Fluctuations: in our lives and in physics

Tapan Nayak
NISER & CERN

19 November 2020

Fluctuations: in our lives and in physics

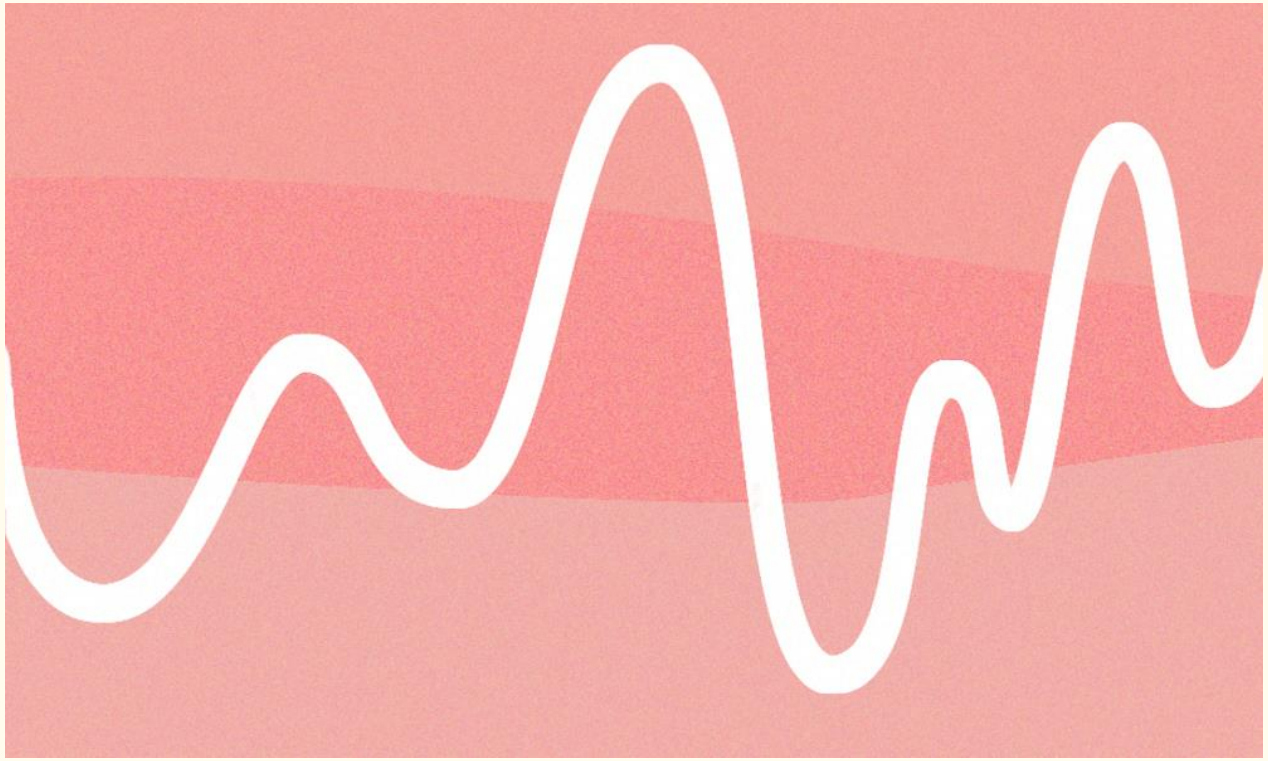
- Fluctuations in Life
- Fluctuations in thoughts
- Fluctuations in feelings
- Fluctuations in our moods
- Fluctuations in our daily activities
- Fluctuations in performance
- Fluctuations in our consciousness
- Population dynamics
- Heartbeat patterns

- Fluctuations in Financial markets
- Fluctuations in climate ….
- Fluctuations in the time of covid
- …..

- Statistical fluctuations: thermal flctuations in a thermodynamic variable
- Condensed matter physics
- Quantum fluctuations
- Vacuum fluctuation in space
- Phase transitions, ….

- Cosmic Microwave background radiation
- Quark-gluon plasma: Event-by-event fluctuations

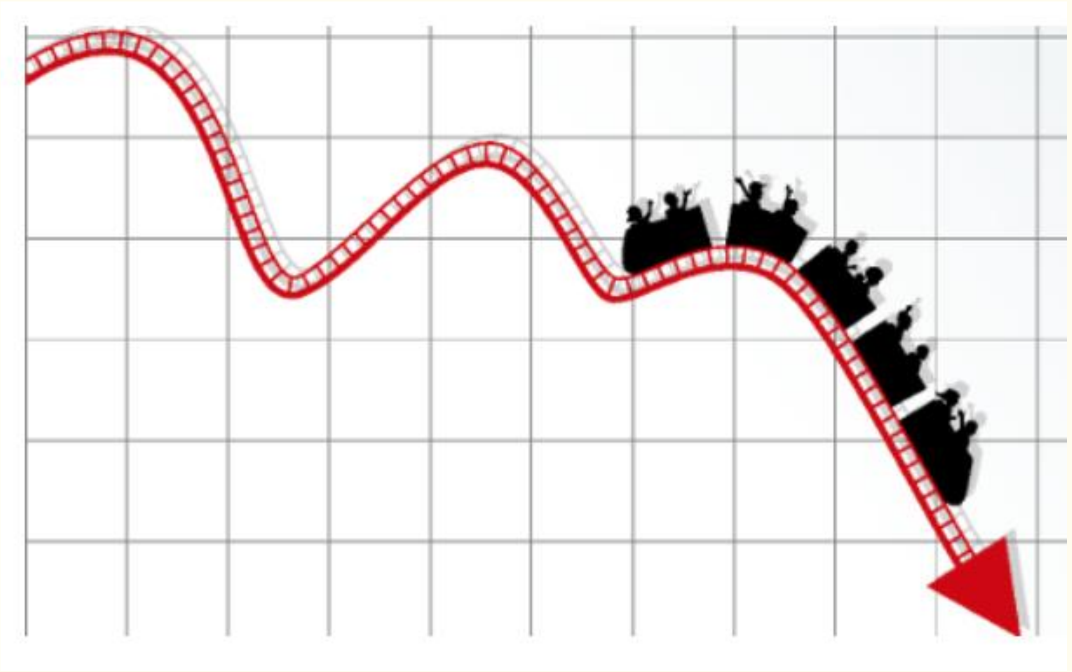
Fluctuations: mood swings



Fluctuations: in Economics

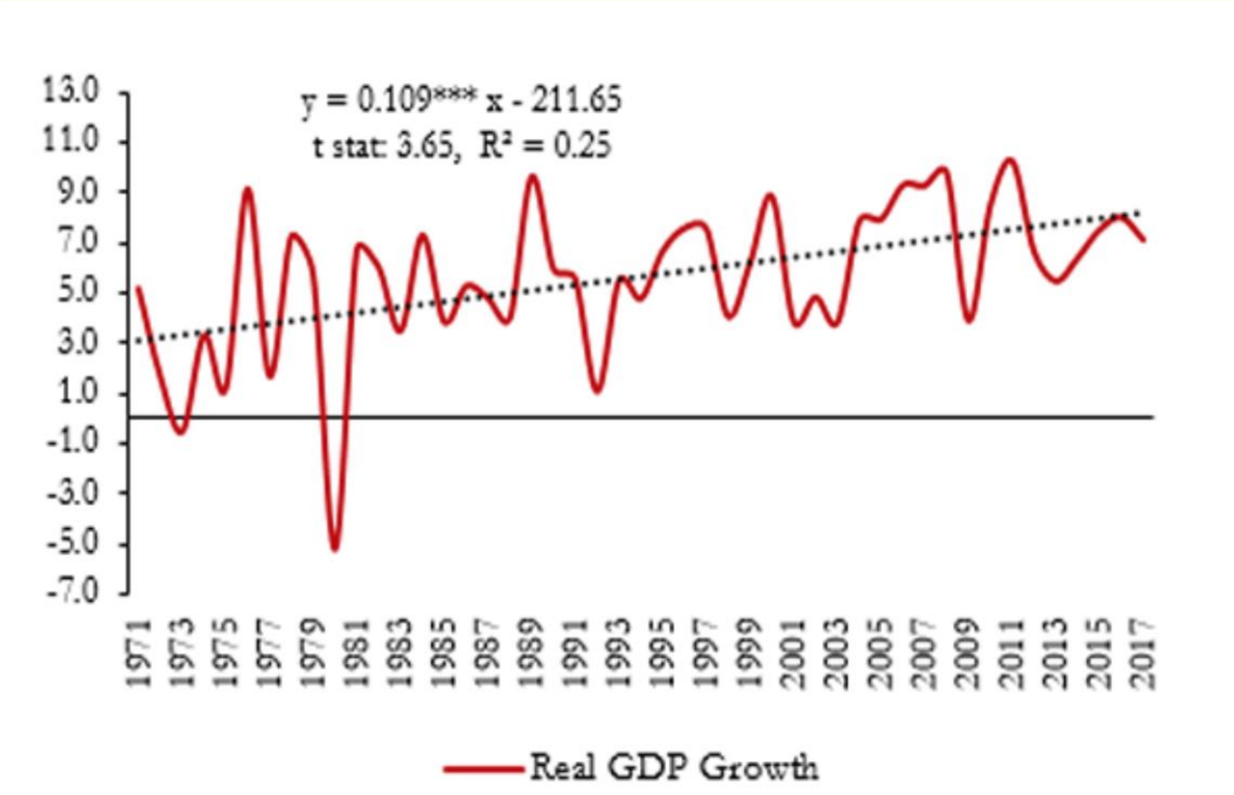
- Economic fluctuations are irregular and unpredictable
- Economic activity fluctuates from time to time: day to day, year to year
- Short term fluctuations vs long term fluctuations
- Short tem fluctuations are viewed as deviations from the continuing long term trends.
- In short term, increase in the prices of commodities may increase/decrease the availability.
- A decrease in the level of prices tends to decrease the quantity of goods and services.
- Shift in economic terms from labour issues, capital, natural resources, calamities, pandemics, advancement in technology

