Parton Distribution Functions

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- introduction and overview
- LHC benchmark cross sections: how do the various sets compare?
- issues and outlook

in collaboration with Alan Martin, Robert Thorne and Graeme Watt

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introduction and overview

parton distribution functions

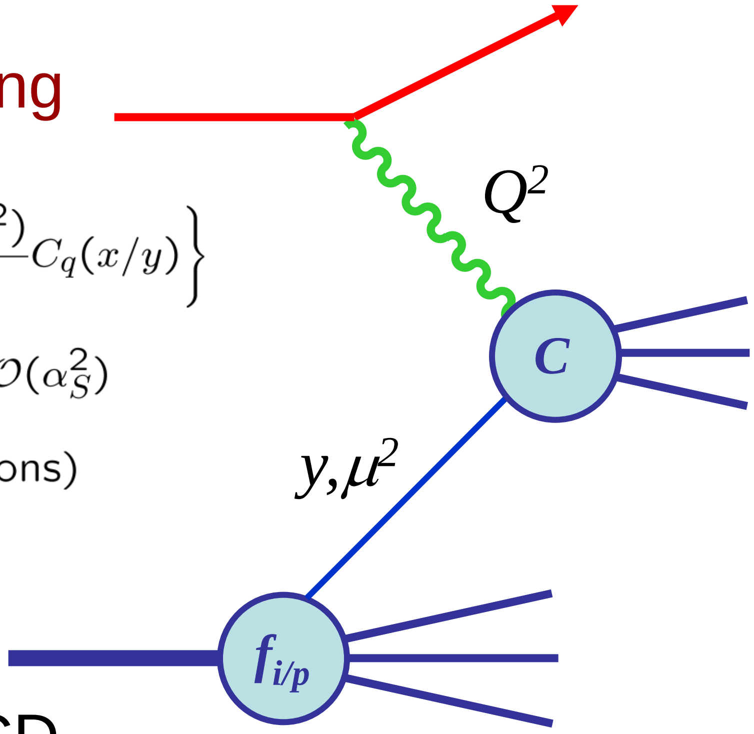
$$f_{i/A}(x, Q^2)$$

- introduced by Feynman (1969) in the *parton model*, to explain Bjorken scaling in deep inelastic scattering data; interpretation as probability distributions

- according to the QCD *factorisation theorem* for inclusive hard scattering processes, universal distributions containing long-distance structure of hadrons; related to parton model distributions at leading order, but with logarithmic scaling violations (DGLAP)

- key ingredients for Tevatron and LHC phenomenology

for example, in Deep Inelastic Scattering



$$\begin{aligned}\frac{1}{x}F_2^{lp}(x,Q^2) \;&=\; x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi}C_q(x/y)\right\} \\ &+\; x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2)C_g(x/y) + \mathcal{O}(\alpha_S^2) \\ &+\; \mathcal{O}(1/Q^2) \;\; \text{(higher twist, mass corrections)}\end{aligned}$$

where the scale dependence of the parton distributions is calculable in QCD perturbation theory

$$\mu^2\frac{\partial}{\partial\mu^2}f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi}\sum_j \int_x^1 \frac{dy}{y} f_j(y,\mu^2)P_{ij}(x/y,\alpha_S(\mu^2))$$

Dokshitzer
Gribov
Lipatov
Altarelli
Parisi

… and $f_i(x,\mu_0^2)$ determined from

- lattice QCD (in principle)
- fits to data (in practice) ⇦

how pdfs are obtained*

- choose a factorisation scheme (e.g. $\overline{\text{MS}}$), an order in perturbation theory (LO, NLO, NNLO) and a ‘starting scale’ Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at Q_0, e.g.

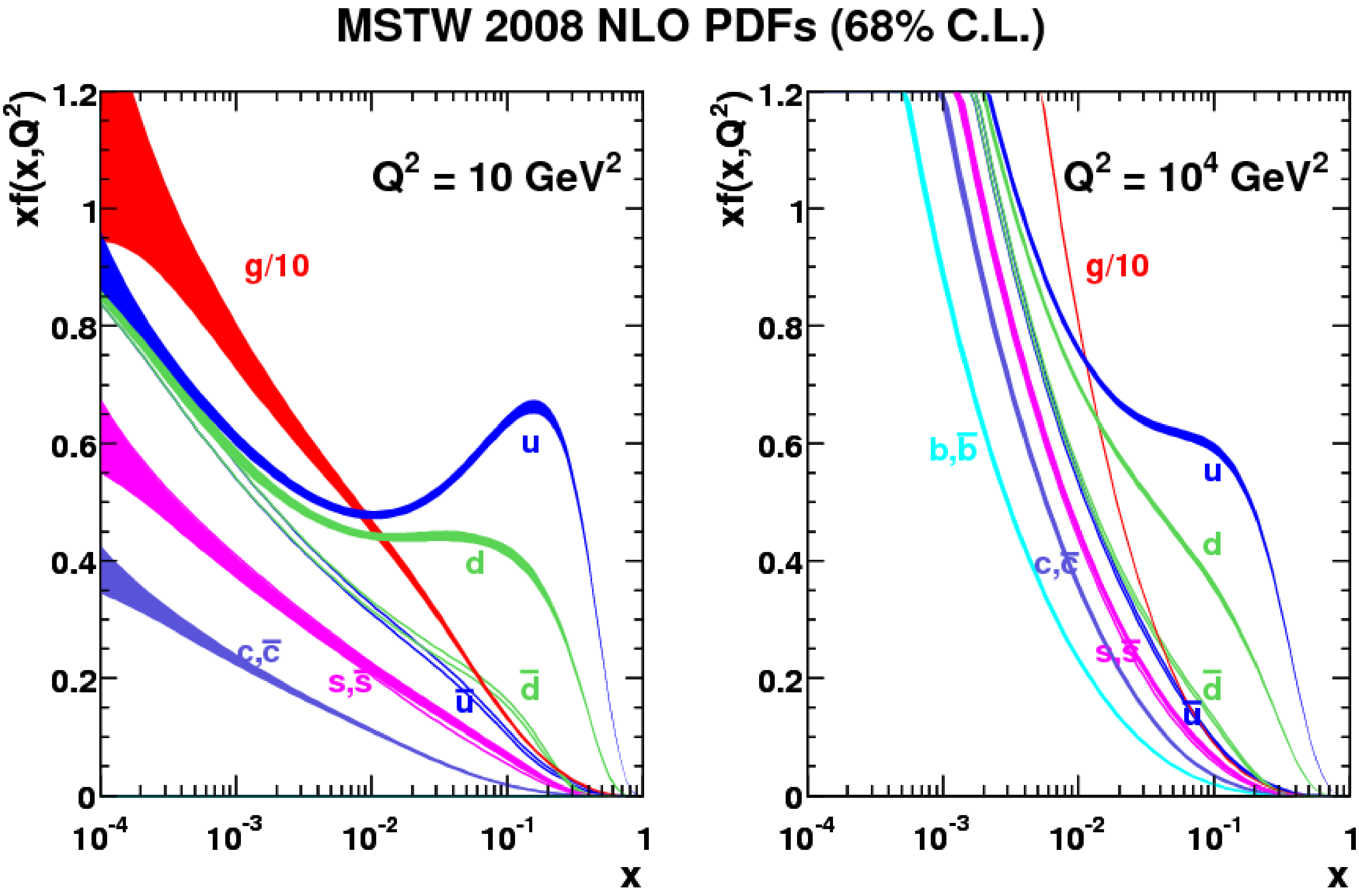
$$f_i(x,Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1-x)^{d_i}$$

- solve DGLAP equations to obtain the pdfs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output ‘global fits’ are available ‘off the shelf”, e.g.

```
SUBROUTINE PDF(X,Q,U,UBAR,D,DBAR,…,BBAR,GLU)
```

input | output

*traditional method


MSTW 2008 NLO PDFs (68% C.L.)
$xf(x,Q^2)$
$Q^2 = 10\ \mathrm{GeV}^2$
g/10
u
d
$\bar{d}$
$\bar{u}$
$s,\bar{s}$
$c,\bar{c}$
$Q^2 = 10^4\ \mathrm{GeV}^2$
$b,\bar{b}$
0
0.2
0.4
0.6
0.8
1
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

the pdf industry

- many groups now extracting pdfs from ‘global’ data analyses (MSTW, CTEQ, NNPDF, …)

- broad agreement, but differences due to
  - choice of data sets (including cuts and corrections)
  - treatment of data errors
  - treatment of heavy quarks (s,c,b)
  - order of perturbation theory
  - parameterisation at Q_0
  - theoretical assumptions (if any) about:
    - flavour symmetries
    - $x \rightarrow 0,1$ behaviour
    - …

HERA-DIS
FT-DIS
Drell-Yan
Tevatron jets
Tevatron W,Z
other

examples of data sets used in fits*

Data set	$N_{\text{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^{\pm}p$ F_2^{charm}	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^{\pm}p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

Data set	$N_{\text{pts.}}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	**2743**

red font = new wrt MRST2006 fit

*MSTW2008

recent global or quasi-global pdf fits

pdfs	authors	arXiv
ABKM	S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others	0908.3128, 0908.2766, …
CTEQ	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, …
GJR	M. Glück, P. Jimenez-Delgado, E. Reya, and others	0909.1711, 0810.4274, …
HERAPDF	H1 and ZEUS collaborations	1006.4471, 0906.1108, …
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1006.2753, 0905.3531, 0901.0002, …
NNPDF	R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others	1005.0397, 1002.4407, 0912.2276, 0906.1958, …

	MSTW08	CTEQ6.6[x]	NNPDF2.0	HERAPDF1.0	ABKM09	GJR08
HERA DIS	✓	✓	✓*	✓*	✓	✓
F-T DIS	✓	✓	✓	✘	✓	✓
F-T DY	✓	✓	✓	✘	✓	✓
TEV W,Z	✓	✓+	✓	✘	✘	✘
TEV jets	✓	✓+	✓	✘	✘	✓
GM-VFNS	✓	✓	✘	✓	✘	✘
NNLO	✓	✘	✘	✘	✓	✓

+ Run 1 only

* includes new combined H1-ZEUS data → 1 – 2.5% increase in quarks at low x (depending on procedure), similar effect on $\alpha_S(M_Z^2)$ if free and somewhat less on gluon; more stable at NNLO (MSTW prelim.)

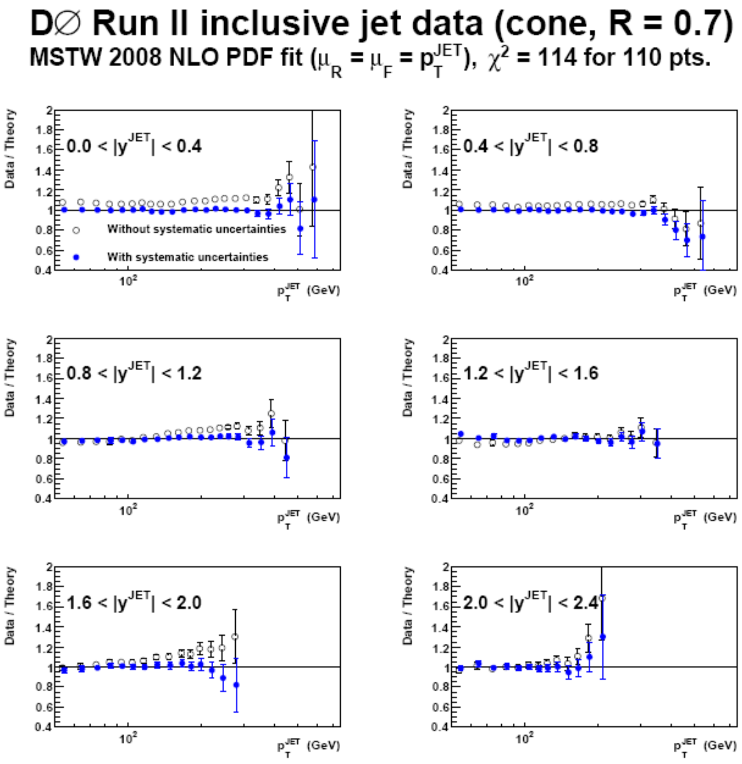
[x] New (July 2010) CT10 includes new combined H1-ZEUS data + Run 2 jet data + extended gluon parametrisation + … → more like MSTW08

impact of Tevatron jet data on fits

- a distinguishing feature of pdf sets is whether they use (MRST/MSTW, CTEQ, NNPDF, GJR,…) or do not use (HERAPDF, ABKM, …) Tevatron jet data in the fit: the impact is on the *high-x gluon*

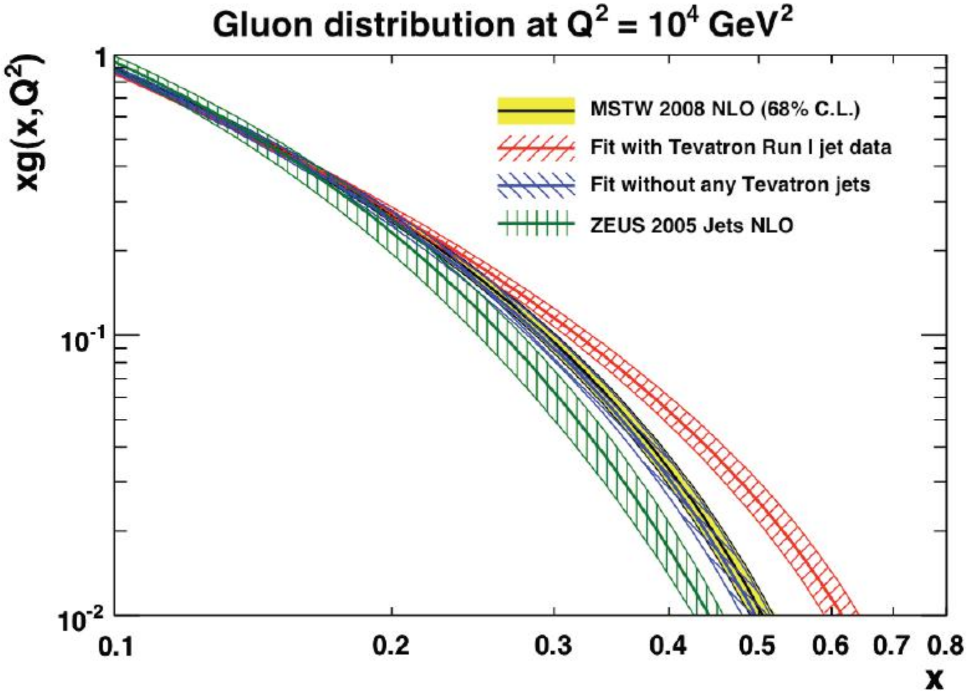
  (Note: Run II data requires slightly softer gluon than Run I data)

- the (still) missing ingredient is the full NNLO pQCD correction to the cross section, but not expected to have much impact in practice [Kidonakis, Owens (2001)]



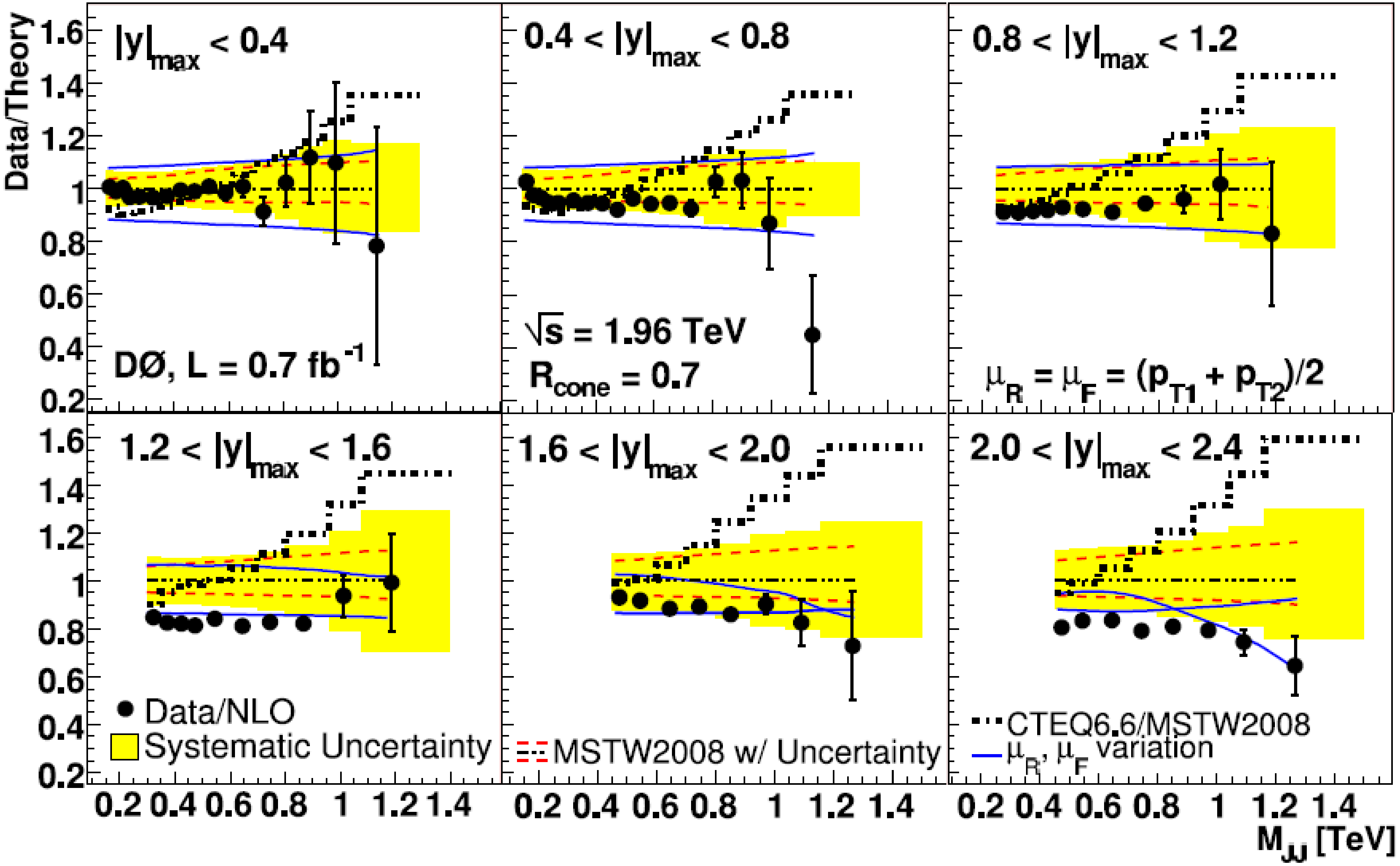
DØ Run II inclusive jet data (cone, R = 0.7)

MSTW 2008 NLO PDF fit ($\mu_R = \mu_F = p_T^{JET}$), χ^2 = 114 for 110 pts.



Gluon distribution at $Q^2 = 10^4$ GeV^2

dijet mass distribution from D0



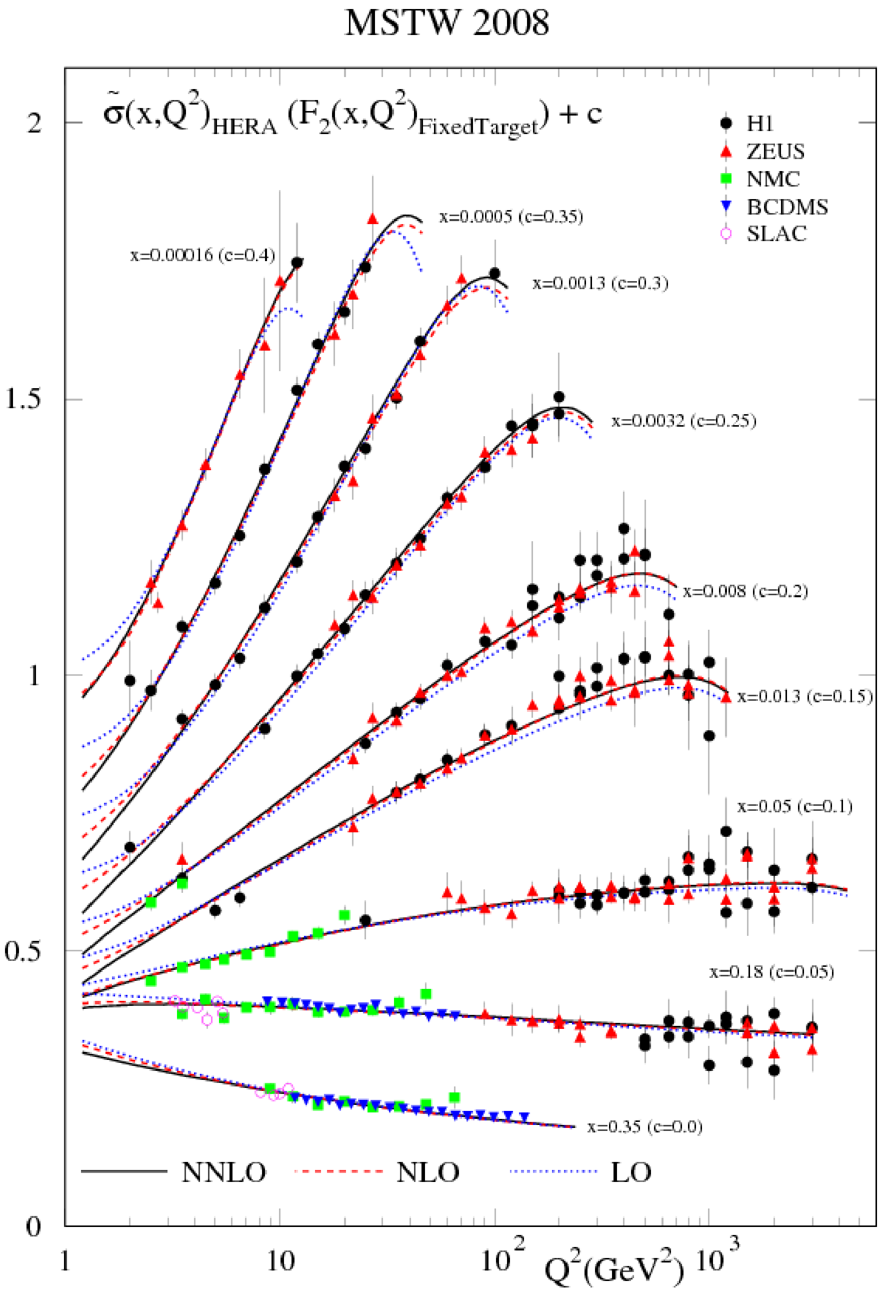
D0 collaboration: arXiv:1002.4594

LO *vs* NLO *vs* NNLO?

in the MSTW2008 fit

χ^2_{global} /dof = 3066/2598 (LO)
2543/2699 (NLO)
2480/2615 (NNLO)

LO evolution too slow at small x;
NNLO fit marginally better than NLO



Note:

- an important ingredient missing in the full NNLO global pdf fit is the NNLO correction to the Tevatron high E_T jet cross section
- LO can be improved (e.g. LO*) for MCs by adding K-factors, relaxing momentum conservation, etc.

pdf uncertainties

- most groups produce ‘pdfs with errors’
- typically, 20-40 ‘error’ sets based on a ‘best fit’ set to reflect 1σ variation of all the parameters* $\{A_i, a_i, \ldots, \alpha_S\}$ inherent in the fit
- these reflect the uncertainties on the data used in the global fit (e.g. $\delta F_2 \approx 3\% \rightarrow \delta u \approx 3\%$)
- however, there are also systematic pdf uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed (see earlier slide)

* e.g. $f_i(x, Q_0^2) = A_i x^{a_i}[1 + b_i\sqrt{x} + c_i x](1 - x)^{d_i}$

determination of best fit and uncertainties

- MSTW08 — 20 eigenvectors. Due to slight incompatibility of different sets (and perhaps to some extent parametrisation inflexibility) use 'dynamical tolerance' with inflated $\Delta\chi^2$ of 5 - 20 for eigenvectors

- CTEQ6.6 — 22 eigenvectors. Inflated $\Delta\chi^2=50$ for 1 sigma for eigenvectors (no normalization uncertainties in CTEQ6.6, *cf.* CT10)

- HERAPDF2.0 — 9 eigenvectors, use $\Delta\chi^2=20$. Additional model and parametrisation uncertainties

- ABKM09 — 21 parton parameters, use $\Delta\chi^2=1$

- GJR08 — 12 parton parameters. Use $\Delta\chi^2=20$. Impose strong theory ('dynamical parton') constraint on input form of pdfs.

Note: NNPDF2.0 create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set → uncertainty determined by spread of replicas. Direct relationship to χ^2 in global fit not trivial.

determination of best fit and uncertainties contd.

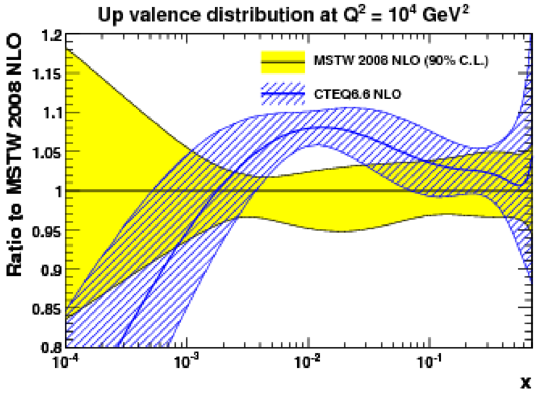
- NNPDF and MSTW (due to extra parameters) have more complicated shape for gluon at smaller x and bigger small-x uncertainty

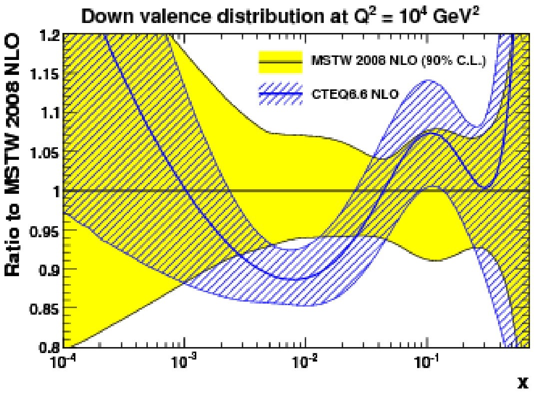
- choice of parametrisation leads to bigger very high-x gluon uncertainty for CTEQ

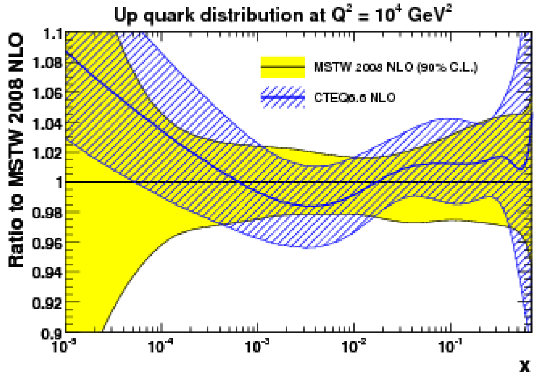
- different theory assumptions in strange quark pdf leads to vastly different uncertainties — MSTW small, NNPDF large; feeds into other ‘light’ quarks

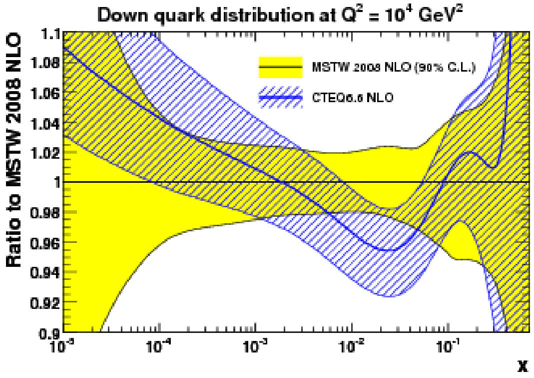
- perhaps surprisingly all get rather similar uncertainties for pdfs and predicted cross sections — see later

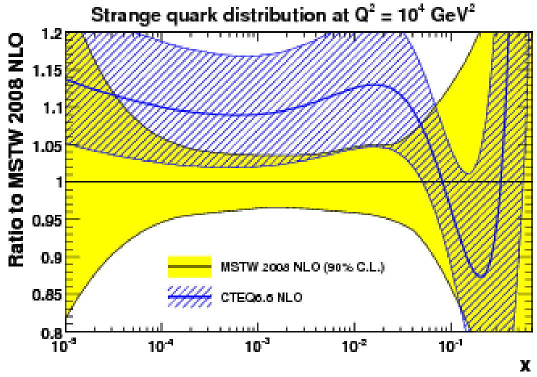
example: MSTW2008(NLO) vs. CTEQ6.6

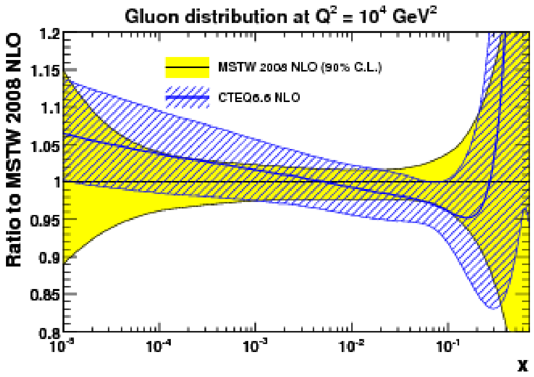

Up valence distribution at $Q^2 = 10^4$ GeV^2


Down valence distribution at $Q^2 = 10^4$ GeV^2


Up quark distribution at $Q^2 = 10^4$ GeV^2


Down quark distribution at $Q^2 = 10^4$ GeV^2


Strange quark distribution at $Q^2 = 10^4$ GeV^2


Gluon distribution at $Q^2 = 10^4$ GeV^2

Note:

