

Electroweak Unification and the Standard Model

Lecture 2

Sreerup Raychaudhuri

TATA INSTITUTE OF FUNDAMENTAL RESEARCH
Mumbai, India

The Lagrangian density

$$\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$$

is manifestly invariant under a **global** U(1) gauge transformation

$$\psi(x) \rightarrow \psi'(x) = e^{-ie\theta} \psi(x)$$

where θ is an arbitrary (real) constant and e is a (real) constant specific to the field...

Easy to show that the Nöther current corresponding to this symmetry is the Dirac current $J^\mu = e \bar{\psi} \gamma^\mu \psi$ and the Nöther charge is just

$$Q = \int d^3\vec{x} j^0 = e \int d^3\vec{x} \bar{\psi} \gamma^0 \psi = e \int d^3\vec{x} \psi^\dagger \psi = e$$

For local U(1) gauge invariance, replace $\mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi$ by

$$\mathcal{L} = i \bar{\psi} \gamma^\mu D_\mu \psi - m \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where, as before, $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Expanding the covariant derivative, we get

$$\begin{aligned} \mathcal{L} &= i \bar{\psi} \gamma^\mu \partial_\mu \psi - e \bar{\psi} \gamma^\mu \psi A_\mu - m \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &= \underbrace{i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi}_{\text{free fermion}} - \underbrace{\frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\text{free e.m.}} - \underbrace{e \bar{\psi} \gamma^\mu \psi A_\mu}_{-eJ^\mu A_\mu} \end{aligned}$$

We will also get Maxwells' equations: $\partial_\mu F^{\mu\nu} = eJ^\nu$

Quantum Electrodynamics (QED)

Once we have Maxwell's equations, we can write

$$\partial_\mu F^{\mu\nu} = eJ^\nu$$

or,

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = eJ^\nu$$

or,

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = eJ^\nu$$

or,

$$\square A^\nu - \partial^\nu (\partial_\mu A^\mu) = eJ^\nu$$

Choose the Lorentz gauge $\partial_\mu A^\mu = 0$ and we recover

$$\square A^\nu = eJ^\nu$$

In static limit, this leads to Coulomb's law and a long-range interaction

Can the photon have a mass?

Then we would have a Klein-Gordon equation: $(\square + M_\gamma^2) A^\nu = eJ^\nu$

coming from a Maxwell equation: $\partial_\mu F^{\mu\nu} + M_\gamma^2 A^\nu = eJ^\nu$

If this is the Euler-Lagrange equation, the Lagrangian density must have an extra *mass term*

$$\mathcal{L}_M = \frac{1}{2} M_\gamma^2 A^\nu A_\nu$$

Under a gauge transformation, $A_\nu \rightarrow A_\nu + \partial_\nu \theta$, and it follows that

$$\mathcal{L}_M \rightarrow \frac{1}{2} M_\gamma^2 (A^\nu + \partial^\nu \theta)(A_\nu + \partial_\nu \theta) \neq \frac{1}{2} M_\gamma^2 A^\nu A_\nu$$

For gauge invariance, we must set $M_\gamma = 0$, i.e. the photon must be massless

gauge invariance \Leftrightarrow long range interactions

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gauge invariance \Leftrightarrow long range interactions

The electromagnetic interaction is not always long-range...

Consider a **superconductor**:

at $T < T_c$, it exhibits the following behaviour:

1. **perfect conductor**, i.e. electric field is thrown out (K.-Onnes)
2. **perfect diamagnet**, i.e, magnetic field is thrown out (Meissner)

i.e. no component of the electromagnetic field can propagate inside a superconductor...

However, at the edges, the fields fall off exponentially:



i.e. $A^\mu \sim e^{-x/\ell} \sim e^{-Mx}$

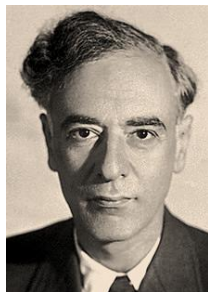
ℓ is the penetration depth ; $M = \ell^{-1}$ is the mass
 \Rightarrow the photon is **massive** inside a superconductor

Somehow, the local $U(1)$ gauge symmetry breaks down inside the superconductor...

