Relativistic Kinematics

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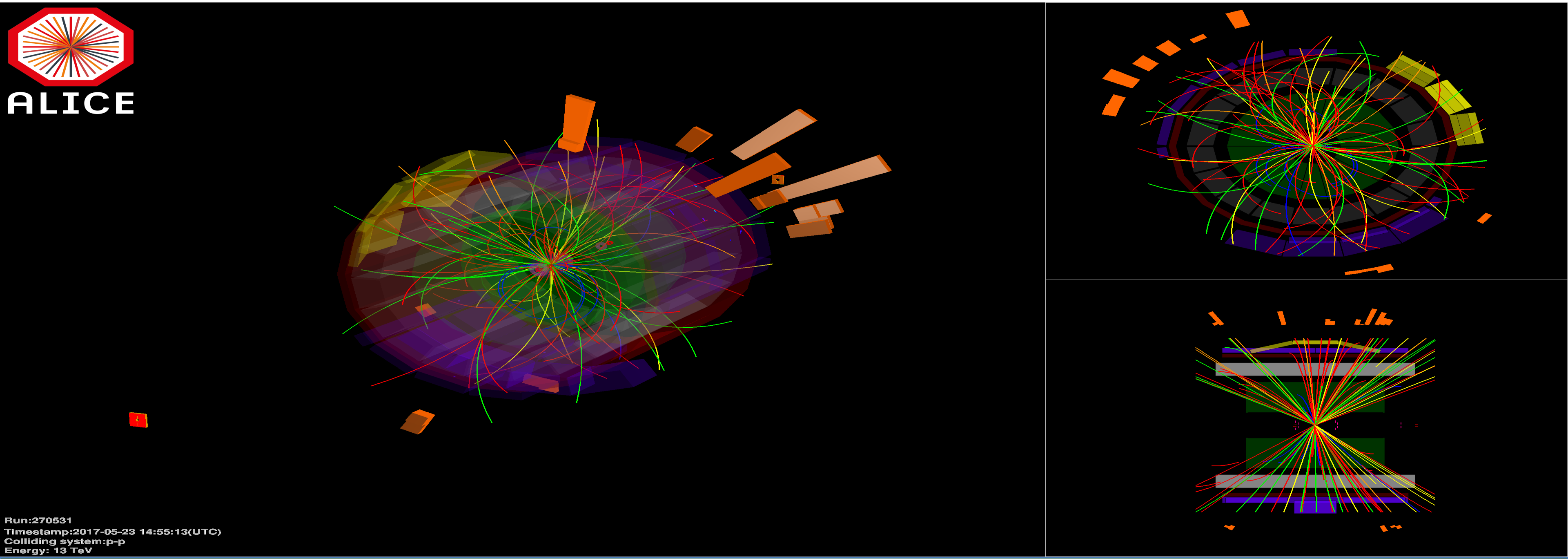


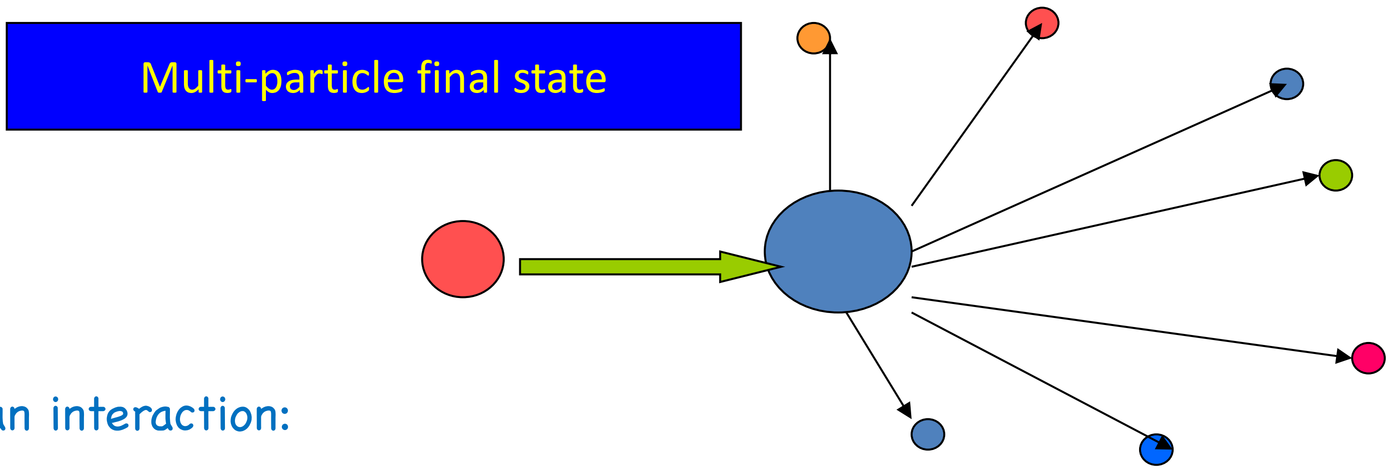
References

- Relativistic Kinematics for Beginners (With Monte Carlo)- Raghunath Sahoo and Basanta K. Nandi (CRC Press- 2020)
- Relativistic Kinematics- Raghunath Sahoo (arXiv:1604.02651)
- Introduction to High Energy Heavy-Ion Collisions- C.Y. Wong
- Ultra-relativistic Heavy-Ion Collisions- R. Vogt
- Introduction to High Energy Physics- D.H. Perkins
- Introduction to Elementary Particles- D.J. Griffiths
- Relativistic Kinematics- R. Hagedorn
- Lecture Notes in Physics (Global Properties in Heavy-Ion Collisions)- M. Kleimat, R. Sahoo, T. Schuster and R. Stock
- Hadrons and Quark-Gluon Plasma- Rafelski and Latessier
- Introduction to Heavy-Ion Collisions- L.P. Tsernai

Disclaimer/Acknowledgement

- This presentation is purely for academic training purpose
- Most of the slides are based on our upcoming book, given in reference-1
- Many plots and ideas come from the excellent textbooks listed on the previous slide
- Lots of good material can nowadays be found by just *Googling*. The authors are then not always recognizable/acknowledgeable.

Estimation of Threshold Energy



Consider an interaction:

- ✓ Projectile A with mass, m_A and energy in the laboratory system, E_{lab} collides with a target, B of mass, m_B at rest ($\mathbf{p_B} = 0$)
- ✓ There are n number of final state particles produced
- ✓ Let them be: C_1, C_2, ………,C_n , with masses, m_{C1}, m_{C2}, ……., m_{Cn}, respectively.

$A+B \rightarrow C_1+C_2+C_3+\ldots\ldots\ldots\ldots+C_n$

Estimation of Threshold Energy

Threshold Energy: energy required to produce the final state particles (any number of) with zero kinetic energy.

Meaning: The particles in the final state are just produced at rest.

The Recipe: (same for any other kinematic problem)

- ❖ Write-down the 4-momenta for the reactants and the products separately
- ❖ Estimate the total 4-momenta before and after the collision
- ❖ Suitably choose the frame of reference for the convenience of calculation
- ❖ Apply the Lorentz invariant property of the scalar product of 4-momenta
- ❖ You are done !

Estimation of Threshold Energy

Step-1: (Write 4-momenta)

4-momenta for particle A : $p_A = (E_A, \mathbf{p_A})$

for particle B : $p_B = (m_B, \mathbf{0})$

For the final state particles, as their kinetic energy is zero at the threshold production, the four-momenta is: $(m_{C1} + m_{C2} + \ldots\ldots + m_{Cn}, \mathbf{0})$.

Step-2: (Estimate the total 4-momenta)

In the laboratory frame, before the collision, $p^{\mu}_{TOT} = (E_A + m_B, \mathbf{p_A})$

In the center-of-mass frame, after the collision,

$$p^{\mu}_{TOT'} = \left(\sum_{i=1}^{n} m_{C_i}, \vec{0}\right) \equiv (M_C, \vec{0})$$

Step-3: (Apply LI property of 4-vector scalar product)

p^2_{TOT} after and before the collision is Lorentz invariant

$$p^{\mu}_{TOT}\ p_{\mu,TOT} = p^{\mu}_{TOT'}\ p_{\mu,TOT'}$$

Estimation of Threshold Energy

Step-4: (Simplify)

$$p^{\mu}_{TOT}\, p_{\mu,TOT} = p^{\mu}_{TOT'}\, p_{\mu,TOT'}$$

$$\Rightarrow \quad (E_A + m_B)^2 - \vec{p_A}^2 = M_C^2$$

$$\Rightarrow \quad E_A^2 + m_B^2 + 2E_A m_B - \vec{p_A}^2 = M_C^2$$

$$\Rightarrow \quad m_A^2 + m_B^2 + 2E_A m_B = M_C^2$$

$$\Rightarrow \quad E_A = \frac{M_C^2 - m_A^2 - m_B^2}{2m_B}$$

$$\Rightarrow \quad \boxed{E_{lab}^{Th} = \frac{(\sum_{i=1}^{n} m_{C_i})^2 - m_A^2 - m_B^2}{2m_B}}$$

Ref: Relativistic Kinematics for Beginners:
Raghunath Sahoo & Basanta K. Nandi
(CRC Press- 2020)

Estimation of Threshold Energy

Example 2.8 In WASA-COSY experiment at Jülich, Germany, proton beam is used on a fixed proton target to produce $\eta\prime$ mesons. Estimate the threshold energy required for the formation of $\eta\prime$: ($pp \to pp\eta\prime$).

Solution

Given masses of the particles are, proton mass, $m_p = 938.27$ MeV and $\eta\prime$ mass, $m_{\eta\prime} = 957.78$ MeV. Using Eqn. 2.47, the threshold energy for $\eta\prime$ production is given by

$$E_{lab}^{Th} = \frac{(\sum_{i=1}^{n} m_{C_i})^2 - m_A^2 - m_B^2}{2m_B}$$

$$= \frac{(2m_p + m_{\eta\prime})^2 - 2m_p^2}{2m_p}$$

$$= \frac{[2 \times 938.27 + 957.78]^2 - 2 \times (938.27)^2}{2 \times 938.27}$$

$$= 3342.67 \text{ MeV} \simeq 3.34 \text{ GeV}$$

It would be interesting to calculate the velocity of the projectile proton to have a feeling of relativistic energy we are working at. We know $E = m\gamma$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, is the Lorentz factor. $\gamma = E/m = 3342.67/938.27 = 3.562$. Solving for the velocity, we get $\beta = 0.95978$, using $\beta = (1 - \frac{1}{\gamma^2})^{\frac{1}{2}}$. This means the projectile proton travels with 95.97835% of the speed of light. What is your guess for the speed of the proton at the Large Hadron Collider, CERN, where the beam energy is 3.5 TeV? If you estimate this, you shall find that the beam proton has a Lorentz factor ~ 3730.27 and it travels with 99.9999964% the speed of light. Now you may be surprised to see that even if the beam energy is almost 1000 times higher, the speed of the proton doesn't change proportionately. What could be the reason? We shall discuss it further in the subsequent chapters. And you will realize that, with change of energy and consequently the speed of the accelerated particle, the relativistic mass of the particle changes and it becomes difficult to accelerate further.

Ref: Relativistic Kinematics for Beginners:
Raghunath Sahoo & Basanta K. Nandi
(CRC Press- 2020)

Collider Vs Fixed Target Experiments

The severity of damage is less.

The damage is much higher !

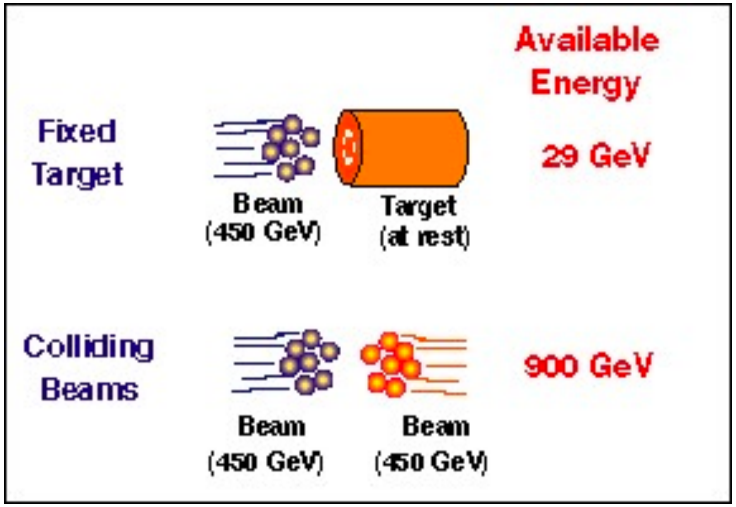
Energy available for particle production:

In laboratory frame (fixed target):

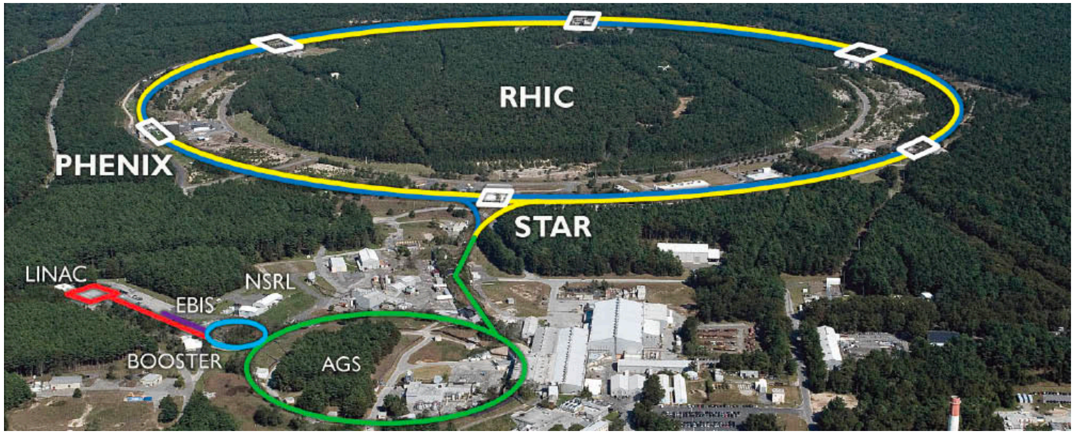
$E_{cm} = \sqrt{(2 m_t E_{beam})}$

In CM frame (collider):

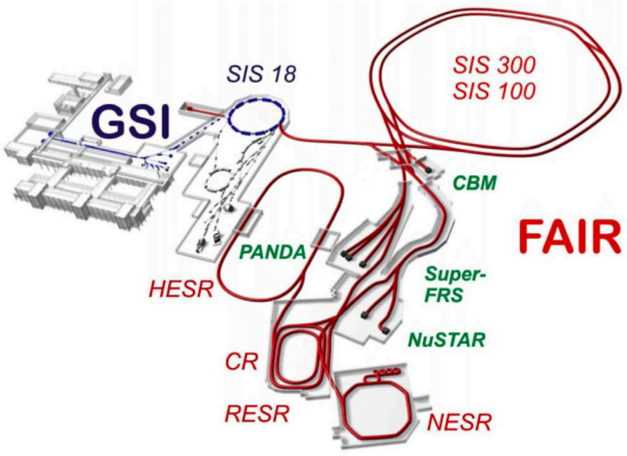
$E_{cm} = (2 E_{beam})$



Collider Vs Fixed Target Experiments



Relativistic Heavy-Ion Collider
Brookhaven National Laboratory, USA



FAIR, GSI, Germany (Coming up)

Large hadron Collider (LHC)
CERN, Geneva, Switzerland

Collider Vs Fixed Target Experiments

- Consider collision of two particles.
- In LS, the projectile of momentum $\boldsymbol{p_1}$, energy E_1 and mass m_1 collides with a target particle of mass m_2 at rest.
- The 4-momenta of particles are: $p_1 = (E_1, \vec{p}_1), \quad p_2 = (m_2, \vec{0})$
- In CM system, $p_1^* = (E_1^*, \vec{p}_1^*) \qquad p_2^* = (E_2^*, -\vec{p}_1^*)$

The square of the total 4-momentum of the system is a conserved quantity.

