

# Electroweak Unification and the Standard Model

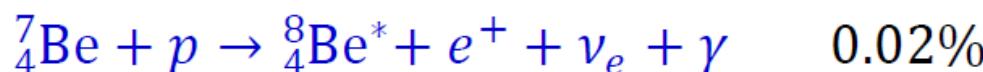
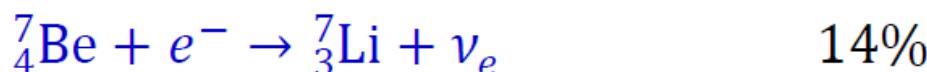
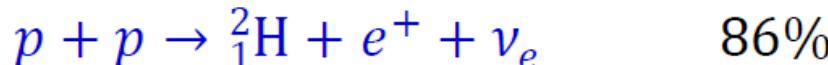
## Tutorial 2

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## Neutrino oscillations

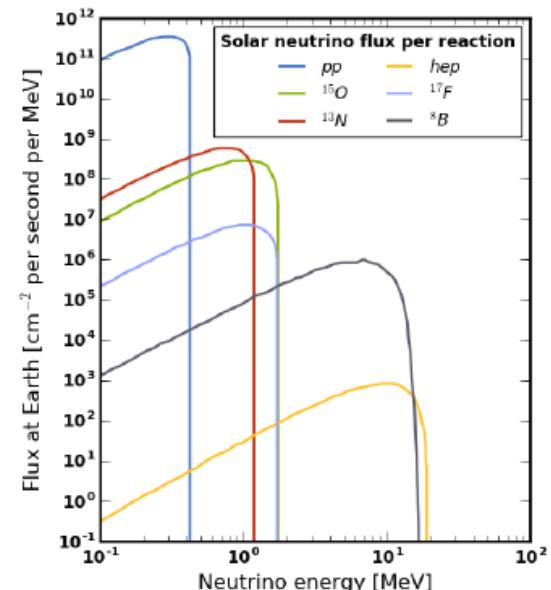
**Solar neutrino puzzle:** neutrinos are produced in the Sun through the fusion reactions



Homestake solar neutrino experiment (1967 – 94) was a big tank with 380 m<sup>3</sup> of C<sub>2</sub>Cl<sub>4</sub> at 1.478 km depth.



The  $\nu_e$  flux was measured by chemical method, measuring the amount of  ${}^{37}_{18}Ar$ , and was always found to be **1/3** of the predicted flux from SSM.



## Standard Solar Model



Raymond  
Davis Jr.

John  
Bahcall

**Atmospheric neutrino puzzle:** neutrinos are produced in the upper atmosphere by cosmic rays, typically from a proton

$$p + N \rightarrow \pi^+ + X$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

If all these neutrinos reach the ground, we should have

$$\frac{N(\nu_\mu)}{N(\nu_e)} = 2$$

Experiments using water Cherenkov counters at the Soudan and Kamiokande mines showed that this ratio  $\approx 1$  and shows a dependence on the neutrino energy.



Takaaki Kajita

Explanation: neutrinos oscillate from one flavour to the other

For 2 neutrinos ( $\nu_e \leftrightarrow \nu_\mu$ ) if the mixed states are

$$\nu_e = \nu_e^0 \cos \theta + \nu_\mu^0 \sin \theta$$

$$\nu_\mu = -\nu_e^0 \sin \theta + \nu_\mu^0 \cos \theta$$



B. Pontecorvo

then the oscillation probability of neutrinos of energy  $E$  at a distance  $L$  is given by

$$P_{\nu_e \leftrightarrow \nu_\mu}(L) = \sin^2 2\theta \sin\left(\frac{L}{4E} \Delta m_\nu^2\right)$$

