Quantum Support Vector Machines in B Meson Continuum Suppression

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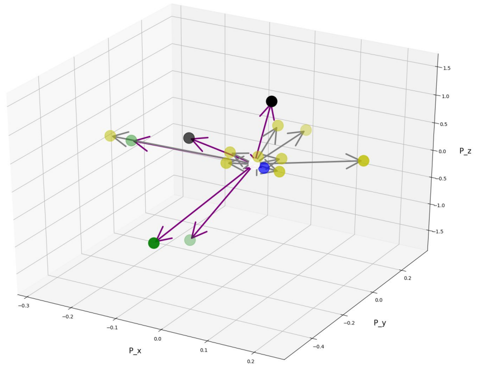
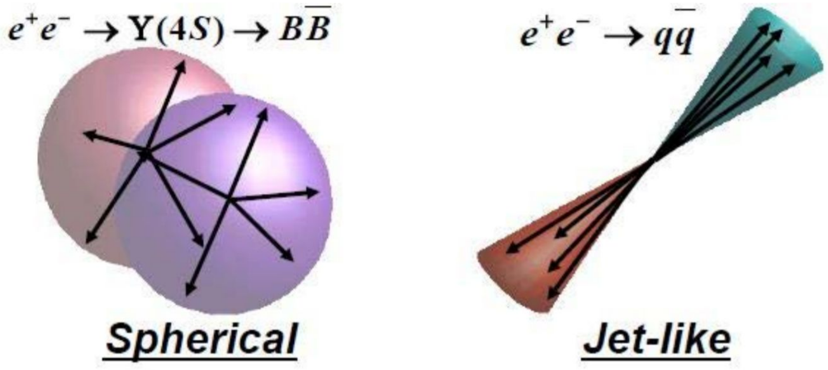
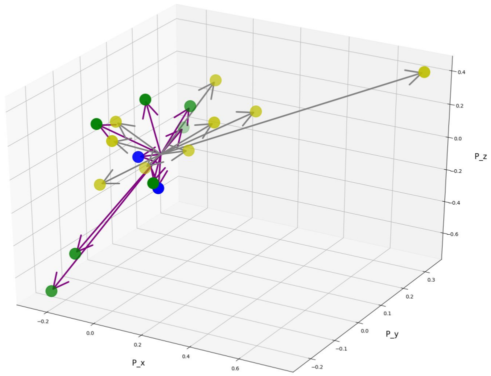
J. Heredge et al, Quantum Support Vector Machines for Continuum Suppression in B Meson Decays
https://arxiv.org/abs/2103.12257

Particle Decay Classification

- Distinguish between B Meson pair and quark pair.
- Constructed variables have been useful, such as Fox-Wolfram Moments.
- Data consists of (p, theta, phi) for each particle in the decay.





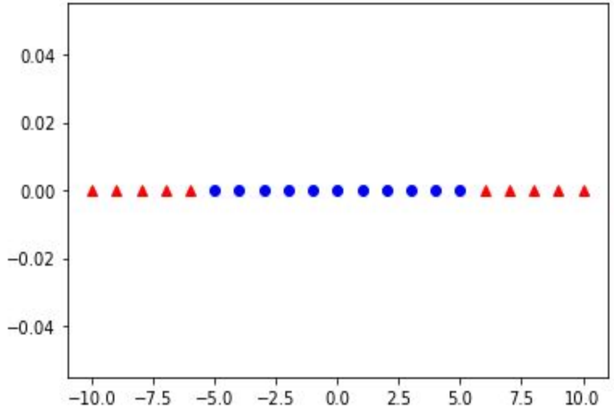


Support Vector Machines (SVM)

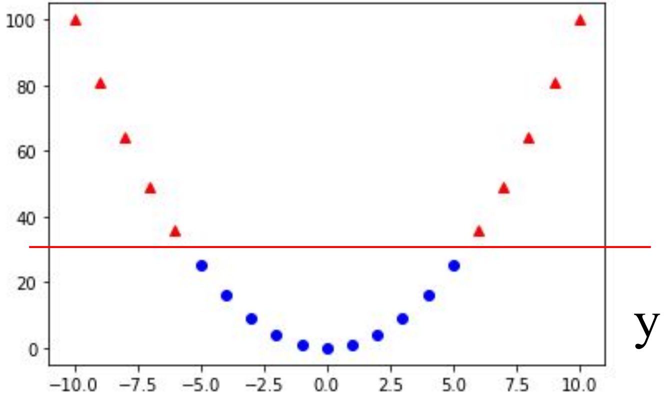
General Idea:

Encode to a higher dimensional space where the classification is easier.

