Gravitational waves from phase transitions: clearing the path between theory and experiment

KAVLI IPMU

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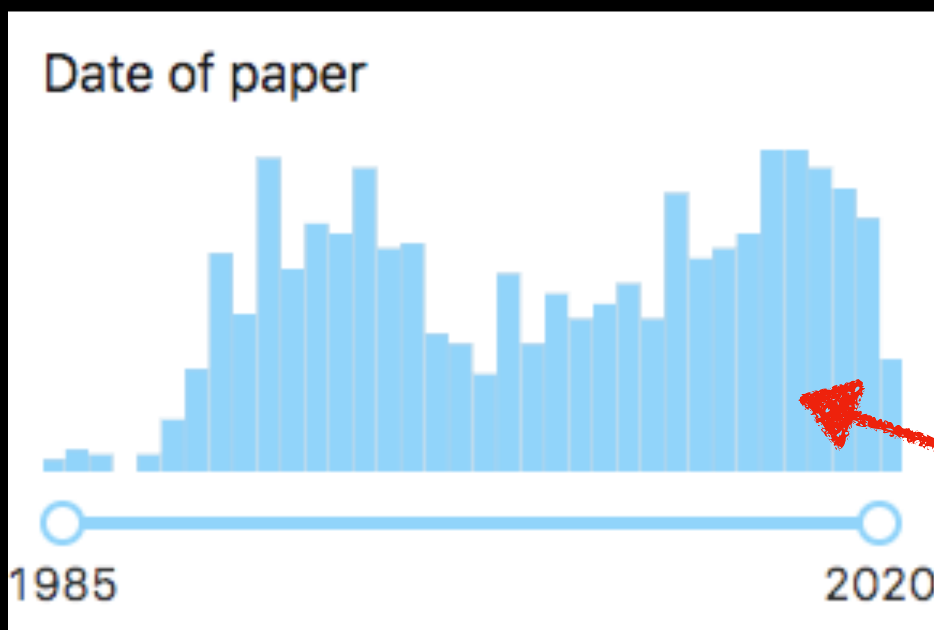
KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Introduction

EWBG papers per year



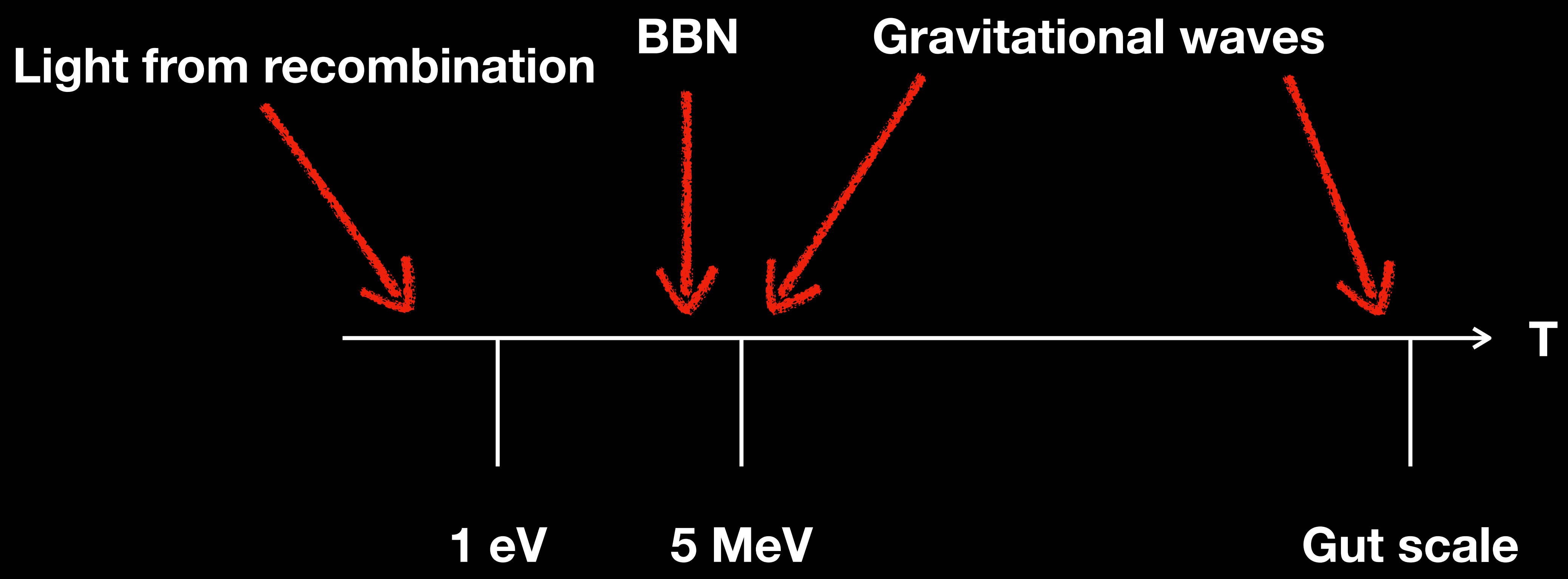
Papers on GWs from PTs

Ligo discovers GWs

source: inspire

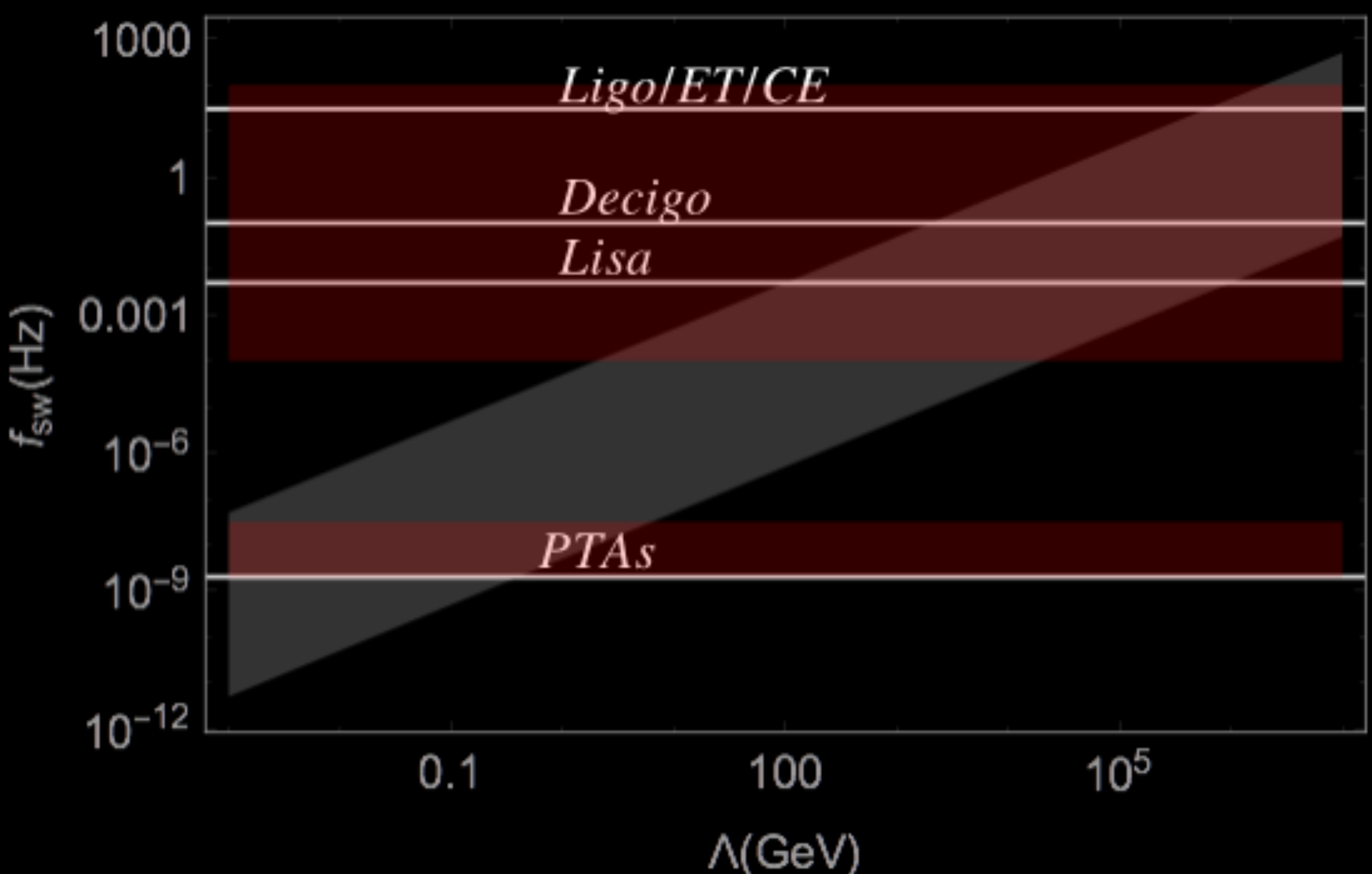
Introduction

1. Universe is transparent to GWs right back to the beginning!



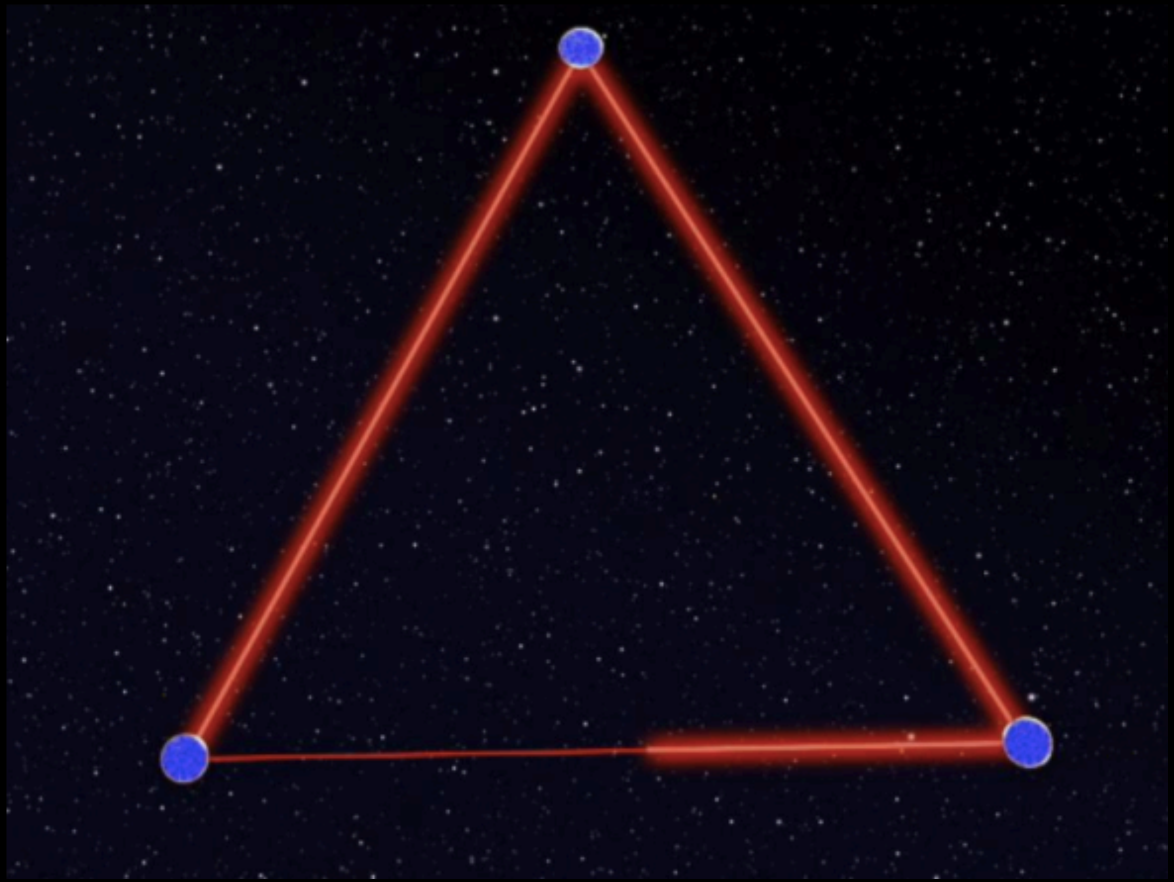
Introduction

2. Probe of BSM



Introduction

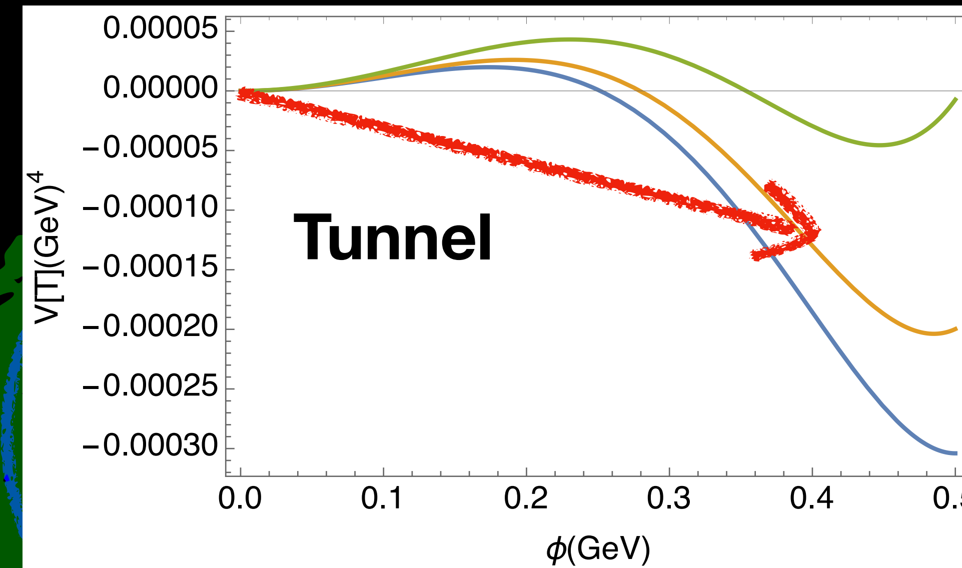
3. The nature of the EWPT is *the* big theoretical question that next generation Experiments can probe

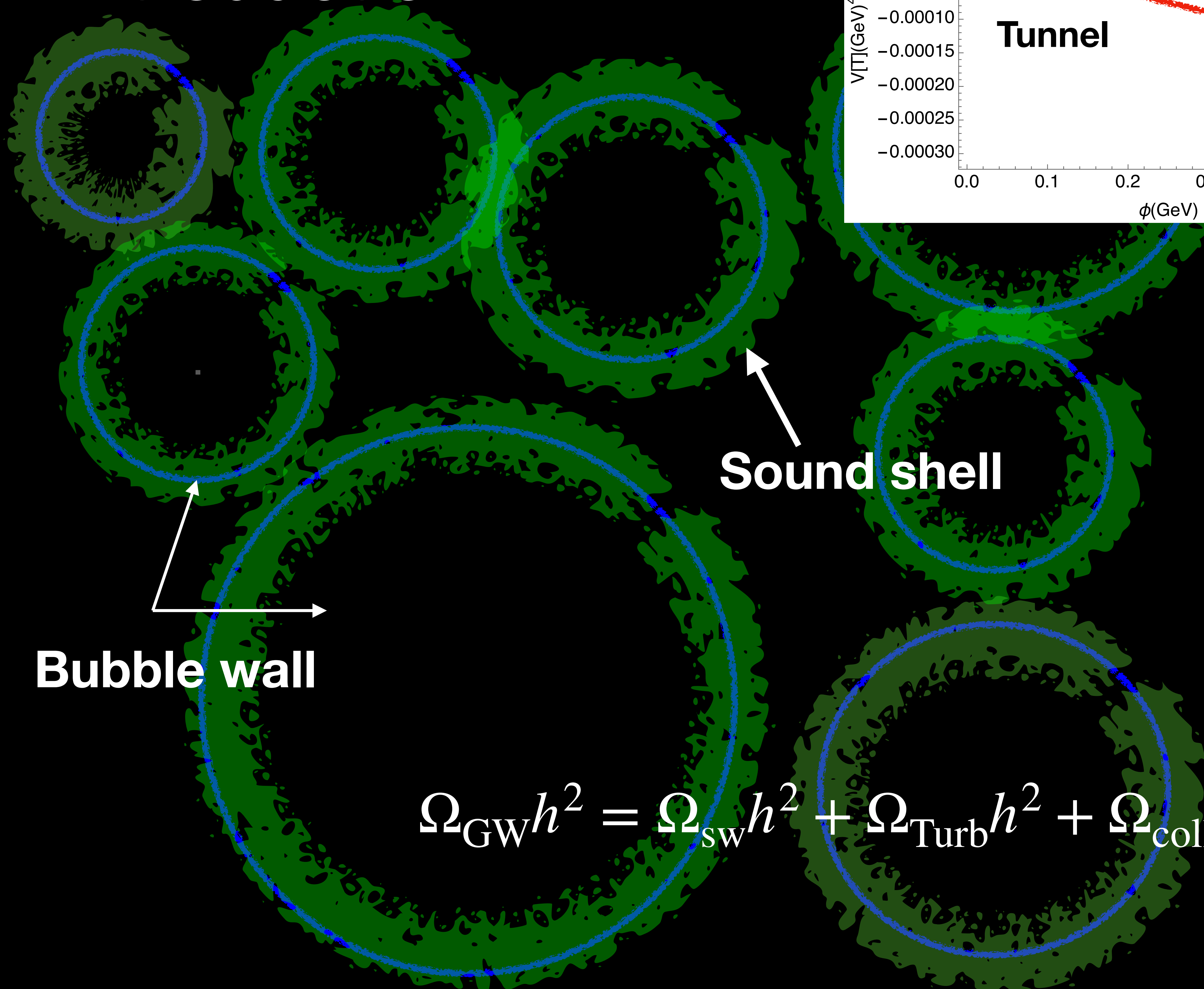


LISA search for primordial GWs
Next generation colliders:
HL-HE-LHC, 100 TeV FCC

Image: 1) Lisa mission 2) Cern

Introduction





$$\Omega_{\rm GW}h^2 = \Omega_{\rm sw}h^2 + \Omega_{\rm Turb}h^2 + \Omega_{\rm coll}h^2$$