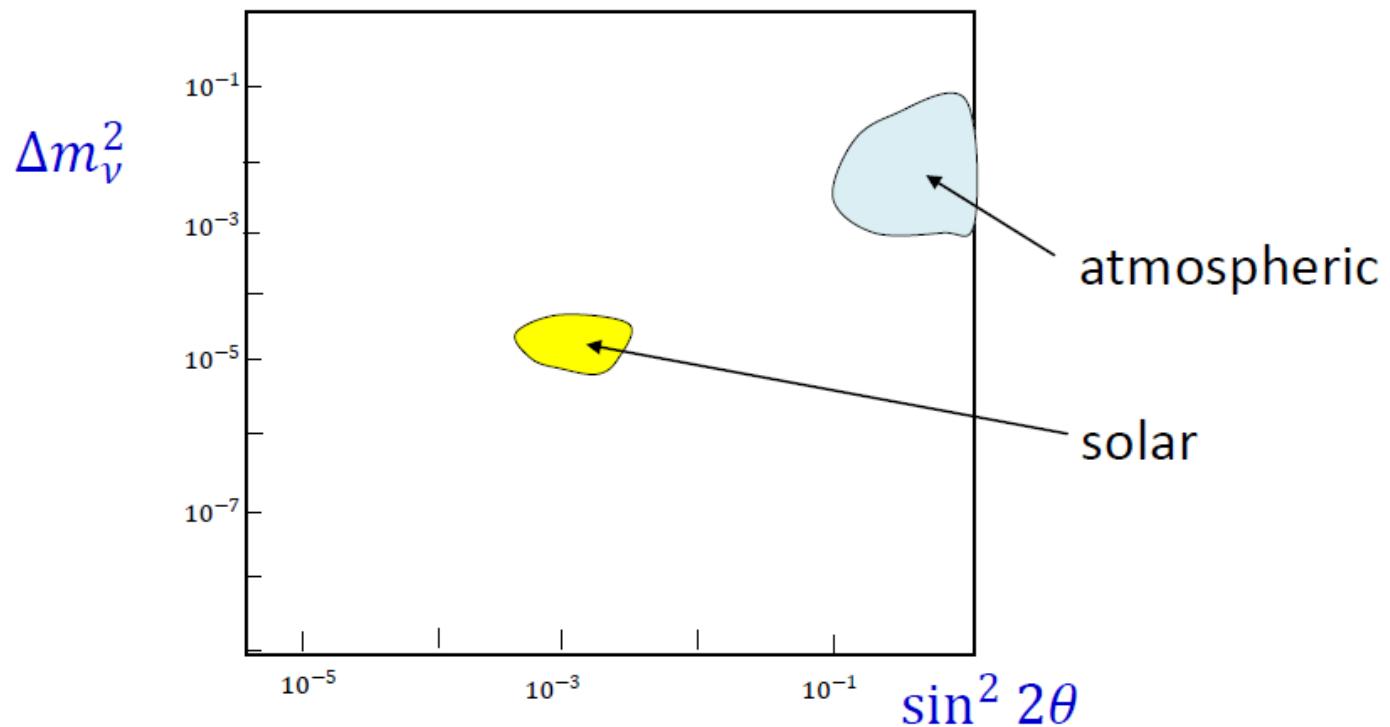
where

$$\Delta m_\nu = |m_{\nu_e} - m_{\nu_\mu}|$$

If the oscillation is nonzero, then  $\Delta m_\nu \neq 0$ , i.e. neutrinos have mass!

$$P_{\nu_e \leftrightarrow \nu_\mu}(L) = \sin^2 2\theta \sin\left(\frac{L}{4E} \Delta m_\nu^2\right)$$

A measurement of  $P_{\nu_e \leftrightarrow \nu_\mu}(L)$  corresponds to a region in the plane of  $\sin^2 2\theta$  and  $\Delta m_\nu^2$  — the so-called MSW plot.



The oscillations of  $\nu_e \leftrightarrow \nu_\mu$  cannot explain both the solar and atmospheric neutrino puzzles. So there must be another neutrino – and that is expected as the  $\nu_\tau$ . Thus we have the simple explanation:

Solar neutrino deficit :  $\nu_e \leftrightarrow \nu_\mu$

Atmospheric neutrino deficit :  $\nu_\mu \leftrightarrow \nu_\tau$

But why not also have  $\nu_e \leftrightarrow \nu_\tau$ , i.e. three-flavour oscillation?

This would be like the flavour mixing in the quark sector.

We cannot see quark oscillations because quarks are confined inside hadrons.