Can't separate in 1D



Separable in 2D

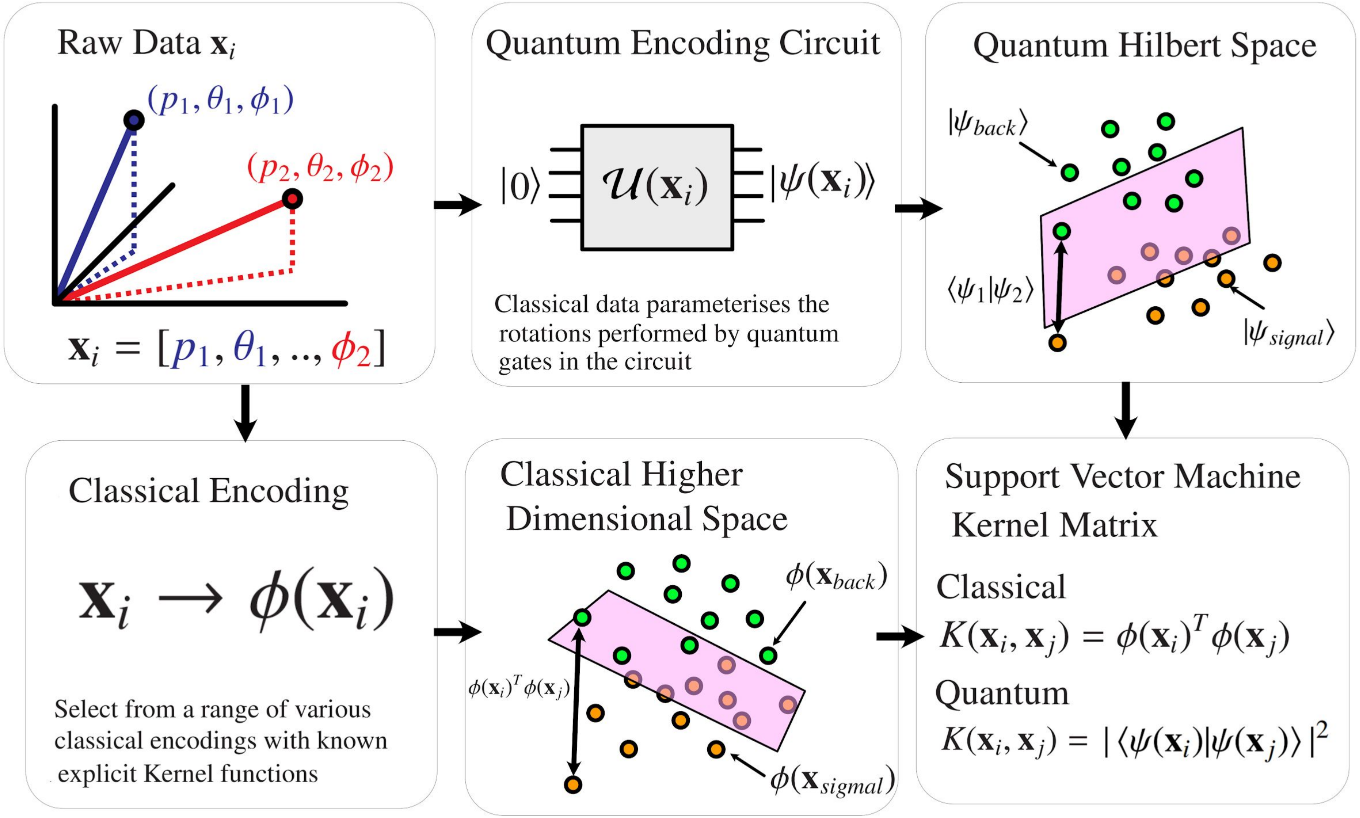


$y = x^2$

Quantum / Classical SVM Comparison

dim = n

n = number of classical inputs



dim = 2^n

If number of qubits = n

Quantum Circuits

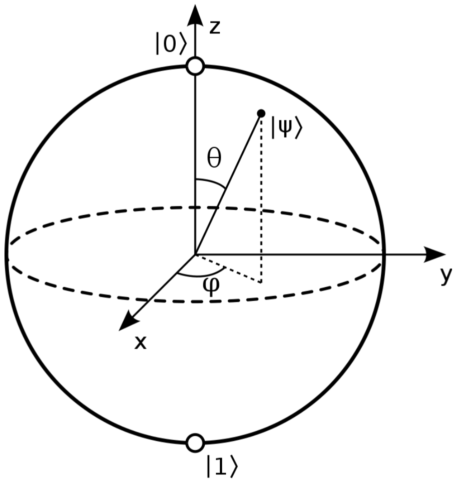
Classical Bit:

$$|0\rangle \quad \text{Or} \quad |1\rangle$$

Quantum Bit (Qubits)