Introduction

It seems a beautiful picture that has inspired the imagination of BSM Phenomenologists

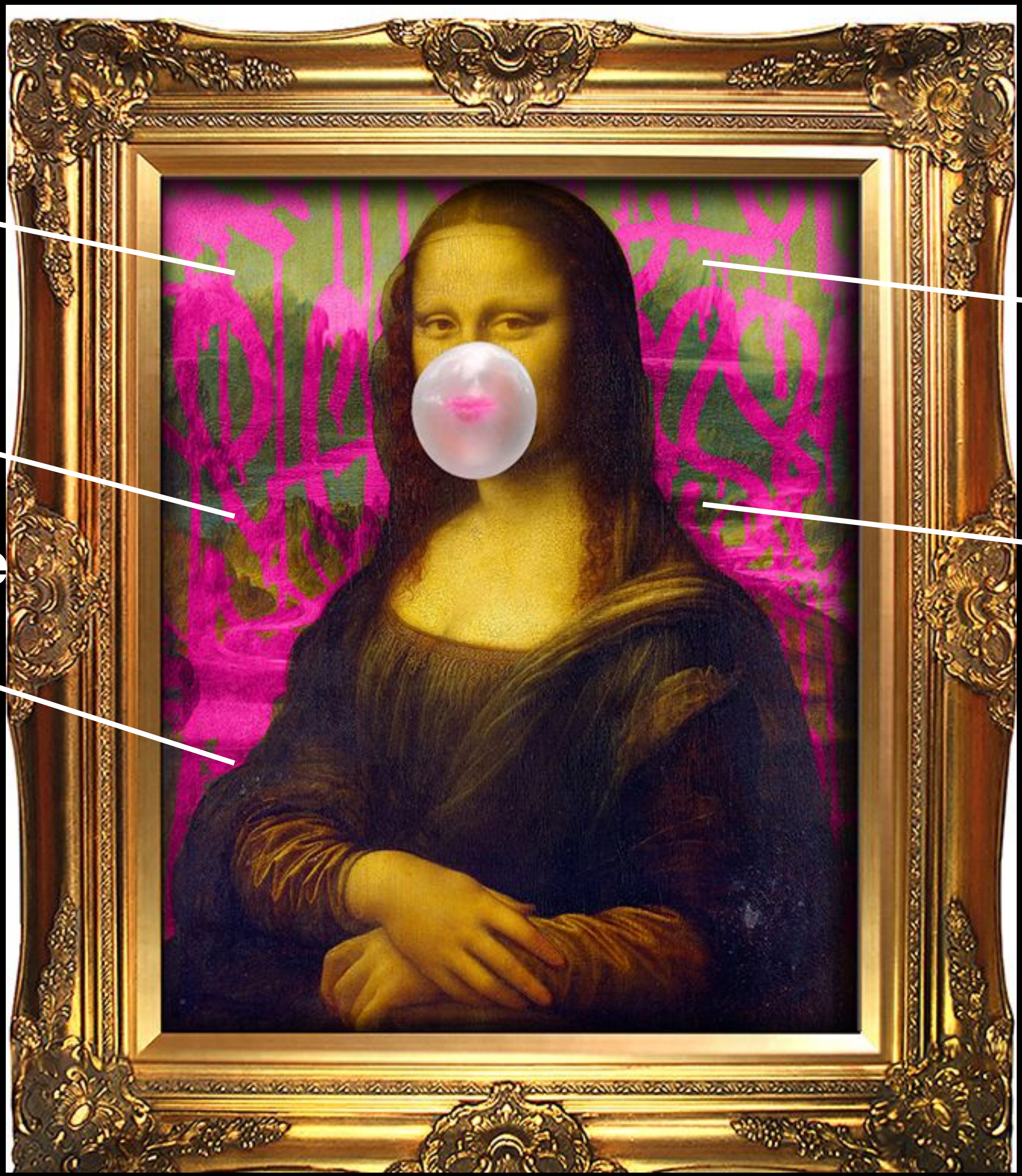
Introduction

Until we take a closer look at the theory

Tachyonic modes

Gauge dependence

Renormalization dependence



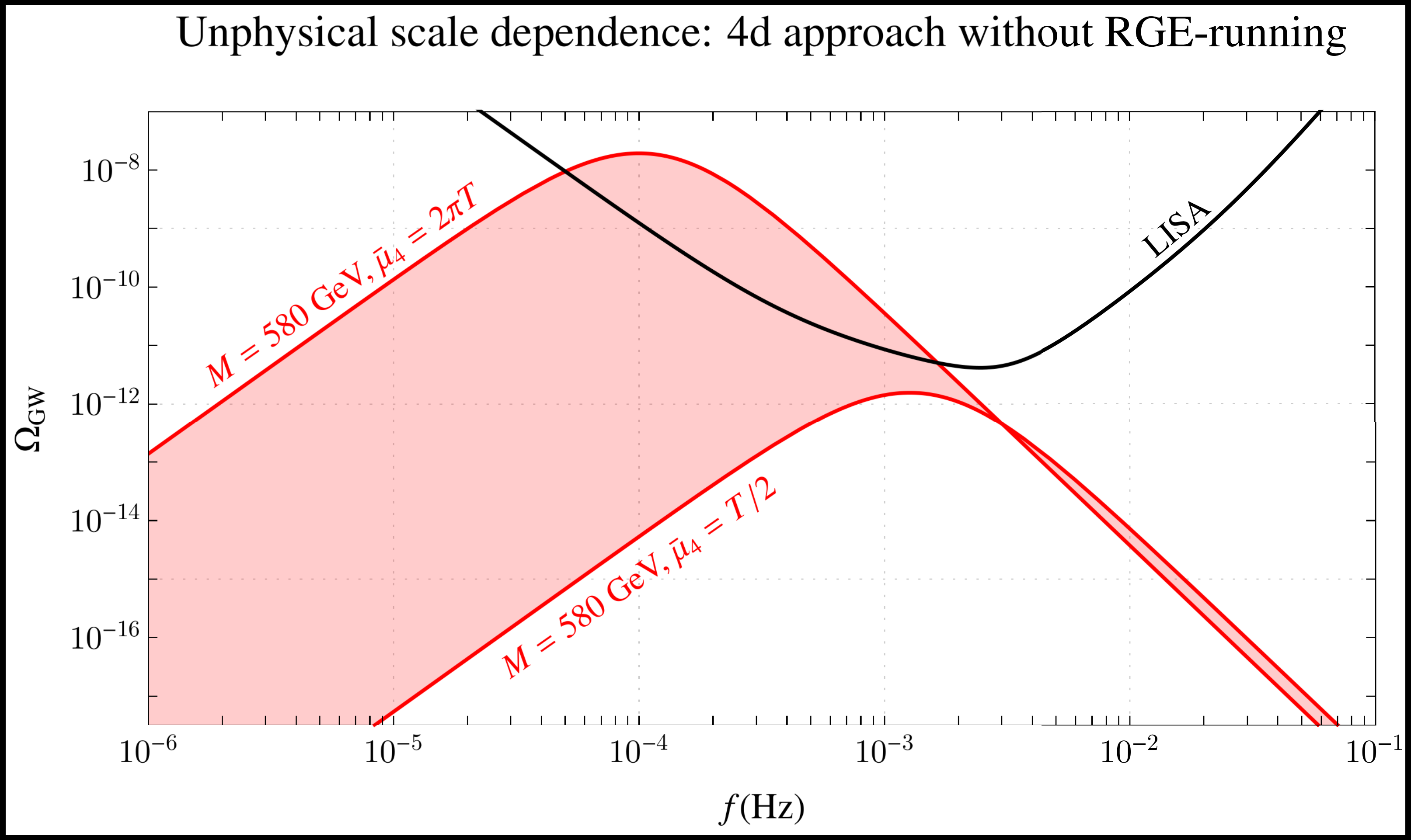
Simulation Uncertainty

Consistent treatment Of inhomogeneous background

Image credit: “studio27”

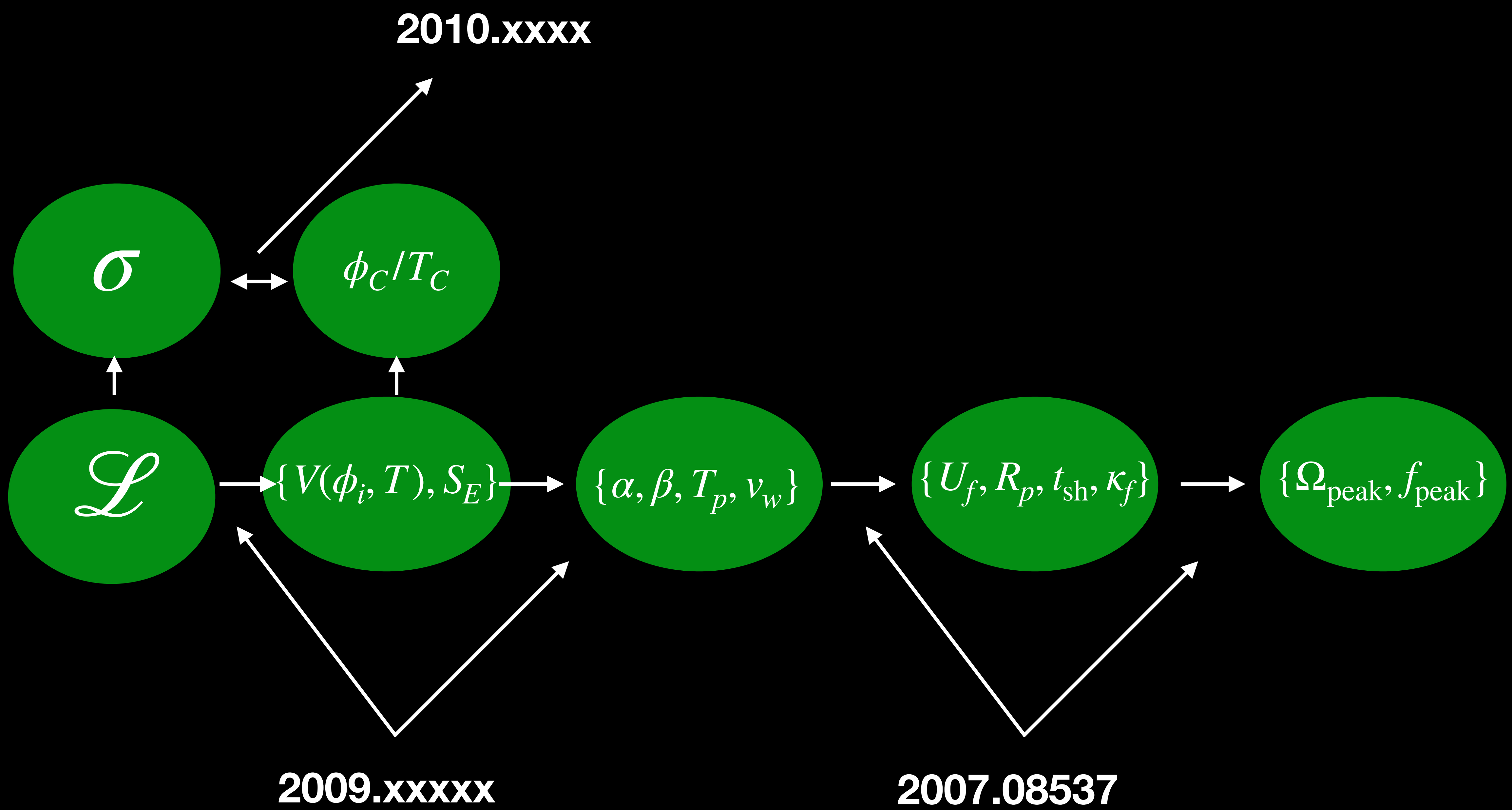
Introduction

Just how bad is this?



Introduction

Theoretical uncertainties from Lagrangian to observables



With Djuna Croon, Oli Gould, Huaike Guo, Phillip Schicho, Andreas Papaefstathiou, Kuver Sinha, Daniel Vagie And Tuomas Tenkanen

Part 1

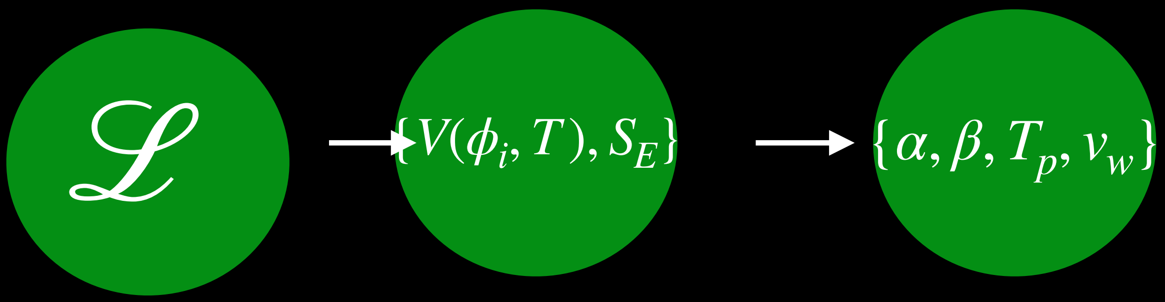
$$\mathscr{L} \longrightarrow \{V(\phi_i, T), S_E\} \longrightarrow \{\alpha, \beta, T_p, v_w\}$$

Problem 1: scale dependence

$$g \to g n_b \sim \frac{gT}{m}$$

$$g \to g(\mu), \quad V_1 \equiv V_1(\mu)$$

Part 1

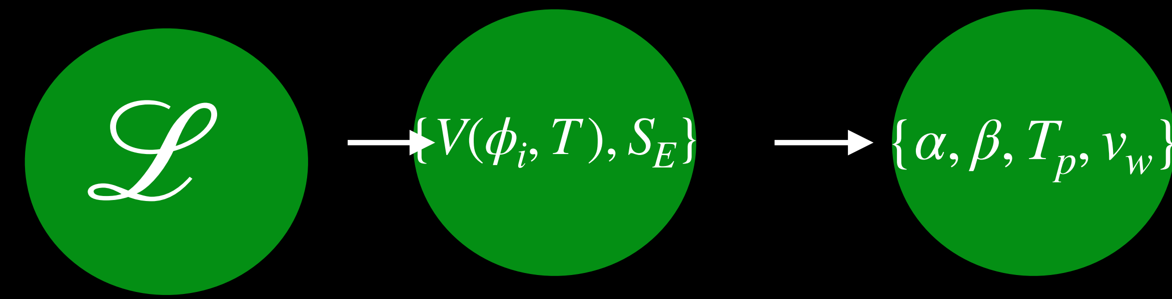


Problem 2: gauge dependence

$V_1(\phi, T)$ depends on m_{GM}

m_{GM} depends on ξ_i

Part 1



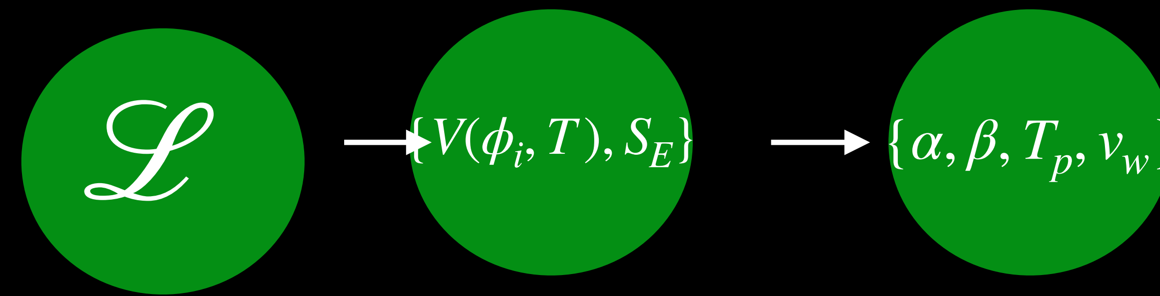
Problem 3: inhomogeneous background

Γ depends on S_E/T

S_E/T depends on V

Catch 22: V derived assuming equilibrium

Part 1



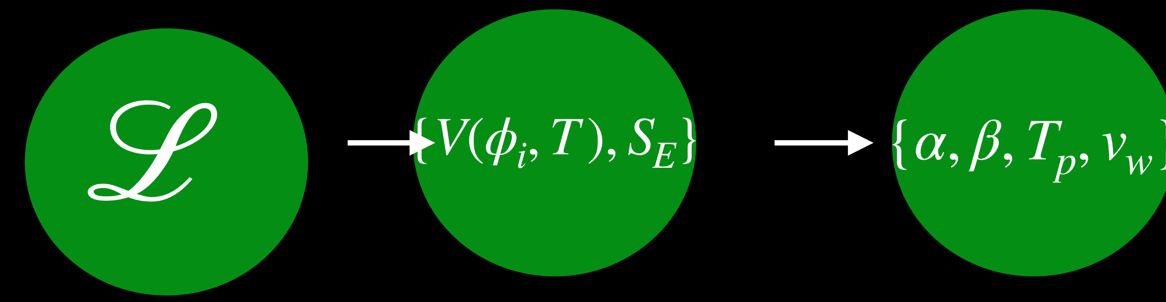
Solution to scale dependence 1: Arnold Espinosa resummation

