Glauber model basics for high energy heavy ion collisions

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November 17, 2020



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- The Glauber model is used to describe nuclear collisions over a wide range of energies.
- It is based on semiclassical picture of nuclear collisions in the impact parameter representation.
- It assumes that the nuclei follow straight line trajectory.
- The nucleus nucleus collisions are obtained in terms of nucleon nucelon interactions with density distributions of the two nuclei as input.

When a beam of particles are sacterred off from a spherically symmetric potential you can expand the plane wave in terms of sum of many partial waves with angular momentum *I*.

Asymptotically, the wave function can be expressed as

$$\Psi(r) = \exp(ikz) + f(\theta, k) \frac{\exp(ikr)}{r}$$
(1)

Here $f(\theta, k)$ is the scattering amplitude given by

$$f(\theta) = f_N(\theta)$$

= $\frac{1}{2ik}\sum_{l=1}^{\infty}(2l+1)(S_l-1)P_l(\cos\theta).$ (2)

The nucleus-nucleus differential cross section as a function of center of mass scattering angle θ is described as

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2,\tag{3}$$

The total reaction cross section in the collision of two nuclei as per the partial wave analysis is given by

$$\sigma^{in} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-|S_l|^2).$$
(4)

Here

- $S_l = \exp(2i\delta_l)$ is called the scattering matrix and δ_l is the nuclear phase shift.
- The factor $(1 |S_l|^2)$ is called the transmission coefficient.
- $|S_l|^2$ is referred to as transparency function or the probability that the projectile undergoes no interaction at a given d.

In a semiclassical approximation, one can write angular momentum l in terms of momentum k and impact parameter b as

$$l+\frac{1}{2}=kb. \tag{5}$$

One can use

$$(2l+1) = 2kb$$
$$\sum_{l} = k \int db$$

We can get the reaction/inelastic cross section in impact parameter representation as

$$\sigma^{in} = 2\pi \int bdb(1 - |S(b)|^2).$$
 (6)

The factor $Tr(b) = 1 - |S(b)|^2$ is nothing but the transmission coeff.

If the two nuclei are assumed to be sharp spheres with radii R_1 and R_2 then

$$Tr(b) = 1 \text{ for } b \le R_1 + R_2 = 0 \text{ for } b > R_1 + R_2.$$
(7)

The total reaction cross section in this case is given by

$$\sigma^{in} = 2\pi \int_0^{R_1 + R_2} bdb = \pi (R_1 + R_2)^2.$$
(8)

This is the well-known geometric formula for the cross section.

The Glauber model: Basic fromalism

- The Glauber model basically describe the nucleus-nucleus interaction in terms of elementary nucleon-nucleon interaction.
- It is based on the assumption that the nucleus travels in a straight line path.
- At high energies this approximation is very good.
- At low energies the nucleus is deflected from straight line path due to Coulomb repulsion.

Consider the collision of a projectile nucleus B on a target nucleus A.

Define $t(\mathbf{b})d\mathbf{b}$ as the probability for having a nucleon-nucleon collision within the transverse area element $d\mathbf{b}$ when one nucleon is situated at an impact parameter **b** relative to another nucleon which is normalized according to

$$\int t(\mathbf{b}) d\mathbf{b} = 1. \tag{9}$$

We define the probability of finding a nucleon in the volume element $d\mathbf{b}_A dz_A$ in the nucleus A at the position (\mathbf{b}_A, z_A) is $\rho_A(\mathbf{b}_A, z_A) d\mathbf{b}_A dz_A$ which is normalized as

$$\int \rho_A(\mathbf{b}_A, z_A) d\mathbf{b}_A dz_A = 1.$$
(10)

Similarly, the probability of finding a nucleon in the volume element $d\mathbf{b}_B dz_B$ in the nucleus B at the position (\mathbf{b}_B, z_B) is $\rho_B(\mathbf{b}_B, z_B)d\mathbf{b}_B dz_B$ which is normalized as

$$\int \rho_B(\mathbf{b}_B, z_B) d\mathbf{b}_B dz_B = 1.$$
(11)