$$|\Phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

Bloch Sphere Representation

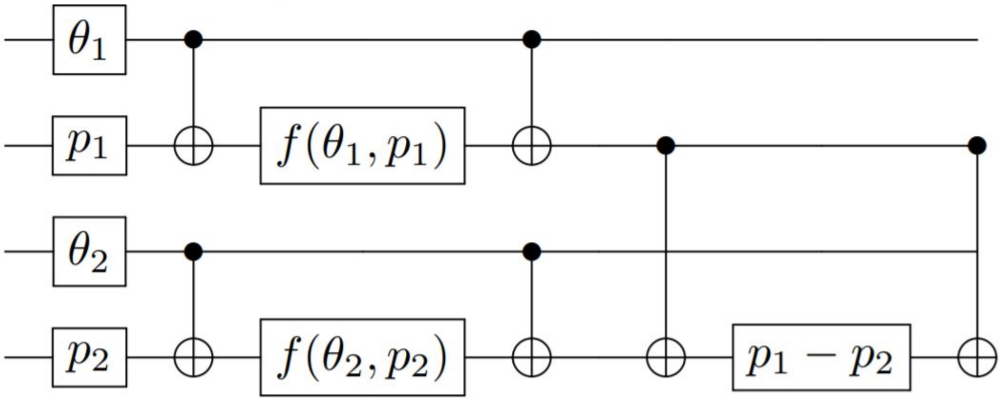


$$|\Phi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

Quantum Circuits

Each line in circuit represents a qubit

Qubits acted upon by a series of gates



Z Rotation Gate

Z Rotation Gate

$$-\boxed{Z_\theta}-$$

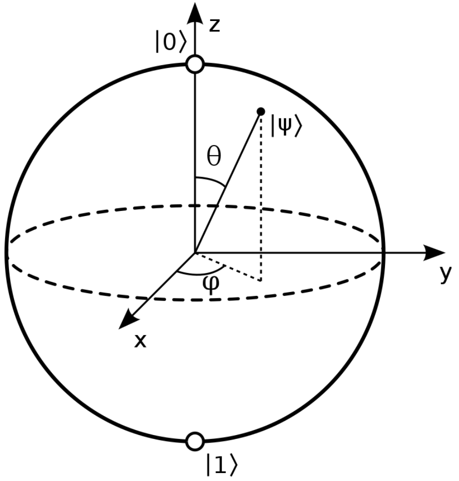
Performs the following action:

$$|\Phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\downarrow$$

$$|\Phi\rangle = \alpha e^{-i\frac{\theta}{2}}\,|0\rangle + \beta e^{+i\frac{\theta}{2}}\,|1\rangle$$

Effectively rotates a single qubit about the Z axis of bloch sphere



$$|\Phi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

Hadamard Gate

Hadamard Gate

Equivalent to rotation by π

about both the X and Z axis.

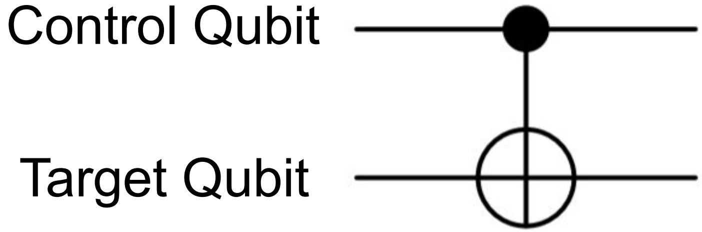
Effectively makes the following transformations:

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Controlled NOT Gate

Controlled NOT
(CNOT) Gate



Operation on 2 qubits

C T

$$|00\rangle \rightarrow |00\rangle$$

$$|01\rangle \rightarrow |01\rangle$$

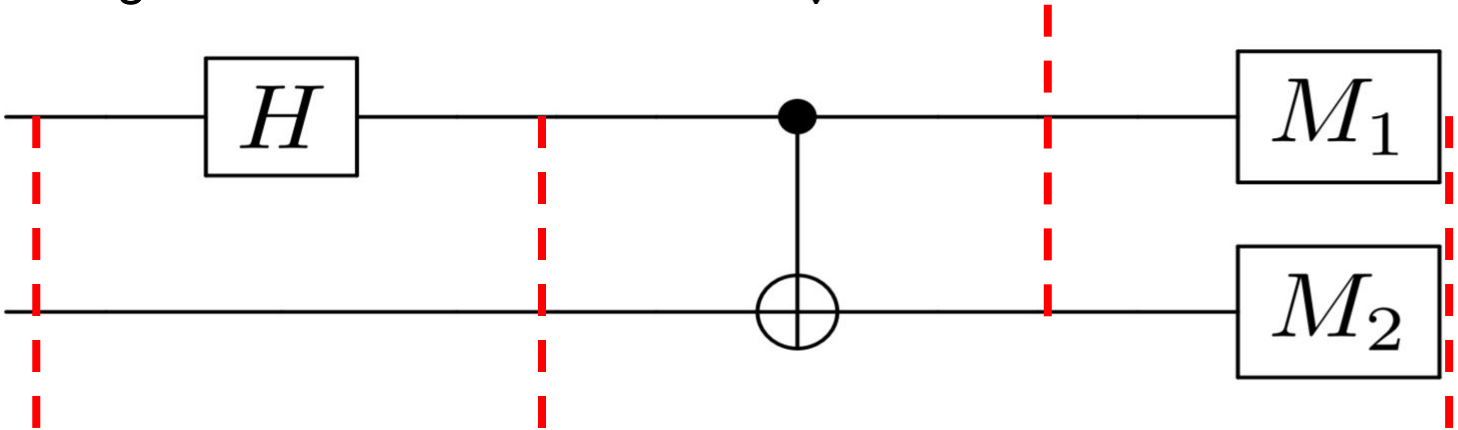
$$|10\rangle \rightarrow |11\rangle$$

$$|11\rangle \rightarrow |10\rangle$$

Example Circuit

Circuit introduces quantum entanglement

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



$|00\rangle$

$$\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$|00\rangle$ or $|11\rangle$

Probability = 0.5

Quantum Support Vector Machine [1]

Encode vector x into
a quantum state

$$|\psi(\mathbf{x})\rangle = U(\mathbf{x})H^{\otimes n}U(\mathbf{x})H^{\otimes n}|0\rangle^{\otimes n}$$

Encoding circuits
(Shown on next slide)

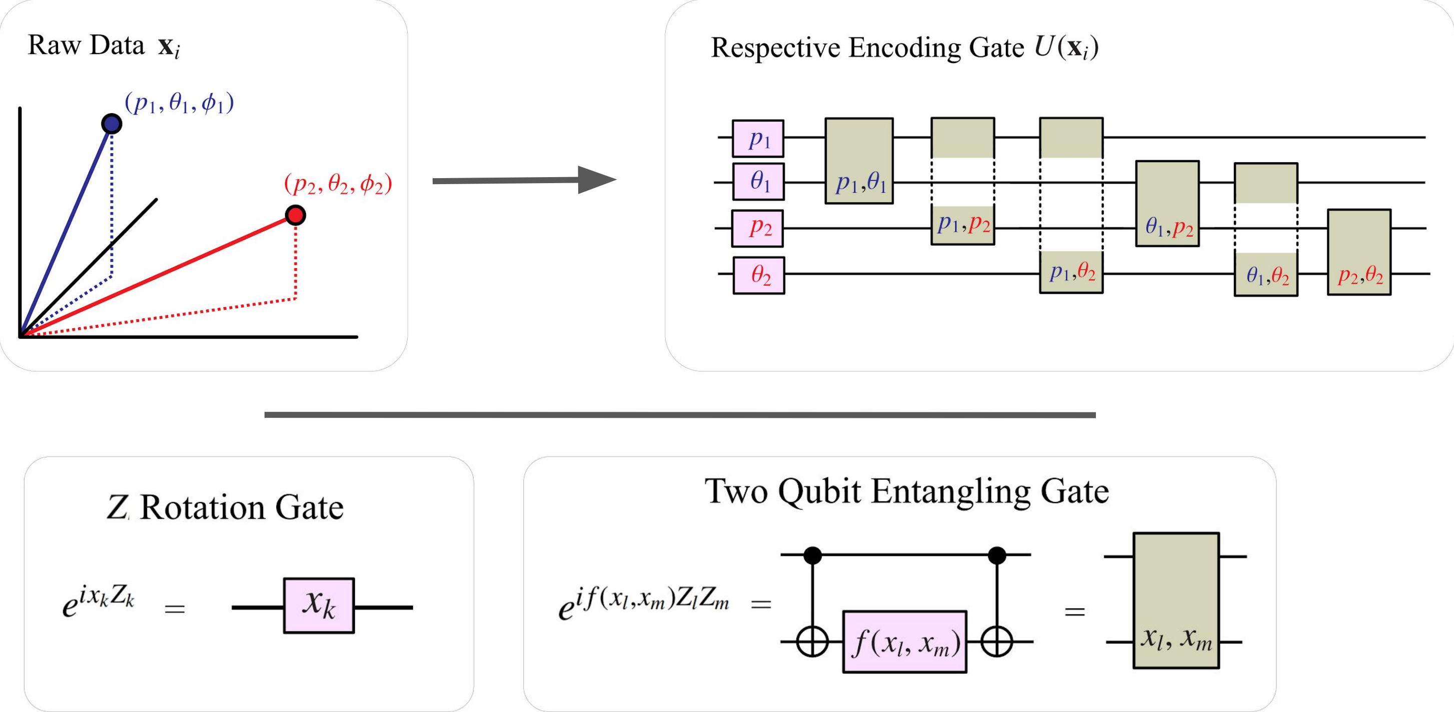
Hadamard gates
acting on n qubits

Initial 0 state

[1] Vojtech Havlicek, et al. Supervised learning with quantum enhanced feature spaces, Nature 567, 209–212 (2019).

