Basket Classifier: Fast and Optimal Restructuring of the Classifier for Differing Train and Target Samples

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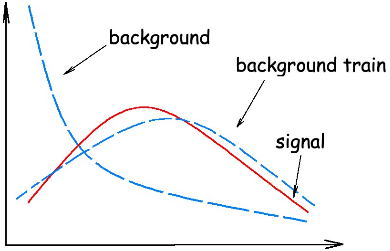
Motivation

Consider a problem of separating π^0 from photons using machine learning algorithms. There is a complication in the problem: the calibration sample is quite different from the real background. Let's formulate it in a more general way: there are two obvious problems of constructing classifier in the case of continuous spectrum:

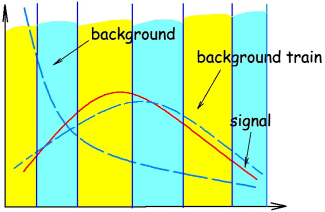
- We want to avoid dependence on the training sample
- We want to train a classifier on the training sample only once, avoiding this procedure in the future when changing spectra



Possible solution

What can we do to avoid these difficulties?

- Let's divide the spectrum into a number of baskets, for each of which we build its own classifier, that maximizes the area under the ROC curve.
- We obtain tolerance to changes in the distribution due to a) their narrowness, b) tolerance of ROC AUC to imbalance classes.
- Now we can solve the problem of maximizing the signal level for a given background level; to do this, we need to select a cut-off threshold in each basket so that for a given amount of background events across all baskets, the sum of signal events is maximum, i.e. solve the optimization problem.



Problem-1

- m_i - number of noise events for the i-th basket,
- n_i - number of signal events for the i-th basket,
- ξ_i - fraction of noise events for the i-th basket,
- α - target signal efficiency,
- f_i - ROC-curve for the i-th basket
- N - number of baskets.

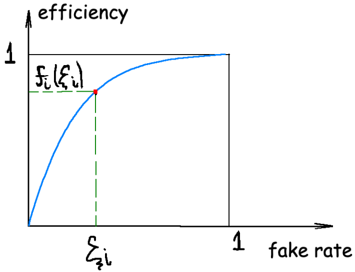
$$\min_{\xi} \quad \sum_{i=1}^{N} m_i \xi_i$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^{N} f_i(\xi_i) n_i}{\sum_{i=1}^{N} n_i} = \alpha$$

Problem-2

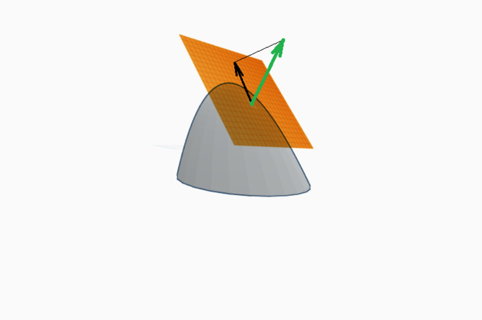
$$\min_{\xi} \quad \sum_{i=1}^{N} m_i \xi_i$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^{N} f_i(\xi_i) n_i}{\sum_{i=1}^{N} n_i} = \alpha$$



Optimization procedure includes 2 steps:

- the projection of the gradient onto the tangent plane
- lowering the vector to the surface of constraints
- repeating the first 2 steps until convergence



Toy example

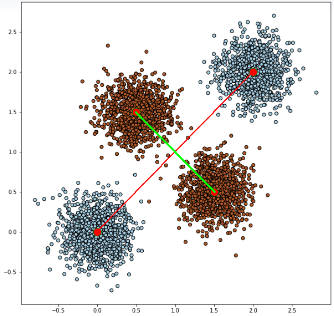
Parameterize 2 families of distributions:

- Distribution 1: two Gaussians with centers at $(-X, X)$ and $(X, -X)$ and fixed variance.
- Distribution 2: two Gaussians with centers at (Y, Y) and $(-Y, -Y)$ and fixed variance.

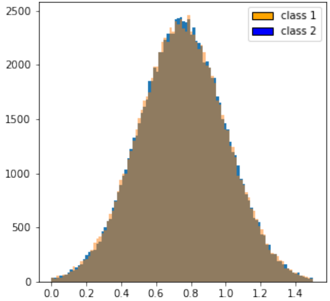
Our goal is to train the procedure on a given sample with parameters X_1 and Y_1 (sample A), and then apply the classifier to another sample with parameters X_2 and Y_2 (sample B), measuring score of the result.

Distributions

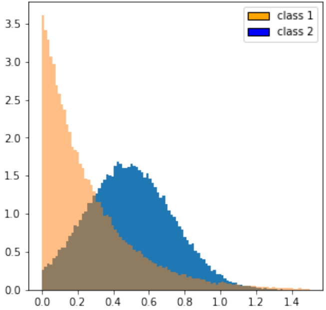
- ...



(a) Toy sample. X and Y define distances represented by red and green lines



(b) X and Y distributions for sample A



(c) X and Y distributions for sample B

Figure: Toy example distributions

Classifiers

- 4 classifier were built in the experiment. The first one was trained on a test sample (sample A), and tested on a target sample (sample B).
- The second one was trained on a target sample and tested on a target sample.
- The third and fourth (with 3 and 7 baskets, respectively) are basket classifiers, which were trained on a test sample and tested on a target sample.
- It is reasonable to expect that we will get the following ranking in terms of quality: the first classifier will demonstrate the worst quality, the second one - the best, and the basket classifiers will be located between them.

Scores

- Let's take a look at the results:

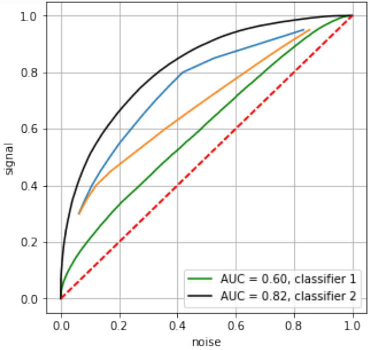


Figure: green line - efficiency for case (1), black line - efficiency for case (2), orange line - basket classifier with 3 baskets, blue - basket classifier with 7 baskets

The graph clearly shows that our assumptions were fully justified. The classifiers are indeed clearly ranked in terms of quality.

Conclusions

- In this paper, we present a concept of basket-based dynamic classifier.
- Such classifier demonstrates a tolerance to significant variations of the spectrum of the target analyzed data from data used for training.
- The procedure of fast adjustment of the basket-based classifier for a given analysis performance is also shown.
- In case of real experiments, such a basket-based classifier may be trained and validated only once in advance of data analyses. Further adjustments to real spectra of particular data analyses does not require re-training if using *a priori* knowledge of shapes of the target data sets.

The End