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The Glauber model: Basic fromalism



Figure: Collision of two nuclei at an impact parameter b

The probability for occurrence of a nucleon-nucleon collision when the nuclei A and B are situated at an impact parameter **b** relative to each other is given by

$$T(b)\sigma_{NN} = \int \rho_A(\mathbf{b}_A, z_A) d\mathbf{b}_A dz_A \ \rho_B(\mathbf{b}_B, z_B) d\mathbf{b}_B dz_B \ t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B) \ \sigma_{NN}.$$
(12)

This can be written in terms of z-integrated densities as

$$T(b)\sigma_{NN} = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \, \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B \, t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B) \, \sigma_{NN}.$$
(13)

Here σ_{NN} is the total/inelastic nucleon nucleon cross section. The collision probability we are talking about is for an inelastic collision. There can be up o $A \times B$ collision. The probability of occurrence of n collisions will be

$$P(n,b) = \binom{AB}{n} (1-s)^n (s)^{AB-n}.$$
 (14)

Here, $s = 1 - T(b)\sigma_{NN}$. The total probability for the occurrence of an inelastic event in the collision of A and B at an impact parameter **b** is

$$\frac{d\sigma_{AB}^{in}}{db} = \sum_{n=1}^{AB} P(n, b) = 1 - s^{AB}.$$
(15)

The total inelastic cross section can be written as

$$\sigma_{AB}^{in} = 2\pi \int bdb \left(1 - s^{AB}\right). \tag{16}$$

From here one can read the scattering matrix as

$$|S(b)|^2 = s^{AB} = (1 - T(b)\sigma_{NN})^{AB}$$
 (17)

In the optical limit, where a nucleon of projectile undergoes only one collision in the target nucleus

$$|S(b)|^2 \simeq \exp(-T(b)\sigma_{NN}AB).$$
(18)

The scattering matrix can be defined in terms of eikonal phase shift $\chi(b)$ as

$$S(b) = \exp(i\chi(b)). \tag{19}$$

Comparing Eq. (18) with Eq. (19), the imaginary part of eikonal phase shift is given by

$$\operatorname{Im}\chi(b) = T(b)\sigma_{NN}AB/2.$$
 (20)

If the ratio of real to imaginary part of NN scattering amplitude is α_{NN} then real part of $\chi(b)$ is

$$\operatorname{Re}_{\chi}(b) = T(b)\alpha_{NN}\sigma_{NN}AB/2.$$

Once we know the phase shift and thus the scattering matrix, we can calculate all the cross sections.

Calculation of T(b) **in momentum space** In the co-ordinate space T(b) is derived as

$$T(b) = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \ \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B \ t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B).$$
(22)

It is a four dimensional integration: two over b_A and two over $b_B.$ It is convenient to write it in momentum space as

$$T(b) = \frac{1}{(2\pi)^2} \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \ \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B$$
$$\exp\left(-i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B)\right) \ f_{NN}(q) d^2q.$$
(23)

Here $f_{NN}(q)$ is the q dependence of NN scattering amplitude given by

$$t(\mathbf{b}) = \frac{1}{(2\pi)^2} \int e^{-i\mathbf{q}\cdot\mathbf{b}} f_{NN}(q) d^2_{a} q_{a} = \frac{24}{3} \left(24 \right)_{a} d^{47}$$

$$T(b) = \frac{1}{(2\pi)^2} \int \exp(-i\mathbf{q}.\mathbf{b}) \rho_A^z(\mathbf{b}_A) \exp(i\mathbf{q}.\mathbf{b}_A) d\mathbf{b}_A$$

$$\rho_B^z(\mathbf{b}_B) \exp(-i\mathbf{q}.\mathbf{b}_B) d\mathbf{b}_B f_{NN}(q) d^2 q$$

$$= \frac{1}{(2\pi)^2} \int e^{-i\mathbf{q}.\mathbf{b}} S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(q) d^2 q$$

$$= \frac{1}{2\pi} \int J_0(qb) S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(q) q dq. \qquad (25)$$

Here $S_A(q)$ and $S_B(-q)$ are the fourier transforms of the nuclear densities.

$$J_0(qb) = (1/2\pi) \int \exp(-qb\cos\phi) d\phi$$
 (26)

is the cylindrical Bessel function of zeroth order.