Encoding Gate

Encoding classical data into a quantum state



Kernel Matrix for All Events

Quantum encoding

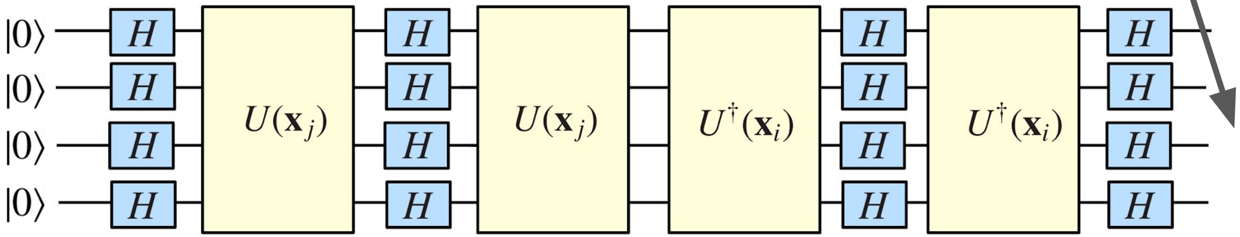
$$|\psi(\mathbf{x})\rangle = \mathcal{U}(\mathbf{x})\,|0\rangle^{\otimes n} = U(\mathbf{x})H^{\otimes n}U(\mathbf{x})H^{\otimes n}\,|0\rangle^{\otimes n}$$

Kernel Estimation

$$|\langle\psi(\mathbf{x}_i)|\psi(\mathbf{x}_j)\rangle|^2 = |\langle 0^{\otimes}|\mathcal{U}^{\dagger}(\mathbf{x}_i)|\mathcal{U}(\mathbf{x}_j)|0^{\otimes}\rangle|^2$$

