Electroweak Unification and the Standard Model

Lecture 3

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Why is all this relevant?

Weak Interactions

Pauli's neutrino hypothesis

Physikalisches Institut
der Eidg. Technischen Hochschule
Gloriastr.
Zürich

Zürich, 4 December 1930

Dear Radioactive Ladies and Gentlemen,
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the 'wrong' statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the 'exchange theorem' of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have the spin $\frac{1}{2}$ and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses.—The continuous β-spectrum would then become understandable by the assumption that in β-decay, a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant……

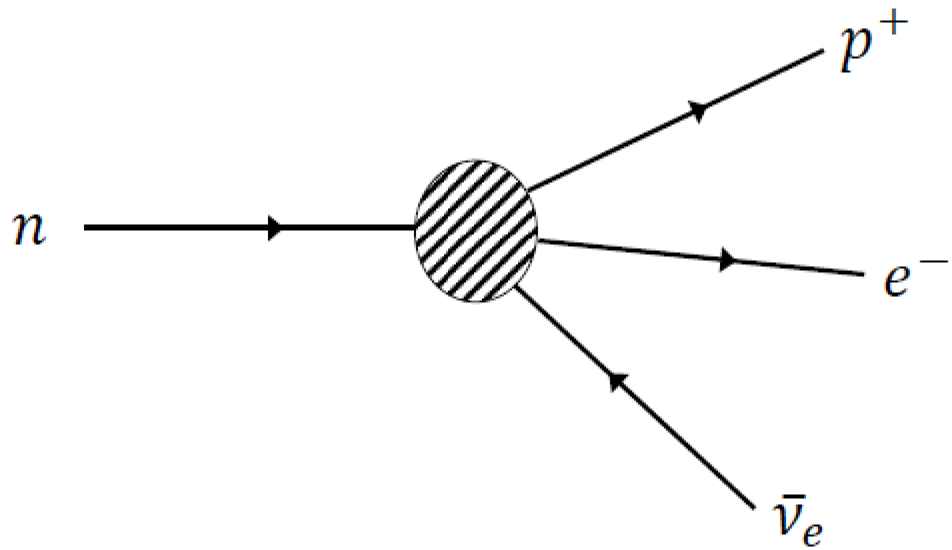
So, dear Radioactives, examine and judge it.—Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December.—With my best regards to you, and also Mr Back, your humble servant,

W Pauli

Wolfgang Pauli
(1900 – 1958)

Fermi's theory of beta decay

$$^{1}_{0}n \;\rightarrow\; ^{1}_{1}p + ^{0}_{-1}e + ^{0}_{0}\bar{\nu}_e$$



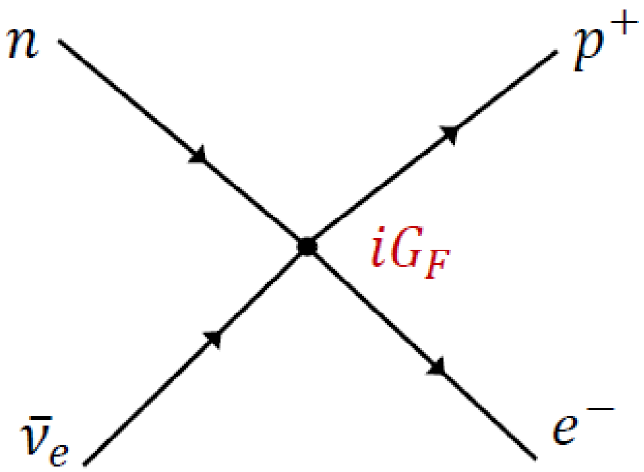
The decay must take place through weak interactions ($\tau = 887$ s).

Can we write down an interaction vertex?

First attempted by Fermi (1934)

Denote the Dirac fields: $\Psi_n = n, \;\; \Psi_p = p, \;\; \Psi_e = e$ and $\Psi_{\bar{\nu}_e} = \bar{\nu}_e$

Fermi's first attempt: try a four-fermion vertex



Weak interaction Hamiltonian:

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\,\overline{p}n\;\bar{e}\nu_e$$

dimension of G_F is M^{-2} : Fermi coupling constant

Simplest possible form of a four-fermion coupling

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 4G_F^2 M_n M_p E_e^2 (1 - \cos\theta_{e\bar{\nu}})$$

i.e. the electron and the antineutrino should tend to come out back-to-back...

Actual experiment showed that, instead, the electron and the antineutrino tended to come out *in the same direction*!

