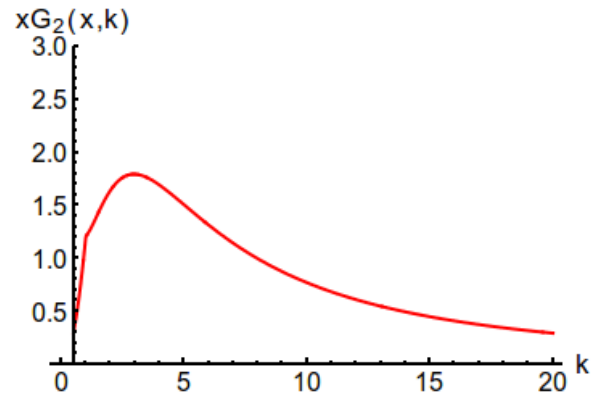


$$\gamma + g \rightarrow \bar{q}q$$

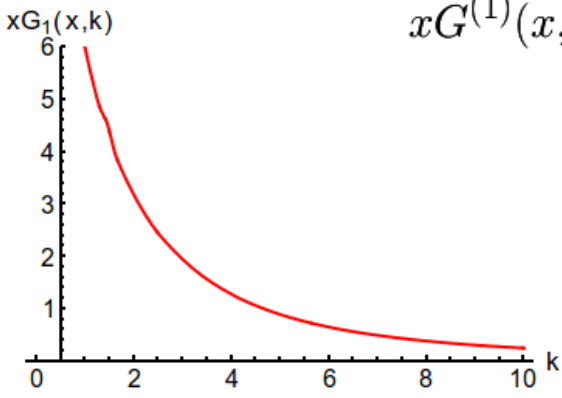
$$u\sigma_{AA \rightarrow 2jet+X} = \int dx_\gamma \frac{dN_\gamma}{dx_\gamma} d\sigma_{\gamma A \rightarrow 2jet+X}$$

$$d\sigma_{\gamma A \rightarrow 2jet+X} = \sum_{\{q,\bar{q}\}} \int \frac{dx_A}{x_A} \int d^2k_T x_A G_1(x_A, k_T) d\sigma_{\gamma g^* \rightarrow q\bar{q}}(x_A, k_T)$$

Kotko, Kutak, Sapeta, Stasto, Strikman '16



dipole gluon density



Weizacker-Williams gluon density

$$xG^{(1)}(x, \mathbf{k}_t^2) \propto \int \frac{d^2\mathbf{x}}{(2\pi)^2} e^{-i\mathbf{k}_t \cdot \mathbf{x}} \frac{(1 - S_A(x, \mathbf{x}))}{\mathbf{x}^2}$$