But we can see meson oscillations (FCNC at one-loop level) like

$$K^0 \leftrightarrow \bar{K}^0$$

$$D^0 \leftrightarrow \bar{D}^0$$

$$B^0 \leftrightarrow \bar{B}^0$$

$$s\bar{d} \leftrightarrow \bar{s}d$$

$$c\bar{u} \leftrightarrow \bar{c}u$$

$$b\bar{d} \leftrightarrow \bar{b}d$$

Three-flavour oscillations in the neutrino sector were actually seen at Daya Bay reactor neutrino experiment in 2012

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \mathbb{P} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

### Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{bmatrix}.$$

$$\theta_{12} \simeq 33.6^\circ \quad \theta_{23} \simeq 47.2^\circ \quad \theta_{13} \simeq 8.5^\circ \quad \delta_{CP} \simeq 200^\circ - 280^\circ$$

In the quark sector, the largest angle is about  $13^\circ$

But neutrino masses in the Standard Model require a  $\nu_R \dots$

In that case, we are back to

$$\mathcal{L}_{\text{Yuk}}^{\text{leptons}} = f_e \bar{L}_L \Phi e_R + f_\nu \bar{L}_L \tilde{\Phi} \nu_R + \text{H. c.}$$

for one generation, and for 3-generations

$$\mathcal{L}_{\text{Yuk}}^{\text{leptons}} = \sum_{i,j=1}^3 f_e^{ij} \bar{L}_L^i \Phi e_R^j + f_\nu^{ij} \bar{L}_L^i \tilde{\Phi} \nu_R^j + \text{H. c.}$$

As in the quark sector, after symmetry breaking, we will have

$$\mathcal{L}_{\text{Yuk}}^{\text{leptons}} = \sum_{i,j=1}^3 \frac{v}{\sqrt{2}} f_e^{ij} \bar{L}_L^i e_R^j + \frac{v}{\sqrt{2}} f_\nu^{ij} \bar{L}_L^i \nu_R^j + \text{H. c.}$$

leading to mass matrices

$$\begin{aligned}\mathcal{L}_{\text{Yuk}}^{\text{leptons}} &= \sum_{i,j=1}^3 M_e^{ij} \bar{e}_L^i e_R^j + M_\nu^{ij} \bar{\nu}_L^i \nu_R^j + \text{H. c.} \\ &= \bar{\mathcal{E}}_L \mathbb{M}_e \mathcal{E}_R + \bar{\mathcal{N}}_L \mathbb{M}_\nu \mathcal{N}_R + \text{H. c.}\end{aligned}$$

Exactly as in the quark sector, we will have to diagonalise these mass matrices

$$\mathcal{E}_L \rightarrow \hat{\mathcal{E}}_L = \mathbb{V}_L^e \mathcal{E}_L \quad \mathcal{E}_R \rightarrow \hat{\mathcal{E}}_R = \mathbb{V}_R^e \mathcal{E}_R$$

$$\mathcal{N}_L \rightarrow \hat{\mathcal{N}}_L = \mathbb{V}_L^\nu \mathcal{N}_L \quad \mathcal{N}_R \rightarrow \hat{\mathcal{N}}_R = \mathbb{V}_R^\nu \mathcal{N}_R$$

