



Grid-based minimization at scale: Feldman-Cousins corrections for light sterile neutrino search

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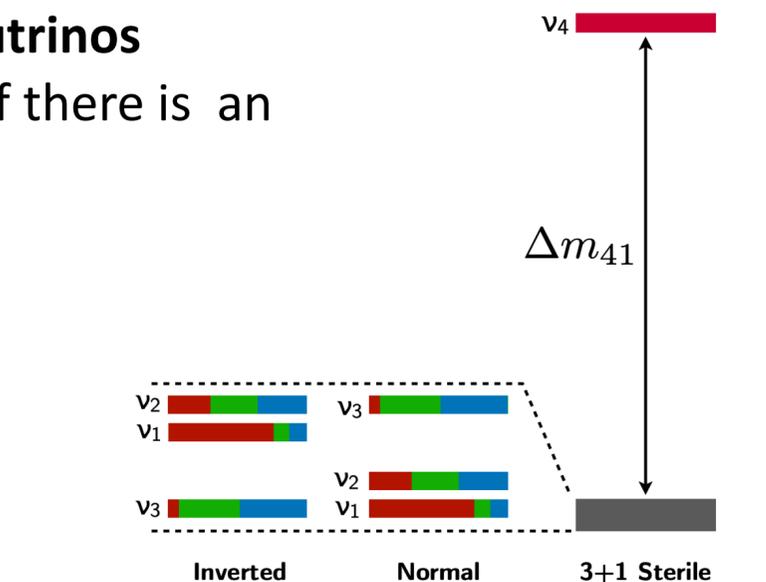
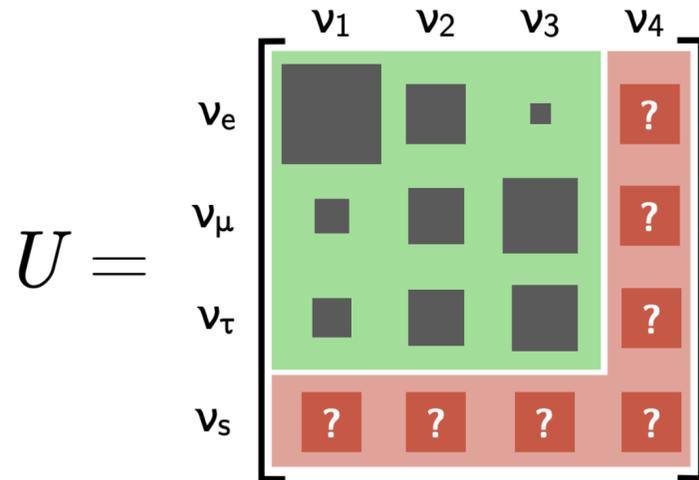
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Physics Problem: Light Sterile Neutrino Search

- Neutrino oscillation is a well-established phenomenon in particle physics
 - » can be used to probe the existence of additional neutrino species beyond the three known flavors, called **sterile neutrinos**
- Minimal extension to the Three-Neutrino Paradigm is if there is an additional sterile neutrino (**3+1 paradigm**)



$$P_{\alpha \rightarrow \beta} = \sin(2\theta)^2 \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)^2$$

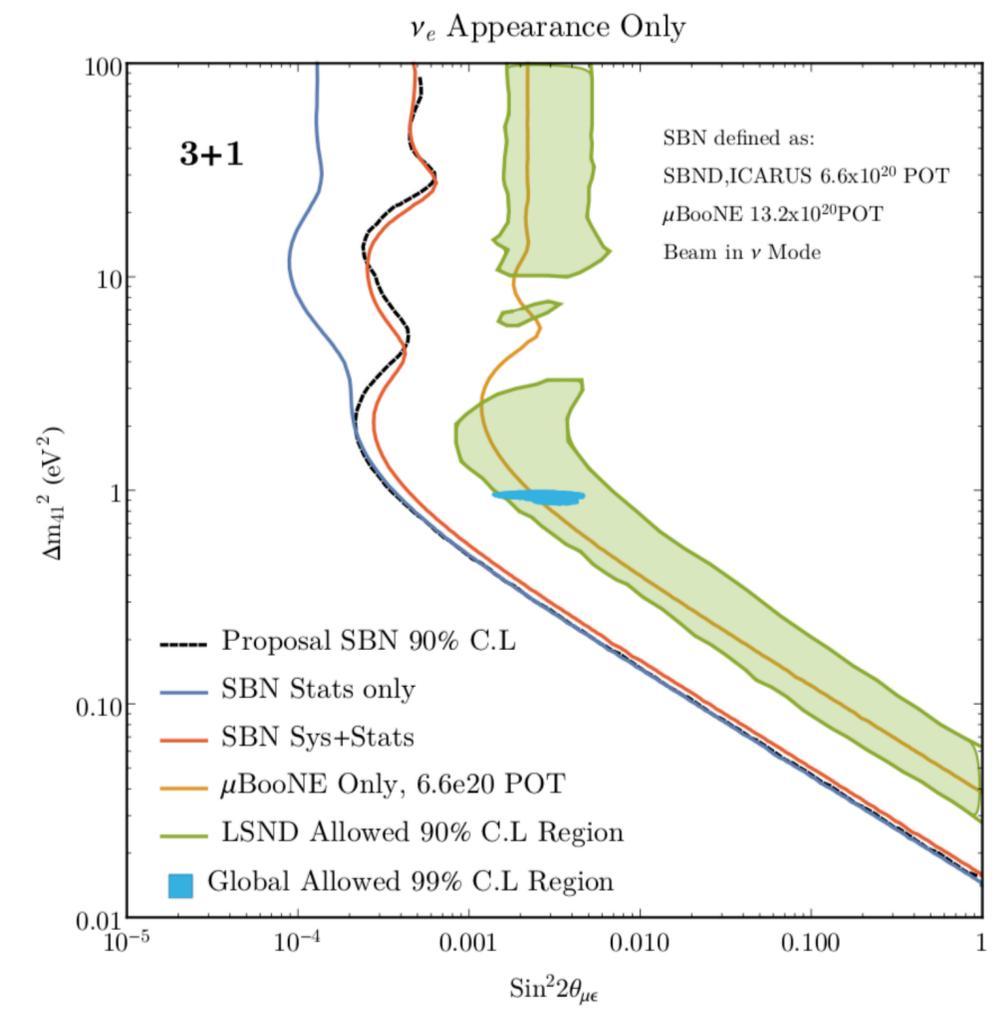
Δm^2 : frequency of oscillation
 $\sin^2(2\theta)$: amplitude of oscillation