CTEQ error bands comparable with MSTW 90%cl set (different definition of tolerance)

CTEQ light quarks and gluons slightly larger at small x because of imposition of positivity on gluon at Q_0^2

CTEQ gluons slightly larger at large x - only Run 1 jet data in fit

→ implications for 'precision' LHC cross sections (later)

pdfs and $\alpha_S(M_Z^2)$

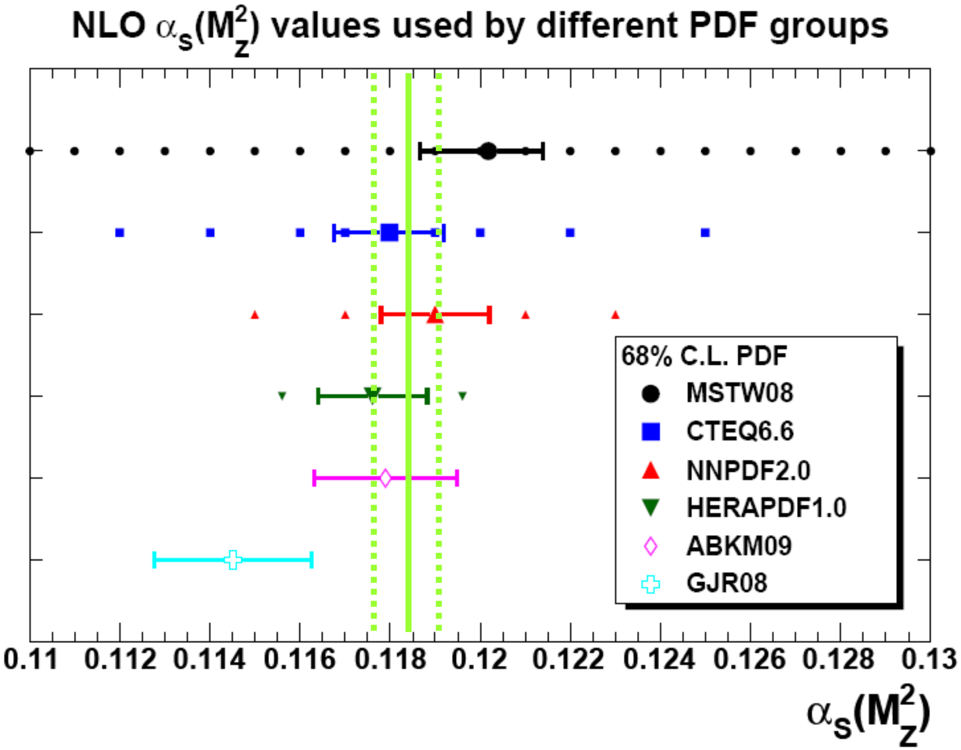
- MSTW08, ABKM09 and GJR08: $\alpha_S(M_Z^2)$ values and uncertainty determined by global fit
- NNLO value about 0.003 – 0.004 lower than NLO value, e.g. for MSTW08

$$\alpha_S^{\overline{MS},NLO}(M_Z^2) = 0.1202^{+0.012}_{-0.015}$$

$$\alpha_S^{\overline{MS},NNLO}(M_Z^2) = 0.1171^{+0.014}_{-0.014}$$

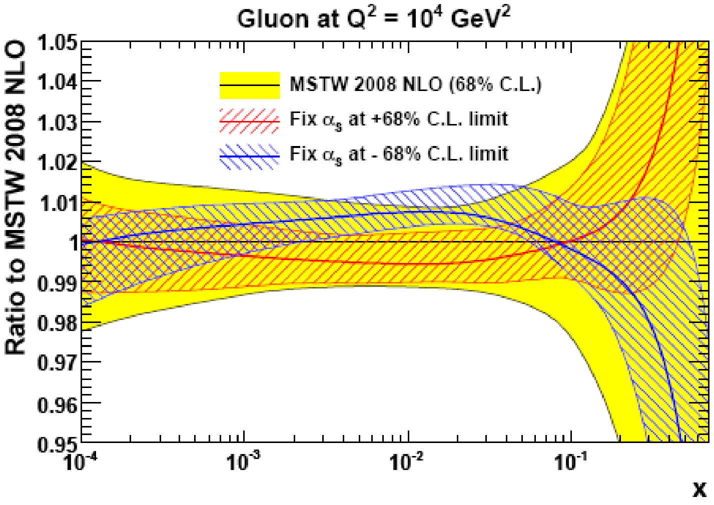
- CTEQ, NNPDF, HERAPDF choose standard values and uncertainties
- world average (PDG 2009)

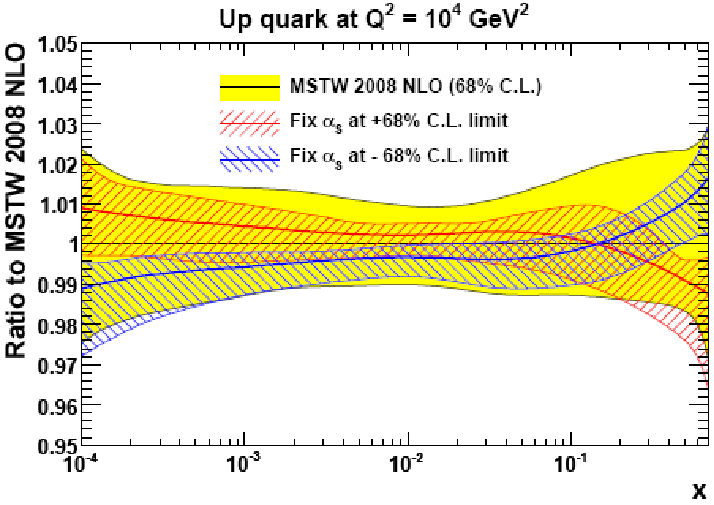
$$\alpha_S^{\overline{MS}}(M_Z^2) = 0.1184 \pm 0.0007$$



- note that the pdfs and α_S are correlated!
- e.g. gluon – α_S anticorrelation at small x and quark – α_S anticorrelation at large x

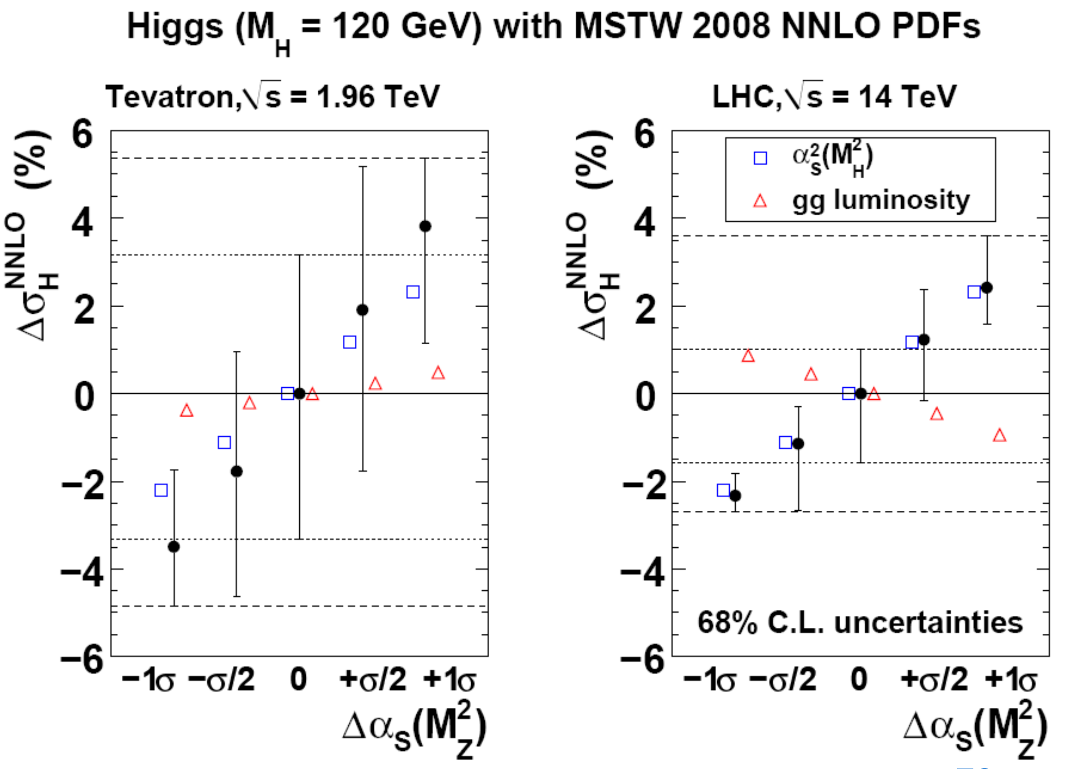
α_S - pdf correlations



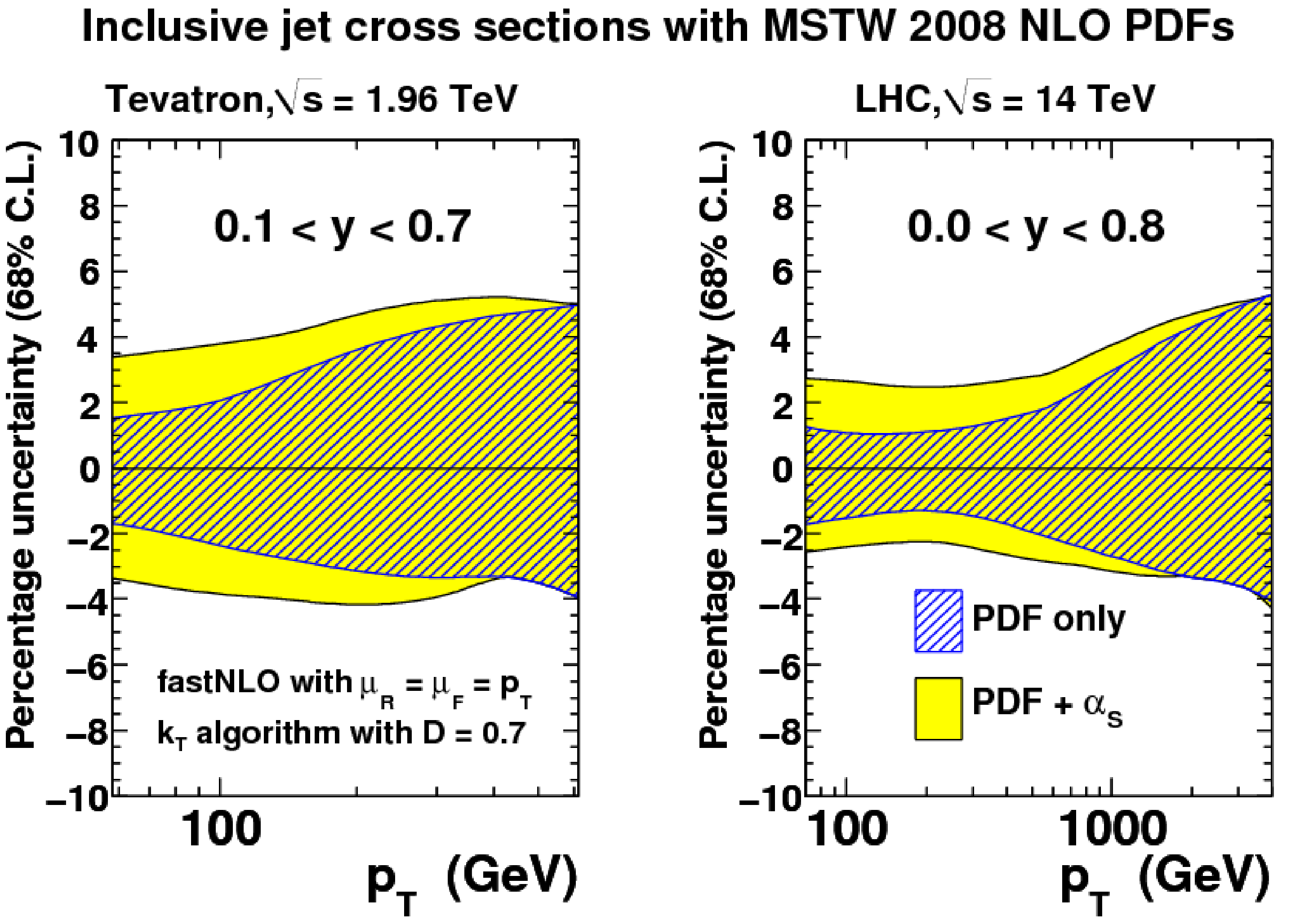


MSTW: arXiv:0905.3531

- care needed when assessing impact of varying α_S on cross sections ~ $(\alpha_S)^n$



pdf + α_S uncertainties in jet cross sections



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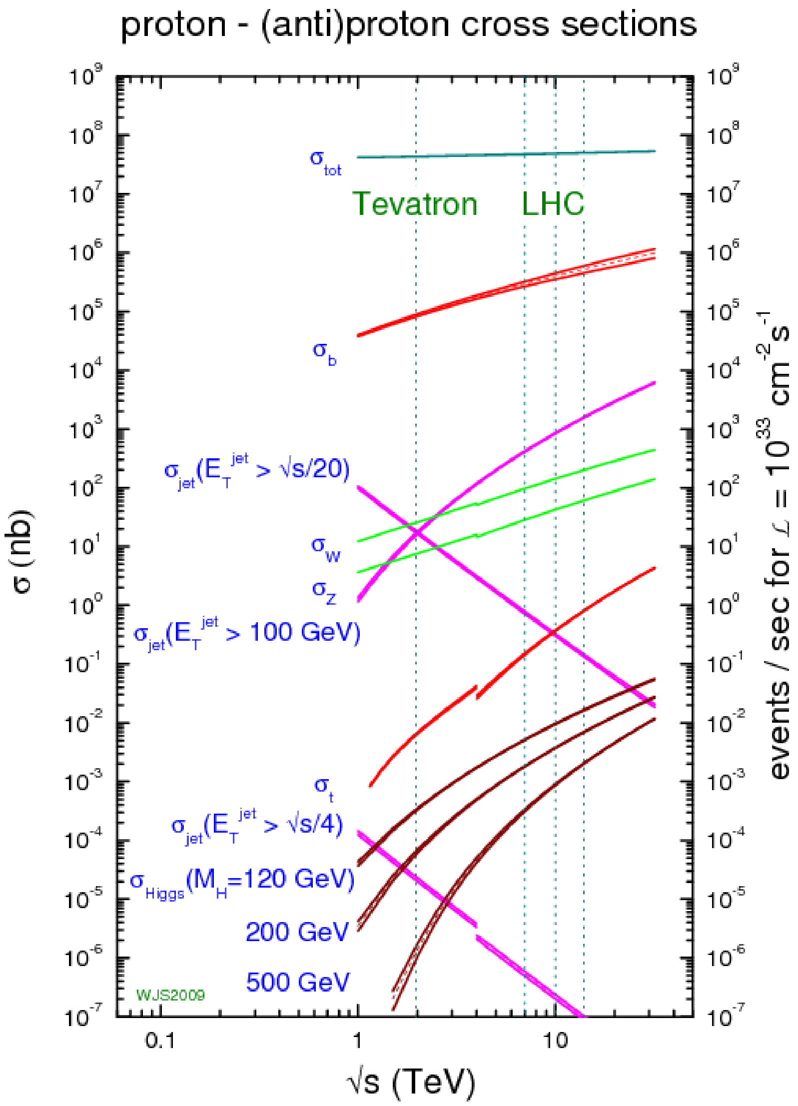
LHC benchmark cross sections

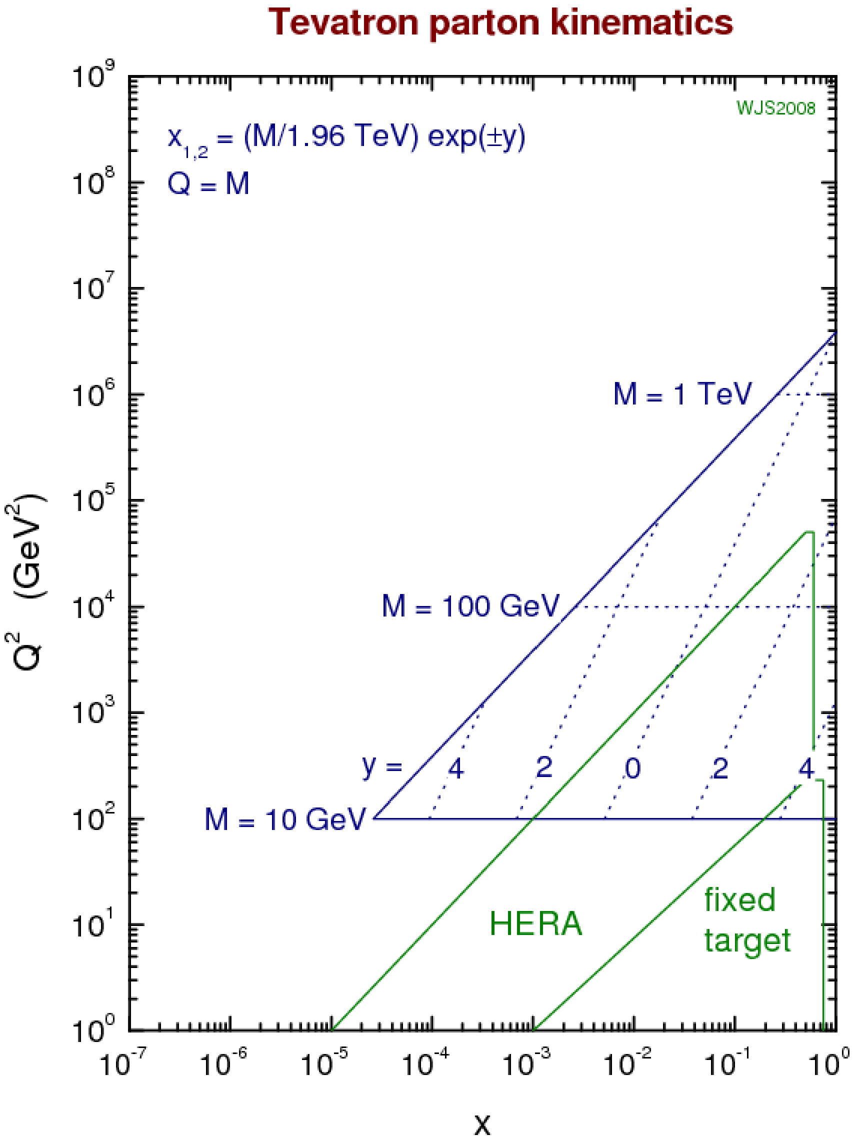
precision phenomenology at LHC

- LO for generic PS Monte Carlos
- NLO for NLO-MCs and many parton-level signal and background processes
- NNLO for a limited number of ‘precision observables’ (W, Z, DY, H, …)

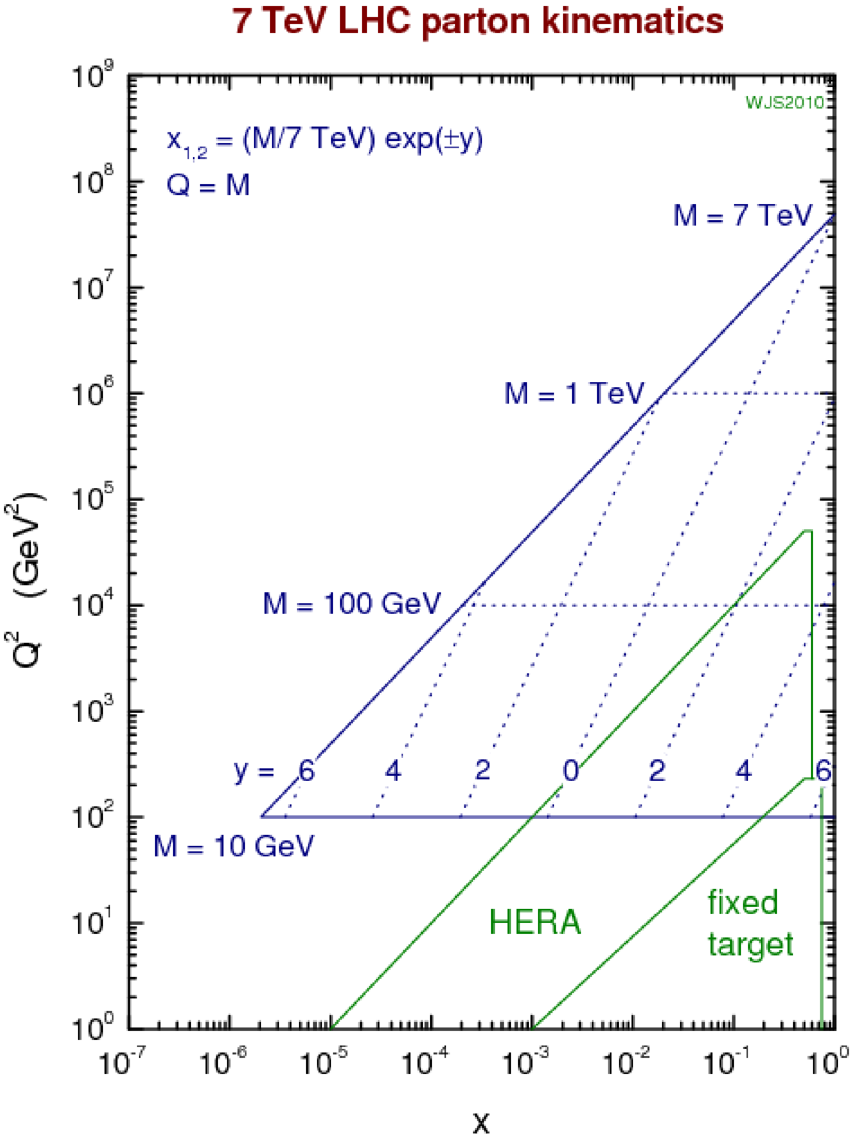
+ E/W corrections, resummed HO terms etc…



$$\delta\sigma_{th} = \delta\sigma_{pdf} \oplus \delta\sigma_{HO} \oplus \delta\sigma_{param} \oplus \ldots$$



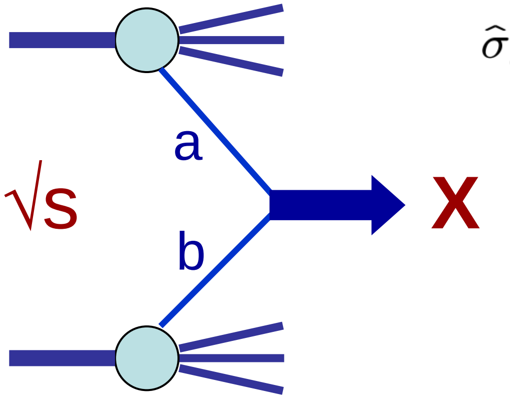
Tevatron parton kinematics



7 TeV LHC parton kinematics

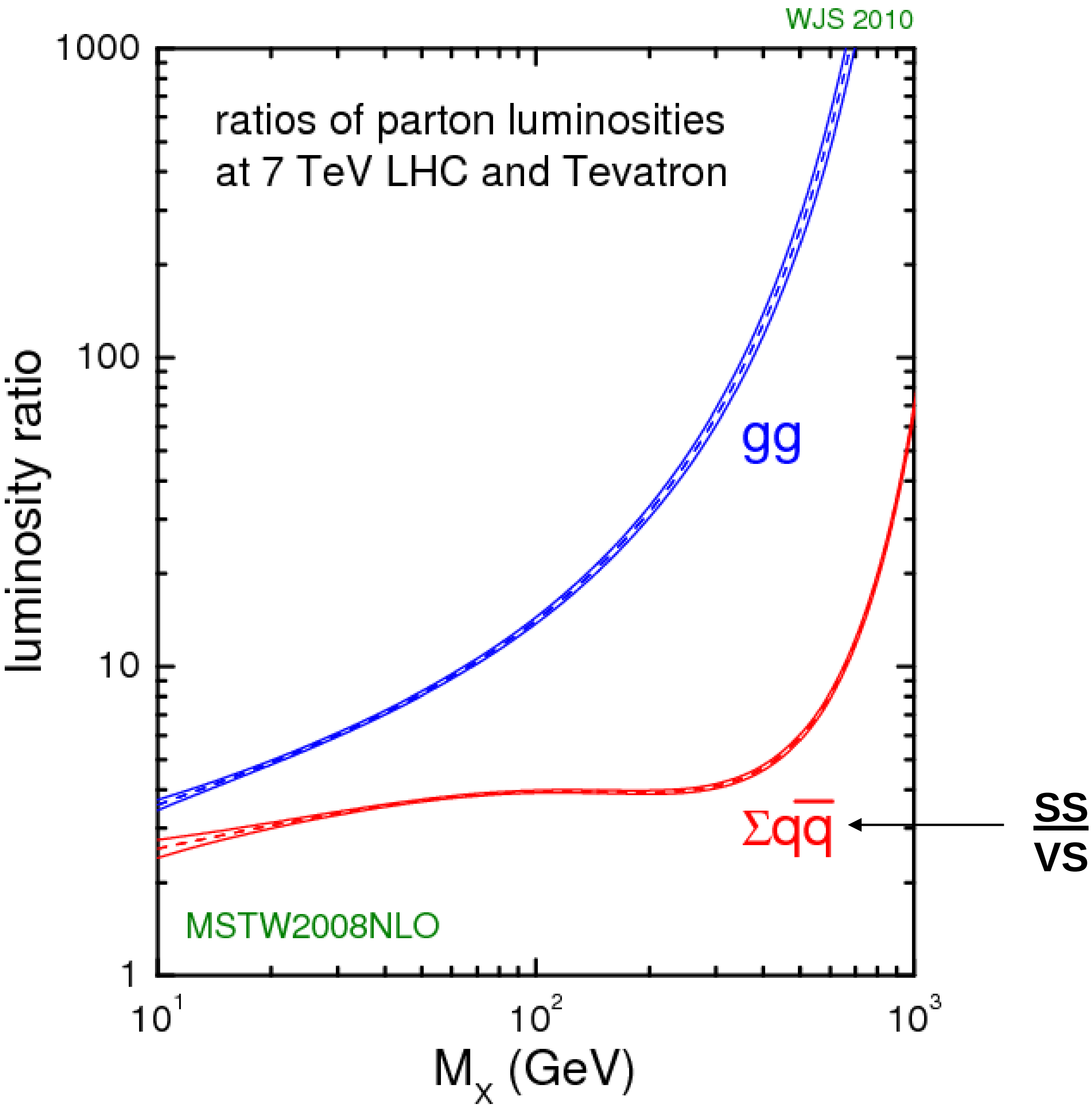
parton luminosity functions

- a quick and easy way to assess the mass, collider energy and pdf dependence of production cross sections