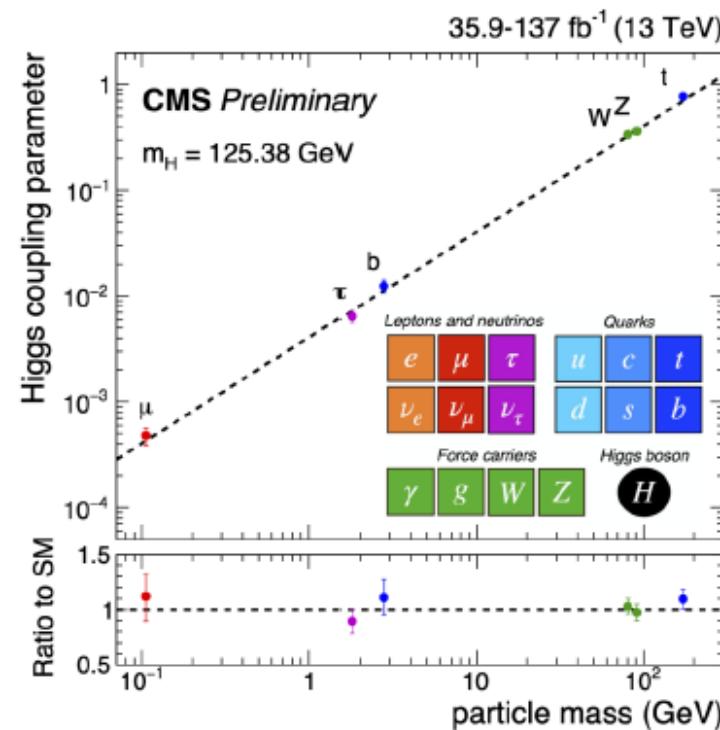
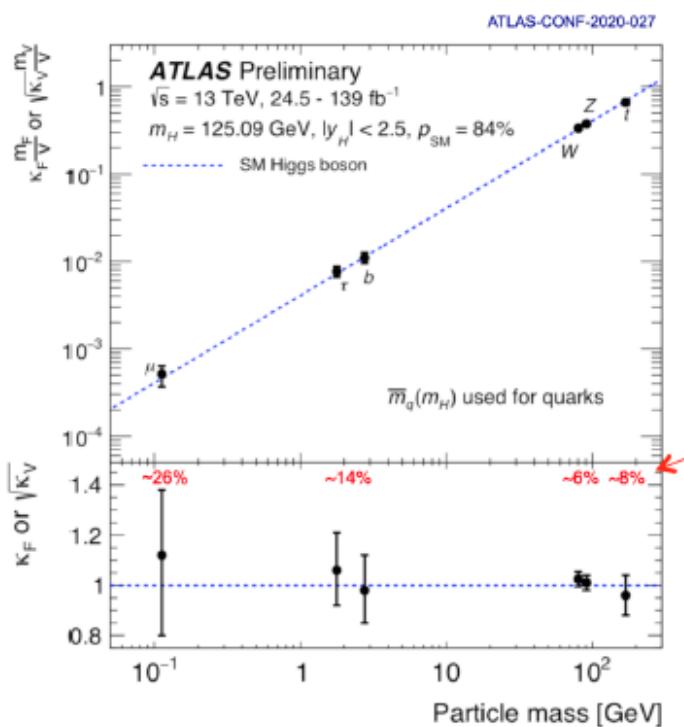
leading to c.c. interactions

$$\mathcal{L}_{\text{cc}}^{(\ell)} = \frac{g}{2\sqrt{2}} \hat{\mathcal{N}}_L \gamma^\mu \mathbb{P} \hat{\mathcal{E}}_L W_\mu^+ + \text{H. c.} \quad \mathbb{P} = \mathbb{V}_L^\nu \mathbb{V}_L^{e\dagger}$$

However...