Do we understand this phenomenon? **Yes.**

- It was first explained by Landau and Ginzburg in 1937 for a non-relativistic theory (which applies to superconductors).
- It was extensively applied in condensed matter systems by Philip Anderson in the 1950's
- It was worked out for a relativistic theory by Englert & Brout (1964) and independently, by Peter Higgs (1964).

The phenomenon is called **spontaneous symmetry-breaking**, but a better name (Coleman) is **hidden symmetry**...



Lev Landau
(1908 – 1968)
– pioneering work
in QM and QFT



Vitaly Ginzburg
(1916 – 2009)
– superconductivity
and plasma theory

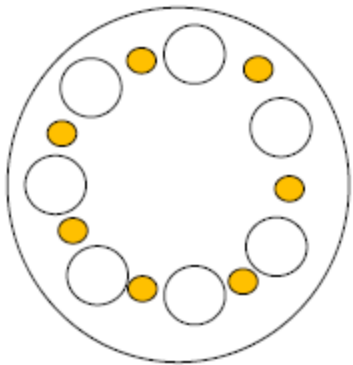


Philip Anderson
(1923 – 2020)
– pioneer of
condensed matter
physics

Hidden symmetry

This arises when the action of a system has a particular symmetry, **but the ground state does not...**

Example 1: Salam's banquet



People are sitting to dinner at a round table. Each has a plate in front and a glass on either hand.

Before the meal, there is perfect symmetry between left glasses and right glasses.

The first person to pick up a glass makes a random choice, say the left glass...

...now everyone must pick up the left glass...

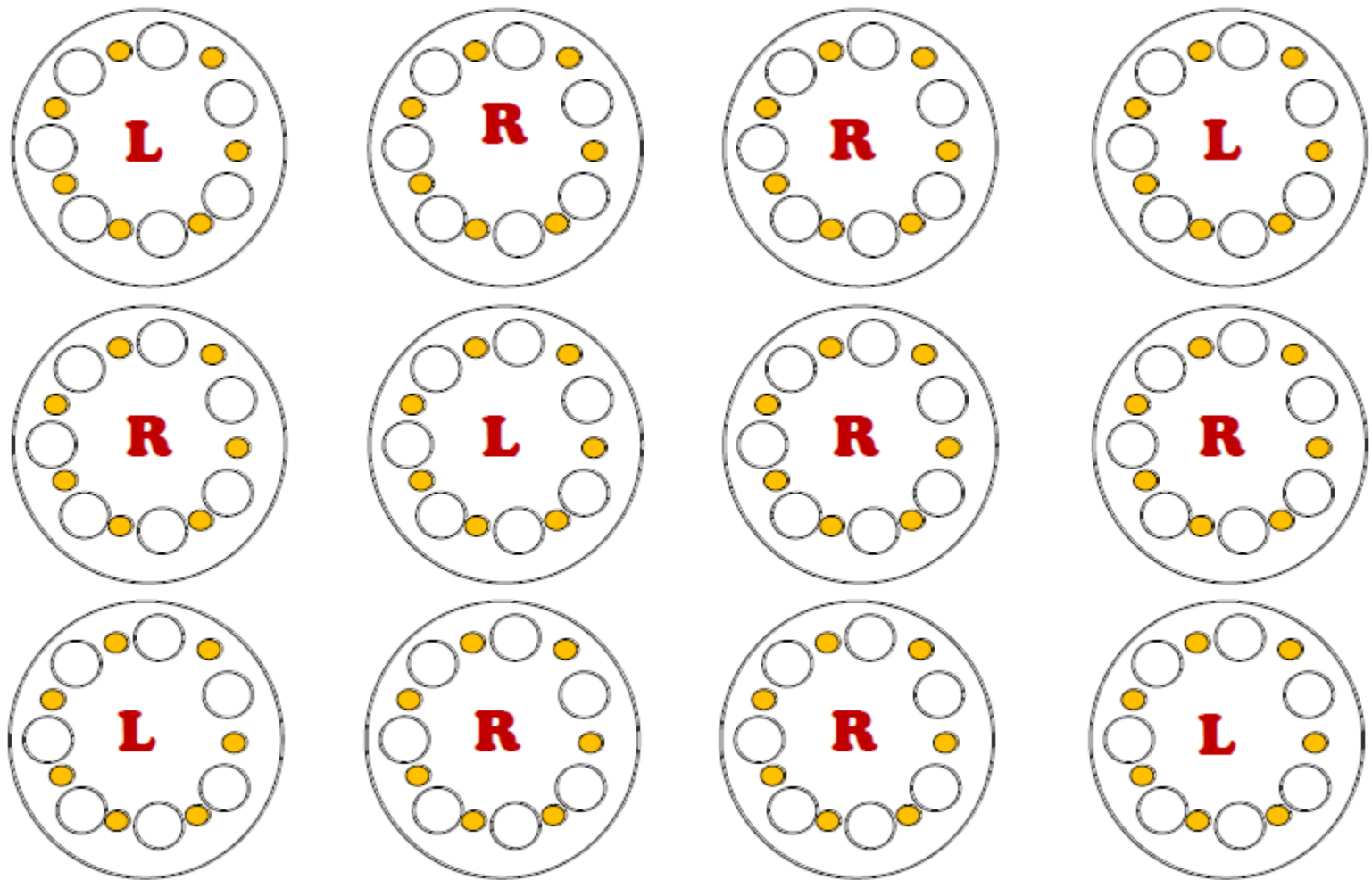
During the meal, there is no symmetry...