$$m \to m + \Pi$$

$$V_1 \to V_1 + V_{\text{Daisy}}$$

$$V_{\text{Daisy}} = -\frac{T}{12\pi}\left([m^2 + \Pi]^{3/2} - m^3\right)$$

Part 1



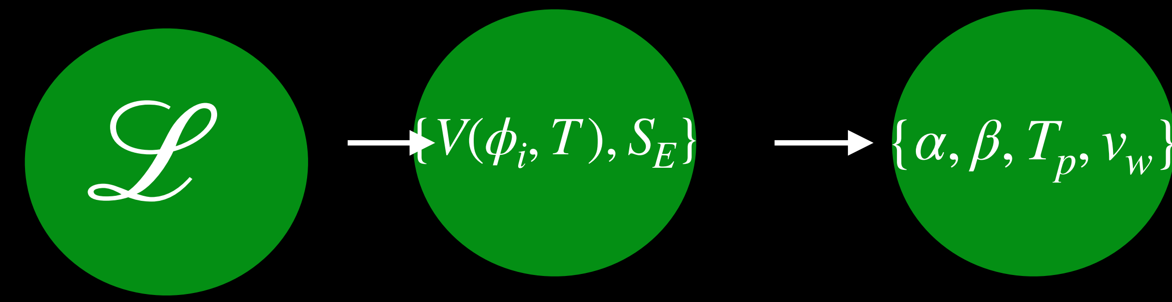
Solution to gauge dependence 2: hbar expansion

$$V = V_0 + \hbar V_1(\phi_0) + \cdots$$

Daisies appear at 2nd order in $\hbar$

Patel, Ramsey-Musolf: 1101.4665

Part 1

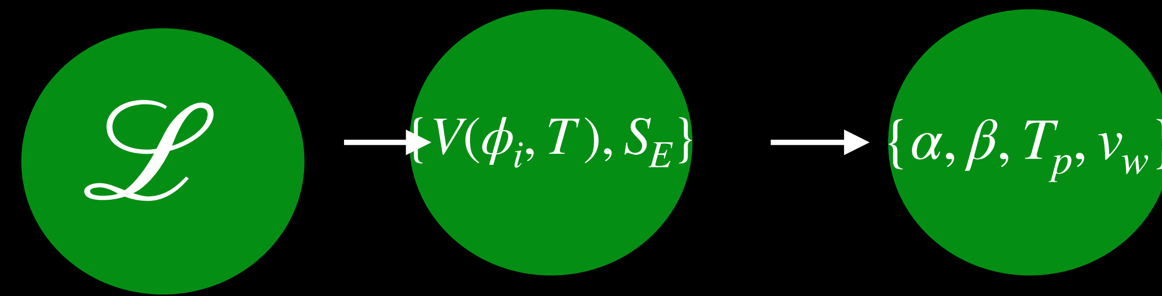


Solution to both 3: Dimensional reduction

$$\Delta \sim \frac{1}{p^2 + m^2 + 2\pi n T}$$

Same as a Kaluza Klein theory with a compactified dimension of size $1/T$!

Part 1



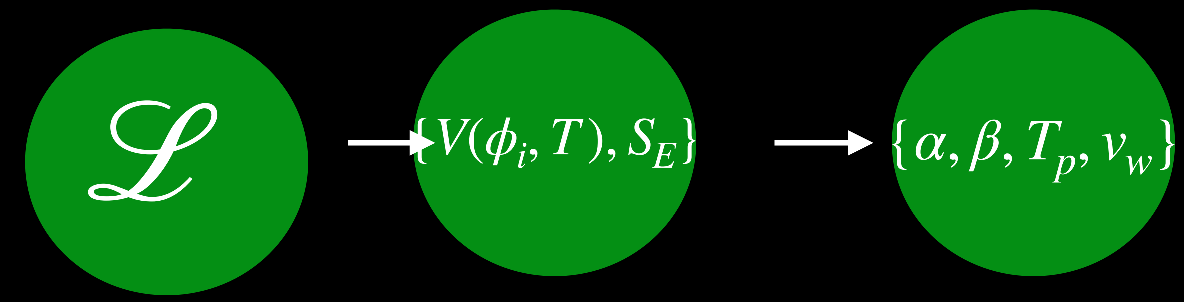
Start: $(d+1)$**-dimensional SMEFT**

Scale	Validity	Dimension	Lagrangian	Fields	Parameters
Hard	πT	$d+1$	$\mathcal{L}_{\text{SMEFT}}$ (2.7)	$G_{\mu\nu}, F_{\mu\nu}, H_{\mu\nu}, \phi, \psi_i$	$\mu_h^2, \lambda, c_6, g, g', g_s, g_Y$
		$\downarrow$ *Integrate out* $n \neq 0$ *modes and fermions*			
Soft	gT	d	$\mathcal{L}_{3d}$ (B.27)	$G_{ij}, F_{ij}, H_{ij},$	$\mu_{h,3}^2, \lambda_3, c_{6,3}, g_3, m_D,$
				A_0, B_0, C_0, ϕ_3	$g_3', m_D', g_{s,3}, m_D''$
		$\downarrow$ *Integrate out temporal adjoint scalars* A_0, B_0, C_0			
Ultrasoft	g^2T/π	d	$\bar{\mathcal{L}}_{3d}$ (B.26)	$G_{ij}, F_{ij}, H_{ij}, \bar{\phi}_3$	$\bar{\mu}_{h,3}^2, \bar{\lambda}_3, \bar{c}_{6,3}, \bar{g}_3, \bar{g}_3', \bar{g}_{s,3}$

End: d**-dimensional Pure Gauge**

$$\phi_{3\text{d}}^2 = \frac{1}{T}\left[1 + \Pi_\phi'(0) - \delta Z_\phi\right] \phi_{4d}$$

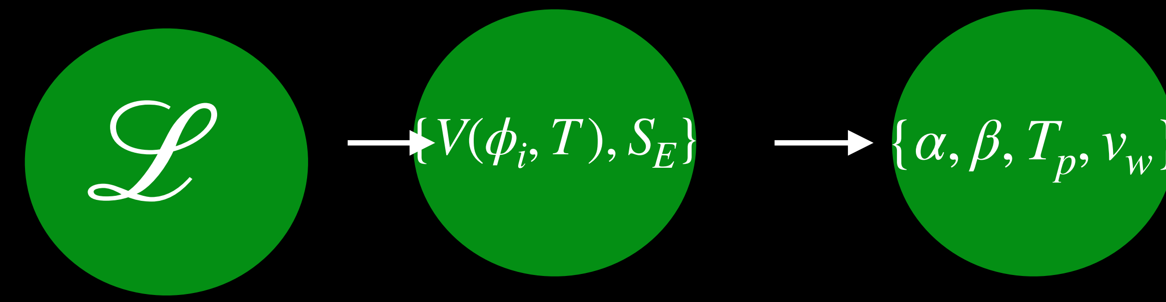
Part 1



Resulting 3d effective theory:

- Includes a less ad -hoc resummation automatically
- Is manifestly gauge invariant
- Easy to go to 2 loops
- Allows other ways of calculating the prefactor of the nucleation rate

Part 1



Recall Γ **involves a catch 22: you need to use the background to derive the backgorund!**

The hierarchy between scales allows a different expansion in 3d

$$\Gamma = \int d^3x \left[V_{\rm IR} + \frac{1}{2}(\partial\phi_{\rm IR})^2 + \sum_n C_n \Lambda_{\rm IR}^2 \left(\frac{\Lambda_{IR}}{\Lambda_{\rm UV}}\right)^{2n} \phi_{\rm IR} \left(\frac{\partial}{\Lambda_{IR}}\right)^{2n} \phi_{\rm IR} \right]$$

Langer 1969, 1974

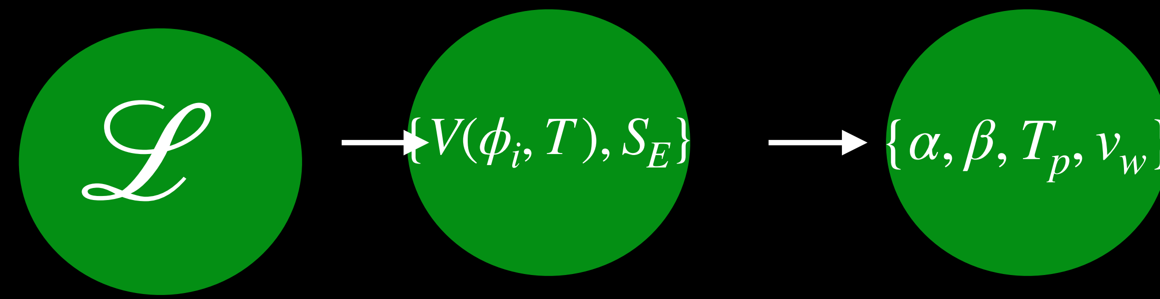
Part 1

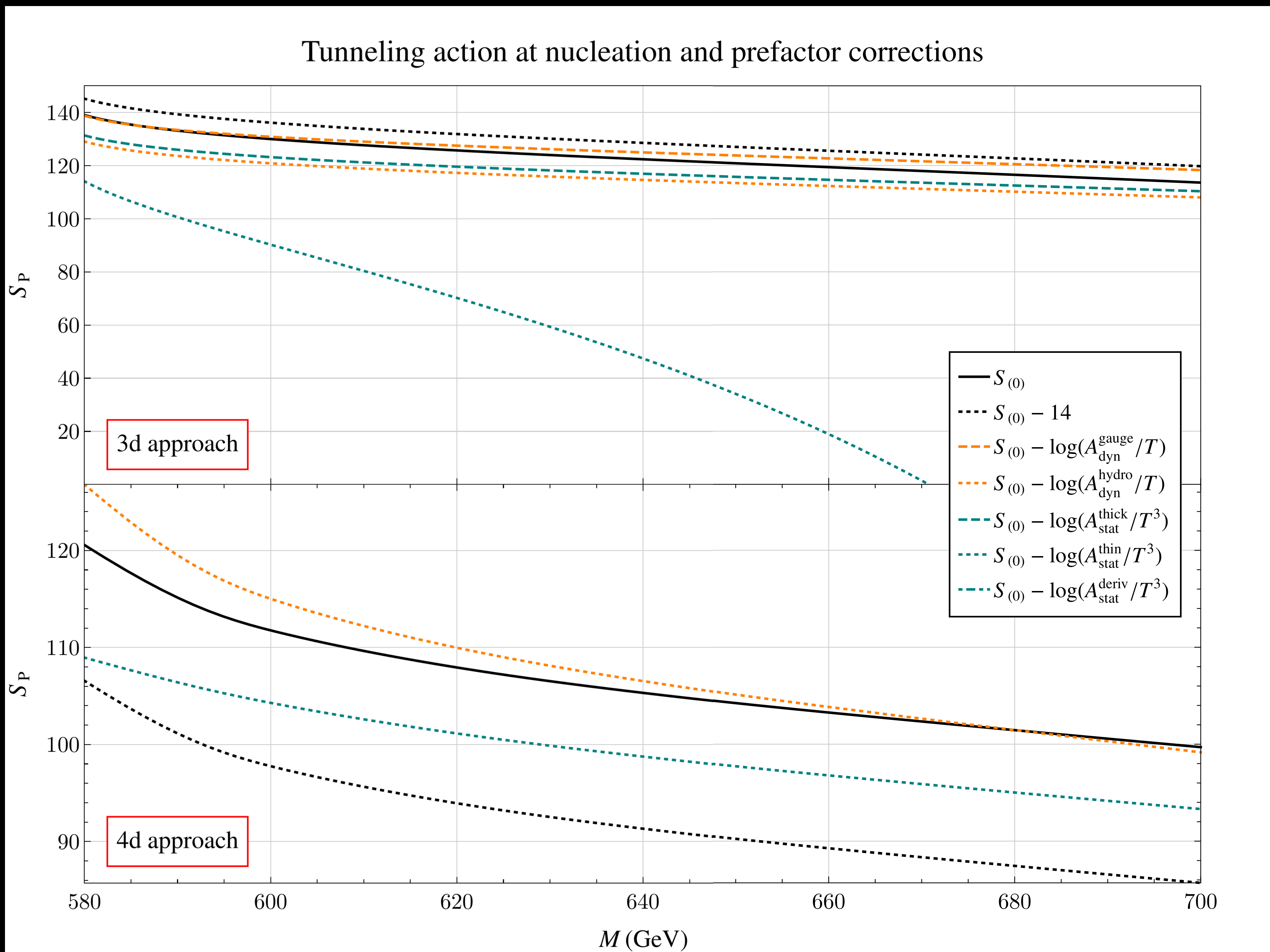
$$\mathscr{L} \longrightarrow \{V(\phi_i, T), S_E\} \longrightarrow \{\alpha, \beta, T_p, v_w\}$$

