Lessons for quantum cosmology from anti-de Sitter black holes

Michal P. Heller

aei.mpg.de/GQFI

based on **2007.04872** with

Alice di Tucci

&

Jean-Luc Lehners

Three-Line Summary:

it is the right time to study
gravitational path integrals using hep-th tools

the field seems ripe for discoveries

Setting the context

Beyond general relativity

This talk takes the hep-th perspective

From this view, quantum gravity is understood the best in anti-de Sitter universes as a quantum field theory on the asymptotic boundary

The two most promising directions to probe beyond general relativity:

black holes

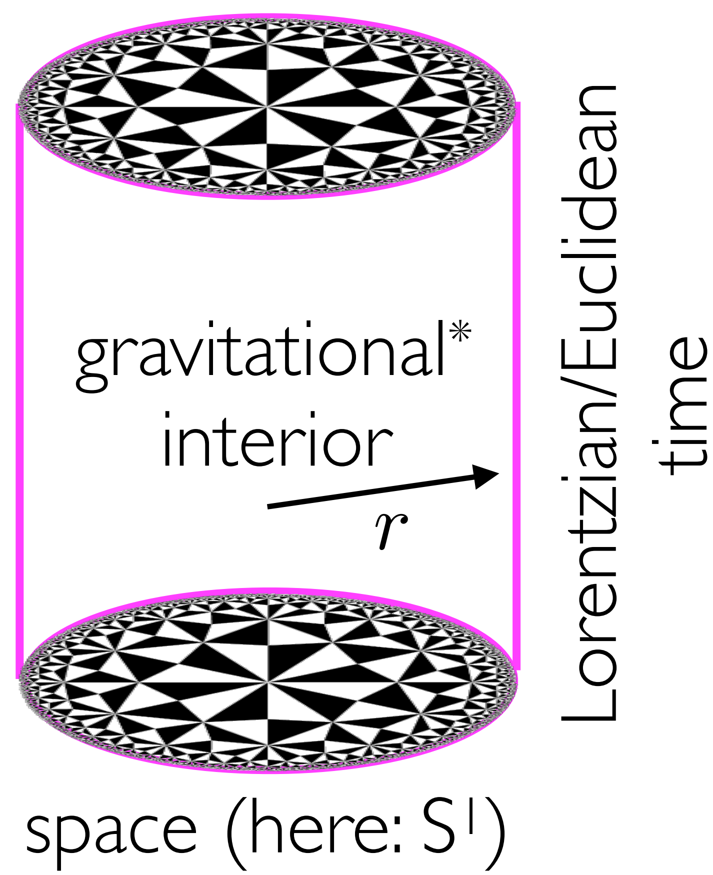
early universe cosmology
(beyond AdS/CFT)

This talk: corroborating novel features of quantum cosmology using the robust interpretation of black holes in AdS/CFT as thermodynamic objects

How does AdS/CFT work?

AdS has a timelike (Lorentzian) or spacelike (Euclidean) asymptotic boundary and variational problem in GR requires specifying boundary conditions there

AdS3:



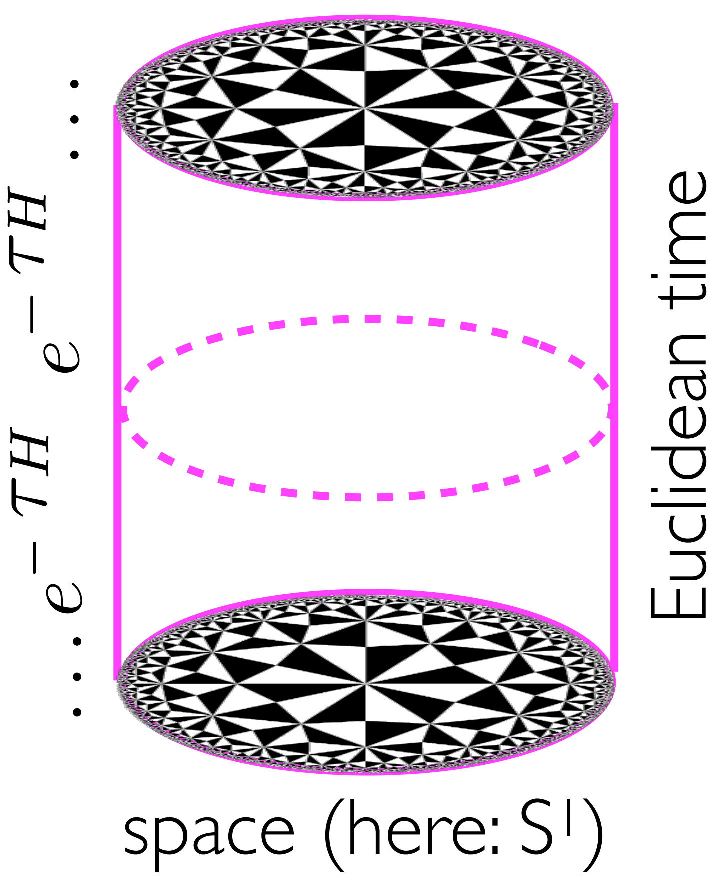
AdS/CFT: the asymptotic boundary is a non-dynamical spacetime in which dual quantum field theories* live (here CFT_2) in the path-integral sense

Pawel Caputa later today is going to tell us how to make a dual quantum field theory sense of a situation in which the boundary is <u>not</u> asymptotic

CFT vacuum as seen from the AdS side

The most natural way to generate $|0\rangle$ QM is via $e^{-\tau H}|\text{generic}\rangle\Big|_{\tau\to\infty}$

AdS/CFT picture:



AdS/CFT identity: $Z_{CFT}[\delta_{\mu\nu}] = Z_{AdS}[g_{ab}|_\partial = \delta_{\mu\nu}] \approx e^{-S_{GRAV}[g_{ab}|_\partial=\delta_{\mu\nu}]}$

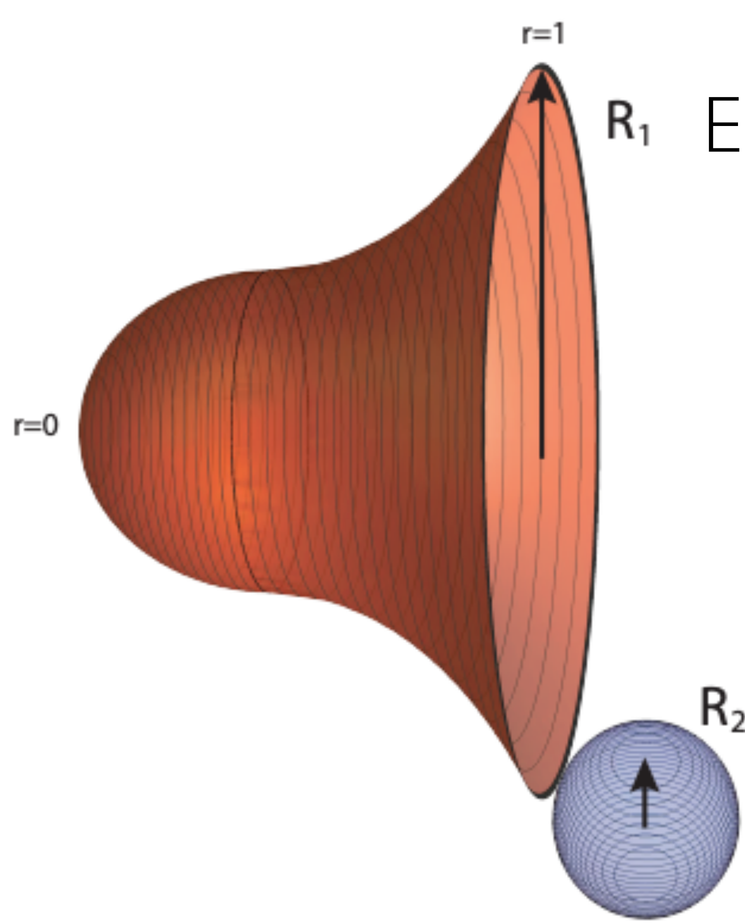
quantum gravity — semi-classical saddle

Basics of black holes in AdS/CFT

For a thermal state:

$$\mathrm{tr}\, e^{-\beta H} = Z_{CFT}[S^1 \times \ space] = Z_{AdS}[g_{ab}|_\partial = S^1 \times \ space] \approx e^{S_{GRAV}[g_{ab}|_\partial = S^1 \times \ space]}$$