Abdus Salam (1926 – 1996)
– electroweak unification

Has the symmetry really been destroyed?

No: if we consider an ensemble of systems...

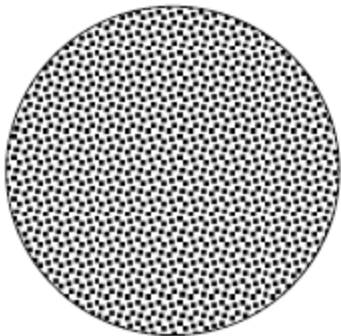


the symmetry reappears!

Hidden symmetry

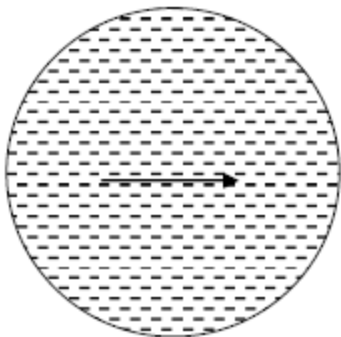
This arises when the action of a system has a particular symmetry, **but the ground state does not...**

Example 2: Ferromagnet below Curie temperature



Above the Curie temperature, all the domains are in **random directions...** **obeys rotation invariance...**

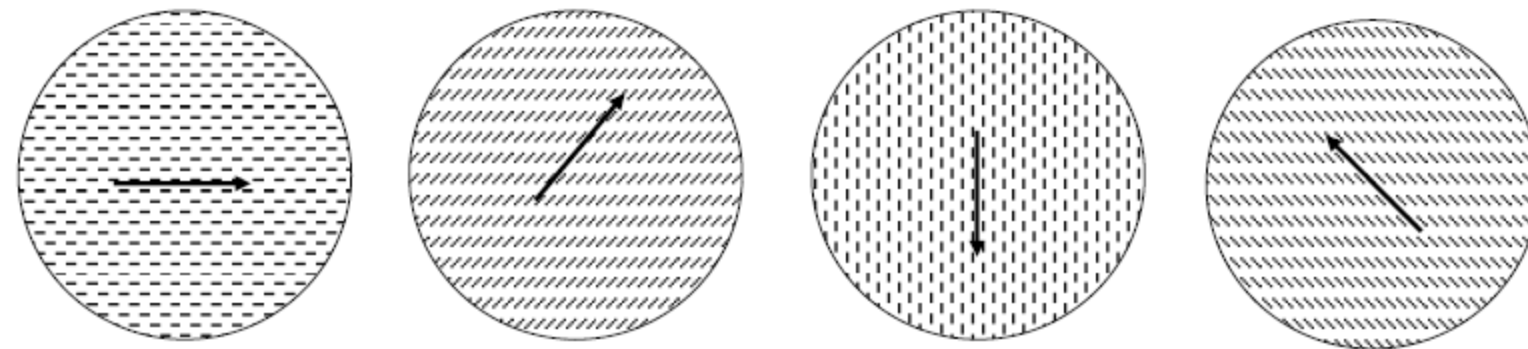
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Below the Curie temperature, all the domains are aligned parallel to a **particular direction...** magnetic field measurement will show a preferred direction, i.e. **rotation invariance is lost**