Benchmark model for analysis: SMEFT with a single operator

$$\mathscr{L} = \mathscr{L}_{\text{sm}} + \frac{1}{M^2}(H^\dagger H)^3$$

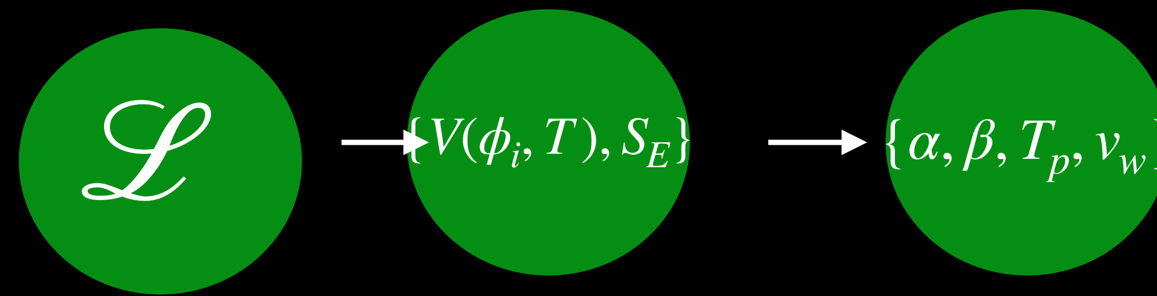
Part 1



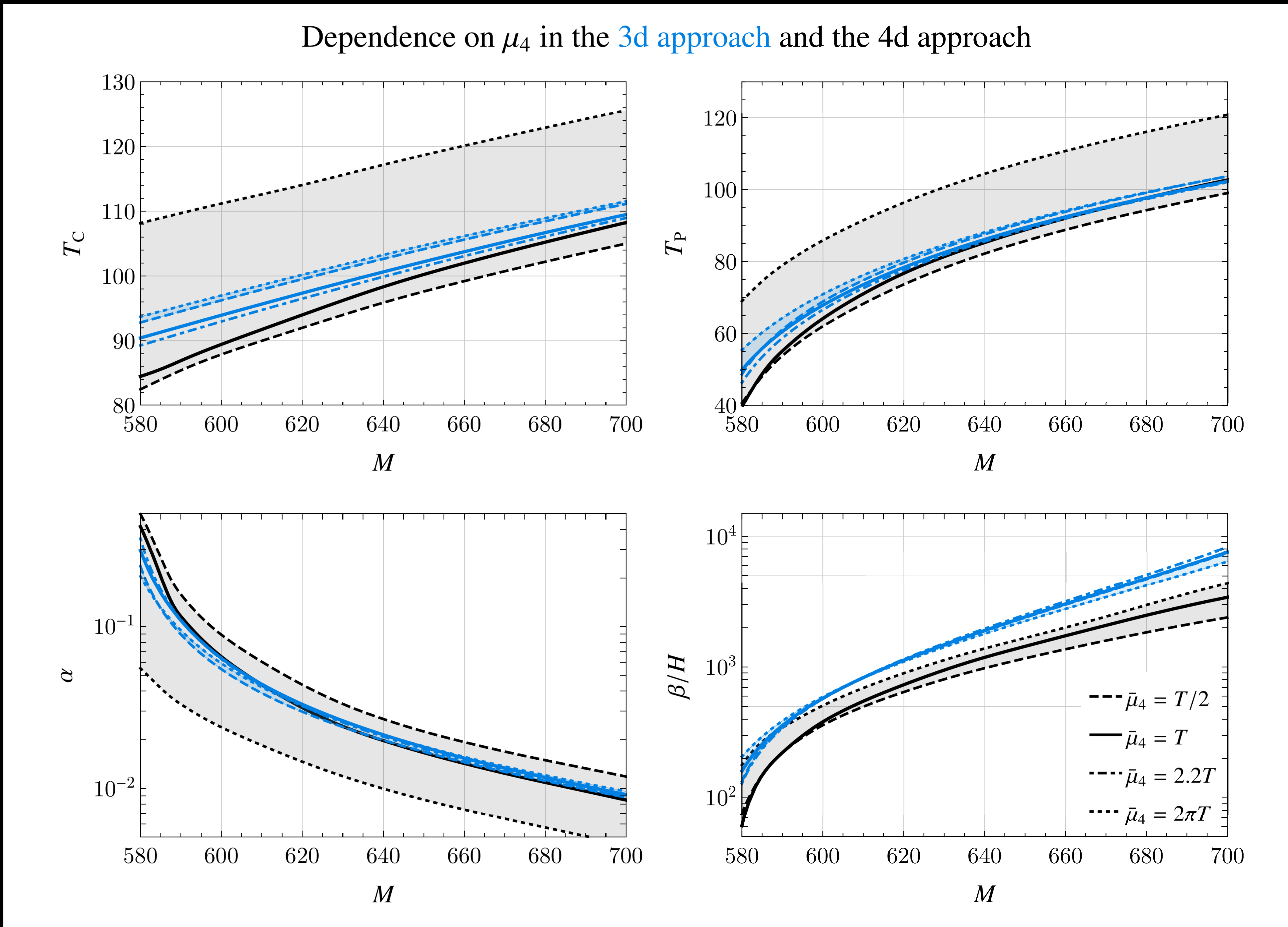


$$\frac{S}{T_p} = 131 + \log[A/T^4] + \cdots$$

Part 1

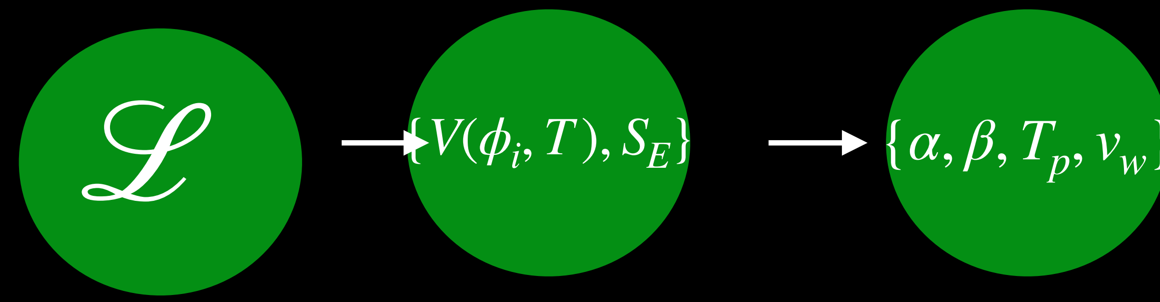


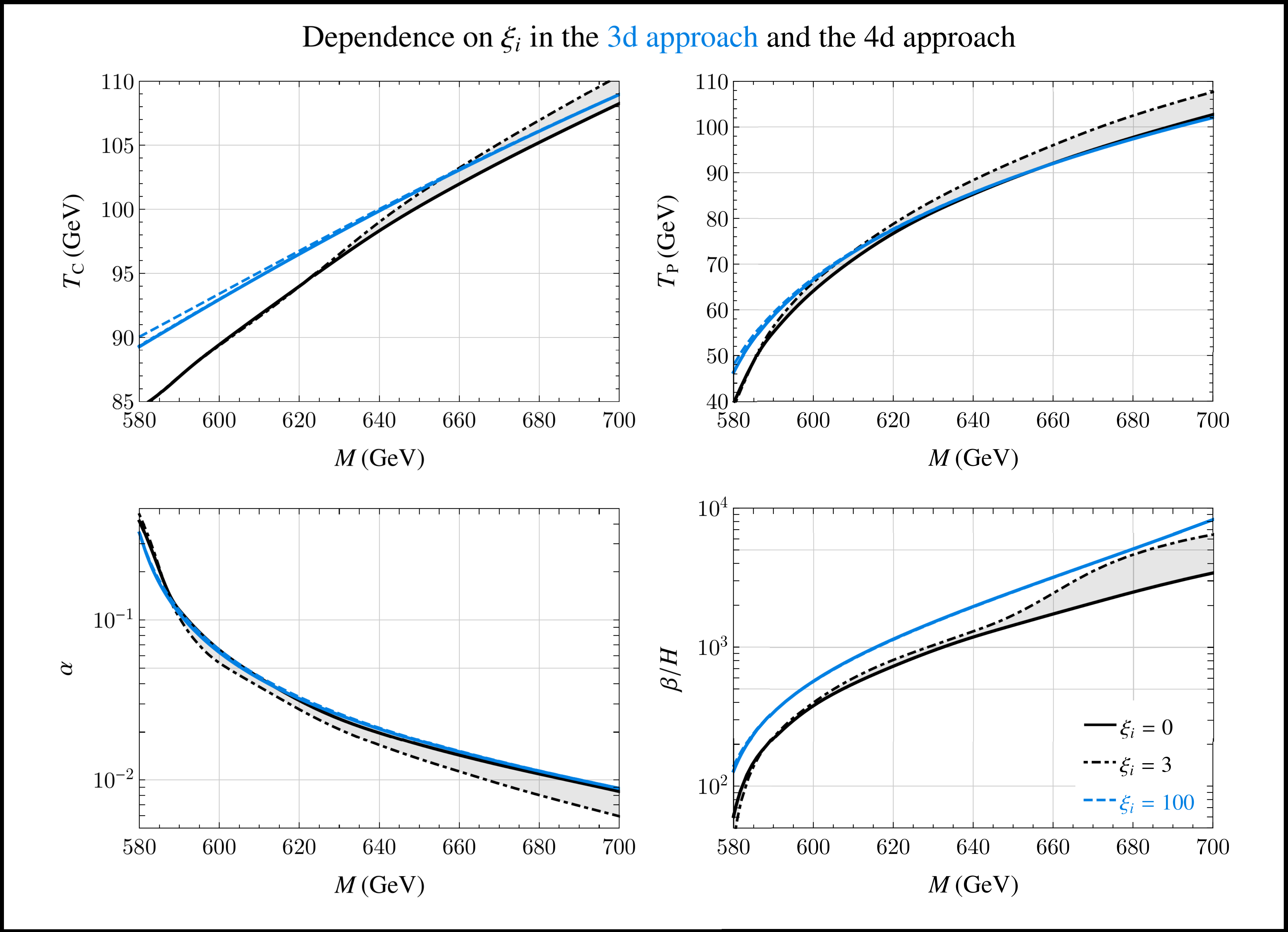
$\mathscr{L}$
$\{V(\phi_i, T), S_E\}$
$\{\alpha, \beta, T_p, v_w\}$



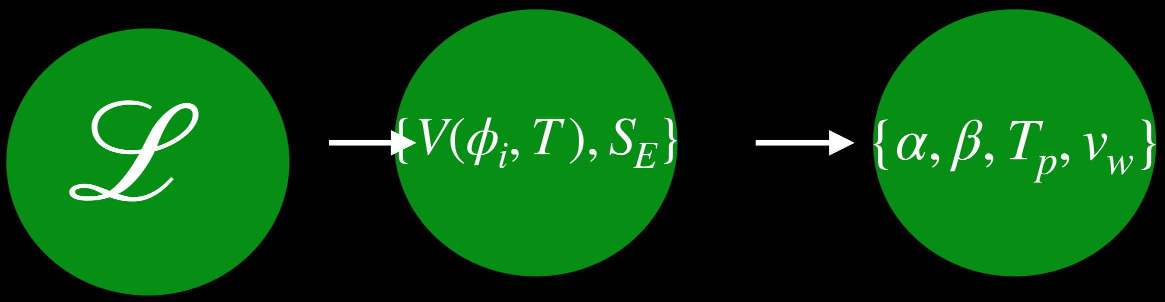
Dependence on μ_4 in the 3d approach and the 4d approach
T_C
T_P
α
β/H
M
580 600 620 640 660 680 700
$\bar{\mu}_4 = T/2$
$\bar{\mu}_4 = T$
$\bar{\mu}_4 = 2.2T$
$\bar{\mu}_4 = 2\pi T$

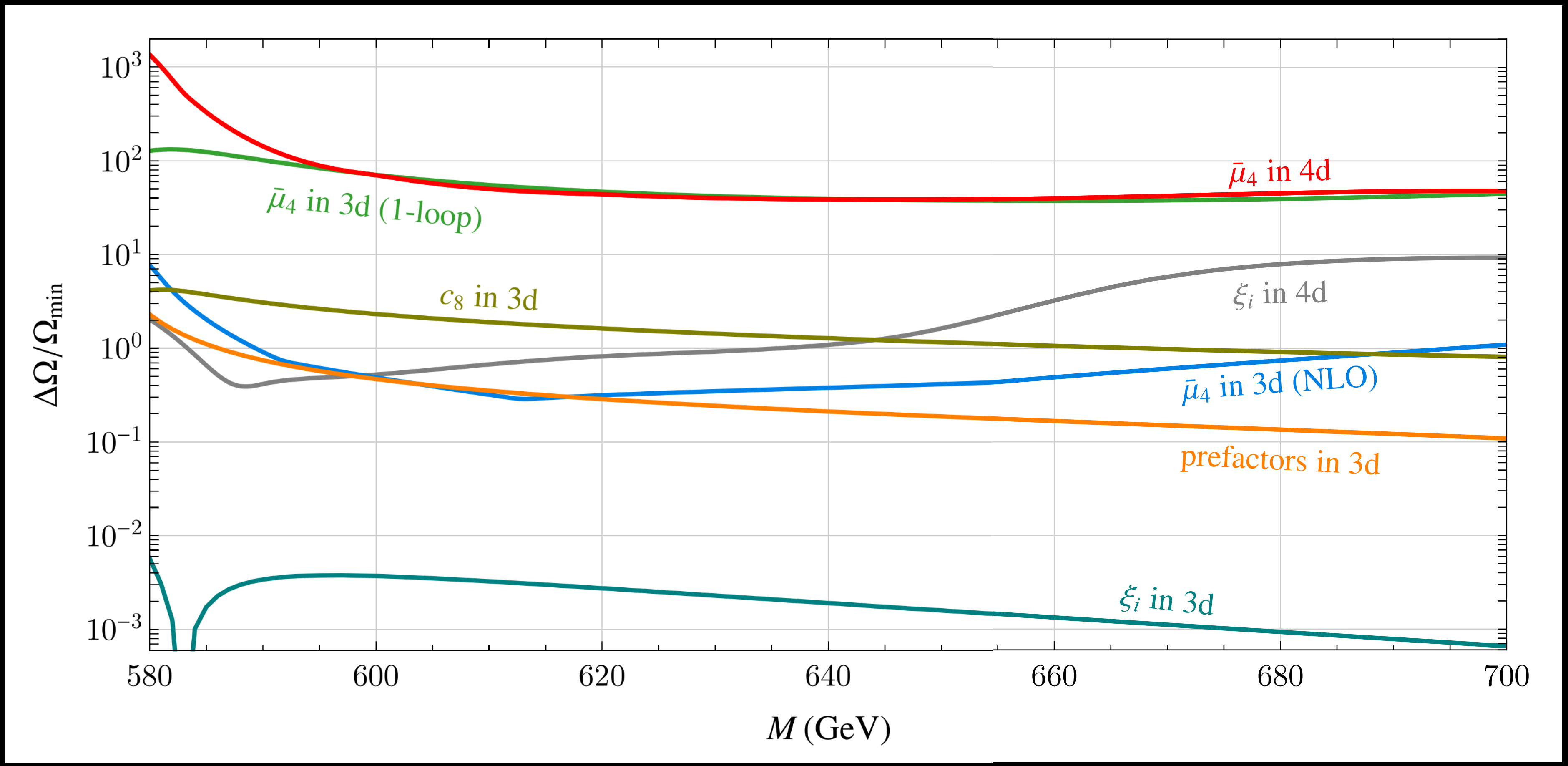
Part 1


$\mathscr{L}$
$\{V(\phi_i, T), S_E\}$
$\{\alpha, \beta, T_p, v_w\}$


Dependence on ξ_i in the 3d approach and the 4d approach
T_C (GeV)
T_P (GeV)
α
β/H
M (GeV)
$\xi_i = 0$
$\xi_i = 3$
$\xi_i = 100$

Part 1





Open question: is it the inclusion of 2-loop effects or the superior resummation that makes NLO 3d superior?

Part 1

$\mathscr{L} \longrightarrow \{V(\phi_i, T), S_E\} \longrightarrow \{\alpha, \beta, T_p, v_w\}$

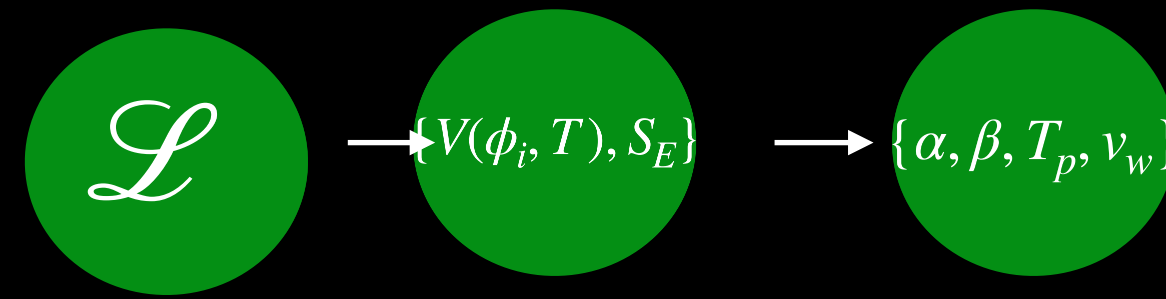
Why the error is so big

$$\alpha \sim \frac{\Delta V}{T^4} \sim T^{-8} \qquad \beta \sim T^{\gamma}, \gamma \sim O(1)$$

$$\Omega_{\rm GW} \sim \alpha^2 \beta^{-2}$$

Dramatic amplification of uncertainties!

Part 1



Summary part 1: despite various prescriptions available only 3d brings theoretical uncertainties under control

Dominant theoretical uncertainty is the unphysical scale dependence

this was shown in SMEFT but any BSM theory with large scale heirachies/large couplings will have even larger uncertainties

Part 2

$\{\alpha, \beta, T_p, v_w\} \longrightarrow \{U_f, R_p, t_{\text{sh}}, \kappa_f\} \longrightarrow \{\Omega_{\text{peak}}, f_{\text{peak}}\}$

$$T^{\mu\nu} = \cdots + (e+p)U^\mu U^\nu \qquad \leftarrow U^\mu = \gamma(1, \mathbf{v}/a)$$

Sound shell model $\rightarrow$ total source is linear superposition of single bubble contributions

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon, \quad e_+ = a_+T_+^4 + \epsilon$$

$$p_- = \frac{1}{3}a_-T_-^4, \quad e_- = a_-T_-^4$$

Bag model $\rightarrow$ pressure/energy inside $=$ pressure/energy/outside $+$ bag constant

Part 2

$\{\alpha, \beta, T_p, v_w\}$ → $\{U_f, R_p, t_{\rm sh}, \kappa_f\}$ → $\{\Omega_{\rm peak}, f_{\rm peak}\}$

$$T^{\mu\nu}_{;\mu}\Big|_{\rm field} = \delta^\nu$$

$$T^{\mu\nu}_{;\mu}\Big|_{\rm fluid} = \delta^\nu \longrightarrow \xi = R_b/t$$

Match v at the bubble boundary

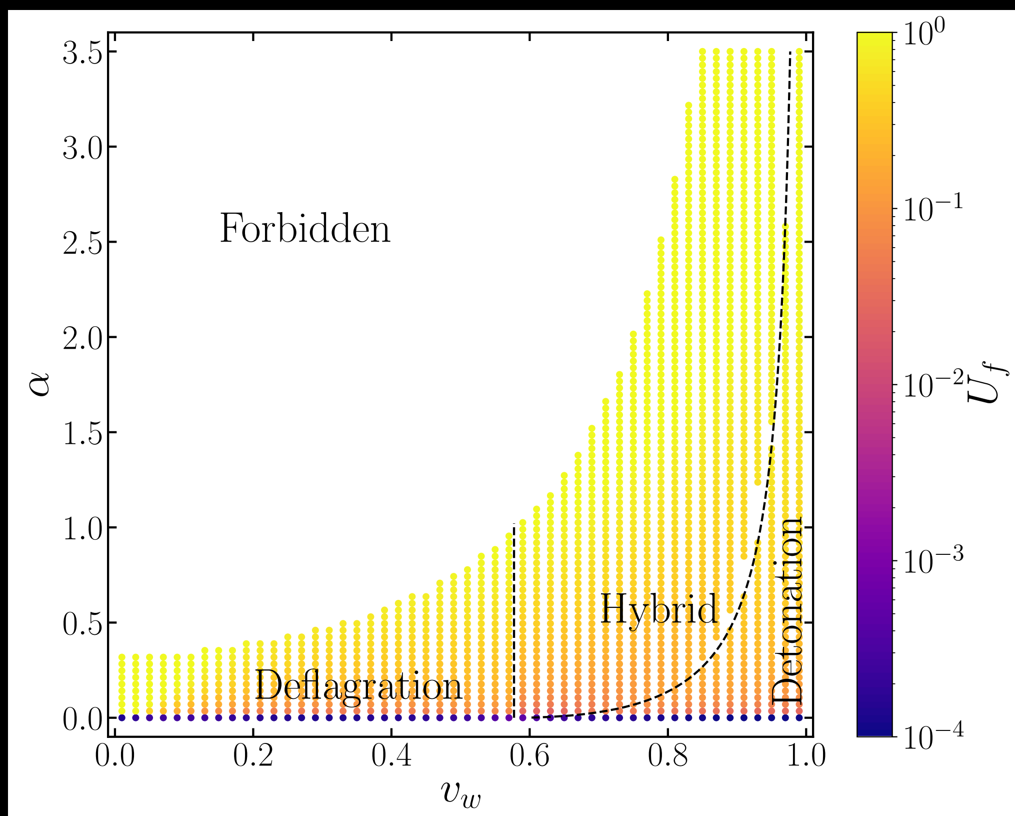
$$2\frac{v}{\xi} = \frac{\gamma^2}{c_s^2}(1 - v\xi)\left[\left(\frac{\xi - v}{1 - \xi v}\right)^2 - 1\right]\partial_\xi v$$