- In CM system,

$$(p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2$$
$$= (E_1 + E_2)^2$$
$$= {E_{cm}}^2 \equiv s$$

√s is the total energy in the CM, which is the invariant mass of the CM system.

- In LS,

$$(p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1 m_2$$

Collider Vs Fixed Target Experiments

Hence, $\boxed{E_{cm} = \sqrt{s} = \sqrt{m_1^2 + m_2^2 + 2E_{proj}m_2}}$

It is evident here that the CM frame with an invariant mass of $\sqrt{s}$, *moves in the LS in the direction of* $\boldsymbol{p}_1$ *with a velocity corresponding to the Lorentz factor:*

$$\gamma_{cm} = \frac{E_1 + m_2}{\sqrt{s}}$$

$$\Rightarrow \sqrt{s} = \frac{E_{lab}}{\gamma_{cm}}$$

This is because $E = \gamma m$ and

$$y_{cm} = cosh^{-1}\gamma_{cm}$$

For a collider with head-on collision (θ = 180^0),

$$s = E_{cm}^2 = m_1^2 + m_2^2 + 2\left(E_1E_2 + |\vec{p}_1||\vec{p}_2|\right)$$

For relativistic collisions, m_1, m_2 << E_1, E_2

$$E_{cm}^2 \approx 4E_1E_2$$

Collider Vs Fixed Target Experiments

For two beams crossing at an angle θ,

$$E_{cm}^2 = 2E_1E_2\left(1+\cos\theta\right)$$

The CM energy available in a collider with equal energies (*E*) for new particle production rises linearly with *E i.e.*

$$\boxed{E_{cm} \simeq 2E}$$

For a fixed target experiment the CM energy rises as the square root of the incident energy:

$$\boxed{E_{cm} \simeq \sqrt{2m_2E_1}}$$

As in LS, $(p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1m_2$

Hence the highest energy available for new particle production is achieved at collider experiments e.g. at SPS Fixed-target experiment to achieve a CM energy of 17.3 AGeV the required beam energy is 158 AGeV.

Collider Vs Fixed Target Experiments

Note:

- More often the collision energy is expressed in terms of nucleon-nucleon (*NN*) CM energy.
- In NN CM frame, two nuclei approach each other with the same boost factor γ.
- The NN CM is denoted by $\sqrt{s_{NN}}$ and is related to the CM energy by:

$$\boxed{\sqrt{s} = A\sqrt{s_{NN}}}$$

Homework for beginners:

Write a ROOT based C++ program to convert LS energy to CM energy for A+B type of collisions. This will be always required to study the excitation function of any observable or comparison of data with low energy fixed-target experiments.

Homework for Advanced Learners:

Write a MC program in ROOT to use the 3-dim Lorentz boosting for the invariant mass reconstruction of

$$\rho \rightarrow e^{+} + e^{-}$$

Collider Vs Fixed Target Experiments

Exercise:

For ^{7}Be+^{9}Be interactions, with the given beam energies, check the following table.

p_{beam} (A GeV/c)	$\sqrt{s_{NN}}$ (GeV)
19	6.1
30	7.6
40	8.8
75	11.9
150	16.8

Collider Vs Fixed Target Experiments

$\boxed{\sqrt{s} = A\sqrt{s_{NN}}}$ This is for a symmetric collision with A number of nucleons in each nuclei.

- ✓ The colliding nucleons approach each other with energy $\sqrt{s_{NN}}/2$ and with equal and opposite momenta
- ✓ The rapidity of the *NN* CM is $y_{NN} = 0$
- ✓ Taking $m_1 = m_2 = m_p$, the projectile and target nucleons are at equal and opposite rapidities:

$$\boxed{y_{proj} = -y_{target} = cosh^{-1}\frac{\sqrt{s_{NN}}}{2m_p} = y_{beam} = ln\left(\frac{\sqrt{s_{NN}}}{m_p}\right)}$$

Example: Suppose two identical particles, each with mass m and kinetic energy *T*, collide head-on. What is their relative kinetic energy, *T´* (i.e. K.E. of one in the rest frame of the other). Apply this to an electron-positron collider, where K.E. of electron (positron) is 1 GeV. Find the K.E. of electron if positron is at rest (fixed target). Which experiment is preferred, a collider or a fixed target expt.?

Given pp maximum beam energy, how to estimate the achievable energy for nuclear collisions (pA, AA)

- An accelerator is capable of accelerating protons upto a beam energy of E_p
- What will be the maximum CM energy for pA and AA collisions?

- ✓ In heavy-ion collision, the atom is fully stripped off with electrons
- ✓ The nucleus has Z number of protons and N number of neutrons: (A-Z)
- ✓ Neutral particles can't be accelerated independently (neutron here)
- ✓ The charged state of the nucleus is responsible for obtaining required energy in the acceleration process
- ✓ Fully stripped means the charged state equals to the total number of protons inside the nucleus
- ✓ Neutrons inside the nucleus are just "free riders " in the process
- ✓ We need to spend energy of the protons to accelerate the neutrons
- ✓ This means the center-of-mass energy for heavy-ion collisions is just not Z-times of the p + p collision energy – But a reduced value

Given pp maximum beam energy, how to estimate the achievable energy for nuclear collisions (pA, AA)

- ✓ For head-ion heavy-ion collisions, $\sqrt{s_{\rm NN}} \simeq 2\sqrt{E_1 E_2}$
- ✓ For a collision of two different nuclei with charges (atomic number) Z_1, Z_2 and atomic masses A_1 , A_2

$$E_1 = E_{\rm proton}(Z_1/A_1)$$

$$E_2 = E_{\rm proton}(Z_2/A_2)$$

- ✓ Now, $\sqrt{s_{\rm NN}} \simeq 2E_{\rm proton}\sqrt{\frac{Z_1 Z_2}{A_1 A_2}}$

$$\Rightarrow \boxed{\sqrt{s_{\rm NN}} \simeq \sqrt{s_{\rm pp}}\sqrt{\frac{Z_1 Z_2}{A_1 A_2}}}.$$

- ✓ Here, $\sqrt{s_{pp}}$ is the CM energy of pp collisions

- ✓ For symmetric collisions with atomic number and mass– (Z; A), this formula becomes:

$$\boxed{\sqrt{s_{\rm NN}} \simeq \sqrt{s_{\rm pp}}\,\frac{Z}{A}}$$

Given pp maximum beam energy, how to estimate the achievable energy for nuclear collisions (pA, AA)

Table 6.1 Table of collision energy, $\sqrt{s_{\rm NN}}$ for p+A and A+A collisions at the LHC, as estimated from the corresponding $p+p$ data taken till 2018. The center-of-mass energies with *-marks represent data already taken.

$\sqrt{s_{pp}}$ (TeV)	0.9	2.36	2.76	5.02	7	8	13
p+Pb	0.57	1.48	1.74	3.16	4.4	5.02*	8.16*
Pb+Pb	0.35	0.93	1.09	1.98	2.76*	3.17	5.14*
Xe+Xe	0.37	0.97	1.14	2.07	2.88	3.3	5.44*

Ref: Relativistic Kinematics for Beginners:
Raghunath Sahoo & Basanta K. Nandi
(CRC Press- 2020)

- For symmetric collisions: $\boxed{\sqrt{s_{\rm NN}} \simeq \sqrt{s_{\rm pp}}\,\frac{Z}{A}}$

- General Formula: $\boxed{\sqrt{s_{\rm NN}} \simeq \sqrt{s_{\rm pp}}\sqrt{\frac{Z_1 Z_2}{A_1 A_2}}}$

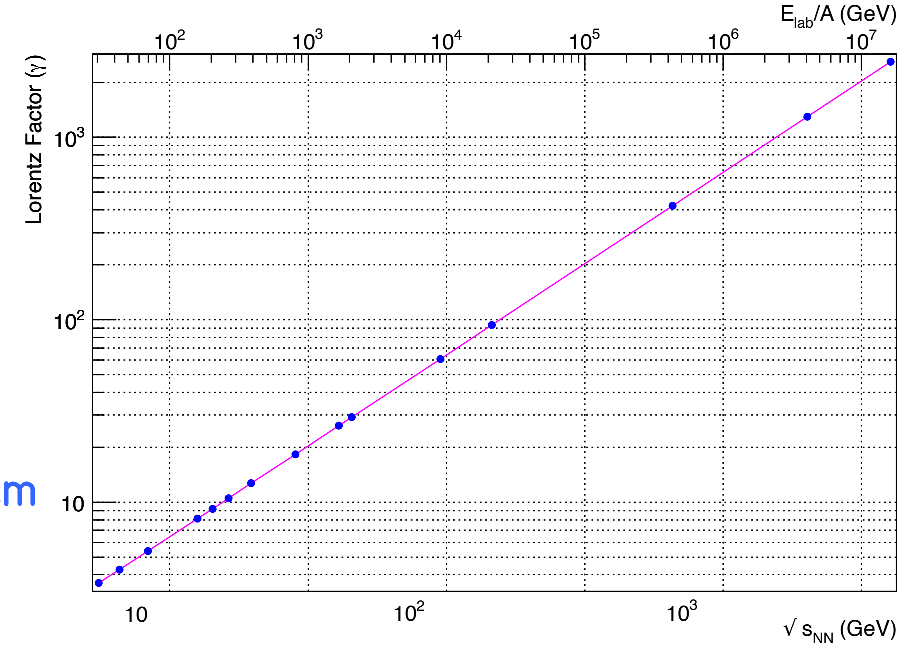
Lorentz factor

Lorentz Factor: $$\gamma = \frac{E}{M} = \frac{\sqrt{s}}{2Am_p}$$

$$= \frac{A\sqrt{s_{NN}}}{2Am_p} = \frac{\sqrt{s_{NN}}}{2m_p}$$

$$= \frac{E^{CM}_{beam}}{m_p}$$

E, M are energy and mass of CM.

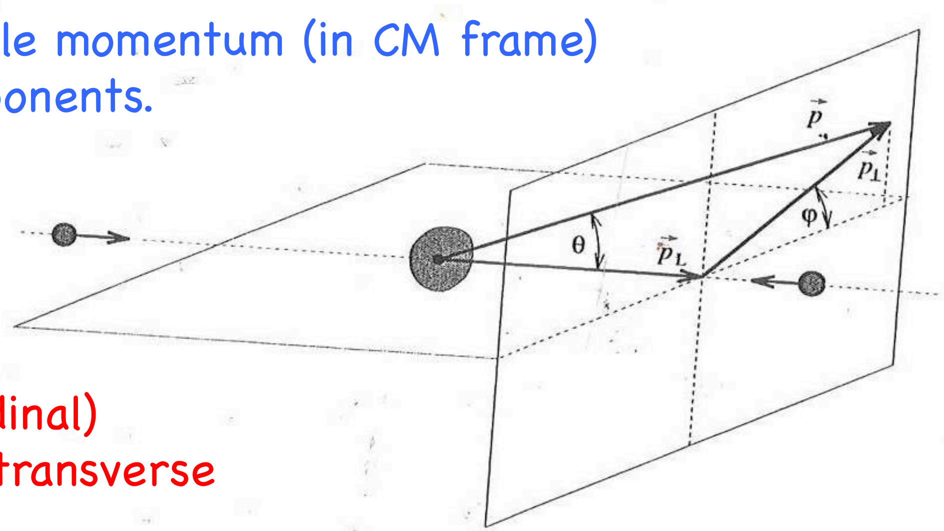


If we assume mass of proton to be ~ 1 GeV, the Lorentz factor is of the order of beam energy in CM system for a symmetric collision.

For asymmetric collision kinematics, please refer to my lecture note/Book.

Experiment: Detector Coordinate System

Schematic decomposition of particle momentum (in CM frame) into parallel and longitudinal components.



Pic: Hadrons and Quark-Gluon Plasma- Rafelski and Latessier

- Beam direction: z-axis (longitudinal)
- Perpendicular to it (xy-plane): transverse

- In a collider, a particle is emitted from the interaction point making a polar angle θ with the collision point.
- Various components of momenta are determined by the tracking detectors.

The particle rapidity is given by:

$$y = tanh^{-1} v_z = tanh^{-1} \frac{p_z}{E} = tanh^{-1} \frac{p_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}$$

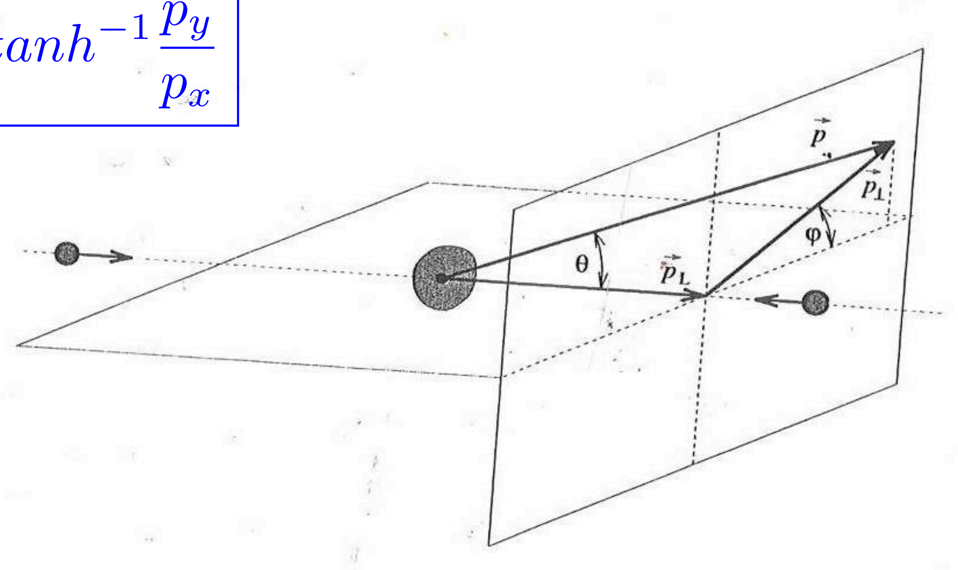
Experiment: Detector Coordinate System

And the azimuthal angle is:

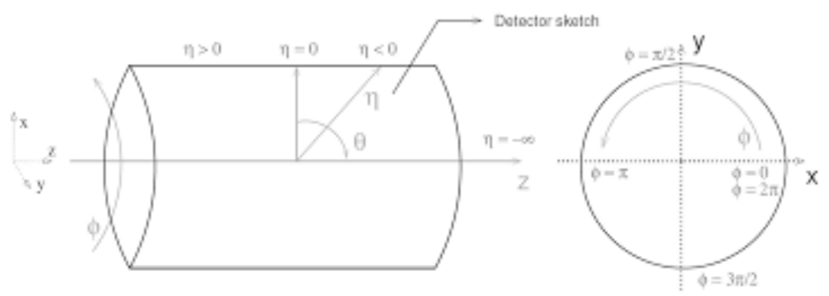
$$\phi = tanh^{-1}\frac{p_y}{p_x}$$

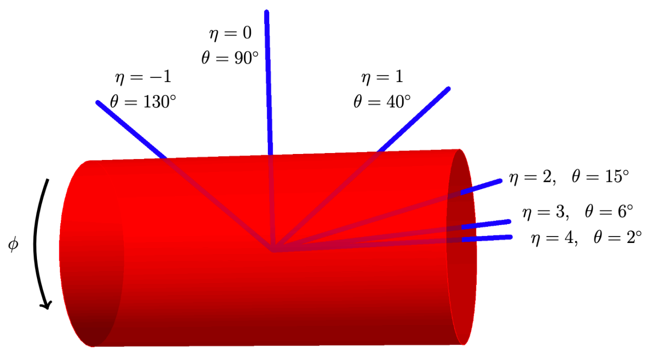
The polar angle is:

$$\theta = cos^{-1}\frac{p_z}{|\vec{p}|} = tan^{-1}\frac{\vec{p}_T}{p_z}$$



A detector plane is spanned by (η, ϕ) with η decreasing while going away from the beam axis in annular rings and ϕ is scanned making an angle with the beam axis and increasing it anti-clock-wise.





