Quark Gluon Plasma: some conceptual issues

Jane Alam

Variable Energy Cyclotron Centre

[jane@vecc.gov.in]

@ ALICE-India School 2020, IIT Indore, Nov.5-20,2020

November 12, 2020

Plan:

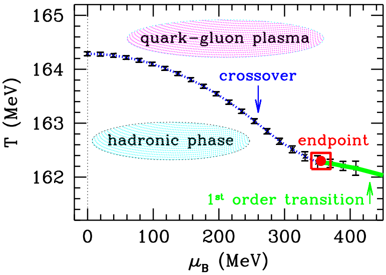
-Why the study of QGP is important?
-Relativistic Kinematics
-Introduction quark structure of proton
-What is QGP and can it be created in Laboratory?
-How QGP evolves in space time?
-Hadrons in medium
-Signals of QGP
-Hadronic
-Electromagnetic: Photons and Dileptons
-Strangeness
-J/ψ
-Jet Quenching

The Goal is to differentiate the following scenarios

(i) Nucleus + Nucleus $\rightarrow$ Hadronic Matter

(ii) Nucleus + Nucleus $\rightarrow$ [Hadronic Matter]*

(iii) Nucleus + Nucleus $\rightarrow$ QGP $\rightarrow$ Mix. Phase $\rightarrow$ Hadronic Matter.



Z. Fodor and S.D. Katz, JHEP **0404** (2004) 050.

Why the physics of QGP is important?

The micro second old baby universe has undergone such a transition.
Only phase transition in the early universe which can be recreated in the laboratory. $T_{EW} \sim 10^3 T_c^{QCD}$
Natural objects like the core of neutron star may contain such a phase of matter.
Condensed matter physics in a new (non-abelian) domain with 15 "electrons" (5 flavours (top too heavy) $\times$3 colours=15 quarks) and 8 "photons".
Nuclear Desert (Nucleus Size 10^{-12} cm, Neutron Stars 10^5 cm).

Is there a strongly interacting system of size few cm or meter?
Generation of mass.

– **Relativistic Kinematics**

- **Natural Unit:**

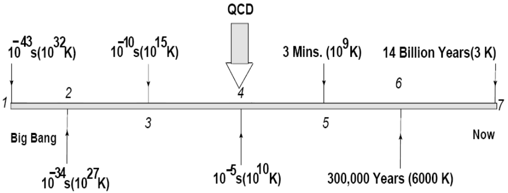
In **S.I.** unit the physical quantities chosen are **Length, Mass and Time** and measure them in the unit of **Meter, Kilogram and Seconds** respectively.

In **Natural (or unnatural ?)** unit the corresponding physical quantities are **Action, Velocity and Mass** - which are measured in the unit of $\hbar$, velocity of light in vacuum **c** and **MeV** respectively, *i.e.* $\hbar = 1$ and c=1.

$\hbar = 1.05 \times 10^{-34}$ **Jule-sec,** $c = 3 \times 10^{8}$ **meter/sec**
$\Rightarrow \hbar c = 1 = 3.162 \times 10^{-26}$ **Jule-meter.**

Putting 1 MeV$= 1.6 \times 10^{-13}$ **Jule** and **1 meter**$= 10^{15}$ **fm.**
$\hbar c = 1 = 197.32$ **MeV fm = 0.197 GeV fm.**
197.32 MeV-fm =1 $\Rightarrow 1 MeV = (197.32 fm)^{-1}$**.**
1 sec $= 1.5 \times 10^{21}$ **MeV**$^{-1}$ **.**

Evolution of the universe:



1: Dawn of time : All the forces of nature are indistinguishable

2: Strong interaction decouples; $W^{\pm}$, Z, γ were equally abundant

3: EM and Weak interactions separated

4: Protons/neutrons appear

5: Light Nuclei formed(He , Li)

6: Atoms formed

No direct way to look at the time before 300,000 years

Nuclear collisions at high energy can create the micro second old universe.

We shall also assume $K_B = 1$, where K_B is the Boltzmann Constant. $1MeV = 1.16 \times 10^{10}\,\text{deg}$ **K**

Example

Number density of Bosons (Fermions) at temperature T

$$\begin{aligned} n &= \frac{g}{(2\pi)^3} \int d^3p \frac{1}{\exp(E-\mu)/T \pm 1} \\ &= g\frac{\zeta(3)}{\pi^2} T^3 \ (\mu = 0). \end{aligned}$$

For $T \sim 3\,\text{deg K}$, $\quad n_\gamma \sim 410/cm^3$

- Kinematics:

Lorentz Transformation:

$$p^\mu = (E, \vec{p}),\ \ p_\mu = (E, -\vec{p}),\ \ p^\mu p_\mu = E^2 - \vec{p}^2 = m^2 \tag{1}$$

$$E' = \gamma(E - \beta p_z),\ \ p'_z = \gamma(p_z - \beta E) \tag{2}$$

$$\begin{aligned} E'^2 - p'^2 &= E'^2 - p'^2_x - p'^2_y - p'^2_z \\ &= E^2 - p^2 = m^2 \end{aligned}$$

$\Rightarrow E^2 - p^2 = P^2 = P_\mu P^\mu = m^2$ is an invariant quantity where $P^\mu \equiv (E, p)$ is the four vector.
(Energy)^2 - (Three Momentum)^2 is invariant under Lorentz transformation.

Consider the reaction $1 + 2 \rightarrow 3 + 4$

$$P_1 + P_2 = P_3 + P_4$$

$$\begin{aligned} s &= (E_1 + E_2)^2 - (p_1 + p_2)^2 \\ &= (P_1 + P_2)^2 = (P_3 + P_4)^2 \\ t &= (P_1 - P_3)^2 = (P_4 - P_2)^2, \\ u &= (P_1 - P_4)^2 = (P_3 - P_2)^2 \end{aligned}$$

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

s, t and u are called **Mandelstam variables.**

Applications:

1. For $E_{lab} = 158\text{GeV}/\text{A}$, what is the E_{cm}?

Evaluate s in CM and Lab frame and equate them.

In **CM frame** ($\vec{p_1} + \vec{p_2} = 0$)**:**

$$\begin{aligned} s &= (P_1 + P_2)^2 = (E_1 + E_2)^2 - (p_1 + p_2)^2 \\ &= (E_1 + E_2)^2 = E_{cm}^2 \end{aligned}$$

$\Rightarrow \sqrt{s}$ is CM energy.
In **Lab frame** ($\vec{p}^2 = 0$**, say):**

$$\begin{aligned} s &= (P_1 + P_2)^2 = m_1^2 + m_2^2 - 2P_1.P_2 \\ &= m_1^2 + m_2^2 - 2(E_1 E_2 - p_1.p_2) \end{aligned}$$

if particle 2 is at rest ($p_2 = 0$)

$$s = m_1^2 + m_2^2 - 2E_{Lab} m_2 = E_{cm}^2 \quad (3)$$

For $E_{Lab} = 158$ GeV/A, $E_{cm} = \sqrt{s} = 17.3$ GeV/A

2. What is the energy and momentum of particle 1 in the CM frame

$$\begin{aligned} E_1^{cm} &= E_1^{cm}\frac{(E_1^{cm}+E_2^{cm})}{E_1^{cm}+E_2^{cm}} - \frac{p_1.(p_1+p_2)}{E_1^{cm}+E_2^{cm}} \\ &= \frac{P_1.(P_1+P_2)}{E_1^{cm}+E_2^{cm}} = \frac{s+m_1^2-m_2^2}{2\sqrt{s}} \end{aligned}$$

$p_1^{cm} = \sqrt{E_1^{cm2}-m_1^2} = \frac{\lambda^{1/2}(s,m_1^2,m_2^2)}{2\sqrt{s}}$ where

$$\lambda(x,y,z,) = x^2+y^2+z^2-2(xy+yx+yz) \qquad (4)$$

is called **triangular function** because $\{-\lambda(x,y,z)\}^{1/2}/4$ is the area of a triangle with sides $\sqrt{x}$, $\sqrt{y}$ and $\sqrt{z}$.

Rapidity: If we define $\gamma = coshy$ *i.e.* $\gamma v = sinhy$ and $v = tanhy$

$$E' = Ecoshy - p_z sinhy, \; p'_z = p_z coshy - Esinhy. \tag{5}$$

$$\begin{aligned} y &= tanh^{-1}v = \frac{1}{2} ln \frac{1+v}{1-v} \\ &= \frac{1}{2} ln \frac{E+p_z}{E-p_z} \end{aligned}$$

$$m_T = \sqrt{E^2 - p_z^2} = \sqrt{p_T^2 + m^2}, \; E = m_T coshy, \; p_z = m_T sinhy \tag{6}$$

Note the similarity of Eq.) with co-ordinate rotations in 2D:

$$x' = xcos\theta + ysin\theta, \; y' = -xsin\theta + ycos\theta \tag{7}$$

Pseudo rapidity (η) :

$$
\begin{aligned}
\eta &= \frac{1}{2} ln \frac{p + p_z}{p - p_z} \\
&= \frac{1}{2} ln \frac{1 + cos\theta}{1 - cos\theta} \\
&= -ln(tan\theta/2)
\end{aligned}
$$

For massless particle $y = \eta$.

$$
dy = \sqrt{1 - \frac{m^2}{m_T^2 cosh^2 y}}\, d\eta = \sqrt{1 - m^2/E^2}\, d\eta = \frac{\vec{p}}{E} d\eta \tag{8}
$$

Existence of Quarks

1.Magnetic moment of the proton:

Bohr magneton of proton is

$$\mu_p = \frac{e\hbar}{2m_p c} \tag{9}$$

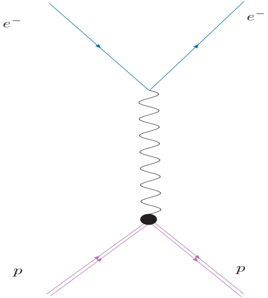
If proton is an elementary particle then the magnetic moment should be one Bohr magneton according to Dirac theory.
However,

$$\mu_p^{Expt} = 2.6 \times \mu_p \tag{10}$$

$\Rightarrow$ First indication that proton is NOT an elementary particle.

What is the dimension of the proton? Take a "photograph" of proton *i.e.* elastic scattering of electron off proton may be used to determine the size of the proton.

2. Elastic scattering e+p→ e +p



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} \left[\frac{G_E^2 + \frac{q^2}{4M^2} G_M^2}{1 + \frac{q^2}{4M^2}} + 2\frac{q^2}{4M^2} G_M^2 tan^2\theta/2\right] \qquad (11)$$

where $G_E(q^2)$ determined from expt.

$$G_E(q^2) = \left(\frac{1}{1 + q^2/\mu^2}\right)^2 \qquad (12)$$

$q^2 = -Q^2 = -4\vec{p}^2 sin^2(\theta/2)$ is the space like momentum transfer.

$G_E(q^2)$ and $G_M(q^2)$ are electric and magnetic form factors of proton. Note that G_E is dipole form factor. The experimental value of $\mu^2 = 0.71$ GeV^2. $G_M(q^2)$ has the same q^2 dependence.

$$
\begin{aligned}
G_E(q^2) &= \int \rho(x) e^{iq.x} d^3x \\
&= \int \rho(x)(1 + iq.x - \frac{(q.x)^2}{2} + ...) d^3x \\
&= \int \rho(x)(1 + iq.x - cos^2\theta q^2 x^2/2 + ...) d^3x
\end{aligned}
$$

Assuming $\rho(x)$ as spherically symmetric and $\int \rho(x) d^3x = 1$ we get

$$G_E(q^2) = 1 - q^2 \frac{< r^2 >}{6} + ... \tag{13}$$

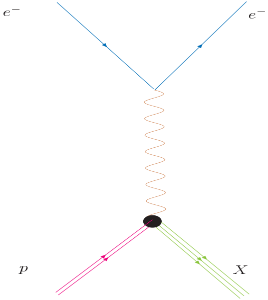
$$< r^2 >= 6(\frac{\partial G_E}{\partial q^2})_{q^2=0} \tag{14}$$

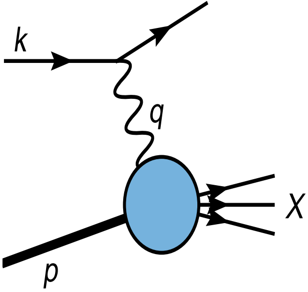
$\sqrt{< r^2 >} = 0.8$ fm.

In fact the dipole form factor has a Fourier transform $\rho(r) \sim e^{-\mu r}$. There is no singularity at $r = 0$ so there is no hard core (no accumulation at $r = 0$) to the proton. This is a non-trivial result. A monopole form factor would have given a Yukawa type distribution $\rho(r) \sim e^{-\mu r}/r$ which is singular at $r = 0$.

3. In-elastic scattering

After determining the size we would like to "see" the internal structure of the proton. Strong evidence for the composite nature of proton came from the DIS (large q^2) experiment.





$e + p \;\rightarrow\; e' + \text{Hadrons}$

$$
\begin{aligned}
q^2 &= (P-P')^2 = P^2 + P'^2 - 2P.P' \\
&= M^2 + M_H^2 - 2p_0 p_0' + 2pp' cos\theta
\end{aligned}
$$

In Lab frame (proton at rest):

$$q^2 = M^2 + M_H^2 - 2Mp_0' \tag{15}$$

Putting $E_0 + M = E + p_0'$

$$q^2 = M^2 - M_H^2 - 2M\nu \tag{16}$$

$\nu = E_0 - E$.

$$q^2 = (P_e - P_{e'})^2 = -2EE_0(1 - cos\theta)$$
$$= -4EE_0 sin^2\theta/2$$

In elastic collisions $M = M_H$, $-q^2 = 2M\nu$ i.e. q^2 and ν related. In experimental term, θ and $(E - E_0)$ are related
$q^2 = -2M\nu = -4EE_0 sin^2(\theta/2)$, $1/E - 1/E_0 = 2\sin^2(\theta/2)/M$

However, for inelastic collisions q^2 and ν are independent and the form factor now has to be replaced by structure functions,

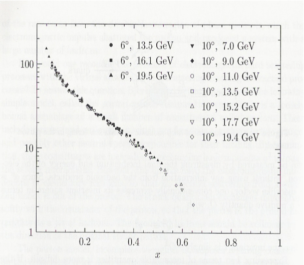
$$\frac{d\sigma}{d\Omega} = (\frac{d\sigma}{d\Omega})_{point}\left[W_2(\nu, q^2) + 2W_1(\nu, q^2)tan^2\theta/2\right] \qquad (17)$$

Measurement of the cross section will enable us to determine the functions $W_{1,2}$.
It is then possible to plot W_1 and W_2 as a function of $\omega = 2M\nu/q^2 = x^{-1}$.