The profile function for the NN scattering can be taken as delta function if the nucleons are point particles.

In general it is taken as a gaussian function of width r_0 as

$$t(\mathbf{b}) = \frac{\exp(-b^2/(2r_0^2))}{2\pi r_0^2}.$$
 (27)

Thus,

$$f_{NN}(q) = \int e^{i\mathbf{q}\cdot\mathbf{b}} t(\mathbf{b}) d\mathbf{b}$$

= $\frac{1}{\pi r_0^2} \int e^{i\mathbf{q}\cdot\mathbf{b}} \exp(-b^2/(2r_0^2)) d\mathbf{b}$
= $\exp(-r_0^2 q^2/2).$ (28)

Here, $r_0 \sim 0.7$ fm is the range parameter and σ_{pp} is the nucleon-nucleon total cross section which is taken as 7.0 fm² at $\sqrt{s} = 5$ TeV.

Density of nucleus: 2pf density

The two parameter fermi density is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-R}{t})},\tag{29}$$

where
$$ho_0 = 3/\left(4\pi R^3(1+rac{\pi^2 t^2}{R^2})
ight).$$

Here

- t is the diffuseness and
- *R*, the half value radius in terms of rms radius *R*.
- It is calculated by $R = 1.19 * A^{1/3} 1.61/A^{1/3}$.





Figure:

For case of Pb, the values are t = 0.55 fm and R = 6.8 fm. $z \to z \to z \to 0.00$



Figure: The density of Pb nucleus versus the distance from the center.

The momentum density can be derived with the Fourier transform

$$S(q) = 4\pi \int j_0(qr) \rho(r) r^2 dr$$
(30)

$$S(q) = \frac{8\pi\rho_0}{q^3} \frac{ze^{-z}}{1 - e^{-2z}} \left(\sin x \frac{z(1 + e^{-2z})}{1 - e^{-2z}} - x\cos x \right).$$
(31)

Where $z = \pi dq$ and x = cq.

The equation for T(b) which is a one dimensional integral can be solved numerically for this density and thus the overlap integral can be extracted 19/47



Figure: The momentum density of Pb nucleus S(q) versus momentum q.

NN scattering parameter

The average σ_{NN} in terms of proton numbers (Z_P and Z_T) and neutron number (N_P and N_T) of projectile and target nuclei is written as

$$\sigma_{NN} = \frac{N_P N_T \sigma_{nn} + Z_P Z_T \sigma_{pp} + (Z_P N_T + N_P Z_T) \sigma_{np}}{A_P A_T}.$$
 (32)

Here, σ_{pp} is the nucleon-nucleon total cross section which is taken as 7.0 fm² at $\sqrt{s} = 7$ TeV.

At high energies, we can assume $\sigma_{pp} = \sigma_{nn} = \sigma_{np}$.

Participant-spectator picture

 The strategy in the field of high energy heavy ion collisions and QGP is to plot the Pb+Pb data as the superposition of the nucleon-nucleon collisions and look for possible departures.

• When two nuclei collide at these energies, the nucleons which come in the overlap region depending on the impact parameter are called participant nucleons and those which do not participate are called spectators.



Figure: Spectator participant picture of heavy ion collision

These spectators and participants nucleon decide how much energy is going in the forward and how much in the transverse direction which can be measured. From these measured energies one can know the impact parameter for a particular event. Let us relook at the total inelastic cross section for a nucleus-nucleus collision as

$$\sigma_{AB}^{in} = 2\pi \int bdb \left(1 - (1 - T(b)\sigma_{NN})^{AB} \right).$$
(33)

- The term $(1 T(b)\sigma_{NN})^{AB}$ gives the probability that in a nucleus nucleus collision none of the nucleons collided with each other.
- For nucleon-nucleon collision A = B = 1 thus $q = (1 T(b)\sigma_{NN})$ gives the probability that two nucleon at an impact parameter b do not collide.