More as if we have $|\mathcal{M}|^2 \propto (1 + \cos\theta_{e\bar{\nu}})$

Fermi's second attempt: try a vertex modelled on e.m. interactions,

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}\, \overline{p}\gamma^{\mu} n \;\; \bar{e}\gamma_{\mu}\nu_e \; = \; \frac{G_F}{\sqrt{2}}\, J^{\mu}_{\text{had}}\; J^{\text{lep}}_{\mu}$$

Current-current form of the weak interaction

With this interaction, the probability for the transition, in the rest-frame of the neutron, comes out to be

$$|\mathcal{M}|^2 \approx 8 G_F^2 M_n M_p E_e^2 (1 + \cos\theta_{e\bar{\nu}})$$

Total decay width (rough estimate):

$$\Gamma_\beta \approx \frac{G_F^2\,\Delta^5}{80\,\pi^3} \approx \frac{1}{887\text{ s}} \qquad \text{where} \quad \Delta = M_n - M_p$$

From this we can estimate

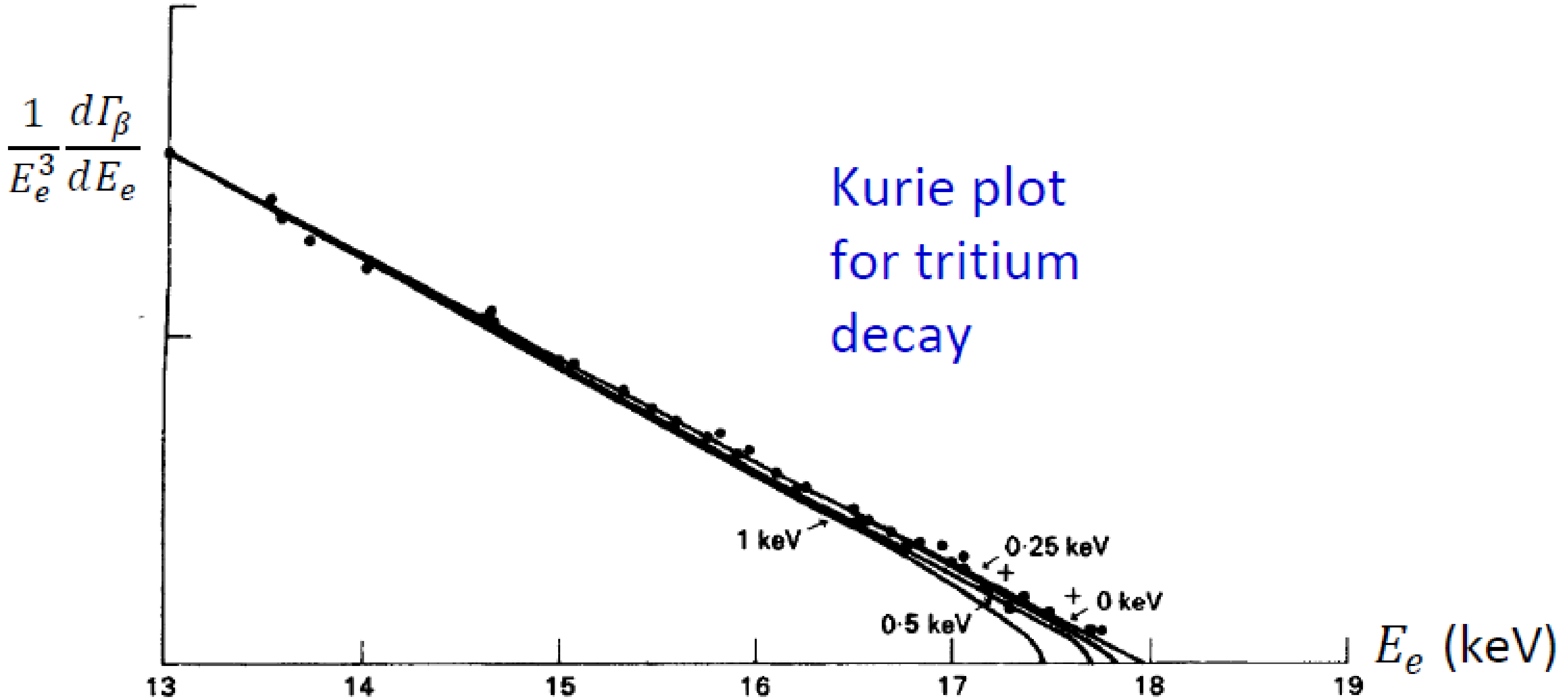
$$G_F \approx 1.8 \times\; 10^{-5} \;\; \text{GeV}^{-2}$$

Given the crudeness of the approximation, this is not a bad estimate...