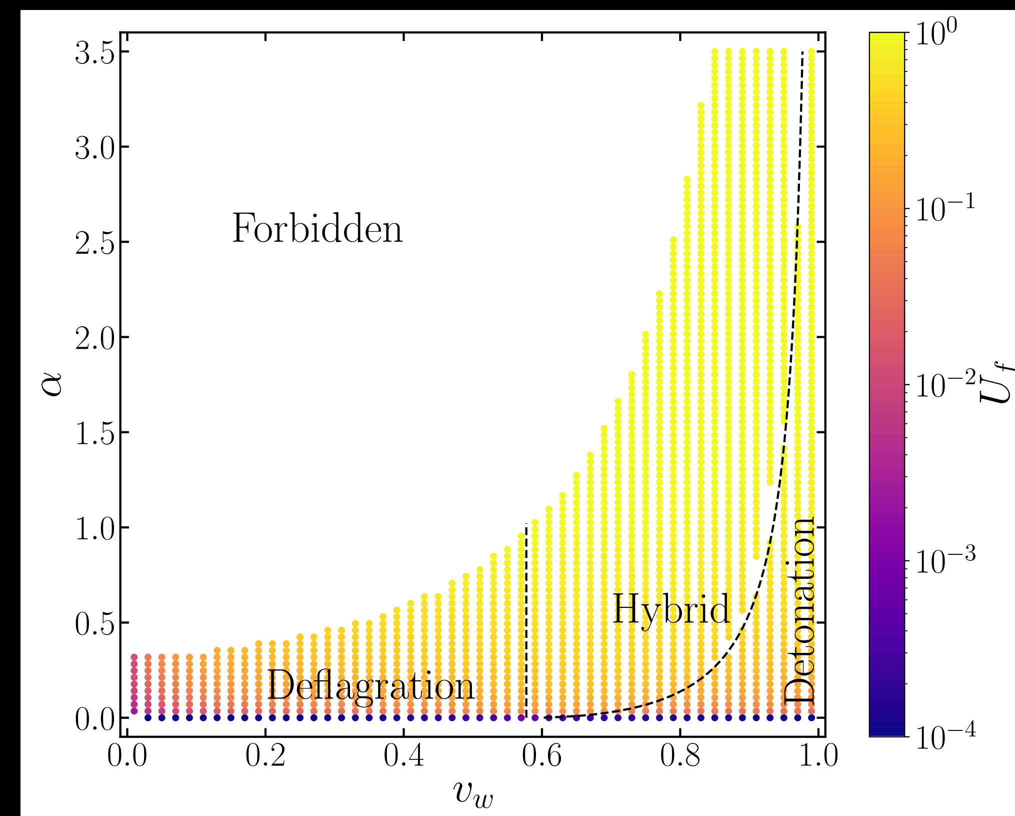
$$\left[\gamma' + \nabla \cdot (\gamma \mathbf{v}) + 3\frac{a'}{a}\gamma\right](e + p) + \gamma e' + \gamma \mathbf{v} \cdot \nabla e = 0$$

Part 2

$\{\alpha, \beta, T_p, v_w\} \longrightarrow \{U_f, R_p, t_{\mathrm{sh}}, \kappa_f\} \longrightarrow \{\Omega_{\mathrm{peak}}, f_{\mathrm{peak}}\}$

Comparing $U_f^2 = \dfrac{3}{4}\kappa_f \alpha$ with numerical solution

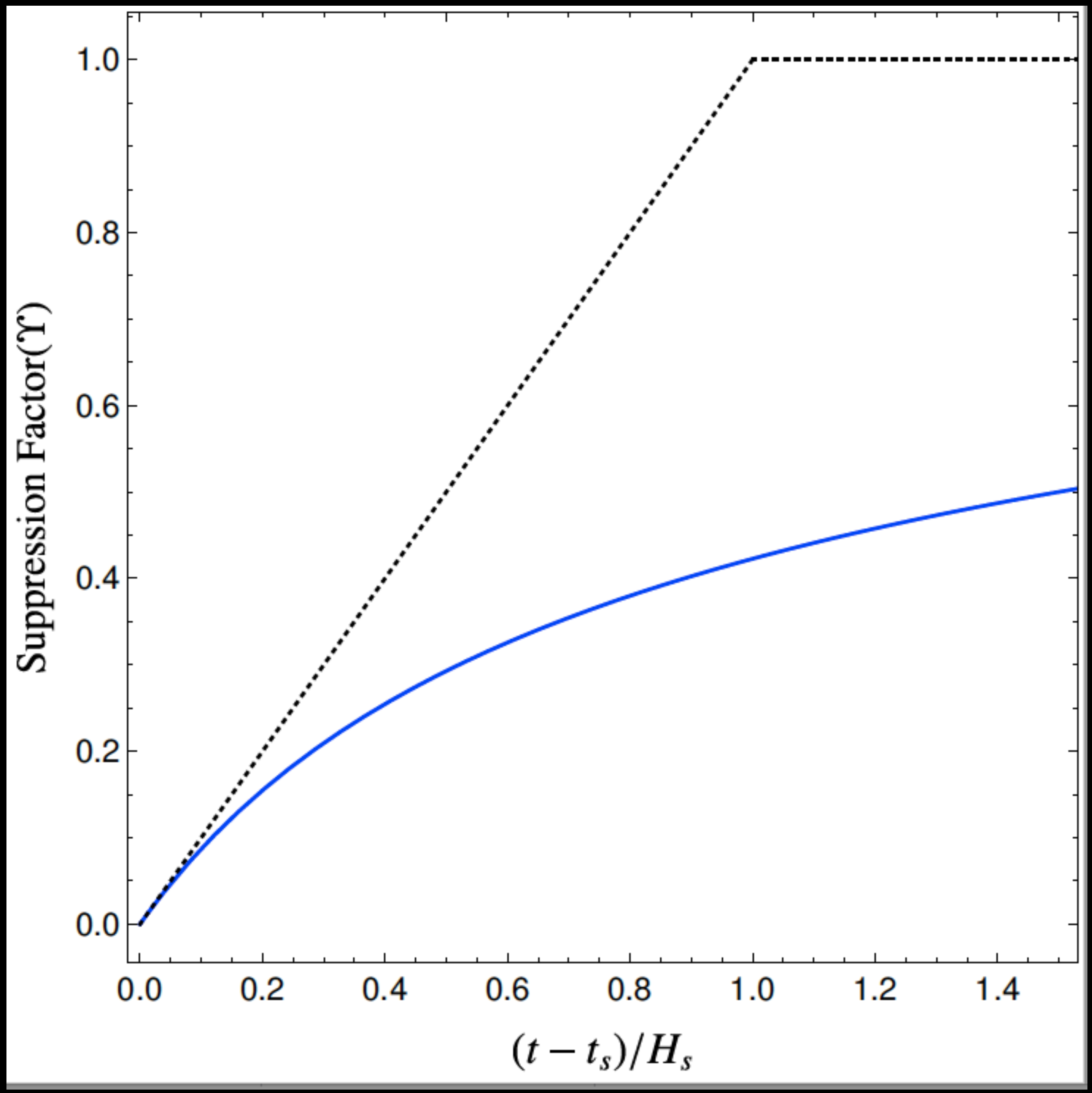




Part 2

$\{\alpha, \beta, T_p, v_w\} \longrightarrow \{U_f, R_p, t_{\rm sh}, \kappa_f\} \longrightarrow \{\Omega_{\rm peak}, f_{\rm peak}\}$

Comparing suppression factor from full calculation in expanding Universe to Ansatz in arXiv: 1903.09642

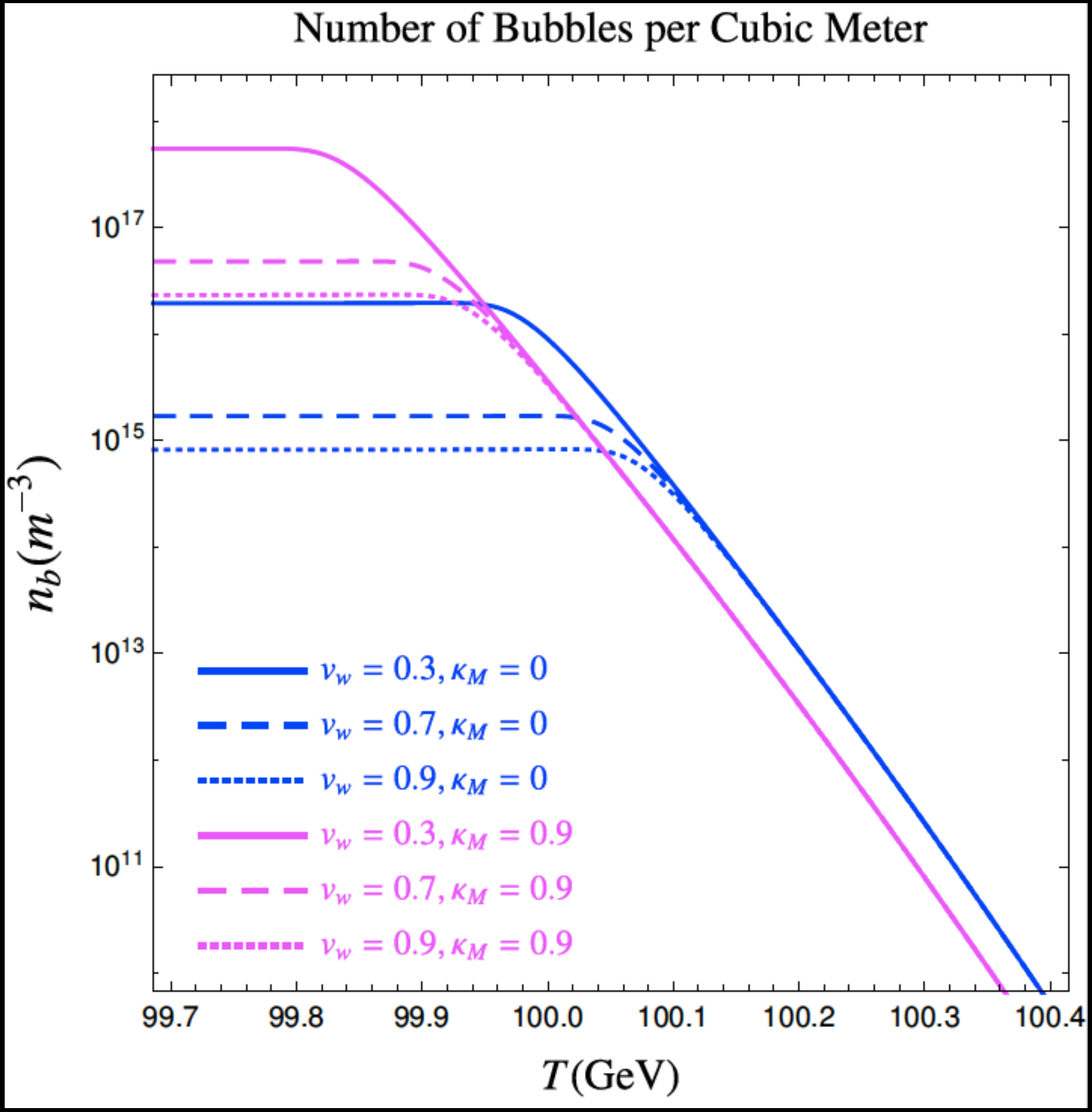


$$\Upsilon = 1 - \frac{1}{\sqrt{1 + t_{sw} H}}$$

Part 2

$\{\alpha, \beta, T_p, v_w\} \rightarrow \{U_f, R_p, t_{sh}, \kappa_f\} \rightarrow \{\Omega_{peak}, f_{peak}\}$

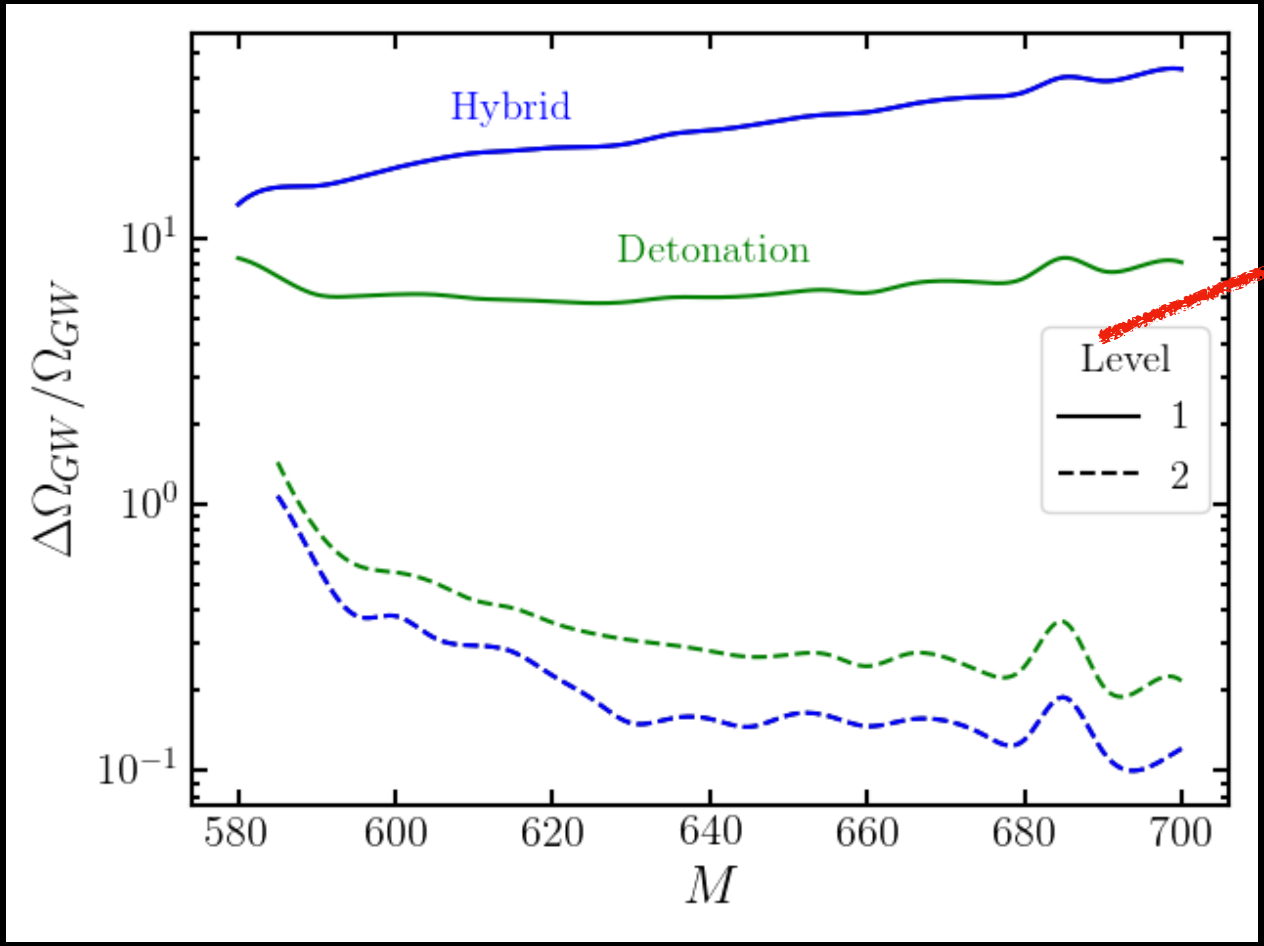
Correct way of calculating mean bubble separation