One of the saddles contributing to the gravity action is an Euclidean AdS black hole. In the following we specialize to gravity in 1+3 D and we have:



boundary
Euclidean time (S^1)

AdS black hole horizon

the boundary becomes asymptotic as proper sizes $R_1, R_2 \to \infty$ with R_1/R_2 fixed

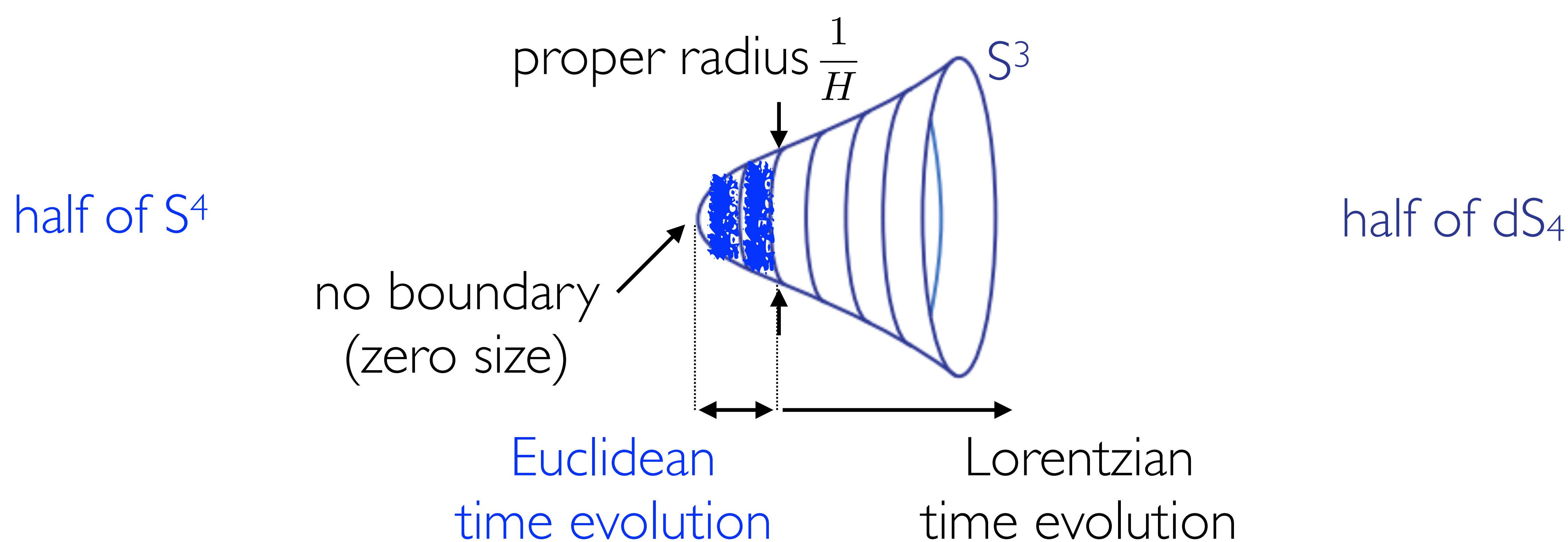
boundary
spatial directions (S^2)

Quantum cosmology 1982

In their pioneering work, Hartle and Hawking followed the "logic" that is robust logic in quantum field theory and so in AdS/CFT:

1) Euclidean time evolution prepares a "ground state"
2) The state can be later evolved in Lorentzian time

