Finite Quantum Gravity

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based on:
arXiv:1407.8036 [hep-th] Nucl. Phys. B **889**, 228 (2014)
arXiv:1503.00261 [hep-th] Nucl. Phys. B **900**, 147 (2015)
arXiv:1605.04173 [hep-th] (click them!)

Problem of infinities in QFT/QG

- present in almost any model of QFT
- motivated research directions on: regularization and renormalization
- caused introductions of new tools: beta functions
- led to new research area: RG flows
- helped in classification of theories: renormalizability and non-renormalizability
- showed the directions for UV-consistent theories: UV-completion
- includes searches for new symmetries or new fundamental interactions

Modification of Gravity

We decided to stay with

- standard rules of Quantum Mechanics
- standard QFT approach with covariant rules of quantization of theories with gauge symmetry
- covariant treatment preserving manifestly Lorentz symmetries and background independence
- quantum field theory (QFT) of gravitational interactions (maybe not classical geometry) of structureless excitations
- perturbative framework of QFT

Therefore we need to MODIFY theory of gravitation.

Einstein's gravity is known to be good physical theory on classical level and for not too big (cosmological) and not too small (microworld) scales.

UV-complete Quantum Gravity

QFT of gravitational interactions (spin 2 fields (on flat background)):

- with self-consistent interactions (covariant theory)
- defined without problems at any (high) energy scale
- 4-dimensional
- only with metric field degrees of freedom
- complete in UV regime
- unitary and with Lorentz invariance (general covariance)
- background independent
- with higher derivatives (based on action different than E-H action)
- with better control over UV-divergences
- (super-)renormalizable or even UV-finite in quantum realm

UV-completion

Motivation:

Let's first quantize matter, put it on curved spacetime background, only later quantize gravitation (Utiyama, De Witt, Shapiro)

Observation:

1-loop off-shell divergences of standard matter theory (with two derivatives) are proportional to R^2 and C^2 on a curved spacetime background. Counterterms needed to be added to the divergent matter effective action are of these types R^2 and C^2 (in $d = 4$) even if the gravitational theory was Einstein-Hilbert Quantum Gravity with R in the action

Conclusion:

These counterterms contain higher derivatives of the background metric. Higher derivatives are inevitable!

Higher Derivative Quantum Gravity

Four-derivative theory (Stelle '77)

$$S_{\rm grav} = \int d^4x \sqrt{|g|} \left(\kappa_4^{-2} R + \alpha_R R^2 + \alpha_C C^2 \right)$$

General higher-derivative theory (Asorey, Lopez, Shapiro '96)

$$S_{\rm grav} = \int d^4x \sqrt{|g|} \left(\lambda + \kappa_4^{-2} R + \sum_{n=0}^{N} \alpha_{R,n} R \Box^n R + \sum_{n=0}^{N} \alpha_{C,n} C \Box^n C \right)$$

Drawbacks:

classical Ostrogradsky instabilities, runaway solutions, presence of massive ghosts with negative residues in the spectrum, rapid decay of putative Gravitational Vacuum

Violation of Unitarity

Non-local Quantum Gravity

Infinitely higher-derivative theory (Kuzmin, Tomboulis, Krasnikov,...)

$$S_{\rm grav} = \int d^4x \sqrt{|g|} \left(\lambda + \kappa_4^{-2} R + R F_R(\Box) R + C F_C(\Box) C \right)$$

Advantages:

- The most general theory describing gravitons' propagation
- Weakly non-local due to non-polynomial functions $F_R(\Box)$ and $F_C(\Box)$
- Unitary (optical theorem is satisfied, and not only)
- Propagator of gravitational modes is highly improved in UV
- In the spectrum only physical massless transverse graviton (spin 2) around flat or MSS background
- Asymptotically Free in UV, like Yang-Mills theory
- Good Quantum Loop Behaviour: renormalizable $\rightarrow$ super-renormalizable $\rightarrow$ UV-Finite

Super-renormalizable Quantum Gravity

Propagator of all quantum modes in UV regime (monomial asymptotics $F_i(\Box) \to \Box^\gamma$) Modesto

$\Pi \sim k^{-(4+2\gamma)}$

Superficial degree of divergence Δ of L-loop graph G

$\Delta = 4L + V[\mathrm{vertex}] - I[\mathrm{propagator}]$

Graviton $h_{\mu\nu}$ and FP ghost fields C_μ are dimensionless $\Rightarrow$ the same maximal number of derivatives in vertices as in propagators in UV

$[\mathrm{vertex}] = -[\mathrm{propagator}] = k^{4+2\gamma}$

Bound on Δ

$\Delta \leqslant 4 - 2\gamma(L-1)$

1-loop Super-renormalizability

For $\gamma \geqslant 3$ only 1-loop divergences survive

Three (four) divergent contributions in the effective action:

$$\Gamma_{\rm div} = \int d^4x \sqrt{|g|} \left(\beta_R R^2 + \beta_C C^2 + \beta_E E\right) + \left(\beta_{\Box R} \Box R\right)$$

Gravitational tensors:

Weyl square: $C^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2$
Gauss-Bonnet: $E = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$