Rapidity

The particle 3-momentum can be decomposed into:

✓ longitudinal (p_z)

✓ transverse ($\mathbf{p}_T$)

For a Lorentz boost along z-direction $\mathbf{p}_T$ will be L.I.

Rapidity is defined as:

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

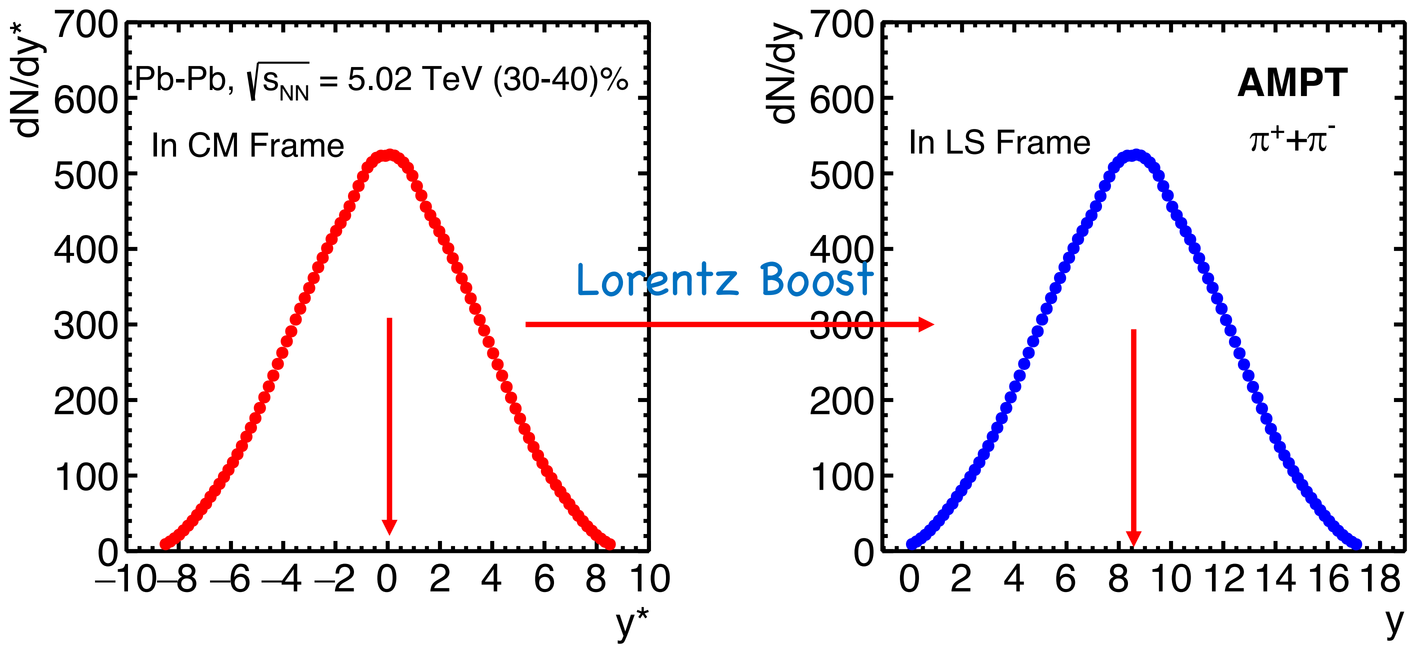
$$= \ln\left(\frac{E+p_z}{m_T}\right)$$

This is a dimensionless quantity being related to the ratio of forward light-cone to backward light-cone momentum.

Rapidity changes by an additive constant under longitudinal Lorentz boost. Shape invariance of rapidity distribution makes it a suitable variable in describing particle production in nucleus-nucleus collisions.

Rapidity

Shape Invariance:



$$y = y^{*} + y_{cm} \qquad y_{cm} = y_b = 8.52$$

For a symmetric collider, $y_{\text{cm}} = \frac{1}{2}(y_a + y_b)$

$$\boxed{y_{\text{beam}} = \pm \ln(\sqrt{s_{\text{NN}}}/m_p) = \pm y_{\text{max}}}$$

Rapidity

- For a free particle, which is on the mass shell, the 4-momentum can be represented by (y, p_T).

- (E, p_z) could be written in terms of (y, p_T) as:

$$\boxed{E = m_T \; cosh \; y \; and \; p_z = m_T \; sinh \; y}$$

- m_T being the transverse mass and defined as $\boxed{{m_T}^2 = m^2 + \vec{p}_T^{\,2}}$

Going from CM to LS system, the rapidity distribution shows a shape invariance with y-scale shifted by an amount equal to y_{cm}.

Rapidity

Rapidity of CM in the Laboratory System:

- ✓ The total energy of the CM system is $E_{cm} = \sqrt{s}$
- ✓ The energy and momentum of the CM in LS are: $\gamma_{cm}\sqrt{s}$ *and* $\beta_{cm}\gamma_{cm}\sqrt{s}$
- ✓ The rapidity of the CM in the LS is:

$$y_{cm} = \frac{1}{2}\ln\left[\frac{\gamma_{cm}\sqrt{s} + \beta_{cm}\gamma_{cm}\sqrt{s}}{\gamma_{cm}\sqrt{s} - \beta_{cm}\gamma_{cm}\sqrt{s}}\right]$$

$$= \frac{1}{2}\ln\left[\frac{1+\beta_{cm}}{1-\beta_{cm}}\right]$$

This is a constant factor for a given Lorentz transformation.

Relationship between Rapidity of a particle in LS and rapidity in CM frame:

- ✓ Let the rapidities of a particle in LS and CM frame respectively are:

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \qquad y^* = \frac{1}{2}\ln\left(\frac{E^* + p_z^*}{E^* - p_z^*}\right)$$

Rapidity

- ✓ For a particle travelling in longitudinal direction, the LT of its energy and momentum components give:

$$\begin{bmatrix} E^* \\ p_L^* \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \cdot \begin{bmatrix} E \\ p_L \end{bmatrix}, \qquad p_T^* = p_T$$

where p_L and p_T are longitudinal and transverse momenta components, which are parallel and perpendicular to β, respectively.

- ✓ The inverse Lorentz transformations on E and p_z give:

$$y = \frac{1}{2}\ln\left[\frac{\gamma\left(E^* + \beta p_z^*\right) + \gamma\left(\beta E^* + p_z^*\right)}{\gamma\left(E^* + \beta p_z^*\right) - \gamma\left(\beta E^* + p_z^*\right)}\right]$$

$$\Rightarrow \boxed{y = y^* + y_{cm}}$$

The rapidity of a particle in the laboratory system is equal to the sum of the rapidities of the particle in CM system and the rapidity of the CM in the LS.

Rapidity

- In other words, the rapidity of a particle in a moving (boosted) frame is equal to the rapidity in its own rest frame minus the rapidity of the moving frame.
- In non-relativistic limit, this is like the subtraction of velocity of the moving frame.

Is it surprising?

Show that the longitudinal velocity β is the non-relativistic realization of rapidity y.

Note:

This simple property of rapidity variable makes it a suitable choice to describe the dynamics of relativistic particles.

Beam Rapidity

- We know: $E = m_T \ cosh \ y,$

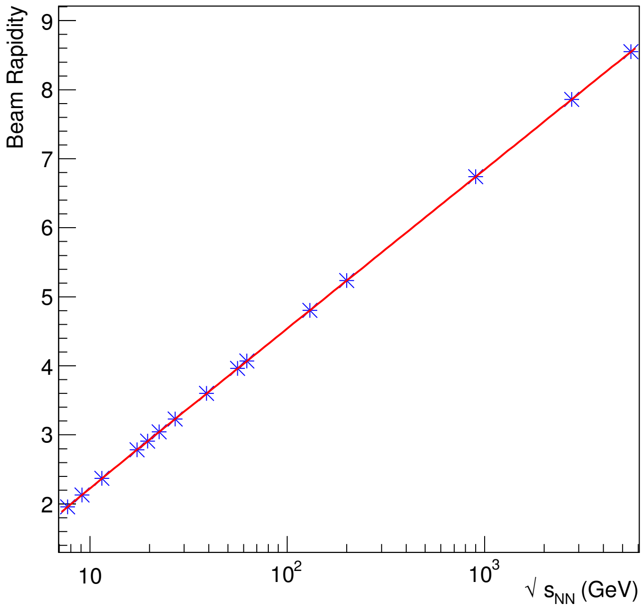
  $p_z = m_T \ sinh \ y$

  $and \quad m_T^2 = m^2 + p_T^2$

- For the beam particle, $p_T = 0$.

- Hence $E = m_b \ cosh \ y_b,$

  $p_z = m_b \ sinh \ y_b$



where m_b = rest mass of the beam particles
y_b = rapidity of the beam particles

$$y_b = \cosh^{-1}\left(E / m_b\right)$$

$$= \cosh^{-1}\left[\frac{\sqrt{s_{NN}}}{2m_p}\right]$$

Beam Rapidity

$\Rightarrow$ $\boxed{y_b \;=\; ln\left[\sqrt{s_{NN}}/m_p\right]}$ Here m_p is the proton mass.

Note that the beam energy $E = \sqrt{s_{NN}}/2$ for a symmetric collider.

Show that: $\boxed{y_b = \mp ln(\sqrt{s_{NN}}/m_p) = \mp y_{max}}$

Use of y_b for limiting fragmentation studies.
- We shall come back to this

Homework:
Express the rapidity of the CM in terms of projectile and target rapidities.

Mid-rapidity in FT and Colliders

- In fixed target experiment (LS), $y_{target} = 0$.

$$y_{lab} = y_{target} + y_{projectile} = y_{beam}$$

- Hence mid-rapidity in fixed-target experiment is given by:

$$y_{mid}^{LS} = y_{beam}/2$$

- In collider experiments (symmetric: CM system),

$$y_{projectile} = -y_{target} = y_{CM} = y_{beam}/2$$

- Hence, mid-rapidity in CM system is given by:

$$y_{mid}^{CM} = (y_{projectile} + y_{target})/2 = 0$$

- This is valid for a symmetric collider only (A+A).
- For a collision like p+A, rapidity shift needs to be taken into account.

- For p+A collisions, the rapidity shift is:

$$\boxed{\Delta y \simeq \frac{1}{2} ln\left[\frac{Z_1 A_2}{Z_2 A_1}\right]}$$

Mid-rapidity in FT and Colliders

Rapidity shift in asymmetric collisions (pA) :

$$\boxed{\Delta y \simeq \frac{1}{2} ln \left[\frac{Z_1 A_2}{Z_2 A_1} \right]}$$

Example:

For p+Pb collisions at LHC, the rapidity shift could be estimated to be 0.465 and need to be taken into account for the comparison of the corresponding spectra with Pb+Pb collisions.

The Maximum Accessible Rapidity

We know

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

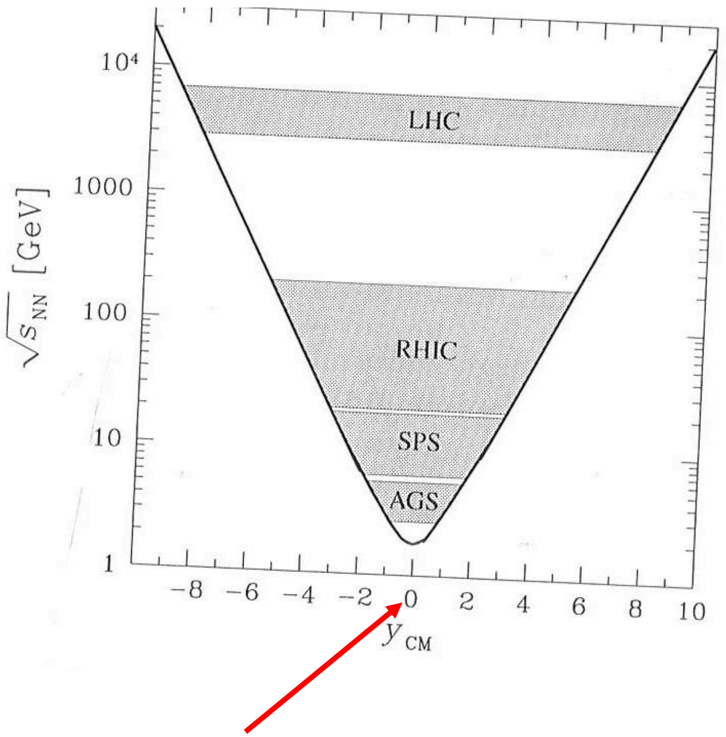
$$\Rightarrow \tanh\ y = \frac{p_z}{E}$$

$$|\,y_{\max}\,| = \ln\left[\frac{E+p_z}{m}\right]$$

$$= \ln\left[\gamma + \gamma\beta\right]$$

$$= \ln\left[\gamma + \sqrt{\gamma^2 - 1}\right]$$

$$= \cosh^{-1}\gamma, \quad if \quad \gamma >> 1$$



- A Lorentz boost β along the direction of the incident particle adds a constant, ln[γ+γβ], to the rapidity.

- Rapidity differences, therefore, are invariant to a Lorentz boost: see homework problem.

The Maximum Accessible Rapidity

Furthermore, $\gamma = \frac{E_{beam}}{m_p}$

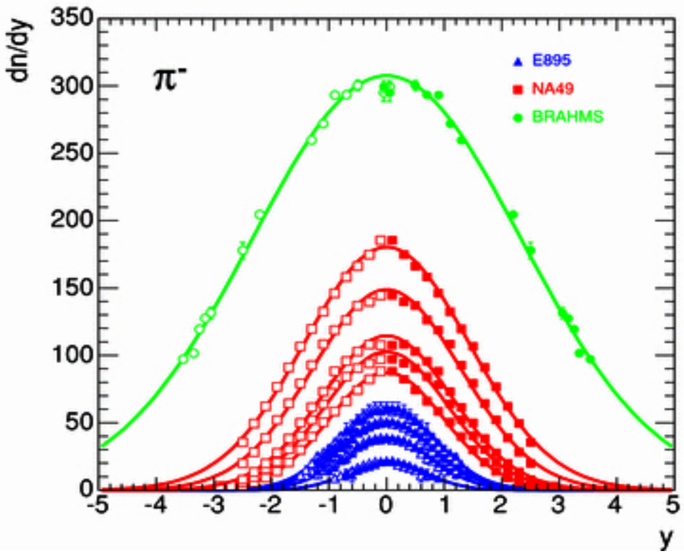
And for symmetric collisions, $E_{beam} = \frac{\sqrt{s_{NN}}}{2} \Rightarrow \gamma = \frac{\sqrt{s_{NN}}}{2m_p}$

Hence, $y_{max} = \cosh^{-1}\left[\frac{\sqrt{s_{NN}}}{2m_p}\right]$

$$= \cosh^{-1}\left[\frac{E_{lab}}{Am_p}\right] \quad \Rightarrow \quad \boxed{y_{max} = y_b = ln\left[\frac{\sqrt{s_{NN}}}{2m_p}\right]}$$

The maximum accessible rapidity is independent of the collision species and only depends on the CM energy (increases with energy).

While running any event generator for event simulation, check if the above formula gives the right result.