Current value: $G_F = 1.166 \times\; 10^{-5} \;\; \text{GeV}^{-2}$

More important:

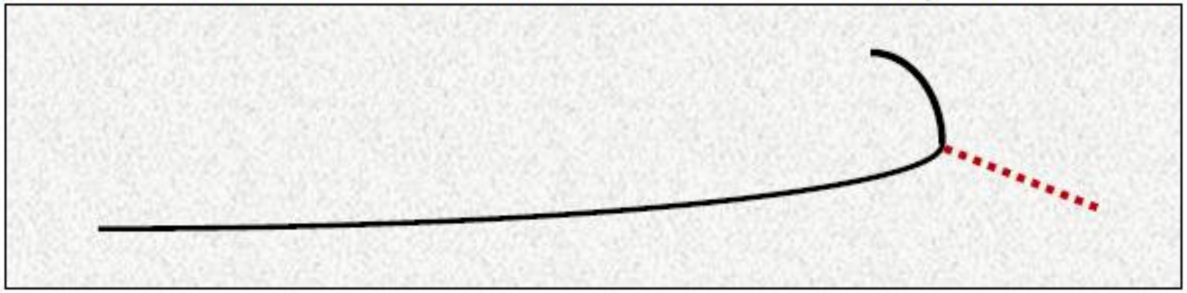
$$\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3} E_e^3 \left[\Delta - E_e\right] \quad \Rightarrow \quad \frac{1}{E_e^3}\frac{d\Gamma_\beta}{dE_e} \approx \frac{G_F^2}{2\pi^3} (\Delta - E_e)$$



Kurie plot for tritium decay

Fermi's theory is spectacularly successful in explaining beta energy spectrum

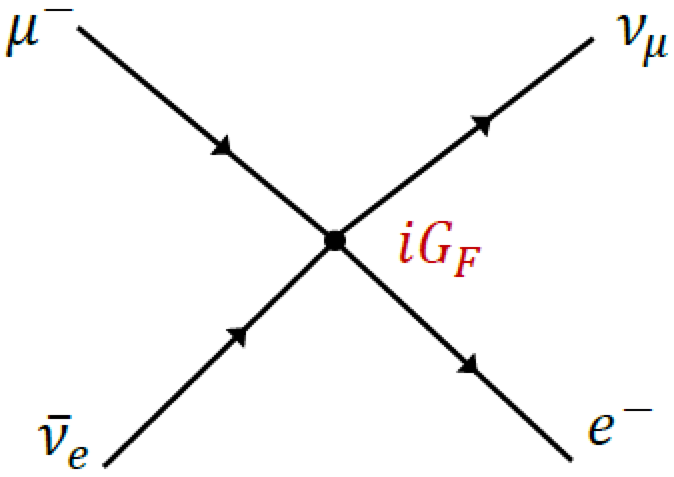
In 1937, the muon was discovered… it decays to electron…



Decay must be through weak interactions (tracks are seen)…

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

Fermi's guess: universality of weak interactions



Use this to calculate the muon lifetime:

$$\tau_\mu \approx \frac{192\,\pi^3}{G_F^2\,M_\mu^5} \approx 2.25 \times 10^{-6}\ \mathrm{s}$$

Spectacular agreement with the experimental value 2.197×10^{-6} s

Vindicates Fermi's hypothesis about universality of weak interactions... today we have many more proofs...

Interestingly, Fermi could have written several forms of the interaction, e.g.

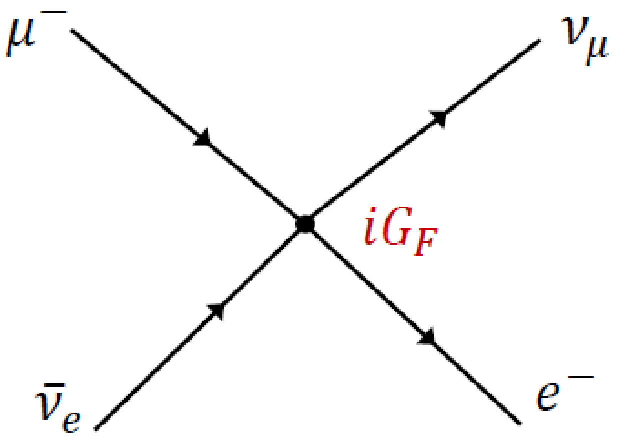
$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} \, \overline{p} \gamma^\mu \gamma_5 n \;\; \bar{e} \gamma_\mu \gamma_5 \nu_e \quad \text{or} \quad \mathcal{H}_I = \frac{G_F}{\sqrt{2}} \, \overline{p} \sigma^{\mu\nu} n \;\; \bar{e} \sigma_{\mu\nu} \nu_e$$

The choice of the vector-vector form turned out to be a stroke of genius, for that is exactly what we predict in the gauge theory of weak interactions – which is what the Fermi theory ultimately leads to...