Has the symmetry really been destroyed?

No: if we consider an ensemble of systems... the symmetry reappears!



If we confine ourselves to the inside of a ferromagnet (Coleman's demon), then rotation invariance will always be violated...



Sidney Coleman
(1937 – 2007)

This is always associated with a phase transition:

i.e. at some high temperature, the symmetry exists
at low temperature the symmetry disappears

in between a flip occurs... critical temperature... \Rightarrow phase transition

How does the superconducting phase break the electromagnetic U(1) gauge invariance?

We discuss the relativistic model, because we will apply the same idea to particle physics problems...

Imagine the interior of the superconductor to have, in addition to the electromagnetic field, a charged scalar field $\varphi(x)$. We have already seen that this leads to a Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - M^2 \varphi^* \varphi$$

In addition to this, let the scalar field have a **self-interaction term**

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - M^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

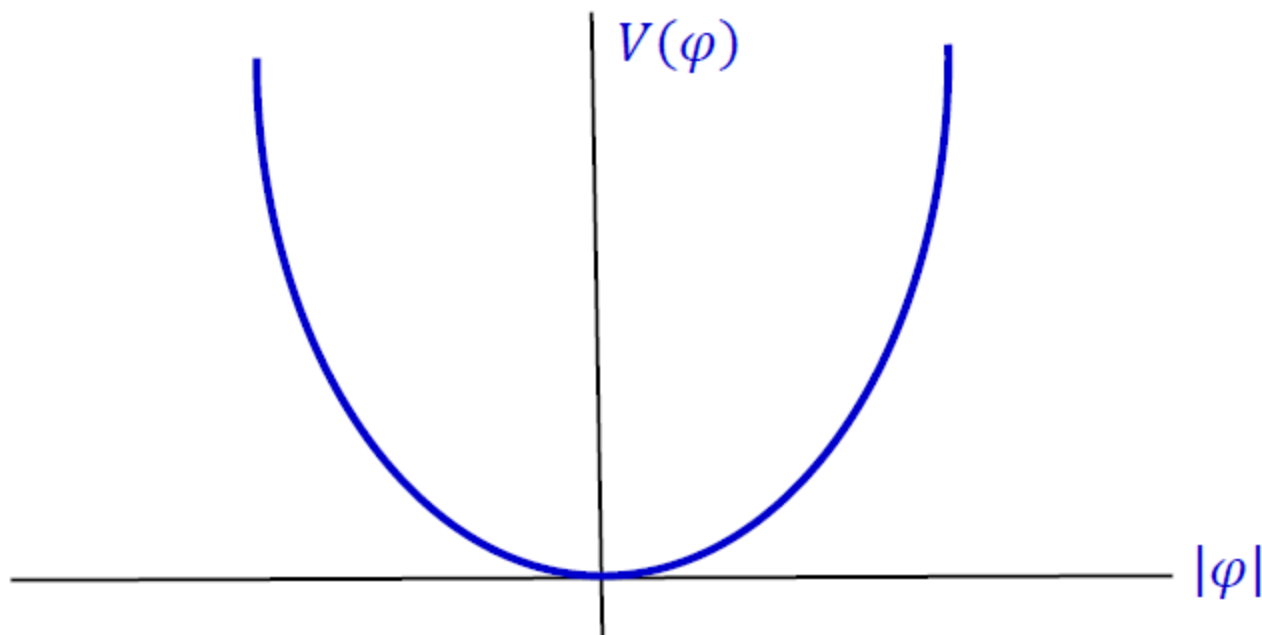
The last two terms can then be thought of as a gauge-invariant potential, i.e. we rewrite