When two nuclei A and B collide the probability of nucleon remaining in the nucleus A will be

$$P_P = q(b)^B \tag{34}$$

and the probability of nucleon remaining in the nucleus B will be

$$P_T = q(b)^A. \tag{35}$$

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The probability of having α participant nucleons from the nucleus A is given by binomial distribution as

$$P(\alpha, b) = \binom{A}{\alpha} (1 - q^B)^{\alpha} (q^B)^{A - \alpha}.$$
 (36)

Similarly,

$$P(\beta, b) = {B \choose \beta} (1 - q^A)^{\beta} (q^A)^{A - \beta}.$$
 (37)

The number of collisions will be

$$P(n,b) = \binom{AB}{n} (1-q)^n (q)^{AB-n}.$$
(38)

The average number of projectile participant and its standard deviation for each impact parameter b is given by

$$<\alpha>=A[1-q(b)^{B}],$$
(39)

$$\sigma_{\alpha}^{2} = A[1 - q(b)^{B}]q(b)^{B} = (40) = (40)$$

The average number of target participant and its standard deviation for each impact parameter b is given by

$$<\beta>=B[1-q(b)^{A}], \tag{41}$$

$$\sigma_{\beta}^{2} = B[1 - q(b)^{A}]q(b)^{A}.$$
(42)

The average number of total participant from both projectile and target for each impact parameter is given by

$$N_{\text{part}} = A[1 - q(b)^{B}] + B[1 - q(b)^{A}].$$
(43)

The average number of N-N collisions is given by

$$N_{coll} = AB[1 - q(b)],$$

= $AB\sigma_{NN}T(b).$ (44)

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Forward energy

The forward energy is measured in the detector called zero degree calorimeter (ZDC). The energy measured in the ZDC is can be related to the number of spectators as

$$\mathsf{E}_{\mathsf{Z}}(\alpha) = (\mathsf{A} - \alpha)\mathsf{E}_0,\tag{45}$$

where E_0 is the projectile energy per nucleon in the lab frame. The cross section of α nucleons participating from projectile is

$$\sigma_{\alpha} = 2\pi \int P(\alpha, b) b db.$$
(46)

The forward energy flow cross section then can be written as

$$\frac{d\sigma}{dE_Z} = \frac{\sigma_\alpha}{E_0},\tag{47}$$

taking into account the discrete nature of the variable α . $\alpha \ge 1$ $\alpha \ge 1$

The average forward energy for a collision is written as

$$E_{Z}(\alpha) = E_{0}(A - \langle \alpha \rangle) = E_{0}A[1 - (1 - q^{B})] = E_{0}Aq^{B}.$$
(48)

Excitation energy

When the nucleons collide at relativistic energies they become excited and produce new particles.

This energy manifests in the measurement of transverse energy and multiplicity.

The (maximum) excitation energy which is related to the number of participants is written as

$$E_{ex} = E_{cm} - m_N(\alpha + \beta). \tag{49}$$

The centre of mass energy for this case is given as

$$E_{cm}^2 = ((\alpha E_A + \beta E_B)^2 - (\alpha \mathbf{P}_A + \beta \mathbf{P}_B)^2), \tag{50}$$

where

$$P_A = \sqrt{E_A^2 - m_N^2}$$
 and
 $P_B = \sqrt{E_B^2 - m_N^2}$.

Here E_A and \mathbf{P}_A are the total energy and momentum of the nucleus A per nucleon and m_N is nucleon mass.

The cross section for having α participants from A and β participants from B is

$$\sigma_{\alpha\beta} = 2\pi \int P(A, \alpha, b) P(B, \beta, b) b db.$$
(52)

The cross sections for all the possible combinations of α and β are calculated and the results are binned to obtain the cross section for each of the excitation energy bin.