However, they applied their logic to cosmology $\int d^4x\sqrt{-g}(R-3\,H^2)$ with the following saddle point emerging:



Quantum cosmology 2020

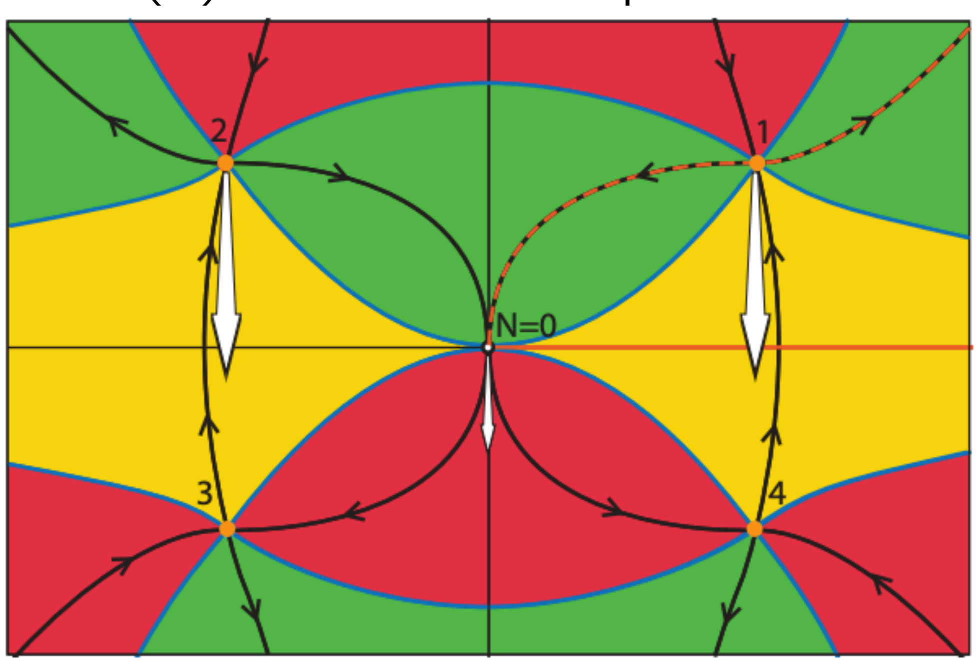
Feldbrugge, Lehners & Turok 1705.00192 ⟶ di Tucci & Lehners 1903.06757

New developments concern quantitative embedding of the no boundary proposal within the minisuperspace path integral model

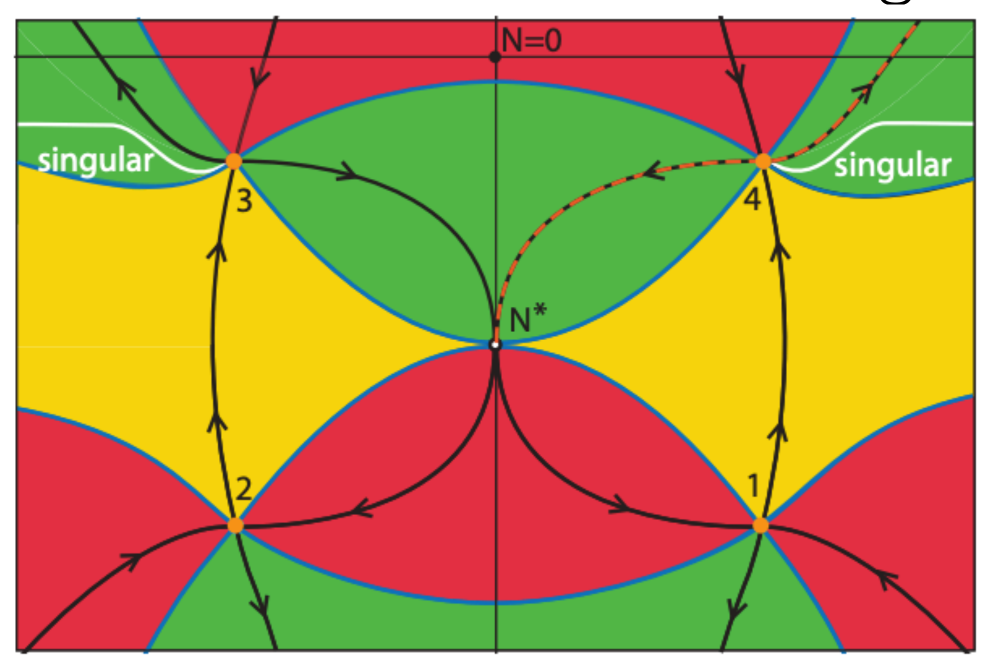
$$ds^2 = -\frac{N^2}{q}dt^2 + q d\Omega_3^2 \quad \text{with} \quad \int_{\mathcal{C}} dN \int_{q(t=0)=0}^{q(t=1)=q_1} Dq\, e^{iS/\hbar}\,,$$