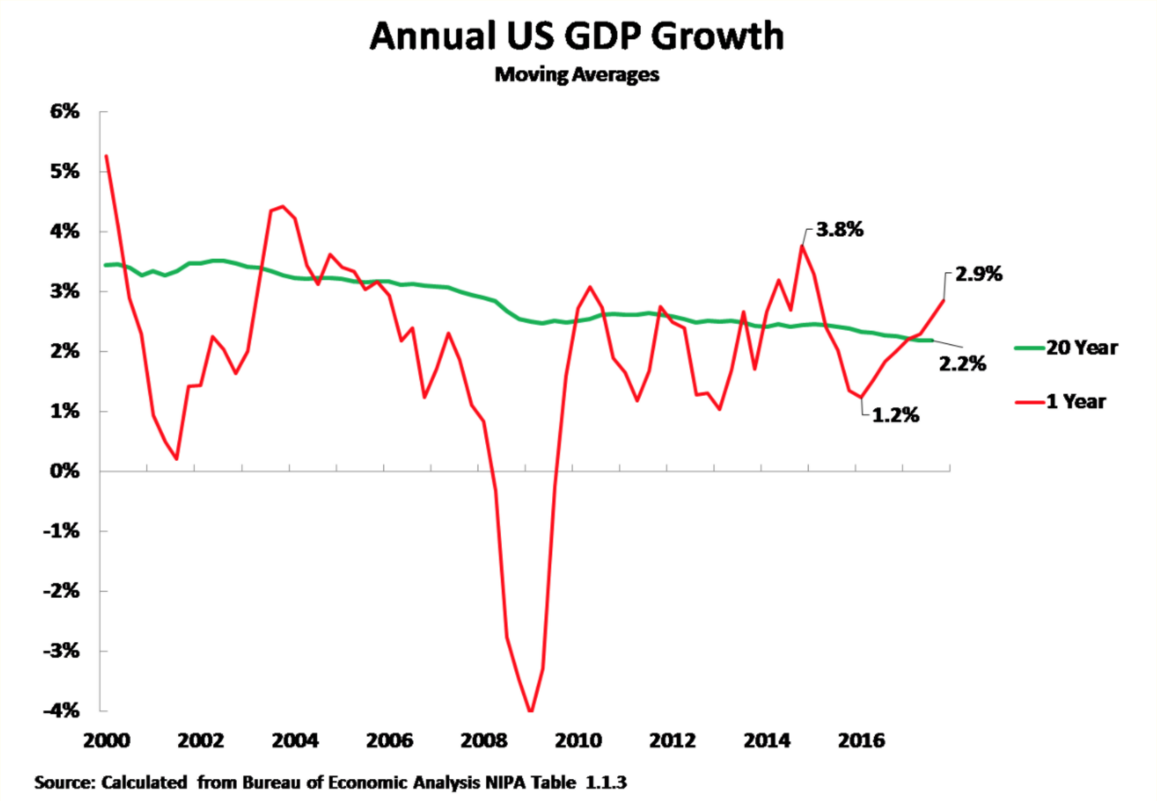
- Forex Market Fluctuations



Fluctuations: in Economics

India

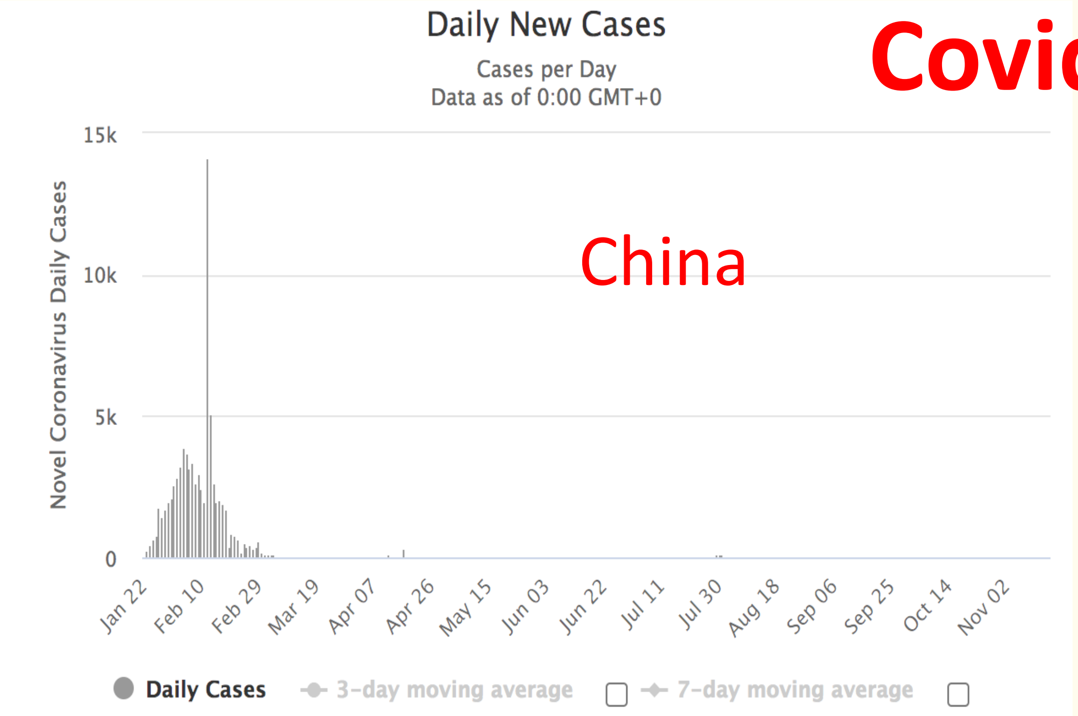


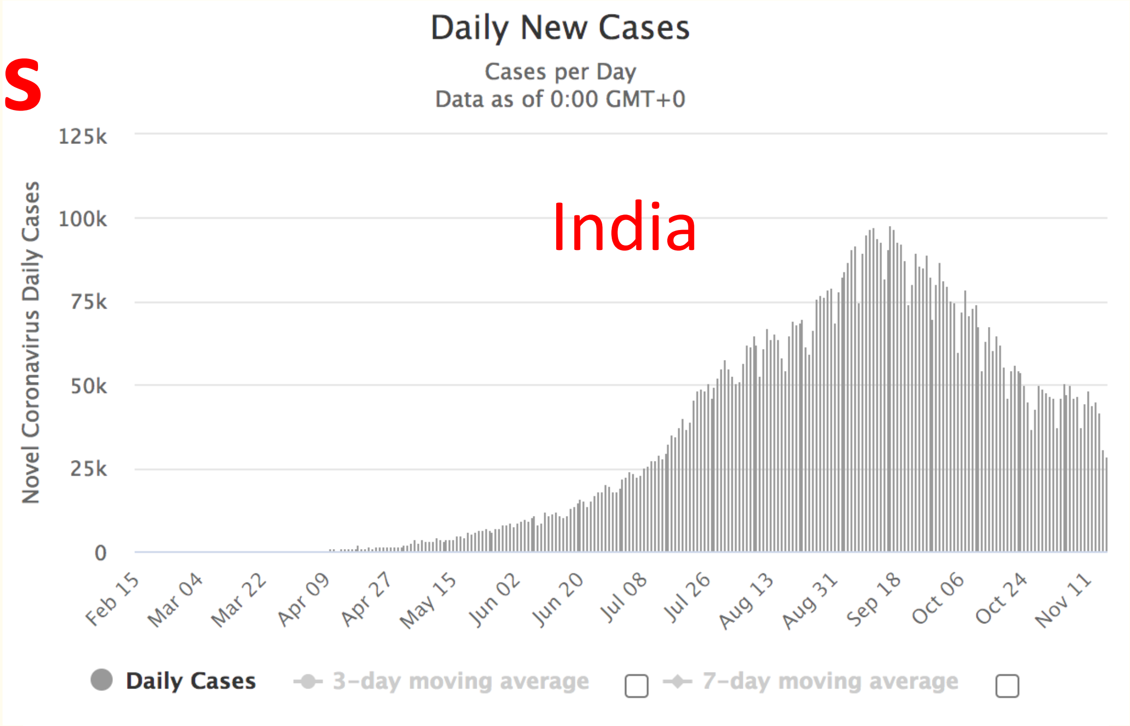


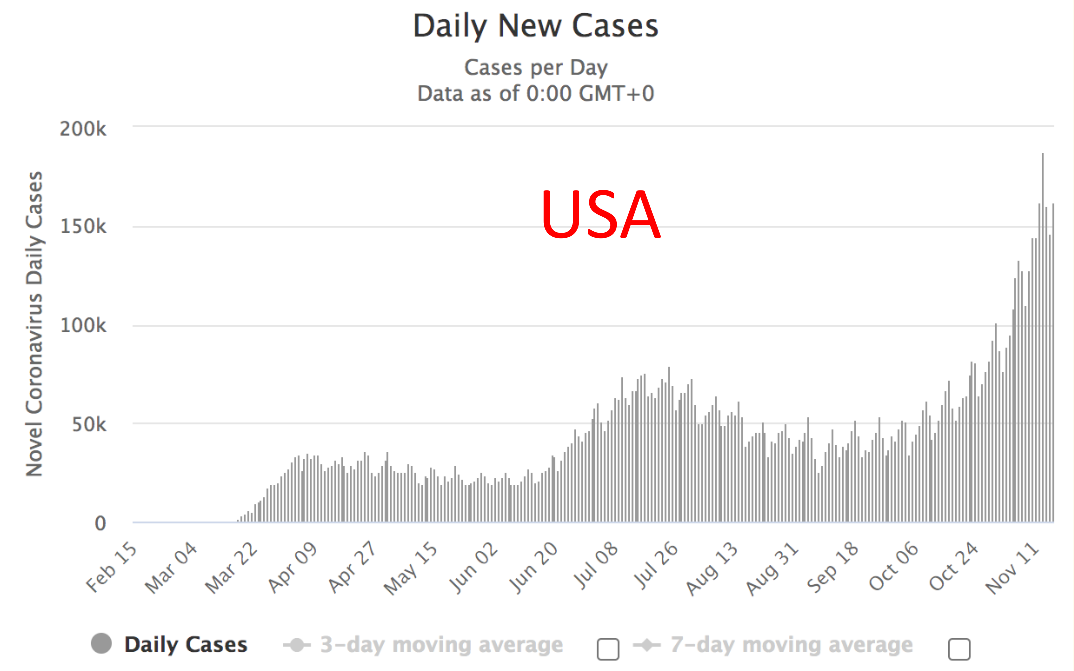
- India's long-term economic growth has steadily accelerated over a fifty-year period, without any prolonged reversals.
- India's rate of growth has become more stable.

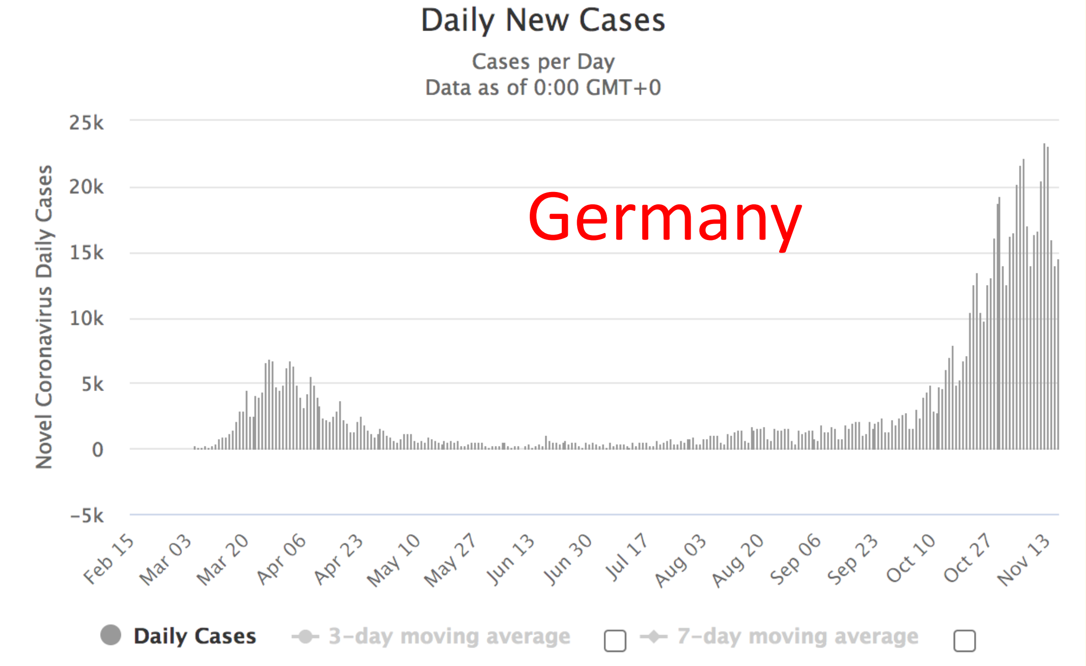
- The U.S. GDP data shows the U.S. economy proceeding on a very predictable track of slow average growth.
- using a moving average to eliminate purely short-term fluctuations, the three-year moving annual average of U.S. GDP growth is 2.1 percent,

Covid cases


Daily New Cases
Cases per Day
Data as of 0:00 GMT+0
China
Novel Coronavirus Daily Cases
15k
10k
5k
0
Jan 22
Feb 10
Feb 29
Mar 19
Apr 07
Apr 26
May 15
Jun 03
Jun 22
Jul 11
Jul 30
Aug 18
Sep 06
Sep 25
Oct 14
Nov 02
Daily Cases
3-day moving average
7-day moving average


Daily New Cases
Cases per Day
Data as of 0:00 GMT+0
India
Novel Coronavirus Daily Cases
125k
100k
75k
50k
25k
0
Feb 15
Mar 04
Mar 22
Apr 09
Apr 27
May 15
Jun 02
Jun 20
Jul 08
Jul 26
Aug 13
Aug 31
Sep 18
Oct 06
Oct 24
Nov 11
Daily Cases
3-day moving average
7-day moving average

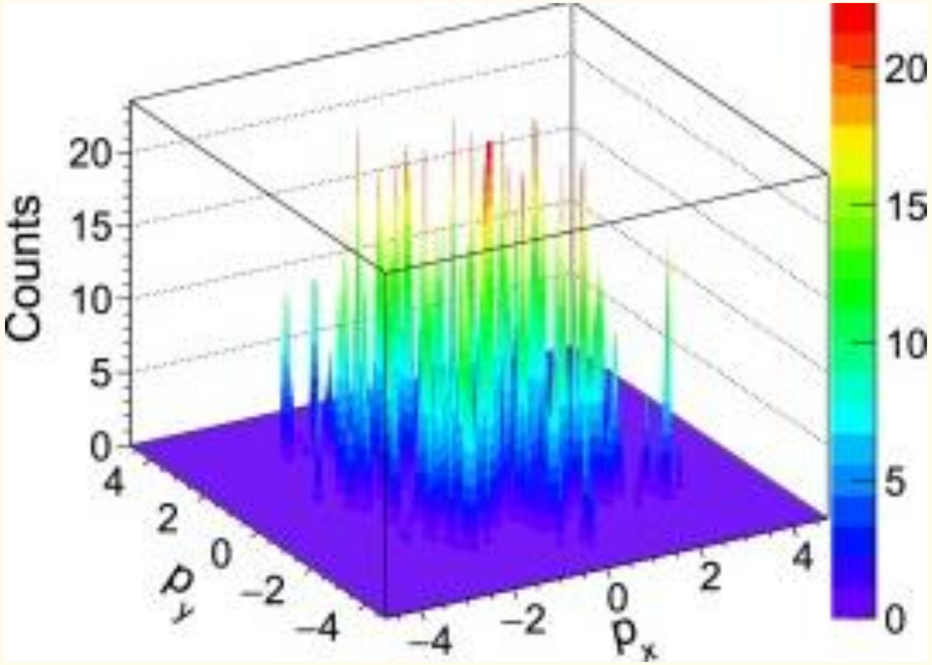

Daily New Cases
Cases per Day
Data as of 0:00 GMT+0
USA
Novel Coronavirus Daily Cases
200k
150k
100k
50k
0
Feb 15
Mar 04
Mar 22
Apr 09
Apr 27
May 15
Jun 02
Jun 20
Jul 08
Jul 26
Aug 13
Aug 31
Sep 18
Oct 06
Oct 24
Nov 11
Daily Cases
3-day moving average
7-day moving average


Daily New Cases
Cases per Day
Data as of 0:00 GMT+0
Germany
Novel Coronavirus Daily Cases
25k
20k
15k
10k
5k
0
-5k
Feb 15
Mar 03
Mar 20
Apr 06
Apr 23
May 10
May 27
Jun 13
Jun 30
Jul 17
Aug 03
Aug 20
Sep 06
Sep 23
Oct 10
Oct 27
Nov 13
Daily Cases
3-day moving average
7-day moving average

Fluctuations: intermittency, fractals, chaos, complexity

A fractal is a fragmented geometrical object that can be subdivided in parts, each of which is a reduced-size copy of the whole. Fractals are generally self-similar and independent of scale (fractal dimension).

Intermittency: In dynamical systems theory: occurrence of a signal that alternates randomly between long periods of regular behavior and relatively short irregular bursts. In other words, motion in intermittent dynamical system is nearly periodic with occasional irregular bursts.



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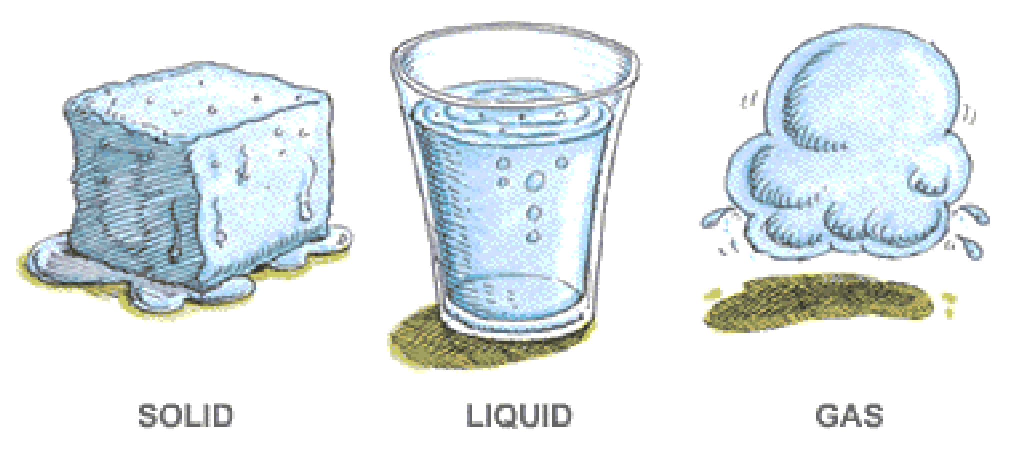
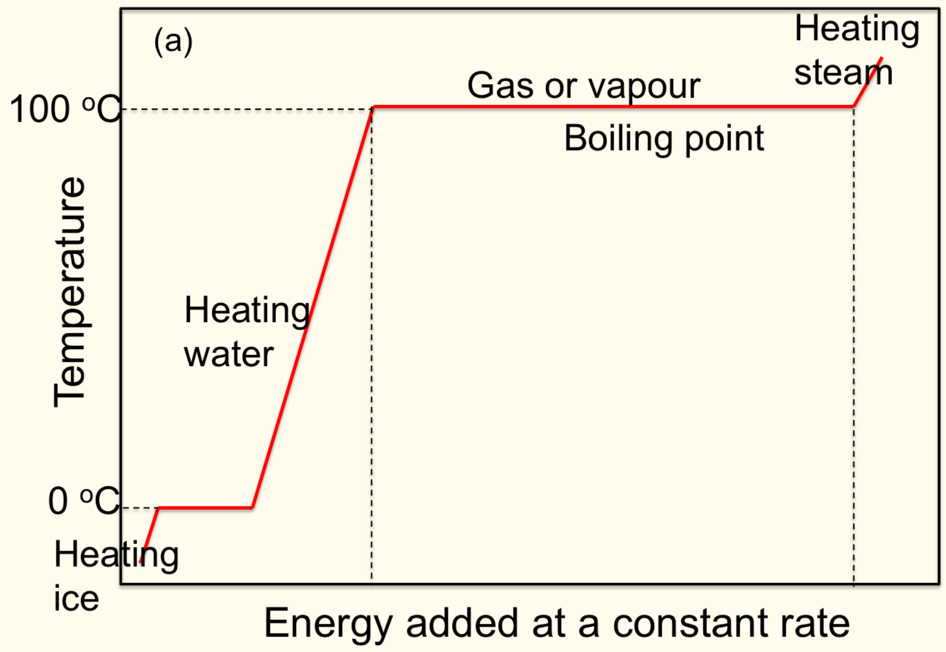
Volume 801, 10 February 2020, 135186



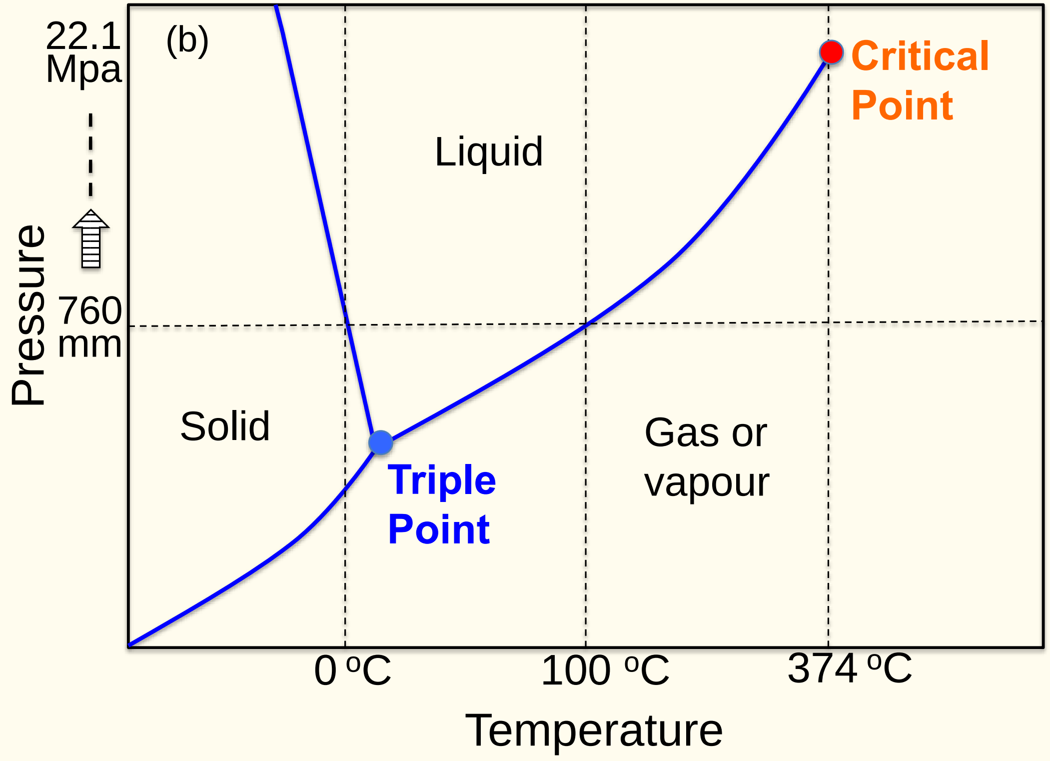
Understanding phase transition

Phase change of water with additional energy to the system at atmospheric pressure



Phase diagram of water:

Two characteristic points: Triple point & Critical point



The triple point is at 0.01°C and 4.58 mm (611.2Pa) and the critical point occurs at 373.946°C and 22.064 Mpa (217.75 atm)

Discrete distributions: binomial

- Biased coin toss
  - N trials
  - probability of success p
  - probability of failure (1-p)
  - $0 \leq p \leq 1$

$$f(n;N,p) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

random variable

parameters

(n is number of experiments)

mean

$$E[n] = \sum_{n=0}^{N} n\, f(n;N,p) = Np$$

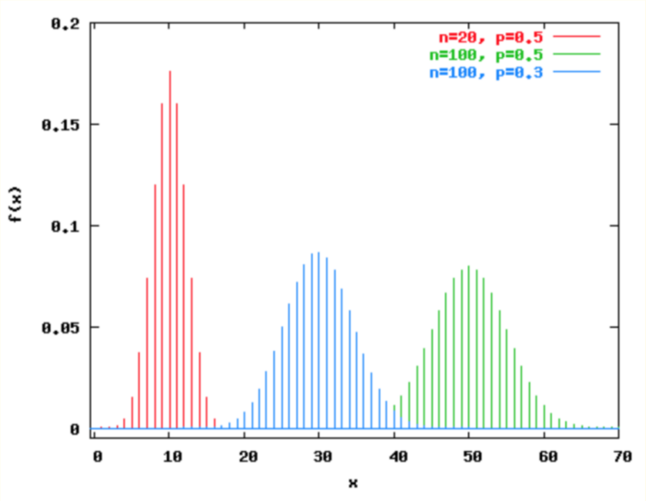
variance

$$V[n] = E[n^2] - (E[n])^2 = Np(1-p)$$

- A binomial distribution is simply the probability of a SUCCESS or FAILURE outcome in an experiment that is repeated multiple times. It has two possible outcomes: For example, a coin toss has only two possible outcomes: heads or tails and taking a test: pass or fail.

