1 From Marc - 2 Sept 2020

P. 2 : I rewrote Mariana's Lagrangian with some minor corrections (e.g. indices) and the connection with Terning's paper (with some constants that we might have to consider).

P. 3: I checked my results sent previously, and fixed my 1/2 factor! Done for visible A only. The red arrows indicate differences with Mariana's equation (1) (from her 12 August notes, here in P. 4)

P. 4 From Mariana's notes sent on 12 August.

PP. 5-6: Pages sent previously, applied to the equations of motion of P. 3. If this is correct, it suggest that Terning's diagonalization basis in Eqs. (4.4) - (4.7) is correct.

Next steps: check equations for B, A_D and B_D and compare with Mariana's results on P. 7 (in particular the unwanted term $\propto B_D^\mu$ in Eq. (2d)). Work out the Feynman rules. MadGraph.

MARC 2 SEPTEMBER 2020 1.

LAGRANGIAN (FROM MARIANA, 12 AUGUST, WITH MINOR CORRECTIONS)

EQS IN TERNING (2018)

$$\mathcal{L} = -\frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^A_{\alpha\beta} F^A_{\mu\nu} + F^B_{\alpha\beta} F^B_{\mu\nu}\right) \qquad (2.4) \times e^2$$

$$+ \frac{n^\alpha n_\mu}{4n^2} \epsilon^{\mu\nu\gamma\delta}\left(F^B_{\alpha\nu} F^A_{\gamma\delta} - F^A_{\alpha\nu} F^B_{\gamma\delta}\right)$$

$$- J_\mu A^\mu - K_\mu B^\mu \qquad (2.4);\ K_\mu \times \frac{e^2}{4\pi}$$

$$- \frac{n^\alpha n^\mu}{2n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^{A_D}_{\mu\nu} + F^{B_D}_{\alpha\beta} F^{B_D}_{\mu\nu}\right) \qquad (4.1)$$

$$+ \frac{n^\alpha n_\mu}{4n^2} \epsilon^{\mu\nu\gamma\delta}\left(F^{B_D}_{\alpha\nu} F^{A_D}_{\gamma\delta} - F^{A_D}_{\alpha\nu} F^{B_D}_{\gamma\delta}\right)$$

$$- J^D_\mu A^\mu_D - K^D_\mu B^\mu_D \qquad J^D \leftarrow \frac{(4.1)}{e_D} \quad K^D \leftarrow (4.1) \times \frac{e_D}{4\pi}$$

$$+ \epsilon e e_D \frac{n^\alpha n^\mu}{n^2} g^{\beta\nu}\left(F^{A_D}_{\alpha\beta} F^A_{\mu\nu} - F^{B_D}_{\alpha\beta} F^B_{\mu\nu}\right) \qquad (4.2)$$

$$- \frac{1}{2} m^2_{A_D} A_{D\mu} A^\mu_D - \frac{1}{2} m^2_{B_D} B_{D\mu} B^\mu_D \qquad (5.1) \quad (5.3)$$

$$F^X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$$

$$J^\mu_{(D)} \equiv e_{(D)} \bar{\psi}_{e(D)} \gamma^\mu \psi_{e(D)} \qquad K^\mu_{(D)} \equiv g_{(D)} \bar{\psi}_{g(D)} \gamma^\mu \psi_{g(D)}$$

is $g_{(D)} = \frac{4\pi}{e^2_{(D)}}$ or $\frac{4\pi}{e_D}$?

EQS OF MOTION W/R A_μ

$$-J^\sigma = -\frac{n_\mu}{n^2}\left(n^\rho \partial_\rho F_A^{\mu\sigma} + n^\sigma \partial_\rho F_A^{\rho\mu}\right)$$

$$+ \epsilon e e_D \frac{n_\alpha}{n^2}\left(n^\rho \partial_\rho F_{A_D}^{\alpha\sigma} - n^\sigma \partial_\rho F_{A_D}^{\alpha\rho}\right)$$

$$+ \frac{n_\mu}{4n^2}\left[2n^\alpha \epsilon^{\mu\nu\rho\sigma} \partial_\rho F^B_{\alpha\nu} - \left(n^\rho \epsilon^{\mu\sigma\gamma\delta} - \underbrace{n^\sigma \epsilon^{\mu\rho\gamma\delta}}_{\emptyset \text{ BY CONTRACTION}}\right)\partial_\rho F^B_{\gamma\delta}\right]$$