Only depends on 2 parameters!

$$\frac{L}{E} \sim \frac{m}{\text{MeV}} : \text{short-baseline}$$

Oscillation probability via this mixing:

$$P_{\alpha\beta} = 4|U_{\alpha 4}|^2|U_{\beta 4}|^2 \sin^2 \left(1.27 \frac{\Delta m_{41}^2 L}{E} \right)$$



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Statistical Coverage for Sterile Neutrino Oscillation Parameter

Compute a χ^2 surface in the $(\Delta m_{41}^2, \sin^2(2\theta))$ oscillation parameter plane:

$$\chi^2(\Delta m_{41}^2, \sin^2 2\theta) = \sum_{i,j} [N_i^{null} - N_i^{osc}(\Delta m_{41}^2, \sin^2 2\theta)] (E_{ij})^{-1} [N_j^{null} - N_j^{osc}(\Delta m_{41}^2, \sin^2 2\theta)]$$

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assuming no oscillation (null
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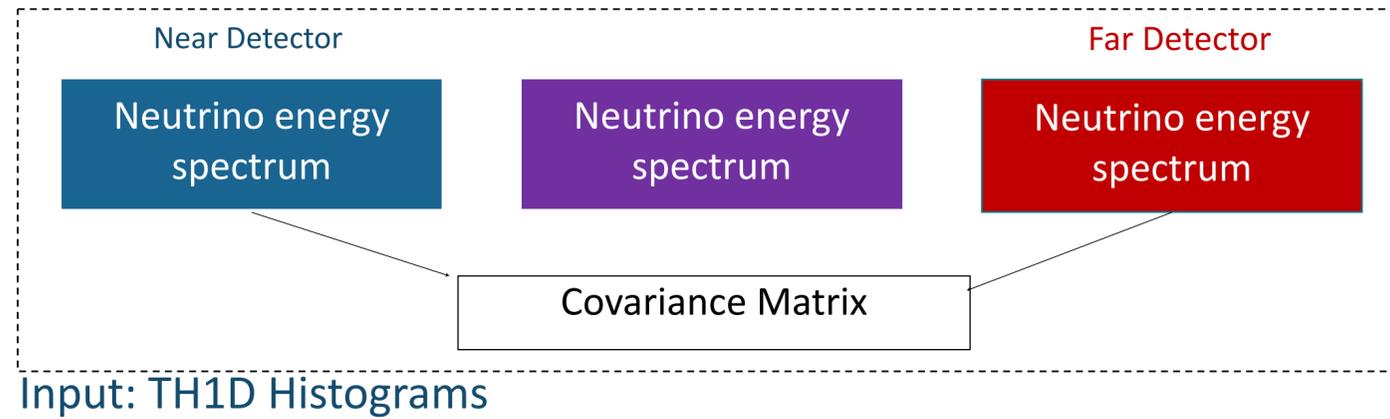
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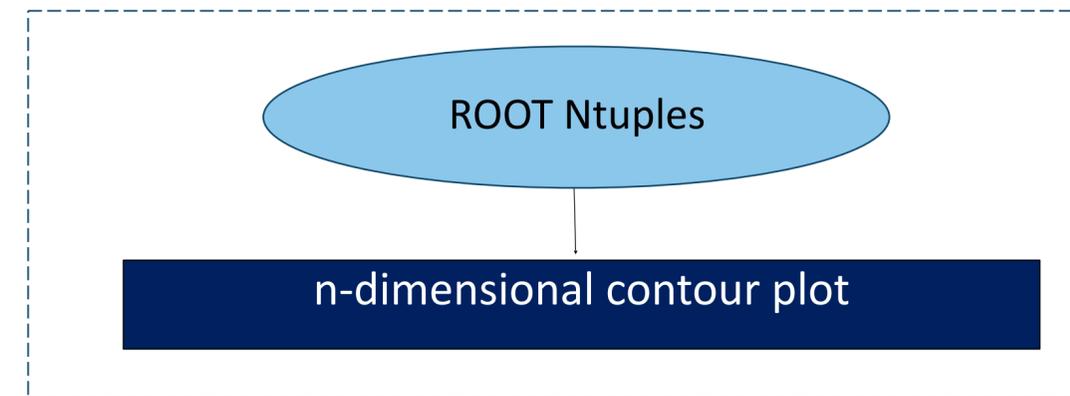
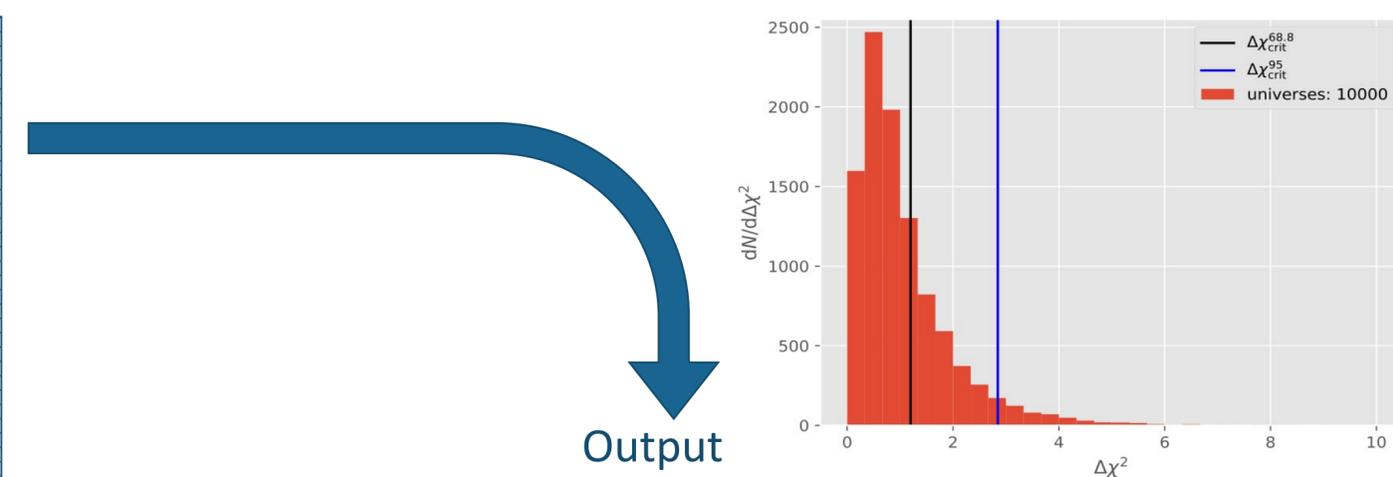
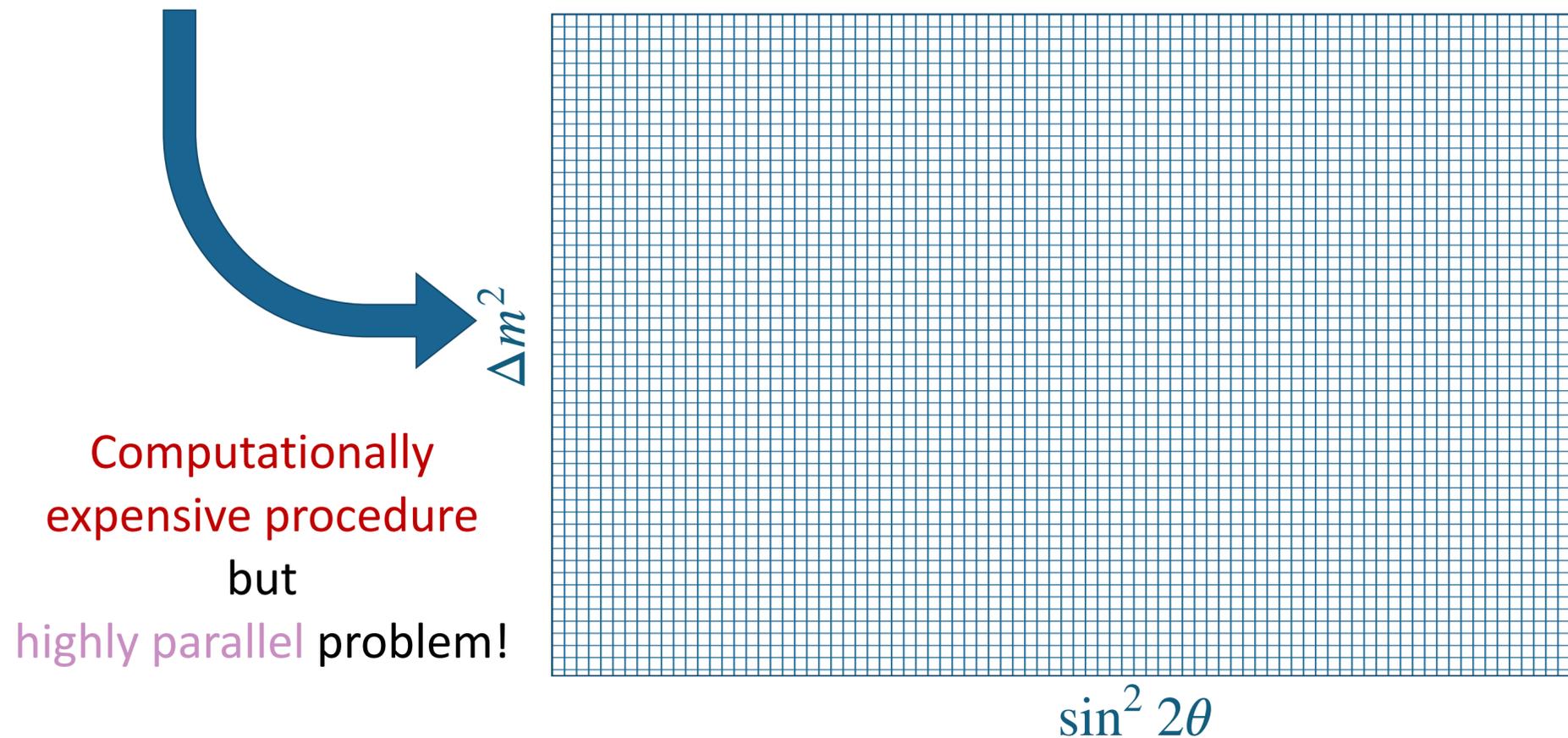
NOT SUFFICIENT FOR THE PROBLEM!