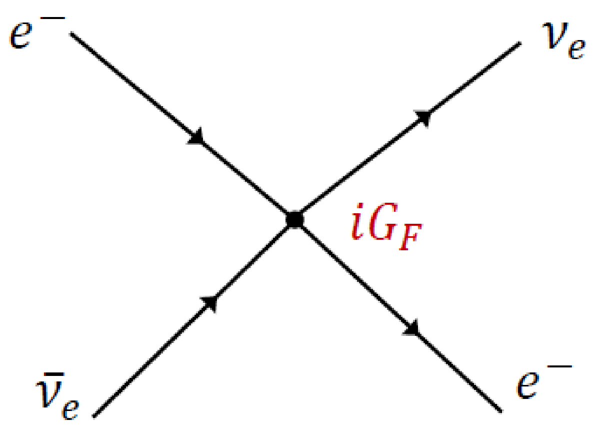
Enrico Fermi (1901 – 1954) was a pioneer of quantum mechanics and is credited with the discovery of Fermi-Dirac statistics, discovering nuclear fission (without realizing it), building the first nuclear reactor and discovering the origin of high energy cosmic rays. He also coined the word 'neutrino'.

Weak scattering processes: the unitarity problem

If we have a vertex



as Fermi postulated, then, by universality, we should also have



and it should be possible to have a scattering process

$$e^- + \nu_e \to e^- + \nu_e$$

Cross-section:

$$\sigma \approx \frac{G_F^2}{\pi} s \left(1 - \frac{M_e^2}{s}\right)$$

where $s = \left(p_e + p_{\nu_e}\right)^2 = E_{\text{cm}}^2$.

Clearly, as $s \uparrow$, $\sigma \uparrow$ ...

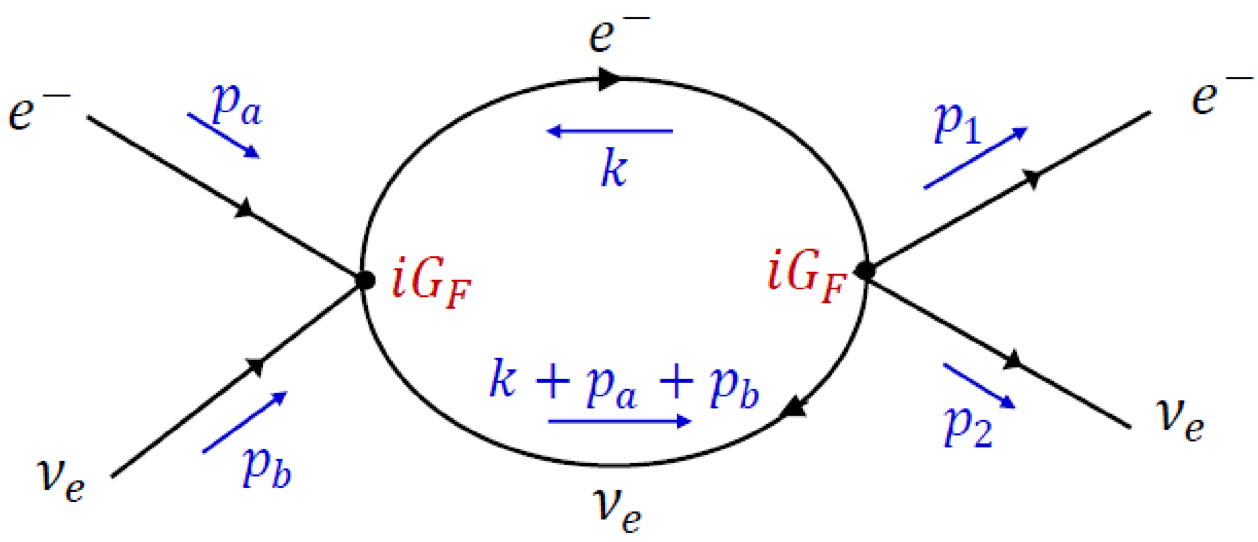
unitarity violation

Perhaps this arises because we took only the LO diagram... ?

... inclusion of higher orders may soften the growth with energy...

...but this leads to a new problem: renormalisability

Consider the simplest one-loop contribution to $e^- \nu_e \to e^- \nu_e$:



The effective coupling due to this would be

$$\frac{iG_F}{\sqrt{2}} \to \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k} - M_e} \frac{1}{\not{k} + \not{p}_a + \not{p}_b}$$

Since k is integrated over all values, the dominant contribution will come from $k \to \infty$, i.e.

$$\frac{iG_F}{\sqrt{2}} \rightarrow \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2}\int_0^\infty \frac{2\pi^2 k^3 dk}{(2\pi)^4}\frac{1}{\cancel{kk}}$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{2}\int_0^\infty \frac{2\pi^2 k^3 dk}{(2\pi)^4}\frac{1}{k^2}$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{16\pi^2}\int_0^\infty k\,dk$$

This extra contribution is quadratically divergent, i.e. if we put a momentum cutoff $k \leq \Lambda$ then,

$$\frac{iG_F}{\sqrt{2}} \rightarrow \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{16\pi^2}\int_0^\Lambda k\,dk$$

$$= \frac{iG_F}{\sqrt{2}} + \frac{(iG_F)^2}{32\pi^2}\Lambda^2$$

If the NLO contribution $\gg$ LO contribution, perturbation theory fails...