Rapidity: Properties

- Rapidity is the relativistic realization of particle velocity
- Rapidity is a dimensionless quantity
- Describes the rate at which a particle moves with respect to a chosen reference point situated on the trajectory of motion

$$y = \tanh^{-1}\beta = \frac{1}{2}\, ln \left[\frac{1+\beta}{1-\beta}\right]$$

- In terms of the energy, E and momentum, p of the particle:

$$\begin{aligned} y &= \tanh^{-1}\beta \\ &= \tanh^{-1}(p/E) \\ &= \tanh^{-1}\left(\frac{\gamma\beta m}{\gamma m}\right) \\ &= \frac{1}{2}\ln\left(\frac{E+p}{E-p}\right) \end{aligned}$$

Rapidity: Properties

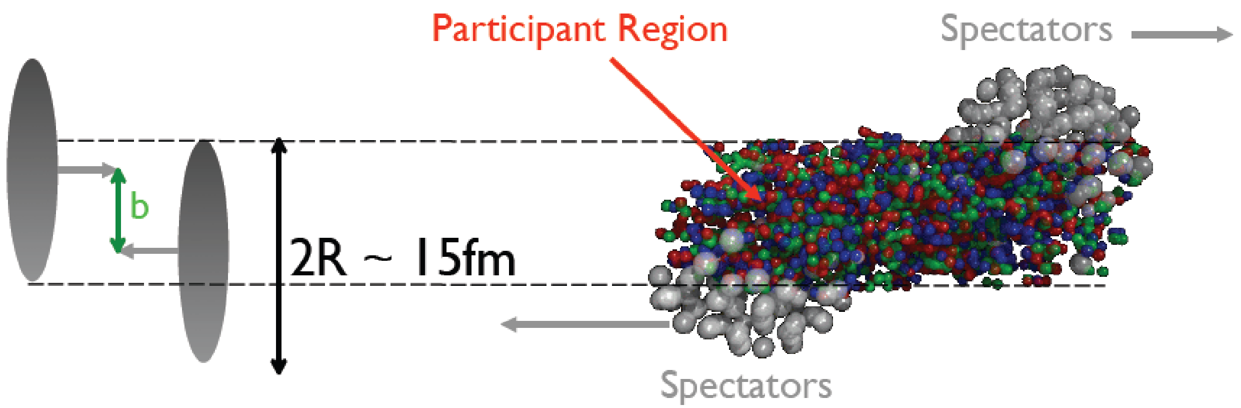
- It is difficult/ not always possible to measure E and/ or p in an experiment
- Energy measurement requires calorimetry or with trackers, the precise identification of secondary particles (PID) with momentum information
- Introduce pseudorapidity: will be discussed
- The notion of positive and negative rapidity is purely a convention
- The positive or negative rapidities correspond to positive or negative velocities of a particle with respect to a chosen axis
- As rapidity has a logarithmic dependence on energy and momentum of a particle, the magnitude of rapidity is rather small, even for the most energetic particle
- The range of rapidity, in principle is: $-\infty \leq y \leq +\infty$

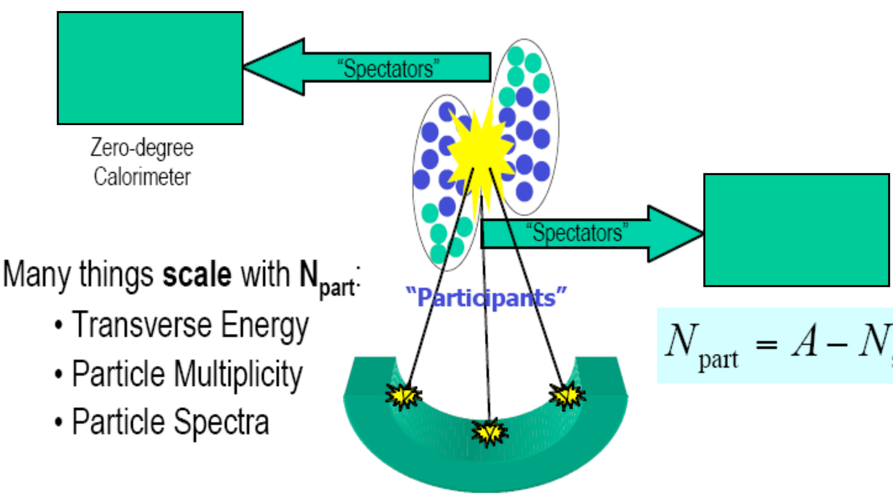
Rapidity: Properties

- In a collider geometry, the rapidity variable makes annular rings on the detector plane, having higher values towards the beam pipe
- Both rapidity, y and the azimuthal angle, ϕ, span the detector plane
- Rapidity formulation simply and naturally incorporates the Lorentz transformation properties of velocity
- The differences in rapidities are Lorentz Invariants
- This is also true for the differential element of rapidity, dy

Collision Geometry

- The ***impact parameter*** (b) determines the number of nucleons that participate in the collision (N_{part})



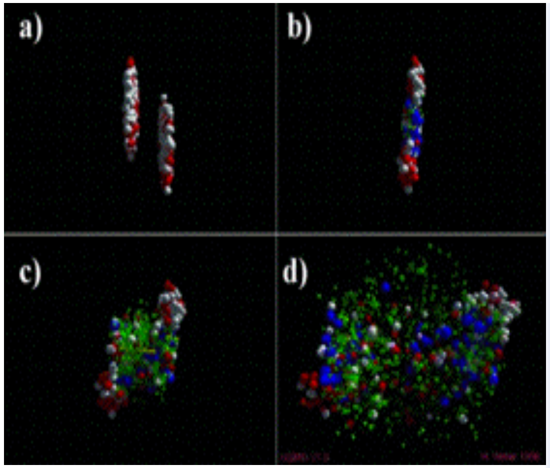


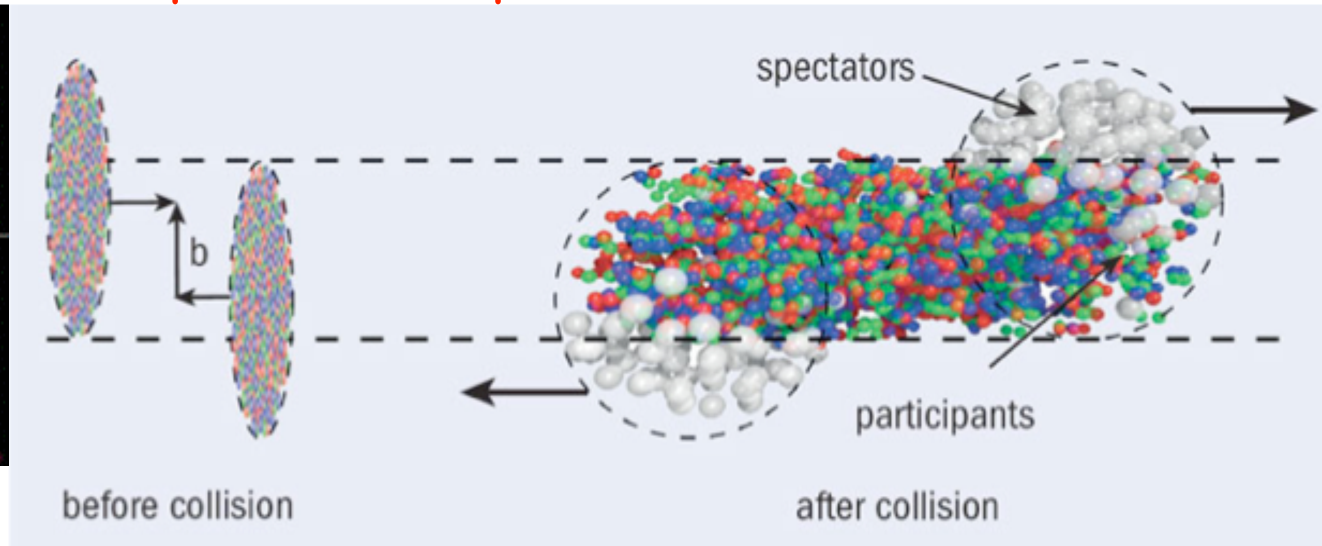
$$N_{part} = A - N_{spec}$$

Small impact parameter
→Many participant nucleons
→Big System (fireball)
→Many produced particles

Large impact parameter
→Few participant nucleons
→Small System (fireball)
→Few produced particles

Participants and Spectators

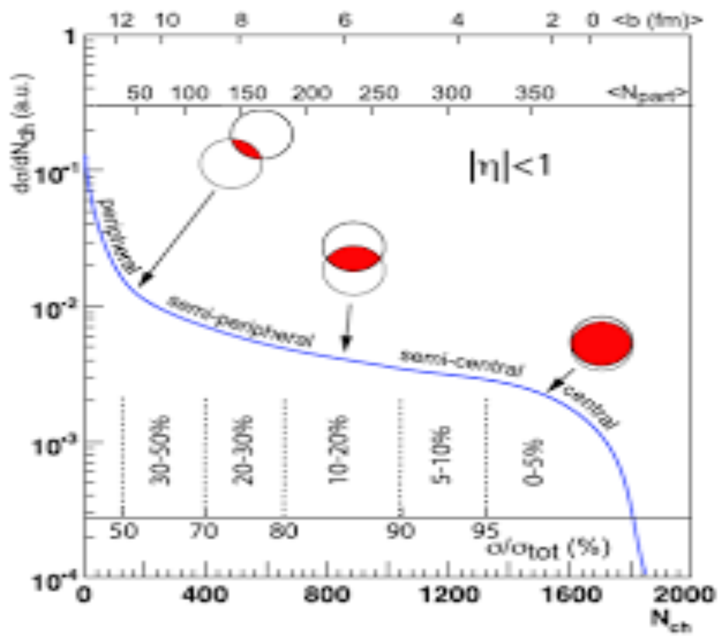




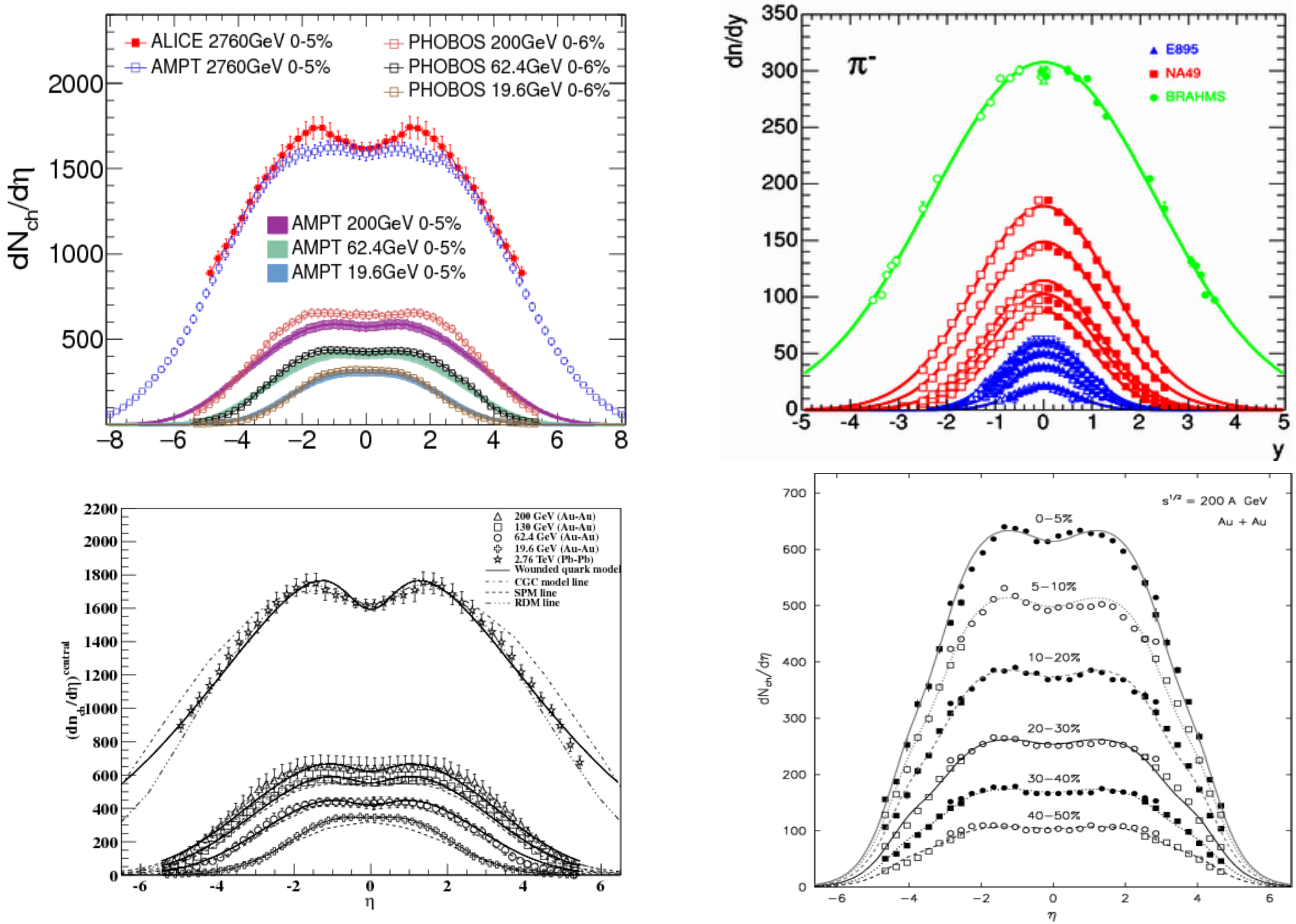
N_{part} : **Number of participant hadrons**
N_{coll} : **Number of binary collisions**

Both are estimated in Glauber model: MC Glauber, Optical Glauber... : See the lectures by P. Shukla

"Global Properties of Nucleus-Nucleus Collisions."
Michael Kliemant, Raghunath Sahoo , Tim Schuster and Reinhard Stock.
The Physics of Quark-Gluon Plasma: Introductory Lectures, Lecture Notes in Physics, Vol. 785, 23-103 (2010). Springer-Verlag Publication



Energy and Centrality dependence of rapidity distribution



Pseudorapidity

- ✓ Assume that the particle is emitted with an angle θ wrt the beam axis.
- ✓ Then we can rewrite the rapidity as:

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

- ✓ At very high energies, when *p* >> *m*

$$y = \frac{1}{2}\ln\left[\frac{p+p\cos\theta}{p-p\cos\theta}\right] = -\ln\left(\tan\theta/2\right) \equiv \eta$$

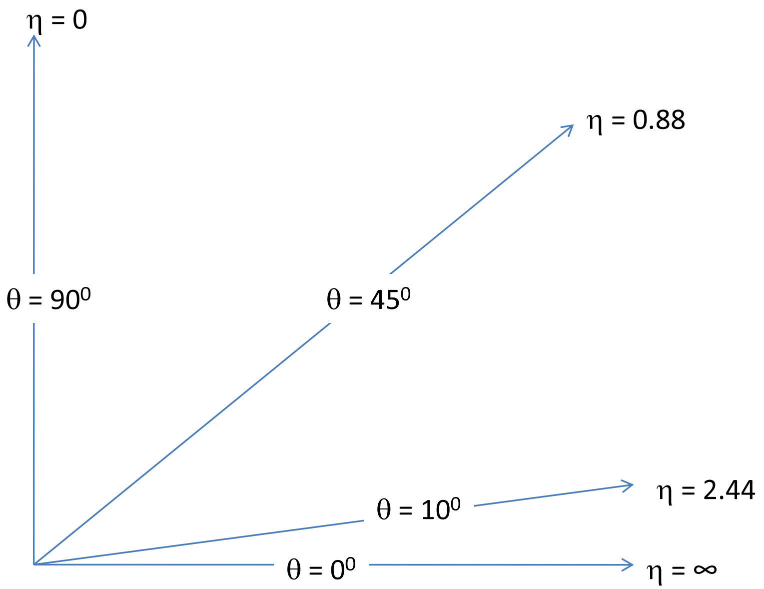
- η is called the pseudorapidity variable and mostly used in collider experiments
- For this, particle identification is not necessary
- Knowing the hit position is good enough