⇒ statistical inaccuracies and serious under-coverage for values of the oscillation parameters that are below the sensitivity of the experiment

Feldman-Cousins Procedure

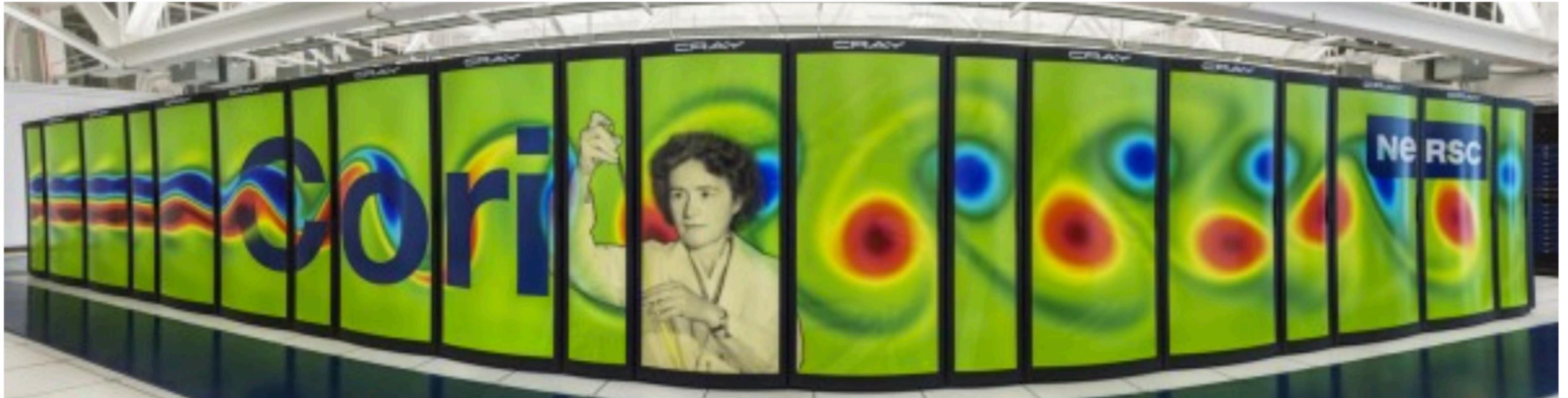


Perform χ^2 calculation at each **oscillation parameter space point (Ngridpoint)** where we statistically fluctuate the oscillated prediction to generate **N number of toy experiments (Nuniv)** – Feldman Cousins procedure