But $\delta \int d^4x \sqrt{|g|} E = 0$

Ways for UV-Finiteness

To get full control over UV-behaviour of the effective action $\Gamma_{\rm eff}$ of the theory beta functions should vanish $\beta_i = 0$

But this should not be the effect of UV-regularization

Kuzmin '89:

1. relations between form-factors: $4F_R + 2F_{\rm Ric} + 4F_{\rm Riem} = 0$ and $F_{\rm Ric} = \beta F_R$ with $\beta \approx -2.392$
2. asymptotically polynomial behaviour of the theory in UV regime
3. the particular irrational value of the exponent: $\gamma \approx 37.22$

FRG community:

In Wilsonian approach the effective action of a fundamental QFT should meet a non-trivial Fixed Point (FP) in the UV regime

UV-Finiteness

Addition of killers operators

$$\Gamma_{\rm kill} = s_1 R^2 \Box^{\gamma-2} R^2 + s_2 C^2 \Box^{\gamma-2} R^2 + s_3 E \Box^{\gamma-2} R^2 + \left(s_4 (\Box R) \Box^{\gamma-2} R^2 \right)$$

Linear structure of beta functions

$$\beta_i = v_i + a_{ij} s_j$$

The matrix a_{ij} is non-degenerate

Condition for UV-Finiteness $\beta_i = 0$

solutions for non-running s_1, s_2, s_3 and s_4 always exist

Conformal symmetry

Finiteness equivalent to vanishing of conformal anomaly

$T \sim \beta$, where T is a trace of pseudo-energy tensor of the quantized gravitational field at one loop

$$T = \beta_C C^2 + \beta_E E + \beta_{\Box R} \Box R = 0$$

$\beta_i = 0 \Rightarrow$ scale-invariance of Green functions $\Rightarrow$ Conformal Invariance

Conformal symmetry in a consistent QFT of gravitational interactions

Conformal QG at UV fixed point of Renormalization Group

Conformal symmetry makes GR-like gravitational singularities conformal gauge-dependent, hence physically unobservable

Why CFT?

Enhancement of symmetries compared to standard QFT

Solution to unitarity problem and unitarity bound

Model QFT with very good behaviour at any energy scale

Basis for doing conformal perturbation theory

Famous examples:

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory
$\mathcal{N} = 8$ conformal supergravity Fradkin, Tseytlin

Why CFT of QG?

Very strong constraint on effective action

if there are no mass scales in the theory

Constraints on anomalous dimensions of operators

Only finite renormalization of couplings

No need for renormalization scale μ (arbitrary)

Idempotency of quantization procedure

Conformally symmetric phase

Basis for study various deformations

Conformal QG

The UV-Finite QG can be made manifestly conformally covariant

Addition of a (spurious) dilaton field ϕ

$$g^{\mu\nu} = \left(\phi^2 \kappa_D^2\right)^{\frac{2}{2-D}} \hat{g}^{\mu\nu}, \quad \phi \to \Omega(x)^{\frac{2-D}{2}} \phi$$

Algebraically "9+1" degrees of freedom in the metric field $\hat{g}_{\mu\nu}$

Crucial in solving problems of classical singularities

Choice of the conformal factor $\Omega^2(x)$ makes the GR singularities (conformal) gauge-dependent, $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$

In real world conformal symmetry is broken

The SSB phase is obtained through the gravitational (conformal) Higgs-like mechanism $\langle \phi^2 \rangle \neq 0$

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(Click them!)

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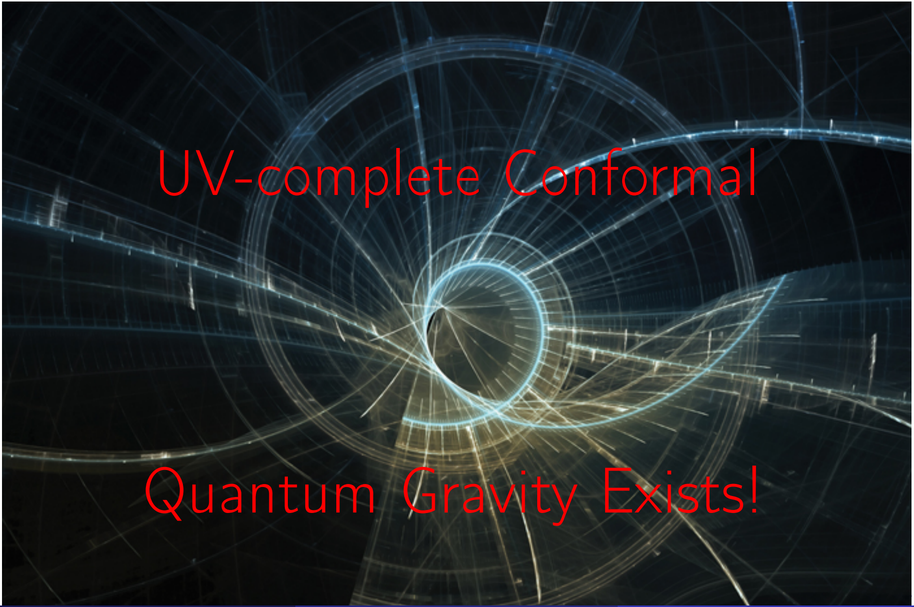
Conclusions




Dziękuję!
Thank you!