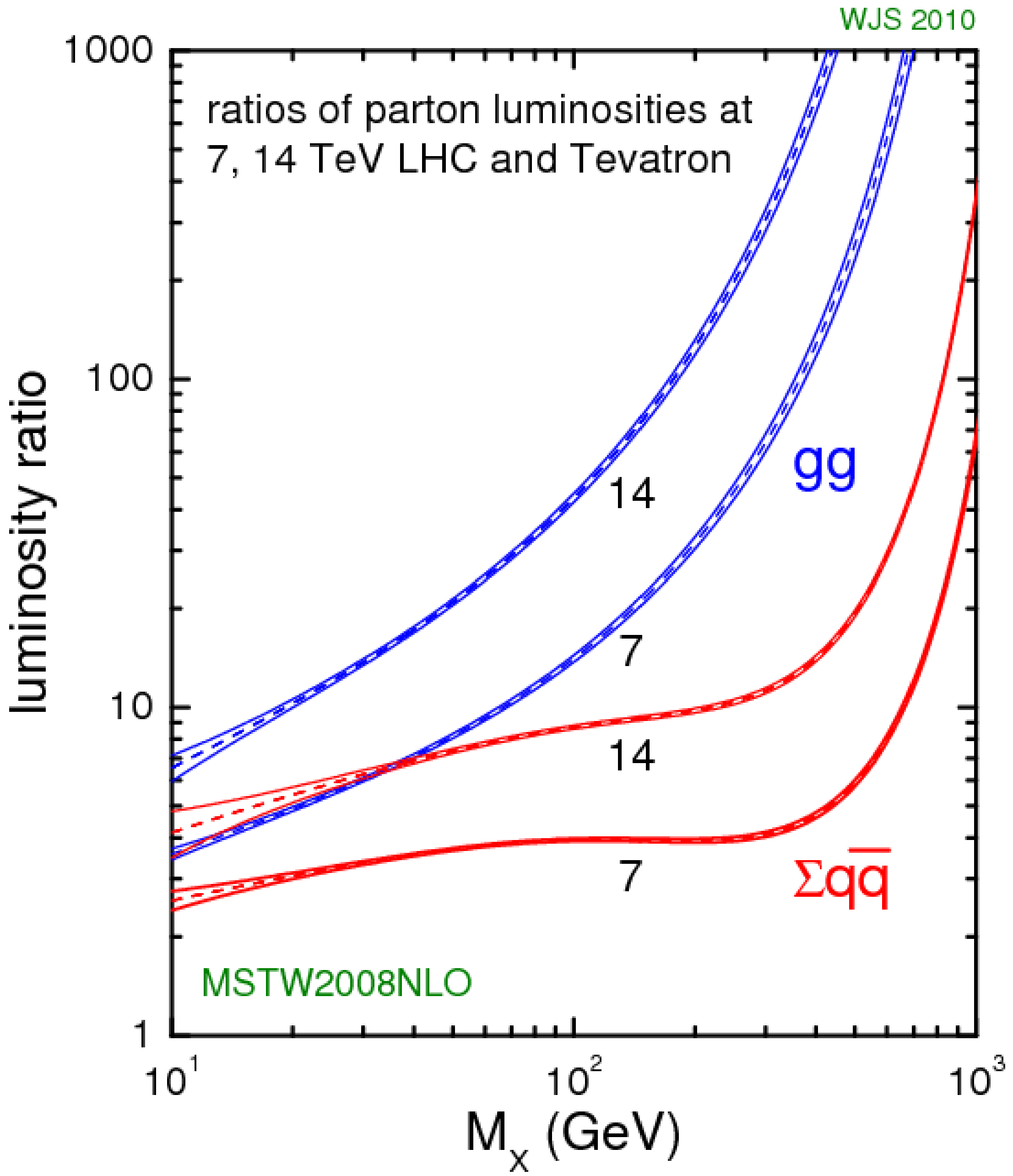


$$\begin{aligned}
\hat{\sigma}_{ab\to X} &= C_X \delta(\hat{s} - M_X^2) \\
\sigma_X &= \int_0^1 dx_a dx_b\, f_a(x_a, M_X^2) f_b(x_b, M_X^2)\, C_X\, \delta(x_a x_b - \tau) \\
&\equiv C_X \left[\frac{1}{s}\frac{\partial \mathcal{L}_{ab}}{\partial \tau}\right] \qquad\qquad (\tau = M_X^2/s) \\
\frac{\partial \mathcal{L}_{ab}}{\partial \tau} &= \int_0^1 dx_a dx_b\, f_a(x_a, M_X^2) f_b(x_b, M_X^2)\, \delta(x_a x_b - \tau)
\end{aligned}$$

- i.e. all the mass and energy dependence is contained in the **X**-independent parton luminosity function in [ ]
- useful combinations are $ab = gg, \sum_q q\bar{q}, \ldots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs



more such luminosity plots available at www.hep.phy.cam.ac.uk/~wjs/plots/plots.html


WJS 2010
ratios of parton luminosities at
7, 14 TeV LHC and Tevatron
gg
14
7
14
7
Σq$\bar{q}$
MSTW2008NLO
luminosity ratio
1000
100
10
1
10^1
10^2
10^3
M_X (GeV)

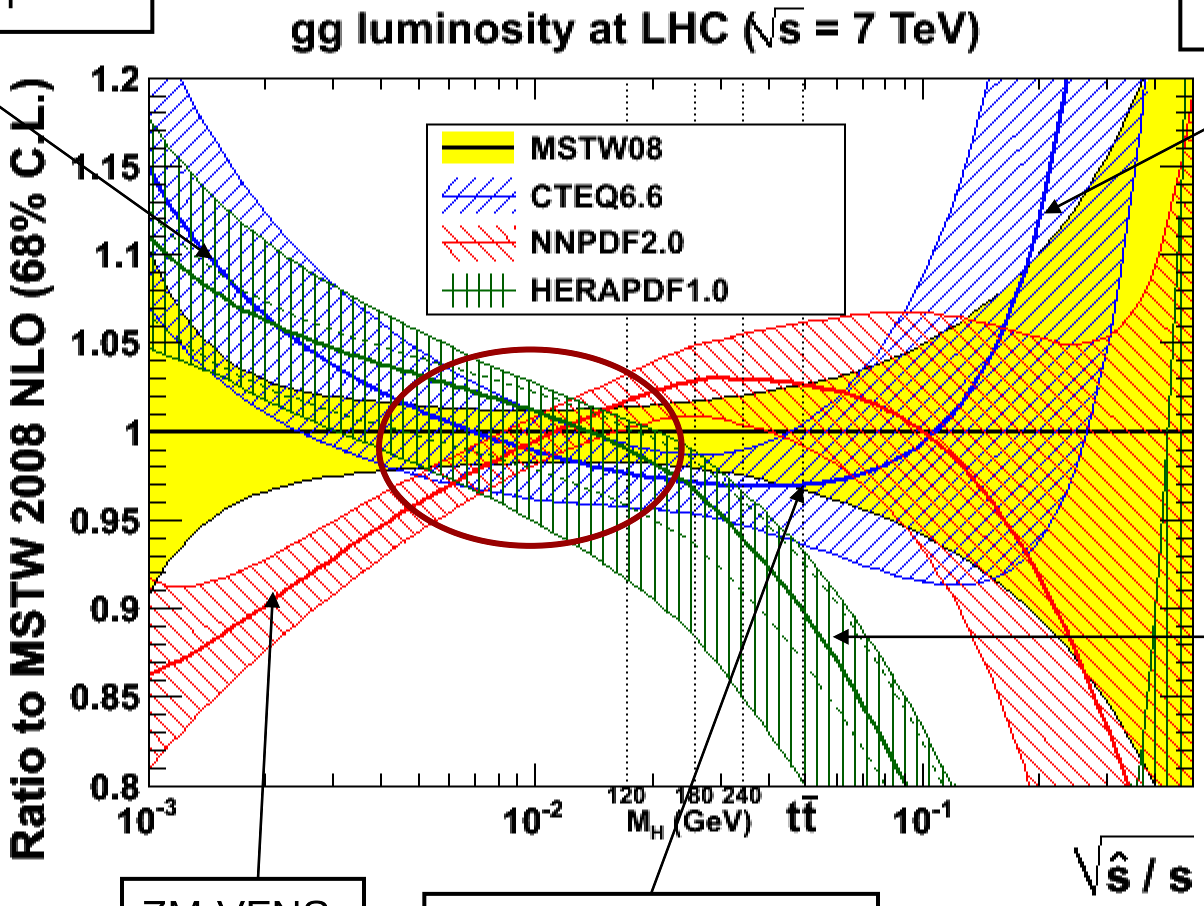
pdfs at LHC – the issues

- high precision cross section predictions require accurate knowledge of pdfs: $\delta\sigma_{th} = \delta\sigma_{pdf} + \ldots$

  → how do the different pdf sets compare?

- can we learn more about pdfs from LHC measurements, e.g.
  - high-E_T jets → gluon?
  - W^+, W^-, Z^0 → quarks?
  - very forward Drell-Yan (e.g. LHCb) → small x?
  - …

parton luminosity comparisons



positivity constraint on input gluon

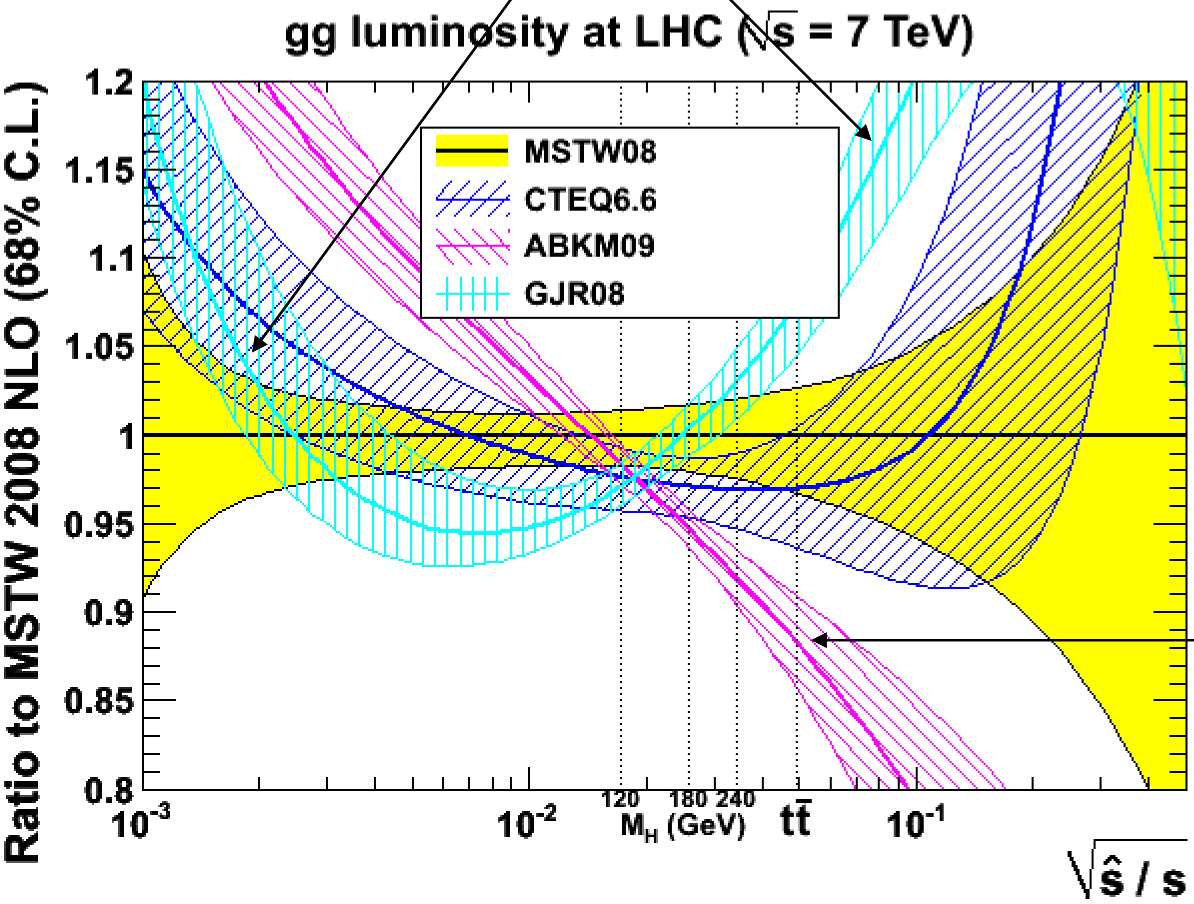
Run 1 vs. Run 2 Tevatron jet data

No Tevatron jet data or FT-DIS data in fit

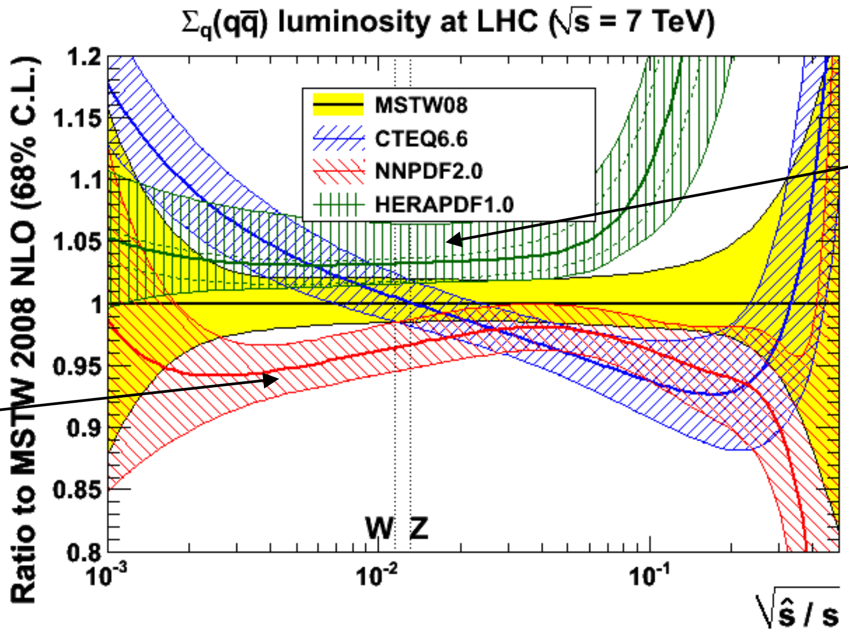
ZM-VFNS

momentum sum rule

Luminosity and cross section plots from Graeme Watt (MSTW, in preparation), available at projects.hepforge.org/mstwpdf/pdf4lhc restricted parametrisation

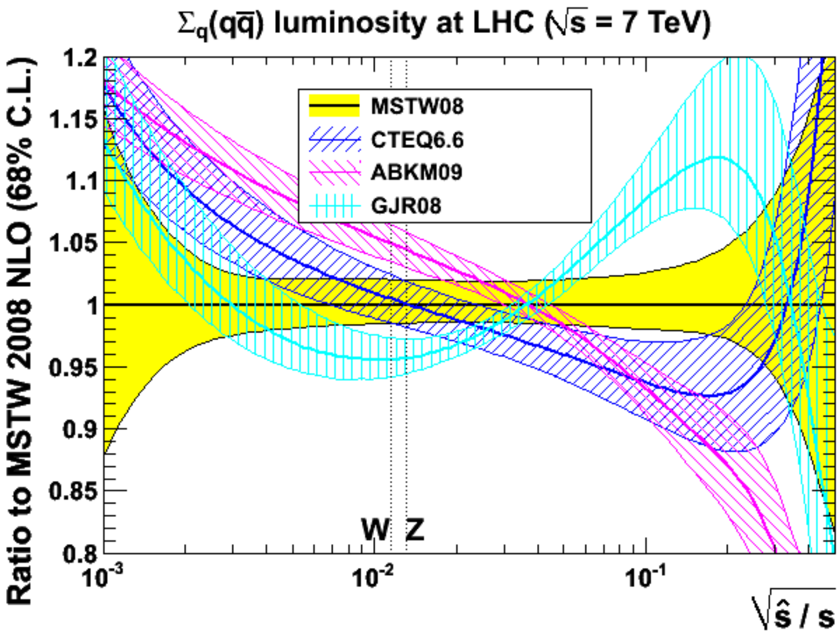


no Tevatron
jet data in fit

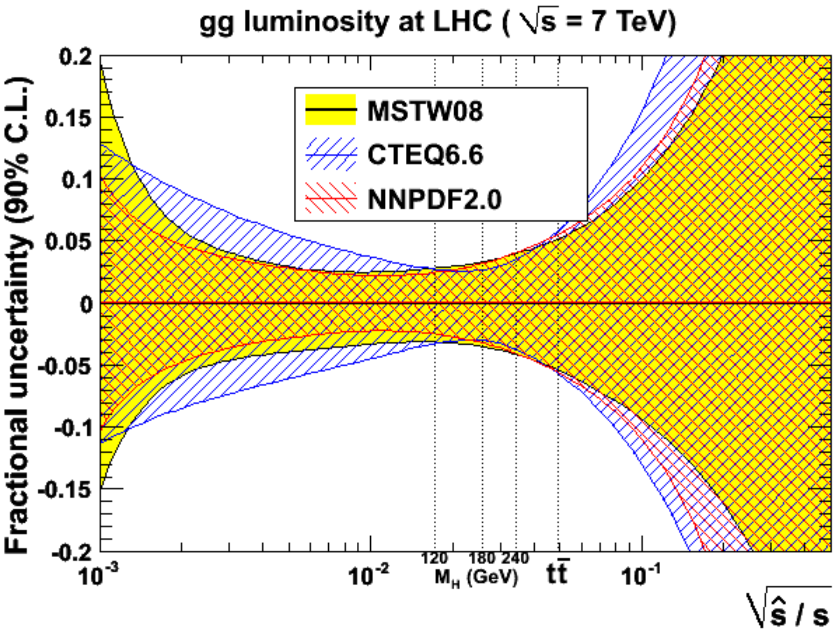


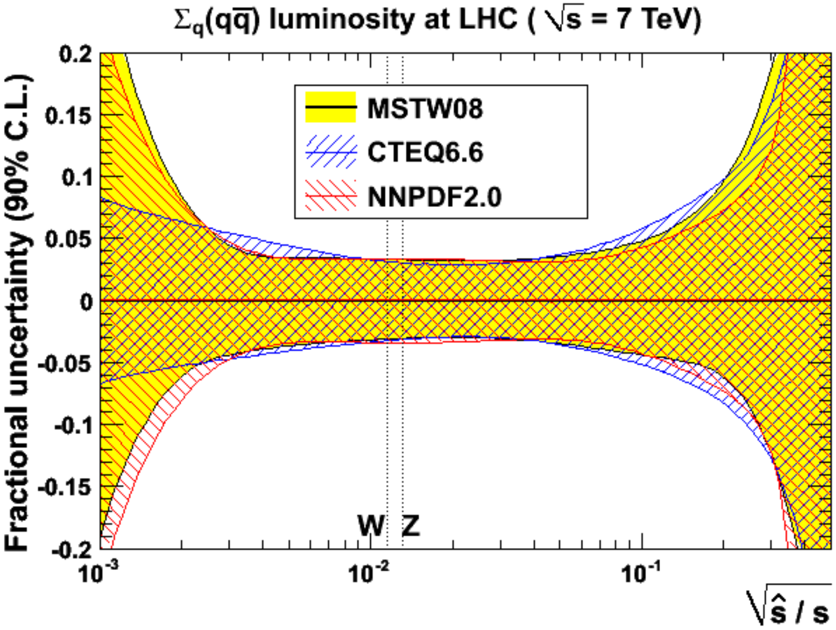
new combined
HERA SF data

ZM-VFNS



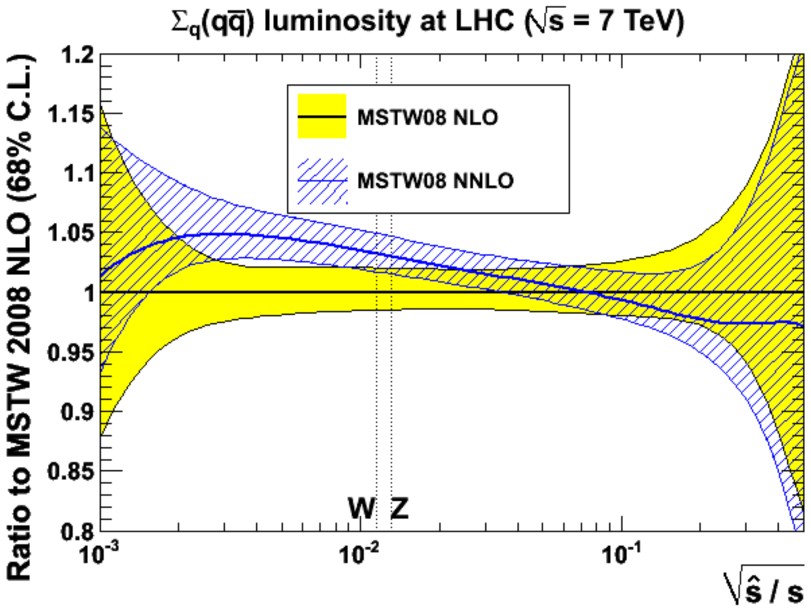
fractional uncertainty comparisons

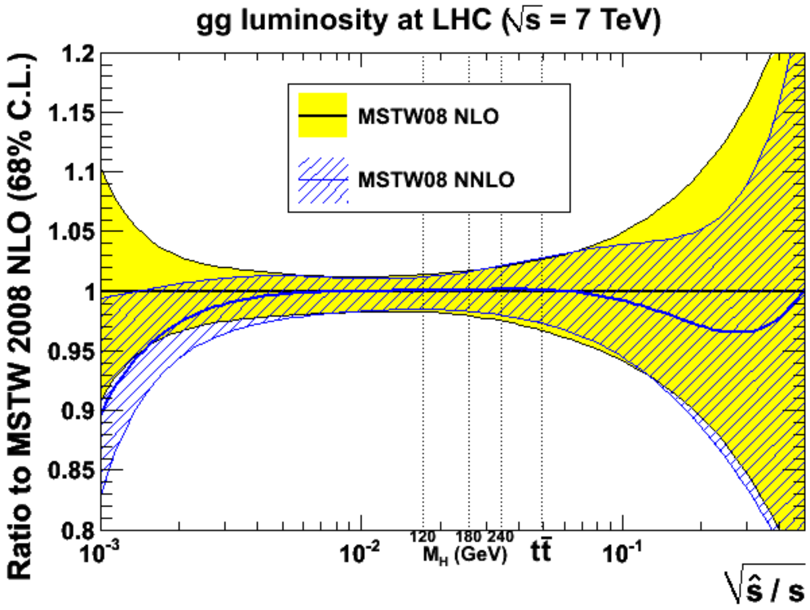




remarkably similar considering the different definitions of pdf uncertainties used by the 3 groups!

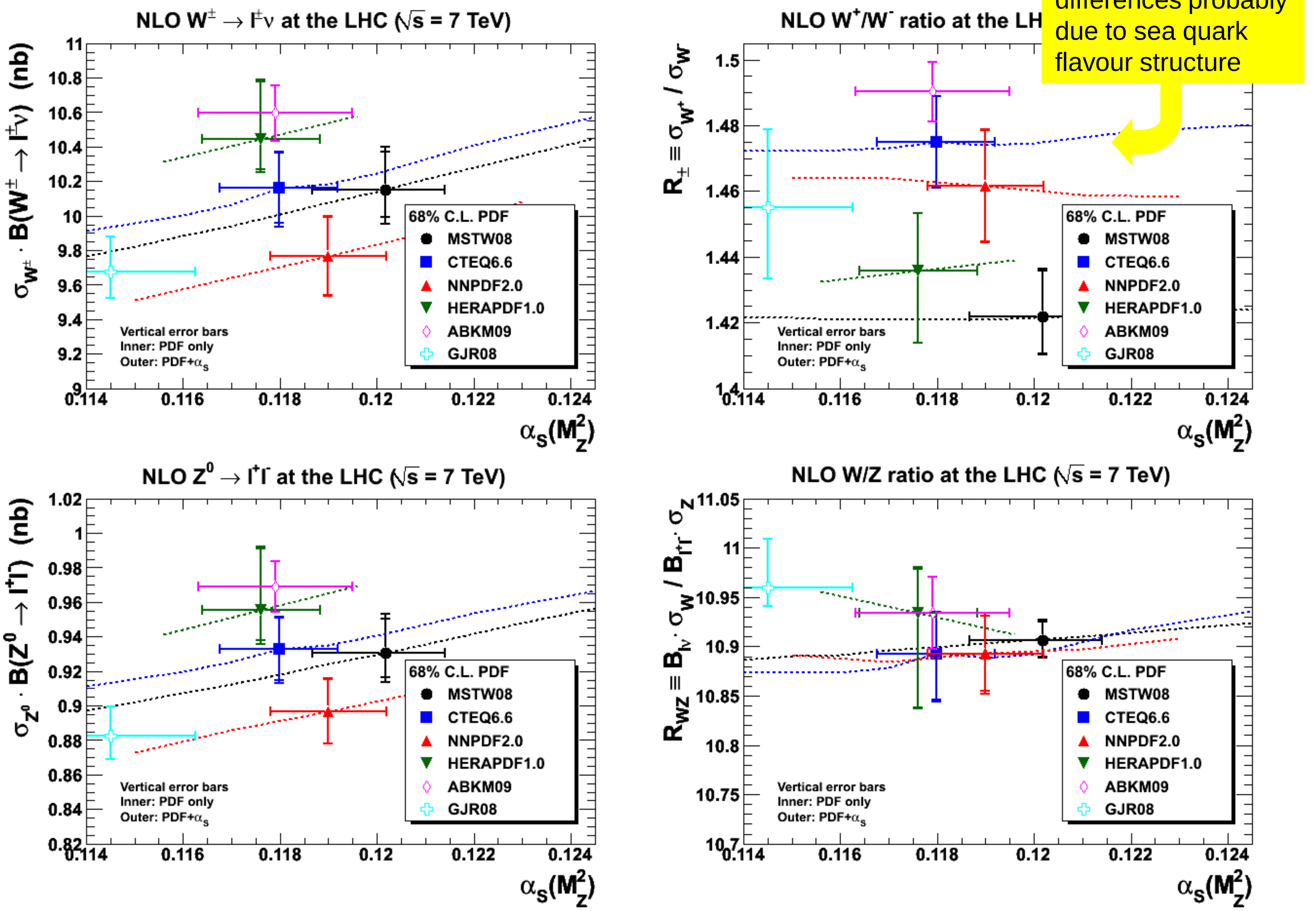
NLO and NNLO parton luminosity comparisons