$$S_A(x, \mathbf{x}) = [S(x, \mathbf{x})]^2$$

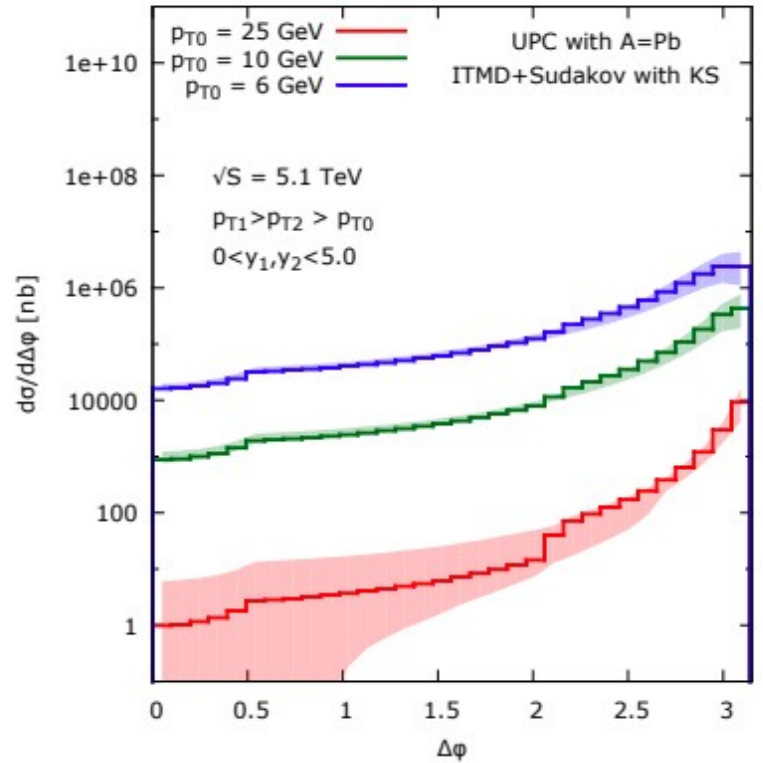
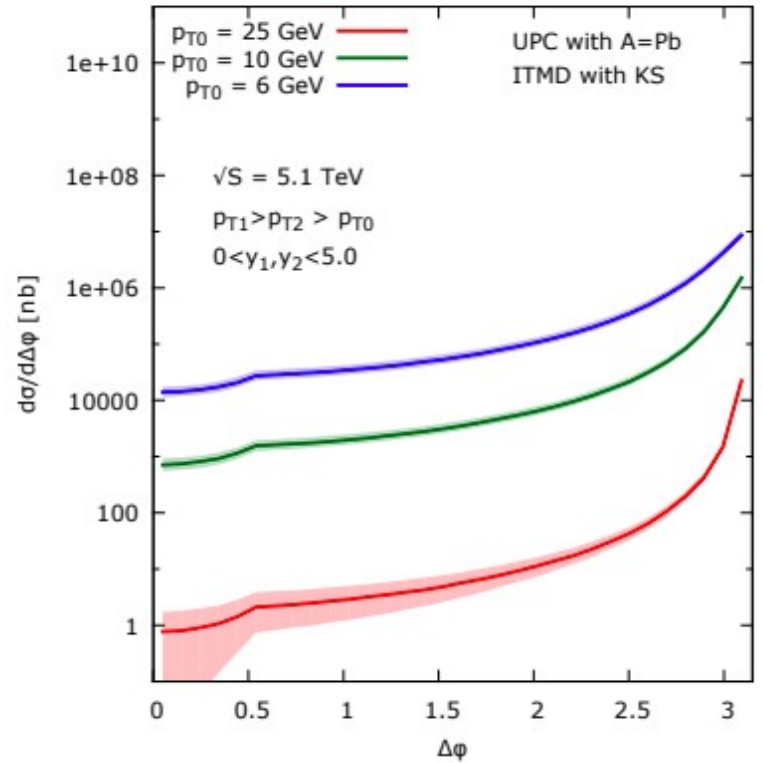
UPC collision of Pb-Pb

$$\gamma + g \rightarrow \bar{q}q$$

P_1

P_2

Kotko, Kutak



photons

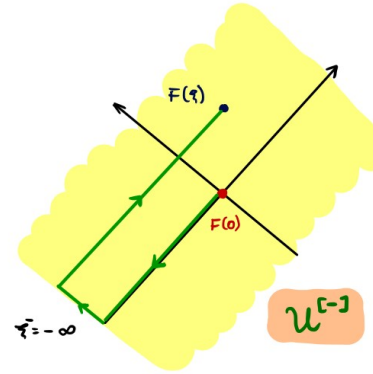
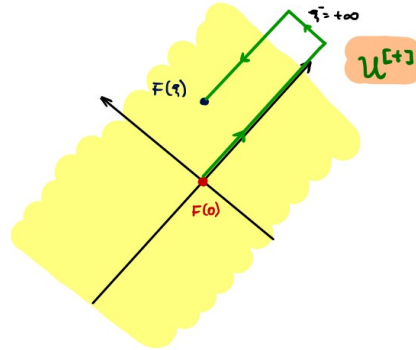
$\rightarrow q\bar{q}(x_A, k_T)$

Summary and outlook

- *We obtained results for cross-section of dijets in central-forward and forward-forward configurations*
- *For central-forward the Sudakov effects were taken into account using b space and model Sudakov*
- *The results for the decorrelations have different shape but both lead to broadening*
- *The results for three jet ITMD show that there is large potential in this observable for saturation search*
- *We have also results with Sudakov for inclusive dijet production in UPC*
- *future applications to dijets at EIC*
- *Possible NLO extension we have already forward Higgs production in hybrid approach*