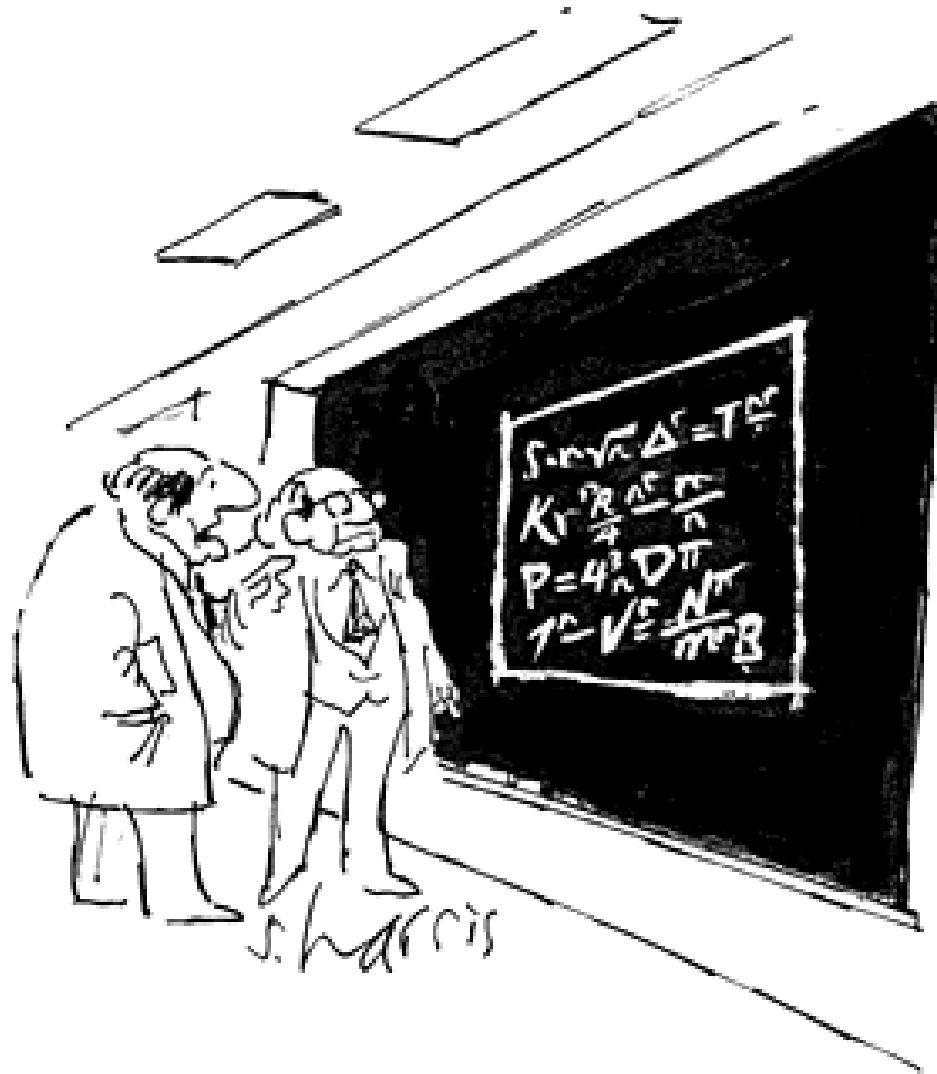
$$m_{\nu_e} < 0.0000000002 \quad m_{\nu_\mu} < 0.00019 \quad m_{\nu_\tau} < 0.00182$$

These come from experimental data e.g. Kurie plots, astroparticle events, cosmological considerations, etc.



$$f_{\nu_e} \sim 10^{-12}$$

Neutrino masses remain a big puzzle in the SM...



"PUTTING A BOX AROUND IT,  
I'M AFRAID, DOES NOT MAKE IT  
A UNIFIED THEORY."



1906	James J. Thomson	discovery of electron
1935	James Chadwick	discovery of neutron
1936	Carl D. Anderson	discovery of positron
1939	Ernest O. Lawrence	invention of cyclotron
1949	Hideki Yukawa	theory of mesons
1950	Charles F. Powell	discovery of pion
1957	Cheng Ning Yang Tao Dai Lee	theory of parity violation
1959	Emilio G. Segre Owen Chamberlain	discovery of antiproton
1960	Donald A. Glaser	invention of bubble chamber
1965	Sin-Itiro Tomonaga Julian Schwinger Richard P. Feynman	development of quantum electrodynamics
1968	Luis A. Alvarez	discovery of hadron resonances
1969	Murray Gell-Mann	development of the quark model

1976	Burton Richter Samuel C.C. Ting	discovery of charm quark
1979	Sheldon L. Glashow Abdus Salam Steven Weinberg	development of the electroweak model
1980	James W. Cronin Val L. Fitch	discovery of CP-violation
1984	Carlo Rubbia Simon van der Meer	discovery of the W and Z bosons
1988	Leon M. Lederman, Melvin Schwartz Jack Steinberger	discovery of the muon neutrino
1990	Jerome I. Friedman, Henry W. Kendall, Robert E. Taylor	proof of the existence of proton structure
1992	Georges Charpak	invention of multiwire proportional counter

1995	Martin L. Perl Frederick W. Reines	discovery of the tau neutrino discovery of the electron neutrino
1999	Gerhardus 'tHooft Martinus J.G. Veltman	proof of renormalisability of the SM
2002	Ray Davis Jr. Masatoshi Koshiba	discovery of solar neutrino oscillations
2004	David J. Gross H. David Politzer Frank Wilczek	theory of asymptotic freedom in QCD
2008	Yoichiro Nambu Makoto Kobayashi Toshihide Maskawa	theory of spontaneous symmetry-breaking theory of CP-violation in the SM
2013	Francois Englert Peter W. Higgs	explanation of the origin of mass
2015	Takaaki Kajita Arthur B. MacDonald	discovery of atmospheric neutrino oscillations

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