$$S \sim \int_0^1 dt\,[-\frac{\dot{q}^2}{4N} + N(1-H^2 q)]$$

Four saddle points ✗ with no boundary being (3) + (4); (1) and (2) unstable to perturbations:



Changing the bc at t = 0 to Neumann (Robin) makes (3) and (4) contribute to the functional integral ✓:

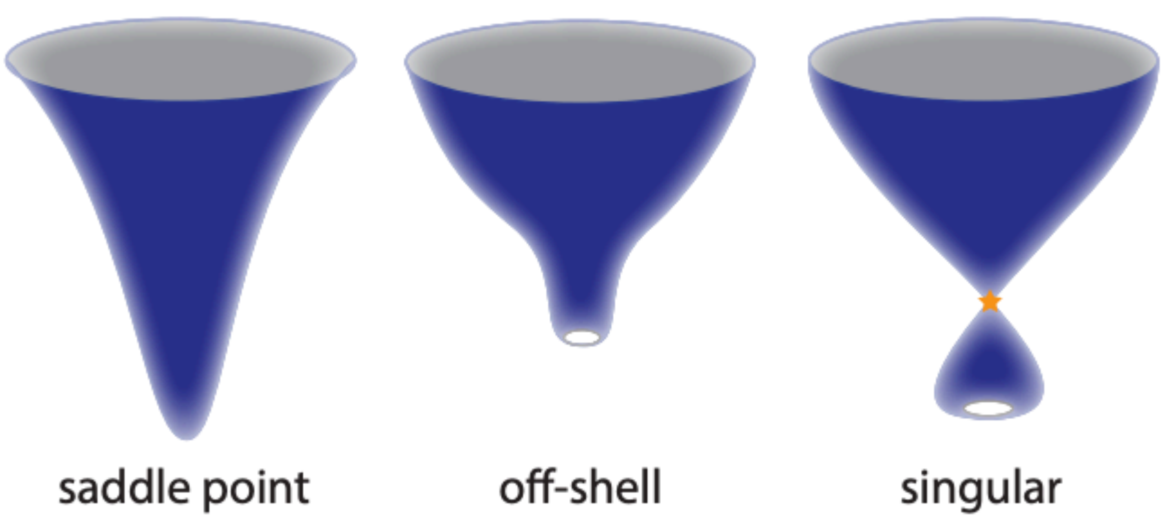


The emerging picture 2020

Feldbrugge, Lehners & Turok 1705.00192 ⟶ di Tucci & Lehners 1903.06757

The minisuperspace path integral needs to be taken over complex metrics

The no-boundary proposal is a consistent saddle point, but off-shell the "geometry" starts with uncertainty in size



The question behind our work

Can one get some further support for the new incarnation of the no-boundary* proposal from the AdS/CFT intuitions and considerations?