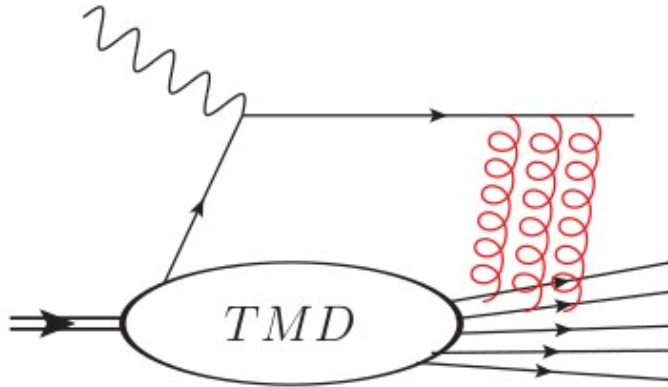
Definition of TMD – gauge links

Two basic structures arise:



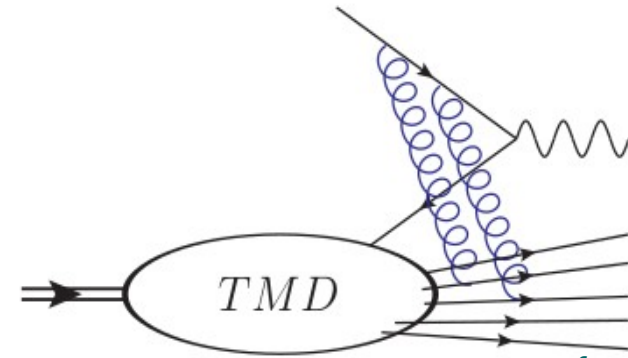
From P. Kotko,

Semi Inclusive DIS



final state interactions

Drell-Yan



initial state interactions

from R. Boussarie
Initial Stages 2019

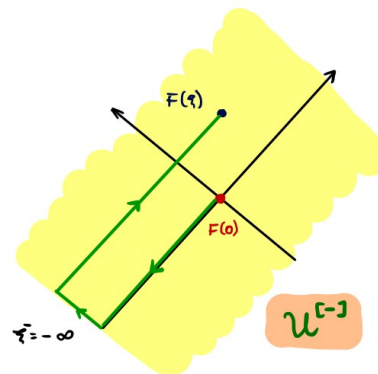
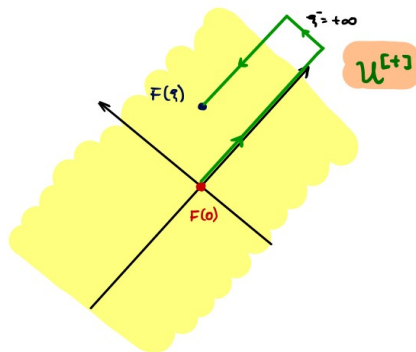
$$\Phi_q^{[+]}(x, p_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle H | \bar{\psi}(0) \mathcal{U}^{[+]} \psi(\xi) | H \rangle$$

C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

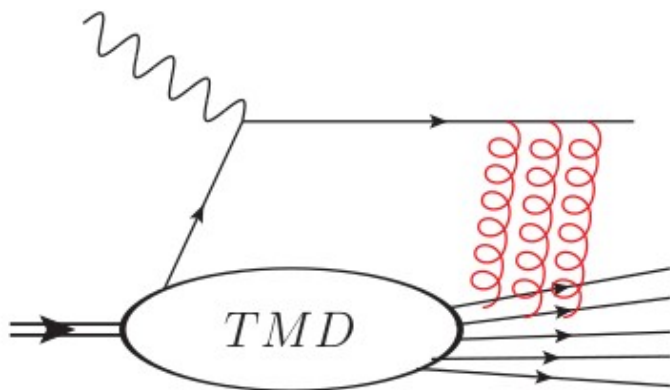
$$\mathcal{U}^{[\pm]} = U_{[(0^-, \mathbf{0}_T); (\pm\infty^-, \mathbf{0}_T)]} U_{[(\pm\infty^-, \mathbf{0}_T); (\pm\infty^-, \infty_T)]} U_{[(\pm\infty^-, \infty_T); (\pm\infty^-, \xi_T)]} U_{[(\pm\infty^-, \xi_T); (\xi^-, \xi_T)]}$$