It is found that at very high ν and Q^2 W_1 and νW_2 are functions of ω only not of q^2 and ν separately, this is known as **Bjorken's scaling.**

$$\begin{aligned} \nu W_2 \rightarrow F_2(x) &= \sum e_i^2 x f_i(x) \\ M W_1 \rightarrow F_1(x) &= \frac{F_2(x)}{2x} \end{aligned}$$

What is x?
$m^2 = (\xi P + q)^2 = \xi^2 P^2 + q^2 + 2\xi P.q = m^2 + q^2 + 2\xi P.q$
$\Rightarrow \xi = q^2/(2P.q) = q^2/(2M\nu) = x.$
So x is the fraction of proton momentum carried by the quark.

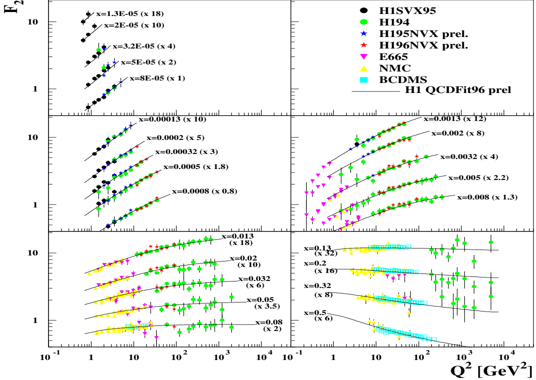


The cross section for DIS:

$$\frac{Q^4}{1+(1-Q^2/(xs))^2}\frac{d^2\sigma}{dxdQ^2} = 2\pi\alpha^2\sum_i e_i^2 f_i(x) \tag{18}$$

The LHS of the above eq. is plotted in the Fig. for $1 < Q^2$ (GeV^2)< 8.
(J. S. Poucher et. al., PRL **32** (1974) 118.)

The presence of free quarks is signaled by the fact that inelastic structure functions ($W_{1,2}$) are independent of q^2 at a given ω. This is equivalent to the onset of $sin^{-4}\theta/2$ behaviour for large momentum transform of Rutherford experiment which has revealed the "point" like structure of atomic nucleus.



History of the discovery (1911) of nucleus by Rutherford through elastic scattering of α on gold foil repeated after almost sixty years through inelastic scattering of electron off proton (during the years 1967-1973) or elastic scattering of virtual photon off quark at large Q^2.

What is the spin of the quark?
For spin 1/2 particles:

$$\left[\frac{\frac{2\nu^2}{q^2}W_2 - W_1}{2W_1}\right]_{q^2,\nu\to\infty} \to 0 \tag{19}$$

For spin zero particles:

$$\left[\frac{\frac{2\nu^2}{q^2}W_2 - W_1}{2W_1}\right]_{q^2,\nu\to\infty} \to \infty \tag{20}$$

4. J/ψ and Υ family

Discovery of two families of mesons that have excitation spectrum similar to Hydrogen atom. The J/ψ and Υ family (November revolution 1974).

Colour

$$\Delta^{++}(J = 3/2) \equiv uuu \Rightarrow \Psi_{\Delta^{+++}} = \Psi_{spin}\Psi_{SU(3)}\Psi_{space} \tag{21}$$

Violation of Pauli exclusion principle !, discard quark model?
Introduce a new quantum number called colour: $\Delta^{++} \equiv u_R u_B u_G$.
[Electron spin in He atom problem plays the role of color in quarks]

$$\Delta^{++} \equiv \, uuu \, \Rightarrow \, \Psi_{\Delta^{+++}} = \Psi_{spin} \Psi_{SU(3)} \Psi_{space} \Psi_{color} \qquad (22)$$

For$| \, \vec{S} \, |= 3/2$, $\Rightarrow L = 0(\vec{J} = \vec{L} + \vec{S})$; space part is symmetric, $\psi_{space} = (-1)^L$; spin part is symmetric; $SU(3)$ flavour, uuu is symmetric. Violation of Pauli exclusion principle!. New quantum numbers called color (red, green, blue) was introduced (Han and Nambu, 1965).

How many colours?

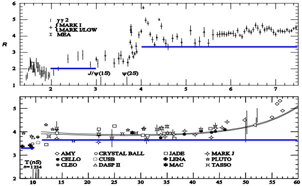
1. Consider $\pi^0 \to \gamma\gamma$

$$\Gamma_{\pi^0 \to \gamma\gamma} = 7.87 \frac{N_c}{3} eV \qquad (23)$$

Experimental value for the above decay width is 7.9 eV, indicating $N_c = 3$.

2. $e^+e^- \rightarrow$ hadrons

$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_q e_q^2$



Data compilation from M. Swartz, PRD **53** (1996) 5268.

Hadrons: *i.e.* mesons and baryons are bound states of $q\bar{q}$ and qqq respectively. Quarks interact via gluon exchange. Gluons also carry colour. Quantum Chromodynamics is the theory for these colour objects.

QED vs QCD

The Lagrangian density for the QED:

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{24}$$

$$D_\mu = \partial_\mu - ieA_\mu \tag{25}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{26}$$

$$F_{\mu\nu}F^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \tag{27}$$

ψ is the fermion field and A_μ represents the gauge field.

Compare with QCD

The Lagrangian density for the QCD:

$$\mathcal{L}_{QCD} = \bar{\psi}^a_F(\gamma_\mu D^\mu - m)\psi^a_F - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \tag{28}$$

$$D_\mu = \partial_\mu - igA^a_\mu T^a \tag{29}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \tag{30}$$

f^{abc} is the structure constant, $[T^a, T^b] = if^{abc}T^c$.

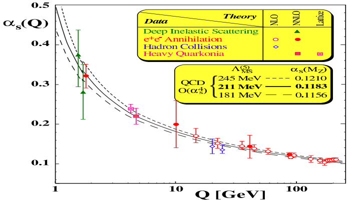
Index a should be summed over the generators of the gauge group. Consider

$$\begin{aligned} F^a_{\mu\nu}F^{a\mu\nu} &= (\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} + gf^{abc}A_{\mu b}A_{\nu c}) \times \\ &\quad (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a} + gf^{ade}A^{\mu d}A^{\nu^e}) \end{aligned}$$

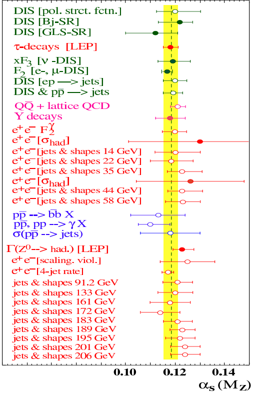
Note that the carrier of electromagnetic force *i.e.* photons do not carry any electric charge as electrons do. However, the carrier of strong force *i.e.* gluons carry color charge as the quarks do. This is the main difference between QED and QCD and the source of complications.

Quarks are always confined inside hadrons. However, at very small distance or large momentum they behave like free particles $\Rightarrow$ asymptotic freedom : Gross & Wilczek and Politzer 1973 (Noble Prize 2004).

There are six quarks, up, down, strange, charm, bottom and top. Quarks interacts via gluon exchange. Each quark has 3 colours and there are 8 colour gluons.



The variation of strong coupling constant with Q.



The value of strong coupling constant from expt.

How the quarks and gluons are distributed inside protons?

Let p be the momentum of the proton and p_g is the fraction of the proton momentum carried by the gluons.

$$\int dx(xp)\{u + \bar{u} + d + \bar{d} + s + \bar{s}\} = p - p_g \tag{31}$$

Dividing by p and defining $e_g = p_g/p$

$$\int dx(x)\{u + \bar{u} + d + \bar{d} + s + \bar{s}\} = 1 - e_g \tag{32}$$

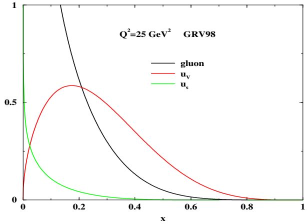
Now integrating over the experimental data on F_2^{ep} and F_2^{en},

$$\begin{aligned} \frac{F_2(x)}{x} &= (2/3)^2\{u^p(x) + \bar{u}^p(x)\} \\ &+ (1/3)^2\{d^p(x) + \bar{d}^p(x)\} \\ &+ (1/3)^2\{s^p(x) + \bar{s}^p(x)\} \end{aligned}$$

Using

$$\int dx F_2^{ep}(x) = \frac{4}{9}e_u + \frac{1}{9}e_d = 0.18$$
$$\int dx F_2^{en}(x) = \frac{4}{9}e_d + \frac{1}{9}e_u = 0.12$$

$e_u = \int_0^1 dx x(u+\bar{u})$
$e_u = 0.36, e_d = 0.18, \Rightarrow\ e_g = 1 - e_u - e_d = 0.46.\ \Rightarrow$ about 50% of the energy is carried by the gluons.



$x = \frac{Q^2}{2M\nu}$, where M is proton mass, ν is energy transfer and $Q^2 = -q^2$.

A simple approach to explain the nature of QCD vacuum (K. Gottfried & V. F. Weisskopf, Concepts of Particle Physics, Vol. 2).

A pair of e^+e^- pops-up in singlet state fron vacuum. The separation between $e^+\&e^-$ and their relative momentum are r and p respectively. The uncertainty principle reads: $r \cdot p \sim 1$. The kinetic energy of the pair is given by, $p \sim 1/r$ (mass is neglected) and the potential energy is given by $-\frac{e^2}{4\pi r}$. The total energy of the pair is given by:

$$E_{e^+e^-} = (1-\alpha)\frac{1}{r} \tag{33}$$

$\alpha = e^2/(4\pi)$. At low energy as $r \to \infty$, $\alpha \sim 1/137$. At small distance α increases, for example at $r \sim 2 \times 10^{-3}$ fm (momentum scale $=$ 100 GeV), $\alpha = 1/128$. At Planck scale, $r \sim 10^{-33}$, $\alpha \sim 1/76$. Therefore, $E_{e^+e^-} = (1-\alpha)/r$ is always positive. Hence, the pair popped-up from the become are not stable.

The pair will be annihilated within a time scale of $1/E_{e^+e^-}$. The QED does not support real pair of e^+e^-.
For QCD scenario is different. The energy of the $q\bar{q}$ popped-up from QCD vacuum is given by,

$$E_{q\bar{q}} = (1-\alpha_s)\frac{1}{r} \tag{34}$$

At short distance $r \to 0$, α_s is very small $\alpha_s << 1$. As r increases α_s also increases. At Planck scale $\alpha_s \sim 0.04$, at EW scale $\alpha_s = 0.118$ and $\alpha_s \sim 1$ at Λ_{QCD} ($r \sim 1$ fm). Therefore, $1-\alpha_s$ decreases and eventually become negative when $r \geq 1$. At large r, however, the potential picks up a term which is proportional to r such that,

$$E_{q\bar{q}} = (1-\alpha_s)\frac{1}{r} + \sigma r \tag{35}$$

$E_{min} < 0$, therefore, the vacuum ($E = 0$) become unstable, therefore, the pair popped-up from vacuum will survive as a real pair for ever.

Chiral symmetry breaking

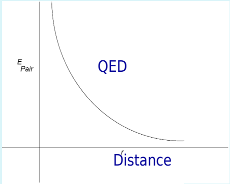
Chiral transition

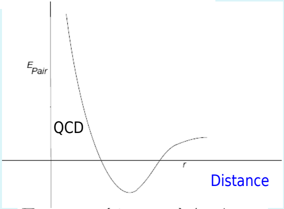
(Gottfried & Weisskopf, Concepts of Particle Physics, Vol-II)

$$q = q_R + q_L$$

$$q_R = \frac{1}{2}[1+\gamma_5]q \quad q_L = \frac{1}{2}[1-\gamma_5]q$$

$$\langle \bar{q}q \rangle = \langle \overline{q_R} q_L \rangle + \langle \overline{q_L} q_R \rangle$$





$$E_{pair} \sim p - \alpha/r$$

$$E_{pair} = (1-\alpha)/r$$

$$\alpha(r \sim 10^{-33}cm) = 1.3 \times 10^{-2}$$

$$\alpha(r \sim 2 \times 10^{-3}fm) = 7.8 \times 10^{-3}$$

$$\alpha(r \sim \infty) = 7.3 \times 10^{-3}$$

$$E_{pair} = (1-\alpha_s)/r + \sigma r$$

$$\alpha_s(r \sim 10^{-33}cm) = 0.04$$

$$\alpha_s(r \sim 2 \times 10^{-3}fm) = 0.118$$

$$\alpha_s(r \sim \infty) \sim Large$$

Quantum Gravity: 10^{-43} sec , GUT: 10^{-36} sec

In QED, if e^+e^-pair is created then it will be unstable because K.E. dominates over the P.E. because α is always $<< 1$.
In QCD, the P.E. can overcome the K.E. of the pair at some r which make the total energy negative and at large r when the linear term (σr) becomes dominant the energy of the pair becomes positive again, producing a negative energy pocket. Therefore, the QCD vacuum can contain real pairs of $q\bar{q}$ and gg.
What is the consequence of the existence of real pairs of $q\bar{q}$ and gg?
Note that the Fermionic quark field q can be written as: $q = q_R + q_L$ and $\bar{q} = \bar{q}_R + \bar{q}_L$, with $q_R = (1+\gamma_5)q/2$ and $q_L = (1-\gamma_5)q/2$
Therefore, mass term of the Dirac Eq.: $\langle\bar{q}q\rangle = \langle\bar{q}_R q_L\rangle + \langle\bar{q}_L q_R\rangle$. In the massless limit, QCD Lagrangian for light quarks has $SU(3)_L \times SU(3)_R$ symmetry.