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - V(\varphi)$$

where

$$V(\varphi) = M^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 = M^2 |\varphi|^2 + \lambda |\varphi|^4$$

If we plot this potential as a function of $|\varphi|$, we will get



with a minimum at $|\varphi| = 0$, i.e. at $\varphi = 0$. No symmetries are broken.

But now, let us consider **another variant** of this theory, viz.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] + M^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2$$

If we try to treat $+M^2 \varphi^* \varphi$ as a mass term, the scalar particle will become a tachyon. Don't try this. Just let $+M^2 \varphi^* \varphi$ be an interaction term. Now, rewrite

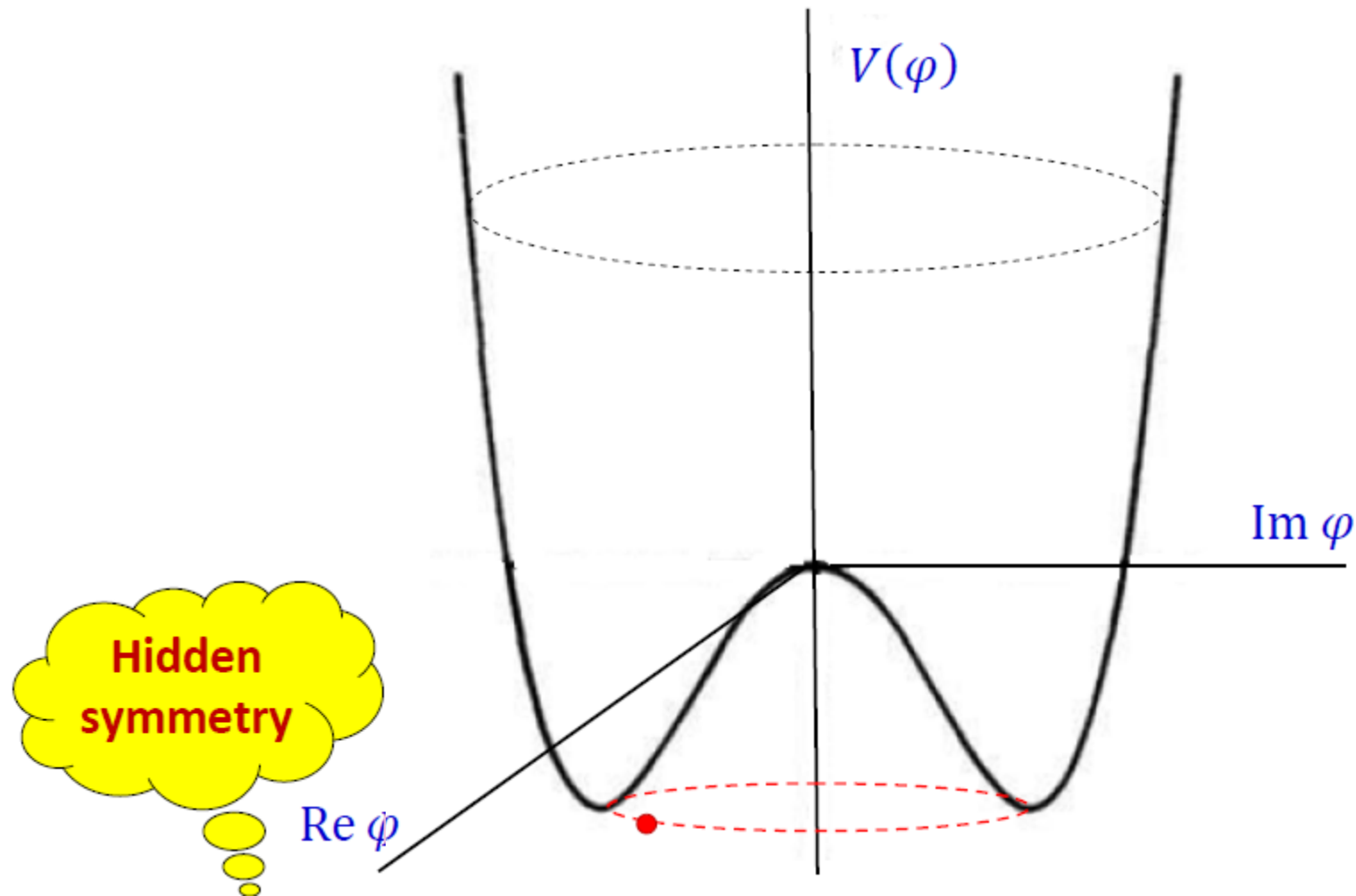
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - V(\varphi)$$