Part 2

$\{\alpha, \beta, T_p, v_w\} \longrightarrow \{U_f, R_p, t_{\rm sh}, \kappa_f\} \longrightarrow \{\Omega_{\rm peak}, f_{\rm peak}\}$

How much does this uncertainty matter?
SMEFT example



Blue$\rightarrow \dfrac{S_E}{T} = 140, \quad \Upsilon = 1, \quad U_f^2 = \dfrac{3}{4}\kappa_f \alpha$

Green$\rightarrow \dfrac{S_E}{T} = 131 + \log[A/T^4] + \cdots, \quad \Upsilon = t_{\rm sw}H, \quad U_f^2 = \dfrac{3}{4}\kappa_f \alpha$

Solid→Hybrid
Dashed→Detonation

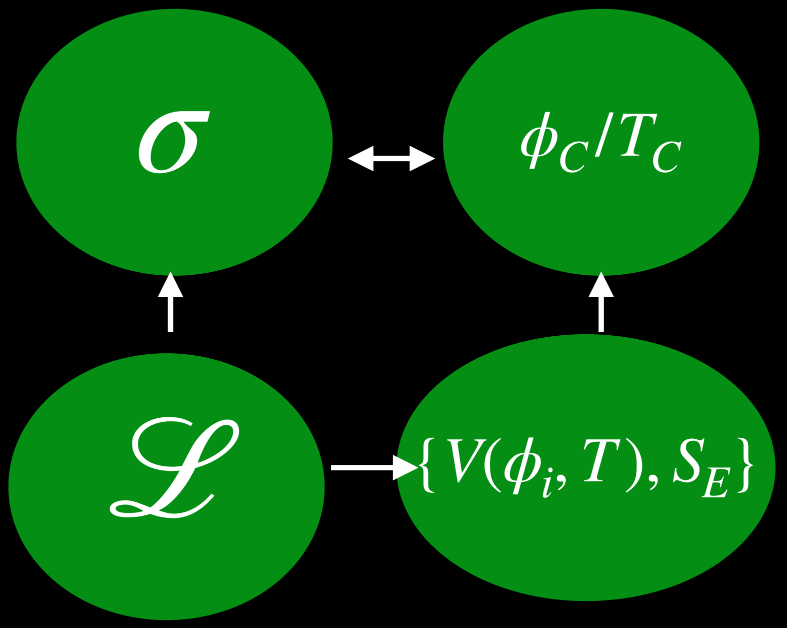
Part 2

$\{\alpha, \beta, T_p, v_w\}$ → $\{U_f, R_p, t_{\text{sh}}, \kappa_f\}$ → $\{\Omega_{\text{peak}}, f_{\text{peak}}\}$

Summary part 2:

- We looked at some limitations in common methods of calculating GW observables
- Uncertainties can be comparable to gauge dependence
- Strongest effect in most visible transitions

Part 3

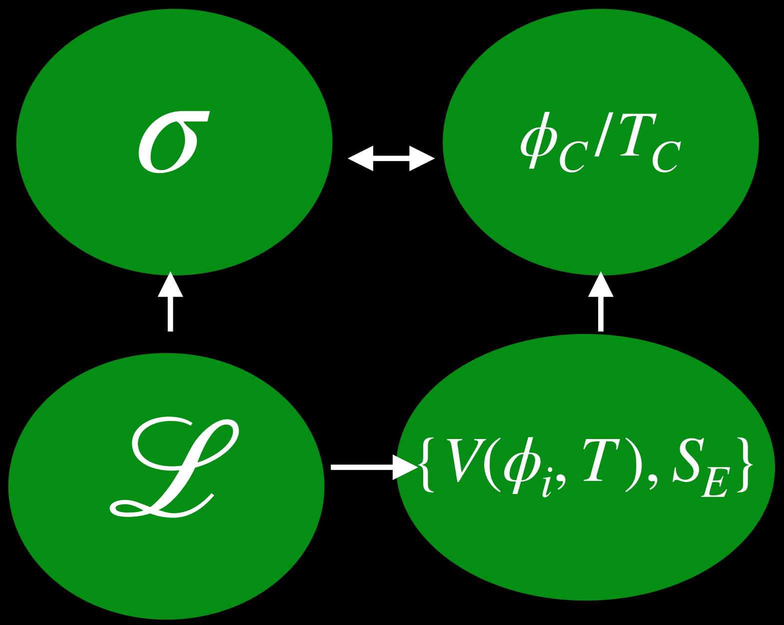


What is a key question the next generation of colliders can answer?

What is the order of the electroweak phase transition? (credit MRM)

Test model: SM+singlet

Part 3

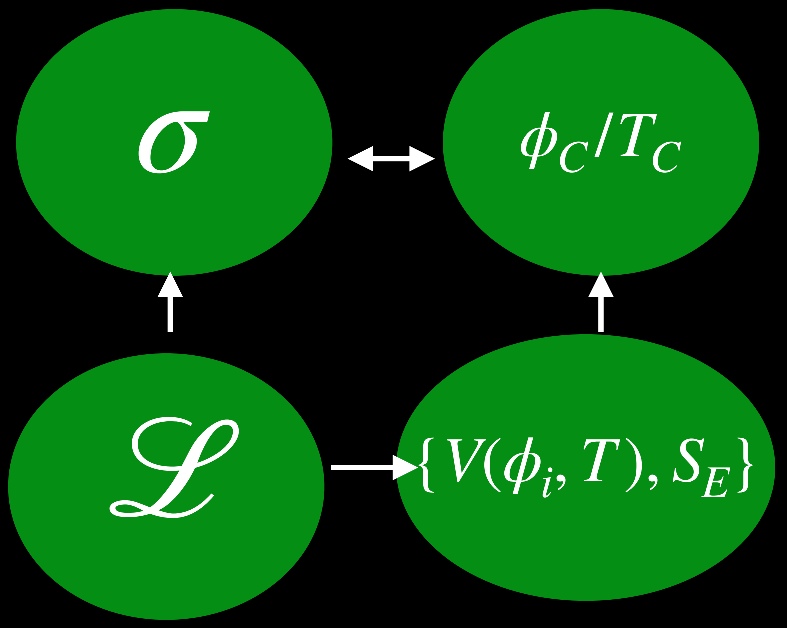


What sort of collider do we need to get a definitive answer on the nature of the EWPT?

To get a viable answer to this need to update what has been done in 2 ways

1. **Complementarity between** $h_2 \to h_1 h_1$ and $h_2 \to ZZ$ **(WW as well)**
2. **Take theoretical uncertainties into account**

Part 3



Philosophy on uncertainties:

1. if you want to say something is a discovery you need to be conservative
2. If you want to say a scenario is ruled out, you need to be liberal

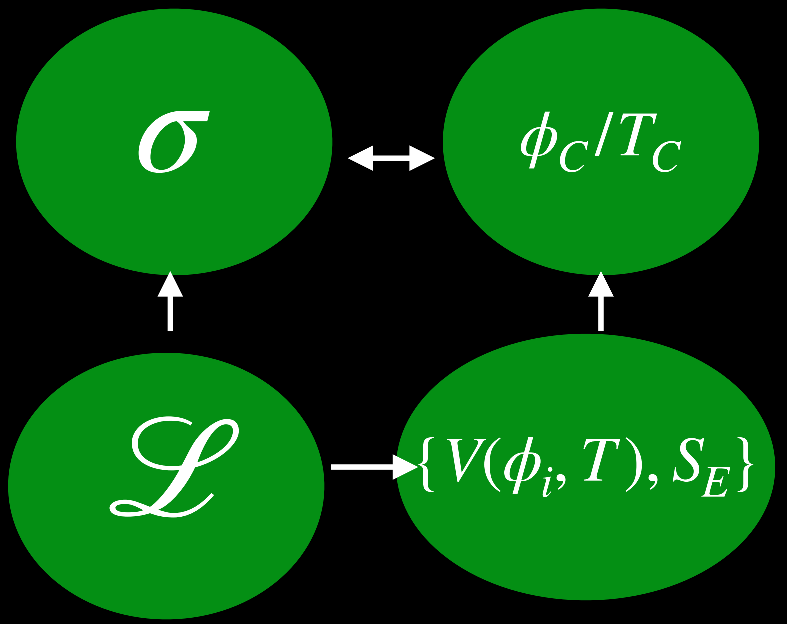
Liberal = 1 point in band gives SFOEWPT

Conservative = all points in uncertainty band give SFOEWPT

Part 3

Tech to do scans in DR is a long way away so Choose your poison:

1. **A.E. at 1 loop with running coupling**
2. **MRM+HP at order $\hbar$**

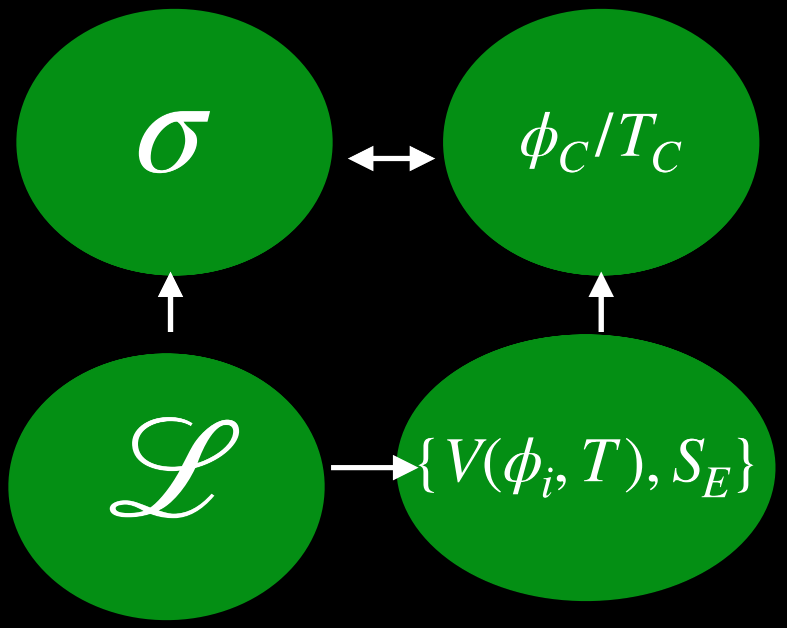


No resummation to $\mathcal{O}(\hbar)$!

Dominant uncertainty is scale dependence so choose A.E.

Need to a) compare with MRM+HP b) improve 4d c) ultimately do DR

Part 3



For now define uncertainty band as follows

1. Derive 1 loop effective potential in *covariant gauge*

2. Elevate all couplings $\lambda \to \lambda(\mu_4)$ where

μ_4 is the RG scale that also appears in the CW potential

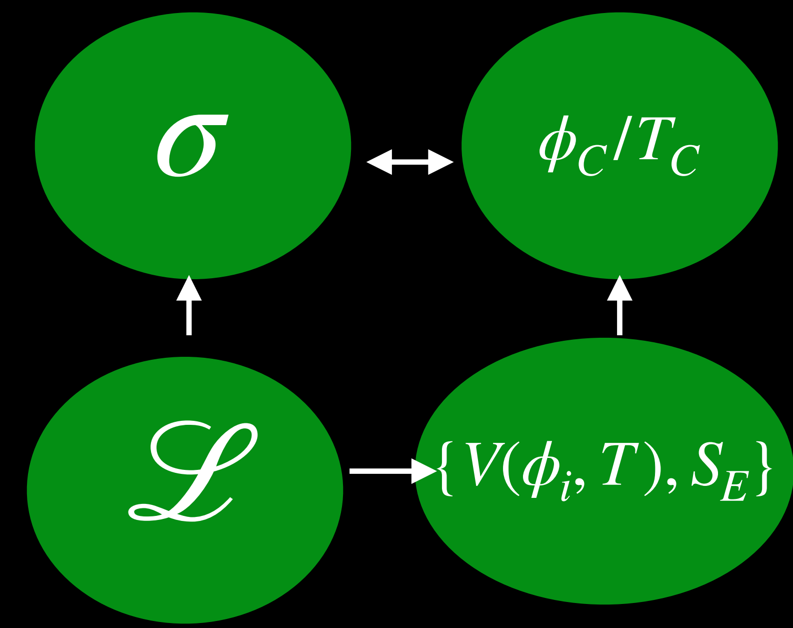
3. Vary $\mu_4 \in [m_Z/2, 5m_Z], \quad \xi_x \in (0,3)$

4. Allow 30 GeV tolerance for zero temperature vev at 1 loop $v_h\Big|_{T=0} = 246\ \text{GeV}$*

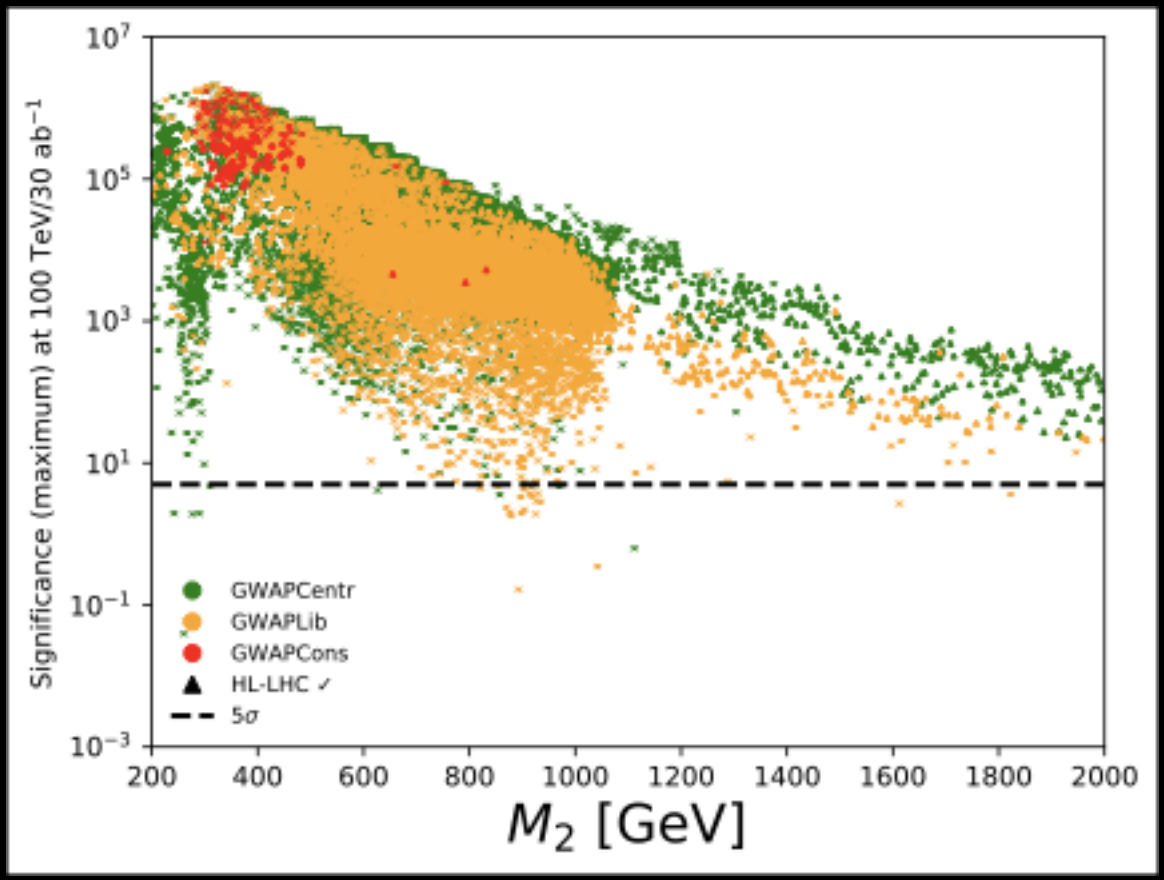
5. Use PHASETRACER for each parameter set