A massless particle moves with the speed of light and its helicity becomes a good quantum number. Since gluon is a vector field a L(eft) quark [R(ight)] can couple with anti-L (anti-R) quark. In QCD vacuum the $\bar{q}q$ pairs are in colour and spin singlet state i.e. $\langle \bar{q}_R q_L \rangle + \langle \bar{q}_L q \rangle$ survives in vacuum. This indicates that if a R quark is injected in vacuum it will annihilate the $\bar{q}_R$ and consequently an observer will be see a q_L, i.e. the vacuum spontenously changes it helicity from R to L. Therefore, it can not move with the velocity of light and hence it has to acquire mass in the vacuum.

Vacuum breaks the symmetry.

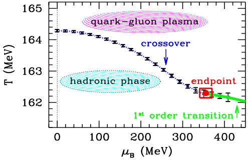
By heating the QCD vacuum (supplying K.E.) the symmetry may be restored at some temperature when the KE overcomes the PE and hence the real $\langle \bar{q}q \rangle$ may disappear from the vacuum. Above a temperature T_c the symmetry will be restored. The order parameter for this transition is $\langle \bar{q}q \rangle$.

How to achieve the temperature where the chiral symmetry is achieved? Accelerate nuclei to very high energy and make collide to convert the KE of the nuclei to thermal energy through multiple random collisions of the quarks forming the nucleons of the nuclei.

QCD at finite density and temperature

In 1975, Collins and Perry predict that at high density $\sim$ few times normal matter density, the properties of the nuclear matter is not governed by the hadronic degrees of freedom but by quarks and gluons degrees of freedom.
Lattice QCD predicts that at high temperature $O(150-200)$ MeV there will be a phase transition from the confined state of hadrons to deconfined state of quarks, anti-quarks and gluons.

It is expected that nucleus-nucleus collisions at ultra-relativistic energies will be able to create such a deconfined state of matter.
QCD phase diagram



What is Plasma?

It is a thermalized state of charge particles with overall charge neutrality where the average K.E. per particle is larger than the interparticle P.E. *i.e.*

$$\begin{aligned} T &>> e^2/r \\ T\,r/e^2 &>> 1 \\ T\,n^{-1/3}/e^2 &>> 1 \\ n^{2/3}\frac{T}{e^2\,n} &>> 1 \\ n\left[\frac{T}{e^2 n}\right]^{3/2} &>> 1 \qquad (36) \\ n\lambda_D^3 &>> 1 \qquad (37) \end{aligned}$$

where $\lambda_D = \sqrt{T/e^2 n} \sim 1/eT$ is called the Debye length. Eq. 37 indicates that the number of particles within a sphere of radius λ_D should be large resulting in screening *i.e.* reducing the interaction for distance $> \lambda_D$.

What is Quark Gluon Plasma (QGP)?

A thermalized state of matter with overall colour neutrality the properties of which are governed by quarks, anti-quarks and gluons which are normally confined within hadrons. In QGP the inter particle interaction between two particles is much less than the K.E. per particle.

The Debye length $\lambda_D \sim 1/gT$ where g is the colour charge. In pQCD the Debye length is given by,

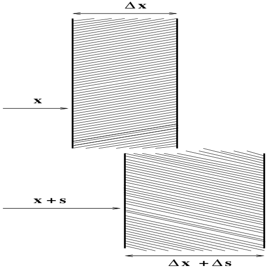
$$\lambda_D = \frac{1}{\sqrt{N_c/3 + N_F/6}\,(gT)} \tag{38}$$

$\lambda_D^{-1} \propto \Pi_{00}(q_0 = 0, \vec{q} \to 0)$

Plasma oscillation (Feynman Lectures on Physics):

Consider a system of charged particle (electron and ions) in thermal equilibrium. If the electrons are moved from the equilibration position there will be an accumulation of electrons in some region resulting in repulsion which will force the electron to wards their original position and gain K.E.

So instead of coming into rest in their original position they will overshoot the mark and oscillate back and forth.



The electron density after the displacement is given by

$$n = \frac{n_0 \Delta x}{\Delta x + \Delta s} = n_0 (1 - \frac{\Delta s}{\Delta x}) \tag{39}$$

The charge density at any point is

$$\rho = -q(n - n_0) = n_0 q \Delta s / \Delta x \tag{40}$$

Solving

$$\frac{\partial E}{\partial x} = 4\pi n_0 q \frac{\partial s}{\partial x} \tag{41}$$

The Eq. of motion

$$\frac{d^2 s}{dt^2} = -(\frac{4\pi n_0 q^2}{m})s = -\omega_p^2 s \tag{42}$$

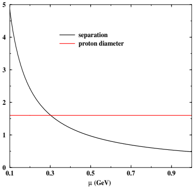
This has oscillatory solution $\sim e^{i\omega_p t}$ with frequency ω_p where ω_p is called **plasma frequency.**

What is the energy density required for the production of QGP?

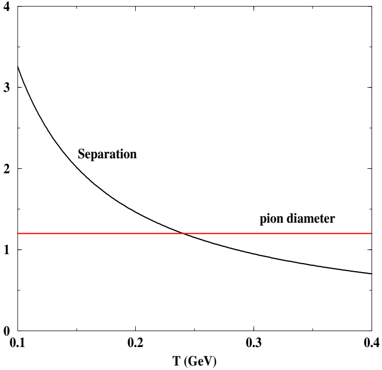
In the collisions of two nuclei a quark will not be associated with its parent proton or neutron if the energy per quark in the struck part (plasma) exceeds the energy per quark inside a free proton or neutron.

$$\rho_c = \frac{m_N}{4\pi R_N^3/3} = 0.5 GeV/fm^3 \quad (43)$$

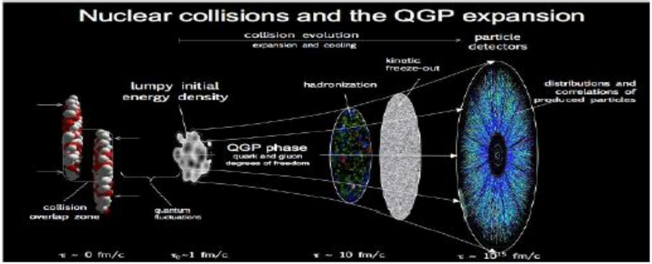
So according to this estimate the energy density should be more than 0.5 GeV/fm^3 $\sim 3-4$ times energy density of nuclear matter (Fig. overlap of protons as function of baryonic chemical potential).



What is the temperature required for QGP formation? At high temperature $> m_\pi$ *i.e.* 140 MeV pion creation will increase the pion density and consequently pions will start overlapping resulting in breakdown of the description of the system in terms of pions. (Fig. overlap of pions as function of temperature).

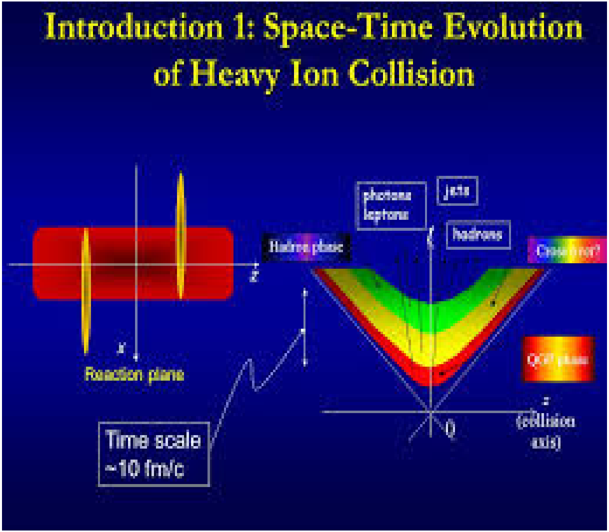


Collision of nuclei at relativistic energies



Kinetic energy of the colliding nuclei converts to thermal energy through random multiple collisions.

Space time evolution of heavy ion collision



T. Hirano, 2004

Thermodynamics
Calculations of energy density:

quarks:

$$\epsilon_q = \frac{g_q}{(2\pi)^3} \int \sqrt{p^2 + m^2} \frac{d^3p}{\exp(E - \mu)/T + 1} \tag{44}$$

for **massless quarks:**

$$\epsilon_q = \frac{g_q}{(2\pi)^3} \int p \frac{d^3p}{\exp(p - \mu)/T + 1} \tag{45}$$

Substitute $x = (p - \mu)/T$

$$\epsilon_q = \frac{g_q T^4}{2\pi^2} \left[\int_{-\mu/T}^{0} dx \frac{(x + \mu/T)^3}{e^x + 1} + \int_0^{\infty} dx \frac{(x + \mu/T)^3}{e^x + 1} \right] \tag{46}$$

Similarly for anti-quarks:

$$\epsilon_{\bar{q}} = \frac{g_q T^4}{2\pi^2} \int_{\mu/T}^{\infty} dx \frac{(x-\mu/T)^3}{e^x+1} \tag{47}$$

Using $\int_{\mu/T}^{\infty} = \int_0^{\mu/T} + \int_{\mu/T}^{\infty}$ and replacing $x \to -x$ in the first integral we get

$$\epsilon_{\bar{q}} = \frac{g_q T^4}{2\pi^2} \left[\int_0^{\infty} dx \frac{(x-\mu/T)^3}{e^x+1} - \int_{-\mu/T}^{\infty} dx \frac{(x+\mu/T)^3}{e^{-x}+1} \right] \tag{48}$$

Adding Eqs. 46 and 48 and using

$$1/(e^x+1) + 1/(e^{-x}+1) = 1, \tag{49}$$

$$\int_0^{\infty} dx x^3/(e^x+1) = 7\pi^4/120 \tag{50}$$

$$\int_0^{\infty} dx x/(e^x+1) = \pi^2/12 \tag{51}$$

$$\epsilon_q + \epsilon_{\bar{q}} = g_q \left[\frac{7}{120} \pi^2 T^4 + \frac{\mu^2 T^2}{4} + \frac{\mu^4}{8\pi^2} \right] \quad (52)$$

For gluons

$$\epsilon_g = g \frac{\pi^2 T^4}{30} \quad (53)$$

Energy density of a thermal system of quarks, anti-quarks and gluons can be written as:

$$\epsilon_{qgp} = \frac{\pi^2}{30} T^4 (16 + \frac{7}{8} \times 12 N_F) \quad (54)$$

when net baryon$= 0$.
$\Rightarrow g_{qgp} = 2 \times 8 + \frac{7}{8} \times 3 \times 2 \times 2 \times N_F$.

Can QGP be created in heavy ion collisions?

Energy density achieved in heavy ion collisions **(Landau's Model):**

$$\begin{aligned}
\epsilon_i &= \frac{E_{cm}}{V} \\
&= \frac{E_{cm}}{\pi R_A^2 2R_A/\gamma} \\
&= \frac{2}{3}\frac{2m_N A}{4\pi R_A^3/3}\frac{E_{cm}}{2m_N A} \\
&= \gamma^2(m_N\rho_0)
\end{aligned} \tag{55}$$

This is enormous !

Two main criticisms for Landau model:

1. Neglecting leading particle effects.
2. Removal of radiation energy due to the deceleration required for complete stopping.

Remedies:
1. During collisions the valence quarks move without much interaction and the energy carried by the gluons are stopped in the collisions volume. This assumption is justifies because gluon-gluon interaction cross-section is larger than quark-quark cross-section. 2. The removal of energy due to decelerated gluons are prohibited due to colour confinement mechanism.

Hwa-Kajantie Model

n

$$dS = 4\frac{\pi^2}{90} g_k T_i^3 \left[\pi R_A^2 \tau_i dy\right] \tag{56}$$

$dS = [\frac{2\pi^4}{45\zeta(3)}]dN$
T_i is initial temperature,τ_i initial thermalization time, R_A is the radius of the colliding nuclei, g_k is the degeneracy of the phase. Connection between multiplicity (dN/dy) and T_i for isentropic expansion.

Glauber model: Estimation of number of collisions and participants

Consider the propagation of EM wave through a medium of refractive index μ. Electric field, $E = E_0 e^{-i(\omega t - k\cdot x)}$. If the wave is traveling through a medium of dimension, d of refractive index μ then the time taken $t = d/v = \mu d/c$ ($\mu = c/v$). Without the medium the time taken $t_0 = d/c$. The delay due to the presence of the medium. $\Delta t = t - t_0 = (\mu - 1)d/c$. The delay due to the presence of medium of width d can be accounted for by replacing t by $t - \Delta t$.

$$E = E_0 e^{-i\omega(t-\Delta t)} e^{ikx} \tag{57}$$

$$E = E_0 e^{-i\omega(t-d(\mu-1)/c)} e^{ikx} \tag{58}$$

Finally,

$$E = E_0 e^{ik(\mu-1)d} e^{i(kx-\omega t)} = e^{i\chi}[E_0 e^{i(kx-\omega t)}] \tag{59}$$

Nuclear Collisions

We know that the total wave function after the scattering,

$$\psi(\rho) = e^{ikz} + \psi_s(\rho) \tag{60}$$

where $\psi_s(\rho)$ is the scattered wave.

$$\psi_s(\rho) = -e^{ik_0\cdot\rho} + e^{ik_0\cdot\rho}e^{i\chi(\rho)} \tag{61}$$

where $e^{ik_0\cdot\rho}$ is the incident wave. The scattered wave can be written as:

$$\psi_s(\rho) = -e^{ik_0\cdot\rho}\Gamma(\rho) \tag{62}$$

implies that the total wave is modified by a multiplicative factor In analogy with the electric field the total wave function can also be written as:

$$\psi(\rho) = e^{ik_0\cdot\rho}e^{i\chi(\rho)} \tag{63}$$

The quantity, $\Gamma(\rho) = 1 - e^{i\chi(\rho)}$, is called the profile function.