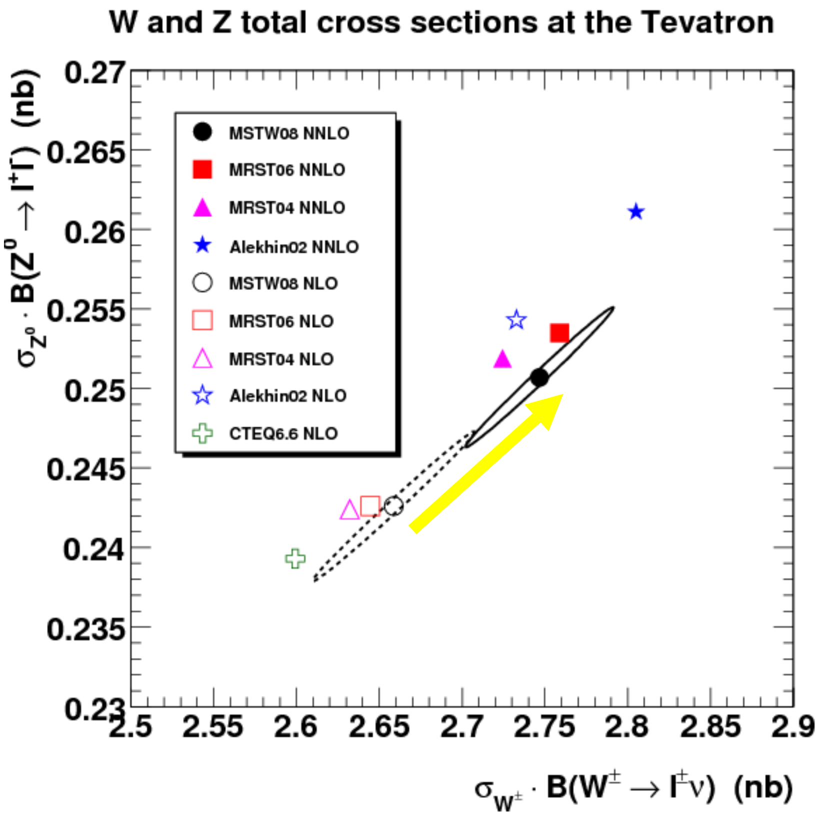
W, Z

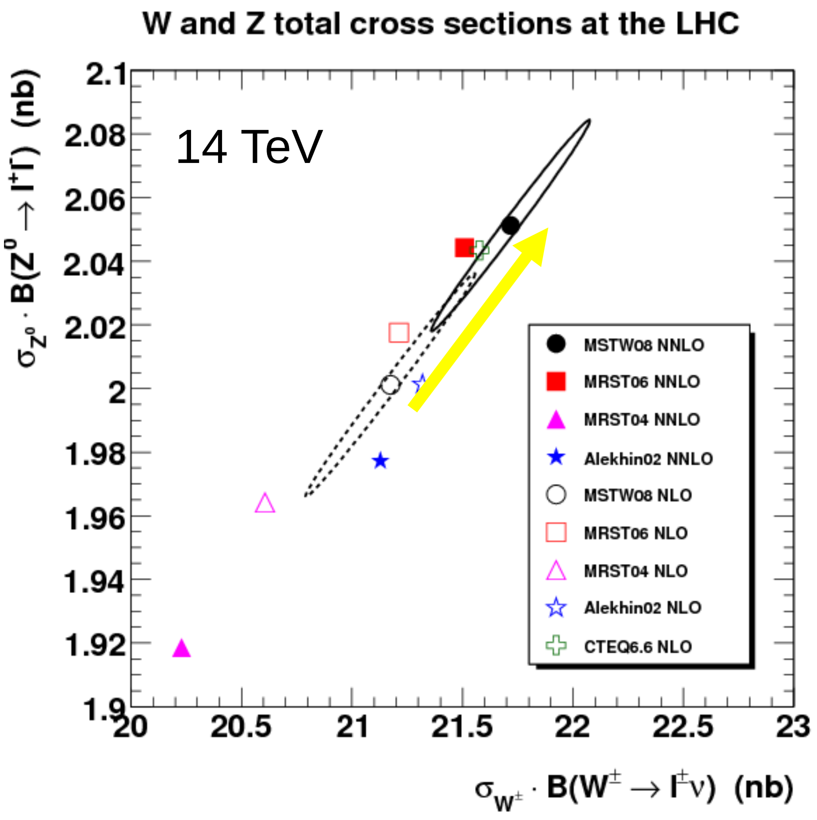
benchmark W,Z cross sections

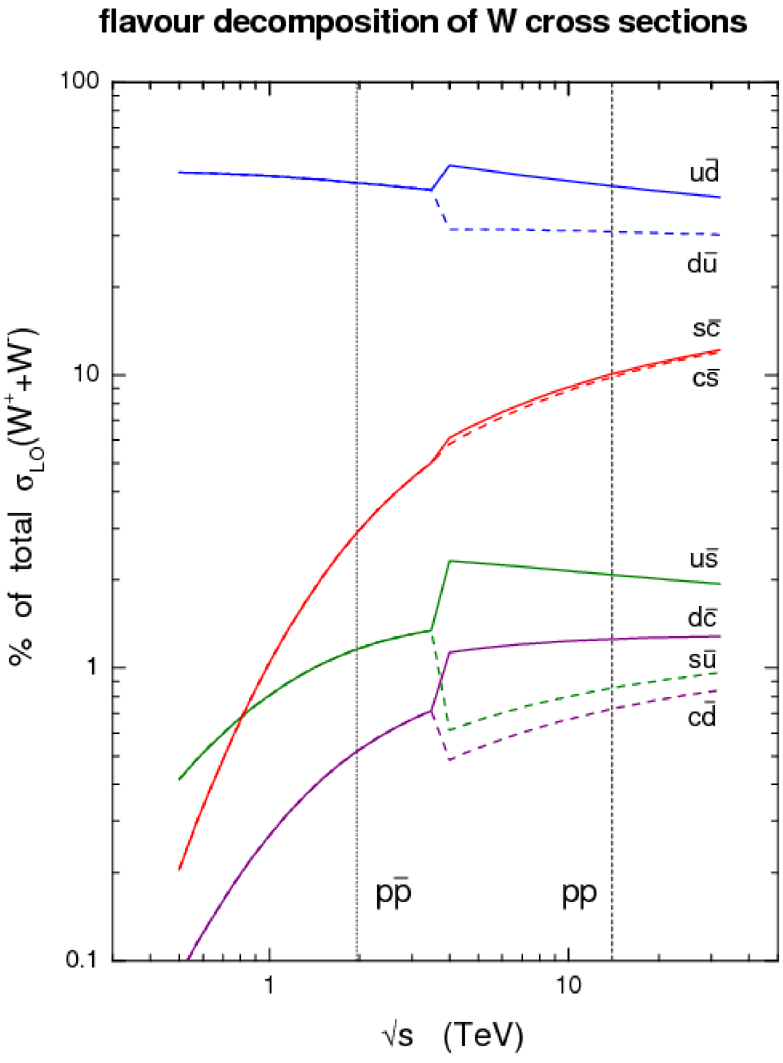


differences probably due to sea quark flavour structure

predictions for σ(W,Z) @ Tevatron, LHC: NLO *vs.* NNLO





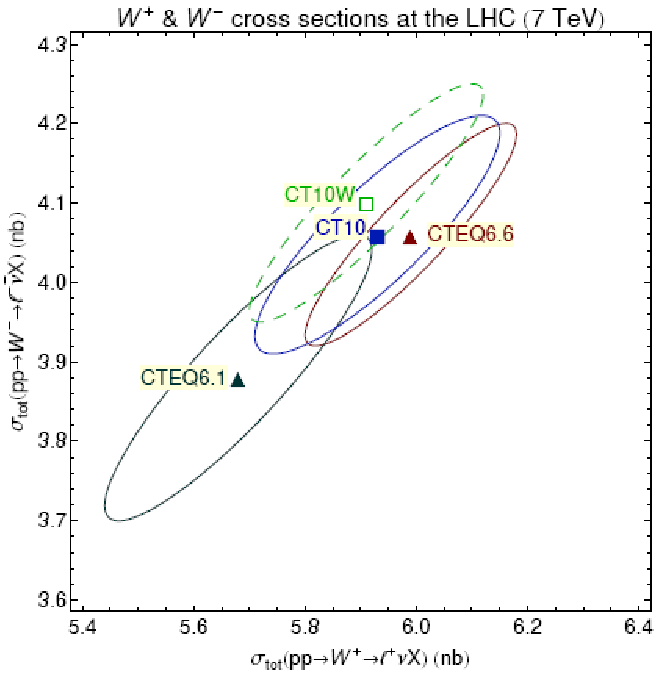


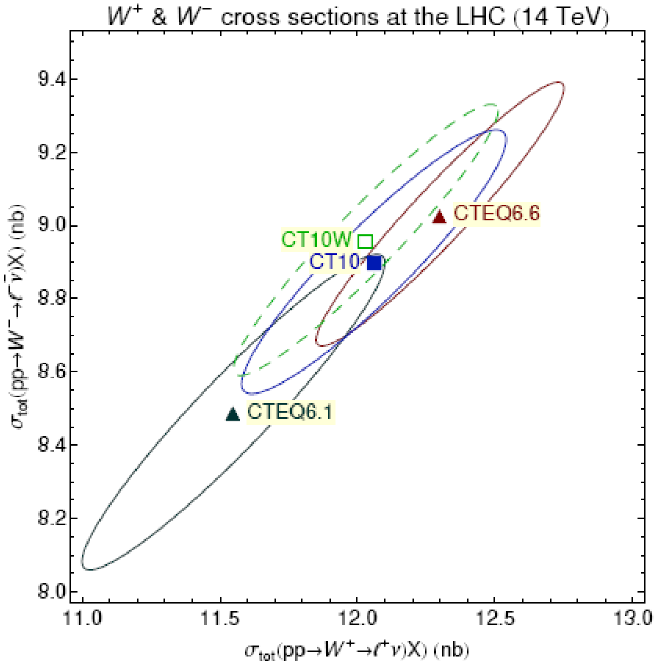
impact of sea quarks on the NLO W charge asymmetry ratio at 7 TeV:

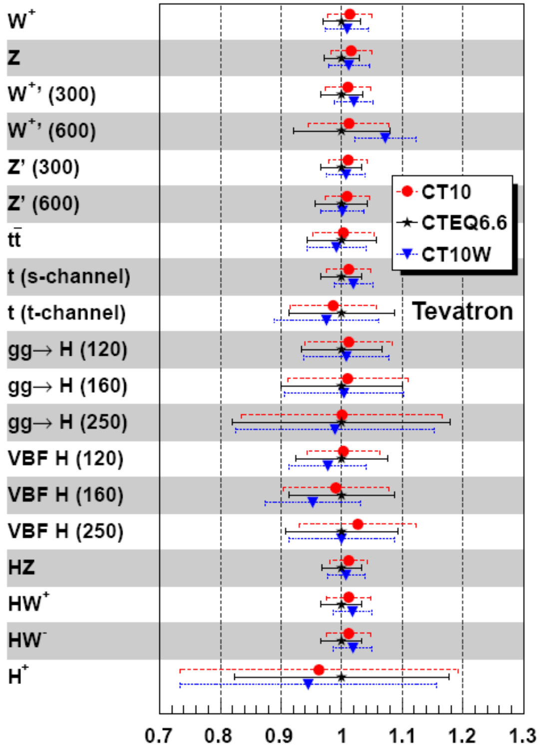
pdfs	R(W^+/W^-)
{udg} only	1.53
{udscbg} = MSTW08	1.42 ± 0.02
{udscbg}$_{sea}$ only	0.99
{udscbg}$_{sym.sea}$ only	1.00

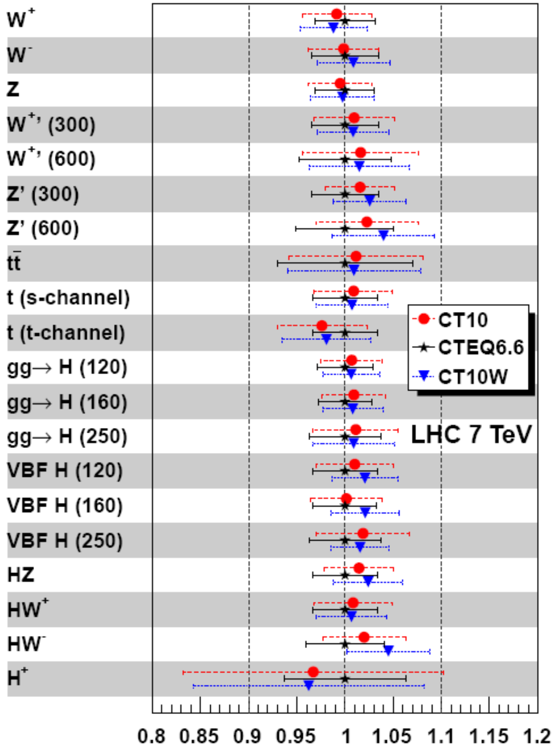
at LHC, ~30% of W and Z total cross sections involves s,c,b quarks

CTEQ6.6 vs. CT10, CT10W (NLO)







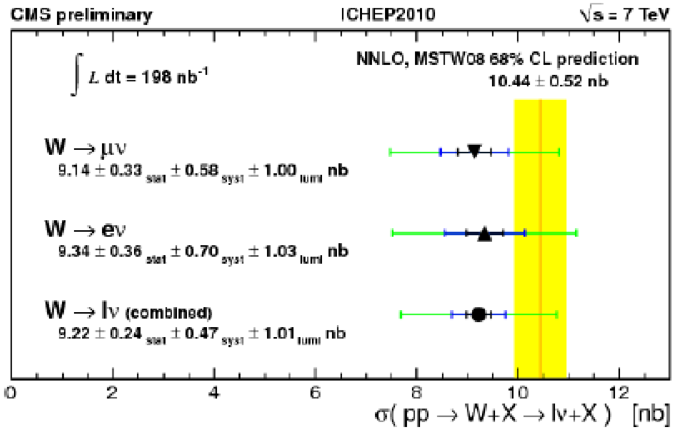


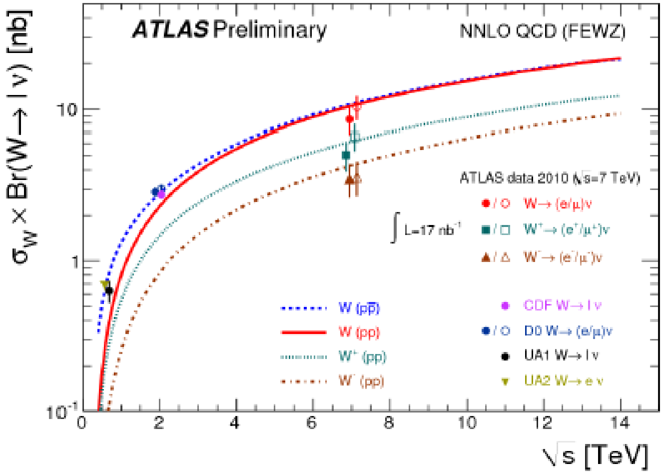
* CT10W: attempt to include recent D0 lepton asymmetry data in global fit → slightly different d/u

see next talks!

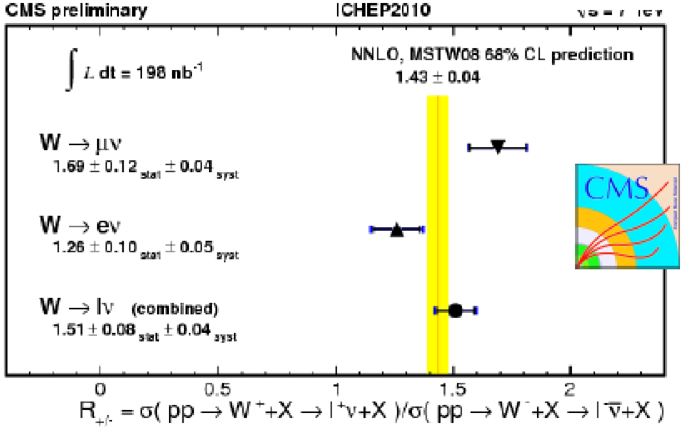
Inclusive W boson measurements: Summary

W→ℓν cross section





Ratio W+/W-



care needed with definition of 'total cross section' in these comparisons

All the results are in agreement with the Standard Model expectations.

using the W^{+-} charge asymmetry at the LHC

- at the Tevatron $\sigma(W^+) = \sigma(W^-)$, whereas at LHC $\sigma(W^+) \sim (1.4 - 1.3)\, \sigma(W^-)$

- can use this asymmetry to calibrate backgrounds to new physics, since typically $\sigma_{NP}(X \to W^+ + \ldots) = \sigma_{NP}(X \to W^- + \ldots)$

- example:

$$gg \to t\bar{t} \to W^+W^-b\bar{b} \to W^\pm(\to l^\pm + \nu) + 4\text{jets}$$

in this case

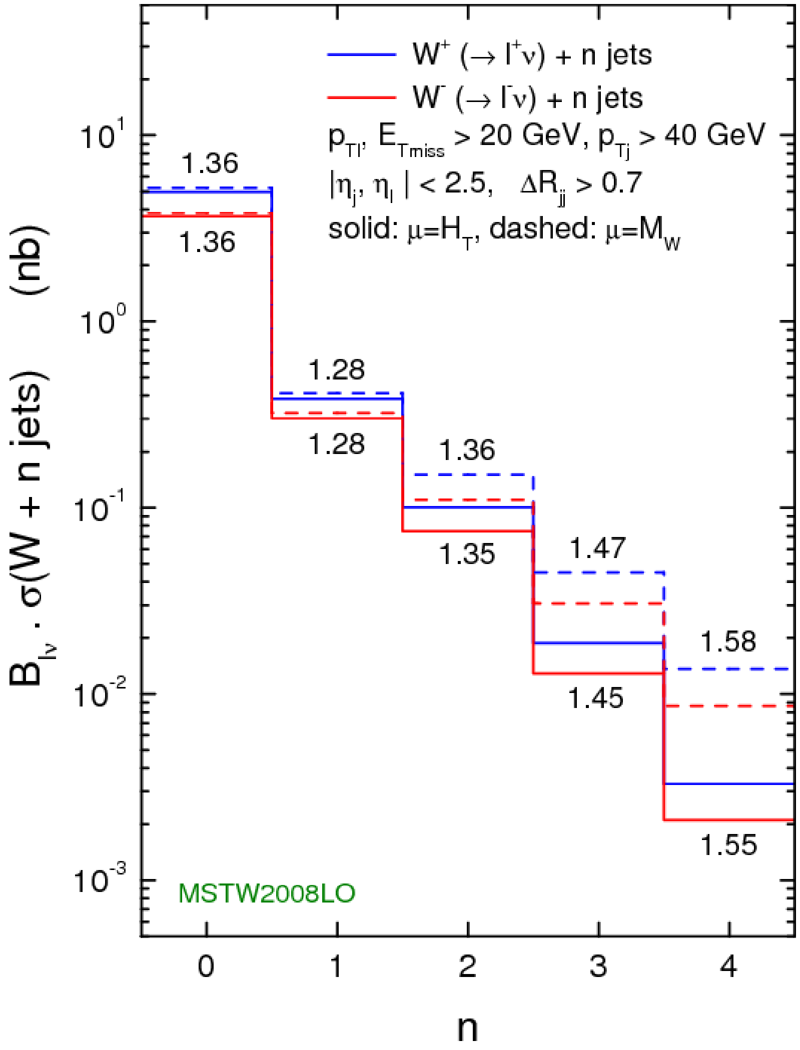
$$\sigma_{\text{signal}}(W^+ + 4\text{jets}) = \sigma_{\text{signal}}(W^- + 4\text{jets})$$

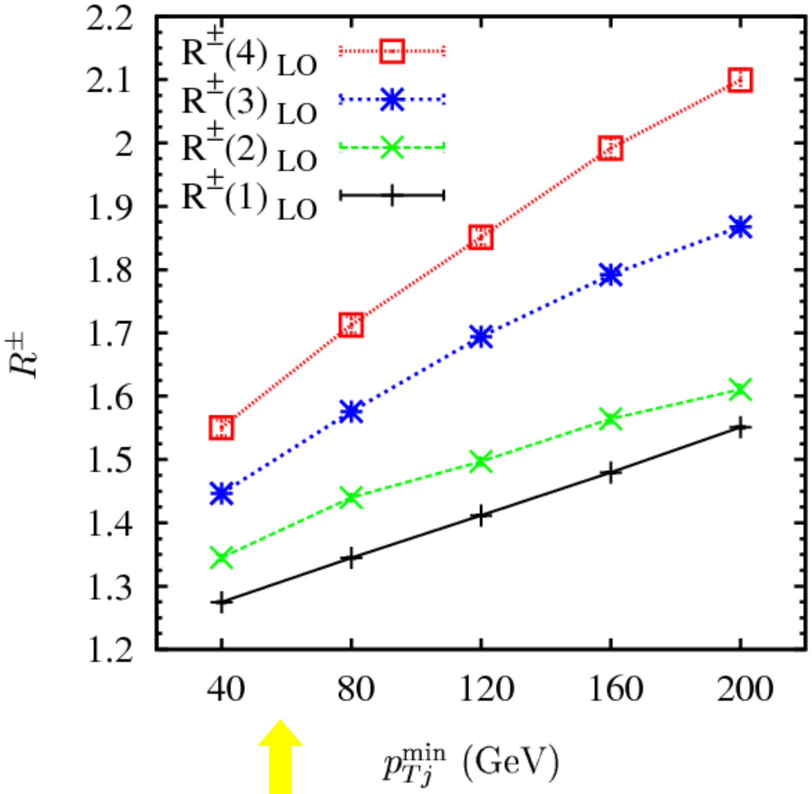
whereas…

$$\sigma_{\text{QCDbkgd}}(W^+ + 4\text{jets}) \neq \sigma_{\text{QCDbkgd}}(W^- + 4\text{jets})$$

which can in principle help distinguish signal and background

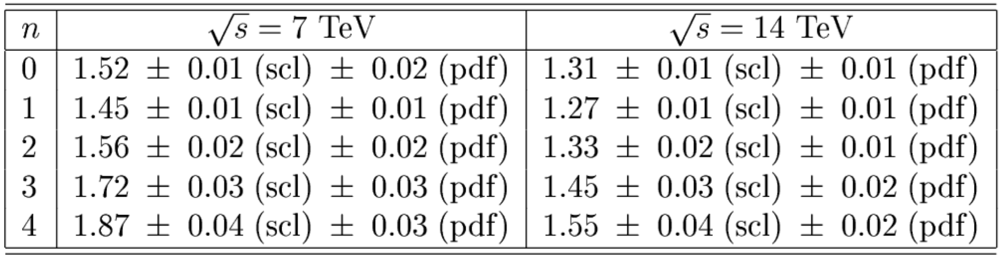
C.H. Kom & WJS, arXiv:1004.3404





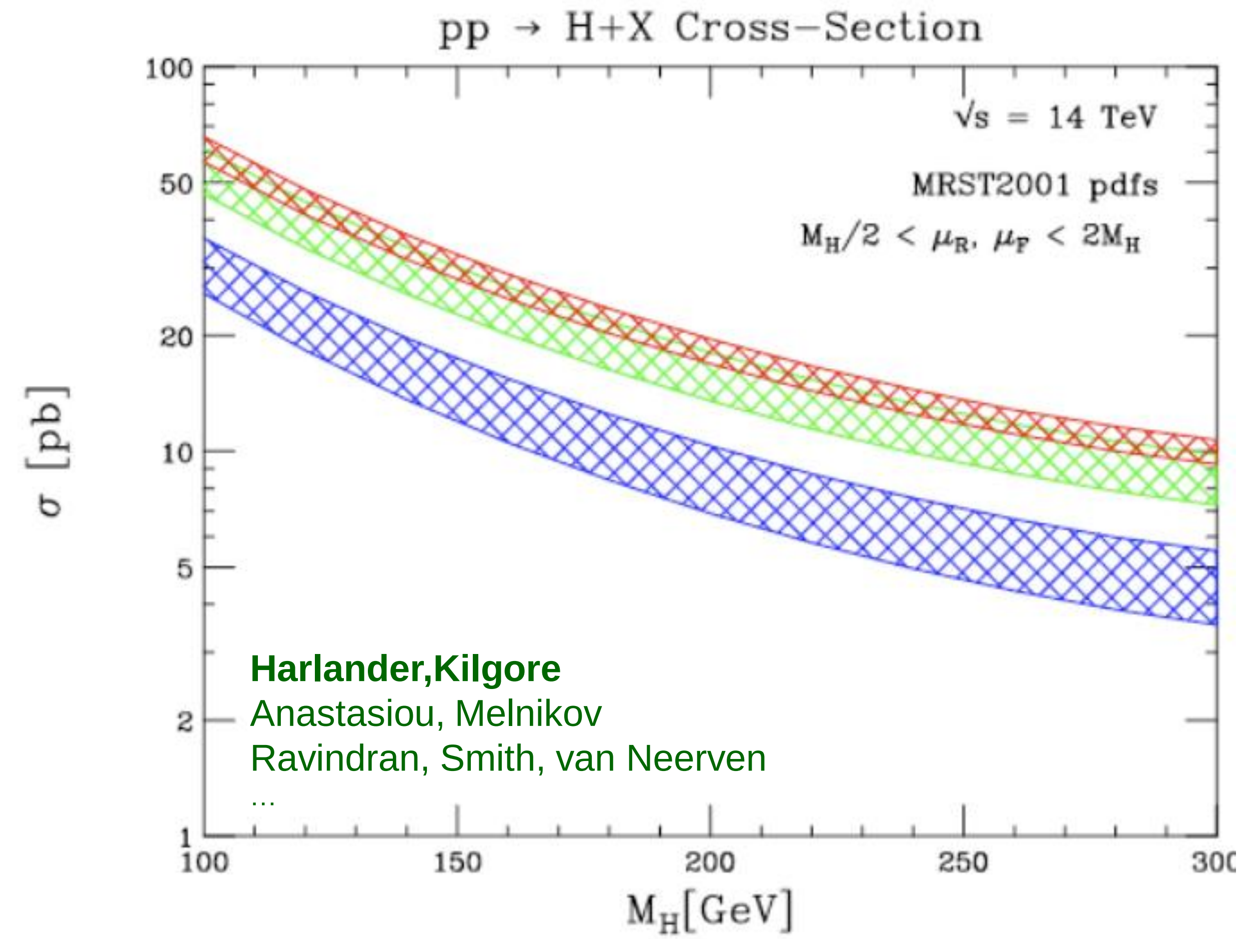
R± increases with jet $p_T^{\min}$

R± larger at 7 TeV LHC



n	$\sqrt{s} = 7$ TeV	$\sqrt{s} = 14$ TeV
0	1.52 ± 0.01 (scl) ± 0.02 (pdf)	1.31 ± 0.01 (scl) ± 0.01 (pdf)
1	1.45 ± 0.01 (scl) ± 0.01 (pdf)	1.27 ± 0.01 (scl) ± 0.01 (pdf)
2	1.56 ± 0.02 (scl) ± 0.02 (pdf)	1.33 ± 0.02 (scl) ± 0.01 (pdf)
3	1.72 ± 0.03 (scl) ± 0.03 (pdf)	1.45 ± 0.03 (scl) ± 0.02 (pdf)
4	1.87 ± 0.04 (scl) ± 0.03 (pdf)	1.55 ± 0.04 (scl) ± 0.02 (pdf)

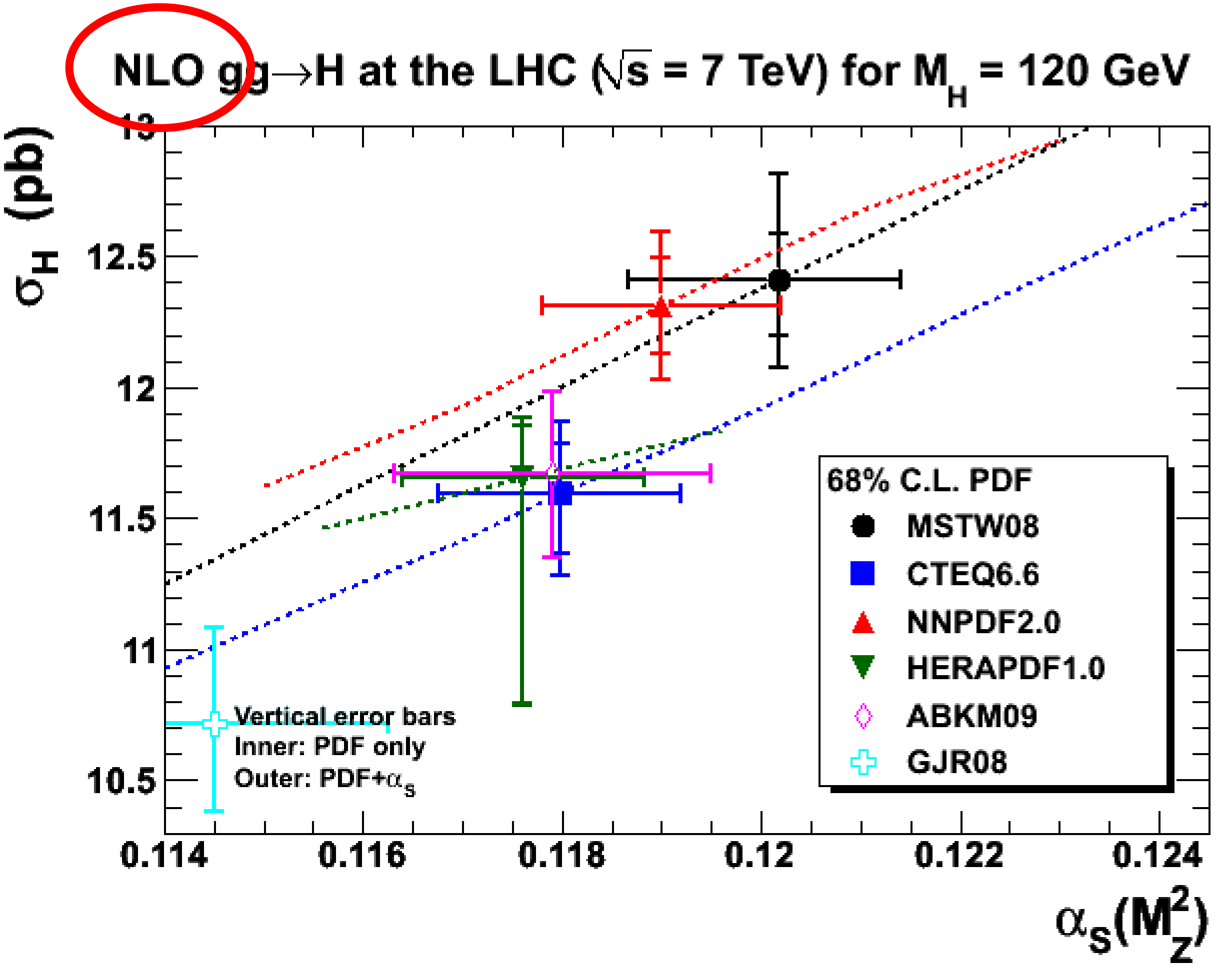
Higgs

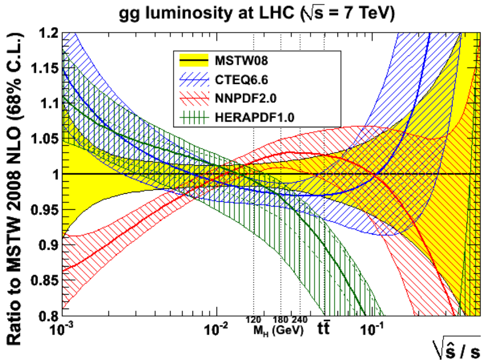


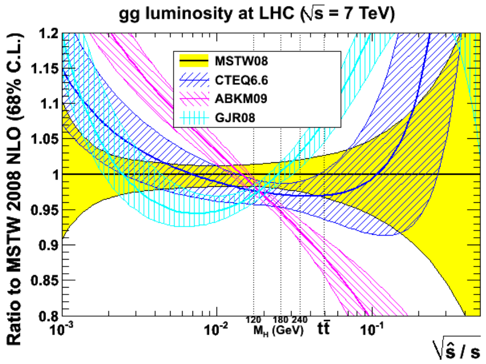
Harlander,Kilgore
Anastasiou, Melnikov
Ravindran, Smith, van Neerven
…

- only scale variation uncertainty shown
- central values calculated for a *fixed* set pdfs with a *fixed* value of $\alpha_S(M_Z)$

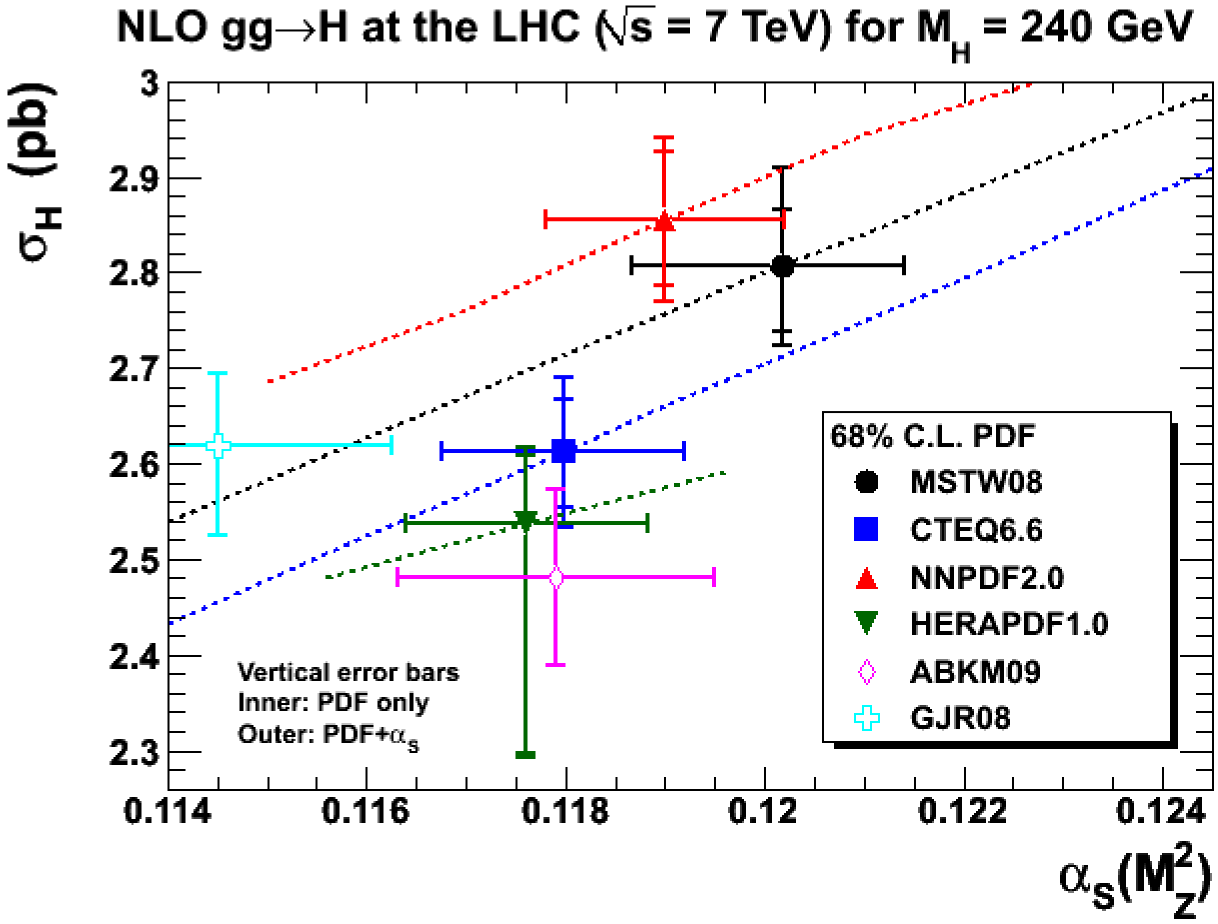
benchmark Higgs cross sections

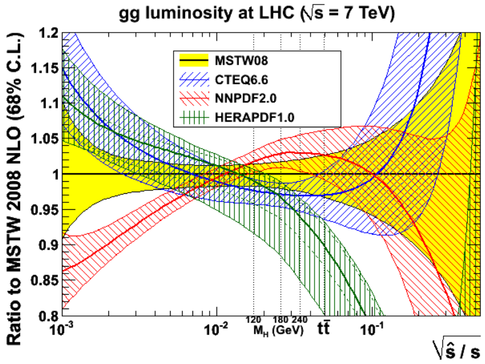


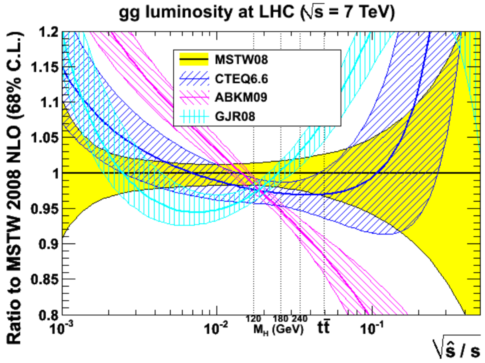




… differences from both pdfs AND α_S !