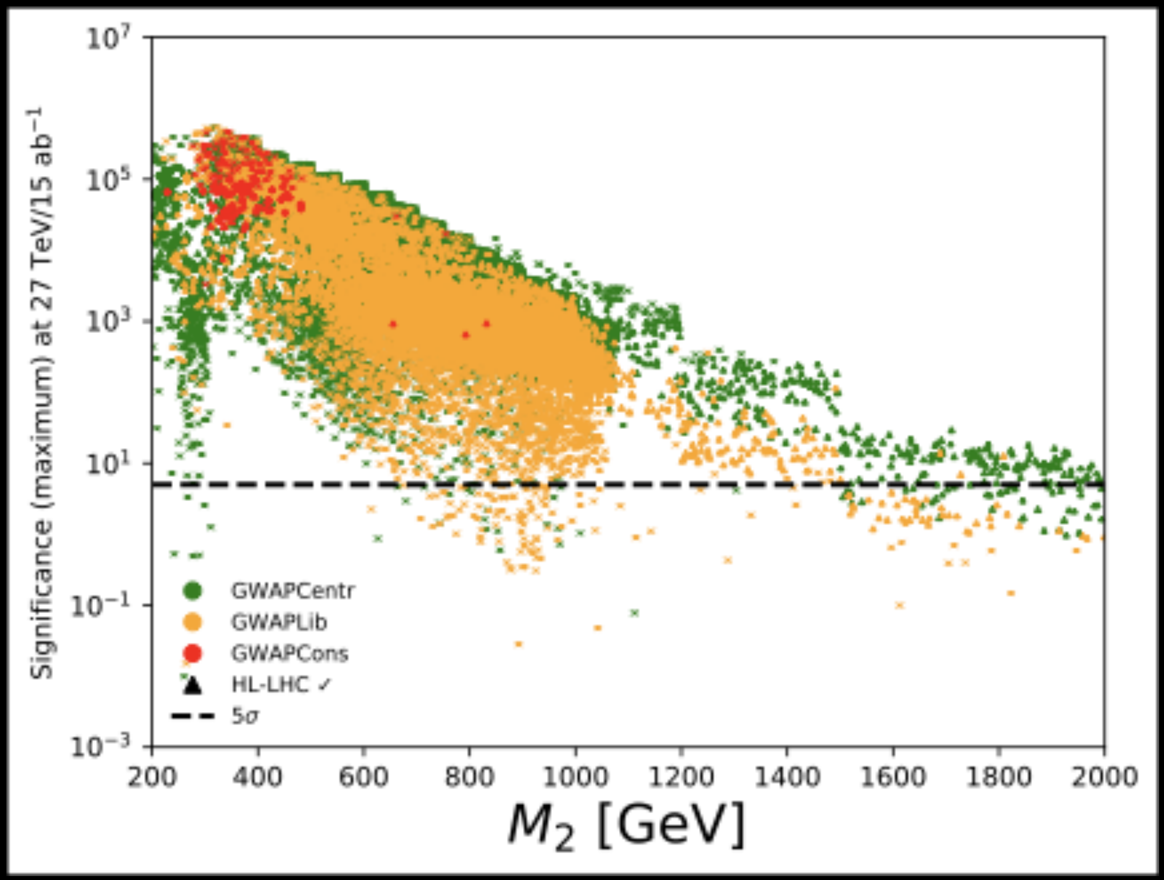
***Will have a different method when results are published. Results not expected to change much**

Part 3

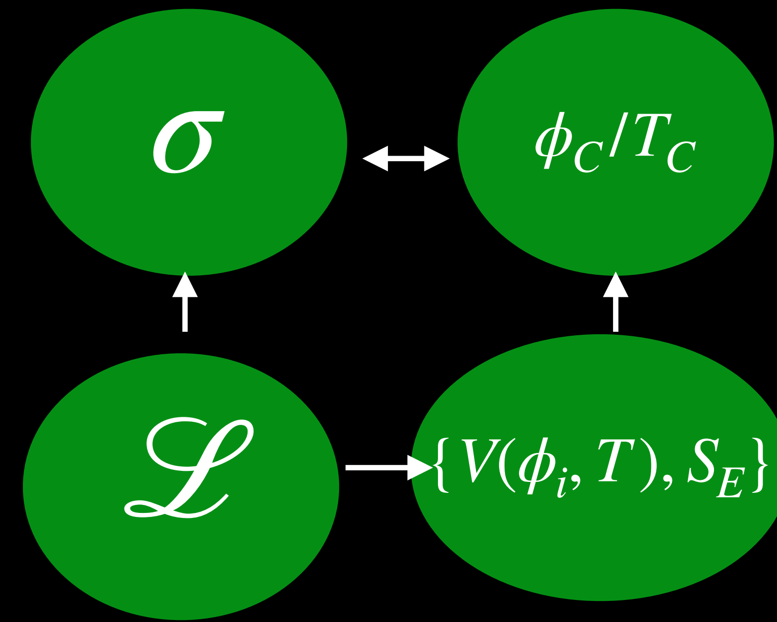


Preliminary results

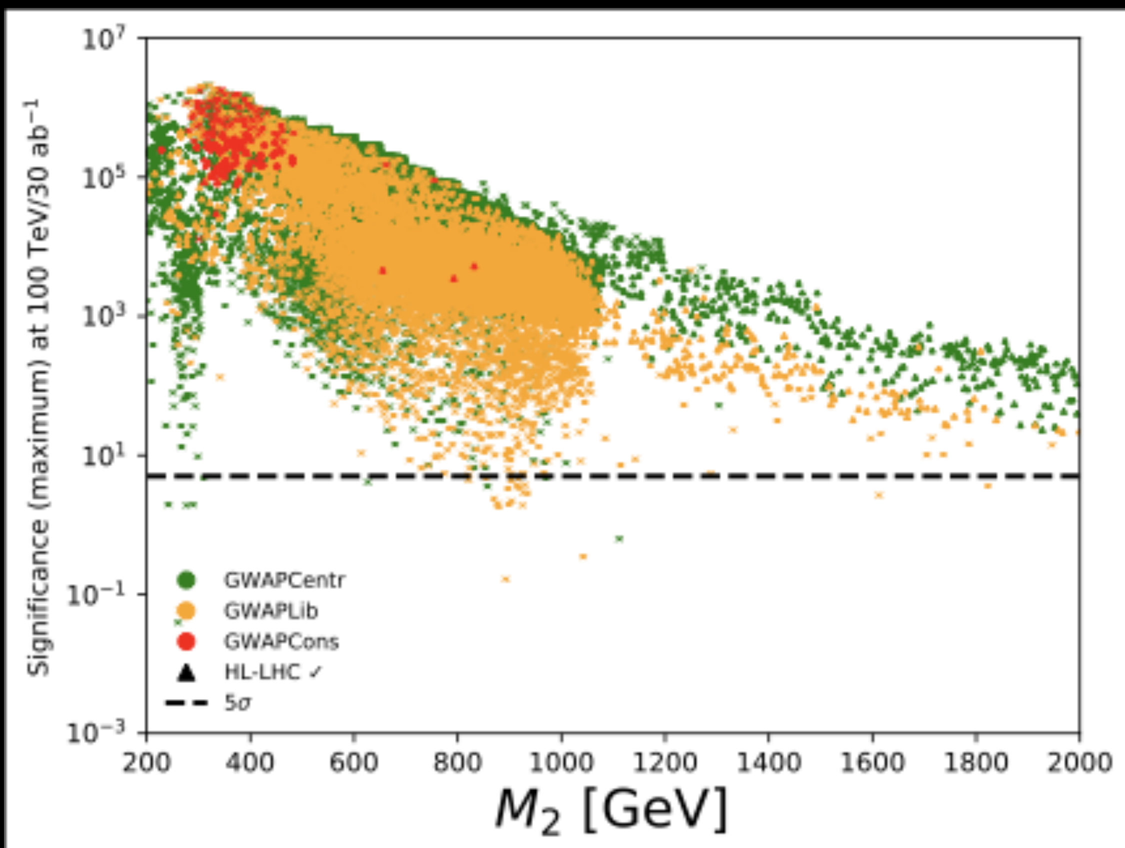




Part 3

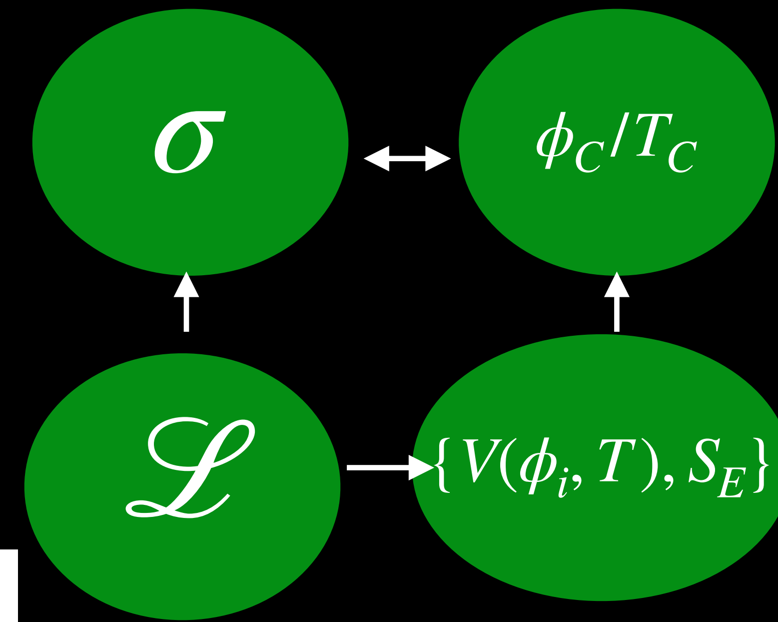


Preliminary results

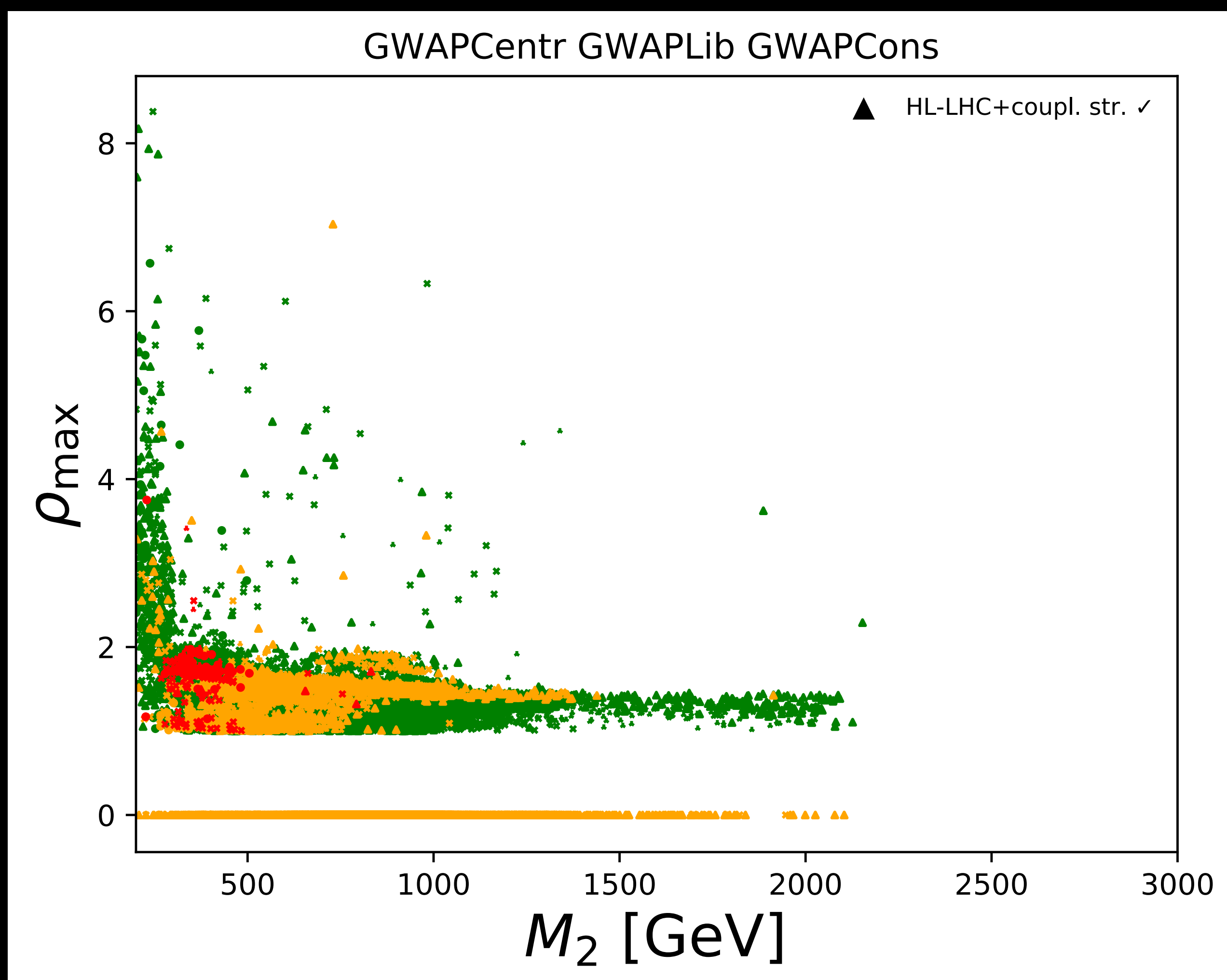


1. Including $h_2 \to ZZ$ significantly improves Reach of collider
2. This is undermined by the very large theoretical uncertainties
3. Even with theoretical uncertainties a dream 100 TeV collider can probe the nature of the EWPT
4. It is possible a 27 collider might be enough (or close)

Part 3



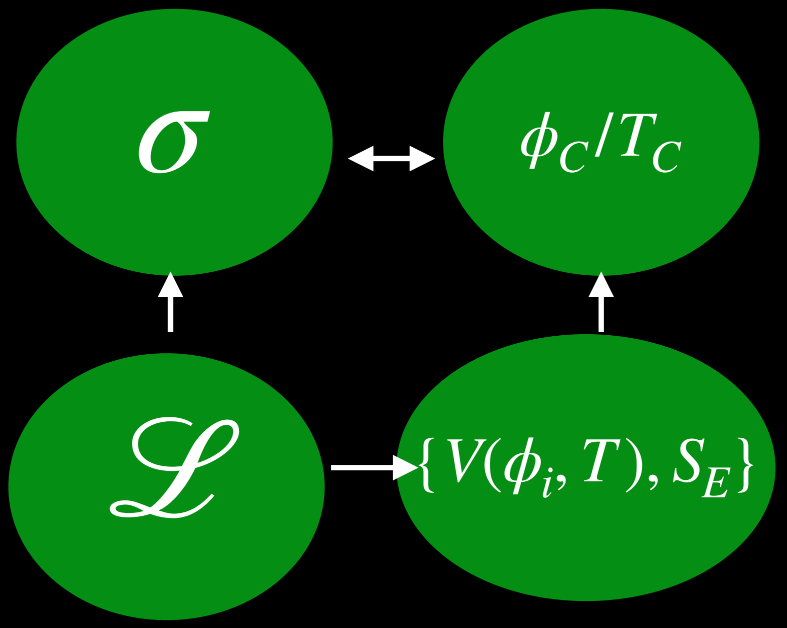
Why isn't it cutting off at high masses?