Glauber Model

If an EM wave traverses n successive absorber then the electric field is given by at the end,

$$E_n = E_0 e^{i\chi_1} e^{i\chi_2} .. e^{i\chi_n} = E_0 e^{i(\chi_1+\chi_2+..\chi_n)} \tag{64}$$

In case of scattering,

$$e^{i\chi_i} = 1 - \Gamma_i(\rho - si) \tag{65}$$

where s_i is the distance of the ith scatterer from the axis. Therefore, the overall factor,

$$e^{i\chi} = e^{i\chi_1} e^{i\chi_2} .... e^{i\chi_A} = \Pi_{i=1,A} \left(1 - \Gamma_i(\rho - s_i)\right) \tag{66}$$

The complete profile function,

$$\Gamma(\rho) = 1 - \Pi_{i=1,A} \left(1 - \Gamma_i(\rho - s_i)\right) \tag{67}$$

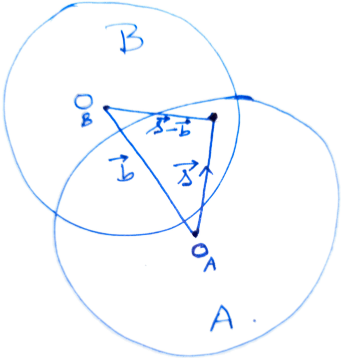
In case of deuteron,

$$\Gamma_d = 1 - (1 - \Gamma_p)(1 - \Gamma_n) = \Gamma_p + \Gamma_n - \Gamma_p \Gamma_n \tag{68}$$

The deuteron profile function is not an addition of neutron and proton profile functions.

Optical Glauber model

- nucleon have high momentum so that they do not get deflected while passing through the nucleus.
- nucleons move independent of each other
- size of the nucleus is larger than the nucleon-nucleon cross section
- nucleons collide inelastically to produce same average number of charged particles in each collisions



Consider the collision of two nuclei, A and B at impact parameter $\vec{b}$. Let $\vec{s}$ is the distance of the colliding nucleon of nucleus A from the centre of A. $\vec{s}-\vec{b}$ is the position of the same nucleon from the centre of B. The prob. per unit area of finding a nucleon in the flux tube of nucleus A is,

$$T_A(\vec{s}) = \int dz_A \rho_A(\vec{s}, z_A) \tag{69}$$

where ρ_A is the prob. per unit volume (normalized to unity) for finding a nucleon at $\vec{s}$,z_A. Similarly we define,

$$T_B(\vec{s}-\vec{b}) = \int dz_B \rho_B(\vec{s}-\vec{b}, z_B) \tag{70}$$

The joint probability per unit are of nucleons being located in the overlapping target and projectile flux tubes respectively in differential area d^2s is given by the product of $T_A(\vec{s})T_B(\vec{s}-\vec{b})d^2s$. The effective overlap area for which a specific nucleon in A can interact with a given nucleon in B, called the thickens function $T_{AB}(\vec{b})$ is given by:

$$T_{AB}(\vec{b}) = \int d^2 s T_A(\vec{s}) T_B(\vec{s} - \vec{b}). \quad (71)$$

The probability that the interaction takes place is $T_{AB}(\vec{b})\sigma_{in}^{NN}$. σ_{NN}^{in} is inelastic cross section for nucleon-nucleon interaction (elastic contribution is neglected because energy loss due to such processes in small).

When nuclei A (with number of nucleon A) and B (with number of nucleon B) collide then the probability of having n number of N+N collision is

$$P(n, b) = \binom{AB}{n} [T_{AB}\sigma_{in}]^n [1 - T_{AB}\sigma_{in}]^{AB-n} \quad (72)$$

The total prob. of an interaction between A and B is:

$$\sum_{n=1,AB} P(n, AB) = \sum_{n=1,AB} \binom{AB}{n} [T_{AB}\sigma_{in}]^n [1 - T_{AB}\sigma_{in}]^{AB-n} \quad (73)$$

The binomial expansion

$$(x+y)^n = \sum_{k=0,N} \binom{N}{k} x^{N-k} y^k = x^N + \sum_{k=1,N} \binom{N}{k} x^{N-k} y^k \quad (74)$$

Therefore,

$$\sum_{k=1,N} \binom{N}{k} x^{N-k} y^k = (x+y)^N - x^N \quad (75)$$

Now we take $x = 1 - T_{AB}\sigma_{in}$ and $y = T_{AB}\sigma_{in}$, $N = AB$, $k = n$. Then

$$\sum_{n=1,AB} \binom{AB}{n} [1 - T_{AB}]^{AB-n} [T_{AB}\sigma_{in}]^n = 1 - [1 - T_{AB}\sigma_{in}]^{AB} \quad (76)$$

Therefore,

$$\sum_n P(n,b) = 1 - [1 - T_{AB}\sigma_{in}]^{AB} \quad (77)$$

The differential cross section,

$$\frac{d^2\sigma_{in}^{AB}}{d^2b} = 1 - [1 - T_{AB}\sigma_{in}]^{AB} \quad (78)$$

A detour to discuss probability function for a binomial random variable. The probability of having m success, in n independent trials and q is the probability of success in a single trial is:

$$P(m; n, q) = \binom{m}{n} q^m (1-q)^{n-m} \tag{79}$$

The average value of m, denoted by $\bar{m}$ is given by,

$$\bar{m} = \sum_{m=0,n} m \binom{m}{n} q^m (1-q)^{n-m} \tag{80}$$

or

$$\bar{m} = \sum_{m=0,n} m \frac{n!}{(n-m)!m!} q^m (1-q)^{n-m} \tag{81}$$

The term with $m = 0$ vanishes,

$$\bar{m} = \sum_{m=1,n} \frac{n!}{(n-m)!(m-1)!} q^m (1-q)^{n-m} \tag{82}$$

Substitute $m - 1 = r$ & $n = s + 1$.

$$\bar{m} = \sum_{r=0,s+1} \frac{(s+1)!}{(s-r)!r!} q^{r+1}(1-q)^{s-r} \tag{83}$$

$$\bar{m} = (s+1)q \sum_{r=0,s} \frac{s!}{(s-r)!r!} q^{r}(1-q)^{s-r} \tag{84}$$

or

$$\bar{m} = (s+1)q = nq(\sum_{r=0,s} \frac{s!}{(s-r)!r!} q^{r}(1-q)^{s-r} = 1) \tag{85}$$

Now apply this result to estimate $N_{coll} = \bar{n}$, average number of collisions,

$$N_{coll} = \sum_{n=1,AB} nP(n,b) = ABT_{AB}(b)\sigma_{in} \tag{86}$$

The number of participants can be calculated as:

$$\begin{aligned} N_{part}(b) &= A \int T_A(s) \left\{1 - \left(1 - \sigma_{in} T_B(\vec{s} - \vec{b})\right)^B\right\} d^2s \\ &+ B \int T_B(\vec{s} - \vec{b}) \left\{1 - \left(1 - \sigma_{in} T_A(\vec{s} - \vec{b})\right)^A\right\}^A d^2s \end{aligned}$$

The N_{coll} and N_{part} are used as inputs to estimate the charge multiplicity, $dN_{ch}/d\eta$ as,

$$\frac{dN_{ch}}{d\eta} = (1 - x) n_{pp} \frac{N_{part}}{2} + x N_{coll} n_{pp} \tag{87}$$

where n_{pp} is the multiplicity measured in pp collisions and $x(1 - x)$ is the fraction of hard (soft) collisions.

Hydrodynamics - effective theory to describe soft physics

Motion of air molecule (nitrogen, say) at room temperature ($T = 300 \circ K$) at atmospheric pressure P). The density, $n = P/k_B T = 1/(35A)^3$ ($1A = 10^{-10}$ metre). This indicates that the intermolecular distance 35A. Therefore at length scale $>> 35A$ air will appear as continuous. The average velocity at $T = 300^\circ K$, $< v >= 475$ m/sec. The mean free path, $\lambda = 587A$ (assuming geometric cross section of nitrogen). The time between two collisions is $\sim 1.2 \times 10^{-10}$ sec. Therefore, for time scale $>> 10^{-10}$ sec air is continuous. This discussion indicates that in fluid dynamics we deal with physics of large time and length scales corresponding to small energy or momentum scales $\Rightarrow$ soft physics.

The density and current of the energy-momentum four vector for a system of n particles. The four momentum (p^μ) density is defined as:

$$T^{\mu 0} = \sum_n p_n^\mu(t)\delta(\vec{x} - \vec{x_n(t)}) \tag{88}$$

(Analogous to electric charge density: $\rho = \sum_n e_n \delta(\vec{x} - \vec{x_n(t)}))$

$$T^{\mu i} = \sum_n p_n^\mu(t) \frac{dx_n^i}{dt} \delta(\vec{x} - \vec{x_n(t)}) \tag{89}$$

(Analogous to electric current density: $J^i = \sum_n e_n \frac{dx_n^i}{dt} \delta(\vec{x} - \vec{x_n(t)}))$

$$T^{\mu\nu} = \sum_n p_n^\mu \frac{p_n^\nu}{E_n} \delta(\vec{x} - \vec{x_n(t)}) \tag{90}$$

where, $p_n^\nu = E_n \frac{dx^\nu}{dt}$ $(\vec{v} = \vec{p}/E)$ For a system in thermal equilibrium, the energy momentum density is given

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3}\, p^\mu \frac{p^\nu}{E} f(\vec{x}, \vec{p}, t) \tag{91}$$

The equation of motion for the fluid:

$$\partial_\mu T^{\mu\nu} = 0 \tag{92}$$

which can be written as:

$$\partial_0 T^{0\nu} + \partial_i T^{i\nu} = 0 \quad (93)$$

(Compare this equation with diffusion equation: $\partial_0 J^0 + \partial_i J^i = 0$ with $J_i(t,x) = -D\partial_i n(t,x)$ given by Fick's law).
For ideal (non-viscous) fluid: $T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + g^{\mu\nu}P$,
$u^\mu = \gamma(1, v)$ ϵ is energy density, P is pressure. For the conservation of baryon,

$$\partial_\mu(n_B u^\mu) = 0 \quad (94)$$

n_B is the net baryon number density. For isentropic expansion we also have,

$$\partial_\mu(su^\mu) = 0 \quad (95)$$

Changing variables (t, z) to (τ, η_s) for (1+1) dimensional expansion,:

$$\begin{aligned} \eta_s &= \frac{1}{2} ln \frac{t+z}{t-z} \\ \tau &= \sqrt{t^2 - z^2} \end{aligned}$$

For a system expanding along longitudinal (z)direction [no transverse expansion, here $(t, x, y, z) \rightarrow (0, 1, 2, 3)$], Eq. 93 boils down to,

$$\partial_0 T^{00} + \partial_i T^{03} = 0 \tag{96}$$

and

$$\partial_0 T^{03} + \partial_i T^{33} = 0 \tag{97}$$

The partial derivatives w. r. t. t and z can be written in terms of τ and η_s as,

$$\frac{\partial}{\partial t} = cosh\eta_s \frac{\partial}{\partial \tau} - \frac{sinh\eta_s}{\tau} \frac{\partial}{\partial \eta_s} \tag{98}$$

and

$$\frac{\partial}{\partial z} = -sinh\eta_s \frac{\partial}{\partial \tau} + \frac{cosh\eta_s}{\tau} \frac{\partial}{\partial \eta_s} \tag{99}$$

Bjorken's boost invariance hydrodynamics (1983)
The experimentally measured multiplicity distribution of particles produced in p+p collisions at high energy shows a plateau when plotted against rapidity. This means that the distribution is

independent of rapidity and hence of Lorentz boost (Frame independence symmetry, Chiu, Sudarshan and Huang, 1975). The observed rapidity distribution can be calculated by convoluting the fluid rapidity distribution obtained from hydrodynamics, $dN/dy(= \frac{45\zeta(3)}{2\pi^4}\pi R^2\tau s)$ with the thermal distribution of the fluid. Assuming the longitudinal velocity component, $v_z = z/t$, one gets, fluid rapidity = space time rapidity. Therefore, $dN/d\eta$ will be independent of η. As discussed before, $dN/d\eta \propto dS/d\eta$, i.e. entropy density is also independent of η. If the baryonic chemical potential of the system is zero then only one independent variable, T (say) can describe the system. As a consequence if entropy density is independent of rapidity, so will be T, then all the other thermodynamic quantities will be independent of η. This fact can be used to simplify the hydrodynamics. Using Eqs.97,98 and 99 and assuming that thermodynamic quantities like ϵ and P are independent of η, we get:

$$\frac{\partial\epsilon}{\partial\tau} + \frac{\epsilon + P}{\tau} = 0 \qquad (100)$$

A simple derivation of Eq. 100

$$dQ = dE + PdV = 0 \tag{101}$$

where $E = \epsilon V$

$$\epsilon dV + V d\epsilon + PdV = 0 \tag{102}$$

$V = \pi R_A^2 \tau$

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = 0 \tag{103}$$

Two (ϵ and P) unknowns can not be determined from a single equation. So we need to supply a relation between P and ϵ, this relationship is called **Equation of State (EOS)**.