Such problems arise in QED as well for e, but there the divergences are logarithmic, i.e. proportional to $\log \Lambda$. Moreover, in every order (NLO, NNLO, NNNLO, ...) we always get a similar logarithmic divergence.

These can be summed up, and the result absorbed into the definition of e -- this process is called renormalisation

In the Fermi theory, however, higher and higher powers of Λ^2 keep coming with higher and higher orders, and there is no scope for renormalisation...

Does this mean that the Fermi theory is wrong?

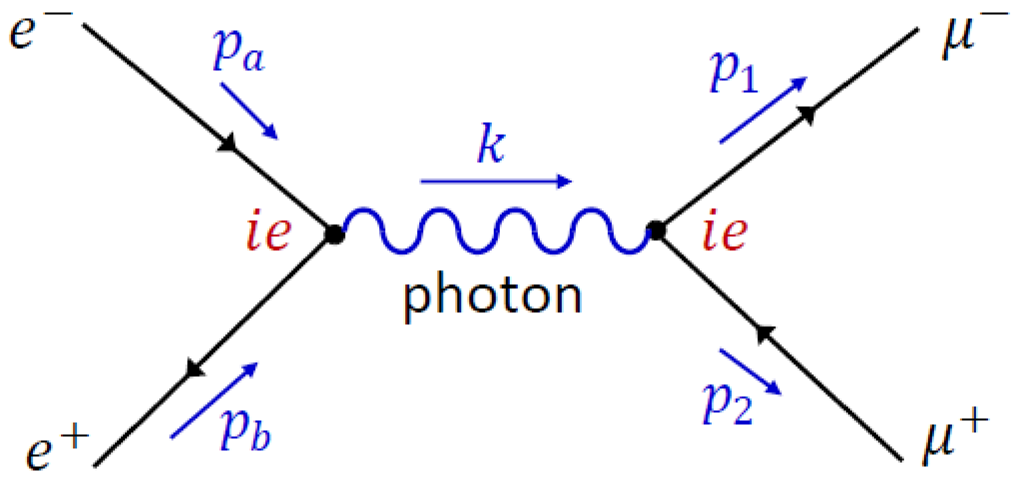
Correspondence Principle: every new theory should reduce to the old theory in the range of parameters where that theory was successful

Fermi theory must be a low-energy effective theory...

Intermediate Vector Bosons (IVB):

Schwinger (1953): if renormalisation is possible in QED, can we make it possible in weak interactions by copying the same form?

Consider the following process in QED: $e^- + e^+ \rightarrow \mu^- + \mu^+$



$$i\mathcal{M} = \bar{v}(p_b)\, ie\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu}}{k^2}\, \bar{u}(p_1)\, ie\gamma^\nu v(p_2)$$
$$= \frac{ie^2}{k^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\;\; \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

Intermediate Vector Bosons (IVB):

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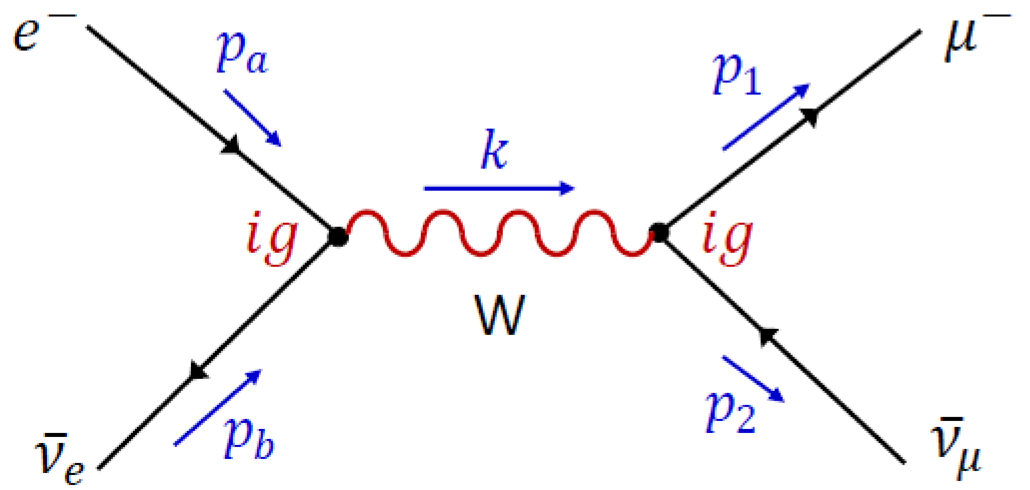

$$i\mathcal{M} = \bar{v}(p_b)\, ig\gamma^\mu u(p_a)\, \frac{-ig_{\mu\nu}}{k^2}\, \bar{u}(p_1)\, ig\gamma^\nu v(p_2)$$
$$= \frac{ig^2}{k^2}\bar{v}(p_b)\,\gamma^\mu u(p_a)\;\bar{u}(p_1)\,\gamma_\mu v(p_2)$$