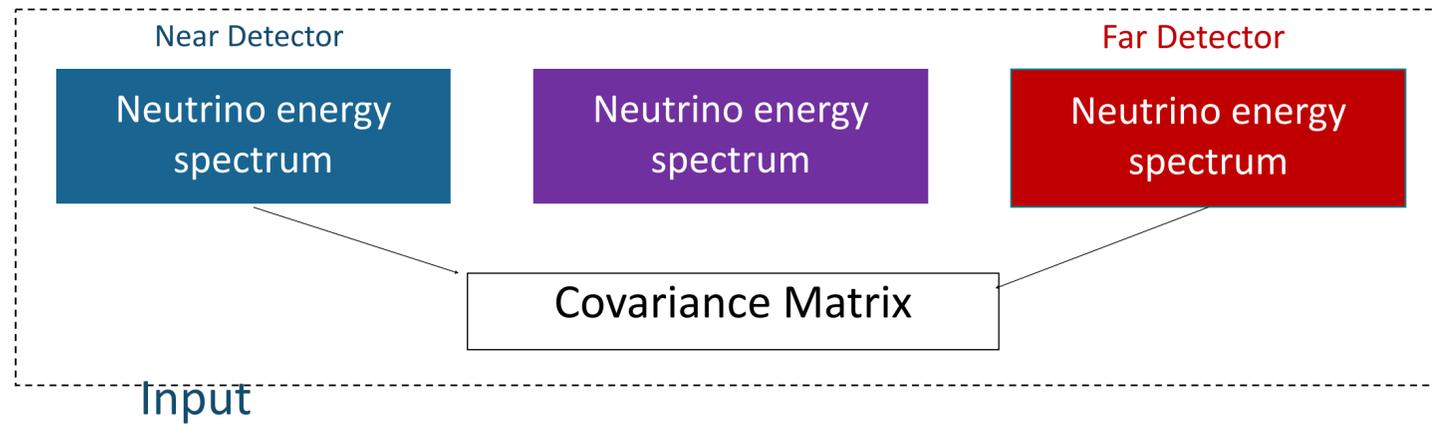


NERSC - National Energy Research Scientific Computing Lawrence Berkeley National Laboratory

Machine	Cori phase 1 (Haswell)	Cori phase 2 (KNL)
CPU	Intel Xeon E5-2698 v3	Intel Xeon Phi 7250
Clockspeed	2.3 GHz	1.4 GHz
Cores per node	32	68
Ranks per core	2	4

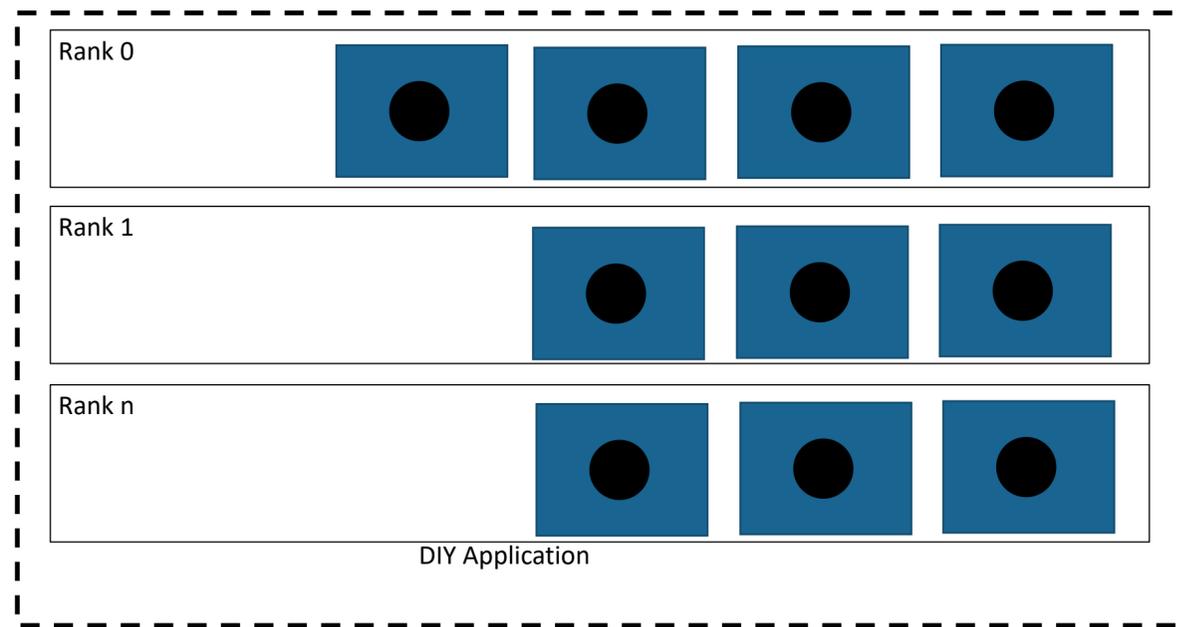


Accelerating Feldman Cousins Procedure on HPC



Perform χ^2 calculation at each **oscillation parameter space point (Ngridpoint)** per **toy experiments (Nuniv)** per rank in a **node** – Feldman Cousins procedure