Proportion of 0000 counts measured over many shots = Kernel Value

Repeat for each event against every other event = Full Kernel Matrix

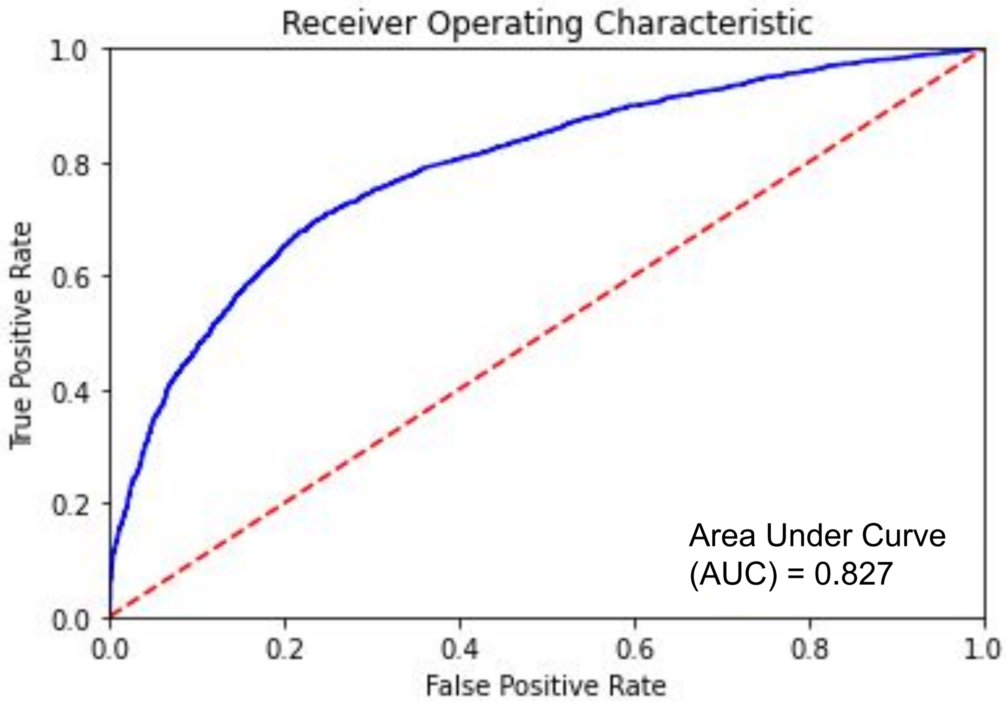


Final Step = Pass Kernel Matrix to Classical SVM

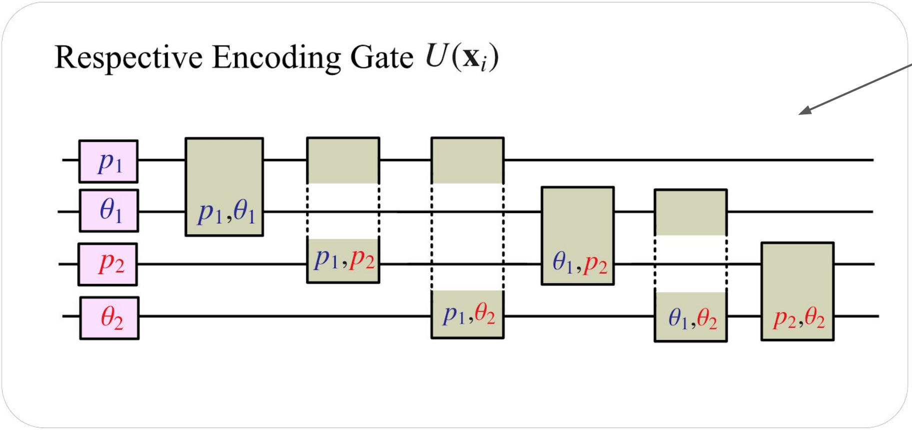
Results - Combinatorial Encoding

Encoding Circuit (70,000 events)	Test AUC
QSVM Combinatorial	0.827

Algorithm (70,000 events)	Test AUC
XGBoost	0.648
RBF kernel	0.866



Multiple Qubit Expansion



This version only explicitly entangles two quibits at a time.

Generalising the circuit to include 3 and 4 qubit interactions increases performance but requires more gates.

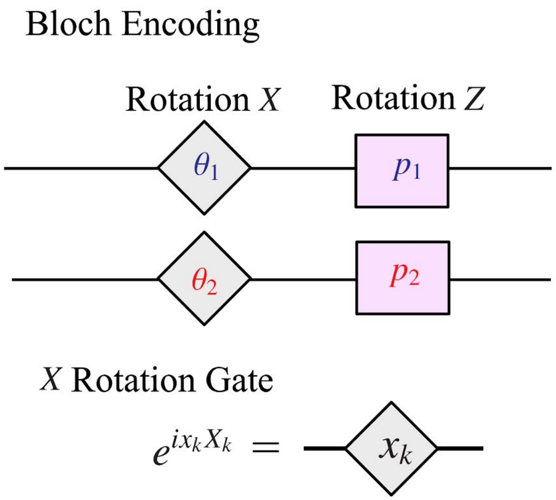
Encoding Circuit (70,000 events)	Test AUC
Combinatorial	0.827
Combinatorial Multi-qubit	0.845

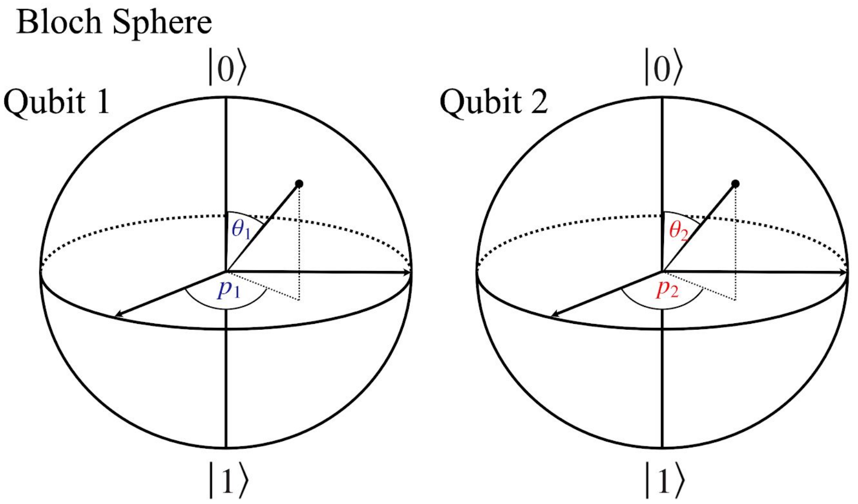
Simple Bloch Sphere Encoding

Each qubit has 2 free parameters

= Encode 2 variables in each qubit

Encoding Circuit (70,000 events)	Test AUC
Combinatorial Multi-qubit	0.845
Simple Bloch	0.861