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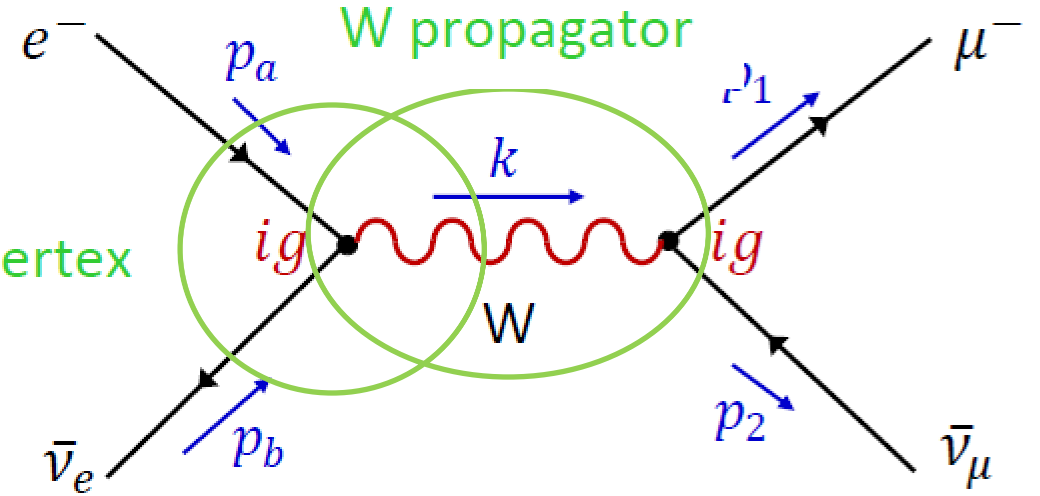


Taking Fermi's idea a step further...

$$i\mathcal{M} = \bar{v}(p_b)\, ig\gamma^\mu u(p_a) \frac{-ig_{\mu\nu}}{k^2} \bar{u}(p_1)\, ig\gamma^\nu v(p_2)$$
$$= \frac{ig^2}{k^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\;\; \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2}$$

<u>Objection:</u> The Fermi coupling constant does not show significant variation with energy as $k^2 \to 0$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2}$$

<u>Schwinger's solution</u>: make the W boson massive

$$\frac{-i g_{\mu\nu}}{k^2} \to \frac{-i g_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}$$

$$i\mathcal{M} = \bar{v}(p_b)\, i g \gamma^\mu u(p_a)\, \frac{-i g_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}\, \bar{u}(p_1)\, i g \gamma^\nu v(p_2)$$