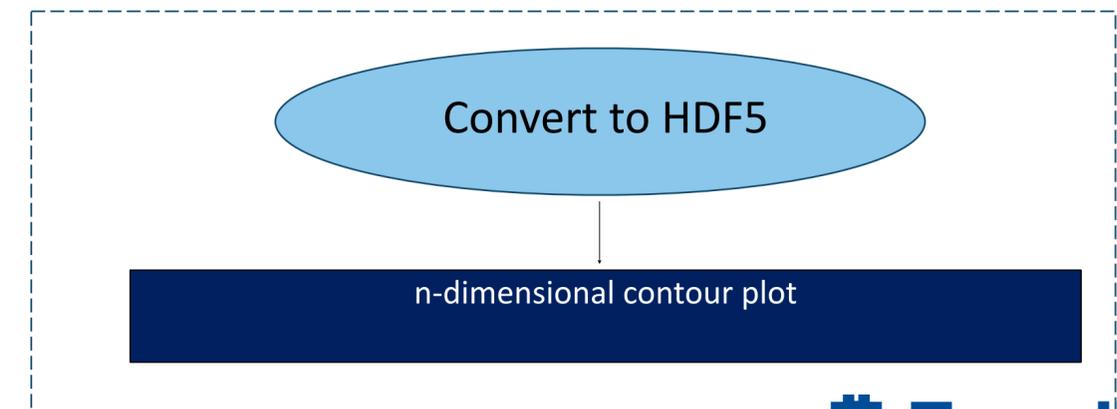
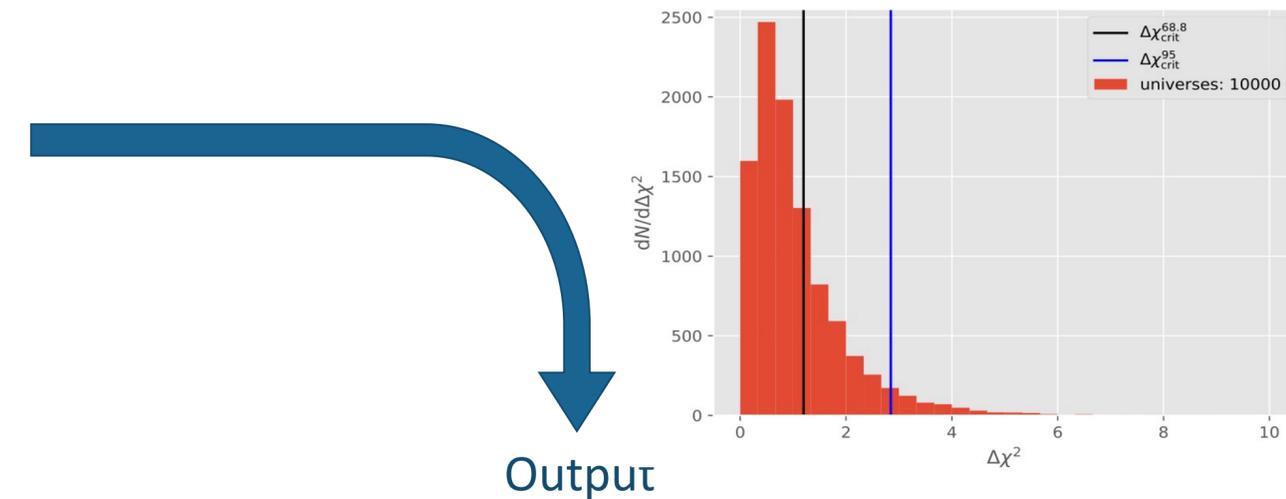
restructured the codes to read the input files only once using MPI



MPI to distribute the work load to all ranks through memory

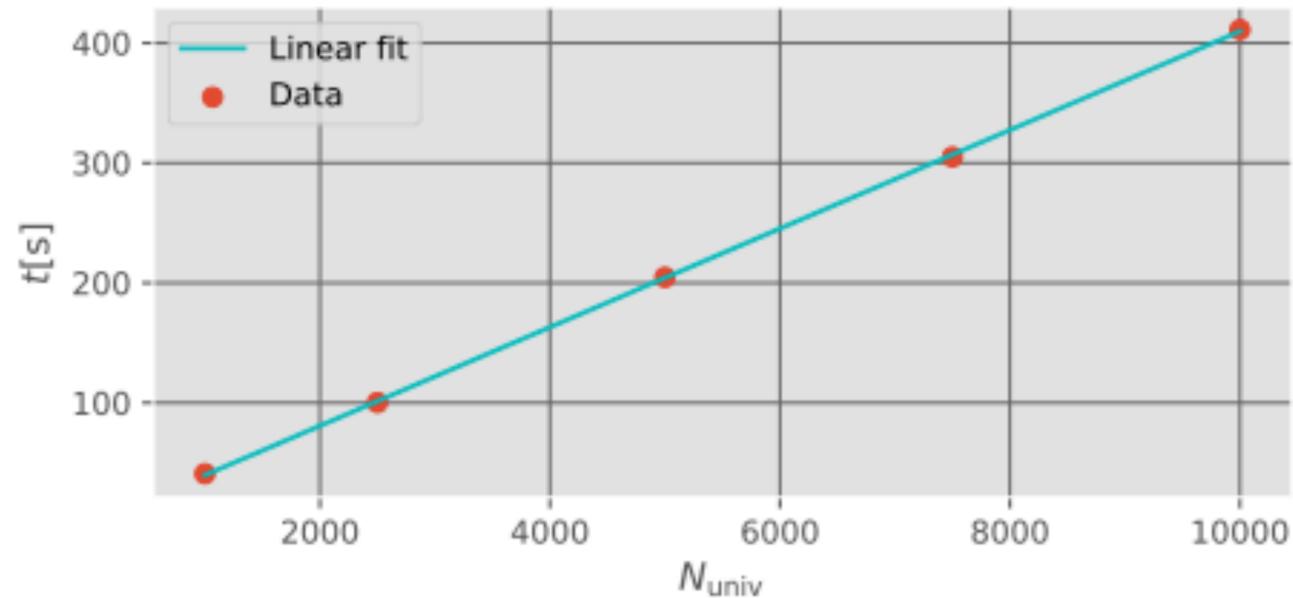
*MPI: Message Passing Interface

replace ROOT histogram objects with types Eigen3 in linear algebra and vectorized calculation to compute χ^2
 » Code is no longer memory-limited, only CPU-limited, allowing up to 350x speedups from original implementation