- In real life: A new drug is introduced for covid. Is there a success or failure?

- A binomial distribution is discrete, that means there is no data point between two points.
- This is different from a normal distribution which has continuous data points.
- For large sample size, the distribution may look like normal distribution.



Discrete distributions: Poisson

- Limiting case of binomial distribution
  - Limit for $p \to 0$
  - Mean successes $Np \to \mu$
  - $n \geq 0$

$$f(n;\mu) = \frac{\mu^n}{n!} e^{-\mu}$$

random variable

parameter

mean

$$E[n] = \sum_{n=0}^{\infty} n \frac{\mu^n}{n!} e^{-\mu} = \mu$$

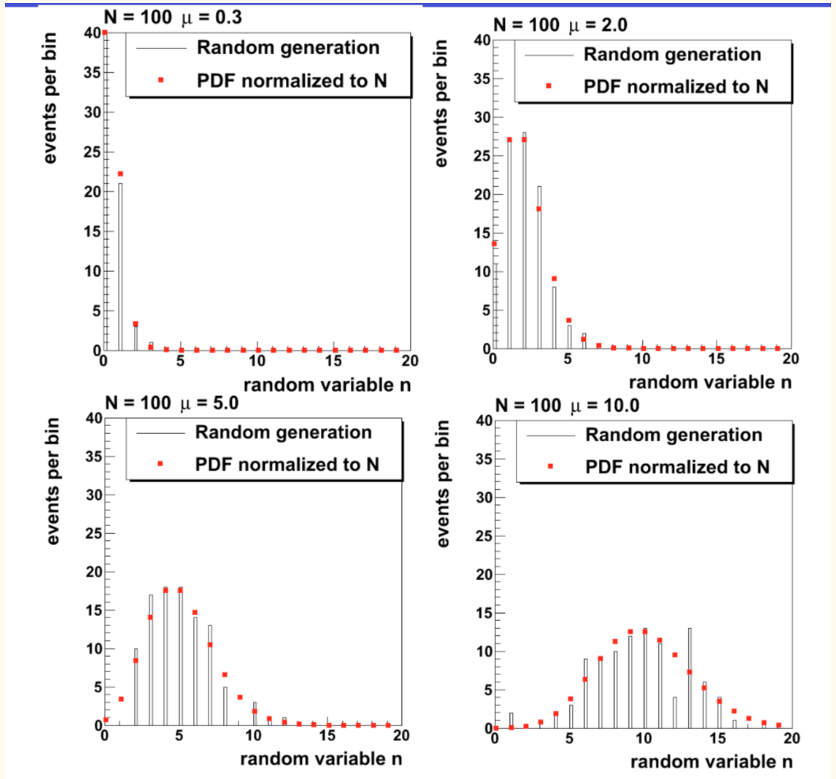
$\sigma = \sqrt{\mu}$

variance

$\sigma/\mu = 1/\sqrt{\mu}$

$$V[n] = E[n^2] - (E[n])^2 = \mu$$

- Events are independent of each other
- The occurrence of one event does not affect/depend on another event
- Two events do not occur at the same time.



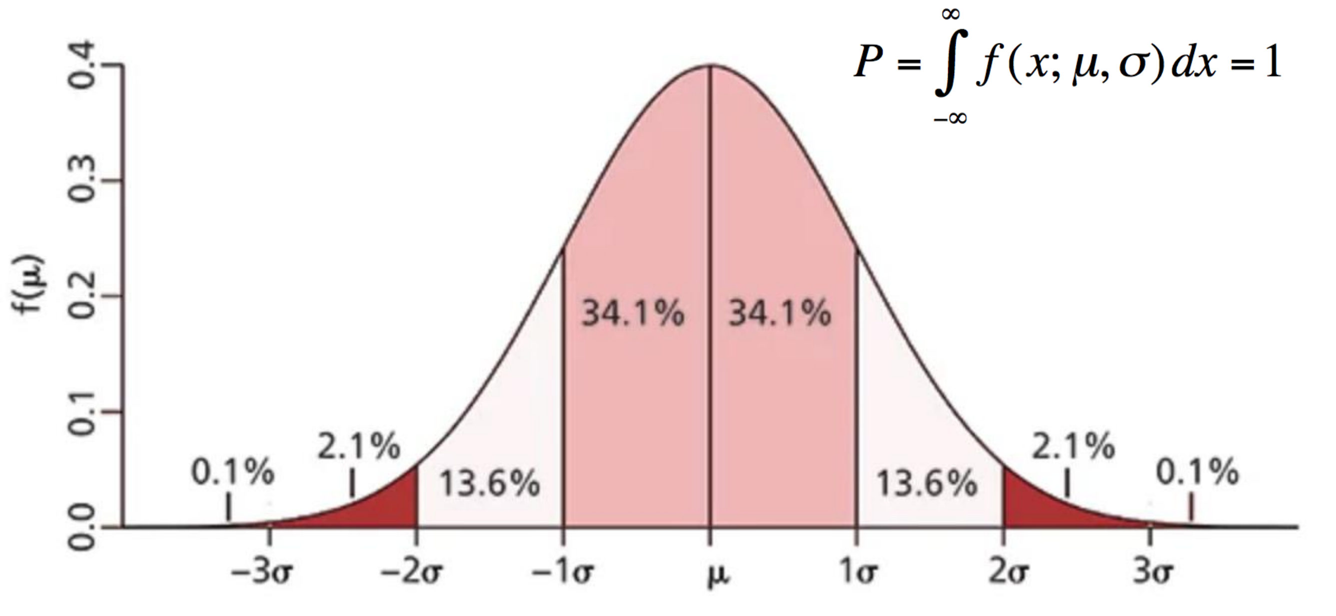
The Poisson is used as an approximation of the Binomial if n is large and p is small.

The distribution approaches Gaussian for values of large mean values (like $\mu \geq 10$) …

Gaussian distribution

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ=0 and σ=1 Normal distribution



Expectation value of x $\quad E[x] = \alpha_1 = \int_{-\infty}^{\infty} x\, f(x)\,dx$

n^{th} moment of a random variable x $\quad \alpha_n = E[x^n] = \int_{-\infty}^{\infty} x^n f(x)\,dx$

n^{th} central moment of a random variable x $\quad m_n = E[(x-\alpha_1)^n]$

mean $\quad \mu \equiv \alpha_1$

variance $\quad \sigma^2 \equiv V[x] \equiv m_2 = \alpha_2 - \mu^2$

root-mean-square $\quad \sigma \equiv \sqrt{V[x]}$

Traditionally, the Gaussian probability distribution is used for a broad quantification of the data set variability in terms of the sample mean and variance.

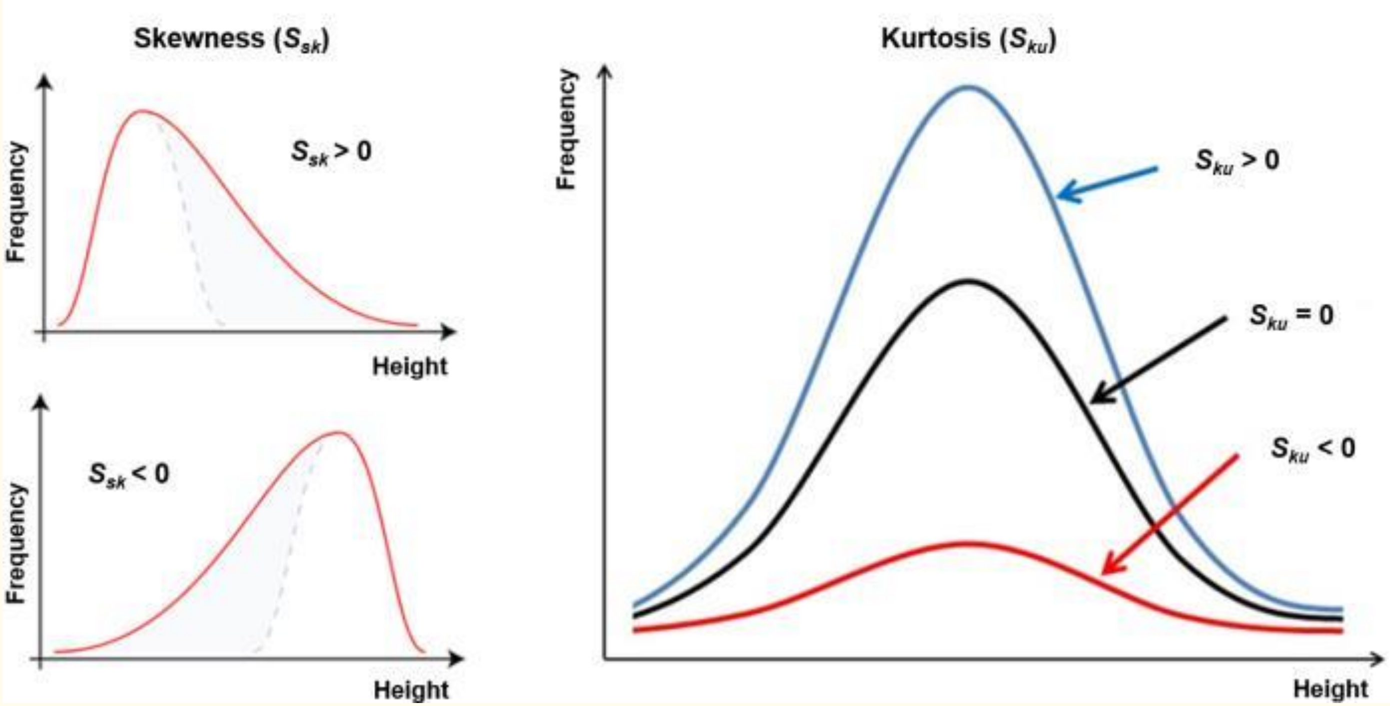
Higher moments: measure of non-Gaussian fluctuations

Mean
$$\overline{x} = \frac{1}{N}\sum_{j=1}^{N} x_j$$

Variance
$$\mathrm{Var}(x_1 \ldots x_N) = \frac{1}{N-1}\sum_{j=1}^{N}(x_j - \overline{x})^2$$

Skewness
$$\mathrm{Skew}(x_1 \ldots x_N) = \frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \overline{x}}{\sigma}\right]^3$$

Kurtosis
$$\mathrm{Kurt}(x_1 \ldots x_N) = \left\{\frac{1}{N}\sum_{j=1}^{N}\left[\frac{x_j - \overline{x}}{\sigma}\right]^4\right\} - 3$$



The kurtosis of the normal distribution is equal to 3. So to normalize the kurtosis with respect to normal or Gaussian distribution, 3 is subtracted and final expression for the kurtosis.

The positive and negative values of the skewness qualitatively show the distribution of the random variable X skewed towards right and left side of the mean of the distribution ….

The kurtosis represents the peakedness and tailed-ness of the distribution

Astronomical probes of our Universe

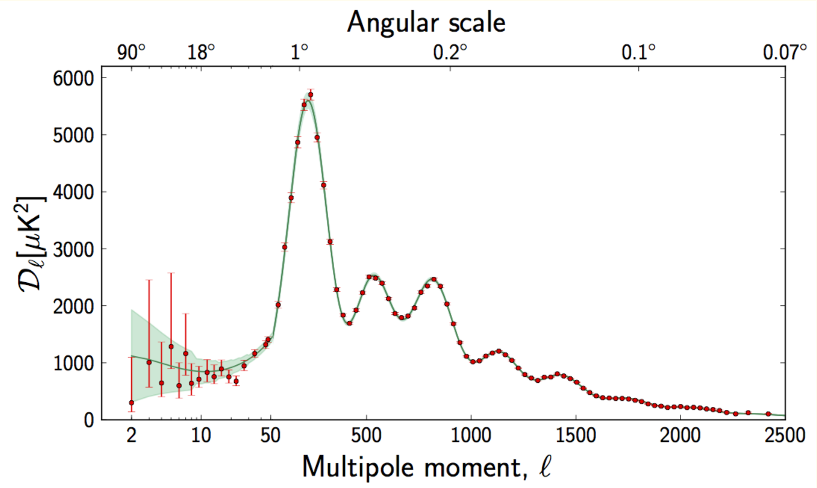
The Nobel Prize in Physics 2006

George Smoot & John Mather

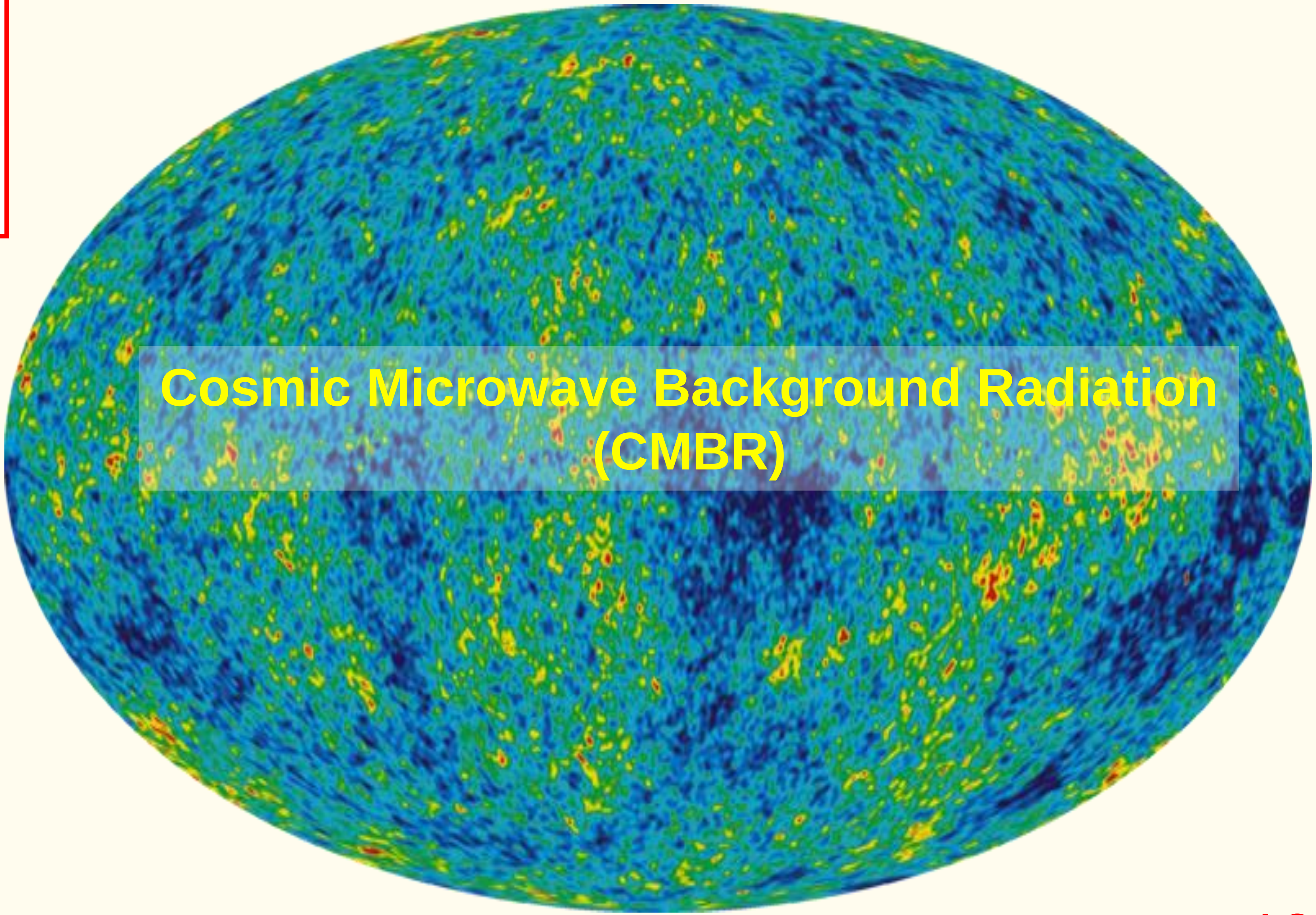
"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"