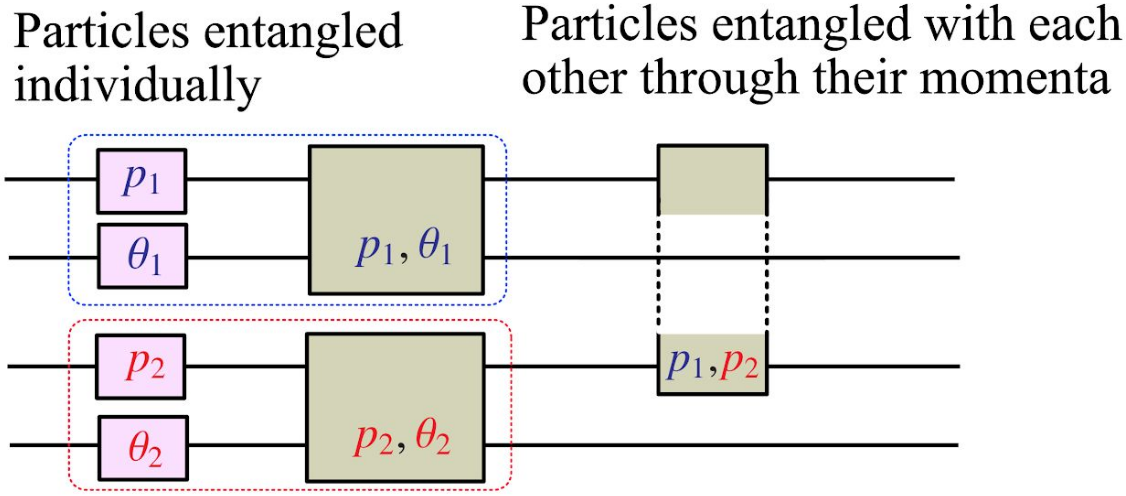




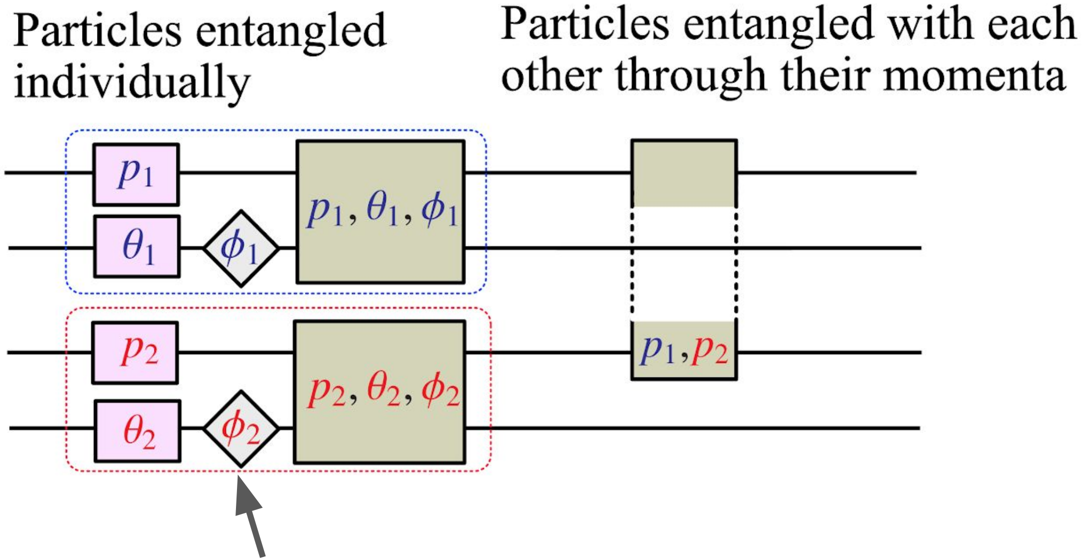
Separate Particle Encoding

First encoding circuit couldn't distinguish particles.

This method also uses less gates in total



Separate Particle Encoding - With Bloch Sphere



Encodes theta and phi variables of particle into the theta and phi angles of the qubit's bloch sphere

Encoding Circuit (70,000 events)	Test AUC
Separate Particle	0.853
Simple Bloch	0.861
Separate Particle + Bloch Sphere	0.877

Result Comparison

These results only used raw particle momentum data.

Classical techniques that use all the data can achieve an AUC 0.93 [2].