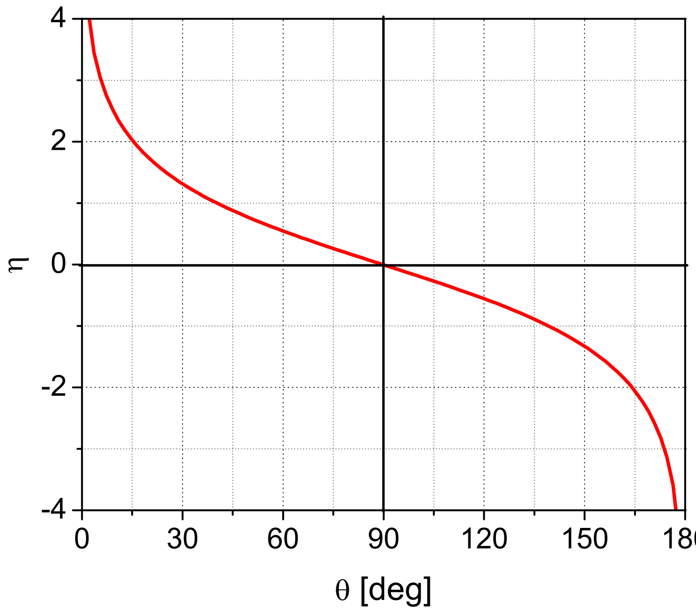
- Hence at very high energies, $\boxed{y \approx \eta = -ln\left[tan(\theta/2)\right]}$

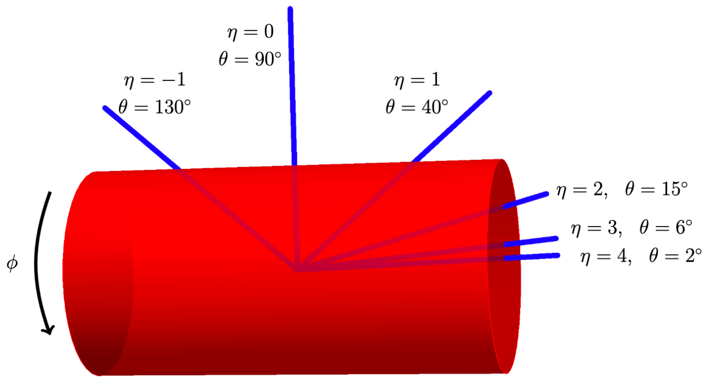
- In terms of particle momentum, η could be rewritten as: $\eta = \frac{1}{2}\ln\left[\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right]$

Pseudorapidity

- Pseudorapidity is defined for any value of mass, momentum and energy
- Could be measured with/without momentum information which needs a magnetic field.





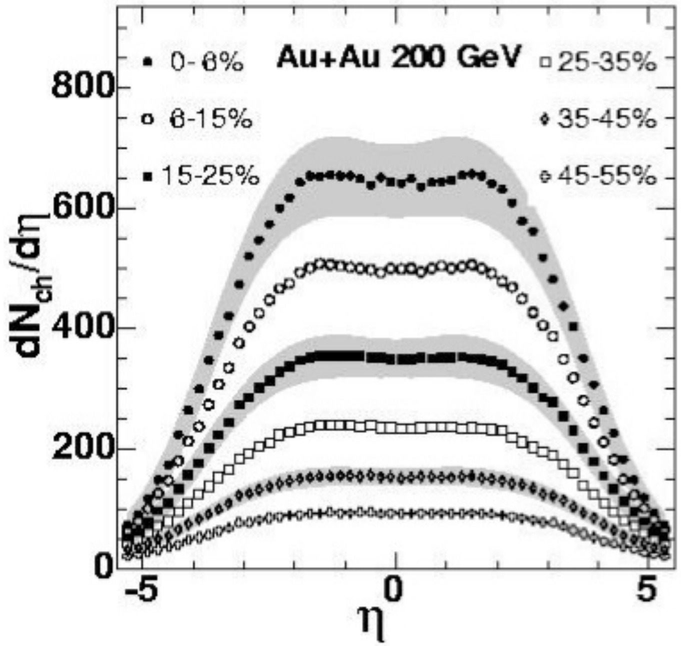


Pseudorapidity

Change of variables from (y, p_T) to (η, p_T)

$$\eta = \frac{1}{2}\ln\left[\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right]$$

$$\Rightarrow e^{\eta} = \sqrt{\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}}, \qquad e^{-\eta} = \sqrt{\frac{|\vec{p}|-p_z}{|\vec{p}|+p_z}}$$



The mid-rapidity $dN_{ch}/d\eta$ for Au+Au collisions at $\sqrt{s_{NN}}$ =200 GeV.

Adding both the equations we get:

$$|\vec{p}| = p_T \cosh\eta$$

Subtracting both the equations we get:

$$p_z = p_T \sinh\eta$$

Using these equations in the definition of rapidity we get:

$$y = \frac{1}{2}\left[\frac{\sqrt{p_T^2\cosh^2\eta + m^2} + p_T\sinh\eta}{\sqrt{p_T^2\cosh^2\eta + m^2} - p_T\sinh\eta}\right]$$

Pseudorapidity

Similarly η can be expressed in terms of y as,

$$\eta = \frac{1}{2}\left[\frac{\sqrt{m_T^2\cosh^2 y - m^2} + m_T \sinh y}{\sqrt{m_T^2\cosh^2 y - m^2} - m_T \sinh y}\right]$$

The distribution of particles as a function of rapidity is related to the distribution as a function of η by the formula:

$$\frac{dN}{d\eta d\vec{p}_T} = \sqrt{1 - \frac{m^2}{m_T^2\cosh^2 y}}\,\frac{dN}{dy d\vec{p}_T}$$

Note:

- In the region y >> 0, dN/dη and dN/dy which are essentially the p_T–integrated values, are the same.
- In the region y ≈ 0, there is a small "*depression*" in dN/dη distribution compared to dN/dy distribution, due to the above transformation. Called "mass dependent suppression".
- At very high energies, where a mid-rapidity plateau is expected for dN/dy distribution, this transformation gives a small dip in dN/dη distribution around η≈ 0.

Pseudorapidity

Note:

- For massless particles like photon, this dip in dN/dη is not expected: could be seen from the above equation.

$$\frac{dN}{d\eta d\vec{p}_T} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN}{dy d\vec{p}_T}$$

- Independent of the frame of reference where η is measured, the difference in the maximum magnitude of dN/dη appears due to the above transformation.

- In CM system, the maximum of the distribution appears at $y \approx \eta = 0$ and the η-distribution is suppressed by a factor:

$$\sqrt{1 - \frac{m^2}{\langle m_T^2 \rangle}}$$

- In LS, the maximum is located around half of the beam rapidity i.e. $\eta \approx y_b/2$ and the η-distribution is suppressed by a factor:

$$\sqrt{1 - \frac{m^2}{\langle m_T^2 \rangle} \cosh^2 (y_b / 2)}$$

which is about unity.

Pseudorapidity

Note:

❖ We know that the shape of the rapidity distribution is frame independent. However, the peak value of η-distribution in the CM frame is lower than its value in LS. This suppression factor at SPS energies is ~ 0.8-0.9.

❖ The Jacobian associated with the conversion of $\eta \leftarrow\rightarrow y$, $J(y,\eta)$ is the multiplier on RHS of the equation, which depends on the momentum distribution of the produced particles.

$$\frac{dN}{d\eta d\vec{p}_T} = \sqrt{1 - \frac{m^2}{m_T^2 \cosh^2 y}} \frac{dN}{dy d\vec{p}_T}$$

❖ $J(y,\eta) = 1$ for $m \ll p$

❖ $\langle p_T \rangle$ increases with collisions energy: $J(y,\eta)$ is smaller at LHC ($J = 1.09$) energies, compared to that of RHIC ($J = 1.25$).

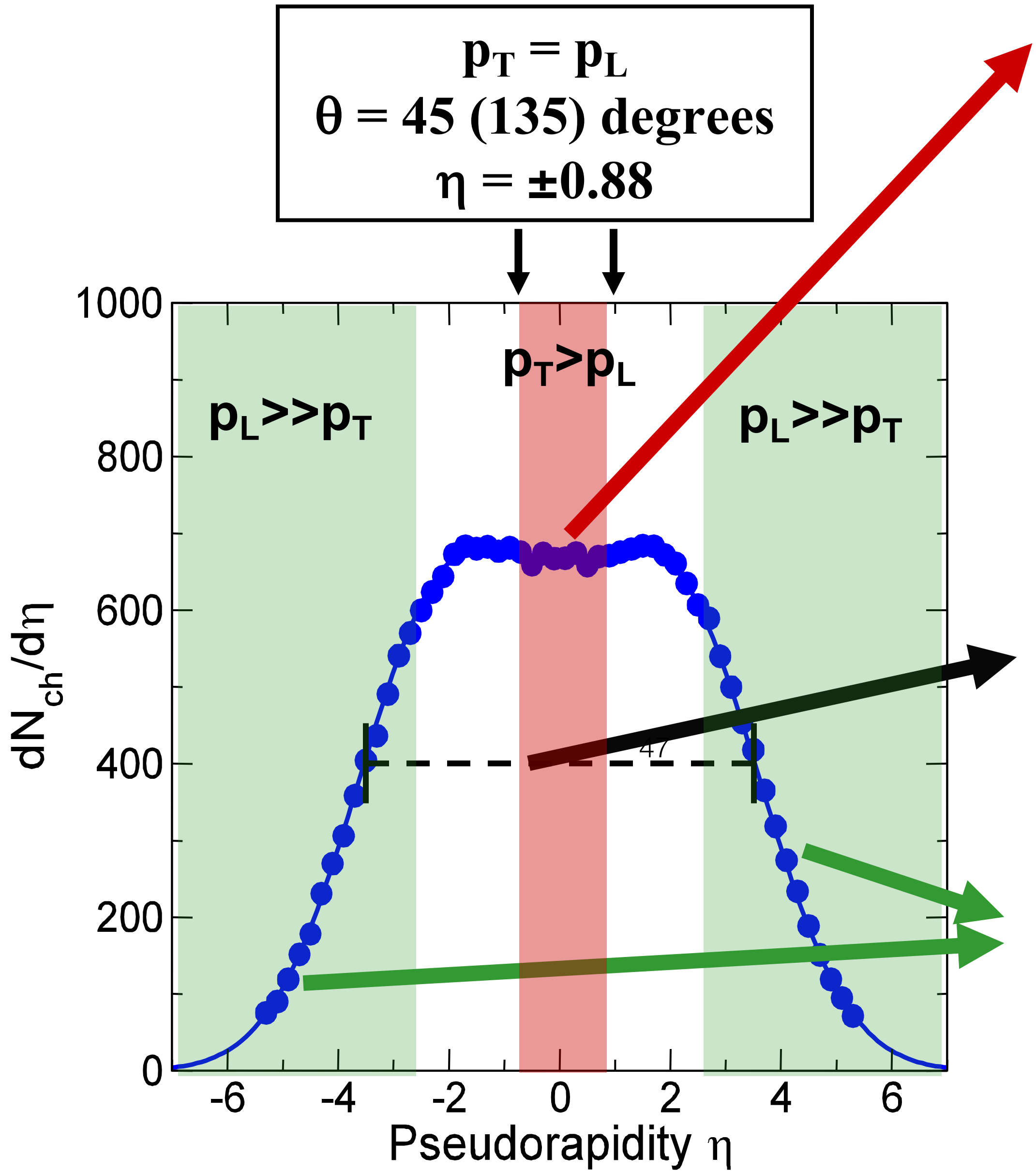
❖ Rewriting the top eqn. and integrating over p_T, one obtains,

$$\frac{dN}{d\eta} = v(y) \frac{dN}{dy}$$

$v(y)$: velocity of the particle which depends on rapidity.

❖ At mid-rapidity, $y = \eta = 0$, $\frac{dN}{d\eta}\Big|_{\eta=0} = v \frac{dN}{dy}\Big|_{y=0}$

Particle Production: Pseudorapidity Distribution



- **Midrapidity peak / plateau**
  - Sensitive to hadroproduction details
  - Related to energy density

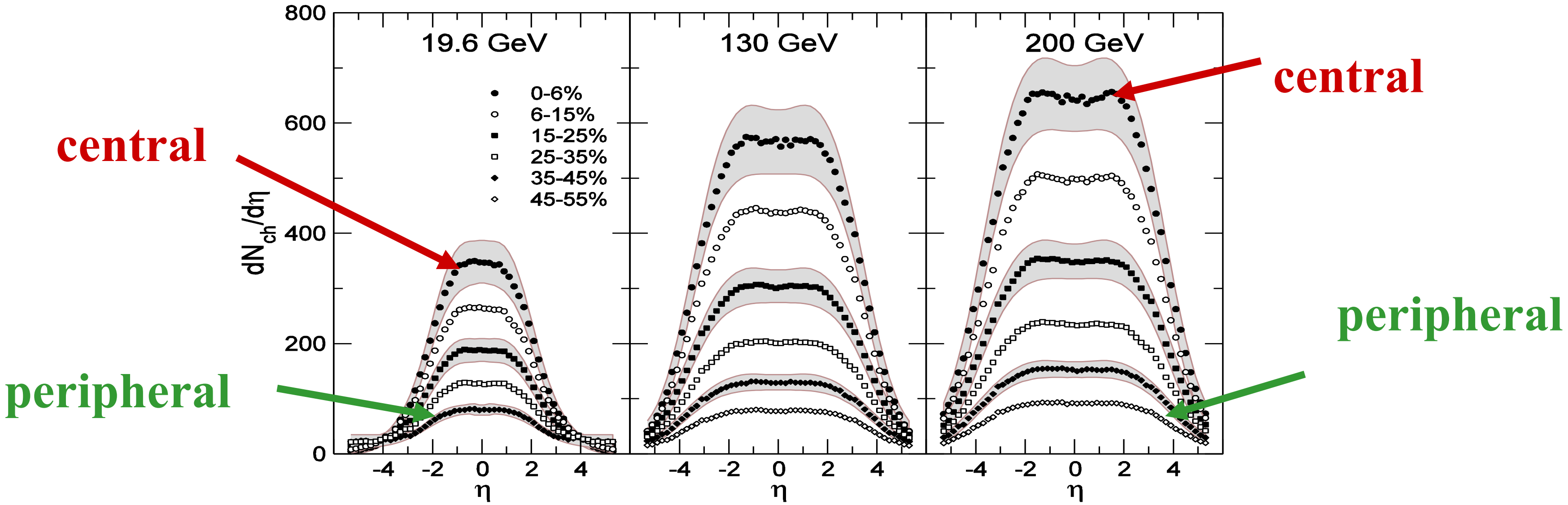
  Bjorken formula (requires a "central-plateau structure" in the y distribution of produced particles)

$$\varepsilon_{BJ} = \frac{\langle m_T \rangle}{Ac\tau_f}\left(\frac{dN}{dy}\right)_{y=0}$$

- **Width of the distribution**
  - Information on longitudinal expansion, stopping vs. transparency, Speed of Sound (EoS)
- **Fragmentation regions**
  - Investigate effects connected with target and projectile fragmentation
  - Hypothesis of Limiting Fragmentation

Particle Production: Pseudorapidity Distribution

- The maximum of pseudorapidity distribution ($dN_{ch}/d\eta \mid_{max}$) at η_{cm}=0:
  - Most frequently used variable to characterize the multiplicity of the interaction
    - Independent of phase space acceptance → allows comparison between different experiments
  - Increases with collision energy ($\sqrt{s}$) and centrality



(Pseudo)rapidity Distributions

- ✓ Rapidity spectra is sometimes parameterized using the following eqn. to extract meaningful physics message.

$$\frac{dN}{dy} = \frac{\langle N \rangle}{2\sqrt{2\pi\sigma^2}} \left\{ \exp.\left[-\frac{1}{2}\left(\frac{y-y_0}{\sigma}\right)^2 \right] + \exp.\left[-\frac{1}{2}\left(\frac{y+y_0}{\sigma}\right)^2 \right] \right\}$$

