Quantum fluctuations of the compact phase space cosmology
– arXiv:2003.08129

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09/26/2020

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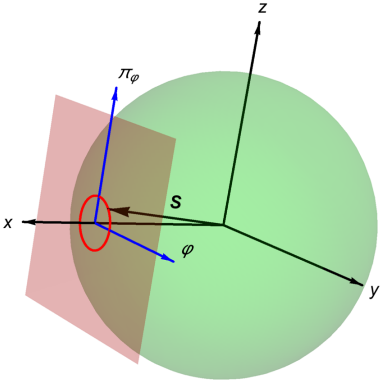
Introduction

–In the big picture we are considering the generalization from flat phase spaces to curved non-linear phase spaces.



Areas with compact phase spaces

- Loop quantum cosmology
- Non-linear sigma models
- semiclassical descriptions of finite dimensional Hilbert spaces.

Physical motivations

- Bounded quantities
- $\dim(\mathcal{H}) < \infty$
- Born reciprocity symmetry

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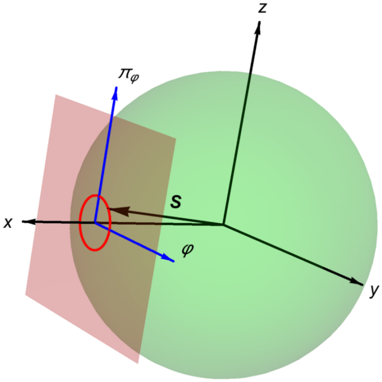

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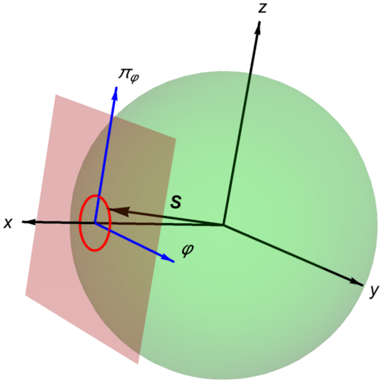

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Progress with nonlinear phase spaces.

– There has been some development in the recent years of applications of the non-linear phase spaces in various areas .

- Cosmological models
  - Compactified minisuperspace models [1904.11338]
  - Cosmological perturbation theory [1704.01934]
- Connections with condensed matter models
  - Spin field correspondence [1612.04355]
  - Heisenberg XXZ model [1708.03207]
- General relativistic spin systems [2008.01729]

–The eventual goal of this program is to consider compactifications of all fields used in theoretical physics.

Compact de-Sitter from ordinary de-Sitter

We need to compactify: $C = NV\left[-\mathcal{H}^2 + \Lambda\right]$. One possible compactification is the following:

$$\boxed{C_s = \frac{N}{L_v}\frac{S_z}{S}\left(-S_y^2 + \Lambda L_v^2\right)}$$

- Polynomial form
- Implementable in condensed matter systems.
- has the correct limiting behavior as $S \to \infty$
- Need to introduce parameters: L_v, L_h

$$S_z = S\frac{V}{L_v}$$

$$S_y = S\sqrt{1 - \frac{V^2}{L_v^2}}\sin\left(\frac{\mathcal{H}}{L_h}\right)$$

$$S_x = S\sqrt{1 - \frac{V^2}{L_v^2}}\cos\left(\frac{\mathcal{H}}{L_h}\right)$$

So that $\{S_i, S_j\} = \epsilon_{ijk} S_k$, and $S = L_v L_h$.

Quantum compact phase space cosmology

–The quantization of the constraint: $C_s = \frac{N}{L_v}\frac{S_z}{S}\left(-S_y^2 + \Lambda L_v^2\right)$ can be done in a straight forward manner just by using the canonical quantization up to some ordering ambiguity.

– From there the dynamics are determined by $\hat{C}_s|\psi\rangle = 0$. The Hilbert space is defined as in any other system with one spin. For $S = 2\hbar$ and taking $\delta = \Lambda/L_h^2$ we have:

$$\hat{C}_s = \hbar^3 \begin{pmatrix} \frac{5}{3} - 12\delta & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & \frac{7}{3} - 6\delta & 0 & 0 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & \sqrt{\frac{3}{2}} \\ 0 & 0 & 0 & 6\delta - \frac{7}{3} & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 12\delta - \frac{5}{3} \end{pmatrix}.$$

Semiclassical extensions

- Analytic results for exact quantum compact phase space cosmology provide some insight into how the model behaves for $S \sim \hbar$.
- Ultimately we want to explore the model in the regime where $S \gg \hbar$, and in this case the exact treatment becomes intractable, because the constraint can't be easily diagonalized.
- The model can still be probed in this regime by implementing semiclassical perturbation theory, to extract features that are most relevant when quantum effects are weak but not ignorable.