Definition of TMD – gauge links

Two basic structures arise:

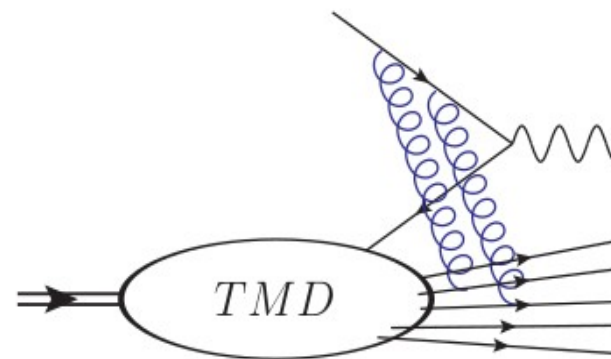


Semi Inclusive DIS



final state interactions

Drell-Yan



initial state interactions

from R. Boussarie
Initial Stages 2019

$$\Phi_q^{[+]}(x, p_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle H | \bar{\psi}(0) \mathcal{U}^{[+]} \psi(\xi) | H \rangle$$

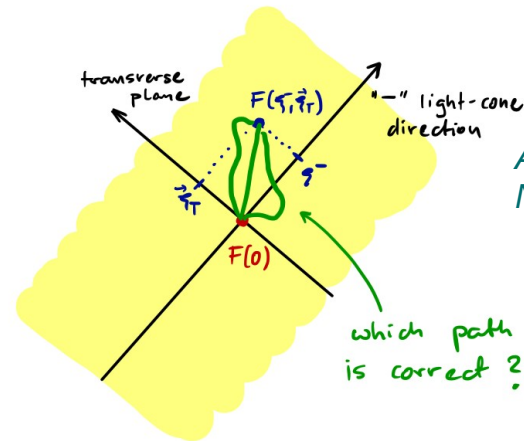
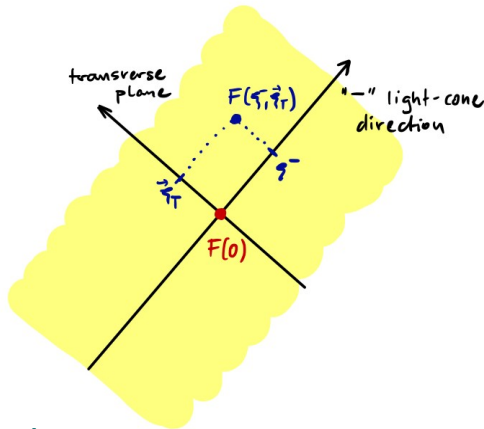
C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

Definition of TMD – gauge links

The formula for TMD gluon density

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \hat{F}^{i+}(\xi^+ = 0, \xi^-, \vec{\xi}_T) \right\} | P \rangle$$