WMAP: Wilkinson Microwave Anisotropy Probe

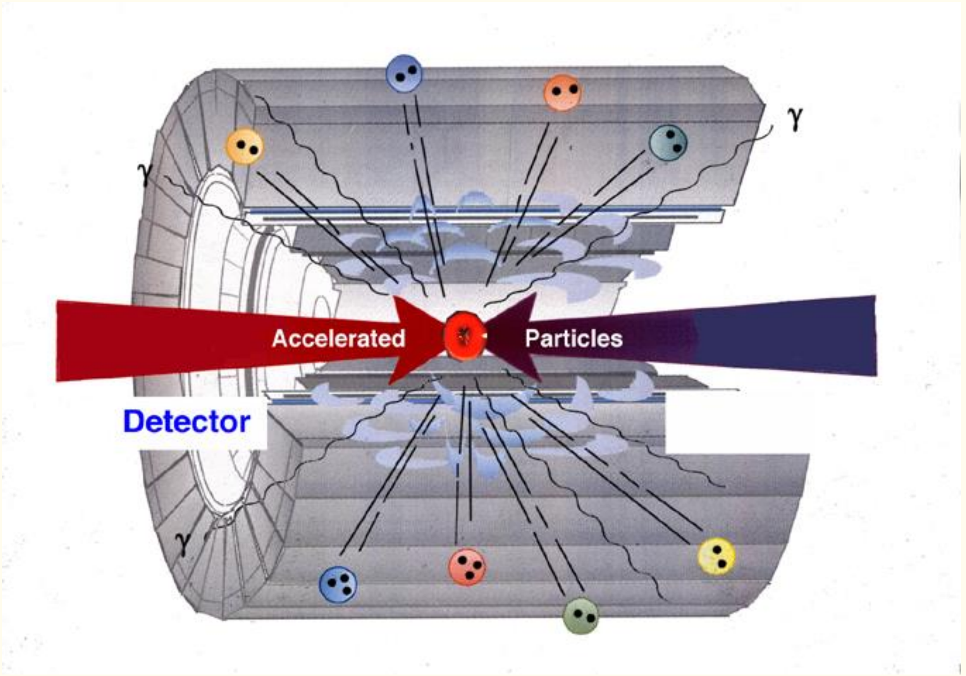
Temperature Fluctuation Spectrum



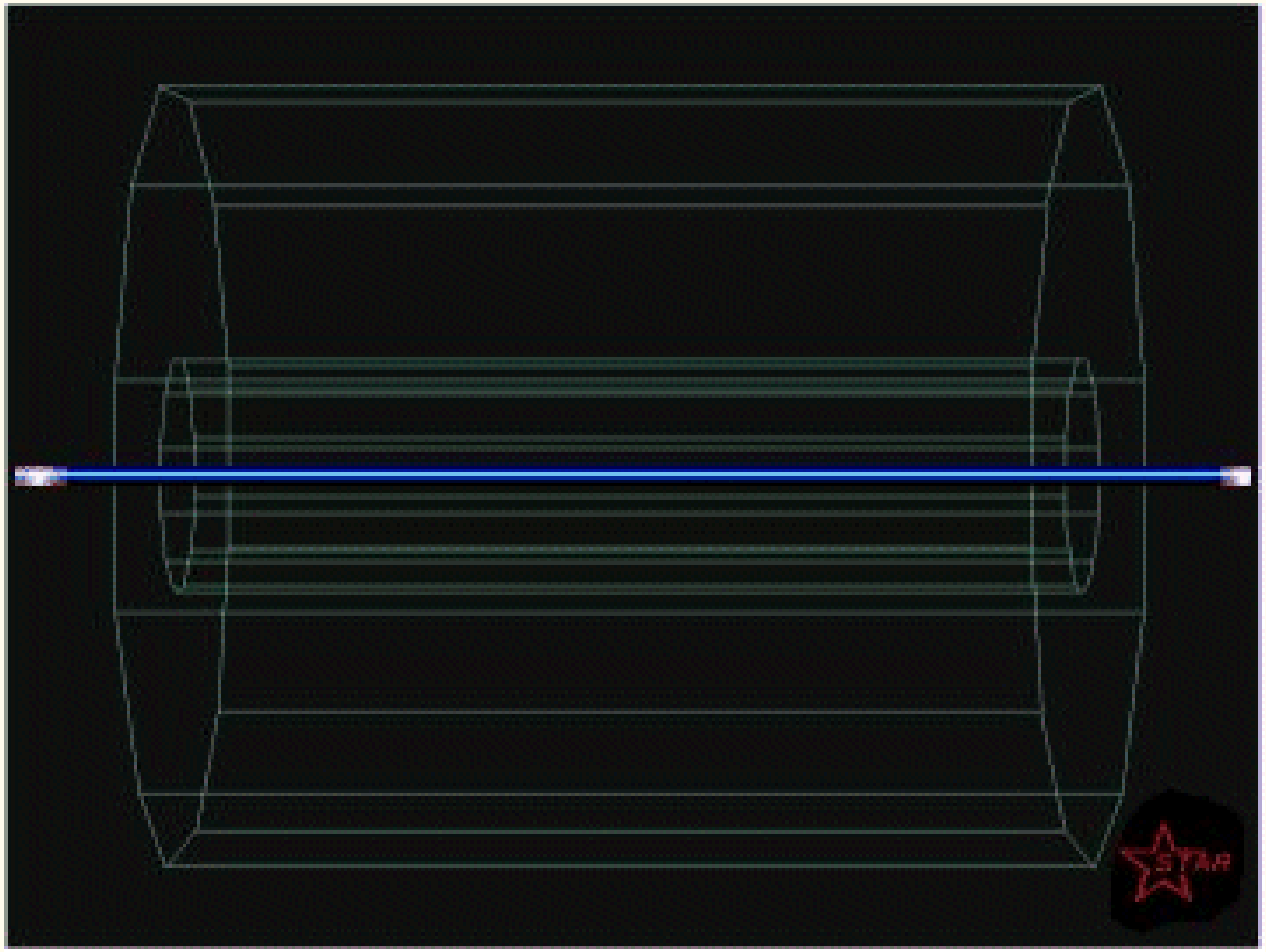
Probes 380,000 years after the Big Bang



Relativistic heavy-ion collisions

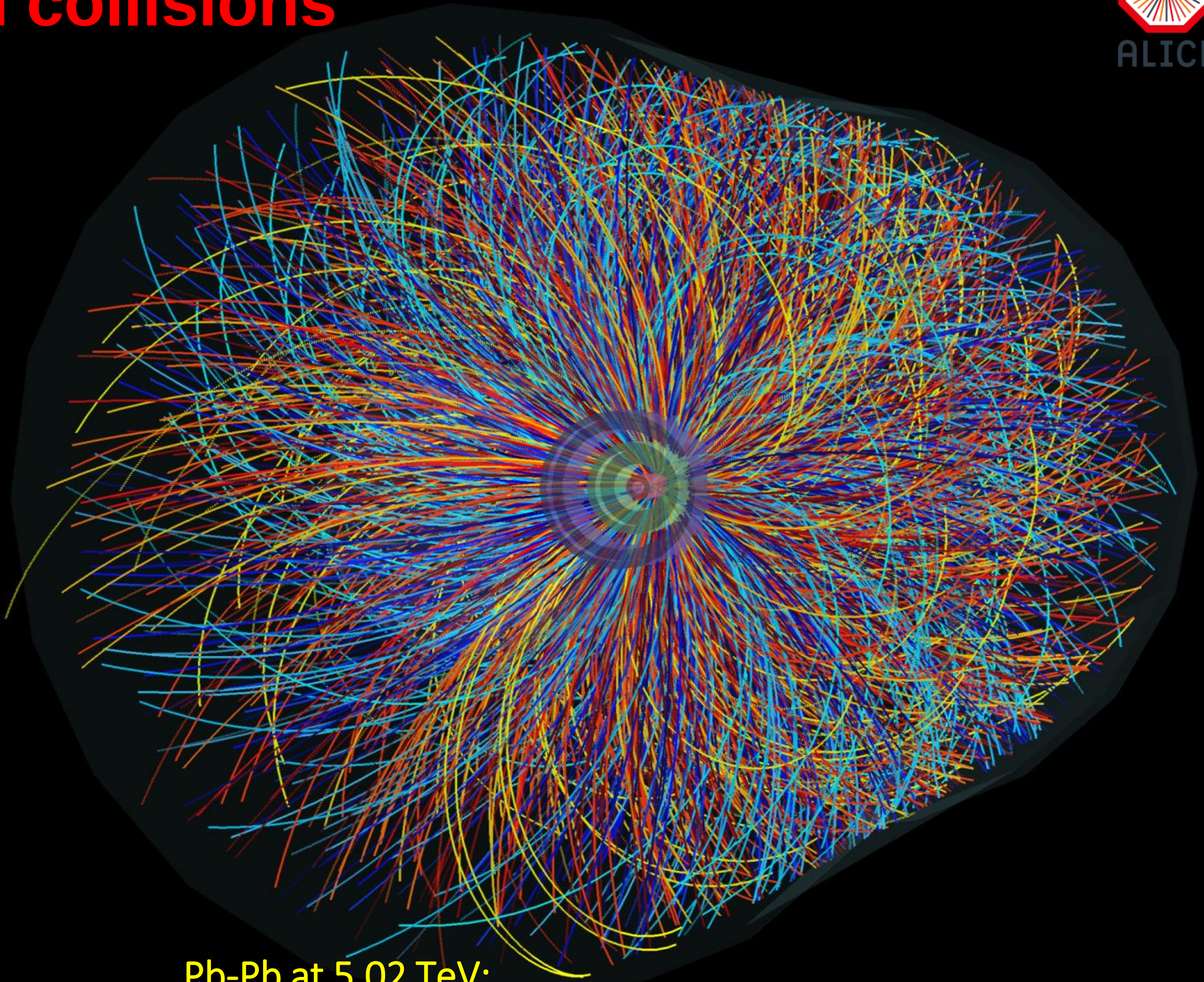


$$E = mc^2$$



Relativistic heavy-ion collisions

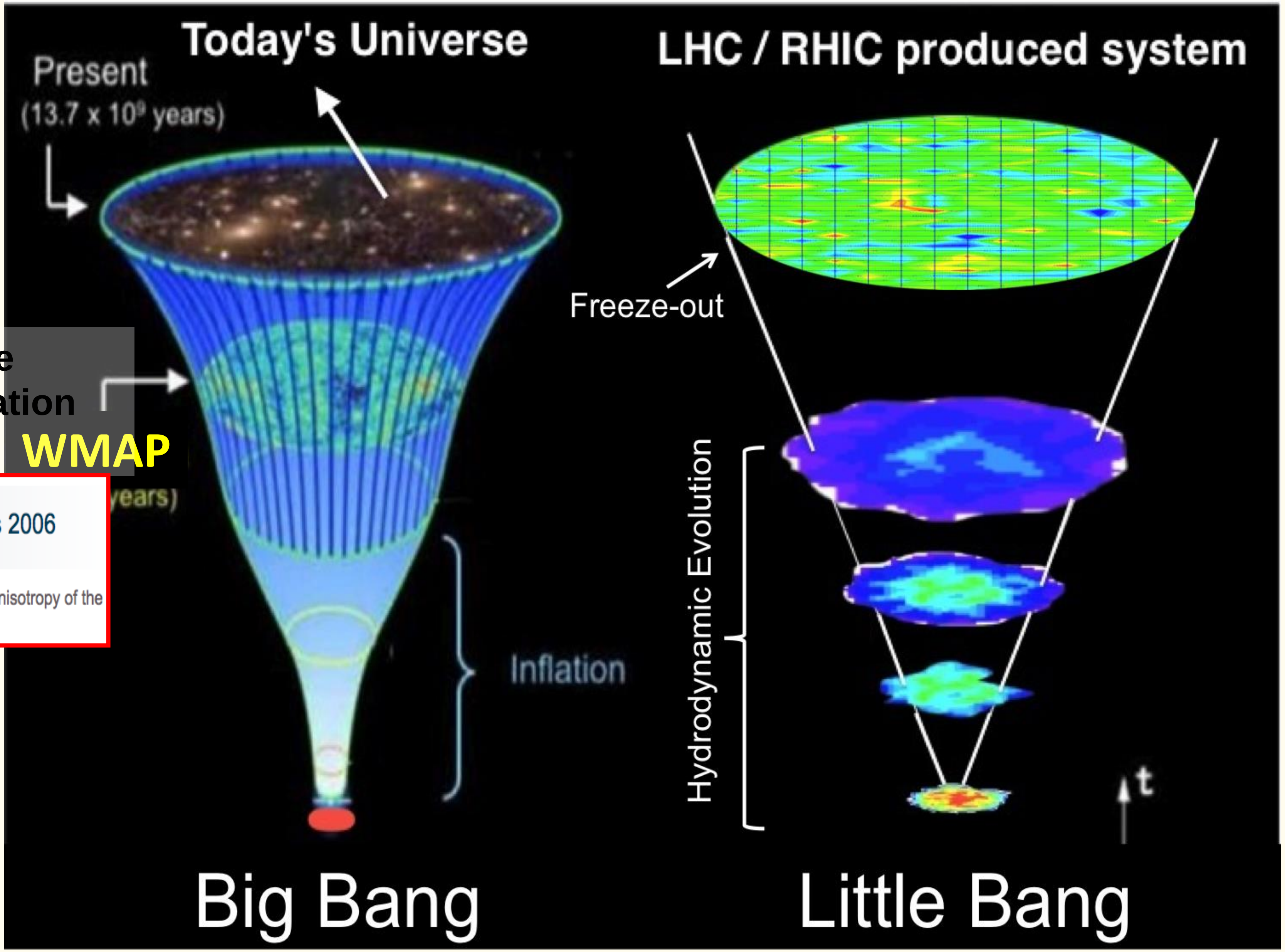
https://home.cern/news/series/lhc-physics-ten/recreating-big-bang-matter-earth

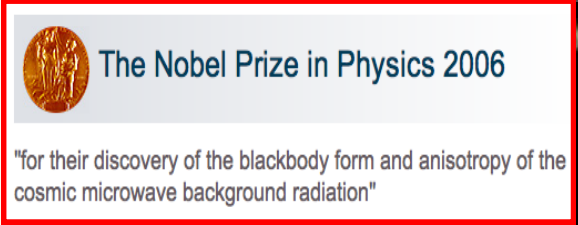


Pb-Pb at 5.02 TeV:
One PeV Collision

The Big Bang and Little Bangs

380,000 years after the Big Bang:
Cosmic Microwave Background Radiation (CMBR)





Temperature Fluctuation

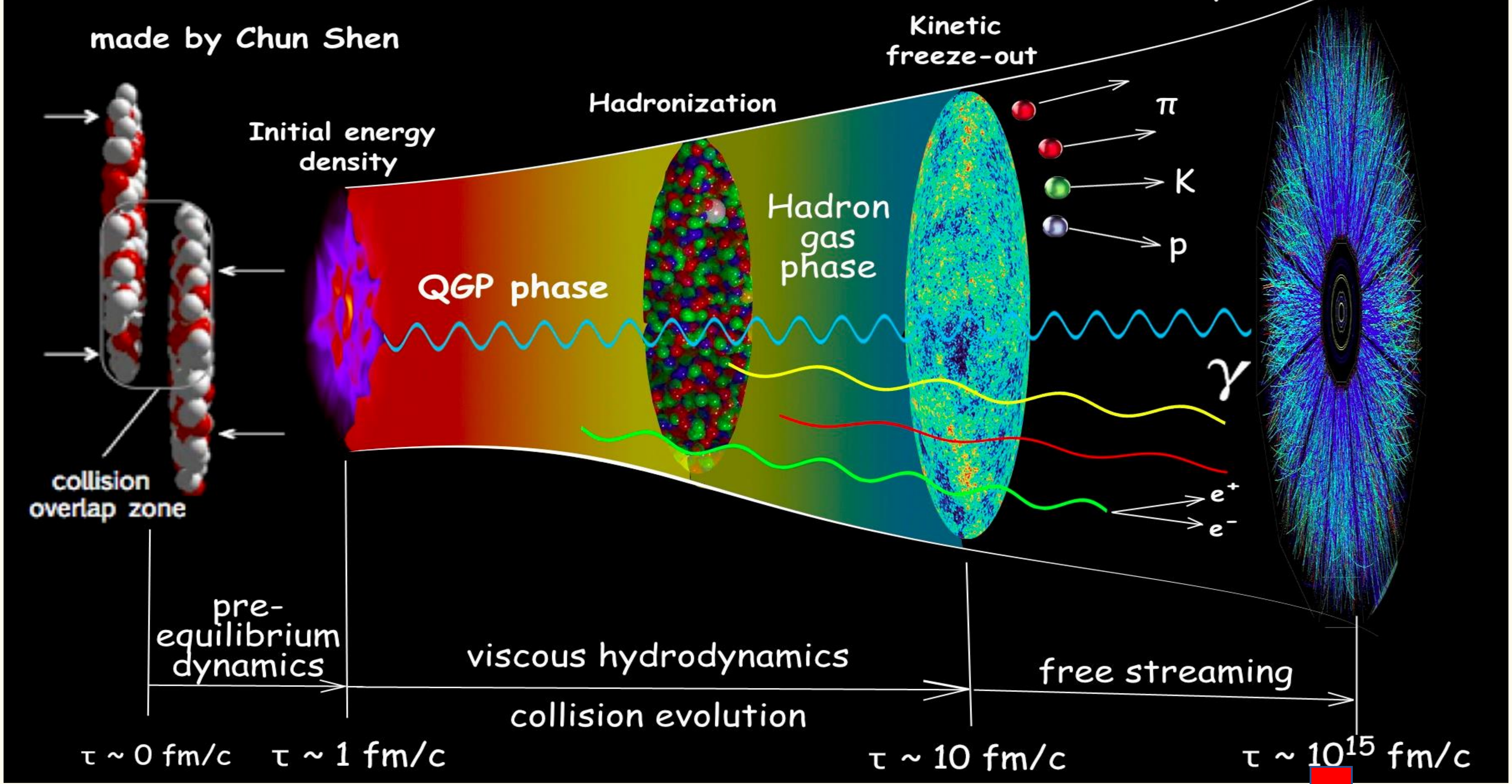
One HUGE Event

Heavy-ion Collisions: Billions of Events (Little Bangs)