Equation of State: Hadronic

$$
\begin{aligned}
\epsilon_h &= 3\frac{\pi^2}{30}T^4 \\
P_h &= 3\frac{\pi^2}{90}T^4 \\
s_h &= \frac{\epsilon + P}{T}
\end{aligned}
$$

QGP

$$
\begin{aligned}
\epsilon_q &= 37\frac{\pi^2}{30}T^4 + B \\
P_h &= 37\frac{\pi^2}{90}T^4 - B \\
s_h &= \frac{\epsilon + P}{T}
\end{aligned}
$$

where B is the bag constant.

Early Universe

Einstein equation in the Robertson-Walker space time,

$$(\dot{R}/R)^2 = 8\pi\epsilon/(3m_{pl}^2) \tag{104}$$

$$d(\epsilon R^3)/dt + PdR^3/dt = 0 \tag{105}$$

governs the evolution of early universe during the QCD phase transition. The characteristic time scale here is $t_c = (3m_{pl}^2/8\pi B)^{1/2} \sim$ few micro second.

Some useful thermodynamic relation

$$TdS = dE + PdV - \mu dN \tag{106}$$

Using

$$N = Vn, \quad E = \epsilon V, \quad S = sV \tag{107}$$

$$dN = ndV + Vdn, \quad dE = \epsilon dV + Vd\epsilon, \quad dS = sdV + Vds \tag{108}$$

$$s = (\epsilon + P - \mu n)/T \tag{109}$$

$$d\epsilon = Tds + \mu dn \tag{110}$$

$$G = E - ST + PV \quad \mu n = \epsilon - sT + P \tag{111}$$

$$\mu dn + nd\mu = d\epsilon - sdT - Tds + dP \tag{112}$$

using $Tds = d\epsilon - \mu dn$

$$dP = sdT + nd\mu \tag{113}$$

For net baryon number $= 0$ we have

$$\begin{aligned} dP &= sdT \\ d\epsilon &= Tds \end{aligned}$$

The velocity of sound is defined as:

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{isentropic} = \left(\frac{\partial lnT}{\partial lns}\right) \tag{114}$$

The velocity of sound is an important parameter which governs the expansion rate and hence the life time of the hot and dense phase.
Assuming $P = c_s^2\epsilon = \epsilon/3$
For the EoS $P = c_s^2\epsilon$, the solution of Eq. 100 is given by:

$$\epsilon = \epsilon_0 \left(\tau_0/\tau\right)^{1+c_s^2} \tag{115}$$

Solving Eq. 103 we get, $\epsilon\tau^{4/3}$ =constant ($c_s^2 = 1/3$) as
$\epsilon \sim T^4 \Rightarrow T^3\tau =$ constant
Suppose the QGP phase at an initial time τ_i at temperature T_i if T_c is the phase transition temperature then the QGP phase ends at a time

$$\tau_q = \frac{T_i^3}{T_c^3}\tau_i \tag{116}$$

For $\tau_i = 1$ fm/c, $T_i = 340$ MeV and $T_c = 170$ MeV $\tau_q = 8$ fm/c. So the QGP life time is
$\tau_{qgp} = \tau_q - \tau_i = 7$ fm/c.
In a first order phase transition scenario the temperature is constant during the mixed phase. The cooling due to expansion is compensated by the latent heat liberated during the transition. However, the entropy density changes. For isentropic expansion, $\pi R_A^2 \tau s =$ constant.
Therefore, for (1+1)dimensional expansion $s\tau =$ constant.

Same equation may be derived by solving

$$\partial_\mu(su^\mu) = 0 \tag{117}$$

where s is the entropy density and $u^\mu = \gamma(1, v)$ is the four velocity of the fluid. In (1+1)dimension

$$\frac{\partial(su^0)}{\partial t} + \frac{\partial(su^z)}{\partial z} = 0 \tag{118}$$

Changing variable:

$$(t, z) \rightarrow (\tau, \eta_s) \tag{119}$$

$\gamma = coshY$ and $\gamma v = sinhY$, η_s and Y are space-time and fluid rapidity respectively.

$$\frac{\partial(s\tau cosh(Y - \eta_s))}{\partial \tau} + \frac{\partial(ssinh(Y - \eta_s))}{\partial \eta_s} = 0 \tag{120}$$

If $Y = \eta_s$ *i.e.* fluid velocity $= z/t$ (called similarity flow) then $s\tau =$ constant.

In the mixed phase, let f_q be the fraction of the QGP and $1 - f_q$ is the hadronic part.

$$s_q \tau_q = f_q s_q \tau + (1 - f_q) s_h \tau \tag{121}$$

$$f_q = \frac{1}{r-1}\left(r\frac{\tau_q}{\tau} - 1\right) \tag{122}$$

$r = g_{qgp}/g_h$ is ratio of degrees of freedom in QGP to hadronic phase. Mixed phase ends at time τ_h when $f_q = 0$ $\tau_h = r\tau_q$.

$$\tau_{mix} = \tau_h - \tau_q = (r-1)\tau_q \tag{123}$$

Indicating that mixed life time increases with r. This means that if at the transition point the decrease in entropy density is large then the increase in the volume should be large for maintaining the total entropy constant and hence the system will remain in the mixed phase for longer time. Similarly the life time of the hadronic phase can be calculated as

$$\tau_f = r\frac{T_i^3}{T_f^3}\tau_i \tag{124}$$

where T_f is the freeze-out temperature, may be determined from the transverse momentum distribution of hadrons produced in the collisions. The life time of the system in a phase transition scenario is

$$\tau_f - \tau_i = (r\frac{T_i^3}{T_f^3} - 1)\tau_i \tag{125}$$

In a 'no phase transition' scenario the life time is

$$\tau_f - \tau_i = (\frac{T_i^3}{T_f^3} - 1)\tau_i \tag{126}$$

Phase transition increases the longevity of the system.

Particle spectra at freeze-out

The hydrodynamics provides the quantities like energy density, pressure, flow etc and in experiments the particles with different momenta are measured. Therefore, a link between the hydrodynamics and the particle spectra is required. The Cooper-Frye formula provides the required link. In a hydrodynamically expanding system the particles decouple from the system when the expansion rate dominates over the interaction rate as a consequence the particles cease to interact (or the mean free path becomes too large for the particle to interact).

The condition, $\tau_{\rm scatt} < \tau_{\rm exp}$ should be satisfied for an expanding system in equilibrium, where $\tau^{j}_{\rm scatt}(T) = (\sum_i \langle v_{ij}\rangle \sigma_{ij} n_j)^{-1}$, is the mean collision time, v_{ij} is the relative velocity, σ_{ij} is the scattering cross section, n_j is the particle density of specie j and $\tau_{\rm exp}(T) = \tau_i(T_i/T)^{1/c_s^2}$ is the expansion time scale. The temperature at which this condition is satisfied is called freeze-out temperature.

At freeze-out, fluid converted to particles. Therefore, the freeze-out condition, $T(\tau, x, y, \eta_s) = T_F$ determines three dimensional space-time surface, σ with surface elements, $d\sigma_\mu$. If j^μ is the current, then the amount of current passing through this surface element is $= j^\mu d\sigma_\mu$. In Milne ($\tau = \sqrt{t^2 - z^2}$, $\eta_s = \frac{1}{2} log \frac{t+z}{t-z}$) $d\sigma^\mu = (\tau dxdyd\eta_s, -\tau d\tau dyd\eta_s, -\tau d\tau dxd\eta_s, -\tau d\tau dxdy)$. The freeze-out condition is specified by freeze-out at proper time, $\tau_f(x, y, \eta_s)$, then $d\sigma^\mu = \tau_f dxdyd\eta_s(1, -d\tau_f/dx, -d\tau_f/dy, -d\tau/d\eta_s)$.

In the present scenario particle current with thermal distribution, $f(\vec{x}, \vec{p})$ in the momentum interval $\vec{p}$ and $\vec{p} + d\vec{p}$ is given by $j^\mu = f(\vec{x}, \vec{p}) d^3 p p^\mu / p_0$ ($d\vec{p} \equiv d^3 p$). The total number of particles,

$$dN = d^3 p \int_\sigma f(x, p) \frac{p^\mu}{p_0} d\sigma_\mu \tag{127}$$

where $p^\mu = (m_T cosh(y - \eta_s), p_x, p_y, m_T sinh(y - \eta_s))]$

$$f(x, p) = \frac{g}{(2\pi)^3} \frac{1}{e^{u \cdot p / T} \pm 1} \tag{128}$$

$E' = u \cdot p$, u^μ is four flow velocity. where $u^\mu = \gamma(1, v_x, v_y, v_z)$

$$\frac{dN}{d^3 p / E} = \frac{dN}{d^2 p_T dy} = \frac{g}{(2\pi)^3} \int f(x, p) p^\mu d\sigma_\mu \tag{129}$$

Signals of Quark Gluon Plasma:
Typical Plasma Size $\sim 10^{-36}$ cm^3, Life Time 10^{-22} sec.
Almost all the signals are "polluted" by background from hadronic matter.

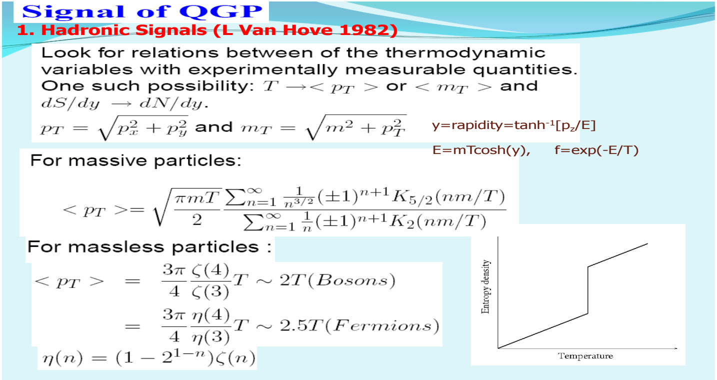
Hadronic Signals to probe equation of state:
The momentum distribution of hadrons emitted from this system, the elliptic flow velocity, HBT radii of the system are some of the quantities which are sensitive to the EOS.
The Step
Look for relations between of the thermodynamic variables with experimentally measurable quantities. One such possibility:
$T \rightarrow < p_T >$ or $< m_T >$ and $S \rightarrow dN/dy$.
For massive particles:

$$< p_T > = \sqrt{\frac{\pi m T}{2}} \frac{\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} (\pm 1)^{n+1} K_{5/2}(nm/T)}{\sum_{n=1}^{\infty} \frac{1}{n} (\pm 1)^{n+1} K_2(nm/T)} \tag{130}$$

For massless particles :

$$< p_T > \;=\; \frac{3\pi}{4}\frac{\zeta(4)}{\zeta(3)} T \sim 2T(Bosons)$$
$$\;=\; \frac{3\pi}{4}\frac{\eta(4)}{\eta(3)} T \sim 2.5T(Fermions)$$

where $\eta(n) = (1 - 2^{1-n})\zeta(n)$



Signal of QGP

1. Hadronic Signals (L Van Hove 1982)

Look for relations between of the thermodynamic variables with experimentally measurable quantities. One such possibility: $T \rightarrow < p_T >$ or $< m_T >$ and $dS/dy \rightarrow dN/dy$.

$p_T = \sqrt{p_x^2 + p_y^2}$ and $m_T = \sqrt{m^2 + p_T^2}$

y=rapidity=$\tanh^{-1}[p_z/E]$

E=mTcosh(y), f=exp(-E/T)

For massive particles:

$$< p_T > = \sqrt{\frac{\pi m T}{2}} \frac{\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} (\pm 1)^{n+1} K_{5/2}(nm/T)}{\sum_{n=1}^{\infty} \frac{1}{n} (\pm 1)^{n+1} K_2(nm/T)}$$

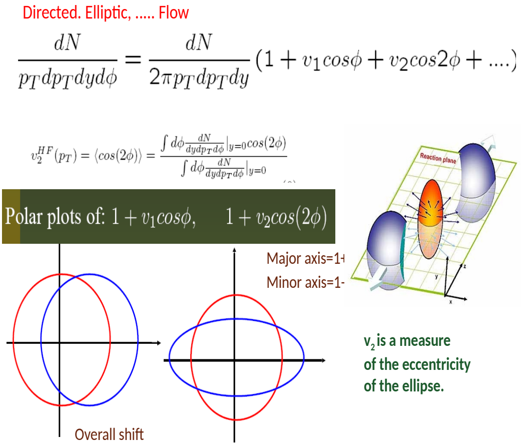
For massless particles :

$$< p_T > \;=\; \frac{3\pi}{4}\frac{\zeta(4)}{\zeta(3)} T \sim 2T(Bosons)$$
$$\;=\; \frac{3\pi}{4}\frac{\eta(4)}{\eta(3)} T \sim 2.5T(Fermions)$$

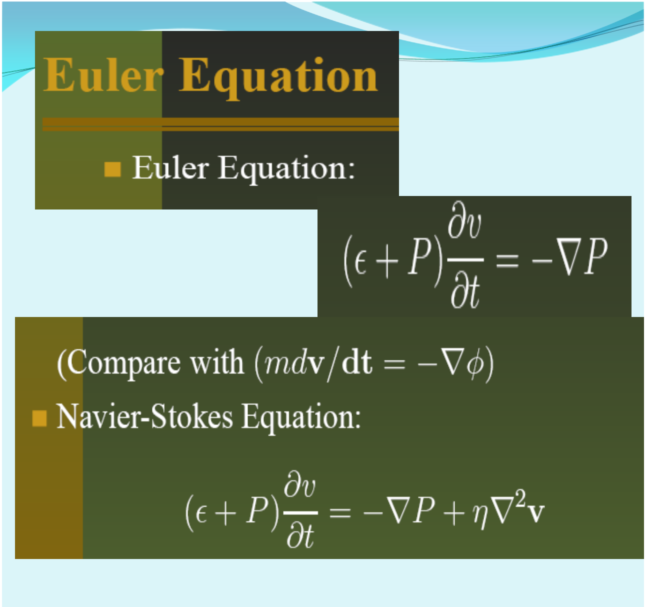
$\eta(n) = (1 - 2^{1-n})\zeta(n)$

Directed, Elliptic, ... Flow

$$\frac{dN}{d^2p_T dy} = \frac{dN}{2\pi p_T dp_T dy}(1 + v_1 cos\phi + v_2 cos2\phi + ...) \qquad (131)$$



Fluid motion - non-relativistic limit



Viscosity

- Viscosity

$$\eta = \frac{1}{3} n < p > \lambda$$

viscosity denotes the ability to transport momentum.