Encoding Circuit (70,000 events)	AUC
Combinatorial Encoding	0.827
Combinatorial Multi-qubit	0.845
Separate Particle	0.853
Simple Bloch	0.861
Separate Particle + Bloch Sphere	0.877

Classical Algorithm (70,000 events)	AUC
XGBoost	0.648
RBF Kernel SVM	0.865

[2] A. Hawthorne-Gonzalvez, M. Sevior, NIM **A 913**, 54 (2019)

Noise Model and Real Device

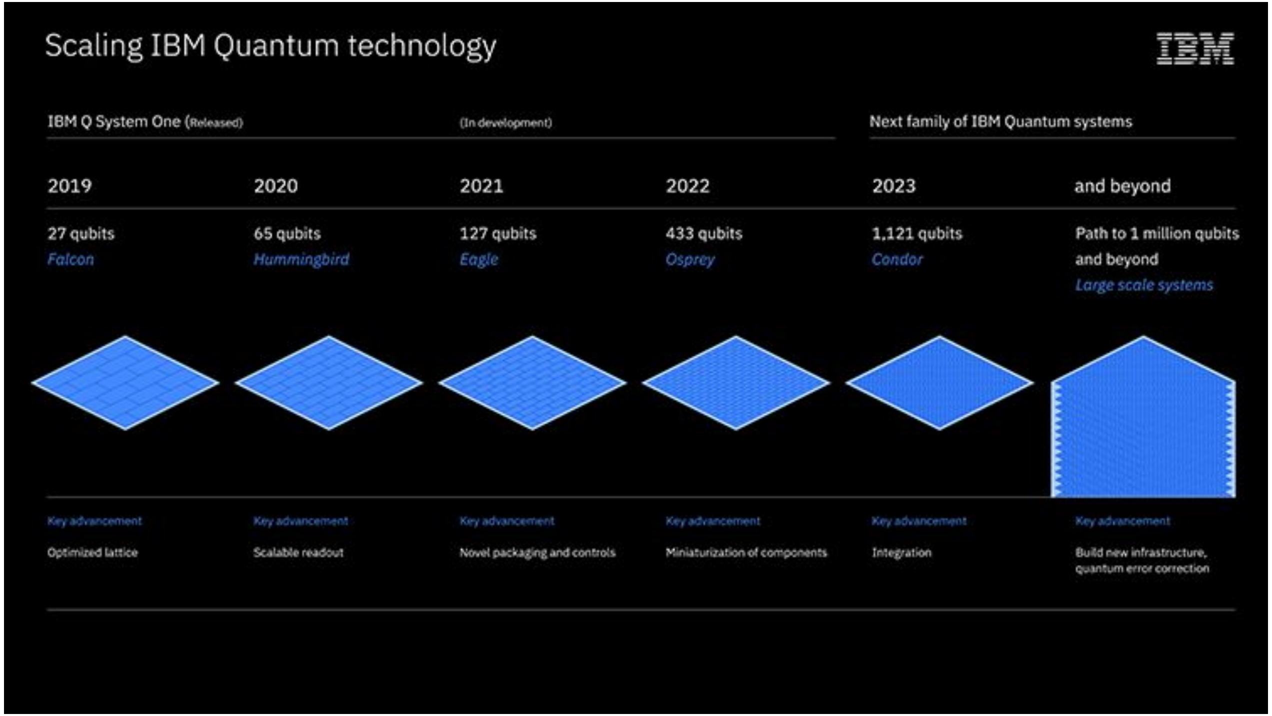
Separate Particle Circuit	Test AUC
Ideal Simulation	0.877
Simulated Noise	0.853
IBMQ Casablanca Device. (60 events)	0.703

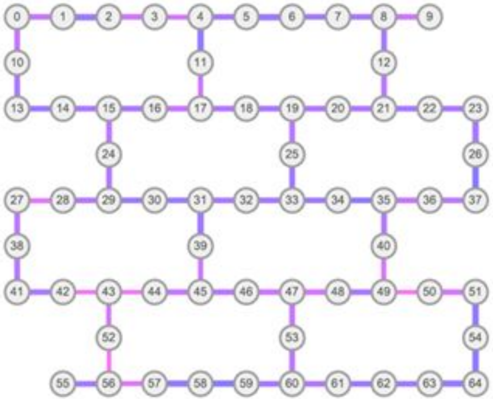
Only a fraction of the dataset was used on real machine due to limited availability

Combinatorial Encoding Circuit	Test AUC
Ideal Simulation	0.827
Simulated Noise	0.472

Combinatorial circuit contains more gates which are a source of quantum error

Quantum Outlook





65 Qubit Device Whole Entanglement [3]

[3] G. Mooney et al, Whole-device entanglement in a 65-qubit superconducting quantum computer, https://arxiv.org/abs/2102.11521

Acknowledgments

The team:

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https://unimelb.edu.au/quantumhub

https://quispace.org

References

[1] Vojtech Havlicek, Antonio D. C´orcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow, and Jay M. Gambetta. Supervised learning with quantum enhanced feature spaces, Nature volume 567, 209–212 (2019)

[2] A. Hawthorne-Gonzalvez, M. Sevior, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 913, 54 (2019)

[3] G. Mooney et al, Whole-device entanglement in a 65-qubit superconducting quantum computer, arXiv:2102.11521