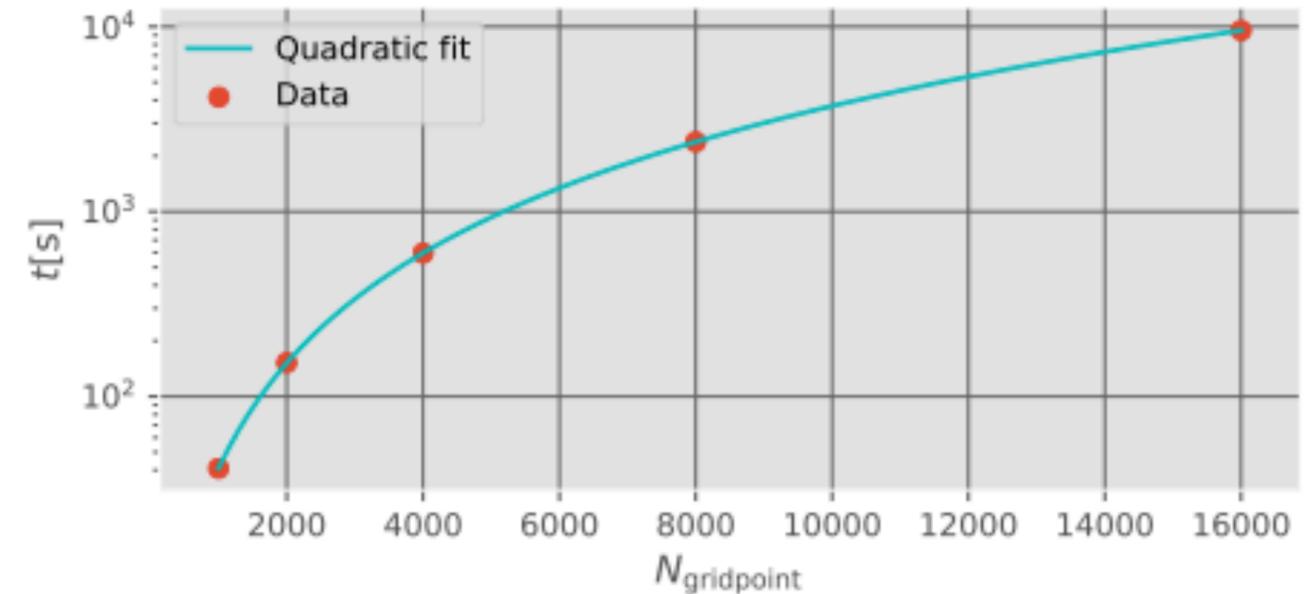


Performance and Scaling

Scalability is important for parallel computing to be efficient



- Fixed number of **Haswell nodes** (and ranks)
- Fix **$N_{gridpoint}$** and varied **N_{univ}** independently
- The data is fit **linearly** and show no visible deviation from linear scaling



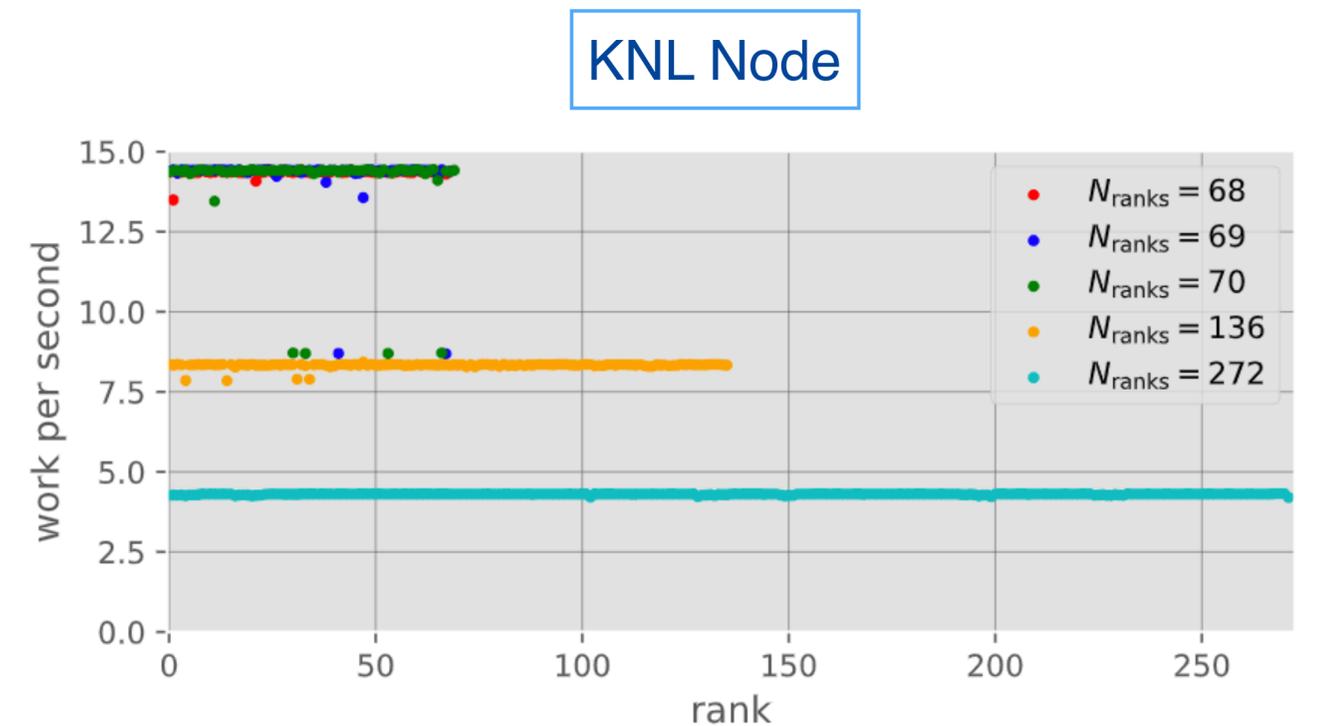
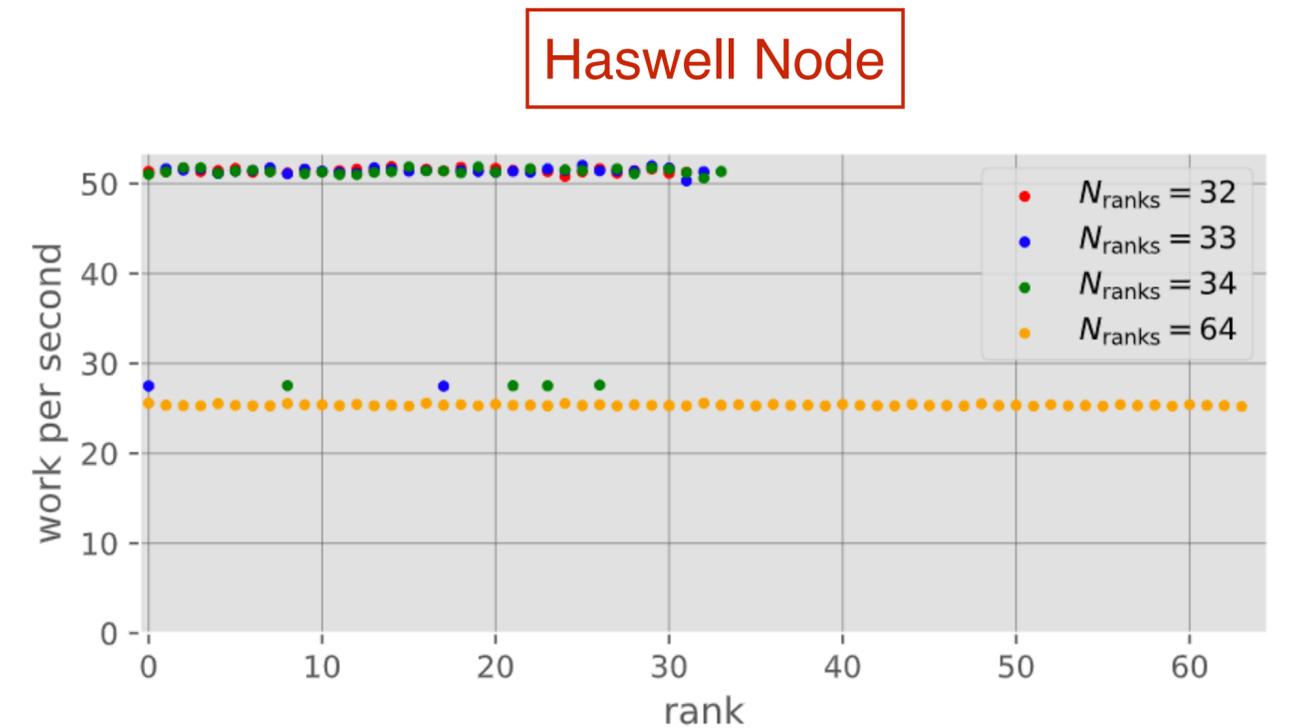
- Fix the **N_{univ}** and compute the program execution time as function of **$N_{gridpoint}$**
- The data is fit **quadratically** and shows that there is no visible deviation from the quadratic scaling