The canonical effective methods

To analyze this canonical system at the semiclassical level, it is most convenient to use the canonical effective methods. That is we formulate the quantum system in term of expectation values and moments rather than states on Hilbert spaces.

$$H_q = \langle \hat{H} \rangle = H_c(\vec{S}) + Q(\vec{S}, \Delta)$$

$$\Delta(S_i^a S_j^b) = \langle \delta \hat{S}_i^a \delta \hat{S}_j^b \rangle_{\text{weyl}} \sim O(\hbar^{(a+b)/2})$$

$$\left\{ \langle \hat{A} \rangle, \langle \hat{B} \rangle \right\} = \frac{1}{i\hbar} \left\langle \left[\hat{A}, \hat{B} \right] \right\rangle$$

$$\dot{F}(\vec{S}, \Delta) = \{F, H_q\}$$

Classical deparametrization

–Given the kinematics we need to analyze the dynamics.

–The classical constraint is

$$C_s = \frac{N}{L_v}\frac{S_z}{S}\left(-S_y^2 + \Lambda L_v^2\right)$$

Deparametrization is non trivial because there are no standard matter sources. Introduce a scalar field such that $p_\phi = \Lambda L_v^2$: $\{\phi, p_\phi\} = 1$.

–Treating ϕ as an internal time, we can deparametrize and treat p_ϕ as a Hamiltonian

$$p_\phi = S_y^2.$$

Semiclassical deparametrization

We can quantize in two different ways; before or after gauge fixing. Quantizing before gauge fixing implies a set of constraints.

$$
\begin{aligned}
C_q &= \langle \hat{C}_s \rangle = S_z \left(S_y^2 - p_\phi \right) + S_z \Delta(S_y^2) \\
&\quad + 2 S_y \Delta(S_z S_y) - \Delta(S_z p_\phi) + O(\hbar^{3/2}) \\
C_{S_i} &= \langle \hat{C}_s \delta \hat{S}_i \rangle = 2 S_y S_z \Delta(S_y S_i) - S_z \Delta(p_\phi S_i) \\
C_{p_\phi} &= \langle \hat{C}_s \delta \hat{p_\phi} \rangle = 2 S_z S_y \Delta(S_y p_\phi) - S_z \Delta(p_\phi^2) \\
C_\phi &= \langle \hat{C}_s \delta \hat{\phi} \rangle = 2 S_z S_y \Delta(\phi S_y) - S_z \Delta(\phi p_\phi)
\end{aligned}
$$

Solving the constraints to leading order gives:

$$
p_\phi = S_y^2 + \Delta(S_y^2) + O(\hbar^{3/2})
$$

Perturbative solutions

–The equations of motion for the the above Hamiltonian are given by the usual Hamilton equations: $\frac{d}{d\phi} = \{\bullet, p_\phi\}$.

–Despite being nonlinear, this system has the constants of motion: S_y, $\Delta(S_y^2)$, $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$. Actually the system can be solved exactly. To the leading order we have...

$$V(\phi) \propto S_z(\phi) = \sqrt{S^2 - S_y^2}\Big[\cos(2S_y\phi) + \ldots$$

$$\ldots + 2S_y\phi\left[\sin(2S_y\phi) - S_y\phi\cos(2S_y\phi)\right]\frac{\Delta(S_y^2)}{S_y^2}\Big]$$

–Perturbative corrections are resonating. Normally this would signal the breakdown of perturbation theory, but here we only consider ϕ in a finite range.

Discussion of Bousso bound 1

It turns out that the parameter $\epsilon = \Delta(S_y^2)/S_y^2$ is related to the total number of degrees of freedom in the system.

$$\epsilon = \frac{\Delta(S_y^2)}{S_y^2} \approx \frac{\Delta(\mathcal{H}^2)}{\mathcal{H}^2} \sim \frac{\Delta(R_H^2)}{R_H^2}$$

On the other hand

$$\frac{\Delta(R_H^2)}{R_H^2} \sim \frac{l_p^2}{1/\Lambda} \sim \frac{l_p^2}{A_H} \leq \frac{1}{N_{\text{dof}}}$$

An independent estimation of ϵ can therefore give us insight into the Bousso bound.

Discussion of Bousso bound 2

–By estimating $\Delta(S_y^2)/S_y^2$ from principles of quantum mechanics, we can obtain an independent estimate of ϵ.
– Considering the state of the universe as a condensate of qubits: $|\psi\rangle = |+ \ldots +\rangle$ and assuming $L_h \sim l_p^{-1}$ we can calculate ϵ.

$$\epsilon \sim \frac{\left\langle \psi \left| \delta \hat{S}_y^2 \right| \psi \right\rangle}{S_y^2} = \frac{\frac{1}{2}\hbar S}{\Lambda L_v^2} = \frac{1}{\Lambda S}$$

Comparing this with our previous estimate we find:

$$N_{\text{bulk}} \sim \frac{S}{\hbar} = \frac{1}{\Lambda^2 l_p^4} \gg \frac{A_H}{l_p^2}$$

Much larger than what would be expected from the Bousso bound.

Conclusion

- Using canonical effective methods Quantum fluctuations of a compact phase space cosmology were analyzed in detail. Semiclassical perturbations were found to be unstable over very long times scales.
- Using the quantum fluctuations, the Bousso bound was analyzed and found to be violated.
    - State of the universe might not be semiclassical in the sense that $\Delta(S_i S_j) \not\sim \hbar$.
    - Compactification scale might not be planckian.
    - Hubble horizon might have a fractal like structure [Barrow, 2020]

Thanks!
Any questions?