- ✓ <N>, σ and y_0 are fitting parameters and σ is the width of the rapidity distribution

- ✓ Landau's energy dependent Gaussian rapidity distribution is given by

$$\frac{1}{\sigma_{in}}\frac{d\sigma}{dy} = \frac{dN}{dy}$$

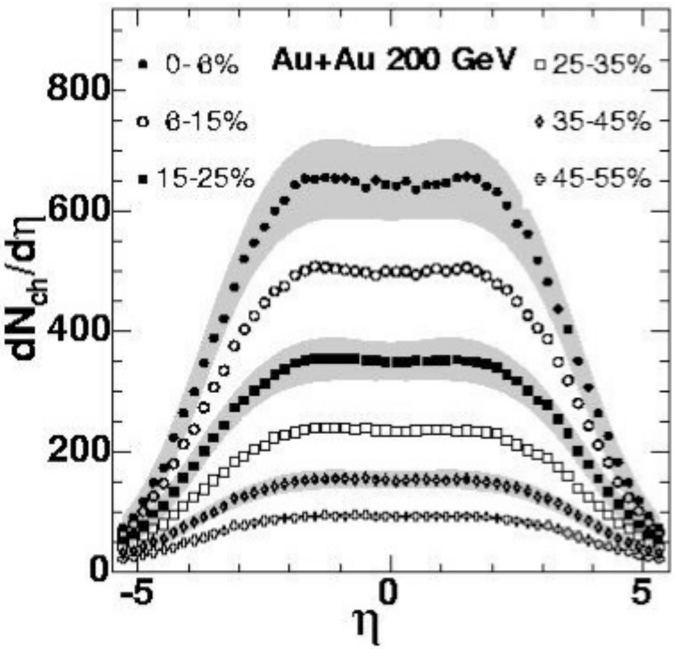
$$= \frac{N}{(2\pi L)^{1/2}} \exp.\left(-y^2/2L\right)$$

where

$$L = \frac{1}{2}\ln\left(s/4m_p^2\right)$$

$$= \ln\gamma$$

$$= \ln\left(\sqrt{s_{NN}}/2m_p\right)$$



(Pseudo)rapidity Distributions

Comparing Landau's equation with the parameterized eqn. we get:

$$\boxed{\sigma_y = \sqrt{ln\left[\sqrt{s_{NN}}/2m_p\right]}}$$

- Width of the rapidity distribution is related to the boost factor.
- The width of the rapidity distribution is related to the longitudinal flow and velocity of sound in the medium.
- Hence could be a useful probe of equation of state of the produced matter.
- With the assumption that the velocity of sound, c_s is independent of temperature, the rapidity density in Landau hydrodynamical picture is given by:

$$\frac{dN}{dy} = K \frac{s_{NN}^{1/4}}{\sqrt{2\pi\sigma_y^2}} \exp.\left(-\frac{y^2}{2\sigma_y^2}\right)$$

K is a normalization factor

where

$$\sigma_y^2 = \frac{8}{3} \frac{c_s^2}{1-c_s^4} \ln\left(\sqrt{s_{NN}}/2m_p\right)$$

(Pseudo)rapidity Distributions

$$\Rightarrow \boxed{\sigma_y^2 = \frac{8}{3}\frac{c_s^2}{1-c_s^4}\, ln\ \gamma}$$

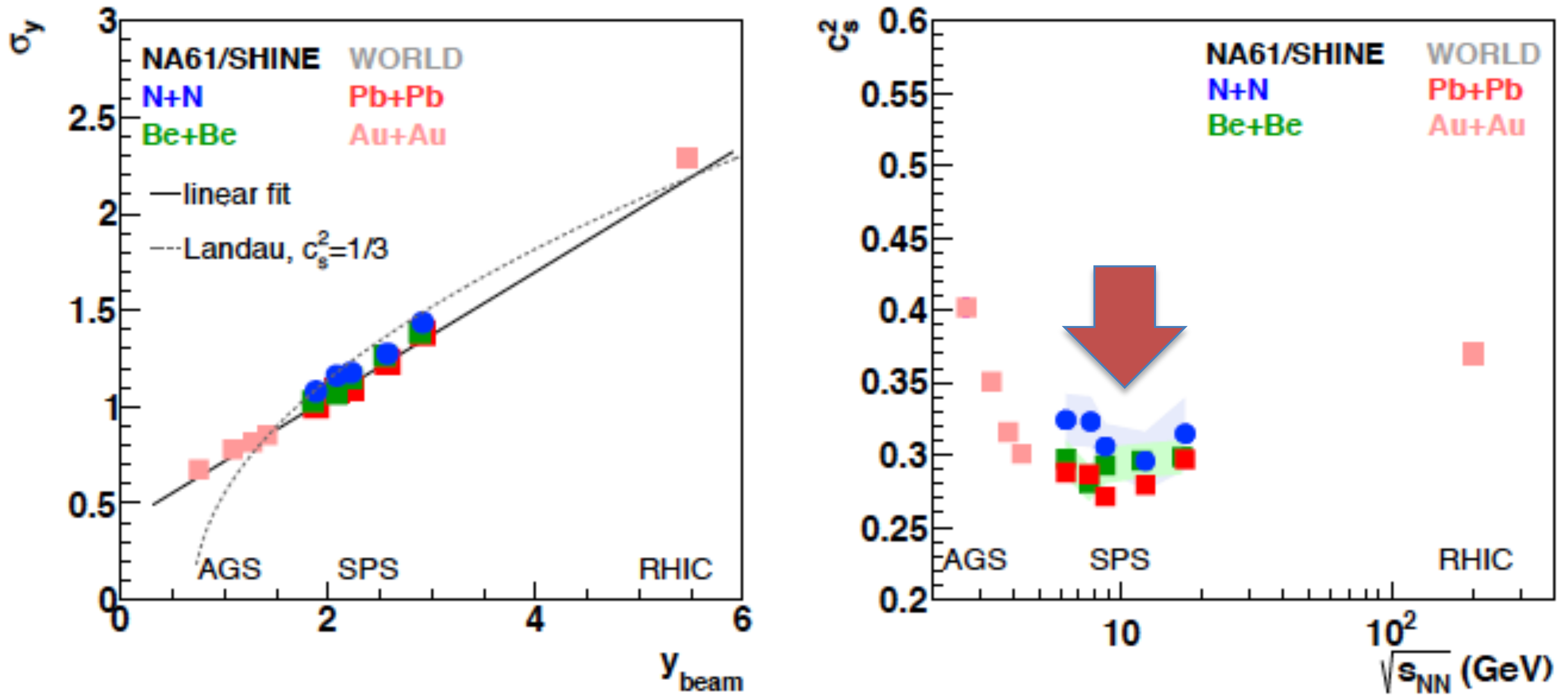
Width of the rapidity distribution helps us in estimating the velocity of sound in the medium and hence in probing the EoS.

- For an ideal gas (Landau's prediction), $c_s^2 = 1/3$.
- For a hadron gas, $c_s^2 = 1/5$ → *expansion of a hadron gas is slower compared to an ideal gas.*
- The Equation of State (EoS) is given by: $$\frac{\partial P}{\partial \varepsilon} = c_s^2$$

where P is the pressure and ε is the energy of the system.

- As the expansion of the matter proceeds as longitudinal and superimposed transverse expansions, a rarefaction wave moves radially inwards with the velocity of sound.

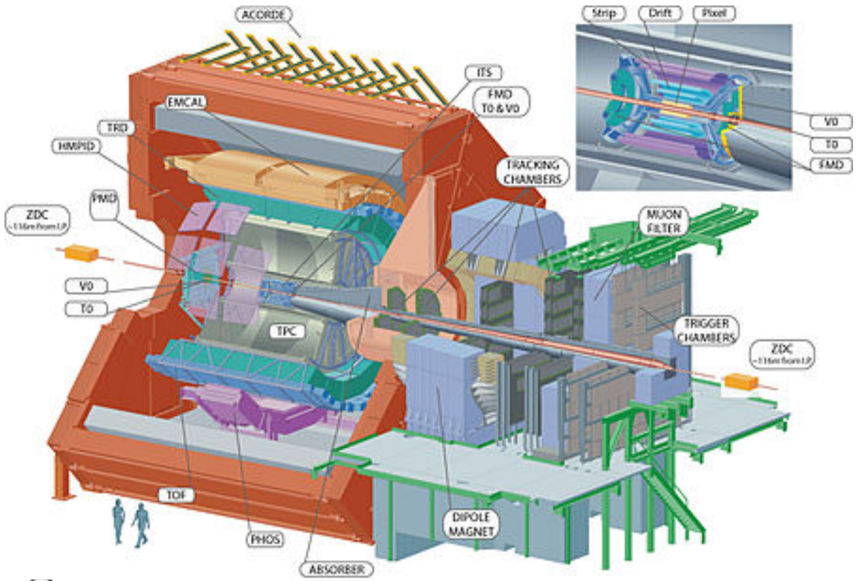
(Pseudo)rapidity Distributions

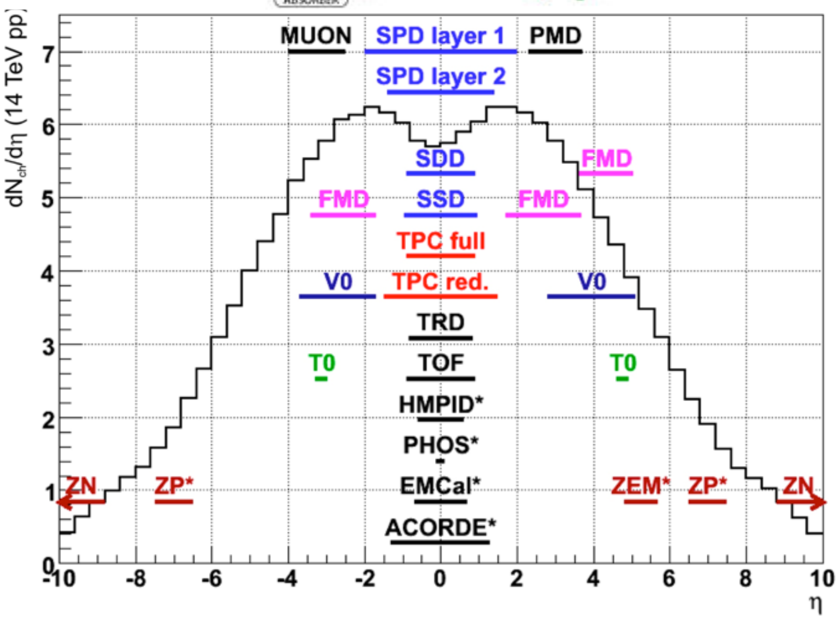


The NA61/SHINE Collaboration : arXiv:2008.06277

The softest point in the EoS could be a signature of deconfinement transition.

Phase Space of ALICE Experiment





- Central Barrel (-0.9 < h < 0.9)
  - tracking, PID (ITS, TPC, TRD, TOF)
  - single arm RICH
  - single arm EM cal (PHOS)
  - Elecromagnetic Calorimeter
- Multiplicity
  - charged (barrel+FMD): (-5.4 < h < 3)
  - photons in PMD (-2.3 < h < -3.7)
- Forward muon arm (2.4 < h < 4.0)
  - absorber, dipole magnet
  - tracking and trigger chambers
- Trigger, timing, luminosity:
  - ZDC, V0, T0, SPT

Invariant Yield

- Cross sections are the probabilities of nuclear reactions expressed by effective areas.

- The cross section, $\boldsymbol{\sigma}$ is measured in the units of barn or millibarn (1 barn = 10^{-24} cm^2 = 100 $fermi^2$, 1 mb = 0.1 $fermi^2$).

- This describes the total yield of a reaction regardless of energies of the emitted particles or of their spatial distributions.

- Differential Cross Section: For example, $d\boldsymbol{\sigma}/dE$ (mb/GeV) or $d\boldsymbol{\sigma}/d\theta$ (mb/radian) are used to study the energy and spatial distributions of the emitted particles, respectively.

- Double Differential Cross Section: For example, $d^2\boldsymbol{\sigma}/dEd\theta$ - at a given E and θ, dN number of particles are emitted into an angular region between θ and $\theta + d\theta$, whose energies lay between E and E + dE.

Invariant Yield

- Subdivisions of nuclear cross sections into smaller parts or in other words making it higher order differential cross section, would require, what information need to be derived from the study/measurement.

- Here, one may consider taking independent variables like v, E, p_T, θ, y, η, etc. to study the reaction cross section.

- σ is a Lorentz Invariant quantity. However, differential cross sections may or may not be invariant. For instance, $d\sigma/dE$ and $d\sigma/d\Omega$.

- As the Lorentz transformation of common differential cross sections are more often cumbersome and difficult, one thus needs the use of invariant cross section.

- We know that the energy, E and the momentum, p taken independently are not Lorentz Invariants.

- However, a suitable combination of E and p i.e. , the energy-momentum four-vector is Lorentz Invariant. The length of E-p four-vector is Lorentz Invariant, as it equals to the rest mass of a particle.

$$\sqrt{E^2 - p^2} = m^2$$

Invariant Yield

$$\frac{d^3\sigma}{dp_x\ dp_y\ dp_z} = \frac{d^3\sigma}{dp^3}$$

where dp^3 is the elementary volume in momentum space.

- As the changes in E and $d^3\boldsymbol{\sigma}/dp^3$ cancel out under Lorentz transformation, $Ed^3\boldsymbol{\sigma}/dp^3$ ($\boldsymbol{\sigma}_{inv}$) is Lorentz Invariant.

- The rapidity variable has the useful property that it transforms linearly under a Lorentz transformation so that the invariant differential single particle inclusive cross section becomes:

$$\frac{Ed^3\sigma}{dp^3} = \frac{Ed^3\sigma}{p_T dp_T dp_L d\phi} = \frac{d^3\sigma}{p_T dp_T dy d\phi},$$

where $dy = \frac{dp_L}{E}$

because: $p_z = m_T \sinh y$,
$dp_z = m_T \cosh y\ dy = p_0\ dy$

Invariant Yield

One can proceed to show that $\frac{d^3p}{E}$ is L.I.

In terms of experimentally measurable quantities, $\frac{d^3p}{E}$ could be expressed as:

$$\frac{d^3p}{E} = d\vec{p}_T dy$$
$$= p_T dp_T d\phi dy$$
$$= m_T dm_T d\phi dy$$

The Lorentz invariant differential cross-section $\frac{Ed^3\sigma}{dp^3} = \frac{Ed^3N}{dp^3}$ is the invariant yield and could be shown in terms of experimentally measurable quantities as:

$$\frac{Ed^3\sigma}{dp^3} = \frac{1}{m_T}\frac{d^3N}{dm_T d\phi dy}$$
$$= \frac{1}{2\pi m_T}\frac{d^2N}{dm_T dy}$$
$$= \frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}$$

To measure the invariant yields of identified particles (known as p_T or m_T -spectra), the above equation is used.

Invariant Yield

$$\frac{Ed^3\sigma}{dp^3} = \frac{1}{m_T}\frac{d^3N}{dm_T d\phi dy}$$

$$= \frac{1}{2\pi m_T}\frac{d^2N}{dm_T dy}$$

$$= \frac{1}{2\pi p_T}\frac{d^2N}{dp_T dy}$$

- To have a statistically significant number in the differential yield, one combines many events of similar nature, e.g., say central events or peripheral events or events of a particular centrality class. This requires at the end, an event normalization, which changes the above equation to:

$$\boxed{\frac{Ed^3N}{dp^3} = \frac{1}{N_{\mathrm{evt}}}\frac{1}{2\pi\ p_T}\frac{d^2N}{dp_T dy}}$$

This formula is used to measure the invariant yields of identified particles experimentally.