Impact parameter and the percentage centrality of collisions

We consider the collision of a projectile nucleus A of mass number A on a target nucleus B of mass number B.

Now Define the total probability for the occurence of an inelastic event in the collision of two nucleus at an impact parameter b is

$$f(b) = 1 - (1 - T_{AB}(b)\sigma_{NN})^{AB}.$$
(53)

The total inelastic cross section can be written as

$$\sigma^{in} = 2\pi \int_0^\infty bdb \left(1 - \left(1 - T_{AB}(b) \sigma_{NN} \right)^{AB} \right).$$
 (54)

The fraction of cross section within b_{min} to b_{max} is given by :

$$F(b_{min}, b_{max}) = \frac{2\pi}{\sigma^{in}} \int_{b_{min}}^{b_{max}} f(b) b db$$
(55)



Figure: f(b) versus b.



Figure: $F(0, b_{max})$ versus b

The average of impact parameter can be calculated for a centrality bin.

$$\langle b \rangle = \frac{2\pi}{F(b_{min}, b_{max})} \int_{b_{min}}^{b_{max}} bf(b)bdb$$
 (56)

The avergae of nuclear overlapiing function can be calculated as a function of impact parameter b:

$$< T_{AB} >= \frac{2\pi}{F(b_{min}, b_{max})} \int_{b_{min}}^{b_{max}} T_{AB}(b)f(b)bdb$$
(57)

The average number of total participant from both projectile and target as a function of impact parameter b is given by :

$$< N_{part} >= \frac{2\pi}{F(b_{min}, b_{max})} \int_{b_{min}}^{b_{max}} N_{part}(b) f(b) b db$$
(58)

The average number of nuclei-nuclei collision as a function of impact parameter b is given by :

$$< N_{coll} >= \frac{2\pi}{F(b_{min}, b_{max})} \int_{b_{min}}^{b_{max}} N_{coll}(b) f(b) b db$$
(59)

Table: Centrality Table

b _{min} – b _{max}	cent1-cent2	< b >	< T(b) >	$< N_{coll} >$
0 - 5.1	0-10	3.38	22.71	1589
5.1 - 7.21	10-20	6.21	13.77	964
7.21 - 8.83	20-30	8.04	8.18	573
8.83 - 10.20	30-40	9.52	4.58	321
10.20 - 11.41	40-50	10.82	2.34	164
11.41 - 12.5	50-60	11.94	1.1	76
12.5 - 13.5	60-70	13.00	0.43	30
13.5 - 14.43	70-80	13.95	0.16	11
14.43 - 15.36	80-90	14.87	0.052	3.7
15.36 - 21.8	90-100	16.21	0.021	1.4

Calculation of T(b): Special cases

In the co-ordinate space T(b) is derived as

$$T(b) = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \ \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B \ t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B). \tag{60}$$

The profile function for the NN scattering in general taken as a gaussian function of width r_0 as

$$t(\mathbf{b}) = \frac{\exp(-b^2/(2r_0^2))}{2\pi r_0^2}.$$
 (61)

It can be taken as delta function if the nucleons are assumed a point particles ($r_0 \rightarrow 0$).

$$t(\mathbf{b}) = \delta^2(b-0).$$
 (62)

 This gives

$$T(b) = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \ \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B \ \delta^2(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B).$$
(63)
$$T(b) = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \ \rho_B^z(\mathbf{b}_A - \mathbf{b})$$
(64)

It is also written as

$$T_{AB}(b) = \int \rho_A^z(\mathbf{s}) d\mathbf{s} \ \rho_B^z(\mathbf{s} - \mathbf{b})$$
(65)

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Figure: Collision of two nuclei at an impact parameter b