LINE 3 is $\frac{n_\mu n^\alpha}{2n^2}\epsilon^{\mu\nu\rho\sigma}\overbrace{\partial_\rho F^B_{\alpha\nu}}^{\partial_\rho\partial_\alpha B_\nu} - \underbrace{\frac{n_\mu n^\rho}{4n^2}\epsilon^{\mu\sigma\gamma\delta}\partial_\rho F^B_{\gamma\delta}}_{\text{RENAMING + PERMUTING INDICES...}}$

$$\Rightarrow \frac{n_\mu n^\alpha}{2n^2}\epsilon^{\mu\nu\rho\sigma}\partial_\rho\partial_\alpha B_\nu + \frac{n_\mu n^\alpha}{4n^2}\epsilon^{\mu\nu\rho\sigma}\partial_\alpha\left(\partial_\rho B_\nu - \partial_\nu B_\rho\right) \quad \circledast$$

IN TERM 1 OF $\circledast$ WITH $\mu=0, \sigma=1$: $\epsilon^{0\nu\rho1}\partial_\rho B_\nu = \partial_3 B_2 - \partial_2 B_3 = F_{32}$

" " 2 " " : $\epsilon^{0\nu\rho1}(\partial_\rho B_\nu - \partial_\nu B_\rho) = \partial_3 B_2 - \partial_2 B_3 - (\partial_2 B_3 - \partial_3 B_2) = 2F_{32}$

ETC. SO THE TWO TERMS IN $\circledast$ ARE EQUAL SO THAT $\circledast$ = LINE 3 is

SIMPLY $\frac{n_\mu n^\alpha}{n^2}\epsilon^{\mu\nu\rho\sigma}\partial_\rho F^B_{\alpha\nu}$. EQ. OF MOTION BECOMES

$$\frac{n_\alpha}{n^2}\left(-n^\mu \partial_\nu F_A^{\alpha\nu} + n^\nu \partial_\nu F_A^{\alpha\mu}\right) + \frac{\epsilon e e_D}{n^2} n_\alpha\left(n^\mu \partial_\nu F_{A_D}^{\alpha\nu} - n^\nu \partial_\nu F_{A_D}^{\alpha\mu}\right)$$
$$- \frac{n_\alpha}{n^2} n^\gamma \epsilon^{\alpha\beta\nu\mu}\partial_\nu F^B_{\gamma\beta} = J^\mu$$

$$\frac{n_\alpha}{n^2}\left(-n^\mu \partial_\nu F_A^{\alpha\nu} + n^\nu \partial_\nu F_A^{\alpha\mu} + n^\gamma \epsilon^{\mu\nu\alpha\beta}\partial_\nu F^B_{\gamma\beta}\right)$$
$$+ \epsilon e e_D \frac{n_\alpha}{n^2}\partial_\nu\left(n^\mu F_{A_D}^{\alpha\nu} - n^\nu F_{A_D}^{\alpha\mu}\right) = J^\mu$$

Equations of motion

VISIBLE

(1)
$$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_A^{\alpha\nu} - n^\nu \partial_\nu F_A^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_B^{\gamma\beta}\right)$$
$$+\frac{\epsilon e e_D}{n^2}\, n_\alpha \partial_\nu\left(n^\mu F_{A_D}^{\alpha\nu} - n^\nu F_{A_D}^{\alpha\mu}\right) = J^\mu$$

(2)
$$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_B^{\alpha\nu} - n^\nu \partial_\nu F_B^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_A^{\gamma\beta}\right)$$
$$-\frac{\epsilon e e_D}{n^2}\, n_\alpha \partial_\nu\left(n^\mu F_{B_D}^{\alpha\nu} - n^\nu F_{B_D}^{\alpha\mu}\right) = K^\mu$$