Invariant Yield

Caveat:

- The invariant cross section and the invariant yield are not necessarily the same, as it is used more frequently in an erroneous way.
- One lands up with a dimension crisis, while trying to equalize the both.

- The connection: When we calculate the differential cross-section from differential yield, we need to divide the integrated luminosity to the differential yield,

$$\boxed{\frac{d^2\sigma}{2\pi\ p_T dp_T dy} = \frac{1}{L_{int}} \frac{d^2 N_{\mathrm{yield}}}{2\pi\ p_T dp_T dy}}$$

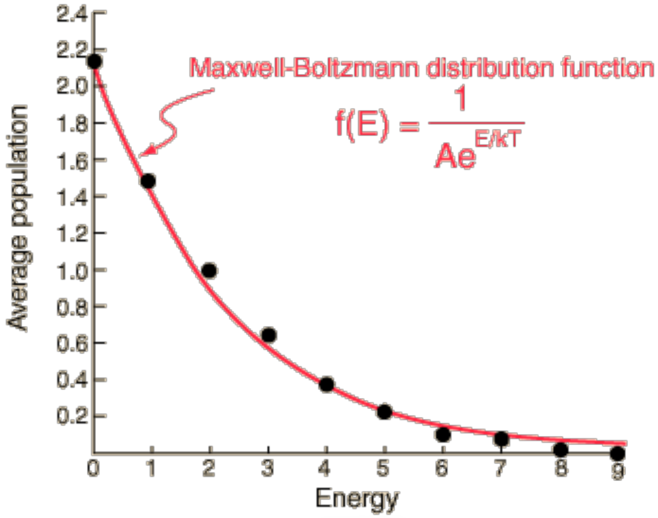
- N_{yield} the number of particles and L_{int} is the integrated luminosity, given by

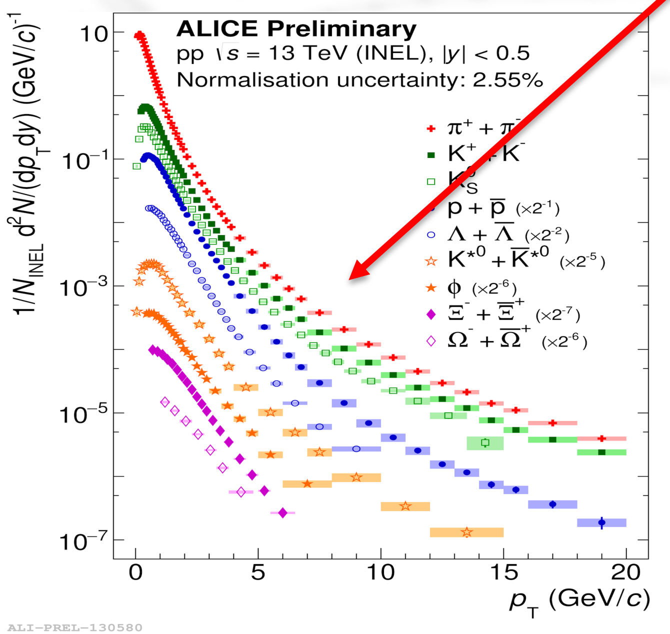
$$L_{int} = \int L_{insta}\ dt$$

L_{insta} is the instantaneous luminosity

Will come back to luminosity later !

Invariant Yield





Boltzmann Distribution:

$$f(p_{\mathrm{T}}) = A p_{\mathrm{T}}\, e^{-m_{\mathrm{T}}/T}$$

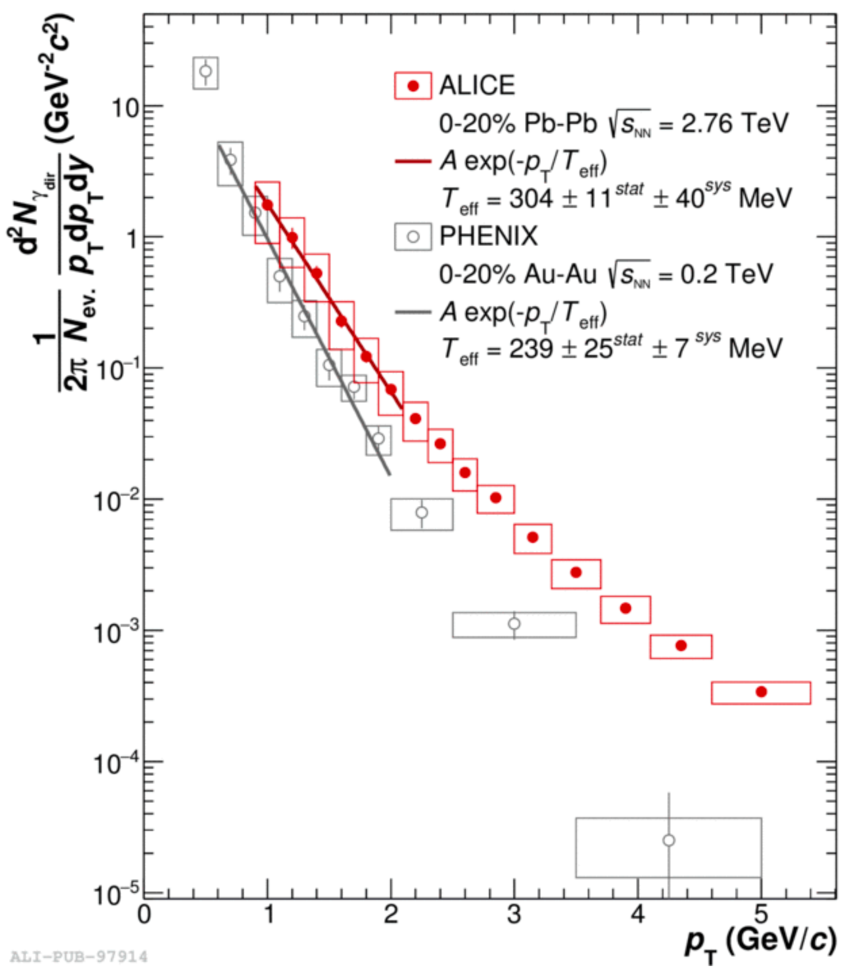
Particle yield exponentially drops down with increase of mass.

Many different functions are fitted to the spectra: Boltzmann, Fermi-Dirac, Bose-Einstein, Tsallis etc.

Note:

- low-p_T part is dominated by soft processes, high-p_T has pQCD contributions
- Spectra deviates from Boltzmann-type
- Follows Levy-Tsallis's distribution

Invariant Yield: Extracting Kinetic FO



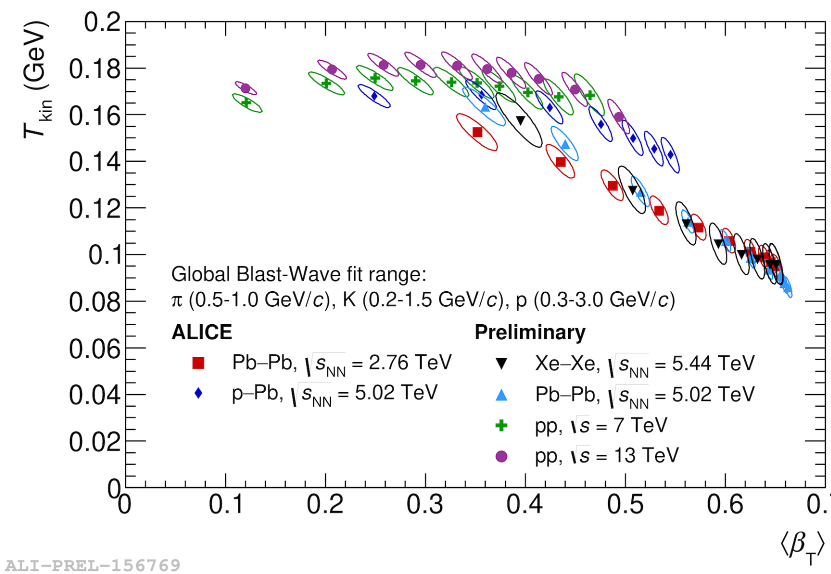
$$T_{\mathrm{eff}} = T_{\mathrm{th}} + 0.5m < \beta >^2$$

Inverse slope

Random thermal motion

Collective radial flow

Hardening of the spectra: higher T_{eff}



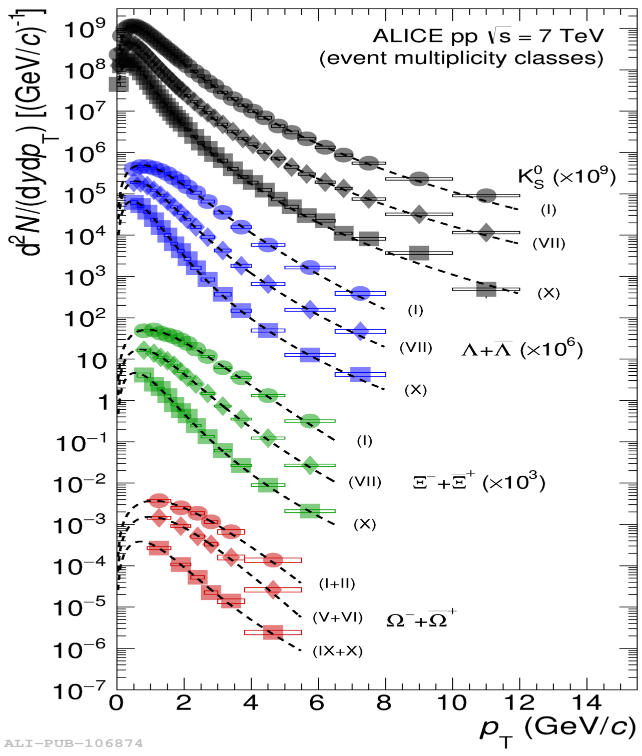
Invariant Yield: Extracting Kinetic FO

(Getting a common FO temperature and radial flow velocity)

- Hydrodynamically motivated Blast Wave Model (assumes thermalization)

$$\frac{dN}{p_T\,dp_T} \propto \int_0^R r\,dr\,m_T I_0\left(\frac{p_T \sinh\rho(r)}{T_{\rm kin}}\right) \times K_1\left(\frac{m_T \cosh\rho(r)}{T_{\rm kin}}\right),$$

- Make a simultaneous fit to the identified particle p_T-spectra (p_T ~ 2-3 GeV/c)



Simultaneous fit:

T_{fo} = 163 ± 10 MeV,

$\langle\beta_T\rangle$ = 0.49 ± 0.02

ALICE: Nature Phys. 13 (2017) 535

Invariant Yield: Extracting Chemical FO

- Experimental measurements are with a low-p_T cut-off
- Fit a well-described function in the range of p_T =0 to p_T^{max}
- Integrate the function (bin counting) in the range to get $dN_{ch}/d\eta$ or for identified particles dN/dy

$$\int_0^{p_T^{max}} \text{Spectra } dp_T = \frac{dN}{dy(\eta)}$$

- Get <mT> or <pT> and use the information to study Bjorken energy density, order of the phase transition (van Hove signal) etc.
- Use GCE for pp high-multiplicity, pA and AA collisions to extract T and μ_B
- Using T and μ_B estimate other thermodynamic observables

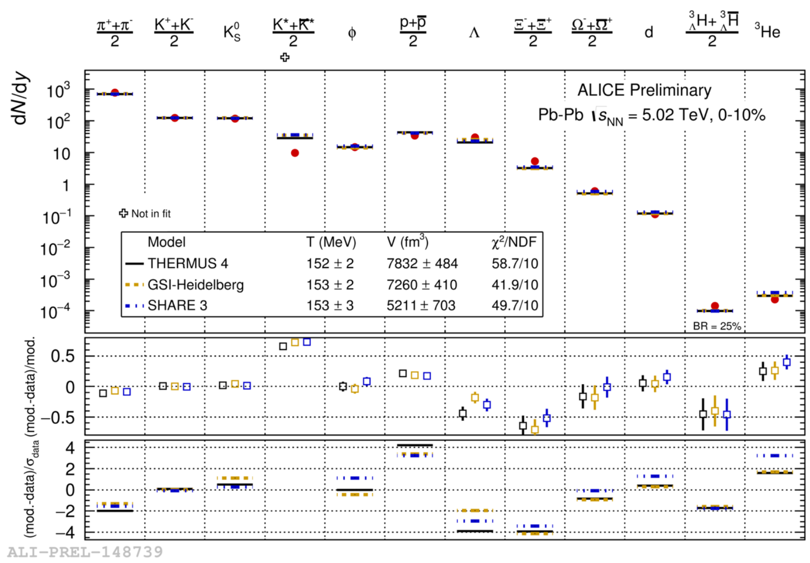
For a thermalized system, the number density of i^{th} particle species in GCE is obtained from the partition function as:

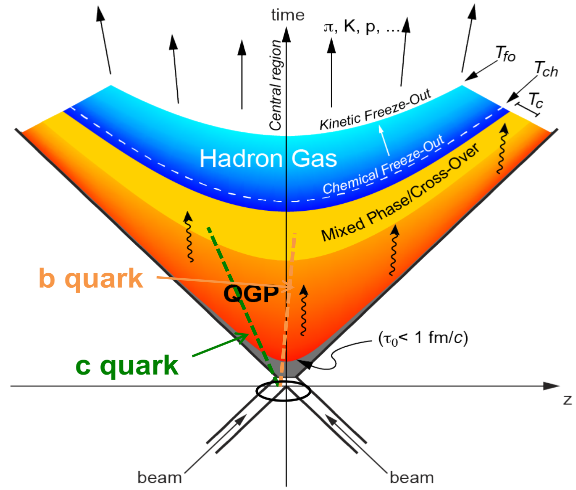
$$n_i = \frac{T}{V}\left(\frac{\partial \ln Z_i}{\partial \mu_i}\right)_{V,T} = \frac{g_i}{(2\pi)^3}\int \frac{d^3p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$$

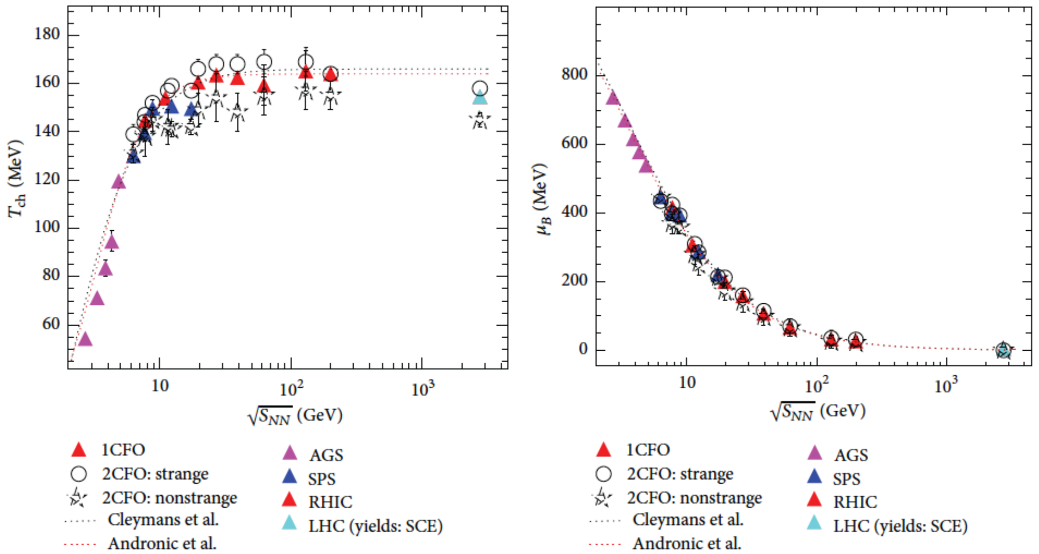
For details: see the lectures by Prof. Jean Cleymans