NLO gg→H at the LHC (√s = 7 TeV) for M_H = 240 GeV
σ_H (pb)
α_S(M_Z^2)
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
0.114
0.116
0.118
0.12
0.122
0.124
68% C.L. PDF
● MSTW08
■ CTEQ6.6
▲ NNPDF2.0
▼ HERAPDF1.0
◇ ABKM09
✚ GJR08
Vertical error bars
Inner: PDF only
Outer: PDF+α_S


gg luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (68% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10^-3
10^-2
10^-1
120 180 240
M_H (GeV)
tt̄
√ŝ / s
MSTW08
CTEQ6.6
NNPDF2.0
HERAPDF1.0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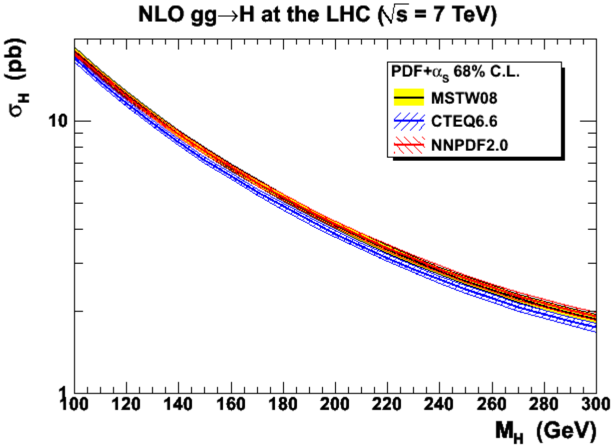
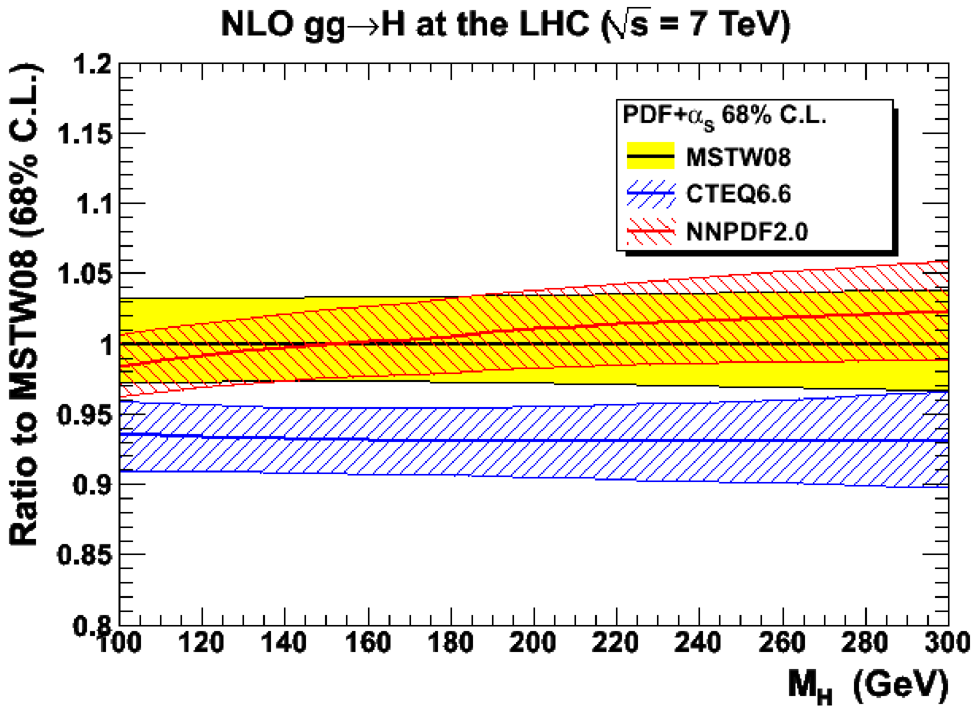
120 180 240
M_H (GeV)
tt̄
√ŝ / s
MSTW08
CTEQ6.6
ABKM09
GJR08



How to define an overall 'best theory prediction'?! See LHC Higgs Cross Section Working Group meeting, 5-6 July, higgs2010.to.infn.it



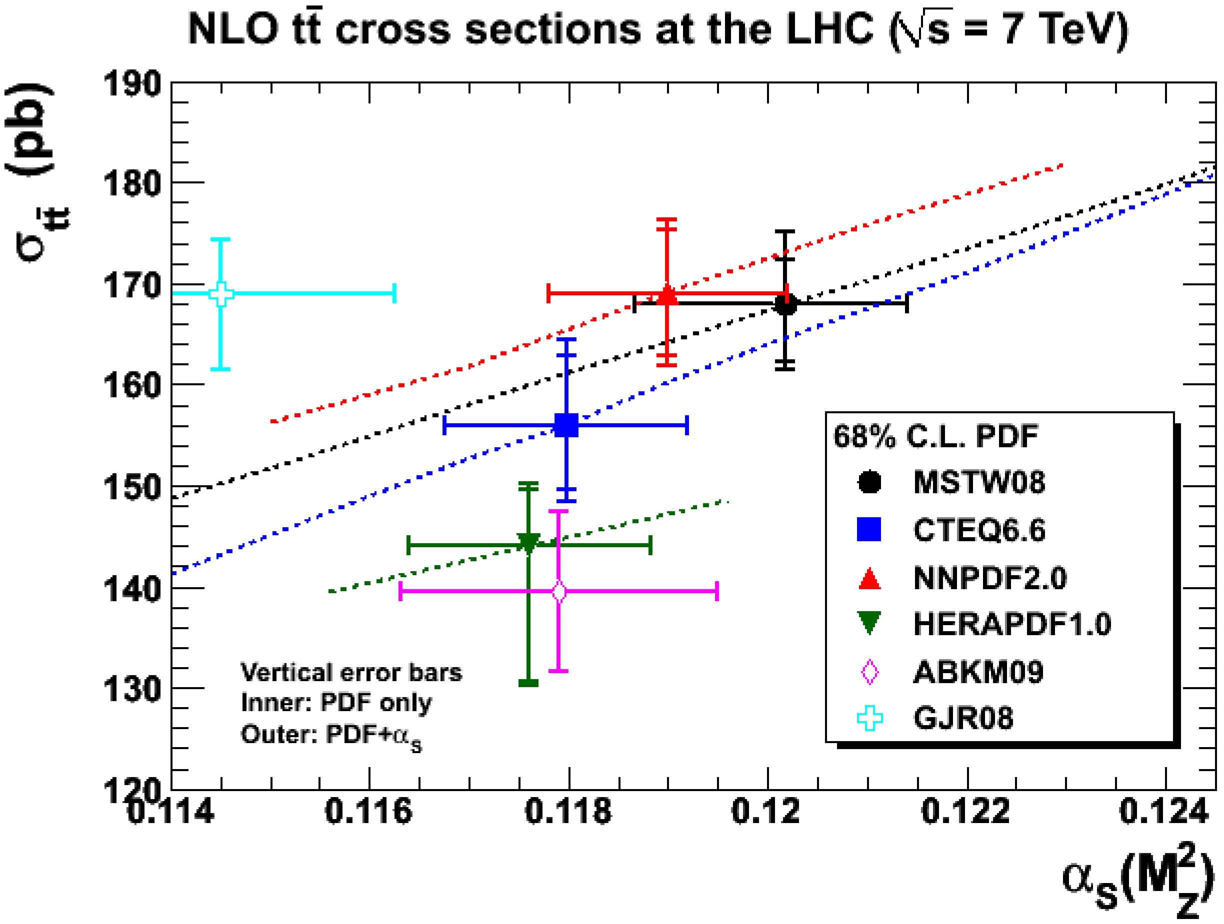
small print

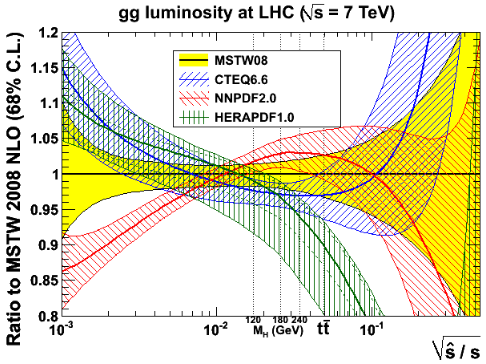
Central predictions use the values of $\alpha_S(M_Z)$ favoured by each PDF group, i.e. 0.1202 for MSTW08, 0.1180 for CTEQ6.6 and 0.1190 for NNPDF2.0. For MSTW08, $\alpha_S(M_Z)$ was determined simultaneously with the PDFs in the global fit. The *experimental* uncertainties on $\alpha_S(M_Z)$ are +0.0012/-0.0015 at 68% C.L The uncertainties on $\alpha_S(M_Z)$ for CTEQ6.6 and NNPDF2.0 are taken to be ±0.0012 at 68% C.L. The combined PDF+α_S uncertainty is calculated following the prescription recommended by each group, i.e. α_S uncertainties are simply added in quadrature for CTEQ6.6, while for NNPDF2.0 the exact prescription is used as explained in arXiv:1004.0962.

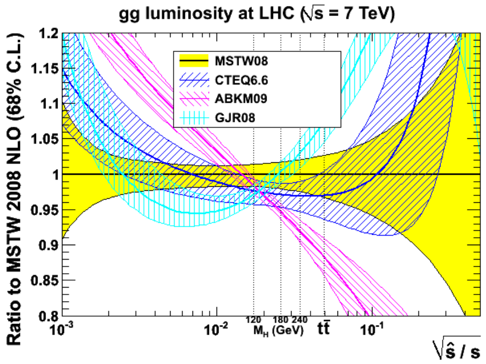
Note: (i) for MSTW08, uncertainty band similar at NNLO
(ii) everything here is at fixed scale $\mu=M_H$!

top

benchmark top cross sections







3

issues and outlook

issues and outlook

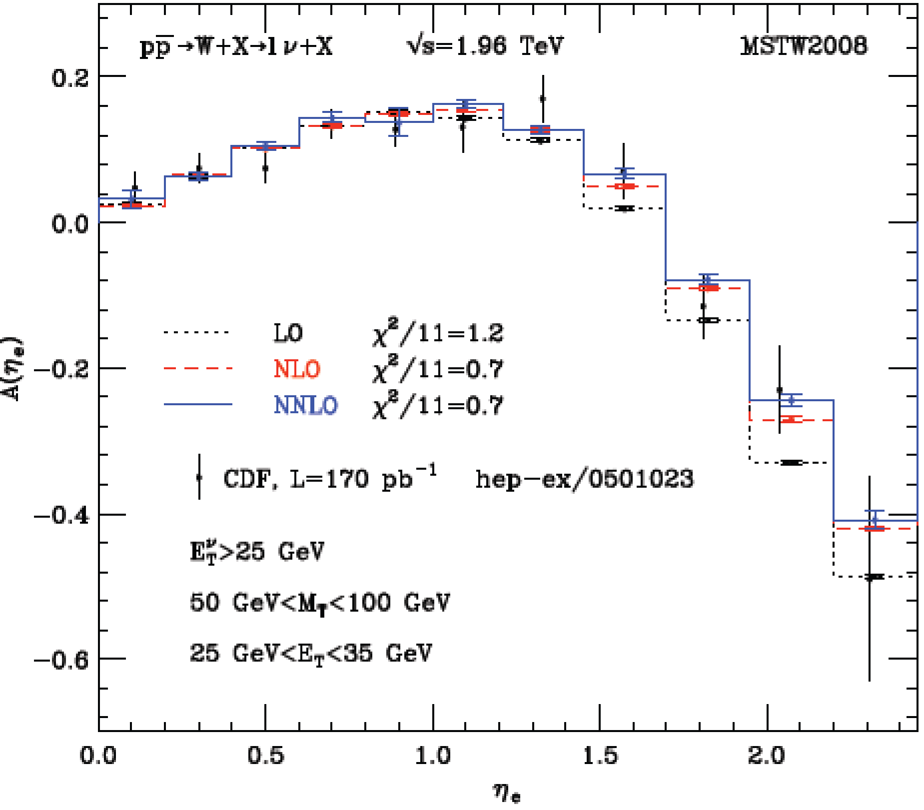
- continuing convergence between the various pdf sets

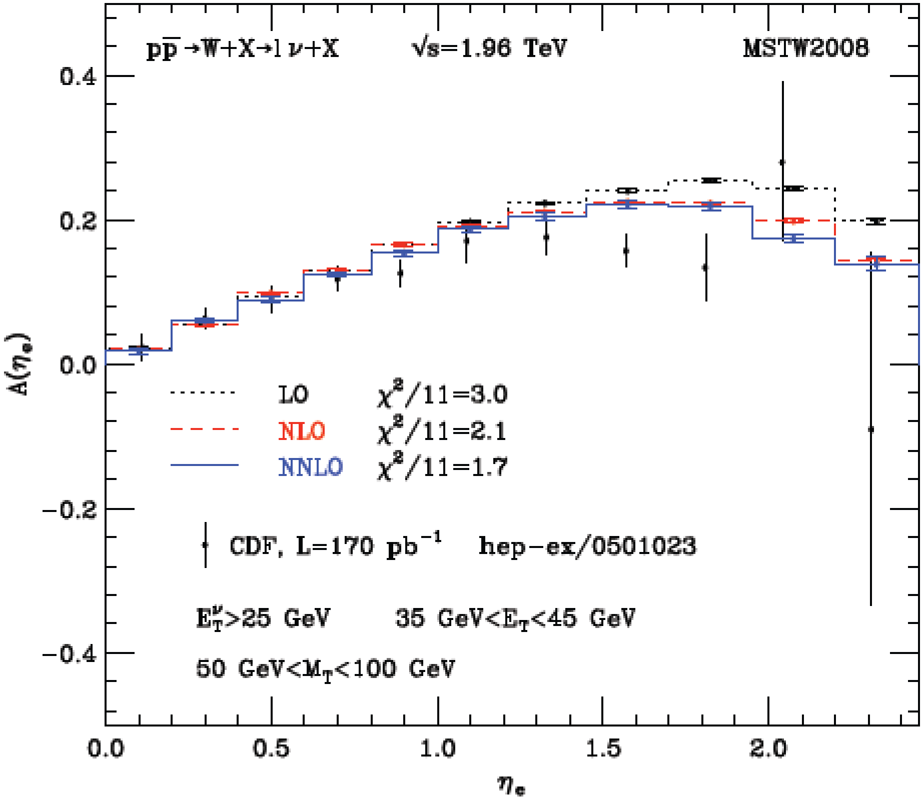
- outstanding issues include:
  - inclusion of combined HERA data (not yet in all fits)
  - difficulty of reconciling Run II Tevatron W asymmetry data
  - proper assessment of uncertainties due to treatment of heavy quark flavours (GM-VFNS optimal but not uniquely defined)
  - beyond NNLO? e.g. influence of $[\alpha_S \ln(1/x)]^n$ contributions
  - ‘QED pdfs’ (MSTW in preparation, *cf.* MRST 2004)

- much discussion (e.g. PDF4LHC workshops) among the pdf groups about how to define a ‘overall best’ theory prediction and uncertainty (be careful with `averaging’ and `envelopes’!)

- eagerly awaiting *precision* cross sections at 7 TeV!

Lepton asymmetry and CDF data



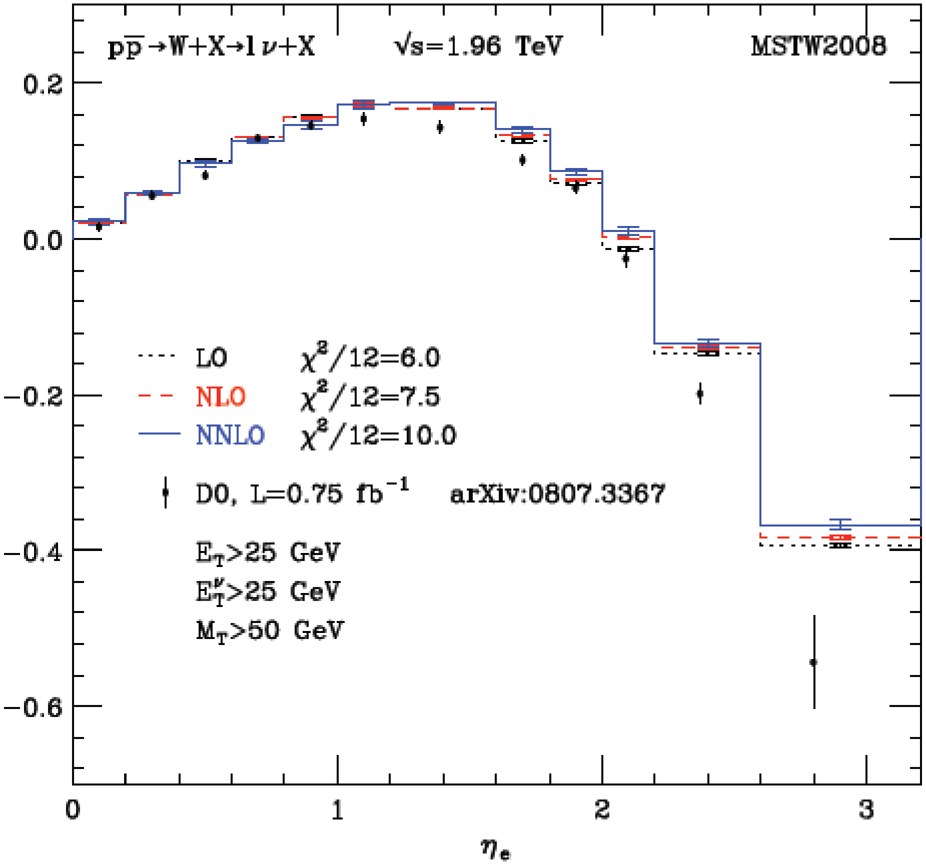


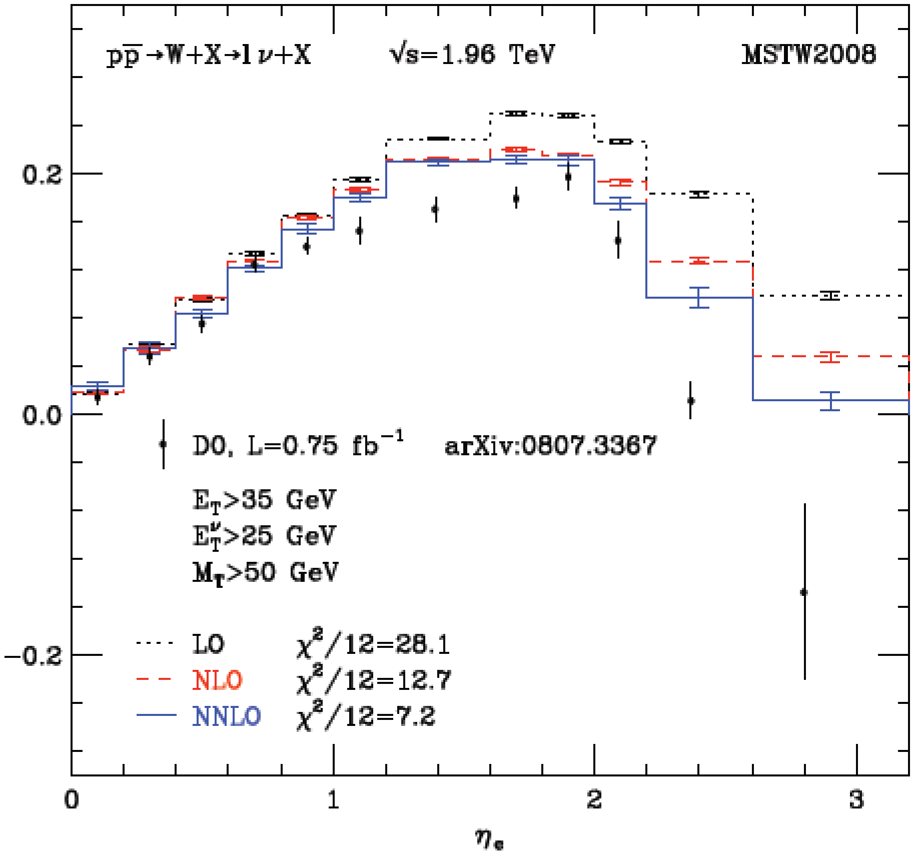
Recent progress in NNLO QCD calculations

Massimiliano Grazzini (INFN, Firenze)

HO10 CERN Theory Institute, 30 june 2010

Lepton asymmetry and new DØ data





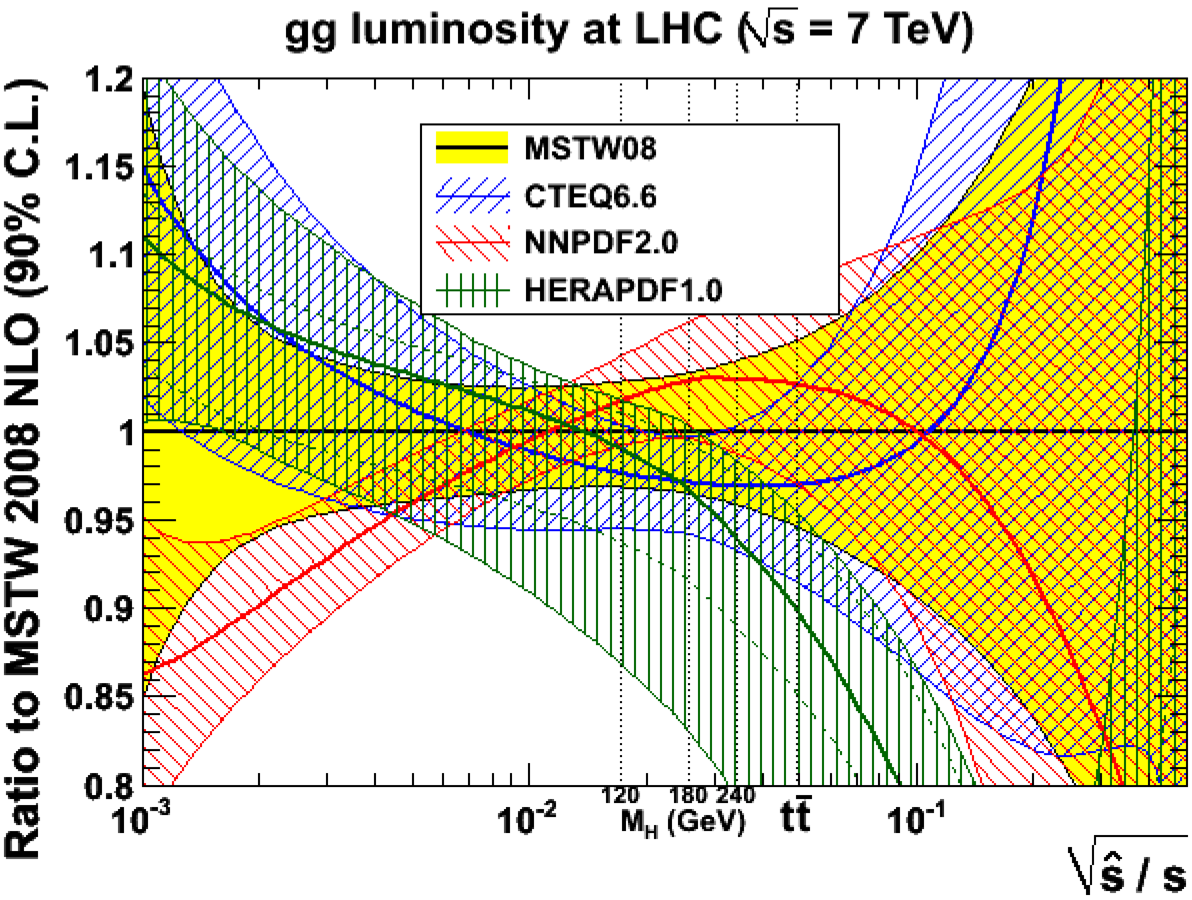
Recent progress in NNLO QCD calculations