=>

Event-by-Event physics: Fluctuations

Relativistic Heavy-Ion Collision



Initial State Fluctuations

Thermal Fluctuations

Hadronization

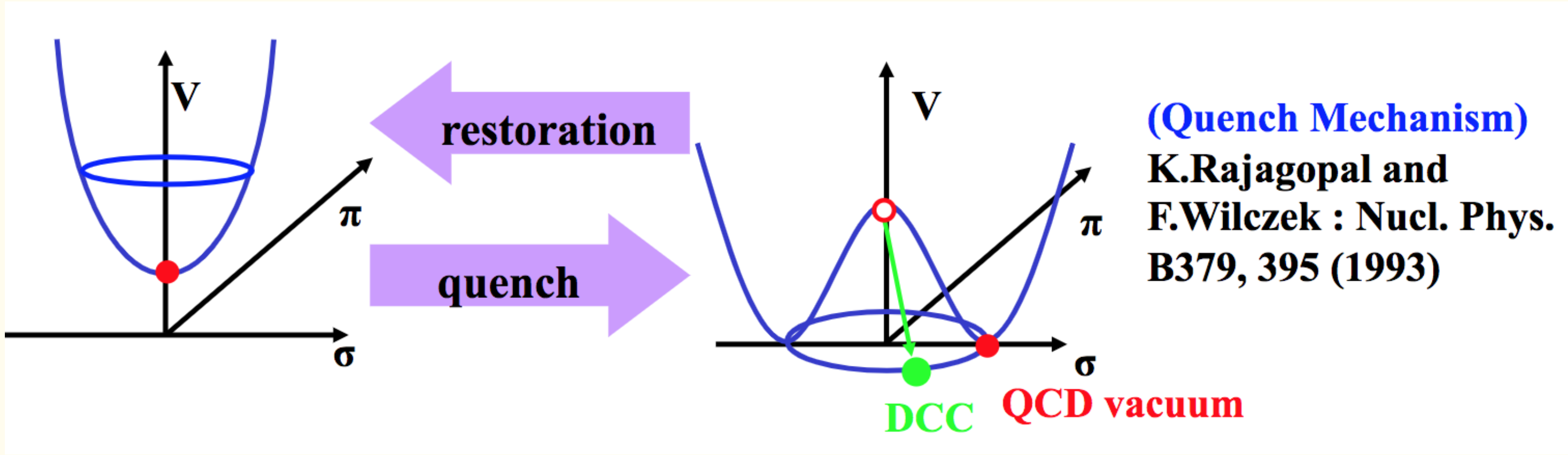
Measurement (Fluctuation-Correlation)

DCC: Disoriented Chiral Condensates

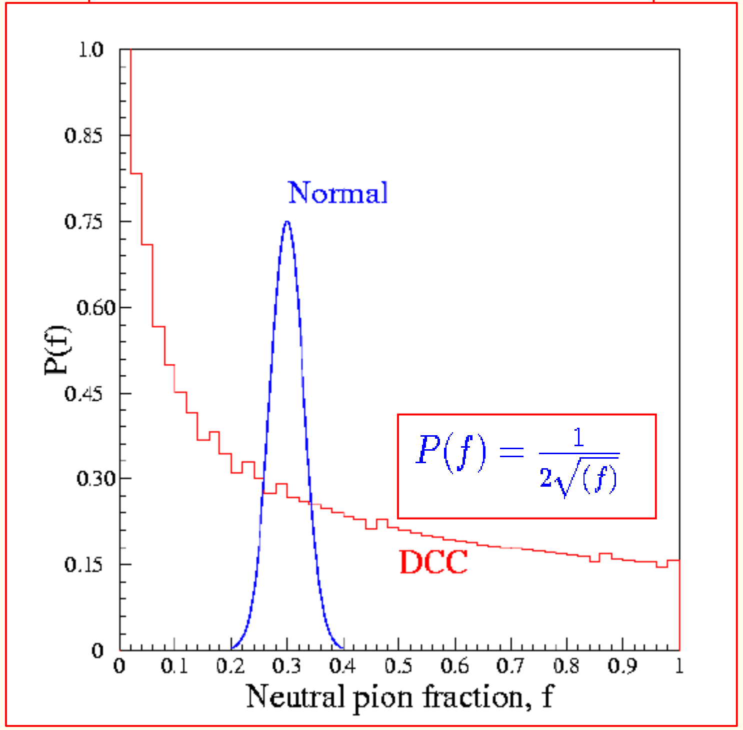
Cosmic ray Centauro events:
Baked Alaska: Bjorken, Kowalski and Taylor
arXiv:hep=ph/9309235

1993

(Chiral symmetry is the fundamental symmetry of QCD for massless quarks, tells about how mass is generated in the ground state when the symmetry is broken)



$$f = \frac{N_{\pi^0}}{N_{\pi^0} + N_{\pi^+} + N_{\pi^-}}$$



- One of the consequences is the **formation of DCC domains**
- **Fluctuation in the number of charged particles to photons**

WA98 Experiment at CERN
(data taking: 1993 – 1996)

Charged to Neutral Fluctuations

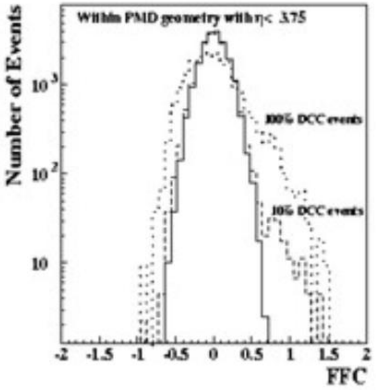
Multiplicity Fluctuations: 2000

Discrete wavelet technique

- ϕ segmented

$$f = \frac{N_{\gamma-like}}{N_{\gamma-like} + N_{ch}}$$

- Obtain FFC of function f



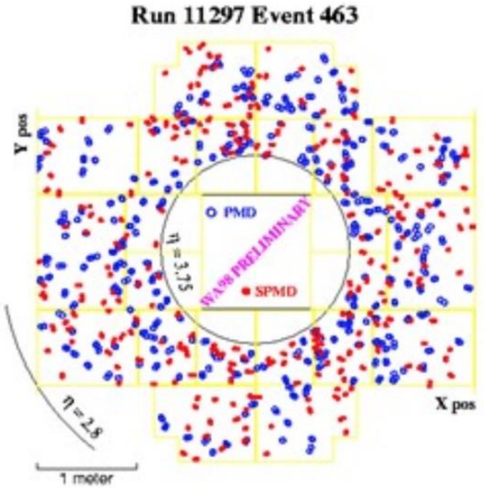


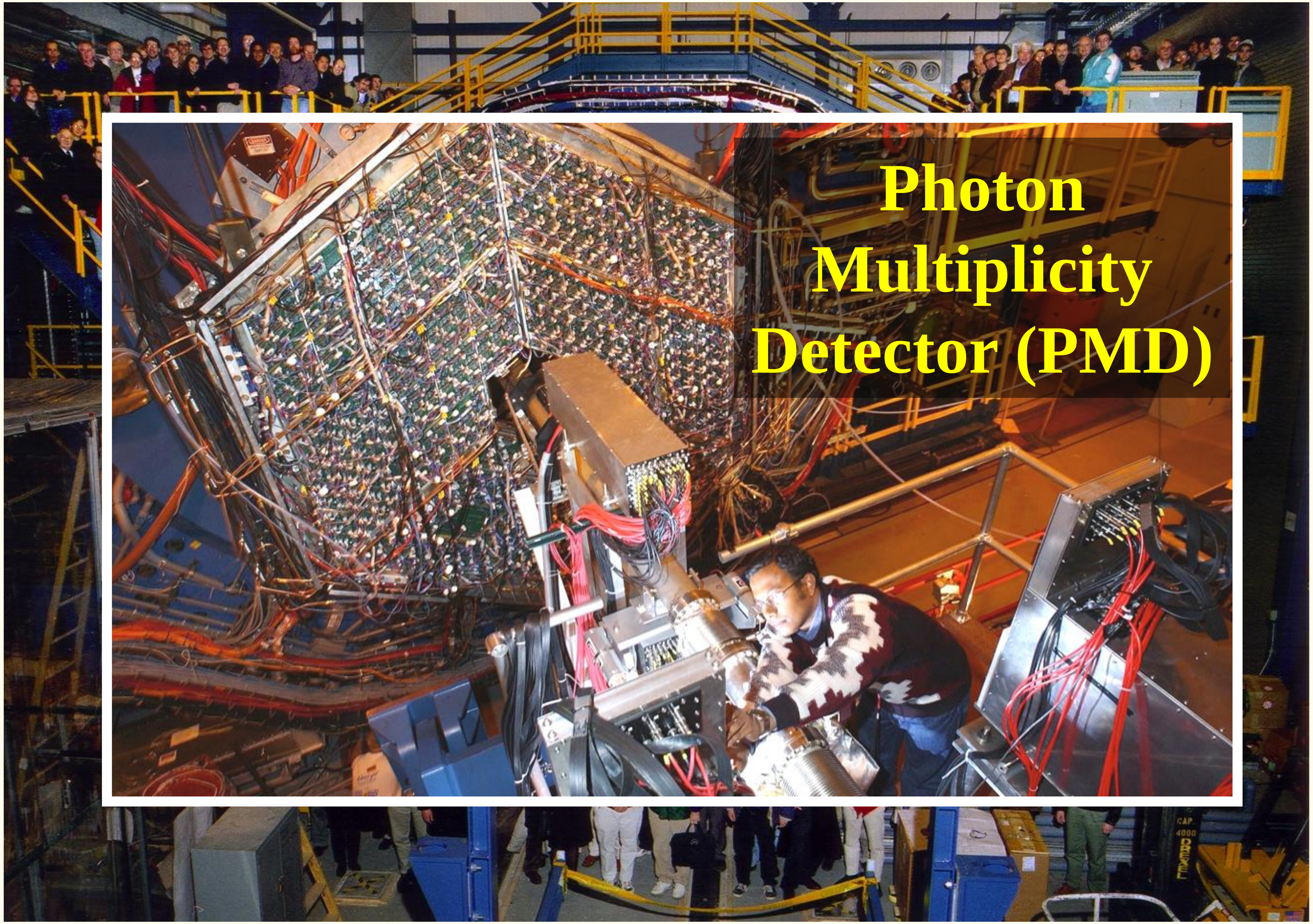
Figure 3. $x-y$ phase space distribution of a normal event from the WA98 experiment on Pb+Pb reactions at 158·A GeV/c.

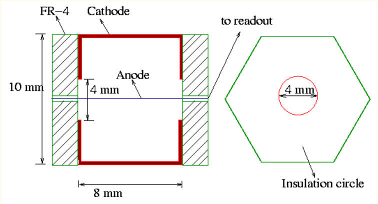
Basanta Nandi
Bedanga Mohanty

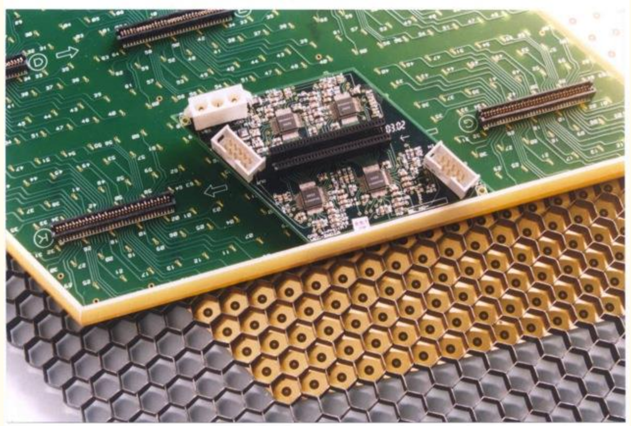
STAR experiment at RHIC, BNL

2000 ....

Year 2001 .....







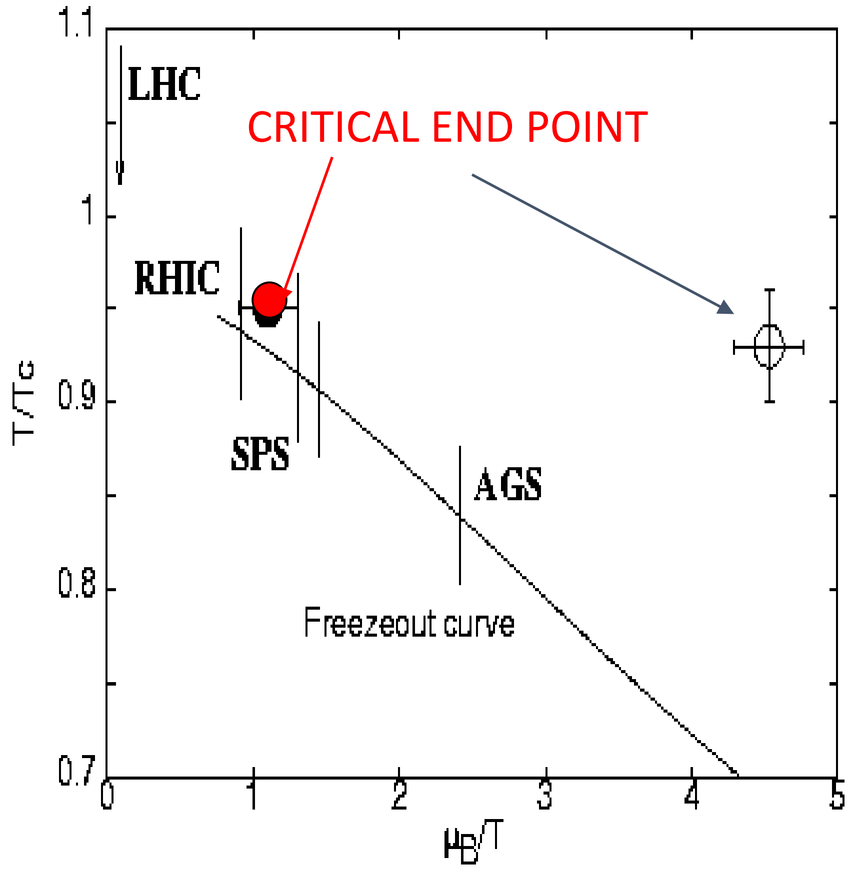
85000 gas cells
4 sq.m. area

RHIC Beam Energy Scan:

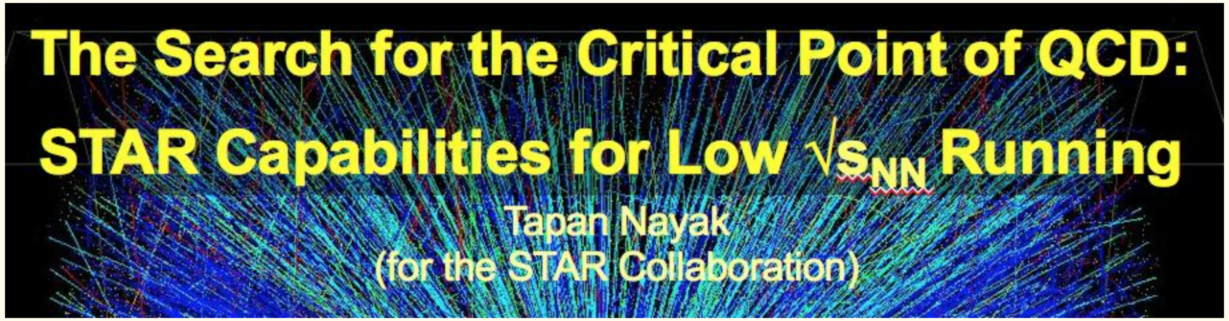
a bit of history

2004 STAR-India Regional Meeting

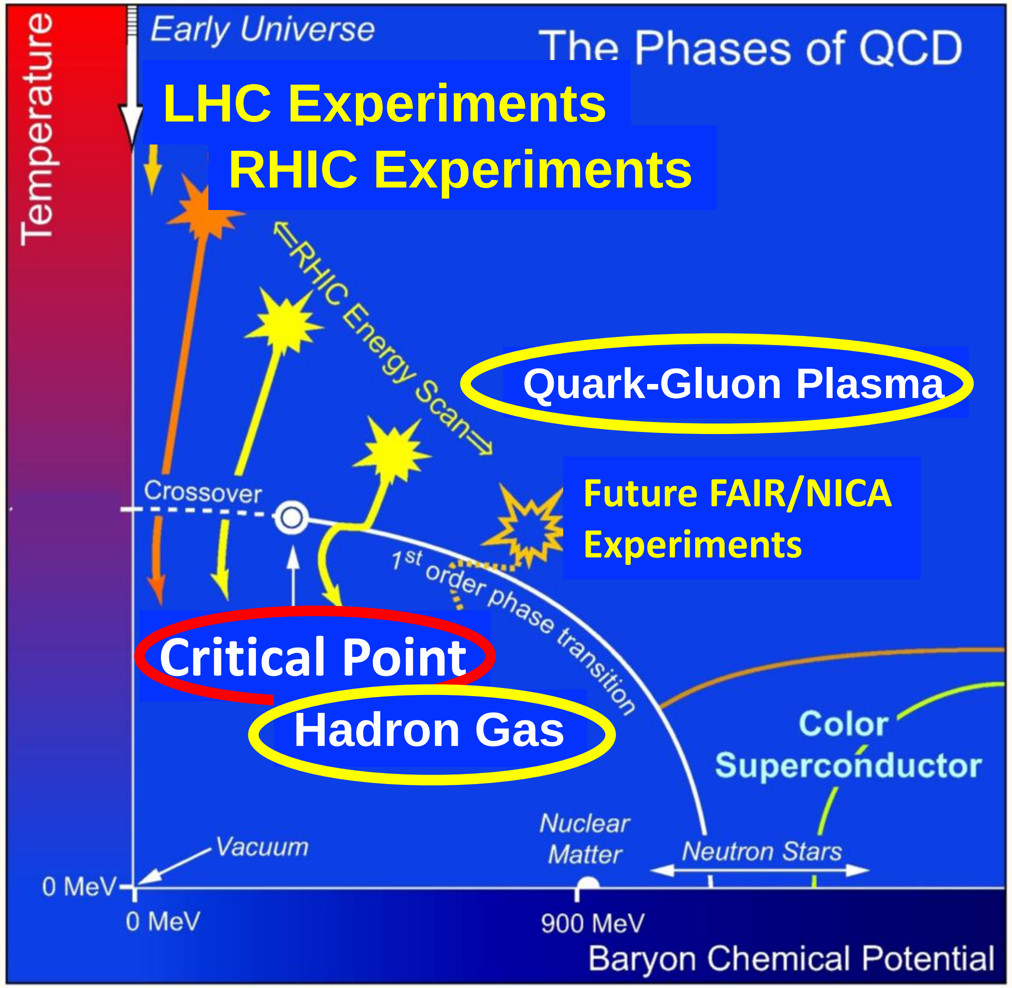
2006 …. Workshop



First Estimations (2006)