Performance and Scaling

Single-node Scaling

- **Goal:** assess benefits gained from using multiple MPI ranks:
 - Select a fixed size problem as function of *Ngridpoint* and *Nuniv*
 - Distribute work among the ranks as evenly as possible
 - Record time spent by the slowest rank (t_{\max}) in the main part of the program for each rank separately.
- **Haswell** attains the best single node scaling when using 32 ranks.
 - No significant improvement in execution time when multi-threading is enabled
 - Oversubscribing 1 and 2 cores in the 33 ranks and 34 ranks respectively, leads to overall worse performance
- Similar performance on **KNL** when using non-integer multiples of the number of ranks (68) but improvements at higher number of ranks

Ranks	68	136	272
t_{\max} [s]	87	75	70
Gain w.r.t. 68 ranks		16%	24%

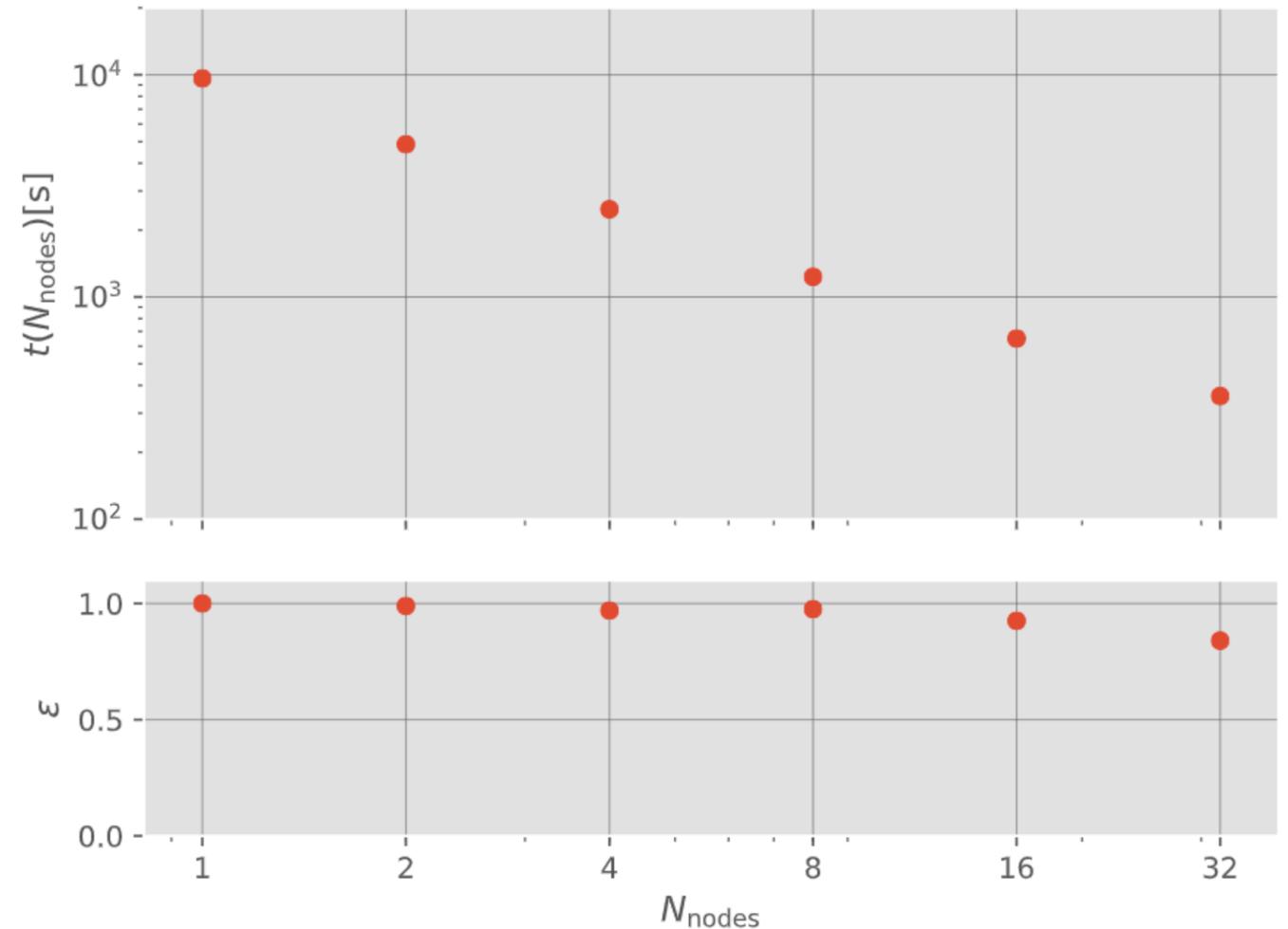


Performance and Scaling

Multi-node Scaling

- Understand scalability of multi node systems to gauge potential performance gain as more computing resources are added
 - Define reasonable large problem of fixed size.
 - Measure the time it takes to complete the main part of the program as a function of the number of used **Haswell nodes** (N_{nodes}).
 - Efficiency of the program execution:

$$\varepsilon = N_{\text{nodes}} \cdot \frac{t(N_{\text{nodes}})}{t(1)},$$

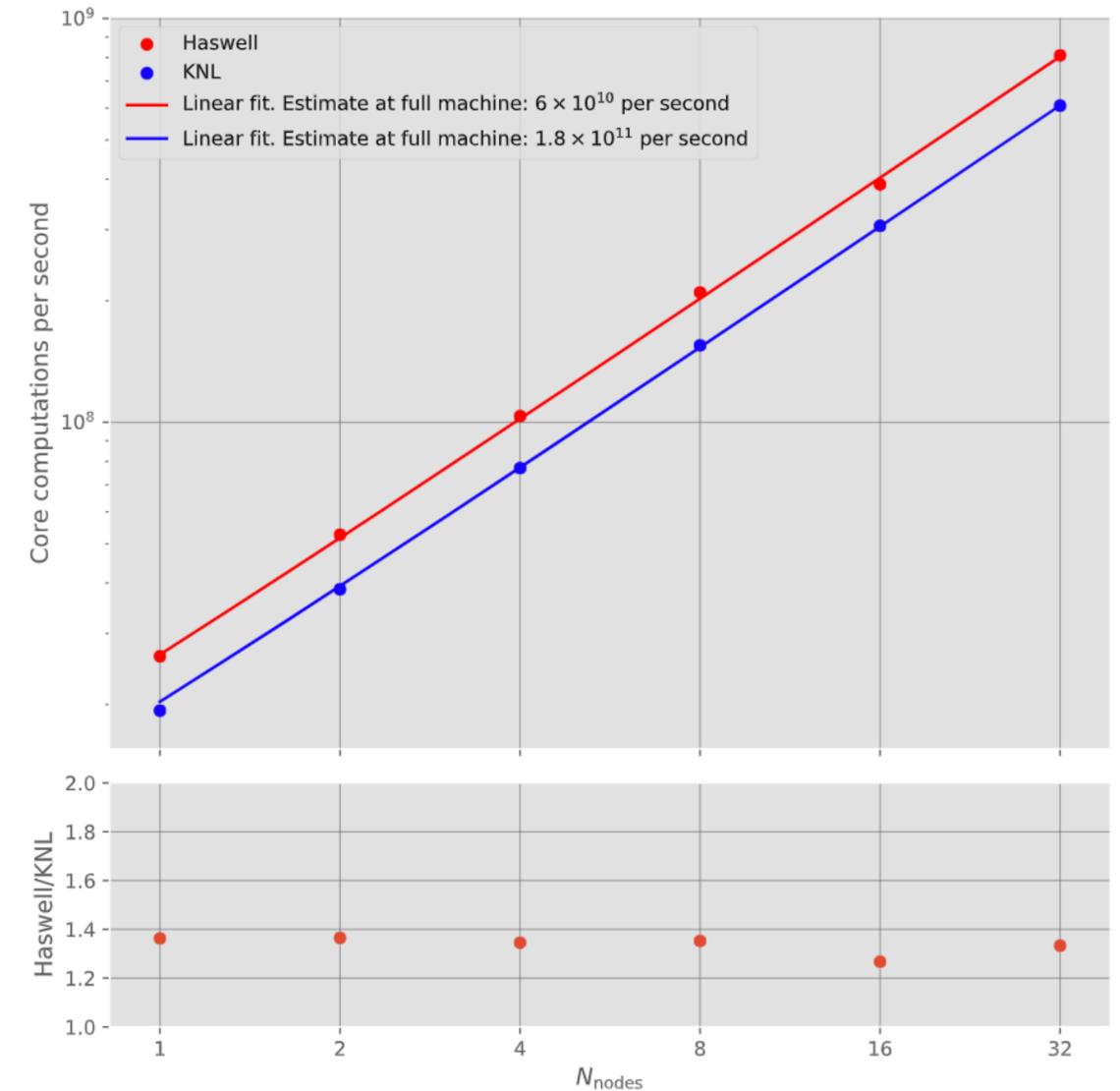


32 ranks per node used in this study

Performance and Scaling

Estimation of what is possible

- Estimate an upper limit of of core computations if the whole machine were available:
 - **Cori phase 1 (Haswell):** estimate an upper boundary of $6 \times 10^{10} \text{ s}^{-1}$ from a linear extrapolation to the entirety of 2388 nodes.
 - **Cori phase 2 (KNL):** estimate an upper boundary of $1.8 \times 10^{11} \text{ s}^{-1}$ when using all 9688 nodes of Cori phase 2 (KNL).
- The linear scaling assumption is optimistic. Should be interpreted as an upper limit of the performance of the program.



Conclusions

- Low statistics and physical boundaries require an empirical approach to calculating a significance
- Adapted a grid-search-based fitting application that calculates Feldman-Cousins corrections to run efficiently on current state-of-the-art processor architectures and systems available at HPC facilities (NERSC computing center).
- Transformed a memory limited serial-execution program into an MPI parallel application that scales up to available compute power of a computing facility.
- Achieved node-level and thread-level parallelism through DIY, and accomplish significant performance improvements by restructuring algorithms to use Eigen3 for matrix multiplications and array manipulations.
- This work was performed in the context of a specific application (SBNFit) designed for short-baseline neutrino oscillation experiments, however, the techniques employed can provide similar benefits in terms of performance and design for broader neutrino oscillation physics program.
- **What's next?**
 - Explore alternative techniques to grid scan procedure, such as utilizing optimizers and approximating discrete binned data into to a continuous function using the Multivariate Functional Approximation (MFA) model (doi: 10.1109/LDAV.2018.8739195.) as well as exploring different linear algebra libraries, such as Kokkos.
 - » Allows to move towards probing higher dimensions in oscillation parameter