where $V(\varphi) = -M^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 = -M^2 |\varphi|^2 + \lambda |\varphi|^4$

This clearly has extrema at

$$\frac{\partial V}{\partial |\varphi|} = 0 \quad \Rightarrow \quad |\varphi| = 0 \text{ (max)}, \sqrt{\frac{M^2}{2\lambda}} \text{ (min)} \equiv \frac{v}{\sqrt{2}}$$

Thus, there are an infinite number of possible ground states $\varphi = v e^{i\alpha}$



Only one of these can be the true ground state... as in a ferromagnet

Let us orient the axes in the complex φ plane such that the ground state (wherever it is) falls along the real axis.

(Just a convenient parametrisation – like choosing the z-axis along a constant magnetic field)

The ground state is now $\varphi_0 = \frac{v}{\sqrt{2}}$. To construct a viable field theory we must expand around this ground state, i.e. $\varphi(x) = \varphi_0 + \varphi'(x)$.

Rewrite the Lagrangian density in terms of this new field $\varphi'(x)$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu(\varphi_0 + \varphi')]^* [D_\mu(\varphi_0 + \varphi')] - V(\varphi_0 + \varphi')$$

Calculate the terms one by one:

$$D_\mu(\varphi_0 + \varphi') = (\partial_\mu + ieA_\mu)(\varphi_0 + \varphi') = \partial_\mu \varphi' + ieA_\mu \varphi' + ie\varphi_0 A_\mu$$

It follows that

$$\begin{aligned}
 & [D^\mu(\varphi_0 + \varphi')]^* [D_\mu(\varphi_0 + \varphi')] \\
 &= [\partial_\mu \varphi' + ieA^\mu \varphi' + ie\varphi_0 A^\mu]^* [\partial_\mu \varphi' + ieA_\mu \varphi' + ie\varphi_0 A_\mu] \\
 &= [\partial_\mu \varphi'^* - ieA^\mu \varphi'^* - ie\varphi_0 A^\mu] [\partial_\mu \varphi' + ieA_\mu \varphi' + ie\varphi_0 A_\mu] \\
 &= \dots + e^2 \varphi_0^2 A^\mu A_\mu
 \end{aligned}$$

Recall $\mathcal{L}_M = \frac{1}{2} M_\gamma^2 A^\nu A_\nu$

Inside a superconductor with a potential as assumed here, **the photon has become massive!**

$$M_\gamma = \sqrt{2} e \varphi_0 = \sqrt{2} e \frac{v}{\sqrt{2}} = ev$$

Since $v = \sqrt{M^2/\lambda}$, M_γ is a manifestation of the scalar self-interactions...