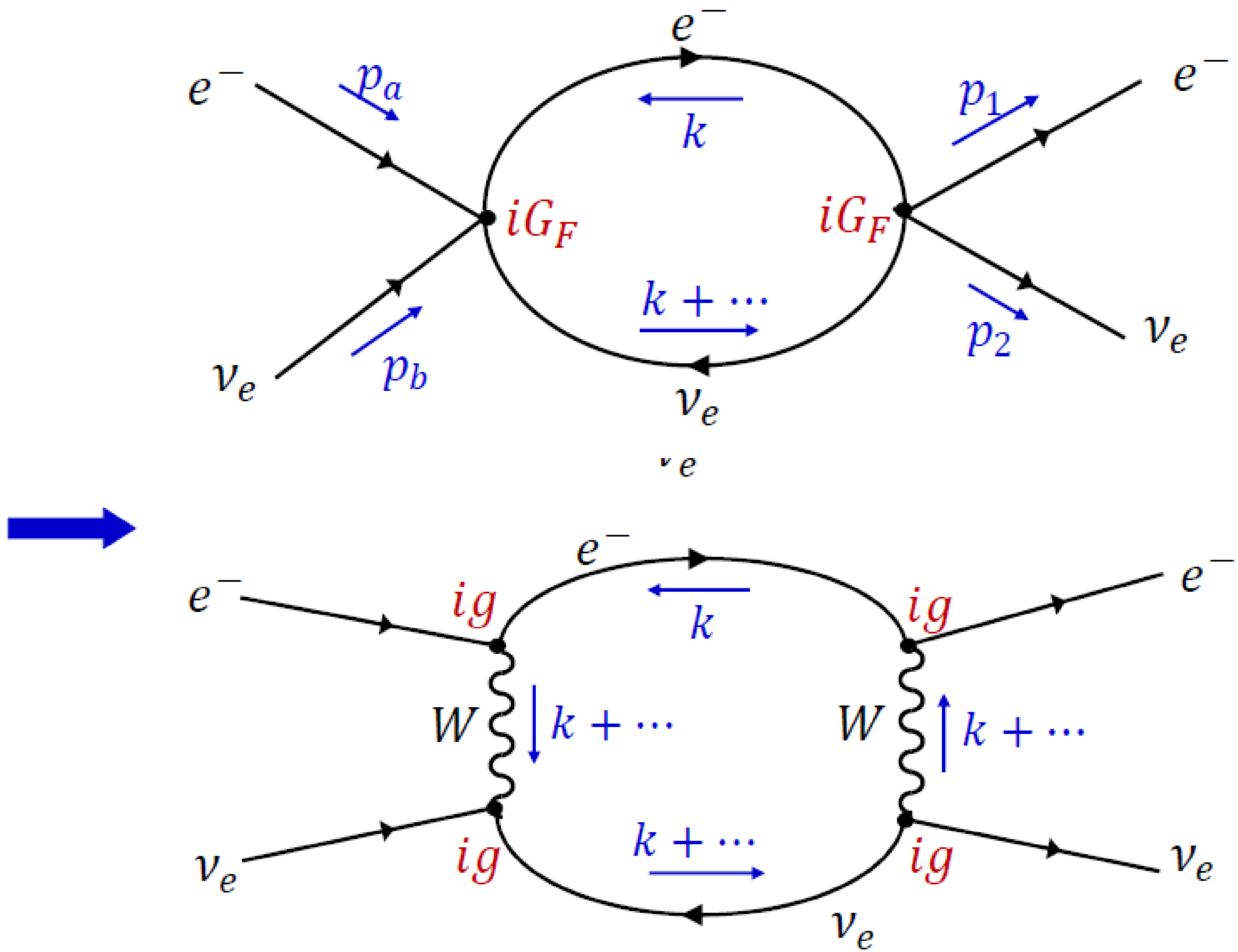
$$= \bar{v}(p_b)\, i g \gamma^\mu u(p_a)\, \frac{-i g_{\mu\nu}}{k^2 - M_W^2}\, \bar{u}(p_1)\, i g \gamma^\nu v(p_2)$$

$$= \frac{i g^2}{k^2 - M_W^2} \bar{v}(p_b)\, \gamma^\mu u(p_a)\; \bar{u}(p_1)\, \gamma_\mu v(p_2)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{k^2 - M_W^2}$$

In the low energy limit, $k^2 \to 0$ we get: $\frac{G_F}{\sqrt{2}} = -\frac{g^2}{M_W^2}$ constant!!

Q. How does this help?



Rewrite the loop integral...

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{\not{k}+\cdots} \rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{1}{k^2-M_W^2} \frac{1}{\not{k}+\cdots} \frac{1}{k^2-M_W^2}$$

$$\propto \int k^3 dk \; \frac{1}{k^2} \; \frac{1}{k^4}$$

$$\propto \int \frac{dk}{k^3} \qquad \text{finite!}$$

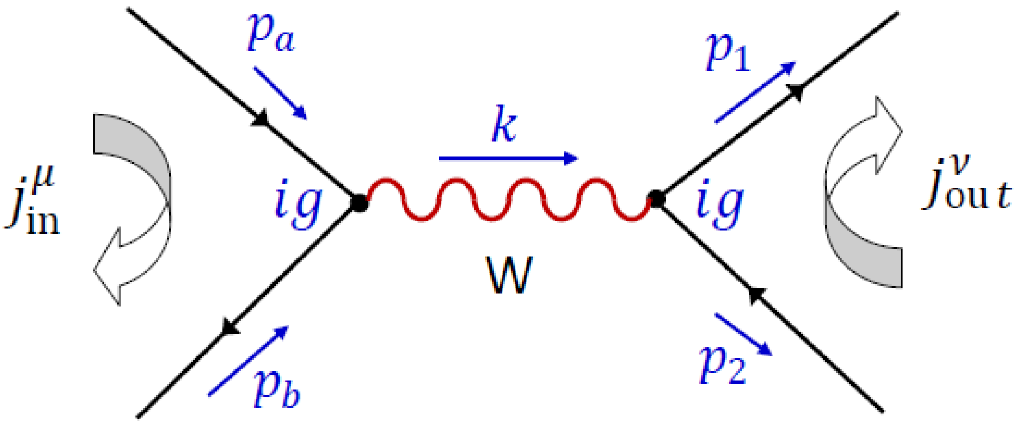
As it would be in QED...

