Electroweak Unification and the Standard Model

Tutorial 1

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Weak Interactions

Parity violation : the $\theta - \tau$ puzzle

Consider the Fermi form of the current-current interaction:

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}}(\overline{p}\gamma^\mu n)(\bar{e}\gamma_\mu \nu_e)$$

Under parity:

$$(\overline{p}\gamma^\mu n) \rightarrow -(\overline{p}\gamma^\mu n)$$
$$(\bar{e}\gamma_\mu \nu_e) \rightarrow -(\bar{e}\gamma_\mu \nu_e)$$

i.e. parity is conserved in the Fermi theory

Before the 1950s, it was thought that parity is as sacred as energy, momentum and angular momentum...

But it was known that some particle are pseudoscalars,

e.g. pions and Kaons have intrinsic parity $P = -1$

This led to the famous $\theta - \tau$ puzzle

Through the early 1950s, cosmic ray experiments showed the existence of two degenerate particles θ and τ, each with mass around 483 MeV and lifetime around 12 ns.

However, it was seen that

$$\theta^+ \to \pi^+ + \pi^0$$

$$\tau^+ \to \pi^+ + \pi^- + \pi^+$$

indicating that $P_\theta = +1$ and $P_\tau = -1$.

Note that the phase space for these decays is very different:

$$M(\theta^+) - M(\pi^+) - M(\pi^0) = 493 - 140 - 135 = 218 \text{ MeV}$$

$$M(\tau^+) - M(\pi^+) - M(\pi^-) - M(\pi^+) = 493 - 3 \times 140 = 73 \text{ MeV}$$

Since the lifetimes are identical, the strength of weak interactions must be different for these different decays $\Rightarrow$ universality is violated

Yang & Lee (1956)


Maybe parity is not conserved in weak interactions, i.e.

$\theta^+ \to \pi^+ + \pi^0$ is really $K^+ \to \pi^+ + \pi^0$

parity-violating channel

$\tau^+ \to \pi^+ + \pi^- + \pi^+$ is really $K^+ \to \pi^+ + \pi^- + \pi^+$

parity-conserving channel

They also showed that none of the earlier experiments had really tested intrinsic parity violation...

Suggested that if the mean value of a party-odd variable, e.g. $\vec{S}.\vec{p}$ could be found to be nonzero, this would be a 'smoking gun' signal for parity violation

Experiment was actually performed by Wu et al (1957)...

Maximal parity violation:

In 1956, Marshak & Sudarshan, and separately, Feynman & Gell-Mann, assumed the parity-violating weak interactions to be of the form

$$\mathcal{H}_I = \frac{G_F}{4\sqrt{2}}\, \overline{p}\gamma^\mu(1-\lambda\gamma_5)n \, . \, \bar{e}\gamma_\mu(1-\lambda\gamma_5)\nu_e$$

Parity is <u>conserved</u> when $\lambda = 0$ (V current), $\lambda \to \infty$ (A current)

Parity is <u>maximally</u> violated when $\lambda = 1$ (V-A currents)

Parity is partially violated for other values of λ....

Rewrite the leptonic current as

$$J^\mu_{\text{lep}} = \bar{e}\gamma_\mu(1-\lambda\gamma_5)\nu_e$$
$$= (1+\lambda)\overline{e_L}\gamma_\mu\nu_{eL} + (1-\lambda)\overline{e_R}\gamma_\mu\nu_{eR}$$




If we can measure the chirality of neutrinos emitted in beta decay, then we should have

$$P(\nu_{eL}) = \frac{(1+\lambda)^2}{(1+\lambda)^2 + (1-\lambda)^2}$$

$$P(\nu_{eR}) = \frac{(1-\lambda)^2}{(1+\lambda)^2 + (1-\lambda)^2}$$

and hence

$$\frac{P(\nu_{eR})}{P(\nu_{eL})} = \left(\frac{1-\lambda}{1+\lambda}\right)^2$$

Goldhaber et al did an experiment in 1957 with the electron capture process

$$e^- + {}^{152}_{63}Eu \rightarrow {}^{152}_{62}Sm^* + \nu_e$$

and found that the neutrino is always left-chiral... It follows that $\lambda = 1.$

Thus the weak interactions do have the form V – A and the W boson vertex for electrons is of the form

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{e}\gamma^{\alpha}(1-\gamma_5)\nu_e W_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^{\alpha}(1-\gamma_5)eW_{\alpha}^{+}$$

We will have similar interactions for the muon

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{\mu}\gamma^{\alpha}(1-\gamma_5)\nu_{\mu} W_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{\nu}_{\mu}\gamma^{\alpha}(1-\gamma_5)\mu W_{\alpha}^{+}$$

and for the nucleons

$$\mathcal{H}_I = \frac{g}{2\sqrt{2}}\bar{p}\gamma^{\alpha}(1-\gamma_5)nW_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{n}\gamma^{\alpha}(1-\gamma_5)pW_{\alpha}^{+}$$

and for the quarks

$$\mathcal{H}_I \approx \frac{g}{2\sqrt{2}}\bar{d}\gamma^{\alpha}(1-\gamma_5)uW_{\alpha}^{-} + \frac{g}{2\sqrt{2}}\bar{u}\gamma^{\alpha}(1-\gamma_5)dW_{\alpha}^{+}$$

Unitarity

S-matrix

The S-matrix is a quantum mechanical operator which connects the initial state to the final state.

$$|f\rangle = \hat{S}|i\rangle$$

Probability amplitude for a scattering process is $\mathcal{S}_{fi} = \langle i|f\rangle = \langle i|\hat{\mathcal{S}}|i\rangle$

S-matrix may be expanded perturbatively as $\mathcal{S} = \mathbb{1} + ig\mathcal{T}$ which means $\mathcal{S}^\dagger = \mathbb{1} - ig\mathcal{T}^\dagger$

Since the S-matrix connects the initial state to the final state, it must be unitary

$$\mathcal{S}^\dagger\mathcal{S} = \mathbb{1} + ig(\mathcal{T} - \mathcal{T}^\dagger) + g^2\mathcal{T}^\dagger\mathcal{T} = \mathbb{1}$$

i.e. unitarity is maintained so long as $\mathcal{T} = \mathcal{T}^\dagger$ and $|\langle g\mathcal{T}\rangle| \ll 1$

Now the cross-section, by a dimensional argument, will vary as $\sigma \sim \frac{1}{s}\int d\Phi\,|\langle g\mathcal{T}\rangle|^2$

If this increases faster than $(\ln s)^2$ then $|\langle g\mathcal{T}\rangle|$ will keep increasing and at some energy It will violate $|\langle g\mathcal{T}\rangle| \ll 1$ i.e. unitarity

Froissart bound