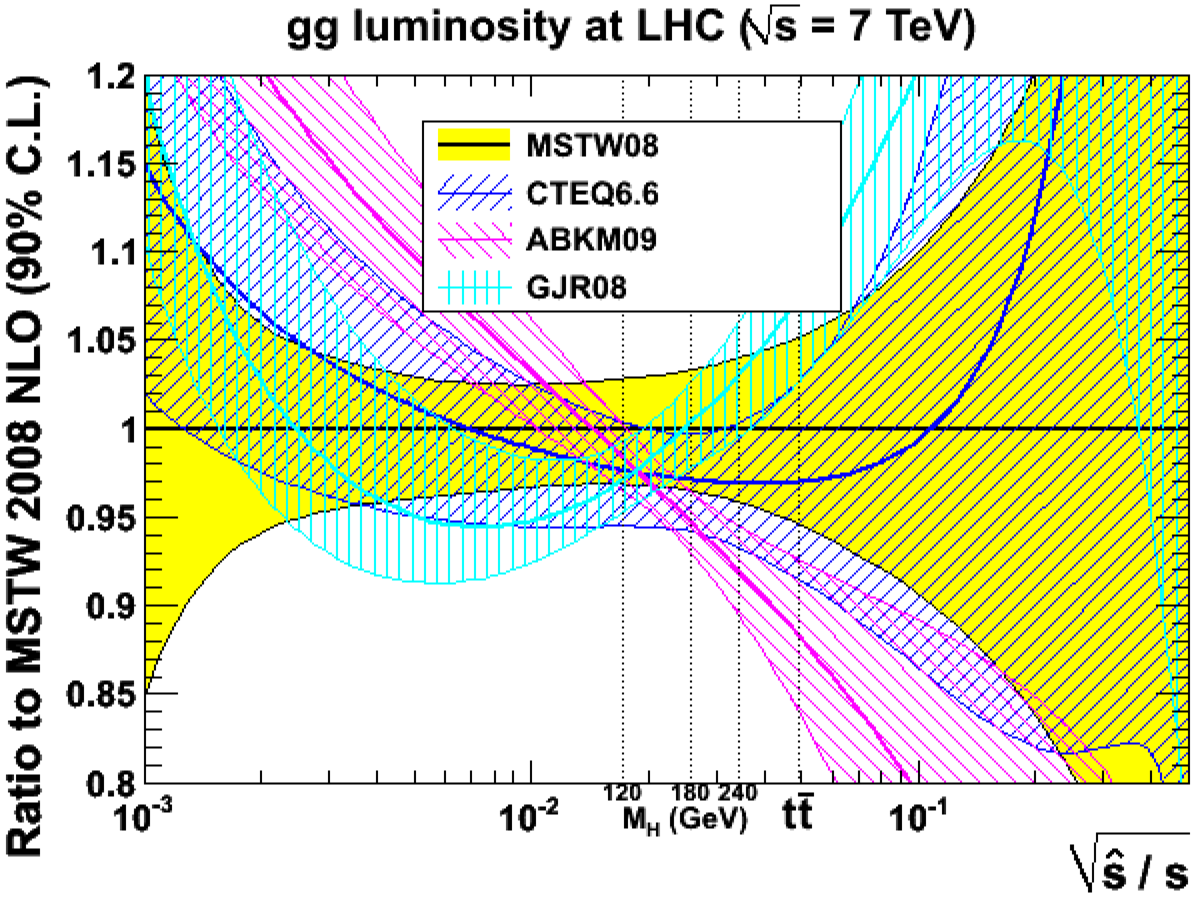
Massimiliano Grazzini (INFN, Firenze)

HO10 CERN Theory Institute, 30 june 2010

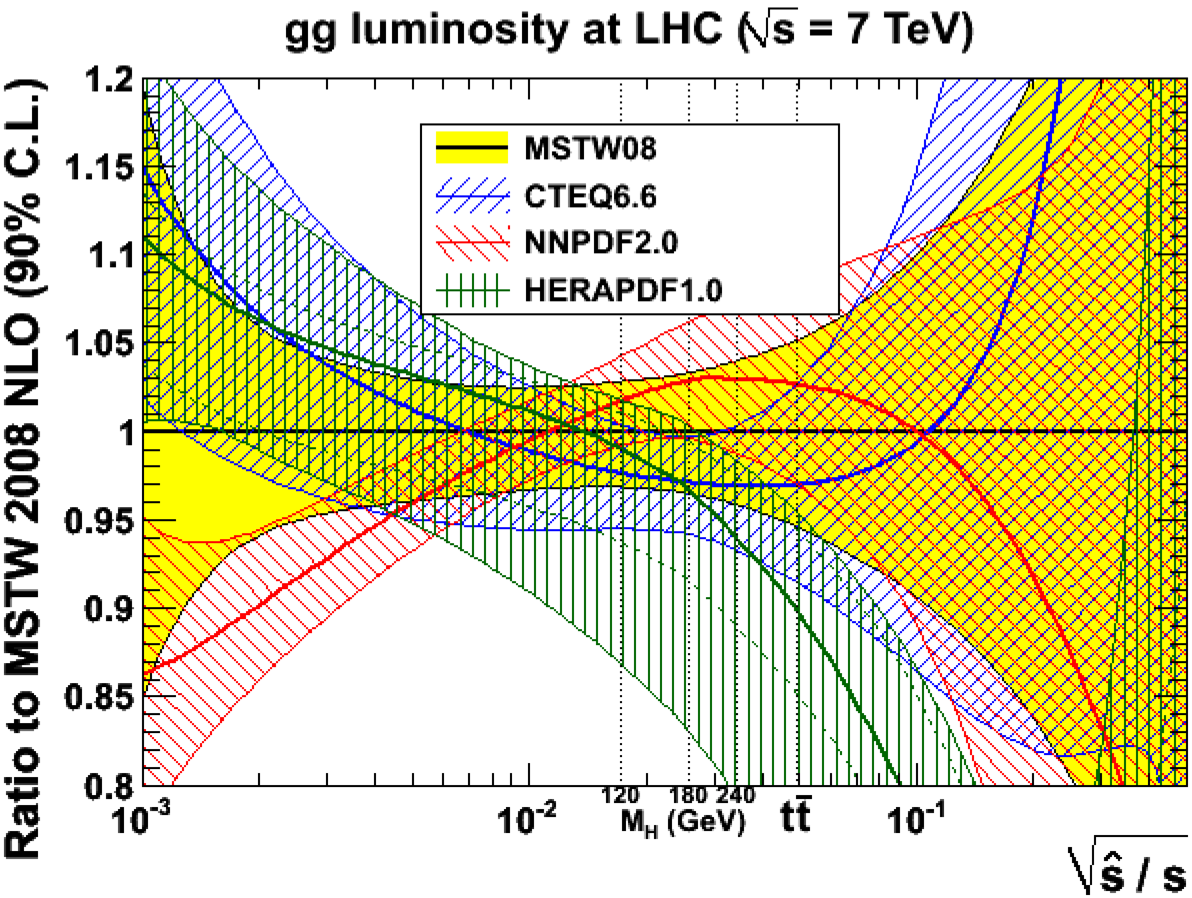
extra slides

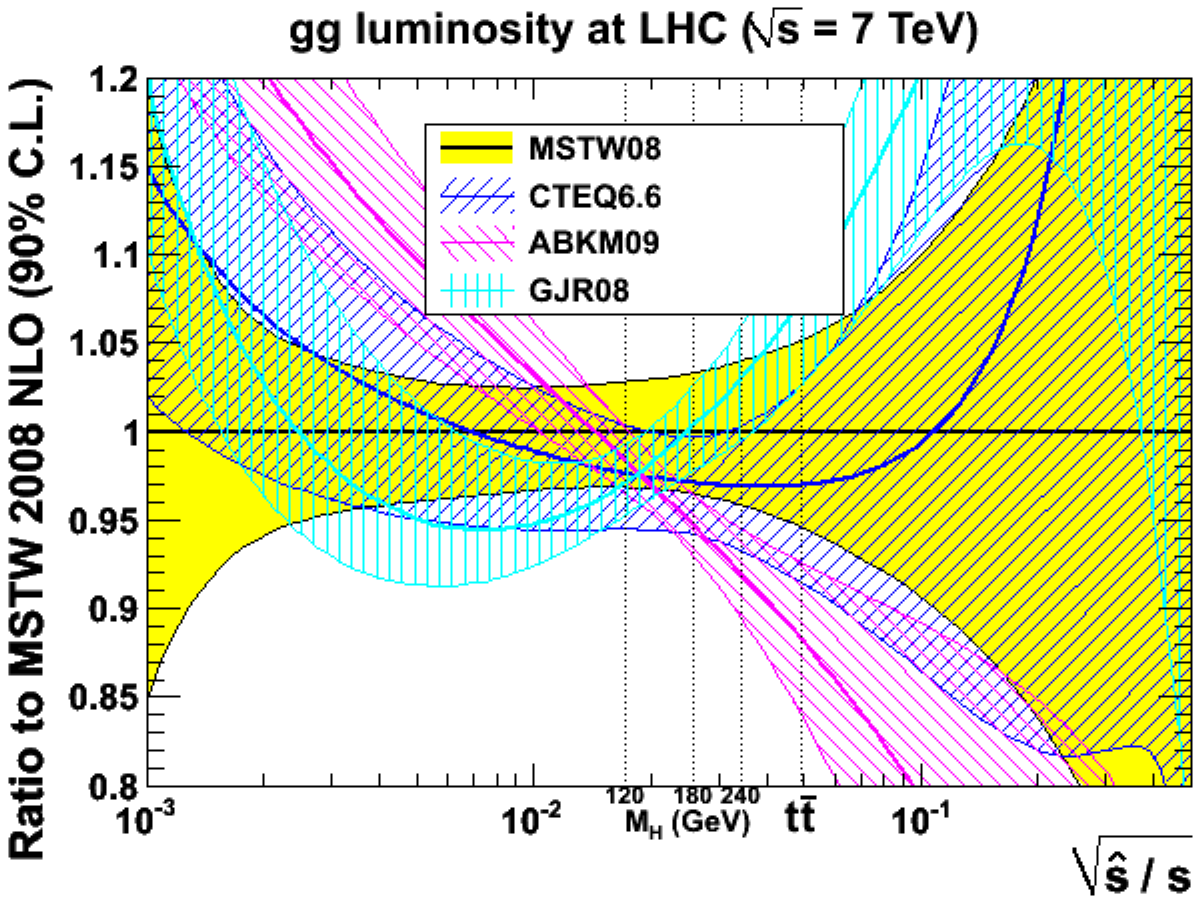
… the same at 90%cl




gg luminosity at LHC (√s = 7 TeV)
Ratio to MSTW 2008 NLO (90% C.L.)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
MSTW08
CTEQ6.6
ABKM09
GJR08
10^{-3}
10^{-2}
10^{-1}
120
180
240
M_H (GeV)
$t\bar{t}$
$\sqrt{\hat{s}/s}$

… the same at 90%cl




gg luminosity at LHC ($\sqrt{s}$ = 7 TeV)
MSTW08
CTEQ6.6
ABKM09
GJR08
Ratio to MSTW 2008 NLO (90% C.L.)
1.2
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0.9
0.85
0.8
10^{-3}
10^{-2}
120
180
240
M_H (GeV)
$t\bar{t}$
10^{-1}
$\sqrt{\hat{s}/s}$

heavy quarks: charm, bottom, …

considered sufficiently massive to allow pQCD treatment: $g \to Q\overline{Q}$

distinguish two regimes:
(i) $Q^2 \sim m_H^2$ include full m_H dependence to get correct threshold behaviour
(ii) $Q^2 \gg m_H^2$ treat as ~massless partons to resum $\alpha_S^n \log^n(Q^2/m_H^2)$ via DGLAP

FFNS: OK for (i) only **ZM-VFNS:** OK for (ii) only

consistent **GM**(=general mass)-**VFNS** now available (e.g. ACOT(χ), RT, BMSN,…) which interpolates smoothly between the two regimes

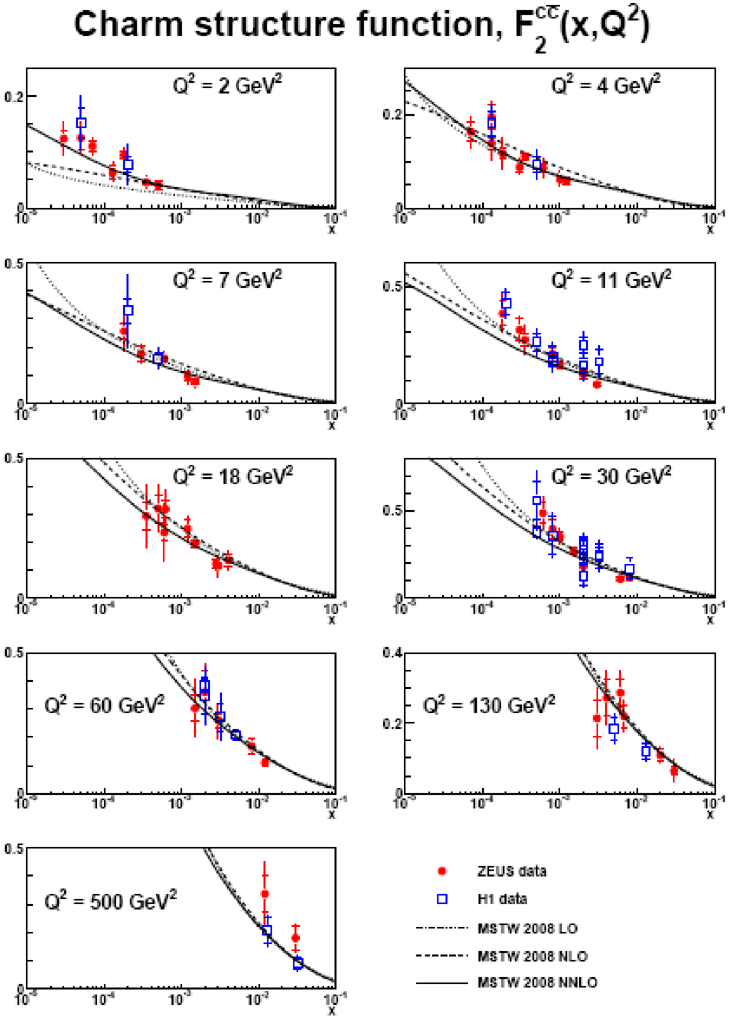
Aivazis, Collins, Olness,Tung; Roberts, Thorne; Buza, Matiounine, Smith, Migneron, van Neerven, …

Note:

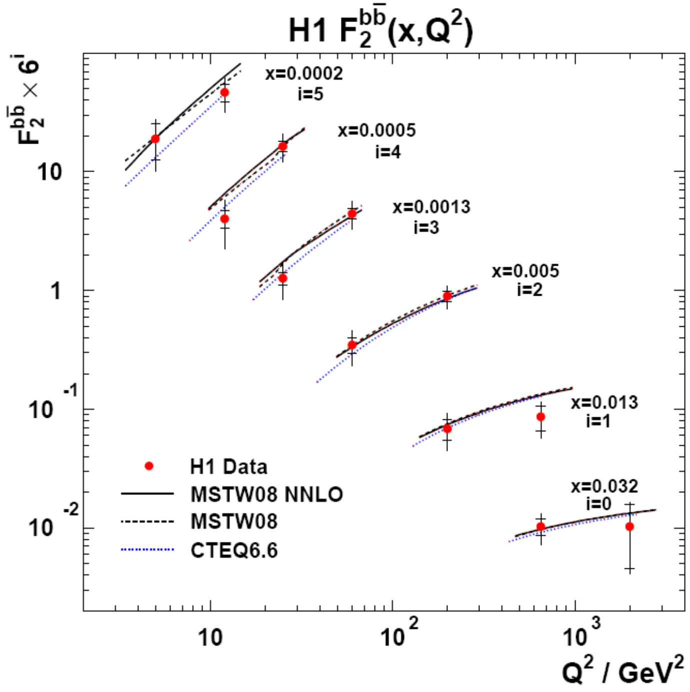
(i) the definition of these is tricky and non-unique (ambiguity in assignment of $O(m_H^2/Q^2)$ contributions), and the implementation of improved treatment (e.g. in going from MRST2004→MRST 2006 or CTEQ 6.1→6.5) can have a big effect on light partons

(ii) the *true* uncertainty on e.g. LHC predictions coming from ambiguities in the heavy quark treatment has yet to be quantified

charm and bottom structure functions



Charm structure function, $F_2^{c\bar{c}}(x,Q^2)$



H1 $F_2^{b\bar{b}}(x,Q^2)$

- MSTW 2008 uses *fixed* values of m_c = 1.4 GeV and m_b = 4.75 GeV in a GM-VFNS

- currently studying the sensitivity of the fit to these values, and impact on LHC cross sections

extrapolation uncertainties

theoretical insight for $x \to 0$:

$f \sim A\,x$

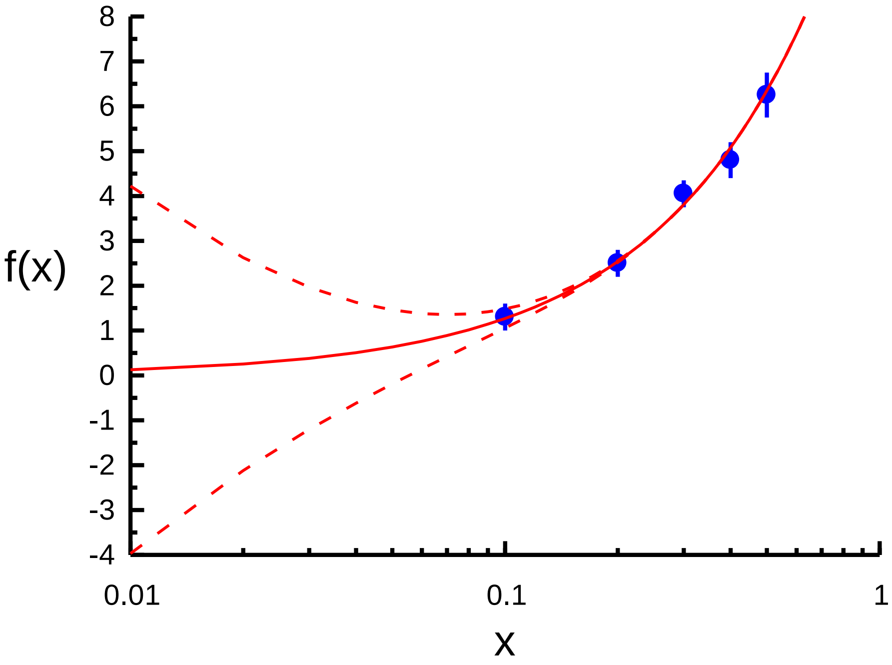
$f \sim A\,x^{\delta}$, $A > 0$

$f \sim A\,x^{\delta}$

no theoretical insight:

$f \sim ???$

…with only sum rules providing a constraint

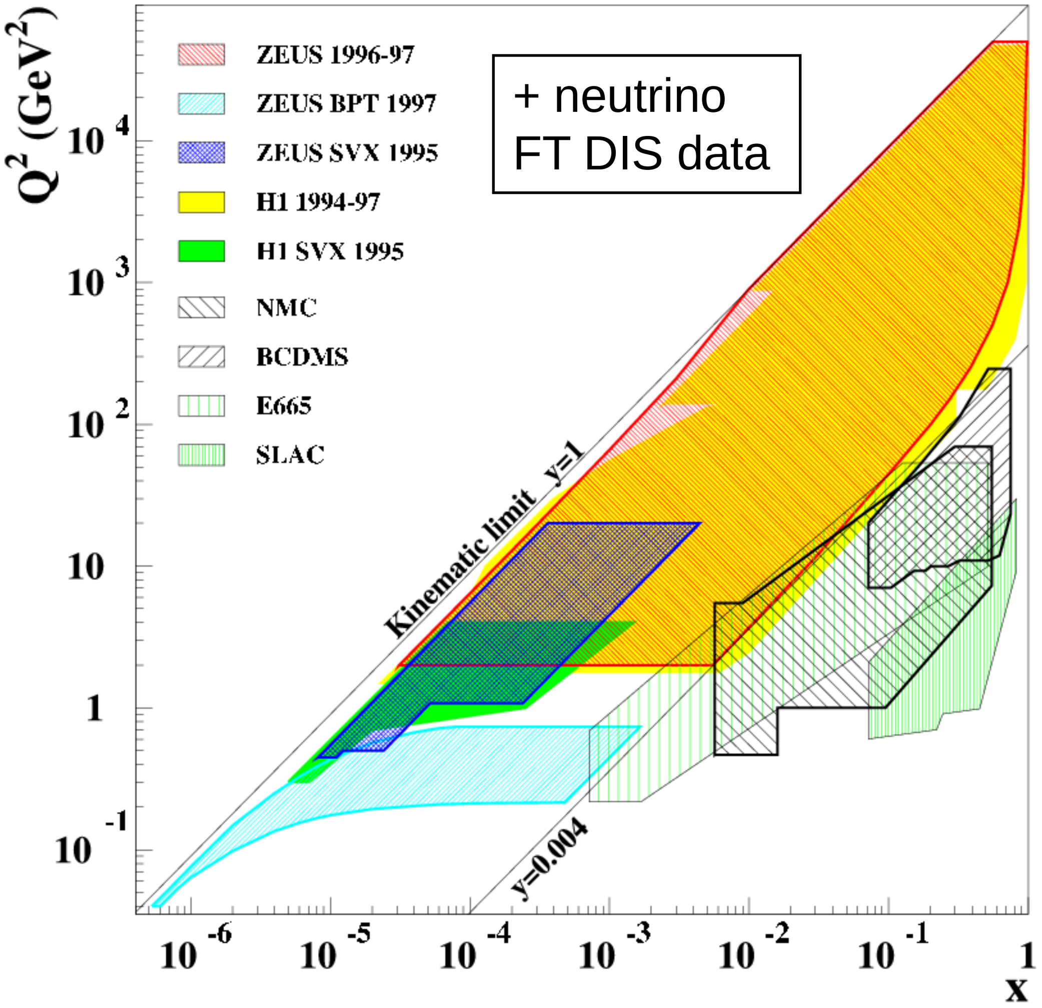


Examples:

(i) the MSTW negative small-x gluon at Q_0

(ii) the NNPDF ‘parameter free’ pdfs at small and large x

summary of DIS data

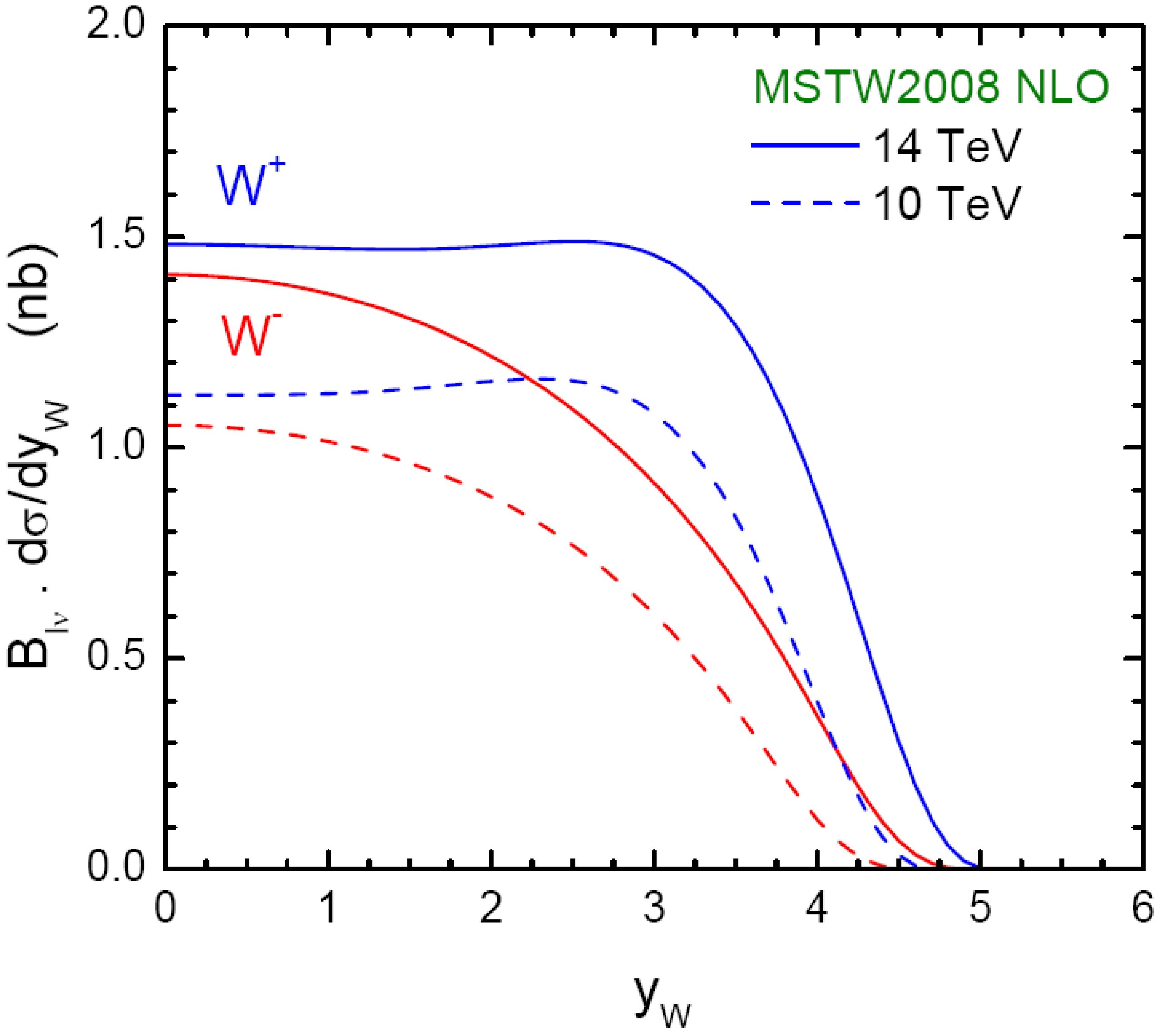


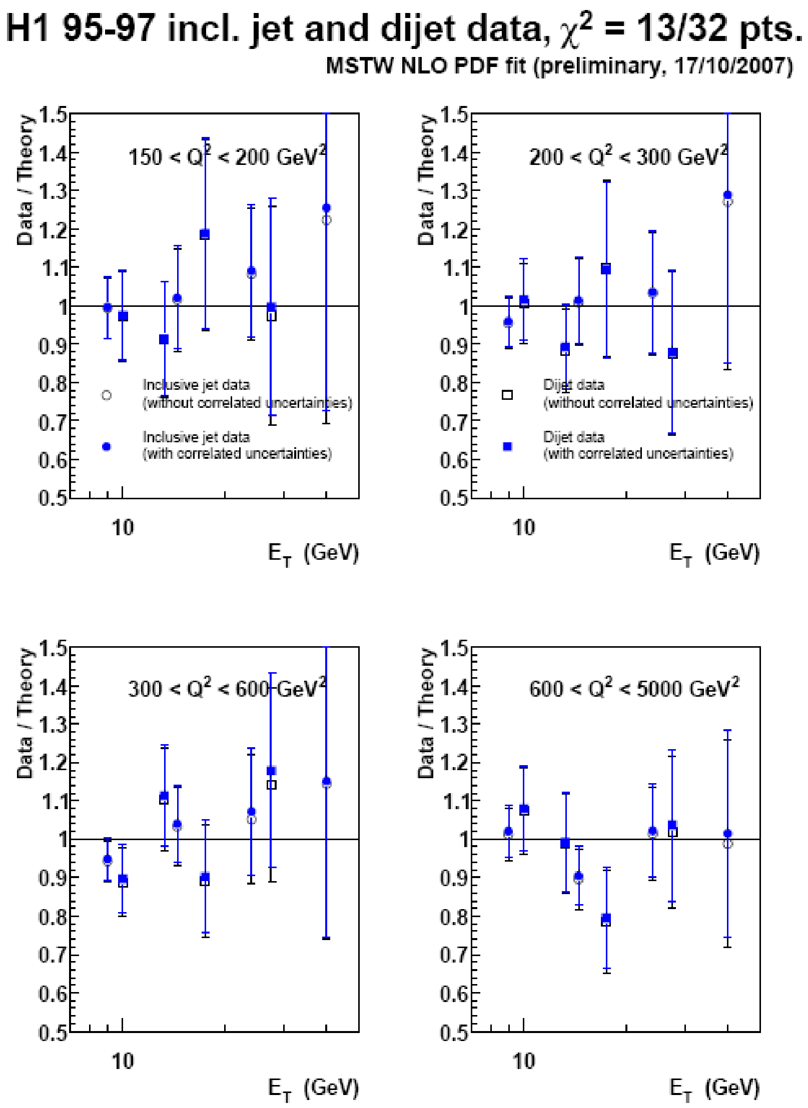
+ neutrino FT DIS data

Note: must impose cuts on DIS data to ensure validity of leading-twist DGLAP formalism in analyses to determine pdfs, typically:

$Q^2 > 2 - 4$ GeV^2

$W^2 = (1-x)/x\ Q^2 > 10 - 15$ GeV^2


MSTW2008 NLO
14 TeV
10 TeV
W^+
W^-
$B_{l\nu} \cdot d\sigma/dy_W$ (nb)
y_W
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6

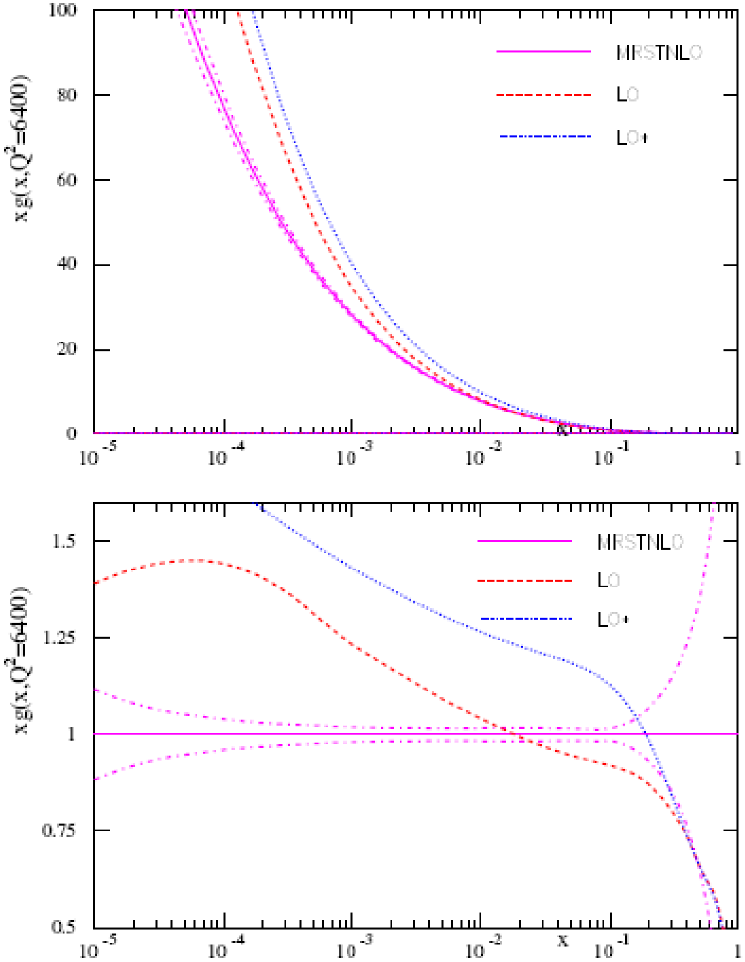


- only in NLO fit (no NNLO correction yet)