DARK

(3)
$$\frac{n_\alpha}{n^2}\left(n^\nu \partial_\nu F_{A_D}^{\alpha\nu} - n^\nu \partial_\nu F_{A_D}^{\alpha\mu} - \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_{B_D}^{\gamma\beta}\right)$$
$$+\frac{\epsilon e e_D}{n^2}\, n_\alpha \partial_\nu\left(n^\mu F_A^{\alpha\nu} - n^\nu F_A^{\alpha\mu}\right) = J_D^\mu + M_{A_D}^2 A_D^\mu A_{D\mu}$$

(4)
$$\frac{n_\alpha}{n^2}\left(n^\mu \partial_\nu F_{B_D}^{\alpha\nu} - n^\nu \partial_\nu F_{B_D}^{\alpha\mu} + \varepsilon^{\mu\nu\alpha}{}_\beta\, n_\gamma \partial_\nu F_{A_D}^{\gamma\beta}\right)$$
$$-\frac{\epsilon e e_D}{n^2}\left(n_\alpha \partial_\nu n^\mu F_B^{\alpha\nu} - n_\nu^{\ \gamma} F_B^{\alpha\mu}\right) = K_D^\mu + M_{B_D}^2 B_D^\mu B_{D\mu}$$

PROCEEDING AS IN MARIANA'S p.3 WITH MY SIGNS BUT WITHOUT 1/2:

$$(1)\quad \frac{m_\chi}{m^2}\left(m^\nu \partial_\nu F_A^{\alpha\mu} - m^\mu \partial_\nu F_A^{\alpha\nu} + \epsilon^{\mu\nu\alpha\beta} m^\gamma \partial_\nu F^B_{\gamma\beta}\right)$$

$$+ \frac{\epsilon e e_D}{m^2} m_\chi \partial_\nu \left(m^\mu F_{A_D}^{\alpha\nu} - m^\nu F_{A_D}^{\alpha\mu}\right) = J^\mu$$

$$J^\mu = \frac{m_\chi}{m^2}\left[\underline{m^\nu \partial_\nu} F_A^{\alpha\mu} - \underline{m^\mu \partial_\nu} F_A^{\alpha\nu} + \underline{\epsilon e e_D\, m^\mu} \partial_\nu F_{A_D}^{\alpha\nu} - \underline{\epsilon e e_D\, m^\nu} \partial_\nu F_{A_D}^{\alpha\mu} + \epsilon^{\mu\nu\alpha\beta} m^\gamma \partial_\nu F^B_{\gamma\beta}\right)$$

$$= \frac{m_\chi}{m^2}\left[m^\nu \partial_\nu \left(F_A^{\alpha\mu} - \epsilon e e_D F_{A_D}^{\alpha\mu}\right) - m^\mu \partial_\nu \left(F_A^{\alpha\nu} - \epsilon e e_D F_{A_D}^{\alpha\nu}\right) + m^\gamma \epsilon^{\mu\nu\alpha\beta} \partial_\nu F^B_{\gamma\beta}\right]$$

FROM TERNING EQS (4.4, 5) $\quad F_A^{\alpha\mu} = (\cos\phi + \epsilon e e_D \sin\phi) F_{\bar{A}}^{\alpha\mu} + (-\sin\phi + \epsilon e e_D \cos\phi) F_{\bar{A}_D}^{\alpha\mu}$

$$F_{A_D}^{\alpha\mu} = \sin\phi\, F_{\bar{A}}^{\alpha\mu} + \cos\phi\, F_{\bar{A}_D}^{\alpha\mu}$$

$$F_B^{\gamma\beta} = \cos\phi\, F_{\bar{B}}^{\gamma\beta} - \sin\phi\, F_{\bar{B}_D}^{\gamma\beta}$$

$$F_A^{\alpha\mu} - \epsilon e e_D F_{A_D}^{\alpha\mu} = \cos\phi\, F_{\bar{A}}^{\alpha\mu} + \cancel{\epsilon e e_D \sin\phi\, F_{\bar{A}}^{\alpha\mu}} - \sin\phi\, F_{\bar{A}_D}^{\alpha\mu} + \cancel{\epsilon e e_D \cos\phi\, F_{\bar{A}_D}^{\alpha\mu}}$$