For a system $\lambda \sim \frac{1}{n\sigma}$

$$\eta = \frac{1}{3} \frac{< p >}{\sigma}$$

viscosity is independent of density !.

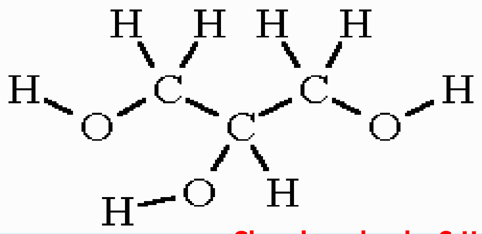
- For a weakly interacting system $\sigma \sim T^{-2}$ and $< p > \sim T$

$$\eta \sim T^3$$

has same dimension as entropy density.

Nature of the fluid: Gas or Liquid ?

- For a gas η increases with temperature
- For a liquid viscosity decreases with temperature. Jiggling with thermal motion helps in unjamimg !



Glycerin molecule: $C_3H_8O_3$

Quantum Mechanics: a particle of momentum $<p>$ can not be localized to a distance smaller than $<p>^{-1} \Rightarrow \lambda \geq <p>^{-1}$ or

$$\eta = \frac{1}{3} n <p> \lambda$$

$$\eta \geq \frac{1}{3} n$$

$s \sim 4n$ where s is the entropy density.

$$\frac{\eta}{s} \geq \frac{1}{12}$$

String theory limit: $\frac{\eta}{s} \geq \frac{1}{4\pi}$

For super fluid He this value is 9 times larger

Strangeness Enhancement:

Consider a nuclear matter with baryon density $\rho = 10\rho_0$. What is the chemical potential of u and d quarks.

For two flavour system $g = 3 \times 2 \times 2 = 12$, $\rho = (n_u + n_d)/3 = 2n_u/3$ $\Rightarrow n_u = 3 \times 10\rho_0/2 = 15 \times 0.16$ fm^3=2.4/fm^3. $\epsilon_F = p_F = \mu$ for massless particles.

$$n_u = \frac{g}{2\pi^2}\frac{p_F^3}{3} \tag{132}$$

$p_F = \mu = 450$. The mass of the s quark ~ 120 MeV.

Pauli exclusion principle prohibits production of $u\bar{u}$ and $d\bar{d}$. $\Rightarrow$ it is less costly for the system to create $s\bar{s}$.

Therefore $K^+(u\bar{s})$, $\Lambda(uds)$ production will be enhanced and $K^-(\bar{u}s)$ production will decrease.

At non-zero temperature, the production of $s\bar{s}$ in QGP through gluon fusion ($gg \to s\bar{s}$) and quark annihilation ($q\bar{q} \to s\bar{s}$) is more favorable in QGP phase than the production of strangeness carrying hadrons K^+K^- in hadronic phase ($\pi^+\pi^- \to K^+K^-$) because of the larger threshold energy required for the production of K^+K^-. The thermal density of strange quarks,

$$n_s = \frac{3 \times 2}{(2\pi)^3} \int d^3p \frac{1}{e^{(\sqrt{p^2+m_s^2}-\mu_s)/T} + 1} \tag{133}$$

where μ_s is the baryonic chemical potential of the strange quark (in heavy ion collisions $\mu_s = 0$ as there are equal number of s and $\bar{s}$). The thermal density of K mesons,

$$n_K = \frac{1}{(2\pi)^3} \int d^3p \frac{1}{e^{\sqrt{p^2+m_K^2}/T} - 1} \tag{134}$$

At a given temperature the abundance of s quarks will be more thank K mesons because K mesons are massive and has lower statistical degeneracy compared to s quarks.

Hadrons in medium

A particle, a ρ meson, say moving in a nuclear medium: the amplitude at a distance z is given by: $\sim e^{-n\sigma z} = e^{-4\pi Imfnz/k}$

$$\psi \sim e^{i2\pi nzf/k} \tag{135}$$

We can also write

$$\psi \sim e^{ikz} \sim e^{i\sqrt{E^2 - m_{eff}^2}z} \tag{136}$$

Writing $m_{eff} = m + \Delta m$ and equating the argument of the exponential function we get

$$\Delta m = -\frac{2\pi nRef}{m} \tag{137}$$

(Rigorous calculations will involve thermal field theory).

Ref may be either positive or negative

$$Ref \sim (2l+1)P_l(cos\theta)sin\delta_l \cos\delta_l \tag{138}$$

Therefore, mass of the hadrons in medium may either increase or decrease depending on the nature of interaction.
The change in the width ($\Delta\Gamma$) of the hadrons

$$\Delta\Gamma = \frac{n}{m}k\sigma \tag{139}$$

(Eletsky and Ioffe, PRL, 1997)

Spontaneous Symmetry Breaking-A Quantum Mechanical Approach

We assume that the vacuum $|0\rangle$ defined as the minimum of the expectation value: $\langle 0|H|0\rangle = H_{min}$ is invariant under the chiral symmetry.

$$Q_R^a|0\rangle = Q_L^a|0\rangle = 0 \tag{140}$$

where,

$$Q_L^a = i\frac{1}{\sqrt{2}}(Q^a - Q_5^a),\;\; Q_R^a = i\frac{1}{\sqrt{2}}(Q^a + Q_5^a) \tag{141}$$

$$Q_5^a = \int d^3x\bar{\psi}\gamma_0\gamma_5\frac{1}{2}\tau^a\psi \tag{142}$$

and

$$Q^a = \int d^3x\bar{\psi}\gamma_0\frac{1}{2}\tau^a\psi \tag{143}$$

Coleman theorem: "a symmetry of the vacuum is the symmetry of the world".

That is $Q_i|0\rangle = 0$ implies, $[Q_i, H] = 0$. The physical state in the spectrum of the H can be classified according to the representation of the chiral group generated by $Q_{R,L}$. Let, ψ be an energy and parity eigenstate,

$$H|\psi\rangle = E|\psi\rangle, \; P|\psi\rangle = |\psi\rangle$$

Consider,

$$HQ_L|\psi\rangle = EQ_L|\psi\rangle, \; HQ_R|\psi\rangle = EQ_R|\psi\rangle,$$

$$PQ_R^L|\psi\rangle = PQ_R^L P^\dagger P|\psi\rangle = Q_L^R|\psi\rangle$$

Therefore,

$$|\psi'\rangle = \frac{1}{\sqrt{2}}(Q_R - Q_L)|\psi\rangle$$

$$P|\psi'\rangle = -|\psi'\rangle, \; P|\psi\rangle = |\psi\rangle$$

Presence of parity degenerate state is not general feature of the hadronic spectrum.
$([\rho(1^-), a_1(1^+)]$ mass in MeV $(770, 1260))$
If massless QCD is a good approximation then,

$$Q_5^a|0\rangle \neq 0 \quad Q^a|0\rangle \neq 0 \tag{144}$$

That is the vacuum is not invariant under the full chiral group. (Pokoroski, Gauge Field Theory).

Dilepton:

QGP:

$$q\bar{q} \rightarrow e^+e^- \tag{145}$$

$$\sigma_q = \frac{20}{3}\tilde{\sigma}(M) \tag{146}$$

where $\tilde{\sigma}(M)$ is the cross section for $e^+e^- \rightarrow \mu^+\mu^-$.

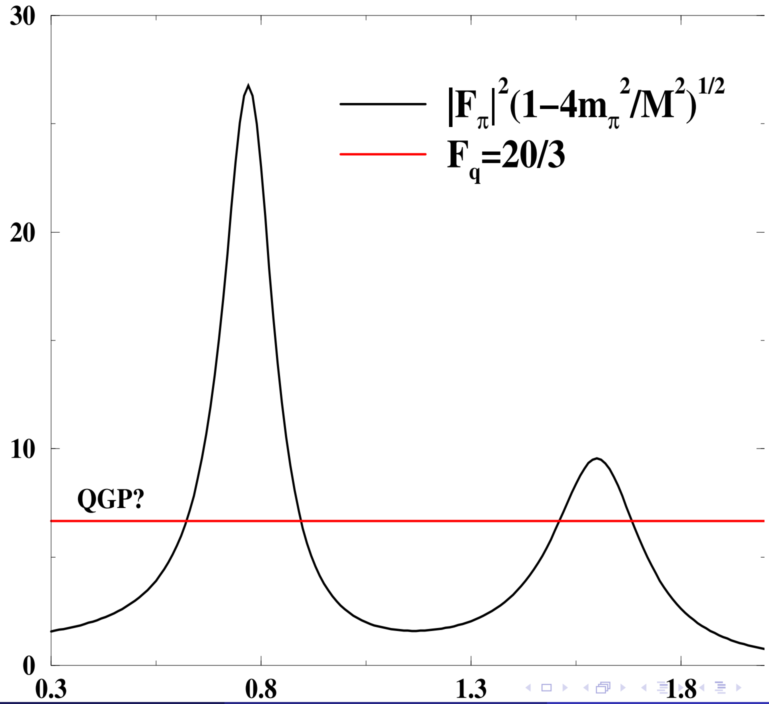
Hadronic:

$$\pi^+\pi^- \rightarrow e^+e^- \tag{147}$$

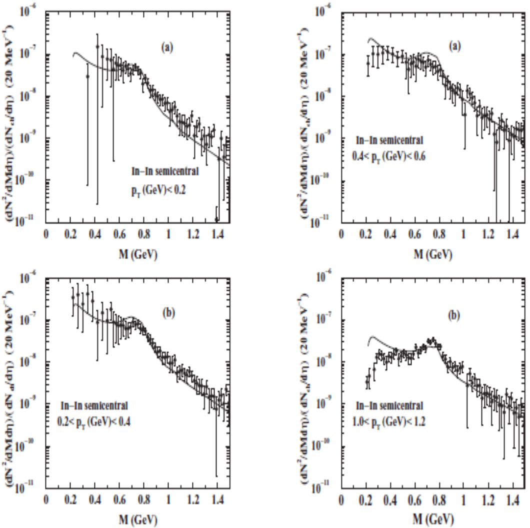
$$\sigma_\pi = |F_\pi(M)|^2(1 - 4\frac{m\_\pi^2}{M^2})^{1/2}\tilde{\sigma}(M) \tag{148}$$

where

$$|F_\pi(M)|^2 = \frac{m_\rho^2}{(m_\rho^2 - M^2)^2 + m_\rho^2\Gamma_\rho^2} \tag{149}$$


30
20
10
0
0.3
0.8
1.3
1.8
$|F_\pi|^2(1-4m_\pi^2/M^2)^{1/2}$
F_q=20/3
QGP?

Muon pairs from In+In collisions @ $\sqrt{s_{NN}}$=17.3 GeV



Data is well described with the broadening of rho meson

Modification of hadronic properties at non-zero temperature and density

Important for chiral symmetry restoration

Hadronic background of QGP signal for vacuum properties of hadrons properties near the QCD phase boundary may be (very) different

Chiral partners: $[\sigma(0^+), \pi(0^-)]$ mass in MeV $(650, 140)$,
$[\rho(1^-), a_1(1^+)]$ mass in MeV $(770, 1260)$,
$[p(1/2^+), N^*(1/2^-)]$ mass in MeV $(938, 1535)$

Can we create a situation where the chiral partner meet each other?

Experimental results on mass shift

• KEK: p (12 GeV) $+$ C, Cu, dilepton data indicate significant shape changes in e^+e^- invariant mass spectrum from $p + C$ to $P + Cu$

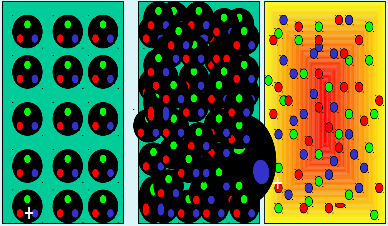
• TAGX: γ (800 - 1200 MeV) $+$ $^3He \longrightarrow m_\rho \sim 642$ MeV

• CHAOS: $\pi^+ A \rightarrow \pi^+ \pi^\pm A'$ at $T_{\pi^+} = 283$ MeV Medium modification is observed in $I = 0$, $J = 0$ (σ meson)

• NA60 Collaboration (Collision of In+In @ $\sqrt{s_{NN}} = 17.3$ GeV. $\Rightarrow$: No significant change in mass but huge broadening has been observed through the invariant mass distribution of $\mu^+\mu^-$ pairs.

QGP at high density and temperature

In 1975, just after the discovery Of the asymptotic freedom (1973), Collins and Perry predict that at high density the properties of nuclear matter is not governed by hadronic but by quarks and gluonic degrees of freedom



A thermalized deconfined state of quarks and gluons is called Quark Gluon Plasma (QGP).

$M = E/c^2 = T/c^2$

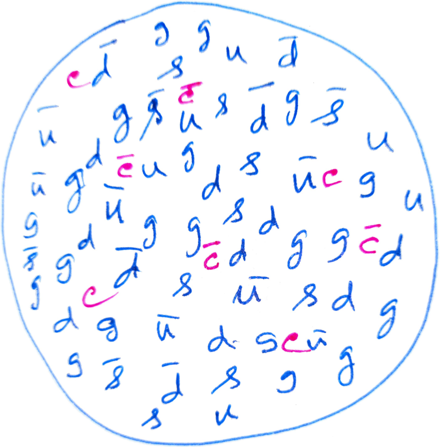
Density required for deconfinement

$\rho_c = \frac{m_N}{4\pi R_N^3/3} = 0.5$ GeV/fm^3 $\rho > (3-4)\rho_0$

Required temperature ~ 154 MeV

J/ψ suppression:

A J/ψ is a bound state (B.S.) of a charm and anti-charm quark.