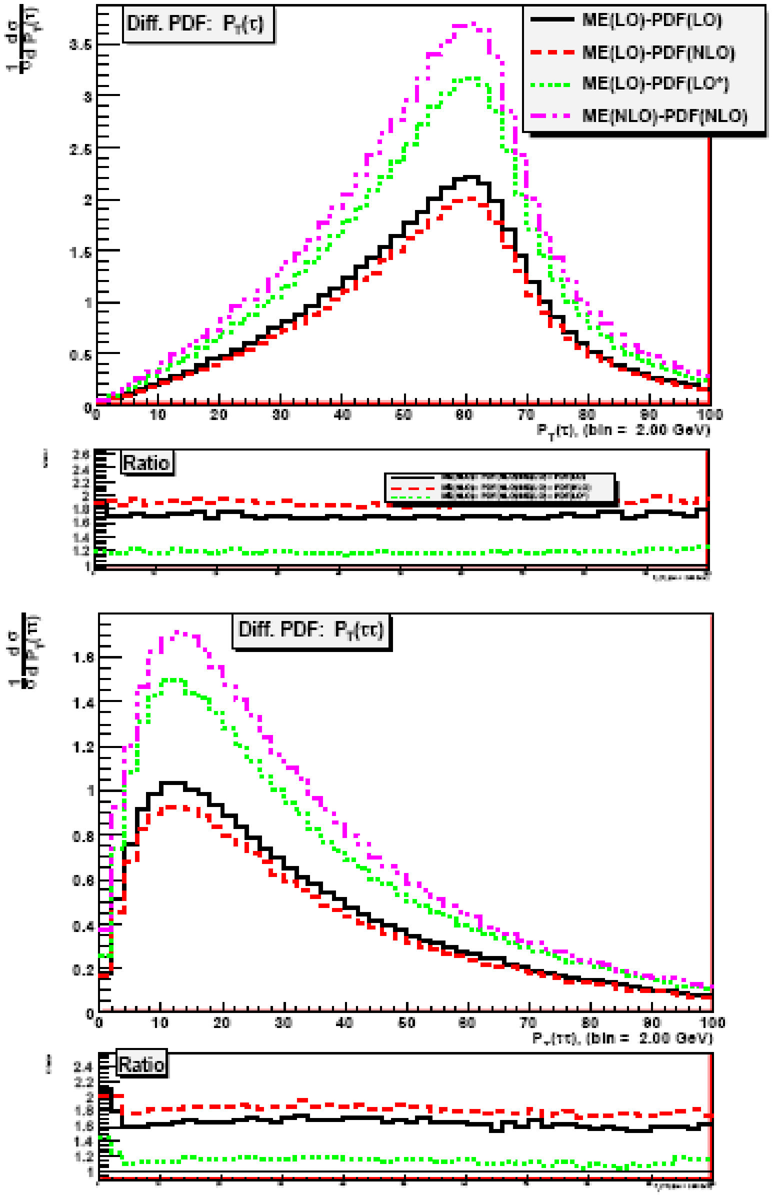
improved LO pdfs

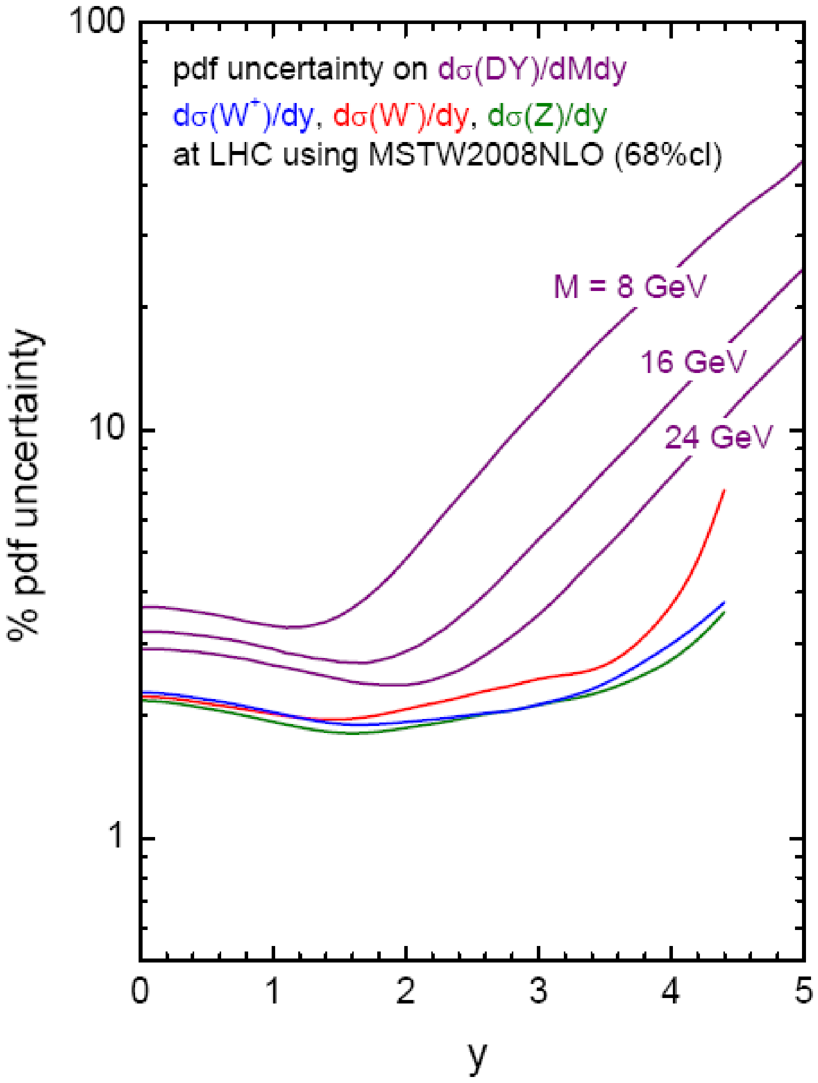
- conventional wisdom is to match pQCD order of pdfs with that of MEs
- but, in practice,
  - σ_{LO} = PDFs(LO) ⊗ ME(LO) can be different from σ_{NLO} = PDFs(NLO) ⊗ ME(NLO), in both shape and normalisation
  - LO pdfs have very poor χ^2 in (LO) global fit (no surprise: NLO corrections at large and small x are significant and preferred by the data)
- momentum conservation limits how much additional glue can be added to LO partons to compensate for missing NLO pQCD corrections (e.g. to get correct evolution rate of small-x quarks)
- therefore relax momentum conservation and redo LO fit; study the impact of this on χ^2, partons and cross sections
- e.g. Thorne & Shertsnev 2007: LO* partons
  - χ^2: 3066/2235 → 2691/2235, momentum conservation: 100% → 113%

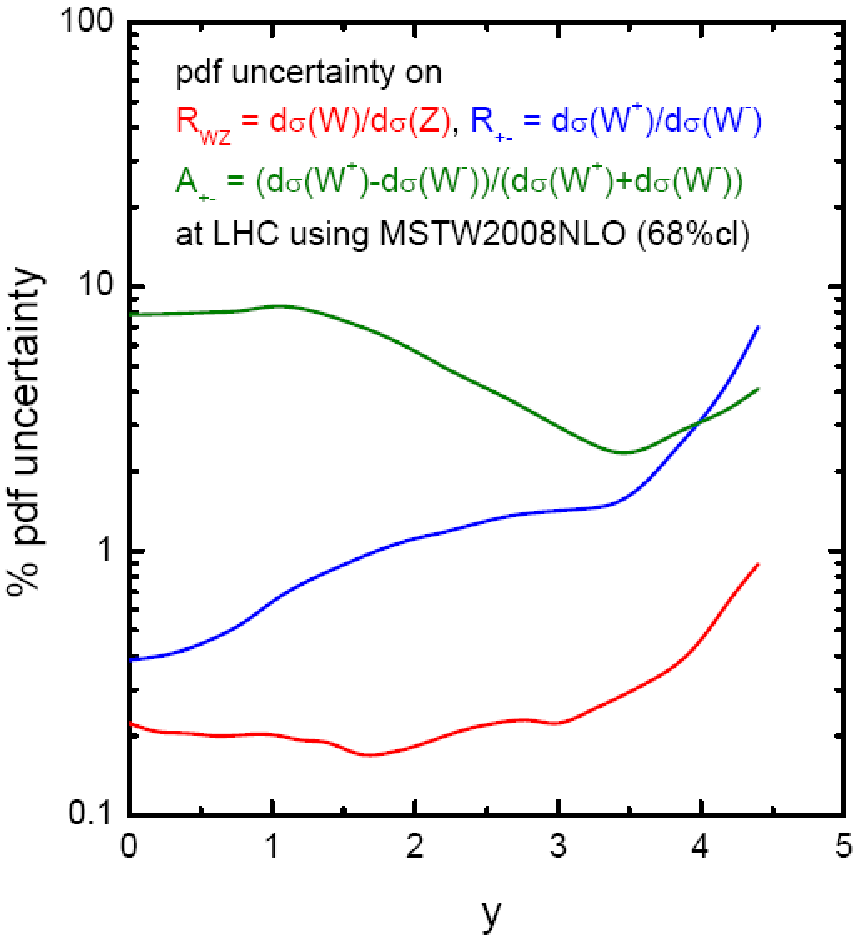
comparison of gluons at high Q^2



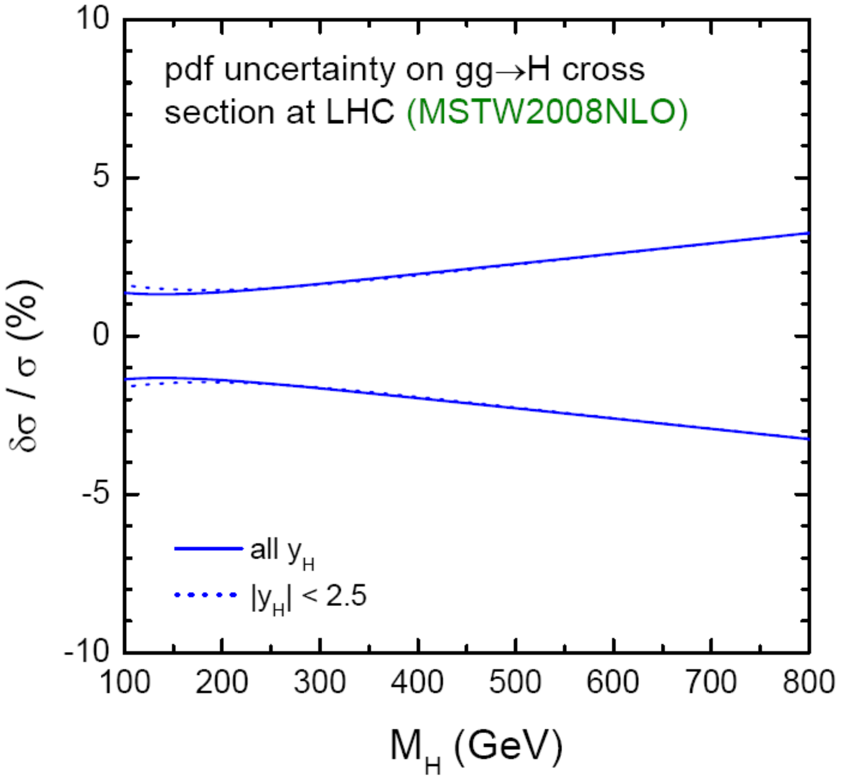
τ transverse momentum distribution in H → τ τ production at LHC



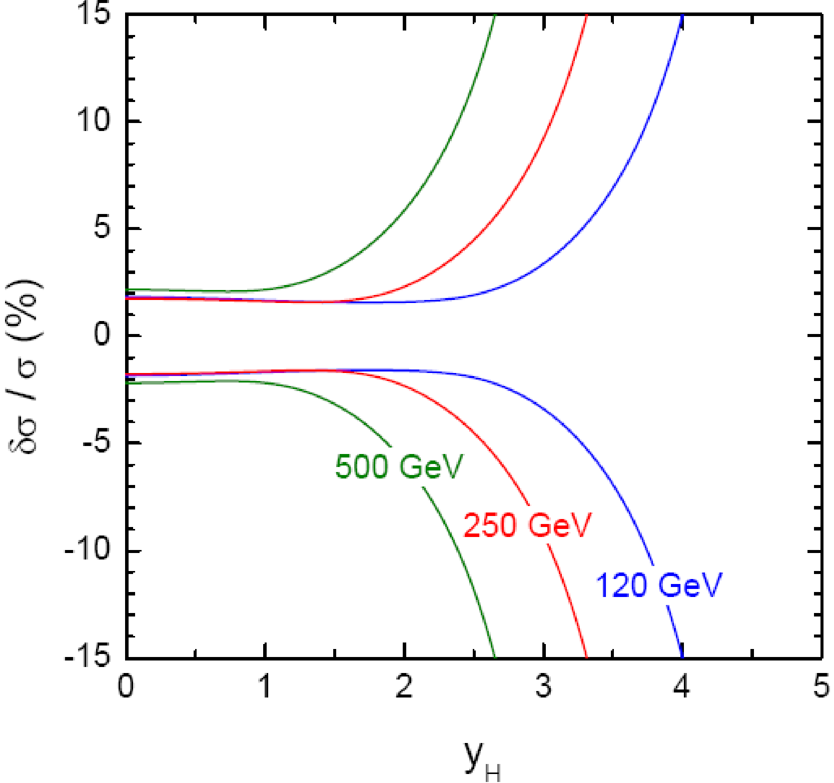

pdf uncertainty on dσ(DY)/dMdy
dσ(W+)/dy, dσ(W-)/dy, dσ(Z)/dy
at LHC using MSTW2008NLO (68%cl)
M = 8 GeV
16 GeV
24 GeV
% pdf uncertainty
100
10
1
0
1
2
3
4
5
y


pdf uncertainty on
R_WZ = dσ(W)/dσ(Z), R+- = dσ(W+)/dσ(W-)
A+- = (dσ(W+)-dσ(W-))/(dσ(W+)+dσ(W-))
at LHC using MSTW2008NLO (68%cl)
% pdf uncertainty
100
10
1
0.1
0
1
2
3
4
5
y

pdf uncertainty on σ(gg→H)



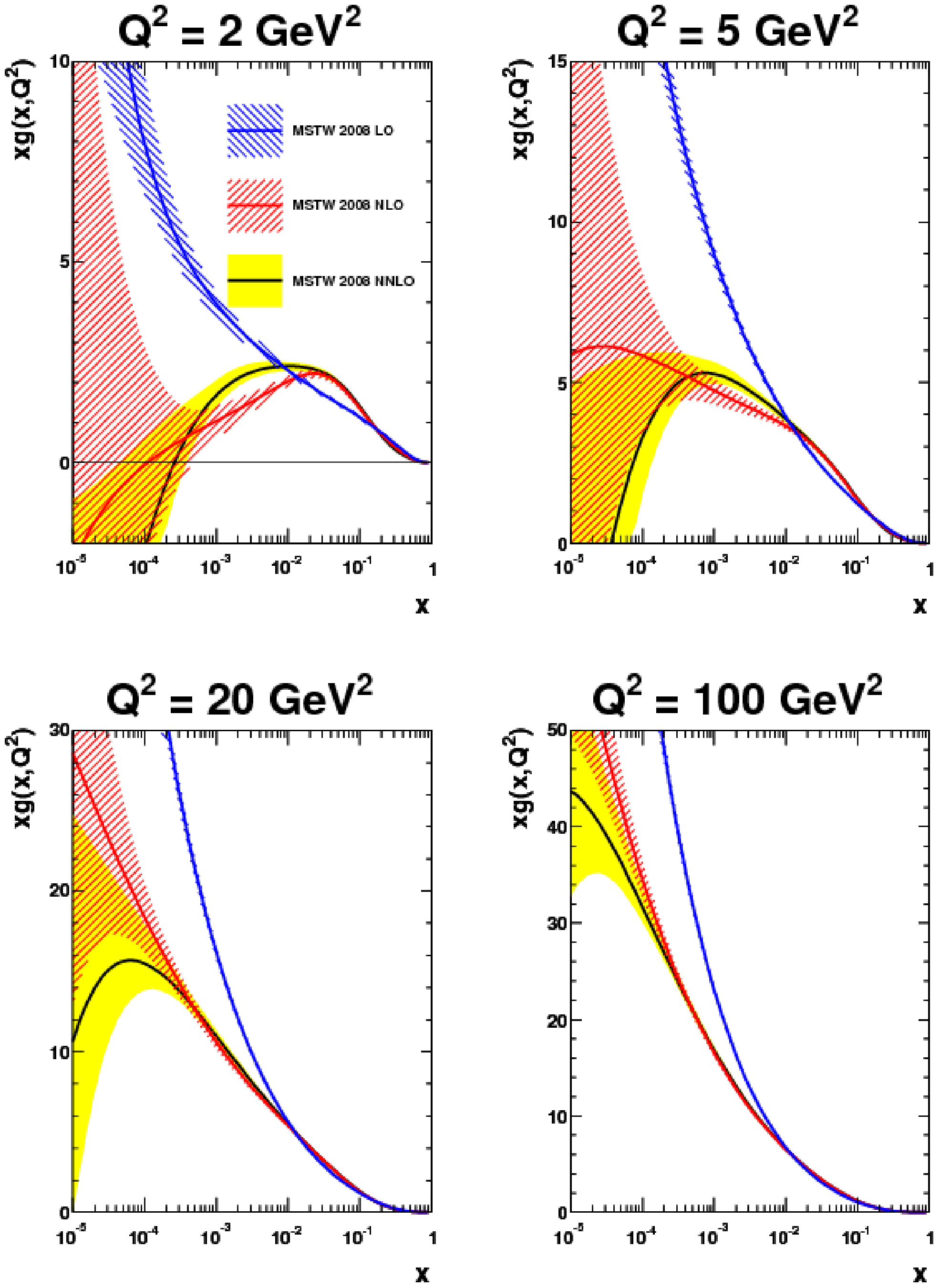
pdf uncertainty on gg→H cross section $d\sigma/dy_H$ at LHC (MSTW2008NLO)



→ typically 2-3% pdf uncertainty, except near edges of phase space comparison of gluons extracted from LO, NLO, NNLO global fits

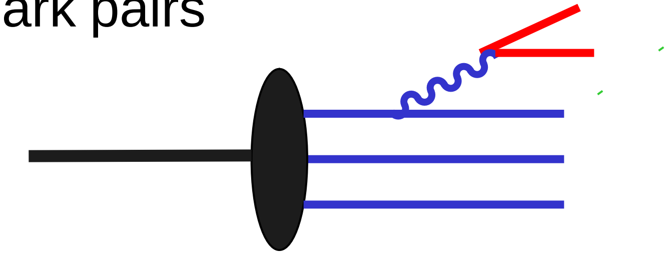
- large positive P_{qg} contributions at small x lead to smaller gluons at higher order

- clear instability at small x,Q^2 , and this is reflected in predictions for F_L (see later)



sea quarks

- the sea presumably arises when ‘primordial‘ valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs

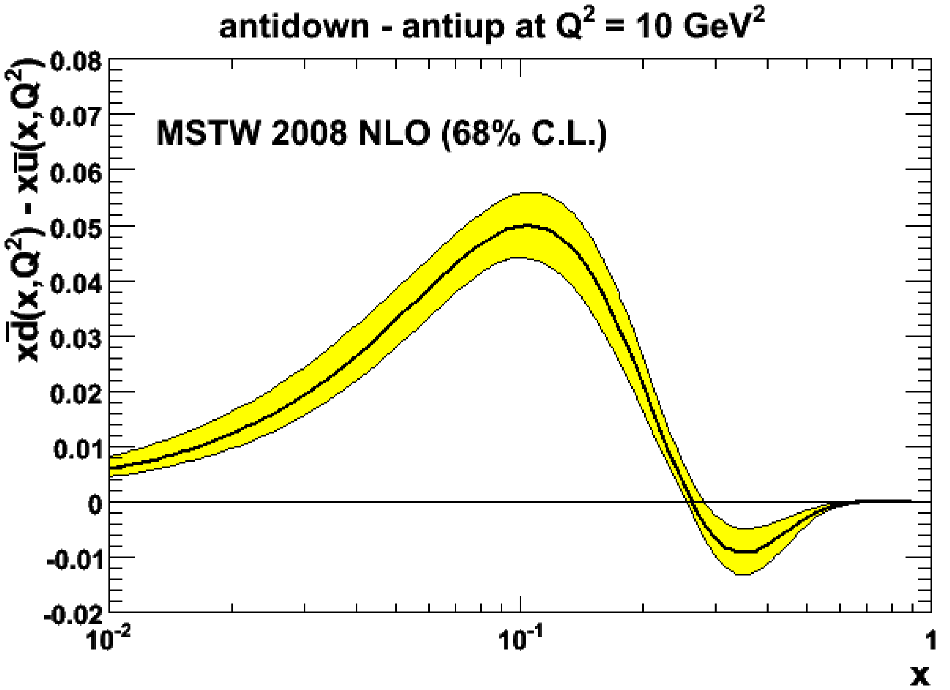


- so we naively expect

$$\bar{u} \approx \bar{d} > \bar{s} > \bar{c} > \ldots$$

- but why such a big d-u asymmetry? Meson cloud, Pauli exclusion, …?

The ratio of Drell-Yan cross sections for *pp,pn* → $\mu^+\mu^- + X$ provides a measure of the difference between the *u* and *d* sea quark distributions



strange

earliest pdf fits had SU(3) symmetry: $s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \bar{u}(x,Q_0^2) = \bar{d}(x,Q_0^2)$

later relaxed to include (constant) strange suppression (cf. fragmentation):

$$s(x,Q_0^2) = \bar{s}(x,Q_0^2) = \frac{\kappa}{2}\left[\bar{u}(x,Q_0^2) + \bar{d}(x,Q_0^2)\right]$$

with $\kappa = 0.4 - 0.5$

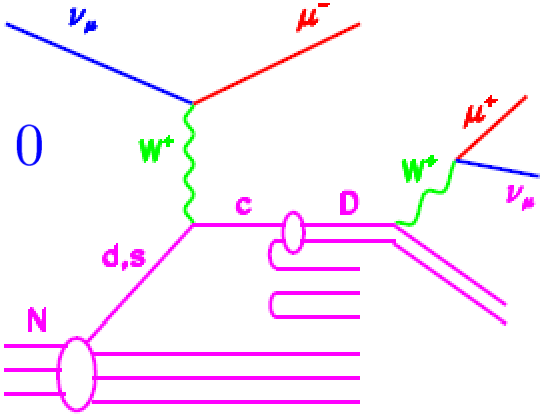
nowadays, dimuon production in υN DIS (CCFR, NuTeV) allows ‘direct’ determination:

$$\frac{d\sigma}{dxdy}(\nu_\mu(\bar{\nu}_\mu)N \to \mu^+\mu^- X) = B_c\,\mathcal{N}\mathcal{A}\,\frac{d\sigma}{dxdy}(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \to c\mu^-(\bar{c}\mu^+)X)$$

in the range $0.01 < x < 0.4$

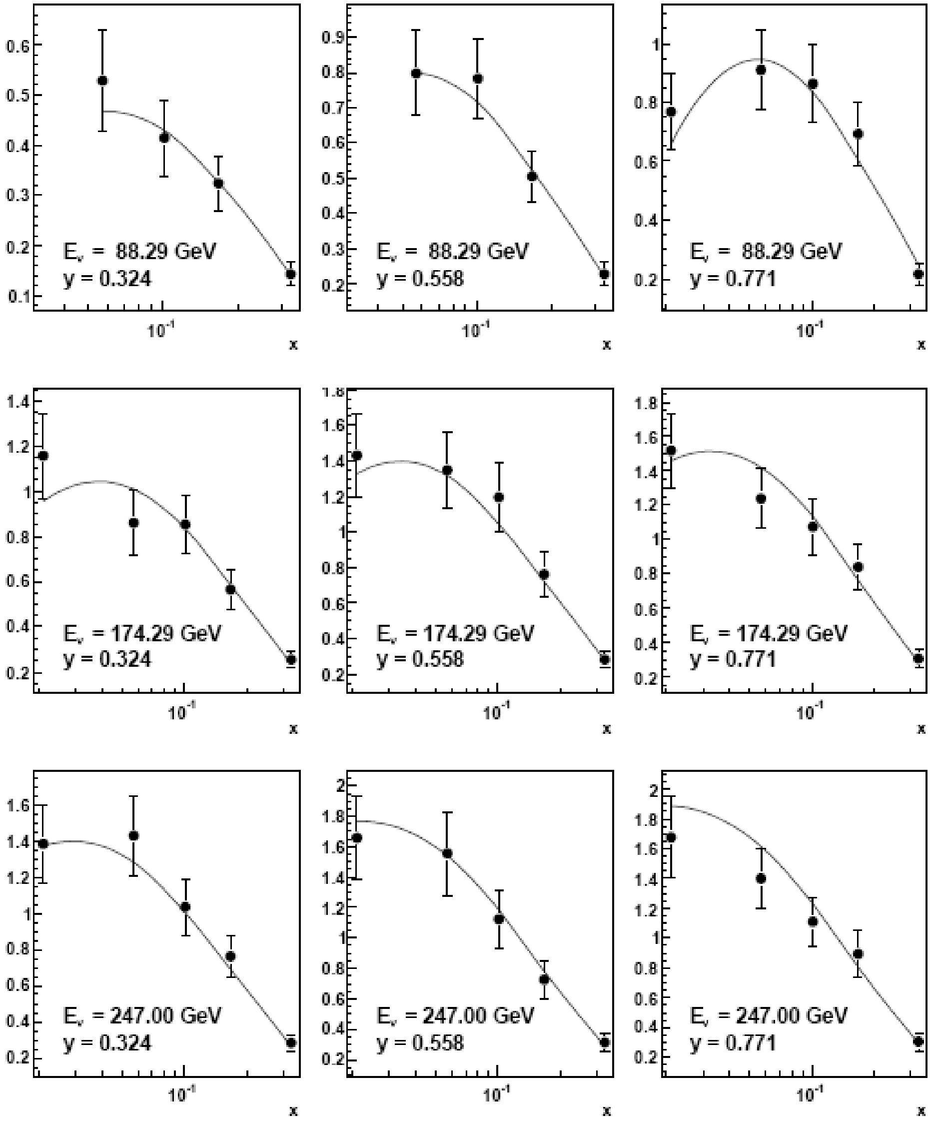
data seem to slightly prefer $s(x,Q_0^2) - \bar{s}(x,Q_0^2) \neq 0$

theoretical explanation?!