$$- \cancel{\epsilon e e_D \sin\phi\, F_{\bar{A}}^{\alpha\mu}} - \cancel{\epsilon e e_D \cos\phi\, F_{\bar{A}_D}^{\alpha\mu}}$$

$$= \cos\phi\, F_{\bar{A}}^{\alpha\mu} - \sin\phi\, F_{\bar{A}_D}^{\alpha\mu}$$

$$J^\mu = \frac{m_\chi}{m^2}\left[m^\nu \partial_\nu \left(\cos\phi\, F_{\bar{A}}^{\alpha\mu} - \sin\phi\, F_{\bar{A}_D}^{\alpha\mu}\right) - m^\mu \partial_\nu \left(\cos\phi\, F_{\bar{A}}^{\alpha\nu} - \sin\phi\, F_{\bar{A}_D}^{\alpha\nu}\right)\right.$$

$$\left. + m^\gamma \epsilon^{\mu\nu\alpha\beta} \partial_\nu \left(\cos\phi\, F_{\bar{B}}^{\gamma\beta} - \sin\phi\, F_{\bar{B}_D}^{\gamma\beta}\right)\right]$$

7

$$J^\mu = \frac{n_\alpha}{n^2}\Big[\cos\phi\left(n^\nu\partial_\nu F_{\bar{A}}^{\alpha\mu} - n^\mu\partial_\nu F_{\bar{A}}^{\alpha\nu} + n^\gamma\epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\bar{B}}^{\gamma\beta}\right)$$

$$- \sin\phi\,\partial_\nu\left(n^\nu F_{\bar{A}_D}^{\alpha\mu} - n^\mu F_{\bar{A}_D}^{\alpha\nu} + n^\gamma\epsilon^{\mu\nu\alpha\beta}F_{\bar{B}_D}^{\gamma\beta}\right)\Big]$$

$$= \cos\phi\,\bar{J}^\mu - \sin\phi\,\frac{e_D}{e}\,\bar{J}_D^\mu$$

FROM TERNING EQ 4.6

WITH SUGGESTS

$$\bar{J}^\mu = \frac{n_\alpha}{n^2}\left(n^\nu\partial_\nu F_{\bar{A}}^{\alpha\mu} - n^\mu\partial_\nu F_{\bar{A}}^{\alpha\nu} + n^\gamma\epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\bar{B}}^{\gamma\beta}\right)$$

$$e_D\bar{J}_D^\mu = e\frac{n_\alpha}{n^2}\left(n^\nu\partial_\nu F_{\bar{A}_D}^{\alpha\mu} - n^\mu\partial_\nu F_{\bar{A}_D}^{\alpha\nu} + n^\gamma\epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\bar{B}_D}^{\gamma\beta}\right)$$

So the (almost) decoupled equations are

(1d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_A^{\alpha\nu} - n^\nu \bar{F}_A^{\alpha\mu} - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_B^{\gamma\beta}\right] = \bar{J}^\mu$$

(2d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_B^{\alpha\nu} - n^\nu \bar{F}_B^{\alpha\mu} + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_A^{\gamma\beta}\right] = \bar{K}^\mu + \epsilon e e_D M_{B_D}^2 B_D^{2\mu}$$

(3d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_{A_D}^{\alpha\nu} - n^\nu \bar{F}_{A_D}^{\alpha\mu} - n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_{B_D}^{\gamma\beta}\right] = \bar{J}^{D\mu} + M_{A_D}^2 A_D^{2\mu}$$

(4d) $$\frac{n_\alpha}{n^2}\partial_\nu\left[n^\mu \bar{F}_{B_D}^{\alpha\nu} - n^\nu \bar{F}_{B_D}^{\alpha\mu} + n_\gamma \epsilon^{\mu\nu\alpha}{}_\beta \bar{F}_{A_D}^{\gamma\beta}\right] = \bar{K}^{D\mu} + M_{B_D}^2 B_D^{2\mu}$$

Perhaps we can think of $\epsilon e e_D M_{B_D}^2 B_D^{\mu 2}$ as some residual mass term from the dark sector into the visible one. (unfortunately it depends on B_D^μ ::)