Probing the QCD phase structure using event-by-event fluctuations



arXiv:0809.3137

Probing the QCD critical point

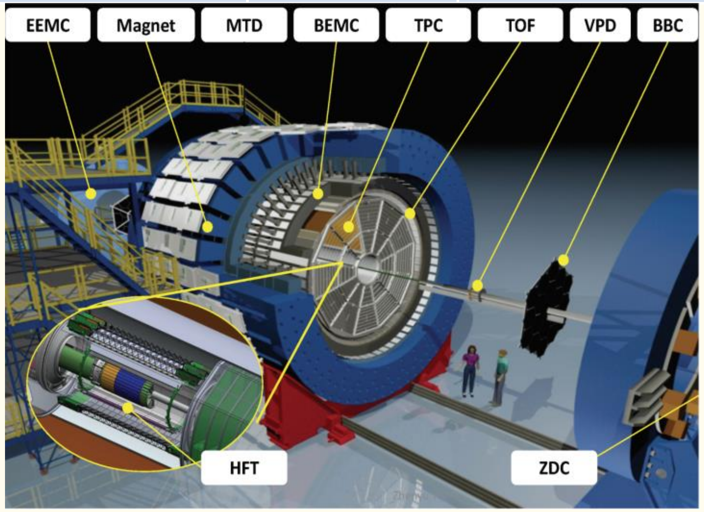
- **Fluctuations of conserved quantities**
  - Onset of QGP phase transition
  - Probing the QCD Critical Point
  - Access to Freeze-out conditions
- **Accessing equation of state**
  - Isothermal compressibility (k_T)
  - Specific heat (c_V)
- **Maps of the Little Bang**

Fluctuations …. Higher moments … starting from 2007

RHIC Beam Energy Scan

BES – I (2010 – 2017)

$\sqrt{S_{NN}}$ (GeV)	m_B (MeV)	Events
200	25	350 M
62.4	73	67 M
54.4		1300 M
39	112	39 M
27	156	70 M
19.6	206	36 M
14.5		20 M
11.5	316	12 M
7.7	422	4 M

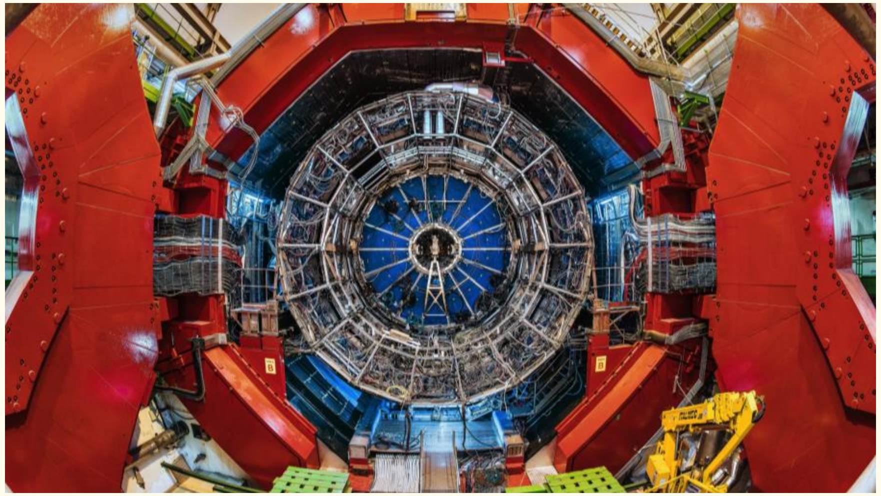


BES – II : ongoing

	$\sqrt{S_{NN}}$ (GeV)	m_B (MeV)
Collider	19.6	205
Collider	14.6	260
Collider	16.7	2235
Collider	11.5	315
Collider	9.1	370
Collider	7.7	420
FXT	7.7	420
FXT	4.5	589
FXT	3.9	633
FXT	6.2	487
FXT	5.2	541
FXT	3.5	666
FXT	3.2	699
FXT	3.0	721

Feb 2020: Fixed Target (FXT) data taking complete: 100M each

LHC: ALICE at Point-2



System	Years	$\sqrt{s_{NN}}$ (TeV)	L_{int}
Pb-Pb	2010, 2011	2.76	~75 mb^{-1}
Pb-Pb	2015, 2018	5.02	~1 nb^{-1}
Xe-Xe	2017	5.44	~0.3 mb^{-1}
p-Pb	2013, 2016	5.02, 8.16	~18 nb^{-1}, ~25 nb^{-1}
pp	2009-2013, 2015-2018	0.9, 2.76, 7, 8, 5.02, 13	>25 pb^{-1}

Bridge between Theory and Experiment

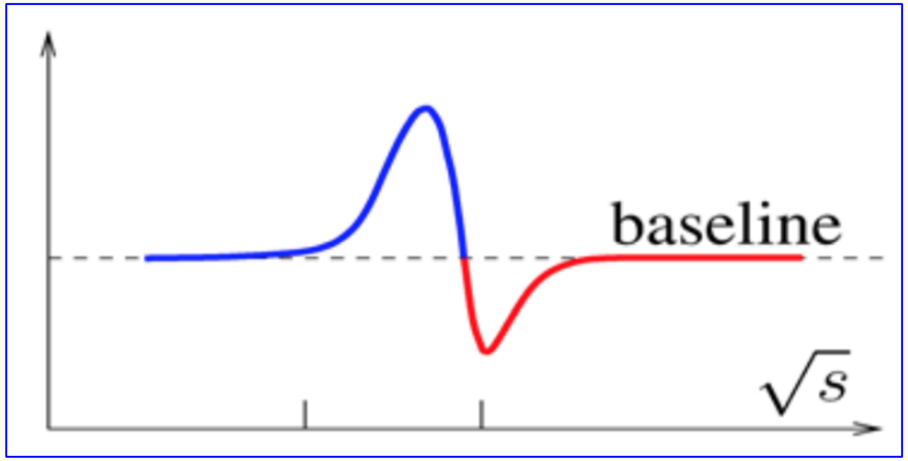
Thermodynamic Susceptibility ⟷ **Moments of the conserved charge distributions**

Q: net-charge
B: net-baryon
S: net-strangeness

$$VT^3\chi_2^Q = <(\delta N_Q)^2>$$

$$VT^3\chi_3^Q = <(\delta N_Q)^3)>$$

$$VT^3\chi_4^Q = <(\delta N_Q)^4)> -3<(\delta N_Q)^2)>^2$$



Non-monotonic behavior as a function of collision energy

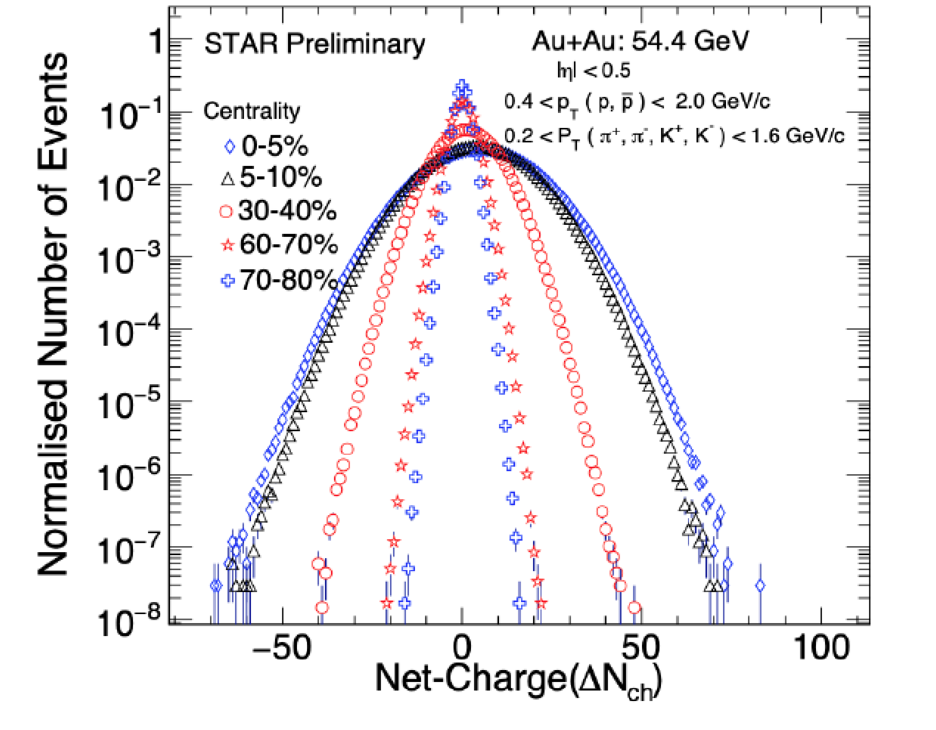
(1) Understanding the phase structure of QCD phase diagram

(2) Search for QCD Critical Point

(3) Critical fluctuations at $\mu_B \approx 0$

Next: Let's see what we get from moments of Q,B,S …..

Q, B, S distributions: Experiment



Lattice calculations: Theory

Stephanov, PRL 107 (2011)

2nd Order Cumulant:

$$\boxed{<(\delta N)^2> \approx \zeta^2}$$

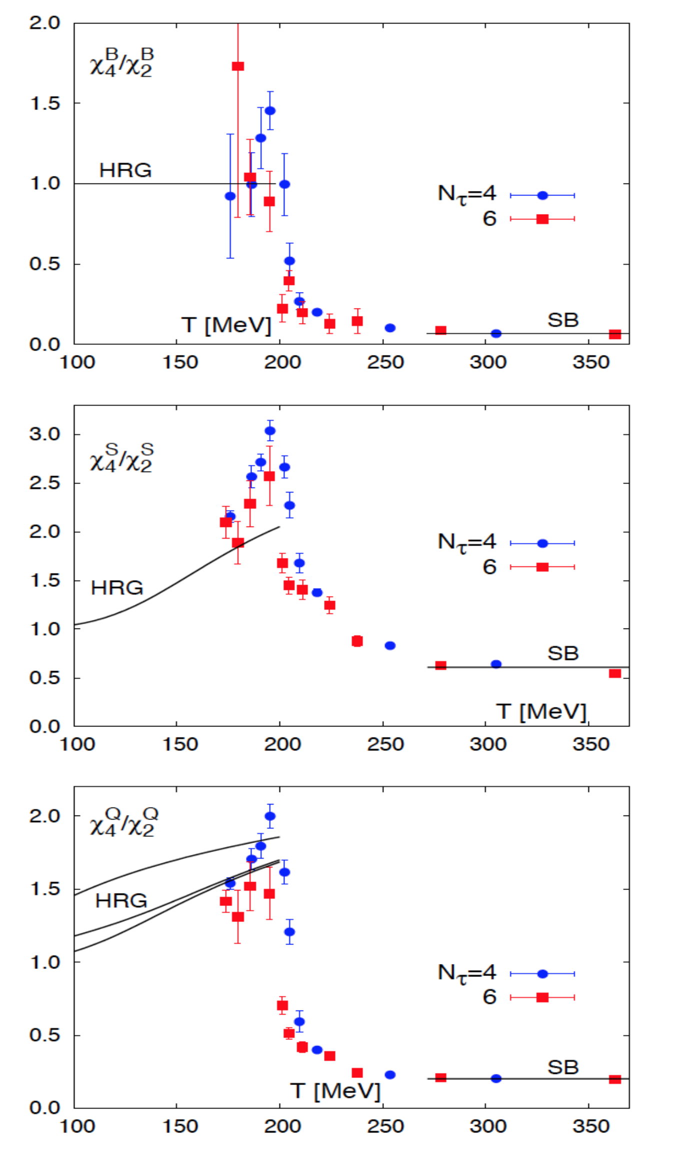
3rd Order Cumulant:

$$\boxed{<(\delta N)^3> \approx \xi^{4.5}}$$

4th Order Cumulant:

$$\boxed{<(\delta N)^4> - 3<(\delta N)^2>^2 \approx \zeta^7}$$

M. Cheng et al. arXiv:0811.1006v2



1st • **Mean** (M) = C_1 = <N>

2nd • **Standard Deviation** (σ) = $C_2 = <(N - <N>)^2>^{1/2}$

3rd • **Skewness** (S) = $\dfrac{C_3}{C_2^{3/2}} = \dfrac{<(N-<N>)^3>}{s^3}$

4th • **Kurtosis** (k) = $\dfrac{C_4}{C_2^{2}} = \dfrac{<(N-<N>)^4>}{s^4} - 3$

+ higher orders

Higher moments of conserved quantities are sensitive to Critical Point induced fluctuations (to correlation length).

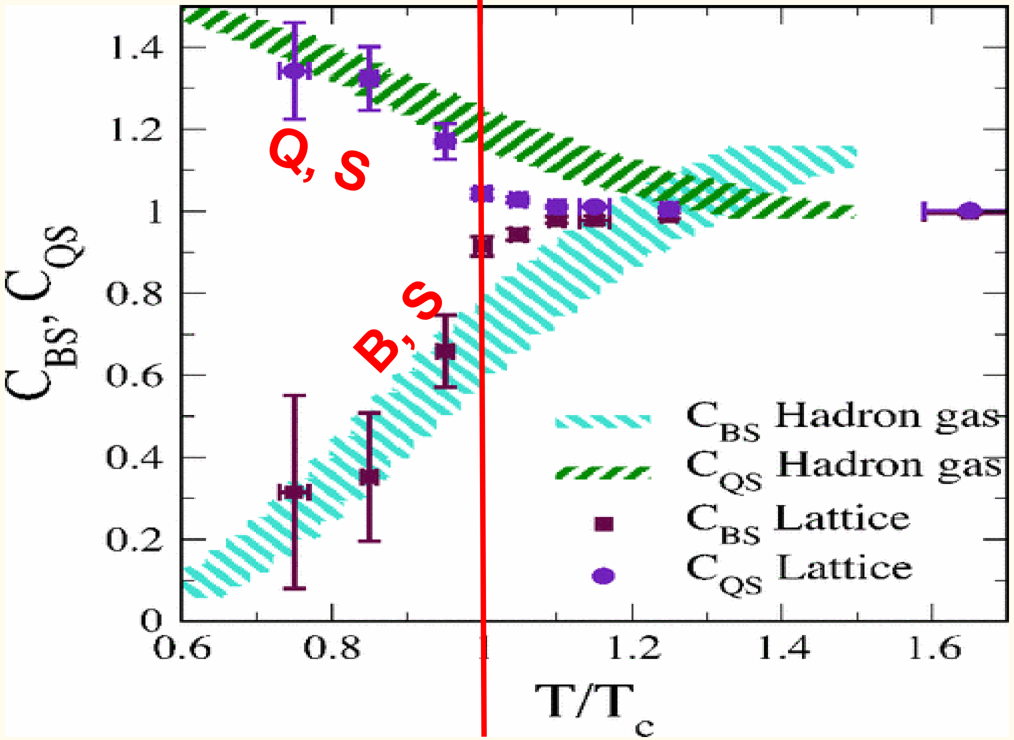
Off-diagonal cumulants of Q,B,S

Diagonal

Off-diagonal /
Cross correlations

$$\begin{bmatrix} \sigma_Q^2 & \sigma_{QB} & \sigma_{QS} \\ \sigma_{BQ} & \sigma_B^2 & \sigma_{BS} \\ \sigma_{SQ} & \sigma_{SB} & \sigma_S^2 \end{bmatrix}$$

LATTICE CALCULATIONS



- **Diagonal:** measures of the **shape**.
  - Diagonal Susceptibilities: $\chi_X^2 = \frac{1}{VT}\sigma_X^2$
  - Variance $c_2 = \sigma^2 = \langle(\delta X)^2\rangle$
- **Off-diagonal:** measures "**correlation**"
  - Off-diagonal Susceptibilities: $\chi_{XY}^2 = \frac{1}{VT}\sigma_{XY}$
  - Co-variance: $c_{1,1} = \sigma^{1,1} = \langle(\delta X)(\delta Y)\rangle$