Our results

MAX PLANCK INSTITUTE
FOR GRAVITATIONAL PHYSICS
(Albert Einstein Institute)

DEUTSCH

INSTITUTE | RESEARCH | CAREER | NEWS ROOM

Home > AEI Staff

Institute staff

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z Ä Ö Ü All

Search employee

Search results for: "heller"

Name	Phone	Email	Room	Groups, Location
Dr. Michal P. Heller Group Leader		michal.p.heller@...	2.18	• Gravity, Quantum Fields and Information • Location Potsdam

Search results for: "lehners"

Name	Phone	Email	Room	Groups, Location
Dr. Jean-Luc Lehners Research Group Leader	+49 331 567-7316	jean-luc.lehners@...	2.16	• Theoretical Cosmology • Location Potsdam

The general idea

We want to do minisuperspace with negative cosmological constant:

$$S \sim \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2}\right] + \ldots$$

with

$$ds^2 = -\frac{N^2}{q(r)}dr^2 + q(r)d\Omega_3^2$$

or

$$ds^2 = -\frac{b(r)}{c(r)}N^2 + dr^2\frac{c(r)}{b(r)}d\tau^2 + b(r)^2 d\Omega_2^2$$

We choose $0 \leq r \leq 1$ and fix the proper size of S^3 and $S^1 \times S^2$ at $r = 1$: this is going to be a boundary, where a dual quantum field theory lives

In an analogy with cosmology, $r = 1$ would be the spatial slice of the universe

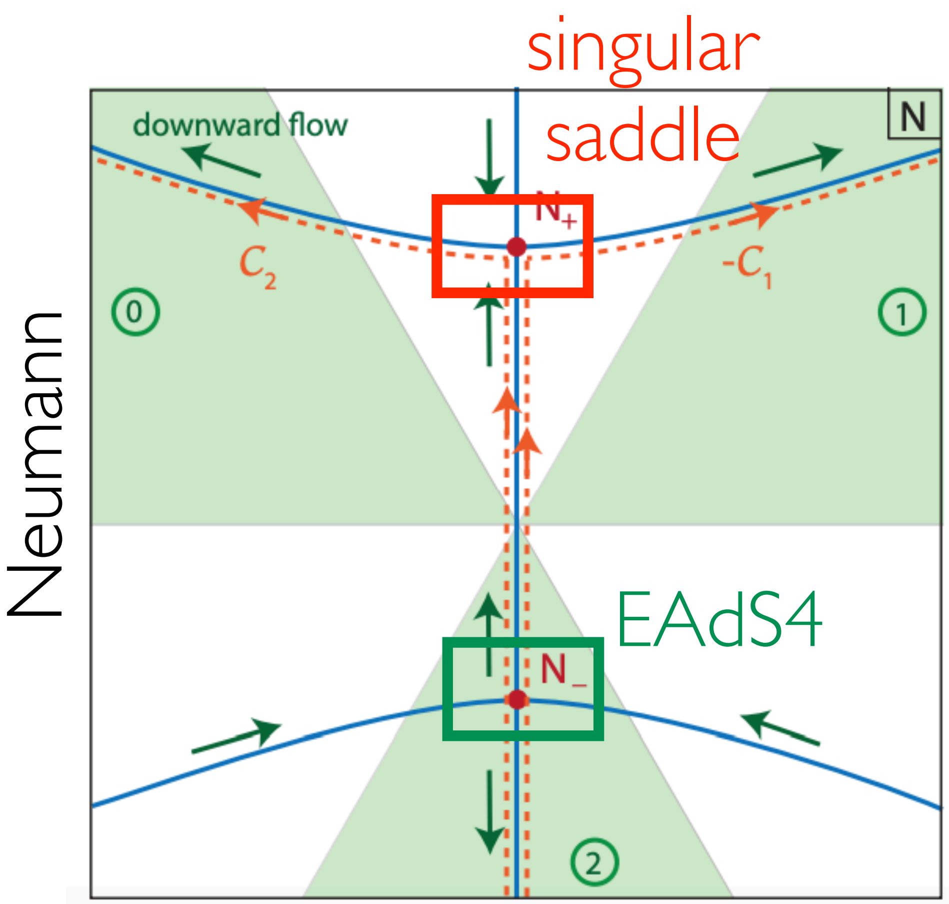
However, in order to evaluate the path-integral, one also needs to specify how q and b with c behave at $r = 0$