The abundances of light quarks and gluons are larger than heavy quarks (charm and bottom) due to Boltzmann suppression. Therefore, formation of $D(c\bar{u}, c\bar{d})$, $\bar{D}(u\bar{c}, d\bar{c})$, $D_s(c\bar{s})$ and $\bar{D}_s(s\bar{c})$ is more than $J\psi(c\bar{c})$.

A B.S. will not survive if the screening radius is less than the Bohr radius of the system.

J/ψ are produced in the initial hard scattering. If QGP is formed then the plasma effects will make the J/ψ unbound. Indicating less J/ψ as compared to the case when QGP is not produced. Therefore, J/ψ suppression can be a signal of QGP formation.

$c\bar{c}$ pair formed in $gg \to c\bar{c}$ and $q\bar{q} \to c\bar{c}$ can not bind inside the plasma.

The Energy of the $c\bar{c}$ system is given by

$$E(r) = \frac{1}{2\mu r^2} - \alpha_{eff}\frac{e^{-r/\lambda}}{r} \tag{150}$$

We have used $p \sim 1/r$, A bound state is possible if $E(r)$ has a minimum,

$$\frac{dE}{dr} \Rightarrow x(1+x)e^{-x} = \frac{1}{\alpha_{eff}\mu\lambda_D} \tag{151}$$

where $x = r/\lambda_D$. There will be no bound state if

$$\Rightarrow \frac{1}{\alpha_{eff}\,\mu\lambda_D} > 0.84 \tag{152}$$

For no screening $\lambda_D = \infty$, the Bohr radius of the of the J/ψ is given by

$$\left[\frac{dE}{dr}\right]_{\lambda_D\rightarrow\infty} = 0 \tag{153}$$

$r_{bohr} = 1/(\mu\alpha_{eff})$ According to Eq. 152, there will be no B.S. for $T > 210$ MeV (we have taken $\alpha_{eff} = 0.5$ and $\mu = 1.5$ GeV).
QCD lattice calculation: B.S. will not survive if $T > 1.5T_c$.

Nuclear shadowing, final state absorption in nucleons etc needs to be estimated reliably.

In QGP a c or $\bar{c}$ finds many more u $\bar{u}$, d $\bar{d}$, s and $\bar{s}$ than $\bar{c}$ or c (because $e^{-m_c/T} << e^{-m_s/T} << e^{-m_{u,d}/T}$) in the thermal bath hence it can form $D(c\bar{u}, c\bar{d})$, $\bar{D}(\bar{c}u, \bar{c}d)$, $D_s(c\bar{s})$ and $\bar{D}(\bar{c}s)$ more easily than J/ψ.

Confined medium - pion gas $f(p) \sim e^{-p/T} \;\Rightarrow < p >= 3T$

Gluon distribution inside hadrons $g(x) \sim C(1-x)^r$

$r = 4$ for proton and $r = 3$ for pions, $x = k/p$ where k is the momentum of the gluon inside pion.

$< x >= 1/5 \;\Rightarrow < k >= 3T/5$.

In deconfined medium

For gluons $f \sim e^{-k/T} \;\Rightarrow < k >= 3T$

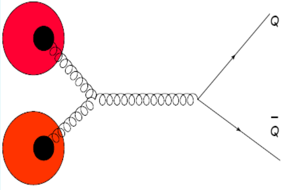
Break-up

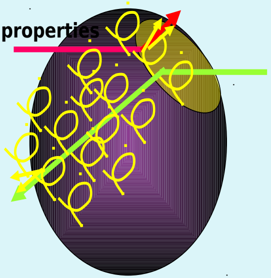
$E_\psi = 2M_D - M_{J/\psi} \sim 640$ MeV. $k > 640$ MeV, implying pion gas should have temperature $3T/5 > 640$ MeV $\Rightarrow T > 1$ GeV.

Heavy quarks as probe of Quark gluon plasma

Dragging heavy flavours in QGP

Why Charm & Beauty? - Early Production
- Do not decide bulk properties





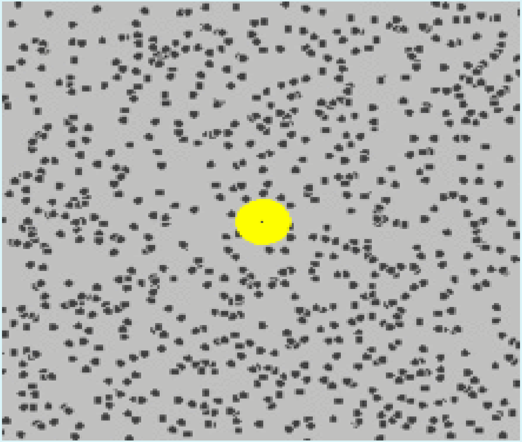
$$g+g\to Q+\bar{Q},\quad g+g\to Q+\bar{Q}+g,$$
$$q+\bar{q}\to Q+\bar{Q},\quad q+\bar{q}\to Q+\bar{Q}+g,$$
$$g+q\to Q+\bar{Q}+q,\quad g+\bar{q}\to Q+\bar{Q}+\bar{q},$$

Relax. Time of HQs ~ (HQ Mass/Temperature) × Relax. Time of Gluons

Evolution of HQs in QGP

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_i}\left[A_i(p)f+\frac{\partial}{\partial p_j}\left[B_{ij}(p)f\right]\right]$$

Motion of heavy quarks in Quark Gluon Plasma ~ Pollen Grains (HQs) in Water (QGP).



Fokker-Planck Equation can be used to describe the evolution of the HQs in the QGP

B. Svetitsky, PRD 1988;
S. Chakraborty and D. Syam, Lett. Nuovo Cim. 1984

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f(x,p,t)=\left(\frac{\partial f}{\partial t}\right)_c$$

- The plasma is uniform ,i.e., the distribution function is independent of x.
- In the absence of any external force, F=0

$$\left(\frac{\partial}{\partial t}\right)f(p,t)=\left(\frac{\partial f}{\partial t}\right)_{c\ c}$$

$$R(p,t)=\left(\frac{\partial f}{\partial t}\right)_c=\int_o d^3k_l\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$ $\longrightarrow$ **is rate of collisions which change the momentum of the charmed quark from p to p-k**

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

Landau Kinetic equation.

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_i}\left[A_i(p)f+\frac{\partial}{\partial p_j}\left[B_{ij}(p)f\right]\right]$$

where we have defined the kernels

$$A_i=\int d^3k\,\omega(p,k)\,k_i \quad \rightarrow \textbf{Drag Coefficient}$$

$$B_{ij}=\int d^3k\,\omega(p,k)\,k_i k_j \quad \rightarrow \textbf{Diffusion Coefficient}$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}\,f'(q)\,v_{q,p}\,\sigma_{p,q\rightarrow p-k,q+k}$$

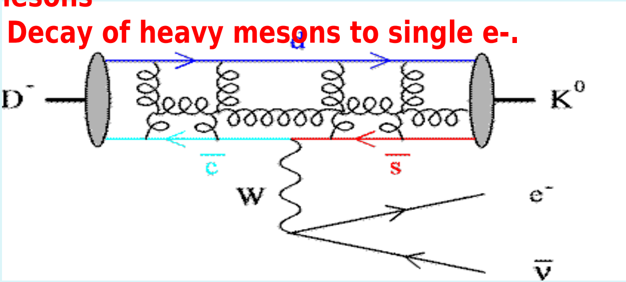
Non -equilibrium Distribution Function $\xrightarrow{\text{replaced}}$ **Equilibrium Distribution Function**

$\Downarrow$

Landau Kinetic Equation $\xrightarrow{\text{reduced}}$ **Fokker Planck Equation**

Inputs

* Initial heavy quark distributions: from pp collisions
* Dissipatative process: collisional, radiative, ...
* c and b fragmentation functions to D, B mesons
* Decay of heavy mesons to single e-.



Drag of Heavy Quarks

$$HQ(p) + QGP(q) \to HQ(p') + QGP(q')$$

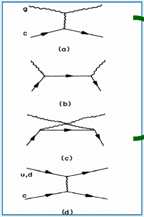
$$\begin{aligned} A_i \; = \; & \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3E_q}f(q)\int\frac{d^3p'}{(2\pi)^3E'_p} \\ & \int\frac{d^3q'}{(2\pi)^3E'_q}\{1\pm f(q')\}\frac{1}{g}\sum\overline{|M|^2} \\ & (2\pi)^4\delta^4(p+q-p'-q')[(p-p')_i] \end{aligned}$$

$$drag \sim \int |M|^2(p-p')....$$

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)B_\perp(p,T) + \frac{p_i p_j}{p^2}B_\parallel(p,T),$$

$$B_\perp(p) = \frac{1}{4}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(\vec{p}.\vec{p}')^2\rangle\rangle}{p^2}\right],$$

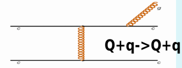
$$B_\parallel(p) = \frac{1}{2}\left[\frac{\langle\langle(\vec{p}.\vec{p}')^2\rangle\rangle}{p^2} - 2\langle\langle\vec{p}.\vec{p}'\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right].$$



$gc \to gc$

Collisional

$qc \to qc$



Radiative

$$F = \frac{k_\perp^2}{\omega^2\theta_0^2 + k_\perp^2} \quad \theta_0 = M/E$$

LPM effects

Insert thermal mass into the internal gluon propagator in the t-channel exchange diagrams to shield the infrared divergence.

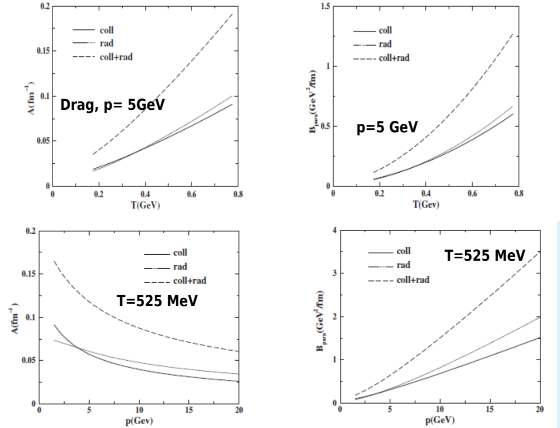
Drag/Diffusion due to collisional and radiative losses

$$|M|^2_{2\to 3} = |M|^2_{2\to 2} \times 12 g_s^2 \frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2y}\right)^{-2},$$

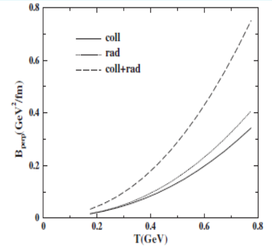
$$X_{\rm eff} = X_{\rm coll} + X_{\rm rad}$$

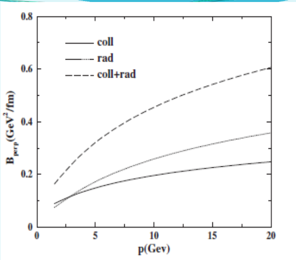
$$X_{\rm rad} = X_{\rm coll} \times \int \frac{d^3k_5}{(2\pi)^3 2E_5} 12 g_s^2 \frac{1}{k_\perp^2} \times \left(1+\frac{M^2}{s}e^{2y}\right)^{-2}[1+\hat{f}(E_5)] \times \theta(\tau-\tau_F)\theta(E_p-E_5).$$

MAZUMDER, BHATTACHARYYA, AND ALAM — PHYSICAL REVIEW D 89, 014002 (2014)



Diffusion due to collisional and radiative losses





Probability current vanishes for del f/delt =0.

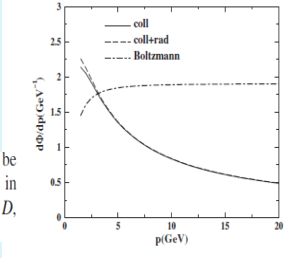
For -

$$f_{eq}^{CQ}(p;T,q) = N \exp[-\Phi(p;T,q)]$$

$$A(p,T) = \frac{1}{p}\frac{d\Phi}{dp}B_{\parallel}(p,T) - \frac{1}{p}\frac{dB_{\parallel}}{dp} - \frac{2}{p^2}[B_{\parallel}(p,T) - B_{\perp}(p,T)].$$

This relation is valid for any momentum of CQ and can be reduced to the well-known Einstein relation $D = \gamma MT$ in the nonrelativistic limit, where $A = \gamma$ and $B_{\perp} = B_{\parallel} = D$, i.e., $B_{ij} = D\delta_{ij}$ and $\Phi = p^2/(2\,\mathrm{MT})$.