Another miracle...

$$\begin{aligned}
 V(\varphi_0 + \varphi') &= -M^2(\varphi_0 + \varphi')^*(\varphi_0 + \varphi') + \lambda[(\varphi_0 + \varphi')^*(\varphi_0 + \varphi')]^2 \\
 &= -M^2\varphi'^*\varphi' + \dots + \lambda[2\varphi_0^2\varphi'^*\varphi' + (2\varphi_0\text{Re}\varphi')^2 + \dots] \\
 &= (-M^2 + 2\lambda\varphi_0^2)[(\text{Re } \varphi')^2 + (\text{Im } \varphi')^2] + 4\lambda\varphi_0^2(\text{Re } \varphi')^2 + \dots
 \end{aligned}$$

Now, $\varphi_0^2 = M^2/2\lambda$

It follows that

$$\begin{aligned}
 V(\varphi_0 + \varphi') &= +4\lambda\varphi_0^2(\text{Re } \varphi')^2 + \dots \\
 &= +\frac{1}{2}4M^2(\text{Re } \varphi')^2 + \dots
 \end{aligned}$$

U(1) symmetry is broken
– both global and local

i.e.

$\text{Re } \varphi'$ has a (real) mass $2M$

$\text{Im } \varphi'$ is massless \Rightarrow Goldstone boson

Goldstone theorem (1962):

To every spontaneously broken continuous global symmetry, there corresponds a massless boson

How to get rid of this massless boson (would induce new long-range interactions otherwise)?

Englert & Brout (1964), Higgs (1964):

Can be done if it is a local symmetry...



Jeffrey Goldstone
(1933 –)

Idea is very simple: parametrise $\varphi(x) = \eta(x) e^{i\xi(x)}$ (polar form)

Consider the unbroken (i.e. gauge invariant) Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + [D^\mu \varphi]^* [D_\mu \varphi] - V(\varphi)$$

where $V(\varphi) = -M^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 = -M^2 |\varphi|^2 + \lambda |\varphi|^4$

At this level, we are free to make any gauge choice we wish...

Make a gauge transformation

$$\varphi(x) \rightarrow e^{-ig\theta(x)}\varphi(x) = \eta(x) e^{i[\xi(x)-g\theta(x)]}$$

We might as well choose a special gauge, since the gauge symmetry is going to be broken anyway...

Choose the gauge function $\theta(x)$ such that

$$\xi(x) - g\theta(x) = 0$$

This is called the **unitary gauge**.

In this gauge, $\varphi(x) = \eta(x)$ and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + [D^\mu\eta]^*[D_\mu\eta] - V(\eta)$$

where $V(\eta) = -M^2\eta^2 + \lambda\eta^4$

The ground state is still at $v/\sqrt{2}$ so we must shift

$$\eta = \frac{v}{\sqrt{2}} + \eta'$$

This will lead to

1. $\mathcal{L}_M = \frac{1}{2} M_\gamma^2 A^\nu A_\nu$ with $M_\gamma = ev$
2. $V(\varphi_0 + \varphi') = +\frac{1}{2} 4M^2 \eta^2 + \dots$ i.e. $M_\eta = 2M$
3. and there is no Goldstone boson...

if we had kept $\xi(x)$ it would have been the Goldstone boson...

Looks like magic!!

How can a degree of freedom of the system vanish?

In the unitary gauge, the photon is massive,
i.e. it has three polarisations.

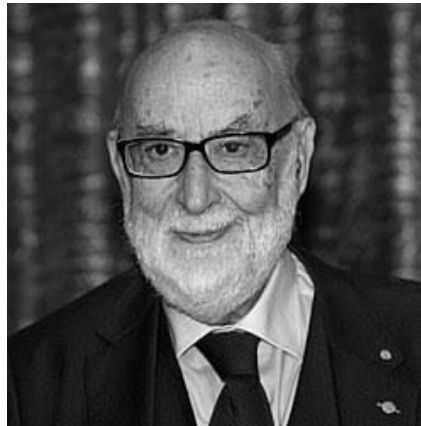
The extra degree of freedom (longitudinal polarisation) which appears here is at the cost of the disappearance of the Goldstone degree of freedom...



Higgs mechanism



Robert Brout
(1928 – 2011)



Francois Englert
(1932 –)



Peter Higgs
(1929 –)



Gerald Guralnik
(1936 – 2014)



Carl Hagen
(1937 –)



Tom Kibble
(1932 – 2016)



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