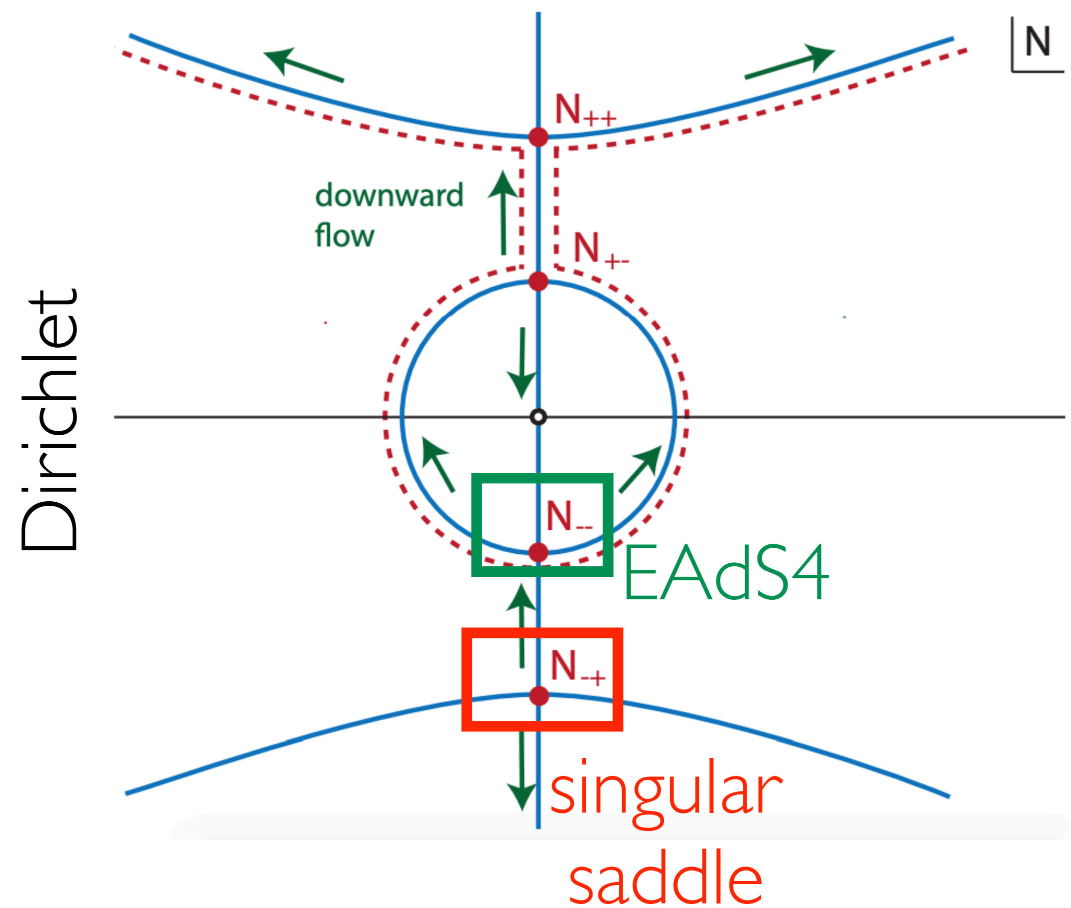
In an analogy with cosmology, $r = 0$ would be the "no-boundary locus"

S^3 minisuperspace with D or N b.c. at r = 0

When evaluating the path integral, we need to supplement the action with the Gibbons-Hawking term at $r = 1$ and for $r = 0$ in the Dirichlet case

$$S \sim N^3 + \dots$$





$$Z \sim e^{-\frac{V_3 l^2}{4\pi G\hbar}}$$

$$Z \sim Ai\left[\left(\frac{3V_3 l^2}{8\pi G\hbar}\right)^{2/3}\right] \qquad (\text{as} \quad R_3 \to \infty)$$