MAZUMDER, BHATTACHARYYA, AND ALAM



$$R_{AA} = \frac{f_{final}(p_T, T_C)}{f_{in}(p_T, T_i)}$$

Nuclear modification factor as a function of transverse momentum

Momentum space Diff. coeff.

$$D = \gamma M T$$

γ **=Drag coeff.**

Since

$$x(t) = \int^t p(t')dt'/M$$

and

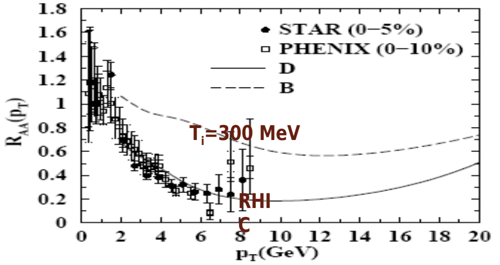
$$\langle x(t)^2 \rangle = 6D_x t$$

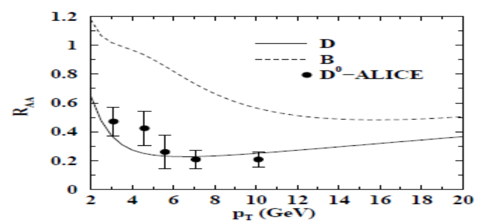
Spatial diff. coeff

$$D_x = \frac{T}{M\gamma}$$

Compare with

$$\lambda = 1/(2\pi T)$$

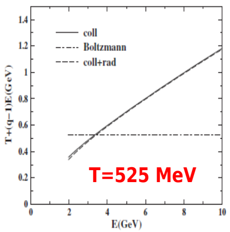


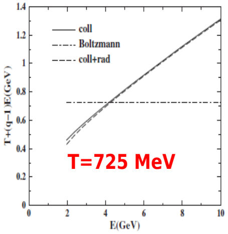


Mazumder, Bhattacharyya, Alam & Das 2012

For Tsallis distribution: $\Phi_{Ts} = \frac{1}{1-q} \ln[1-(1-q)E(p)/T_T],$

$$T_T + (q-1)E = \frac{dE}{dp} \frac{1}{p\frac{A}{B_\parallel} + \frac{1}{B_\parallel}\frac{dB_\parallel}{dp} + \frac{2}{p}(1-\frac{B_\perp}{B_\parallel})}.$$

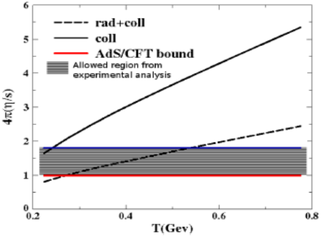




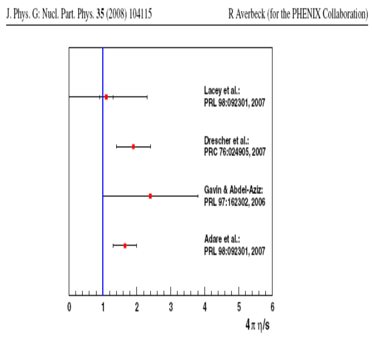
Shear viscosity vs diffusion

$$\eta = \frac{1}{3} n < p > \lambda$$

$$4\pi\frac{\eta}{s} \approx 1.25\pi\frac{T^3}{B_\perp}.$$



Shear viscosity to entropy ratio:



eta/s:
of QGP (RHIC data)

~ of Li^6 atoms [Temperature difference ~10^{19} density differnece 10^{25} with QGP]

~of finite nuclei (Auerbach & Shlomo PRL 2009)

~(1-5)/4pi

Generation of Strong Magnetic Field in Heavy Ion Collision

-Earths magnetic field ~ 0.6 Gauss

-Hand held magnet ~ 100 Gauss

-Super conducting LHC magnet ~ 8.3×10^4 Gauss

- Strongest steady state magnet ~ 4.5×10^5 Gauss

-Surface field of neutron star ~ 10^{12} Gauss

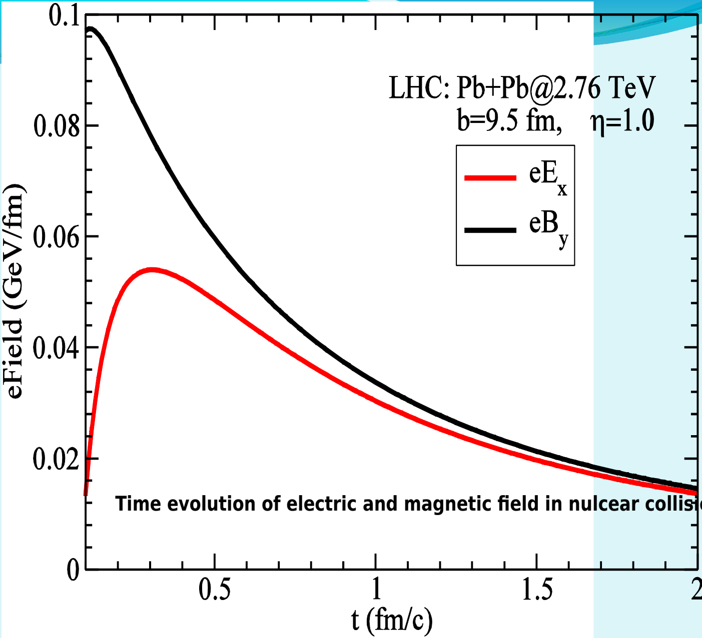
-Critical magnetic field of electron ~ 4×10^{13} Gauss

-Surface field of magnetar ~ 10^{15} Gauss

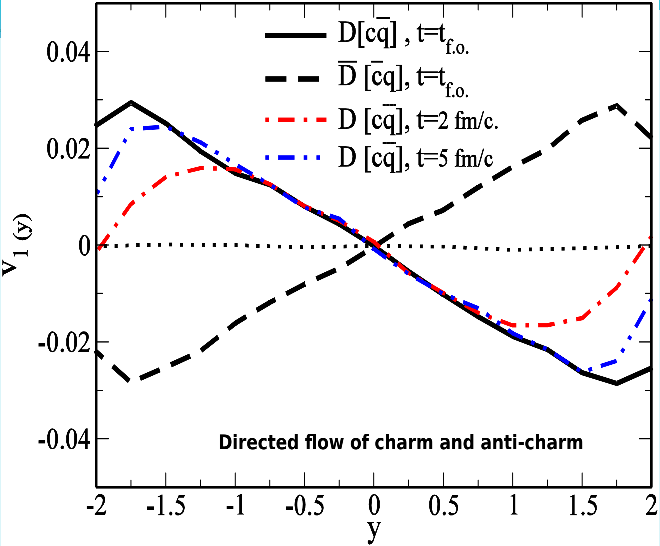
- Heavy ion collisions at RHIC ~ 10^{17} Gauss

-Heavy ion collision at LHC ~ 10^{18} Gauss

(Compiled by K. Itakura)



Time evolution of electric and magnetic field in nulcear collisions



Das, Plumari, Chatterjee, Alam, Scardina and Greco, arXiv: 1608:0223 [nu

QCD critical point

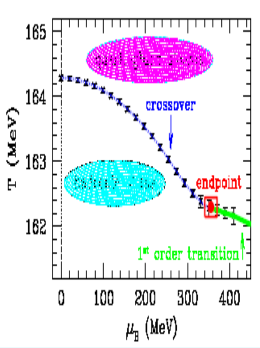
Effects of critical point on the correlation of density fluctuations.

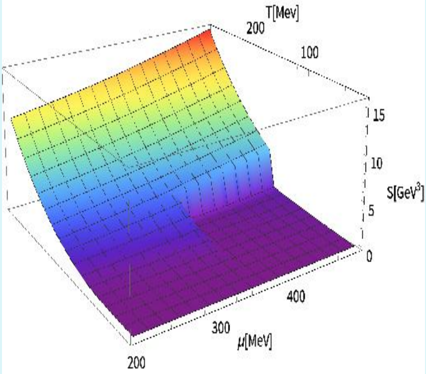
The Equation of State (EoS) of QCD Matter.

The properties of the system in QCD phase diagram is described by T and μ.

The location of critical point at (μ_c, T_c) = (367 MeV, 154 MeV)

Variation of entropy density (S) with μ and T.





The energy momentum and the net Baryon number conservation Eqs. For QGP:

$$\partial_\mu T^{\mu\lambda} = 0, \quad \partial_\mu N^\mu = 0$$

where the energy momentum tensor and the net baryon flux are given by:

$$\begin{aligned} T^{\lambda\mu} &= \epsilon u^\lambda u^\mu + P\Delta^{\lambda\mu} - \frac{1}{3}\zeta\Delta^{\lambda\mu}\partial_\alpha u^\alpha + \frac{1}{9}\zeta\beta_0\Delta^{\lambda\mu}D(\zeta\partial_\alpha u^\alpha) + \frac{\zeta\alpha_0}{3}\Delta^{\lambda\mu}\partial_\alpha\left\{\frac{n\kappa T^2}{\epsilon+P}\nabla^\alpha(\alpha)\right\} \\ &- 2\eta\Delta^{\lambda\mu\alpha\beta}\partial_\alpha u_\beta + 4\eta\beta_2\Delta^{\lambda\mu\alpha\beta}D(\eta\Delta^{\rho\sigma}_{\alpha\beta}\partial_\rho u_\sigma) + 2\alpha_1\eta\Delta^{\lambda\mu\alpha\beta}\partial_\alpha\left\{\frac{n\kappa T^2}{\epsilon+P}\nabla_\beta(\alpha)\right\} \end{aligned}$$

The full charge current (up to second-order in velocity gradient) can be written as,

$$\begin{aligned} N^\mu &= nu^\mu - \frac{n\kappa T}{(\epsilon+P)}\Big[\frac{nT}{(\epsilon+P)}\nabla^\mu\alpha - \beta_1\Delta^{\mu\nu}D\left\{\frac{n\kappa T^2}{(\epsilon+P)}\nabla_\nu\alpha\right\} - \frac{\alpha_0}{3}\nabla^\mu(\zeta\partial_\alpha u^\alpha) \\ &- 2\alpha_1\Delta^{\mu\nu}\partial^\rho(\eta\Delta^{\alpha\beta}_{\rho\nu}\partial_\alpha u_\beta)\Big] \end{aligned}$$

Introduce a small perturbation in the QGP and observe its propagation in presence of critical point (Hasanujjaman et. al. PRC, in press). The dispersion relation for the wave propagating in QGP is given by:

$$a\omega^3 + b\omega^2 + c\omega = 0 \quad \rightarrow \quad \omega(a\omega^2 + b\omega + c) = 0$$

has complex roots.

Waves with wave vector more than k_{th} will dissipate: $\left|\frac{\omega_{\mathfrak{Im}}(k)}{\omega_{\mathfrak{Re}}(k)}\right|_{k=k_{th}} = 1$

The corresponding threshold wavelength is used to define the fluidity,

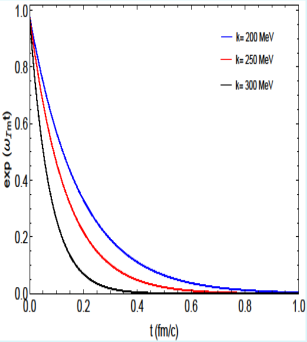
$$\mathcal{F} \sim \frac{\lambda_{th}}{l}.$$

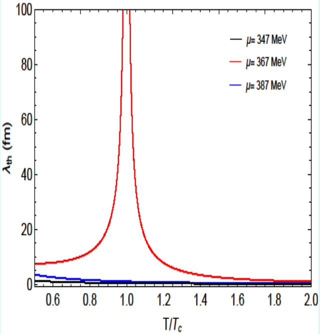
where $l \sim n^{-1/3}$ or s-1/3 [s(n) is entropy (net baryon) density].

Propagation of sound wave in QGP

The sound wave propagating in QGP dissipates. Waves with larger k or smaller wave length dissipate more.

At the critical point (T/Tc=1 and μ=367 MeV). The threshold diverges. The dimensional quantity, $F=\lambda_{th/l}$ called fluidity does also diverge.





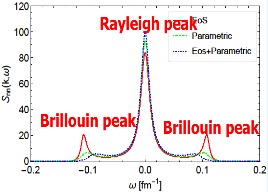
The dynamical spectral structure of density fluctuation evaluated in 2nd order Israel-Stewart dissipative hydrodynamics.

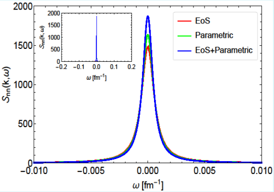
$$S'_{nn}(k,\omega) = \left[M_{11}^{-1} - n_0\left(\frac{\partial s}{\partial n}\right)_T M_{16}^{-1}\right]\langle\delta n(k,0)\delta n(k,0)\rangle$$

$$S_{nn}(k,\omega) = \frac{S'_{nn}(k,\omega)}{\langle\delta n(k,0)\delta n(k,0)\rangle}$$

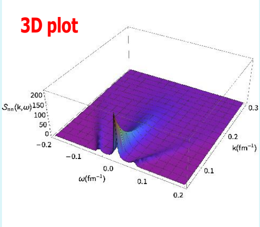
Hasanujjaman et al
arXiv:2008.03931

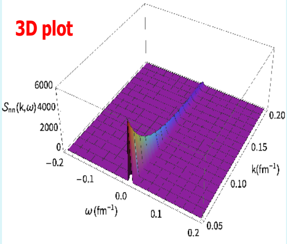
Rayleigh peak is due to entropy fluctuation at constant pressure.
Brillouin peaks are due to pressure fluctuation at constant entropy.





At the critical point the Brillouin peaks merge with Rayleigh peak





Why ....?

Relevant for:

-Early Universe Cosmology

-Compact Astrophysical objects (Neutron Star)

Offers Opportunity to Study:

-Condensed Matter Physics not of Atoms but of Elementary Particles

-Non-abelian Field Theory (QCD) in Thermal Bath

High Temperature & Density – Phase Transition (Chiral/Deconfinement) - Restoration/breaking of symmetries.

Nuclear Collisions at Relativistic Energies – tool to create matter at ultra-high temperature (Early Universe) & density (Neutron Star)

Matter at highest temperature, highest density and lowest viscosity/entropy ratio under the influence of highest electromagnetic field detected so far in the universe.

Why ....?

Relevant for:

-Early Universe Cosmology

-Compact Astrophysical objects (Neutron Star)

Offers Opportunity to Study:

-Condensed Matter Physics not of Atoms but of Elementary Particles

-Non-abelian Field Theory (QCD) in Thermal Bath

High Temperature & Density – Phase Transition (Chiral/Deconfinement) - Restoration/breaking of symmetries.

Nuclear Collisions at Relativistic Energies – tool to create matter at ultra-high temperature (Early Universe) & density (Neutron Star)

Matter at highest temperature, highest density and lowest viscosity/entropy ratio under the influence of highest electromagnetic field detected so far in the universe.