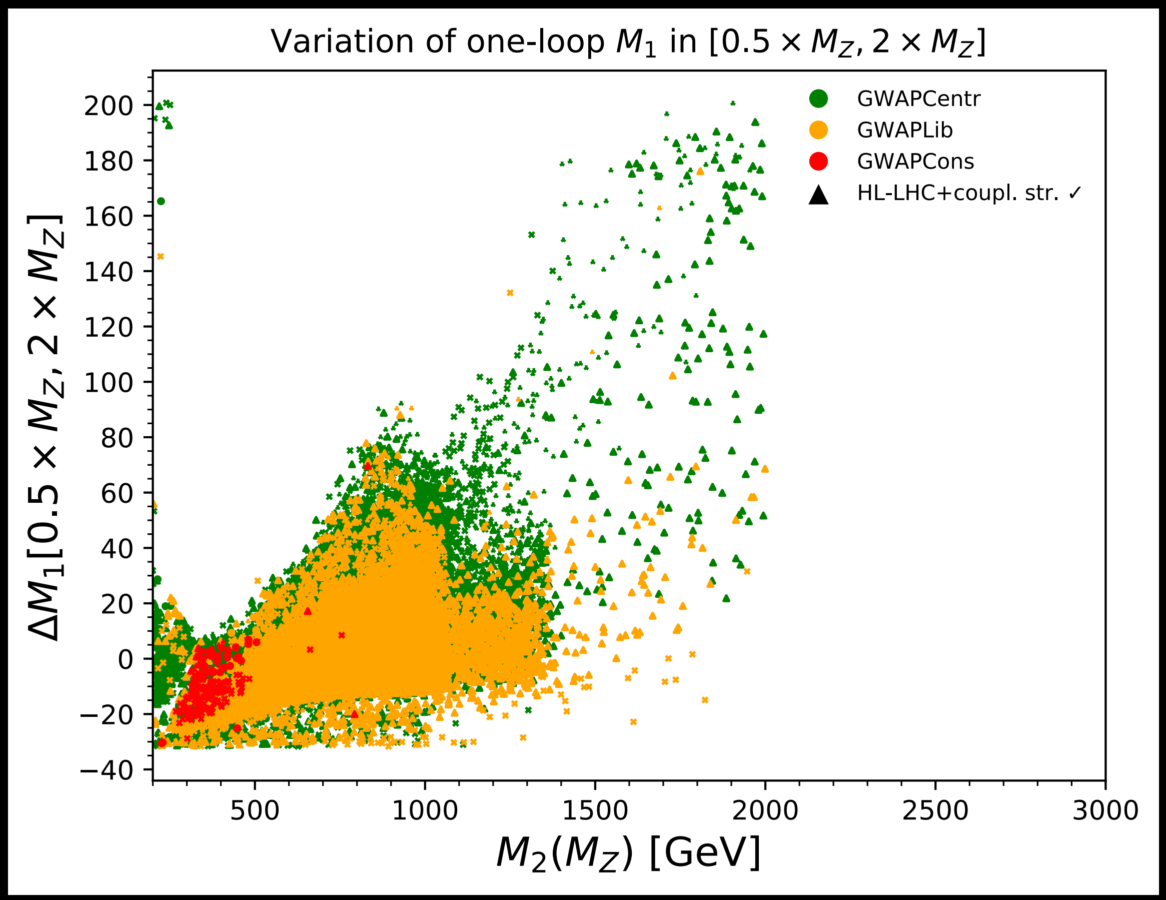


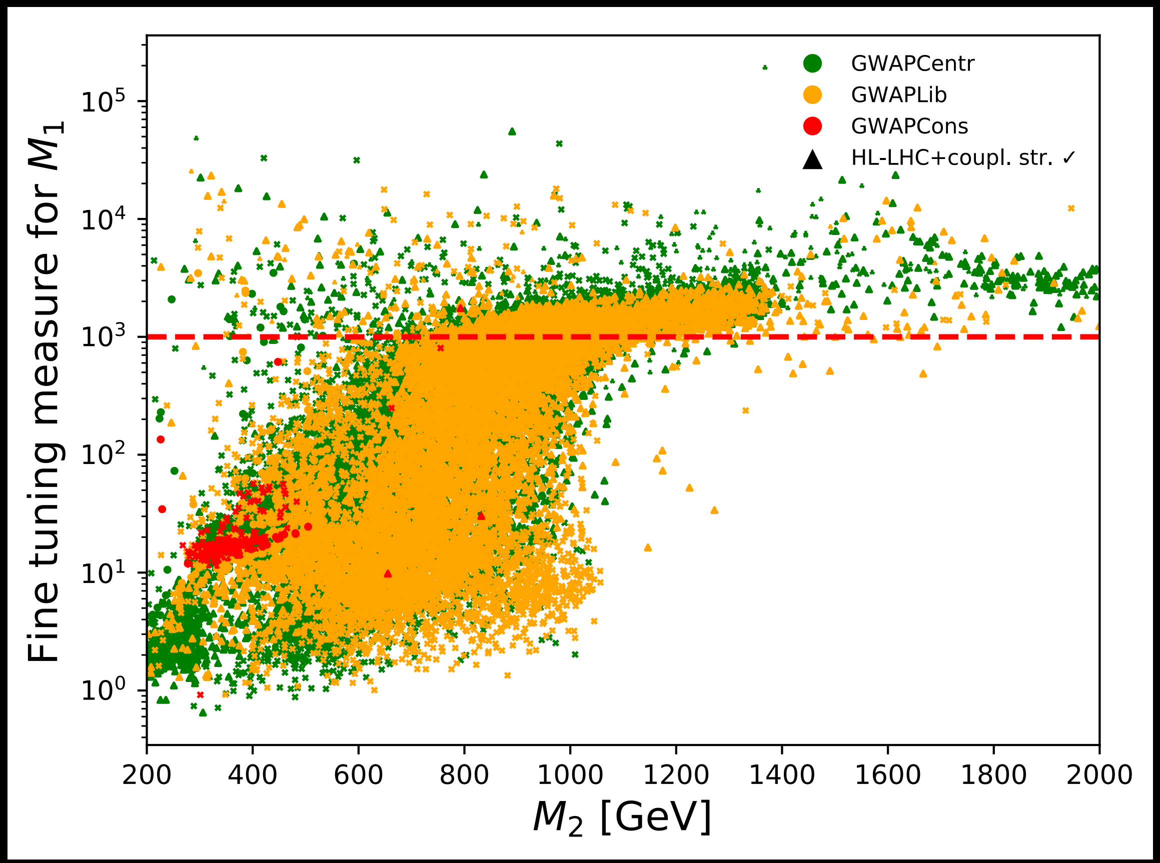
Part 3

Theoretical error grows with mass

But so does fine tuning!



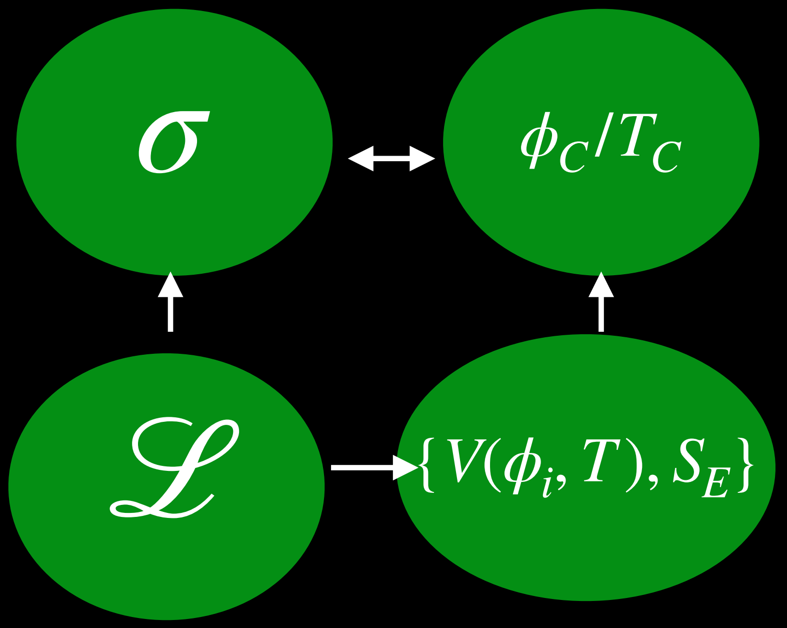


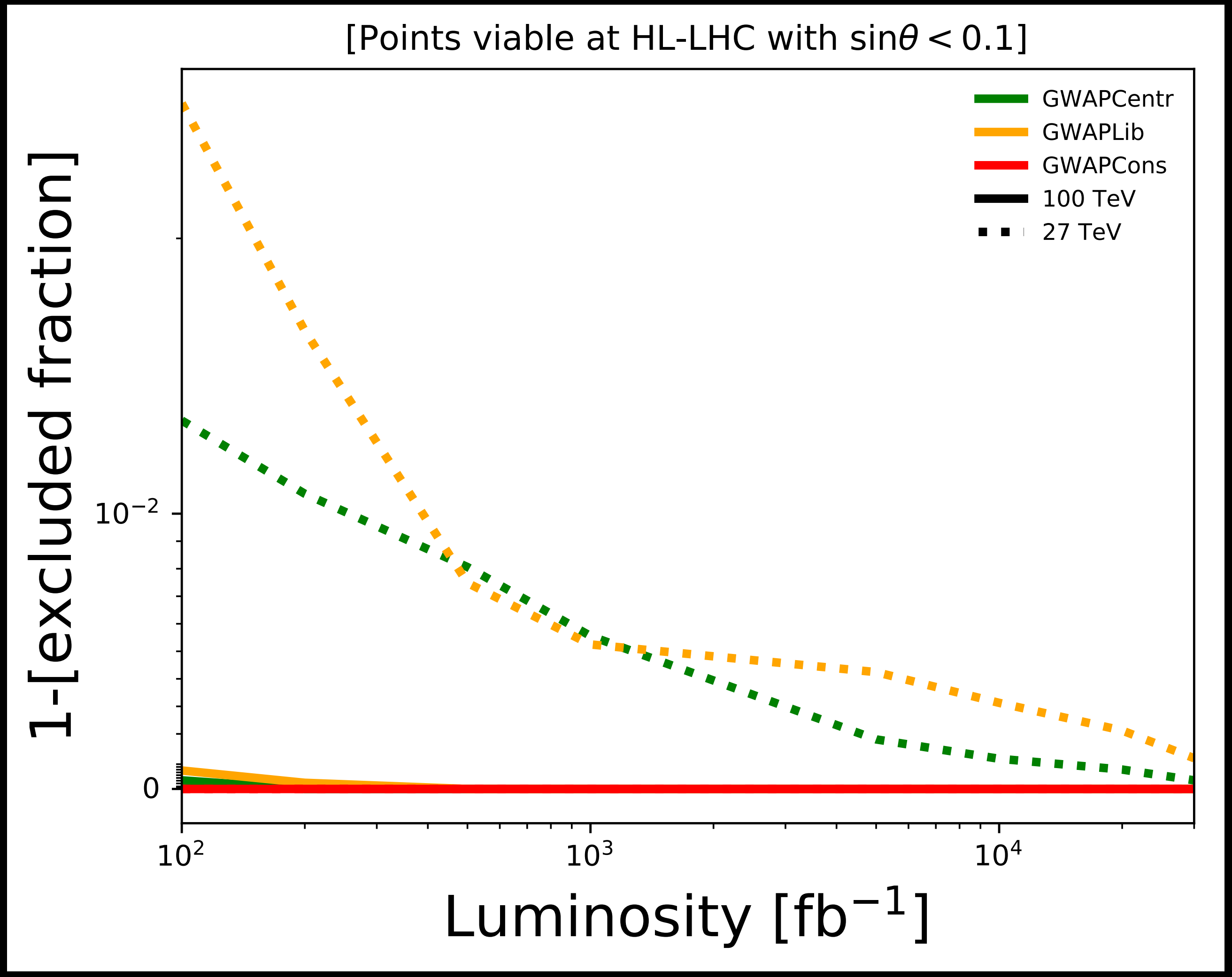


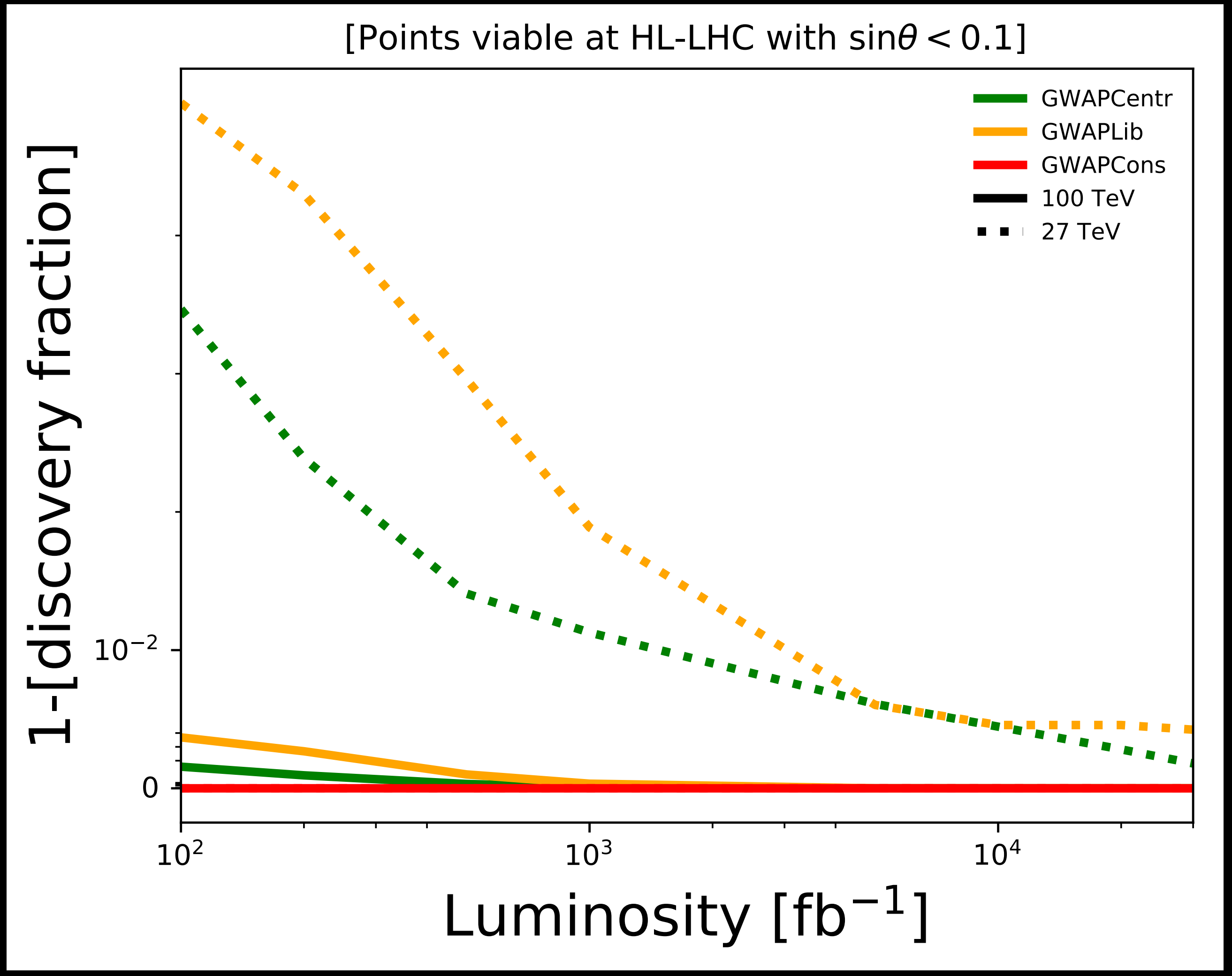
Points just get harder to find at higher mass! Can't rule out a much higher mass!

Part 3

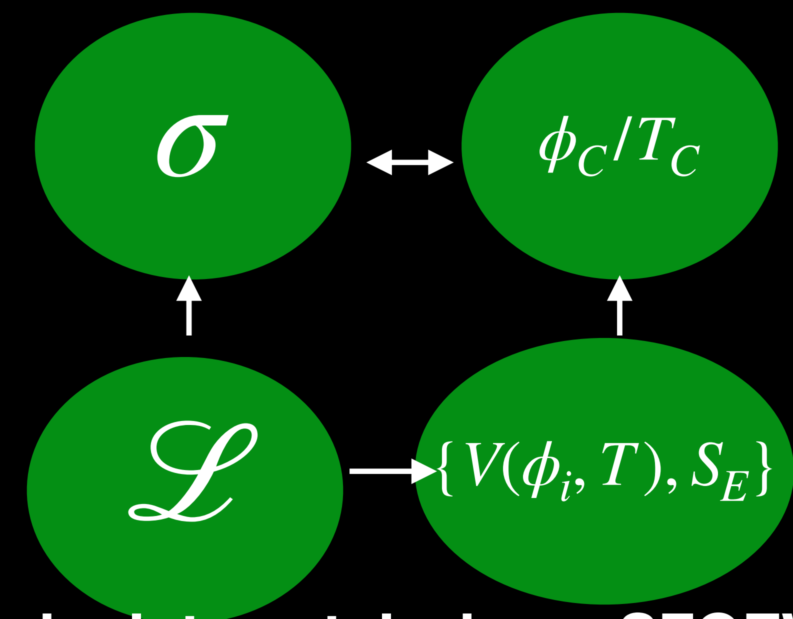
With that caveat what machine do we Need







Summary part 3



1. Theoretical uncertainties qualitatively change The needed collider design to test a SFOEWPT

2. This is more than compensated if one looks At complimentary channels

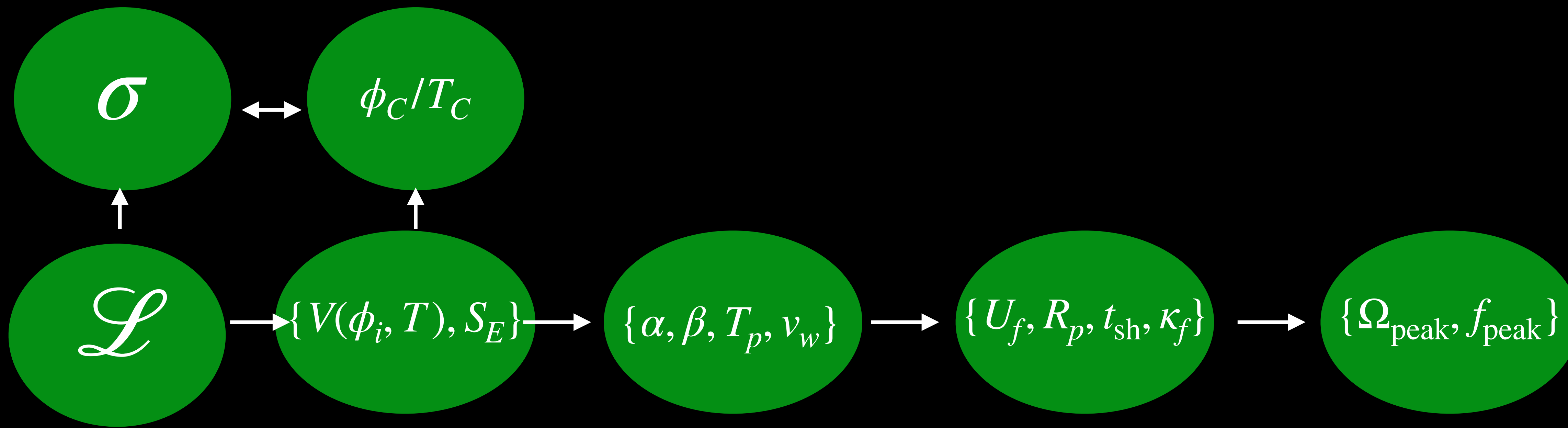
3. We cannot rule out very large (say plank scale) singlets catalysing a SFOEWPT

4. Is the big question for next generation Colliders "Is the Universe natural?" or "what is the nature of the EWPT?"

It turns out you may not be able to decouple these questions!

Upgraded methods of handling the phase transition needed to get a verdict!

Summary



- We have at least a decade before next generation experiments sheds light on the EWPT

- Theory has a large gap to catch up on to interpret these results

- We have provided methods for improving these uncertanties as well as ways of Estimating them

- We have analysed the consequences of theoretical uncertainties on making Conclusions from next generation experiments