The Dirichlet calculation appeared in a different context in 1804.00942 by Caputa & Hirano

Black hole thermodynamics requires N at r = 0

For black hole saddles, the situation is more intricate. However, the important take home from our studies is simple:

If we want the minisuperspace gravitational partition function to be consistent with black hole thermodynamics, i.e.

$$\log Z_{GRAV} \approx \mathcal{S}_{Bekenstein-Hawking} - \frac{1}{T_{Hawking}} M_{black\,hole}$$

then we should not use the Gibbons-Hawking term at $r = 0 \longrightarrow$ Neumann

This example with $S^1 \times S^2$ boundary together with an earlier study of the S^3 boundary show Neumann conditions are not that unusual in minisuperspace.

Analytic continuation in cosmological constant

Let's get back to our AdS example with S^3 boundary and Neumann condition:

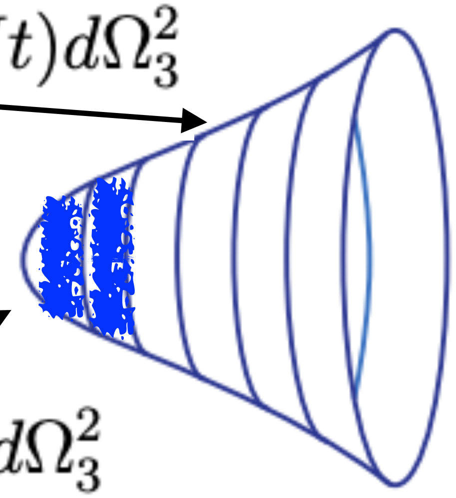
$$ds^2 = -\frac{N^2}{q(r)}dr^2 + q(r)d\Omega_3^2$$

Upon keeping l in all the formulas explicitly, one can take $l = \frac{i}{H}$ and reproduce the cosmological results of di Tucci & Lehners 1903.06757. E.g. saddles:

$$q(r) = H^2 N_\pm^2 r^2 + 2N_\pm i r \qquad N_\pm = -\frac{i}{H^2} \pm \frac{1}{H^2}\sqrt{H^2 R_3^2 - 1}$$

Let's compare with Hartle and Hawking saddle:

$$-dt^2 + \frac{1}{H^2}\cosh^2(Ht)d\Omega_3^2$$



$$d\eta^2 + \frac{1}{H^2}\sin^2(H\eta)d\Omega_3^2$$

for $R_3 \leq \frac{1}{H}$ our saddle is part of S^4

for $R_3 > \frac{1}{H}$ our saddle is complex:

$$D = \int_0^1 dr\sqrt{-\frac{N_\pm^2}{q}} = \frac{\pi}{2\,H} + i\frac{1}{H}\mathrm{arccosh}(H\,R_3) \checkmark$$

Summary and outlook

Summary 2007.04872 with  & 

Alice di Tucci Jean-Luc Lehners

Minisuperspace model is prominent in quantum cosmology in the context of the 1982 no boundary proposal of Hartle & Hawking and its refinements

While the original has problems, adopting Neumann* rather than Dirichlet conditions at the "initial time" and being careful about integration contours solves them while giving the same geometry as the dominant saddle point

Feldbrugge, Lehners & Turok 1705.00192 ⟶ di Tucci & Lehners 1903.06757

What we did is to check if one can see similar Neumann conditions emerging naturally in AdS context and backed up by the dual CFT interpretation

The answer is yes, which shows there are not as exotic as they seemed

Note that our study do not imply that minisuperspace is a way to do quantum gravity backed by AdS/CFT. However, here minisuperspace results could be in principle checked against quantum field theory calculations.

The cosmological considerations I described today, together with reproducing a clear signature of unitarity in black hole evaporation (the Page curve) using the semi-classical picture

, see 2006.06872 by Almheiri et al. for a review, make it an interesting time to study gravity path integrals using hep-th tools.

Thank you