naive definition of gluon distribution



A. Belitsky, X. Ji, F. Yuan
Nucl.Phys. B656 (2003) 165-198

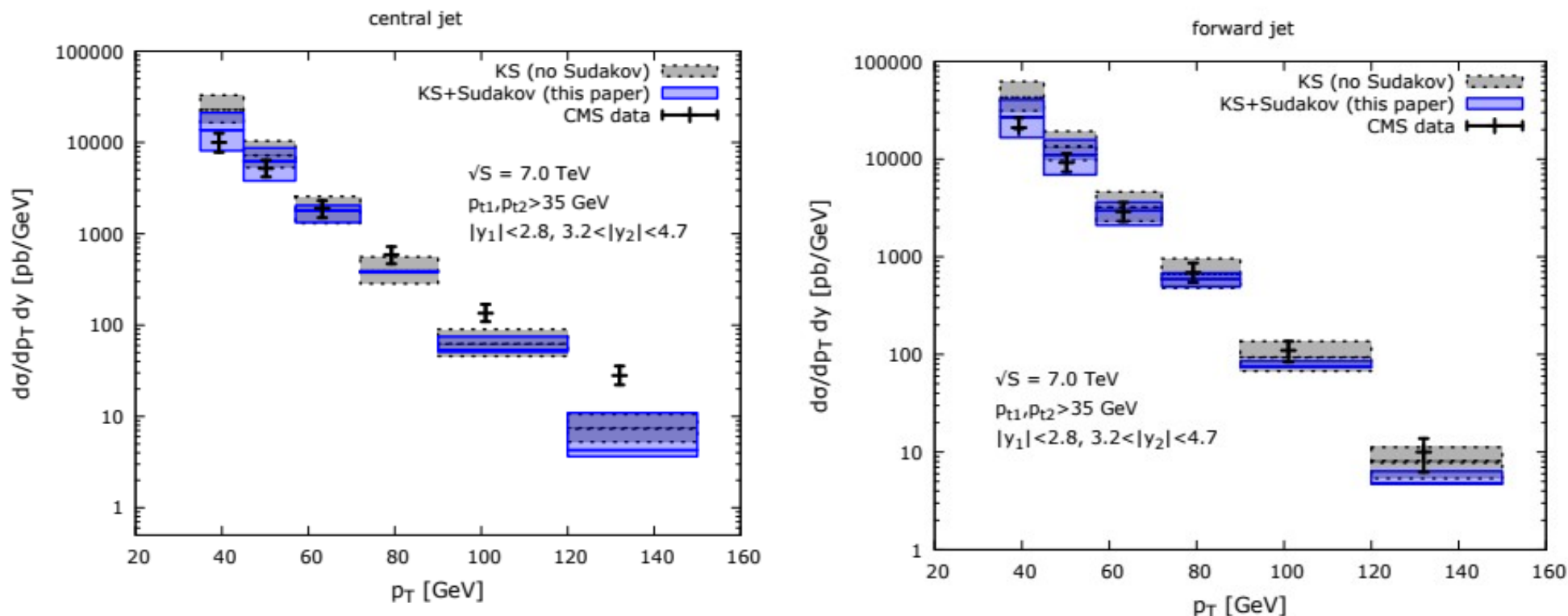
From P. Kotko,

The generalization is achieved via gauge link which accounts for exchange of collinear gluons between the soft and hard parts renders the gluon density gauge invariant....

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P | \text{Tr} \left\{ \hat{F}^{i+}(0) \mathcal{U}_{C_1} \hat{F}^{i+}(\xi) \mathcal{U}_{C_2} \right\} | P \rangle$$

P_T spectra no-Sudakov vs. “b” space Sudakov and uncertainty

van Hameren, Kotko, Kutak, Sapeta
2010.13066



Good rescription of forward jet – probes small k_t part of the gluon density

Central jet requires Monte Carlo parton shower corrections - we do not account for them now since there is no shower available for KS gluons

Calculations have been done using KaTie [van Hameren CPC. 224 \(2018\) 371-380](#) and LxJet [Kotko Monte Carlo programs](#)

BACKUP

$$O = \frac{\sigma}{W} \sum_i w_i F_i^O(X_i)$$

observable total cross-section

function defining observable: cuts etc.

$$O = \frac{\sigma}{W} \left[\sum_i w_i F_i^O(X_i) \Theta(\mu_i > k_{Ti}) + \sum_j w_j F_j^O(X_j) \Theta(k_{Tj} > \mu_j) \right]$$

$$W = \sum_i w_i \quad \text{total weight}$$

$$\bar{O} = \frac{\sigma}{\bar{W}} \left[\sum_i w_i \Delta(\mu_i, k_{Ti}) F_i^O(X_i) \Theta(\mu_i > k_{Ti}) + \frac{\widetilde{W}}{W} \sum_j w_j F_j^O(X_j) \Theta(k_{Tj} > \mu_j) \right]$$

of order 1

$$\bar{W} = \sum_i w_i \Delta(\mu_i, k_{Ti}) \Theta(\mu_i > k_{Ti}) + \frac{\widetilde{W}}{W} \sum_j w_j \Theta(k_{Tj} > \mu_j)$$

modified weight