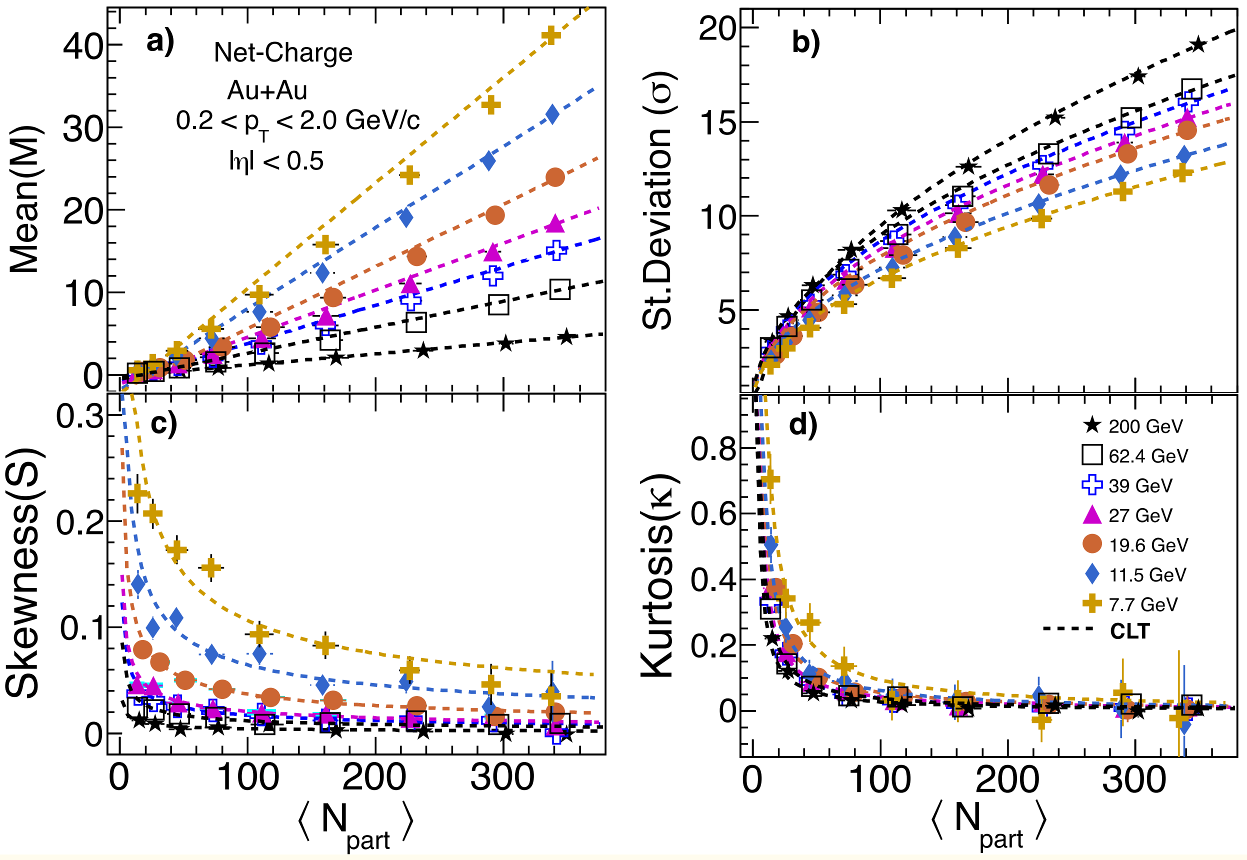
- **Correlated production of hadrons**
- **Onset of phase transition**
- **Estimation of Freeze-out temperature**

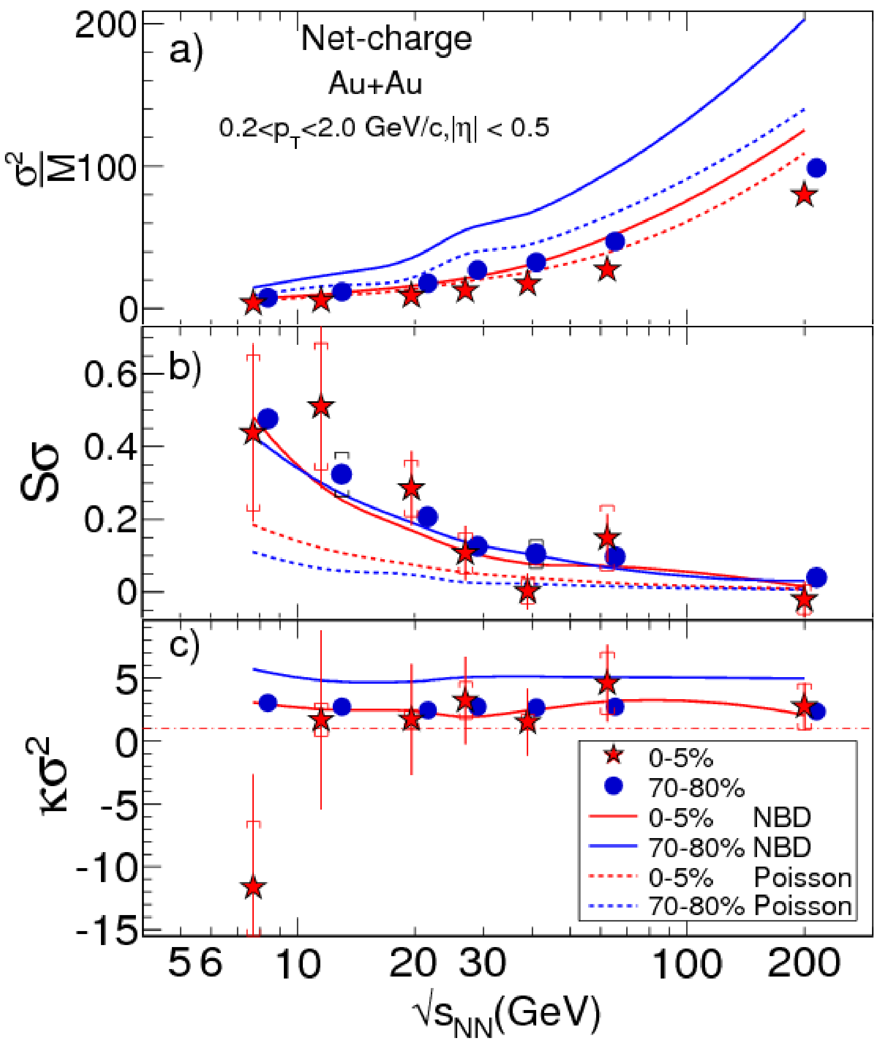
Koch, Majumdar, Randrup PRL 95 (2005), Majumdar, Muller PRC 74 (2006)
Gavai, Gupta PRD 73 (2006), F. Karsch and K. Redlich, PLB 695 (2011)

A. Chatterjee, N. Sahoo, Sandeep C., TN et al., J. Phys G 43 (2016)
R. Bellwied et al., Phys. Rev. D **101** (2020)

Moments of the net-charge (Q) distributions

As a function of centrality for 7.7 GeV to 200 GeV





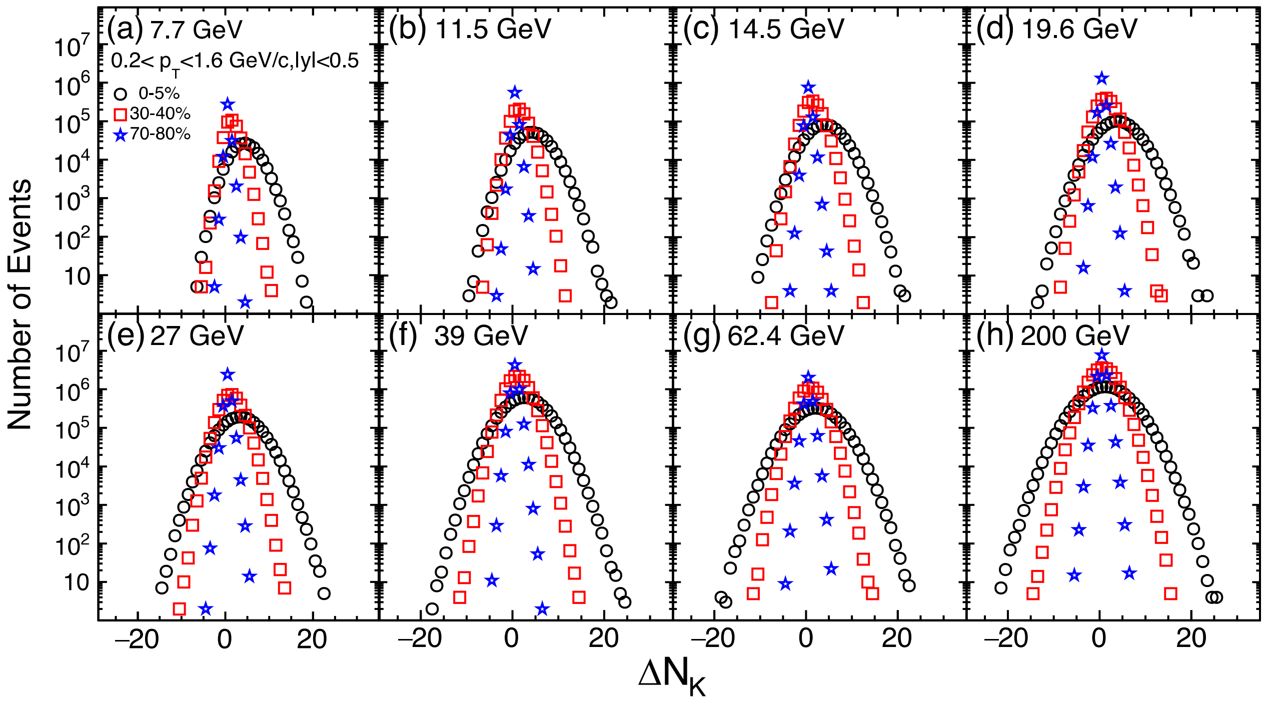
$$\sigma^2 = \langle (N - \langle N \rangle)^2 \rangle$$
$$S = \langle (N - \langle N \rangle)^3 \rangle / \sigma^3$$
$$\kappa = \langle (N - \langle N \rangle)^4 \rangle / \sigma^4 - 3$$

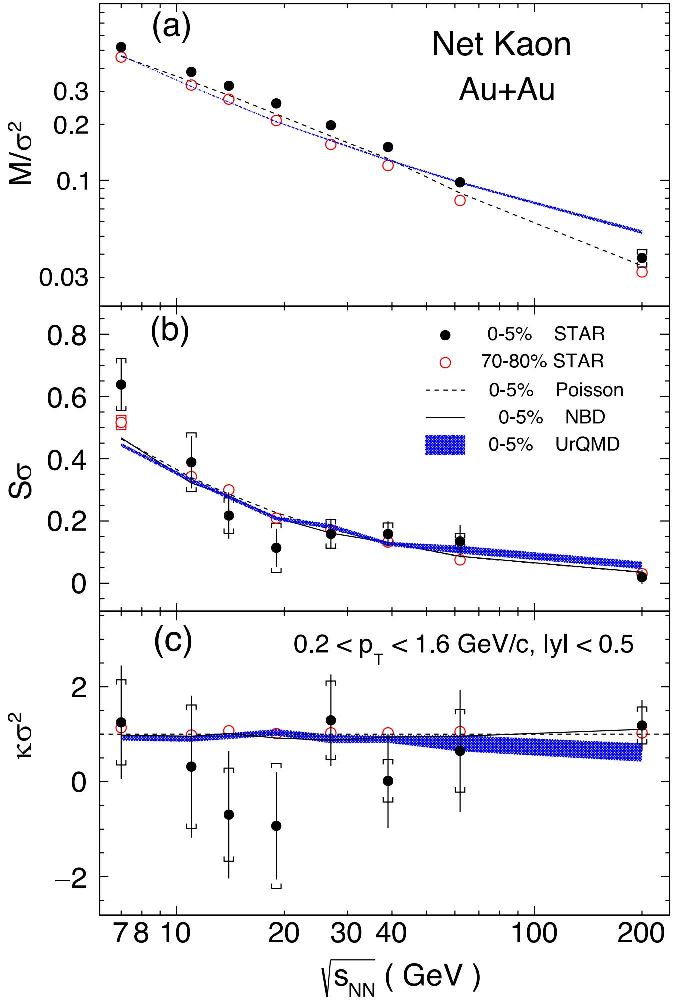
STAR Collaboration PRL 113 (2014)
Nihar R. Sahoo, TN

Within the present uncertainties, **no non-monotonic** behavior has been observed.

Moments of the net-kaon (S) distributions

STAR Collaboration Phys.Lett. B785 (2018) 551-560
A. Sarkar et al.





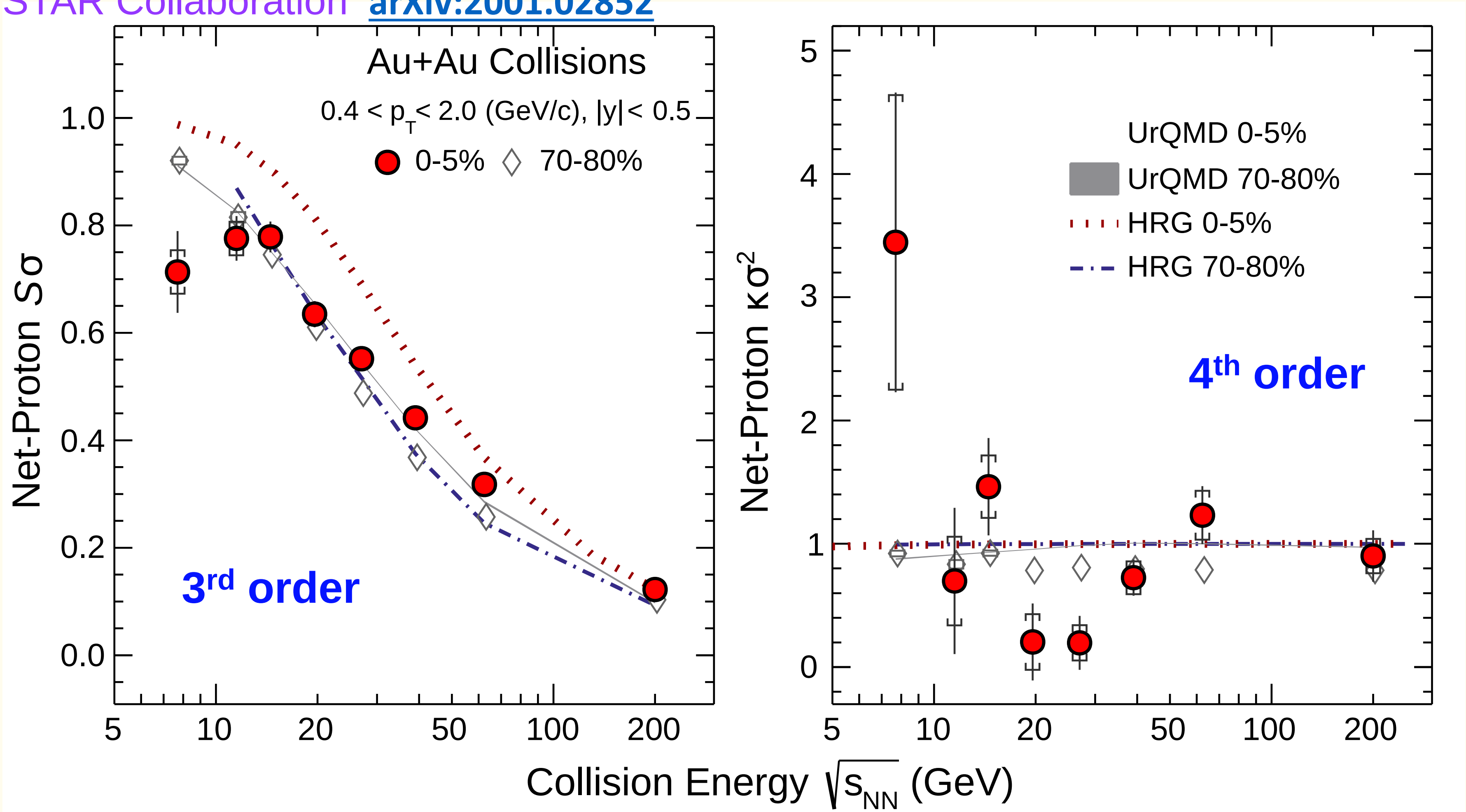
$$\sigma^2 = \langle (N - \langle N \rangle)^2 \rangle$$
$$S = \langle (N - \langle N \rangle)^3 \rangle / \sigma^3$$
$$\kappa = \langle (N - \langle N \rangle)^4 \rangle / \sigma^4 - 3$$

Amol Sarkar

Uncertainties are very large, so difficult to conclude: **no non-monotonic** behavior has been observed.

Moments of the net-proton (B) distributions

STAR Collaboration arXiv:2001.02852



$\kappa\sigma^2$ values go below unity (statistical baseline) and then rise to values above unity with decrease in beam energy. Significance: 3σ

B. Mohanty
TN

Observation of non-monotonic behavior in central collisions

⇒ hint of the location of the Critical Point?