Calculation of T(b): pA collisions

$$T_A(b) = \int \rho_A^z(\mathbf{s}) d\mathbf{s} \ \rho_B^z(\mathbf{s} - \mathbf{b})$$
(66)
$$-\mathbf{b}) = \delta^2(\mathbf{s} - \mathbf{b})$$

Let me replace $\rho_B^z(\mathbf{s} - \mathbf{b}) = \delta^2(\mathbf{s} - \mathbf{b})$

$$T_A(b) = \rho_A^z(\mathbf{b}) \tag{67}$$

Let me also write for pB collisions

$$T_B(b) = \rho_B^z(\mathbf{b}) \tag{68}$$

$$\sigma_A^{in} = 2\pi \int bdb \left(1 - (1 - T(b)\sigma_{NN})^A \right).$$
(69)

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Number of participant : Improved formula

$$\sigma_{AB}^{in} = 2\pi \int bdb \left(1 - (1 - T(b)\sigma_{NN})^{AB} \right). \tag{70}$$

We derived number of participants as

$$N_{\text{part}} = A[1 - (1 - T(b)\sigma_{NN})^{B}] + B[1 - (1 - T(b)\sigma_{NN})^{A}].$$
(71)

The improved formula

$$N_{\text{part}} = A \int d^2 s T_A(\mathbf{s}) \left(1 - (1 - T_B(\mathbf{s} - \mathbf{b})\sigma_{NN})^B \right) + B \int d^2 s T_B(\mathbf{s} - \mathbf{b}) \left(1 - (1 - T_A(\mathbf{b})\sigma_{NN})^A \right)$$
(72)

Note, $q = (1 - T(b)\sigma_{NN})$ gives the probability that two nucleon at an impact parameter *b* do not collide.

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Scaling the pp cross sections to cross sections in AA collisions

Scaling the pp hard events cross sections to hard events cross sections in AA collisions

The inclusive inelastic scattering cross section is given by

$$\sigma_{AB}^{in} = 2\pi \int bdb \left(1 - \left(1 - T_{AB}(b) \sigma_{NN} \right)^{AB} \right).$$
(73)

We can write the hard scattering cross section as

$$\sigma_{AB}^{hard} = 2\pi \int bdb \left(1 - (1 - T_{AB}(b)\sigma_{NN}^{hard})^{AB} \right).$$
(74)

It can be approximated by

$$\sigma_{AB}^{hard} = \int 2\pi b db \, AB \, T_{AB}(b) \sigma_{NN}^{hard}. \tag{75}$$

The hard scattering cross section for minimum bias collisions

$$\sigma_{AB}^{hard} = AB \sigma_{NN}^{hard} \tag{76}$$

Hard scattering yield

$$N_{AB}^{hard}(b) = \sigma_{NN}^{hard} T_{AB}(b).$$
(77)

Number of binary collisions is given by

$$N_{coll}(b) = AB \,\sigma_{NN} \,T_{AB}(b). \tag{78}$$

Thus the hard scattering yield will be

$$N_{AB}^{hard}(b) = \frac{N_{coll}(b)}{\sigma_{NN}} \sigma_{NN}^{hard}.$$
 (79)

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Glauber Monte Carlo approach

- The coordinates of the nucleons are generated as per the density distributions of the two nuclei.
- The impact parameter is generated as per $d\sigma/db = 2\pi b$.
- A nucleon nucleon collision probability is calculated.
- All required quantities are obtained.



Figure: Snapshot of Glauber MC for Au+Au collisions

Summary

- I presented the basic formalism of Glauber Model along with derivation.
- One can calculate most required quantities in Heavy Ion collisions like number of participants, number of collisions with this presn.
- One can calculate centrality and associated it with Npart, Ncoll.
- One can calculate hard cross sections like J/psi production, jet production, photon production etc.
- On can calculate global quantities like multiplicity, transverse energy etc.

Summary

- C.Y. Wong, *Introduction to High Energy Heavy Ion Collisions*, World Scientific, Singapore, 1994.
- P. Shukla, "Glauber model for heavy ion collisions from low-energies to high-energies," nucl-th/0112039.