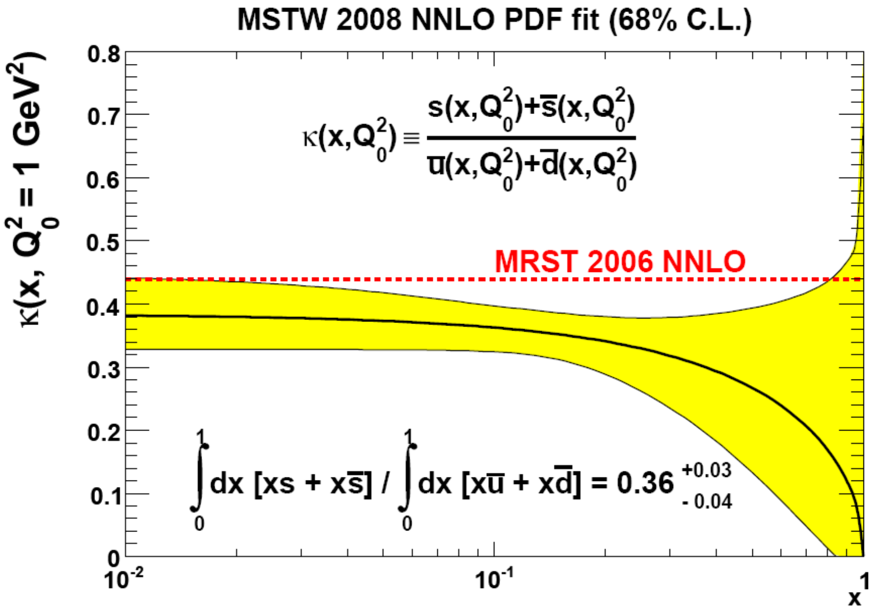


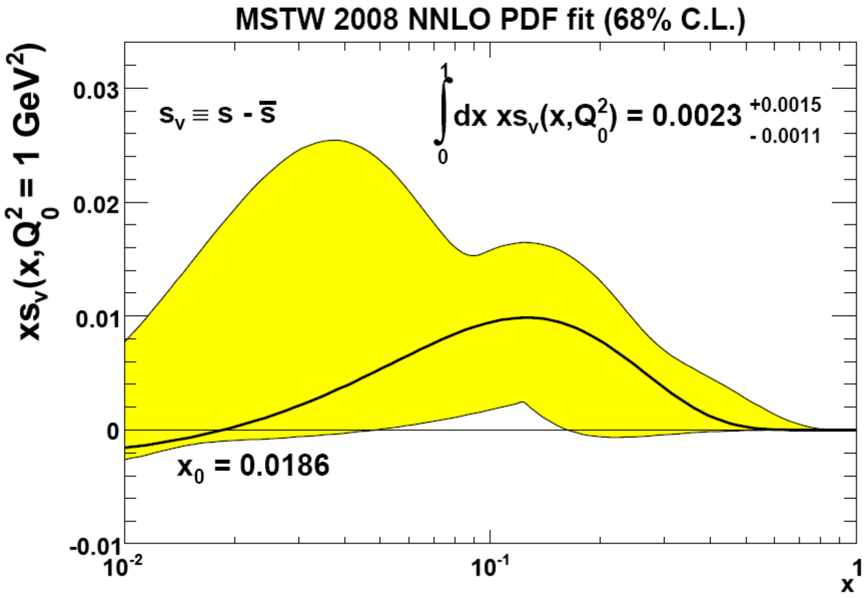
NuTeV $\frac{100\pi}{G_F^2 M_N E_\nu}\frac{d\sigma}{dxdy}(\nu_\mu N \to \mu^+\mu^- X)$ in GeV^{-2}

MSTW 2008 NNLO PDF fit, χ^2 = 13 for 21 DOF



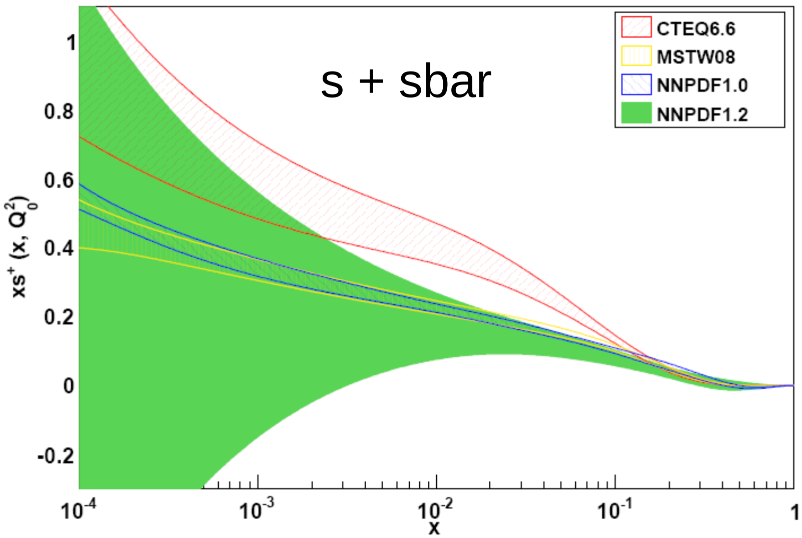
MSTW

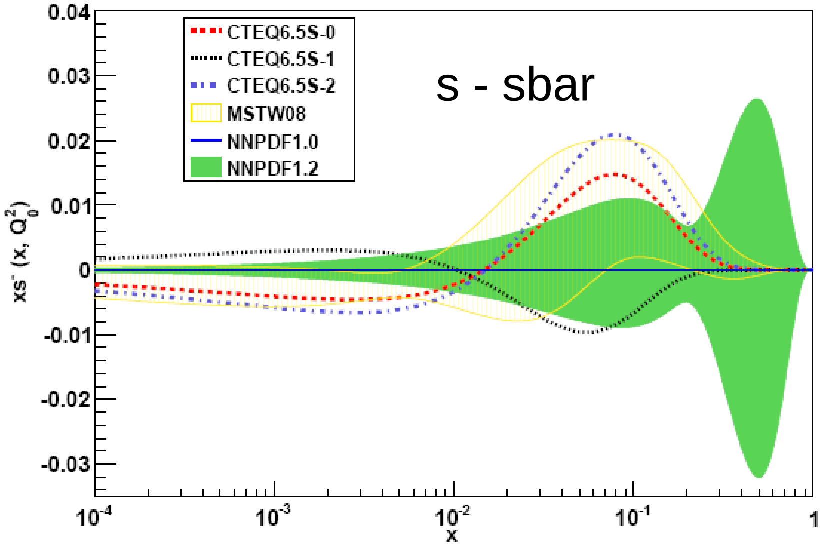

MSTW 2008 NNLO PDF fit (68% C.L.)
$\kappa(x, Q_0^2 = 1\ \mathrm{GeV}^2)$
$\kappa(x,Q_0^2) \equiv \frac{s(x,Q_0^2)+\bar{s}(x,Q_0^2)}{\bar{u}(x,Q_0^2)+\bar{d}(x,Q_0^2)}$
MRST 2006 NNLO
$\int_0^1 dx\,[xs + x\bar{s}] / \int_0^1 dx\,[x\bar{u} + x\bar{d}] = 0.36^{+0.03}_{-0.04}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10^{-2}
10^{-1}
1
x


MSTW 2008 NNLO PDF fit (68% C.L.)
$xs_v(x,Q_0^2 = 1\ \mathrm{GeV}^2)$
$s_v \equiv s - \bar{s}$
$\int_0^1 dx\, xs_v(x,Q_0^2) = 0.0023^{+0.0015}_{-0.0011}$
$x_0 = 0.0186$
-0.01
0
0.01
0.02
0.03
10^{-2}
10^{-1}
1
x

MSTW

strange quark in NNPDF





Note:

MSTW: assume u,d,s quarks have same x^δ behaviour as $x \to 0$

NuTeV $\sin^2\theta_W$ anomaly largely removed

α_S

arXiv:0905.3531 [hep-ph]

NLO	$\alpha_S(M_Z^2)$ (expt. unc. only)
MSTW (this work)	$0.1202\ ^{+0.0012}_{-0.0015}$
CTEQ [2]	0.1170 ± 0.0047
H1 [23]	0.1150 ± 0.0017
ZEUS [48]	0.1183 ± 0.0028
Alekhin [57]	0.1171 ± 0.0015
BBG [58]	0.1148 ± 0.0019
GJR [59]	0.1145 ± 0.0018

NNLO	$\alpha_S(M_Z^2)$ (expt. unc. only)
MSTW (this work)	$0.1171\ ^{+0.0014}_{-0.0014}$
AMP [60]	0.1128 ± 0.0015
BBG [58]	$0.1134\ ^{+0.0019}_{-0.0021}$
ABKM [61]	0.1129 ± 0.0014
JR [62]	0.1158 ± 0.0035

- reasonable consistency between different analyses
- MSTW values slightly higher because of smaller low-x gluon needed for high-p_T Tevatron jet fit

MSTW (2009):
full global NLO and NNLO fit

CTEQ (2008):
full global NLO fit

H1 (2001):
H1 + BCDMS

ZEUS (2005):
ZEUS inc. DIS-JET + photoprodn.

BBG = Blumlein, Bottcher, Guffanti (2006):
non-singlet DIS analysis

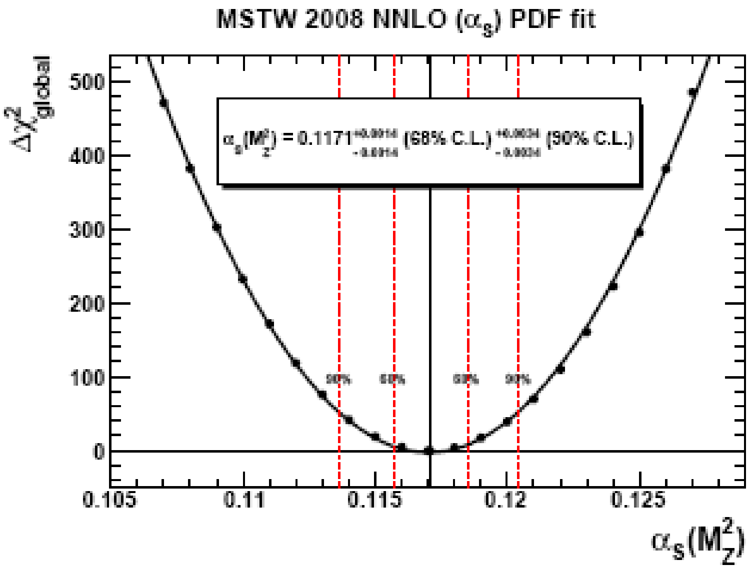
AMP = Alekhin, Melnikov, Petriello (2006):
DIS + DY

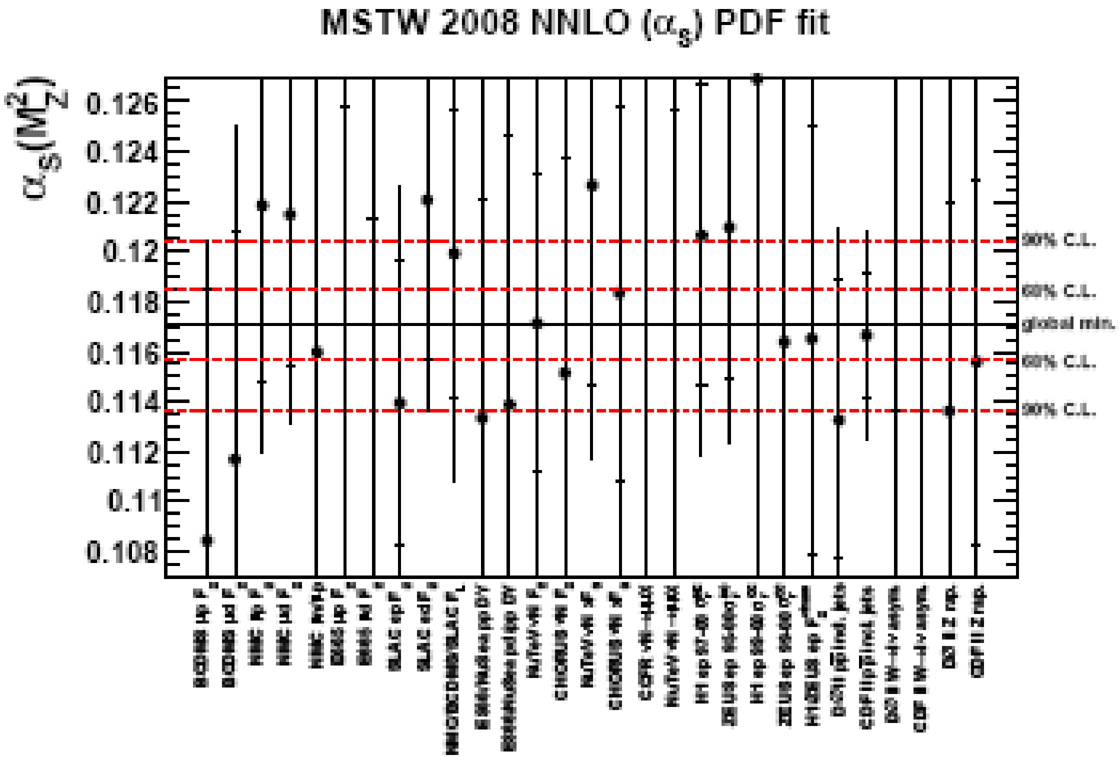
GJR = Gluck, Jimenez-Delgado, Reya (2008):
DIS + DY + Tevatron jet

JR = Jimenez-Delgado, Reya (2009):
DIS + DY

ABKM = Alekhin, Blumlein, Klein, Moch (2009):
DIS + DY

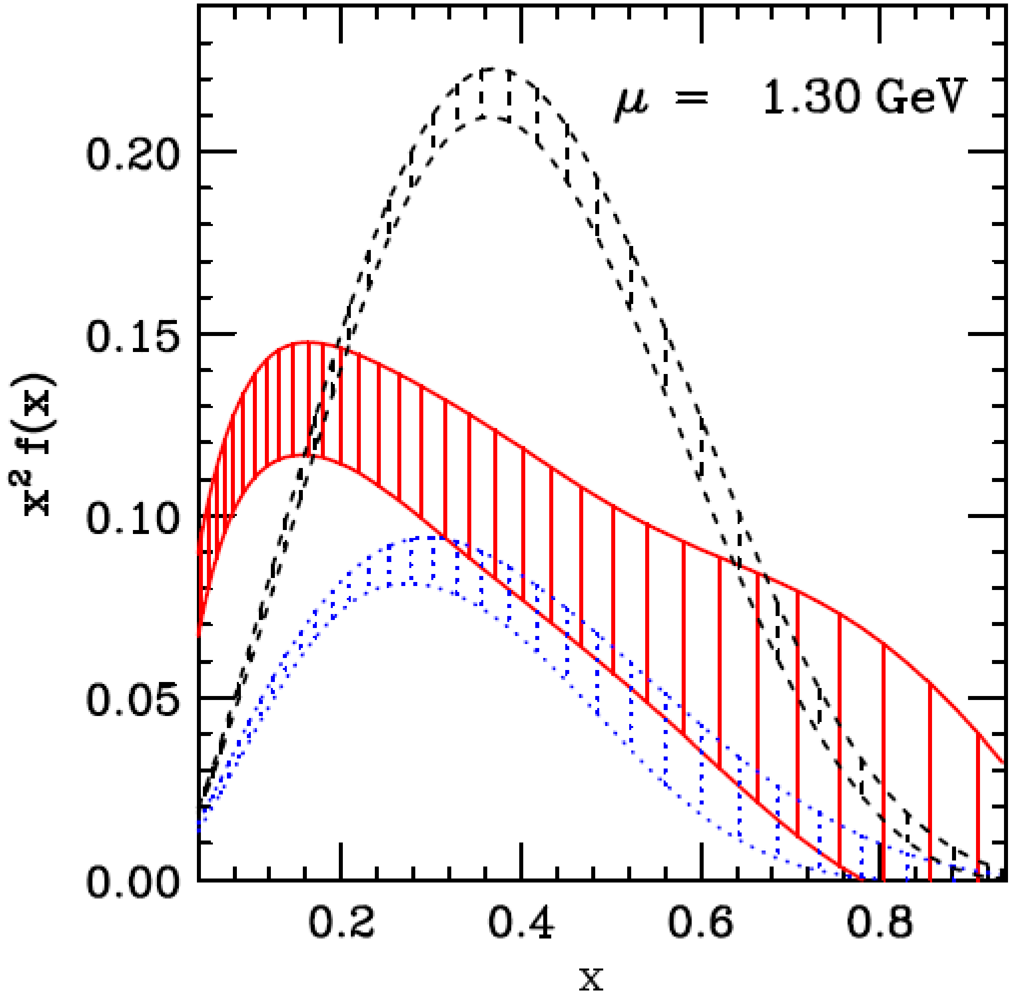
PDG(2008): $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$


MSTW 2008 NNLO (α_S) PDF fit
$\Delta\chi^2_{\text{global}}$
500
400
300
200
100
0
0.105
0.11
0.115
0.12
0.125
$\alpha_S(M_Z^2)$
$\alpha_S(M_Z^2) = 0.1171^{+0.0014}_{-0.0014}$ (68% C.L.) $^{+0.0034}_{-0.0034}$ (90% C.L.)
90%
68%
68%
90%


MSTW 2008 NNLO (α_S) PDF fit
$\alpha_S(M_Z^2)$
0.126
0.124
0.122
0.12
0.118
0.116
0.114
0.112
0.11
0.108
90% C.L.
68% C.L.
global min.
68% C.L.
90% C.L.
BCDMS μp F_2
BCDMS μd F_2
NMC μp F_2
NMC μd F_2
NMC μn/μp
E665 μp F_2
E665 μd F_2
SLAC ep F_2
SLAC ed F_2
NMC/BCDMS/SLAC F_L
E866/NuSea pp DY
E866/NuSea pd/pp DY
NuTeV νN F_2
CHORUS νN F_2
NuTeV νN xF_3
CHORUS νN xF_3
CCFR νN→μμX
NuTeV νN→μμX
H1 ep 97-00 σ_r^{NC}
ZEUS ep 95-00 σ_r^{NC}
H1 ep 99-00 σ_r^{CC}
ZEUS ep 99-00 σ_r^{CC}
H1/ZEUS ep F_2^{charm}
DØ II pp̄ incl. jets
CDF II pp̄ incl. jets
DØ II W→lν asym.
CDF II W→lν asym.
DØ II Z rap.
CDF II Z rap.

CT09G fit to Run I and Run II jet data simultaneously, find much harder gluon (with more flexible parameterisation)

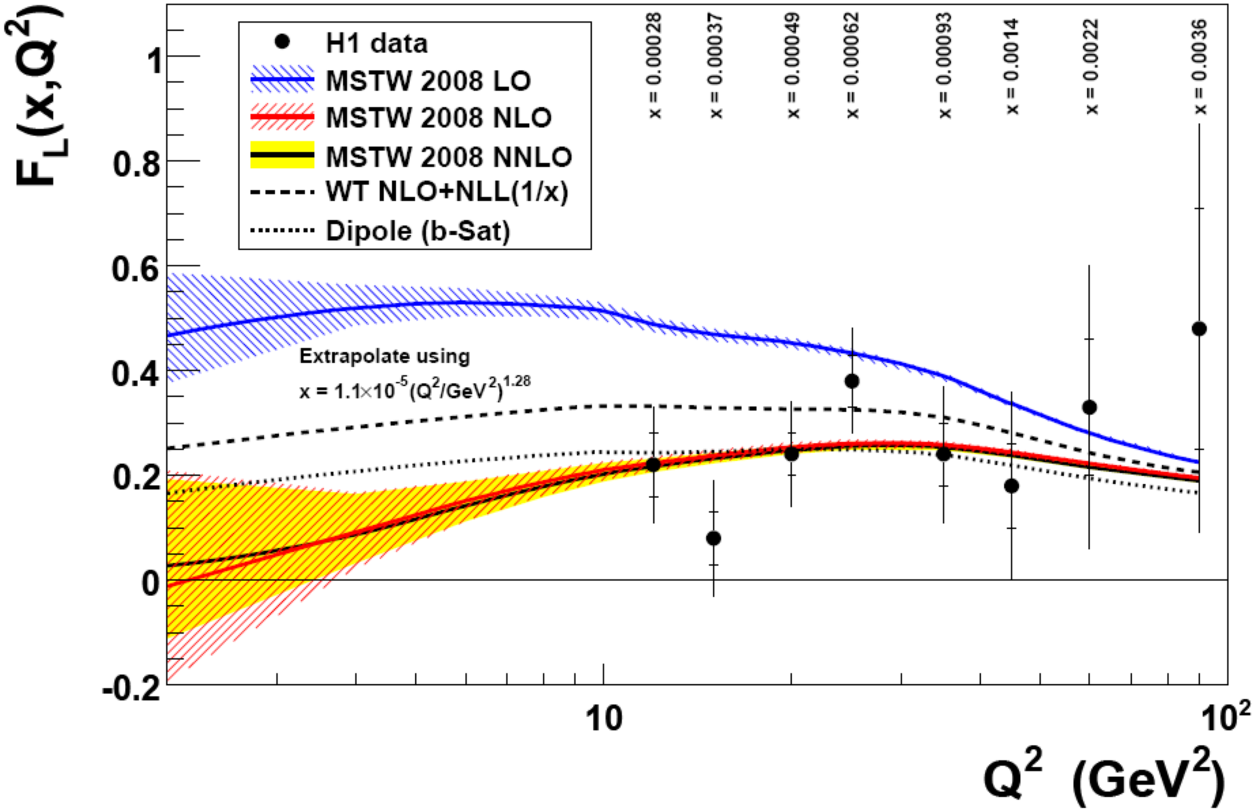
… harder than valence quarks?!

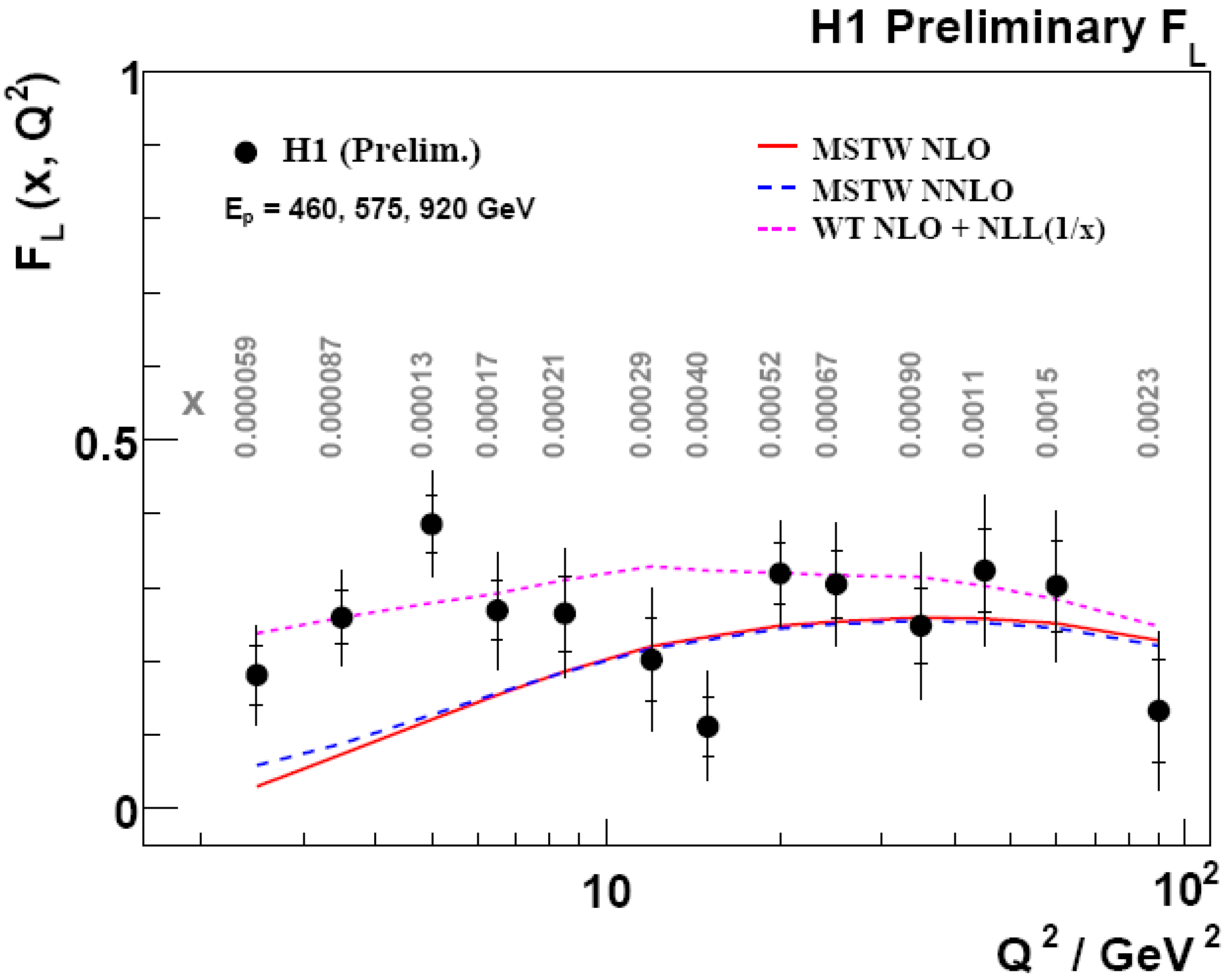


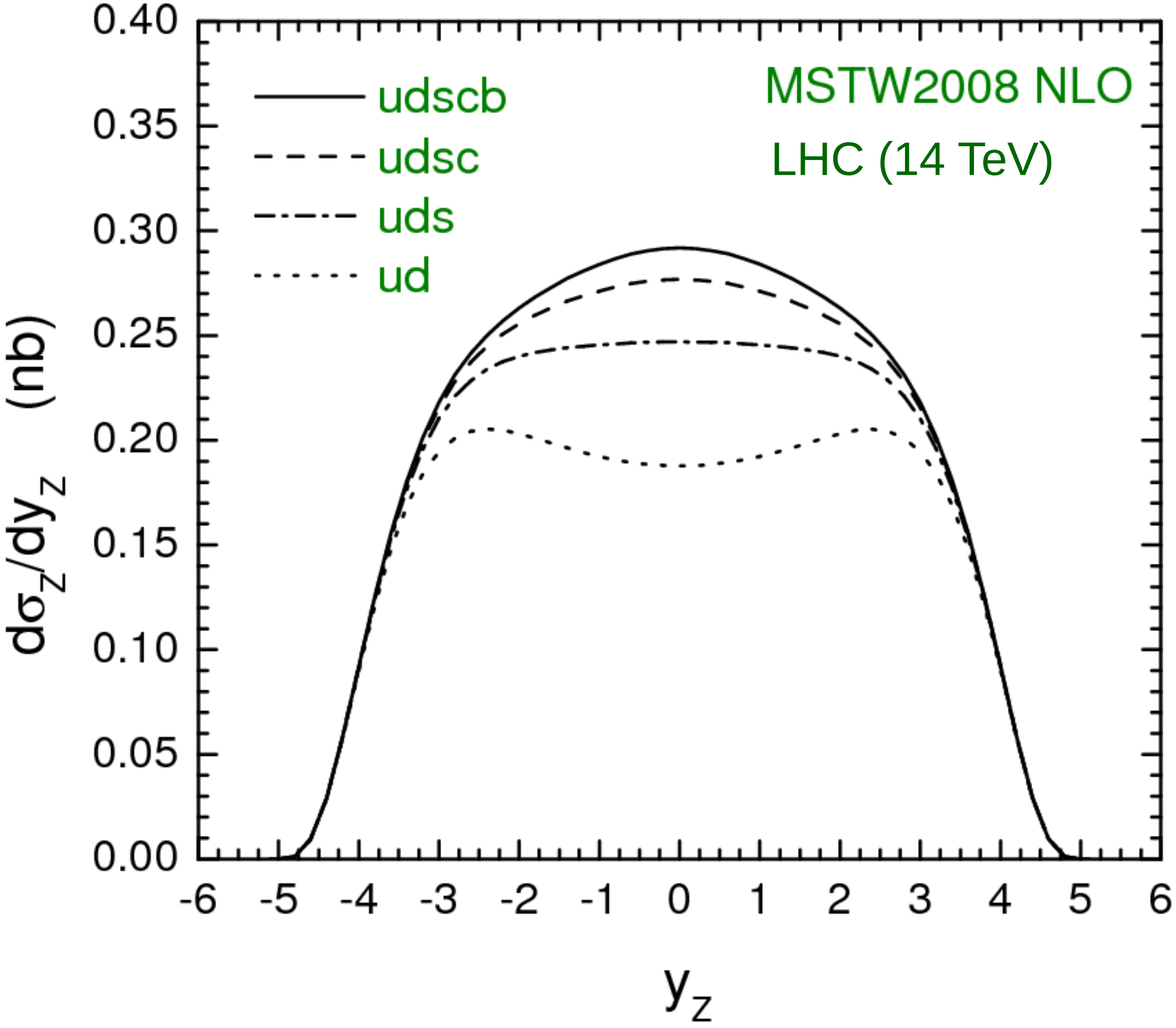
F_L

- an independent measurement of the small-x gluon
- a test of the assumptions in the DGLAP LT pQCD analysis of small-x F_2
- higher–order ln(1/x) and higher-twist contributions could be important

$$\begin{aligned} \frac{\partial F_2}{\partial \ln Q^2} &\simeq \alpha_S P^{qg} \otimes g + \ldots \\ F_L &\simeq \alpha_S C_{Lg} \otimes g + \ldots \end{aligned}$$




H1 Preliminary F_L
$F_L(x, Q^2)$
1
0.5
0
H1 (Prelim.)
E_p = 460, 575, 920 GeV
MSTW NLO
MSTW NNLO
WT NLO + NLL(1/x)
x
0.000059
0.000087
0.00013
0.00017
0.00021
0.00029
0.00040
0.00052
0.00067
0.00090
0.0011
0.0015
0.0023
10
10^2
Q^2 / GeV^2


MSTW2008 NLO
LHC (14 TeV)
udscb
udsc
uds
ud
$d\sigma_Z/dy_Z$ (nb)
y_Z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

LHCb

Unique features

- pseudo-rapidity range 1.9 - 4.9
  - 1.9 - 2.5 complementary to ATLAS/CMS
  - > 2.5 unique to LHCb
- beam defocused at LHCb: 1 year of running = 2 fb^{-1}
- trigger on low momentum muons: $p > 8$ GeV, $p_T > 1$ GeV

→ access to unique range of (x,Q^2)

LHCb

→ detect forward, low p_T muons from $q\bar{q} \longrightarrow \mu^+\mu^-$