Invariant Yield: Extracting Chemical FO



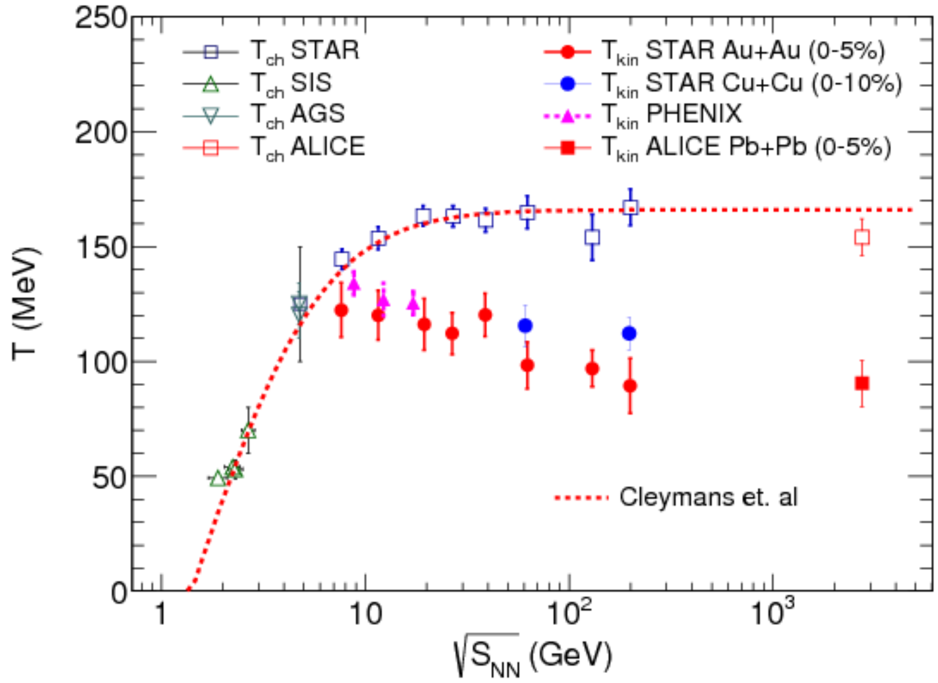


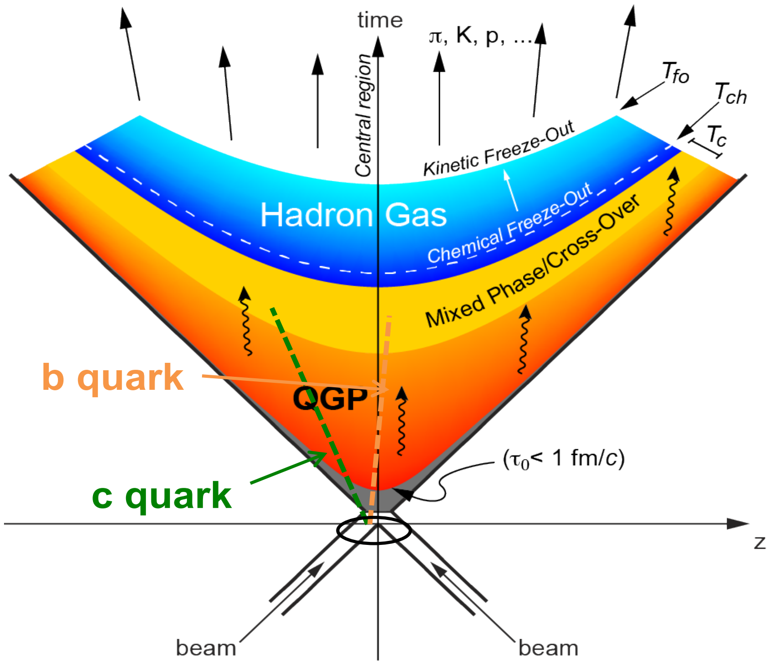
Pb-Pb at 5.02 TeV



Review on KFO & CFO: Sandeep Chatterjee, Sabita Das, Lokesh Kumar, D. Mishra, Bedangadas Mohanty, **Raghunath Sahoo,** and Natasha Sharma
Advances in High Energy Physics, Volume 2015, Article ID 349013 (2015)

Spectra: Indication of a finite Hadronic Phase Lifetime





Review: "Freeze-out Parameters in Heavy-Ion Collisions at AGS, SPS, RHIC and LHC Energies."
S. Chatterjee, S. Das, L. Kumar, D. Mishra, B. Mohanty, Raghunath Sahoo, and N. Sharma, Adv. in High Energy Physics (AHEP) (2015) Vol. 2015, Article ID 349013,

Luminosity

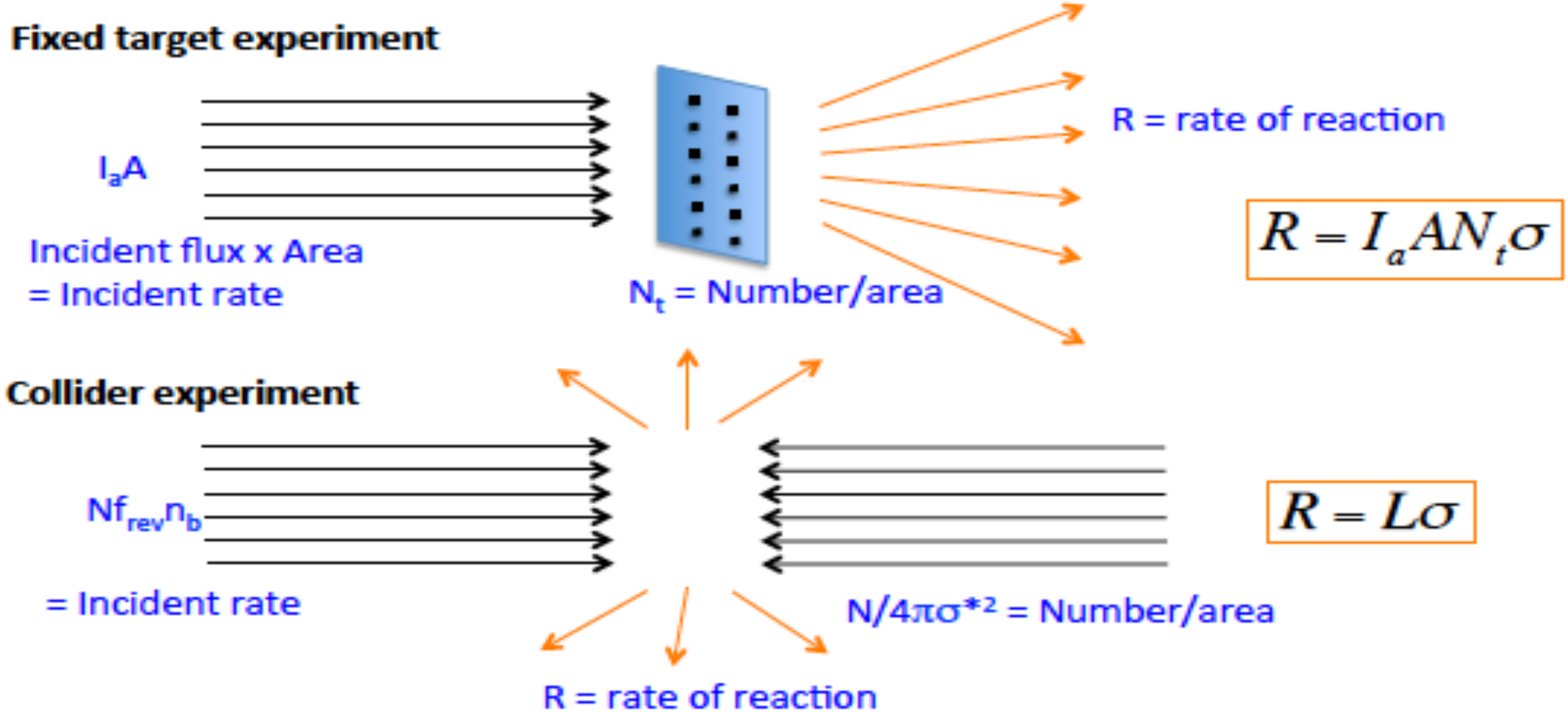
- Luminosity is defined as a quantity which measures the ability of a particle accelerator to produce the required number of interactions.
- This is an accelerator specific parameter.
- The number of useful interactions (events) becomes important especially when rare events with smaller production cross sections (σ_p) are studied.
- The relationship between rate of the interaction and cross section is:

$$\frac{dR}{dt} = \mathcal{L}.\sigma_p$$

$\mathcal{L}$ is in $cm^{-2}s^{-1}$

Luminosity

Cross section, Luminosity, Reaction Rate



$$\mathcal{L} = \frac{N^2 f_{\text{rev}}\, n_b}{4\pi\sigma^{*2}}$$

Depends on f_{rev} revolution frequency
n_b number of bunches
N number of particles/bunch
σ^* beam size or rather overlap integral at IP

4

Luminosity in Fixed Target Experiment

Example 6.10 Luminosity Estimation for a Fixed Target Experiment:
In a fixed target experiment, liquid hydrogen target is used, which is charcterized by a target density, $\rho = 0.072\ gm.cm^{-3}$ with a target thickness of 15 cm. The beam flux is given by $N_{\text{beam}} = 10^{11}$ protons/sec. Estimate the luminosity for this set up.

Solution
Here the number of target protons is given by: $n = 6 \times 10^{23} \times 0.072 \simeq 0.432\ cm^{-3} \times 10^{23}$.
Hence the luminosity is given by:
$\mathcal{L}_{FT} = N_{\text{beam}}\ n.\ l = 10^{11} \times 0.432\ \times 10^{23} \times 15 = 6.48 \times 10^{34}\ cm^{-2}.sec^{-1}$.

Ref: Relativistic Kinematics for Beginners:
Raghunath Sahoo & Basanta K. Nandi
(CRC Press- 2020)

Luminosity and Event Statistics

Example 6.11 Effect of Accelerator Luminosity on Event Statistics
The Large Hadron Collider (LHC) is a 40 MHz particle collider. We consider proton+proton collisions at a center-of-mass energy, $\sqrt{s} = 13$ TeV, with 1.15×10^{11} protons/bunch and 2808 bunches/beam with a beam size of 16 μm at the interaction point. Calculate the luminosity of the LHC machine for this configuration. If you are interested in a rare process having cross section, $\sigma_p = 1$ femto barn ($=10^{-39}$ cm^2), to bound the statistical errors by 2%, how much time the LHC has to take data?

Solution

Here, $n_1 = n_2 = 1.15 \times 10^{11}$ protons/bunch,

$n_b = 2808$ bunches/beam,

$\sigma \equiv$ beam size at the interaction point $= 16\ \mu m = 16 \times 10^{-4}$ cm,

Given LHC is a 40 MHz particle collider $\Rightarrow$ f= 40×10^6 sec^{-1}. Now the luminosity is

Luminosity and Event Statistics

given by

$$\mathcal{L} = \frac{f n_1 n_2 n_b}{4\pi\sigma^2},$$
$$= \frac{40 \times 10^6\ sec^{-1} \times (1.15 \times 10^{11})^2 \times 2808}{4\pi \times (16 \times 10^{-4}\ cm)^2}$$
$$= 46.168 \times 10^{36}\ cm^{-2} sec^{-1}$$

Given $\sigma_p = 1\ fb = 10^{-39}\ cm^2$,

$$\frac{dR}{dt} = \mathcal{L}.\sigma_p$$
$$= 10^{-39}\ cm^2 \times 46.168 \times 10^{36}\ cm^{-2} sec^{-1}$$
$$= 46.168 \times 10^{-3} sec^{-1}$$
$$= 166\ hr^{-1}$$

This means there are 166 events occurring per hour at the LHC. To bound the statistical errors by 2% means,

$$\frac{1}{\sqrt{N}} = 0.02$$
$$\Rightarrow N = \frac{1}{0.02^2} = 2500$$

Hence to bound statistical errors by 2%, LHC needs to run for: $2500/166 \simeq 15$ hours.

Ref: Relativistic Kinematics for Beginners:
Raghunath Sahoo & Basanta K. Nandi
(CRC Press- 2020)

FUTURE CIRCULAR COLLIDER at CERN

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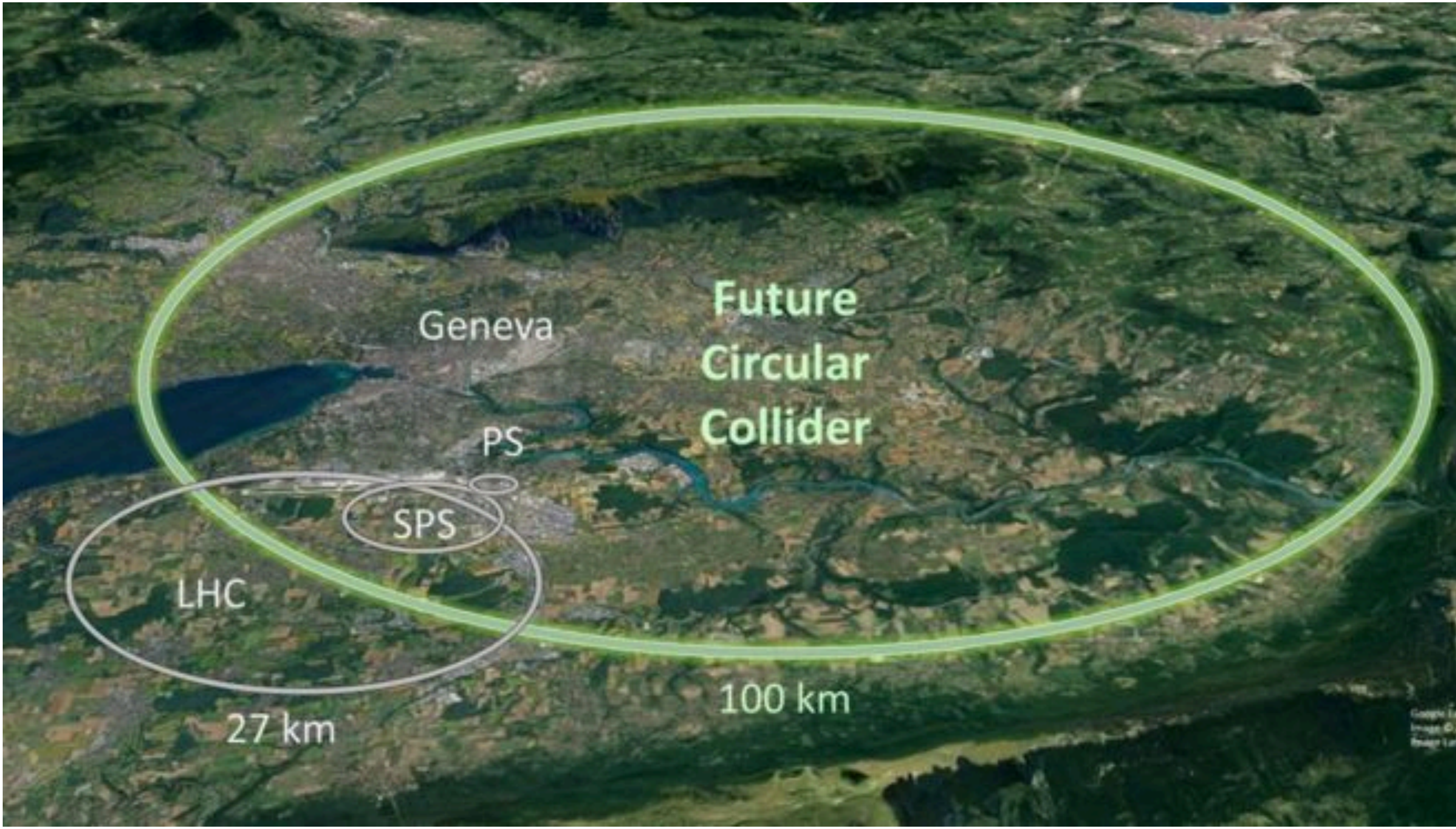


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