But we have cheated... $\frac{-ig_{\mu\nu}}{k^2} \rightarrow \frac{-ig_{\mu\nu}+k_\mu k_\nu/M_W^2}{k^2-M_W^2}$

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{k}} \frac{k_\mu k_\nu}{k^2-M_W^2} \frac{1}{\not{k}+\cdots} \frac{k_\mu k_\nu}{k^2-M_W^2} \propto \int k^3 dk \; \frac{1}{k^2} \; \frac{k^4}{k^4} \propto \int k dk$$

Only way to make IVB work is to get rid of the $k_\mu k_\nu/M_W^2$ term...

What does the propagator couple to?



$$\mathcal{M} \propto j^\mu_{\rm in}\, ig\, \frac{-ig_{\mu\nu} + k_\mu k_\nu / M_W^2}{k^2 - M_W^2}\, ig\, j^\nu_{\rm out}$$

The offending term will go away if $j^\mu_{\rm in} k_\mu = 0$ and/or $k_\nu j^\nu_{\rm out} = 0$

To have conserved currents, there must be a gauge symmetry...

Julian Schwinger (1918 – 1994) was a pioneer of quantum field theory and developed the idea of correlation functions to study interacting fields. He was also the first to realize that neutrinos have more than one flavour.

To have conserved currents, there must be a gauge symmetry...

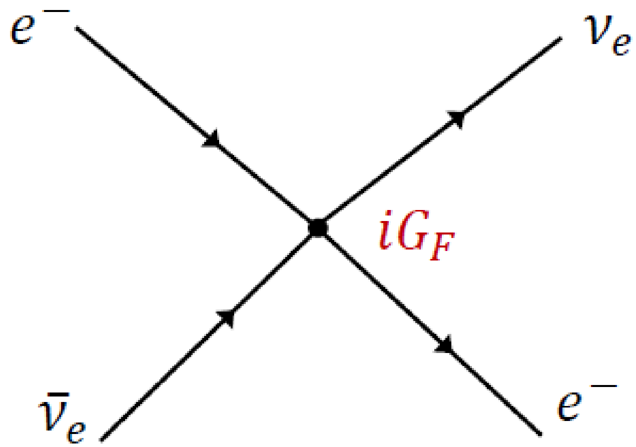
But this cannot be a U(1) gauge symmetry, like QED

Why not? Because the W boson is charged,
i.e. there are two W bosons

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_1^+ + iW_2^+) \qquad W_\mu^- = \frac{1}{\sqrt{2}}(W_1^+ - iW_2^+)$$

i.e. the group of gauge symmetries must have at least <u>two</u> generators

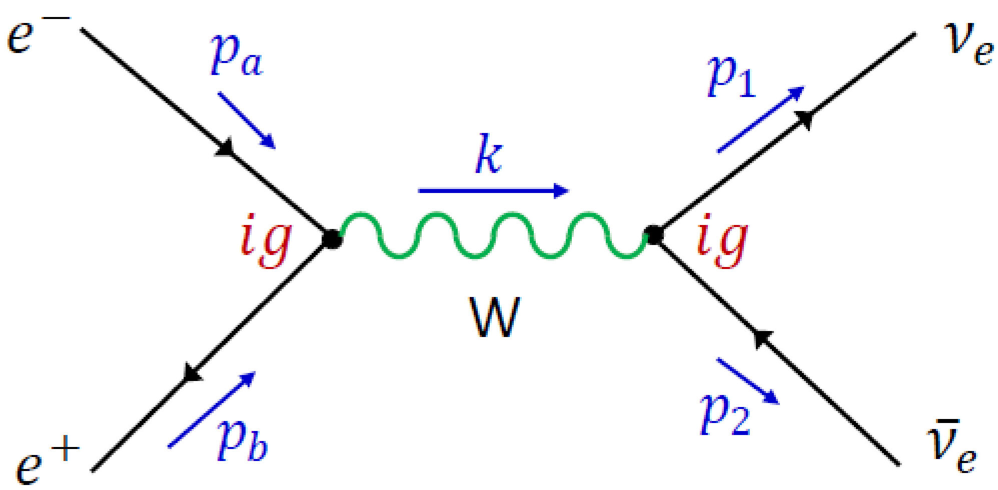
In fact, if we have a four-fermion theory with the vertex



there is nothing, in principle, to prevent a process like

$$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$$

How will this look in the IVB theory?



So, perhaps we have a neutral W boson also – a W_μ^0

This W_μ^0 cannot be the photon because it couples to neutrinos...

i.e. the group of gauge symmetries must have <u>three</u> generators

after U(1), the next unitary group is SU(2), which has 3 generators...