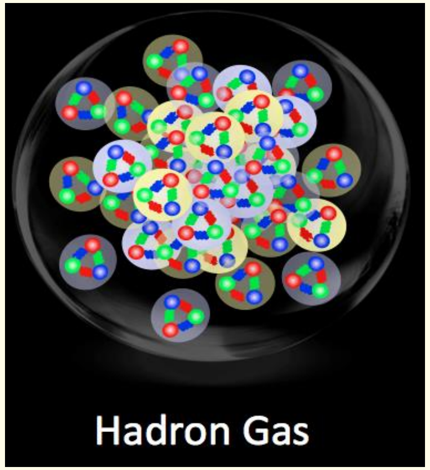
⇒ need to go to higher order moments …..

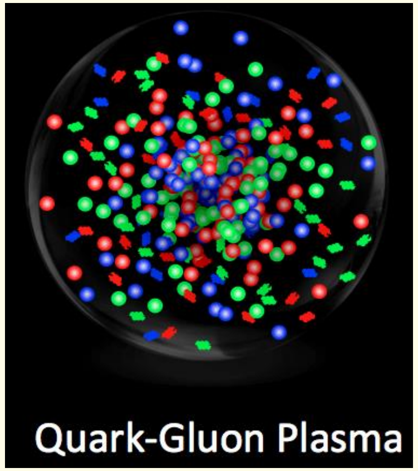
Jeon, Koch. PRL 85 (2000)
Asakawa, Heinz, Müller, PRL 85 (2000)

Net-charge Fluctuations

Net Charge: $Q = N_+ - N_-$

ALICE PRL 110 (2013) 152301
Basanta Nandi, Satyajit Jena, TN …





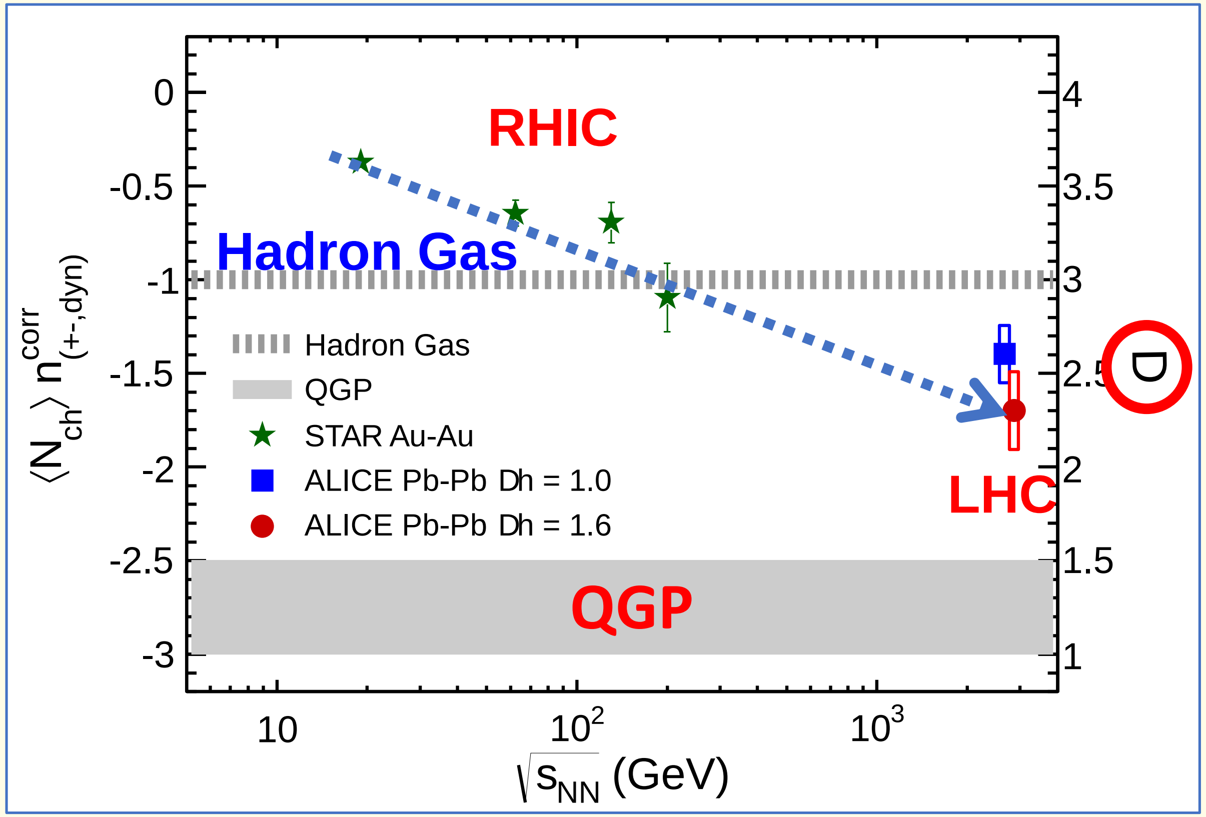
Confined: a few degrees-of-freedom

Deconfined: many degrees-of-freedom

$$n_{(+-,dyn.)} = \frac{\langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle^2} + \frac{\langle N_-(N_- - 1)\rangle}{\langle N_-\rangle^2} - 2\frac{\langle N_+ N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}$$

$$D = 4\frac{\langle(\delta Q)^2\rangle}{N_{ch}} \approx \langle N_{ch}\rangle\langle n_{dyn}\rangle + 4$$

$D \approx$ 3 – 4 for HG and 1 – 1.5 for QGP



Fluctuations are observed below the hadron gas limit!

- **Interplay of QGP and hadron phase**
- **Role of diffusion ????**

Can we access the Equation of State from experimental observables?

Isothermal compressibility

Phys.Lett. B784 (2018) 1-5

Isothermal compressibility expresses how a system's volume responds to a change in the applied pressure.

$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Expected behavior of k_T at the critical point (T_c):

$$k_T \propto \left(\frac{T - T_C}{T_C}\right)^{-\gamma} \propto \varepsilon^{-\gamma}$$

Critical behavior

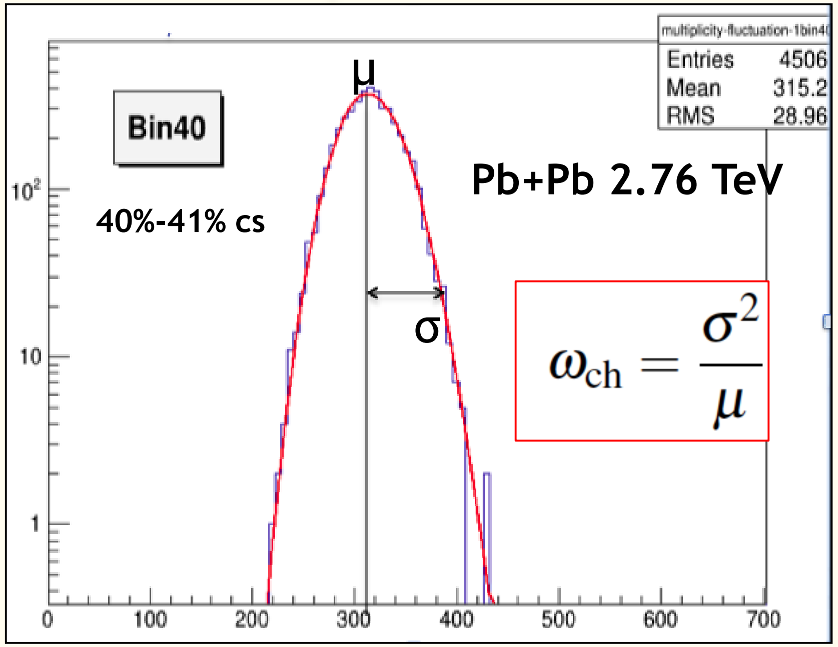
→ power law scaling of k_T

→ increase by an order of magnitude close to the QCD critical point (CP).

Can be measured by experimental observables =>

Isothermal compressibility: Beam Energy Dependence

Multiplicity distributions

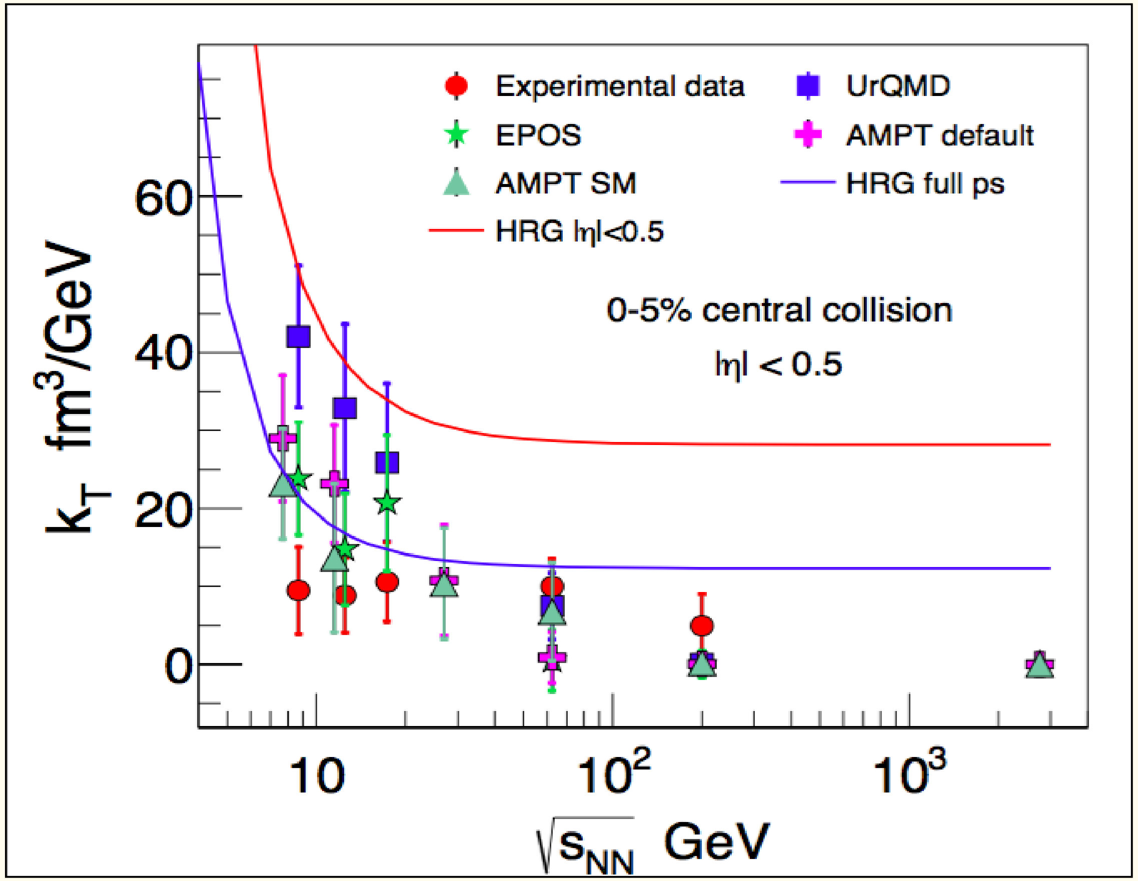


Within GCE, k_T is related to the multiplicity fluctuations:

$$\boxed{\omega_{ch} = \frac{k_B T \mu}{V} k_T}$$

Challenge: to separate statistical fluctuations from Dynamic fluctuations.

Phys.Lett. B784 (2018) 1-5
Mukherjee, Basu, Chatterjee, Chatterjee, Adhya, Thakur, TN



Highly compressed matter at lower energies!

=> Corresponds to RHIC BES, NICA & FAIR energies

Heat capacity

Heat capacity is a response function which expresses how much a system's temperature changes when heat is transferred to it, or equivalently how much δE is needed to obtain a given δT.

- **Heat capacity:** $\boxed{C = \left(\frac{\partial E}{\partial T}\right)_v}$

Landau and Lifschitz

L. Stodolsky, PRL 75 (1995)

Phys.Rev. C94 (2016) no.4, 044901

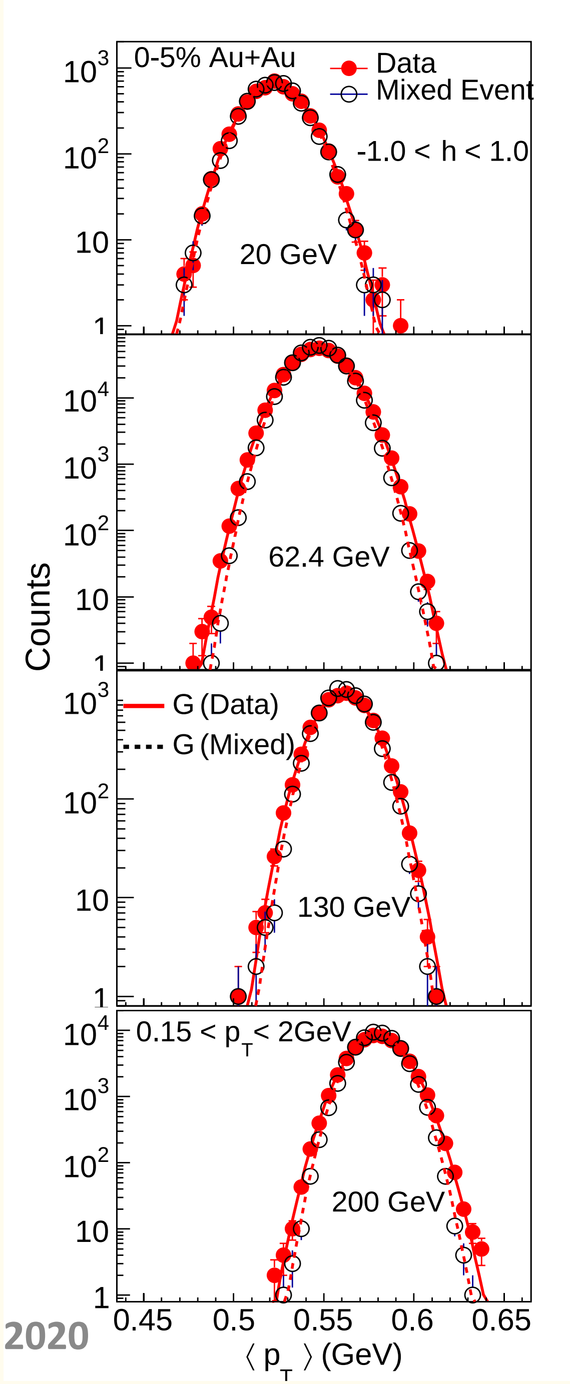
$$C_v = \frac{(\langle E^2\rangle - \langle E\rangle^2)}{\langle T\rangle^2}.$$ **Fluctuation in Energy**

$$\frac{1}{C_v} = \frac{(\langle T^2\rangle - \langle T\rangle^2)}{\langle T\rangle^2}.$$ **Fluctuation in Temperature**

Can be measured by experimental observables =>

RHIC (STAR) data:

STAR, Phys. Rev. C 72 (2005)



Mean p_T → T_{eff}

Fluctuation in Temperature

$$(\Delta T_{\rm eff})^2 = (\Delta T_{\rm eff}^{dyn})^2 + (\Delta T_{\rm eff}^{stat})^2$$

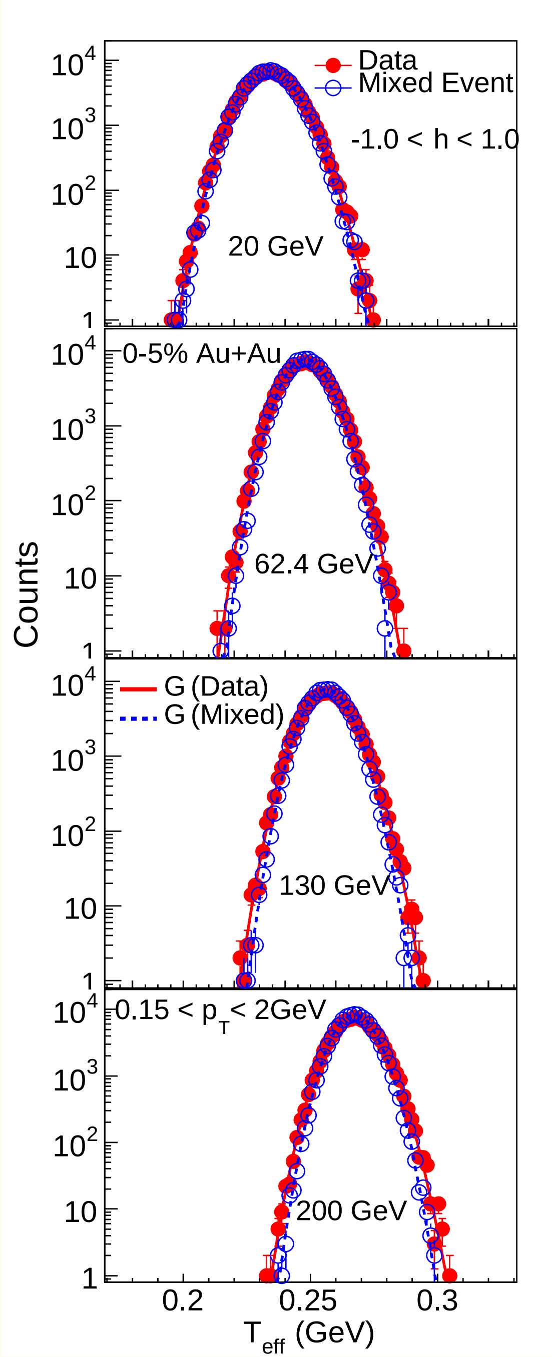
Total fluctuation: Dynamical + statistical

data and mixed events

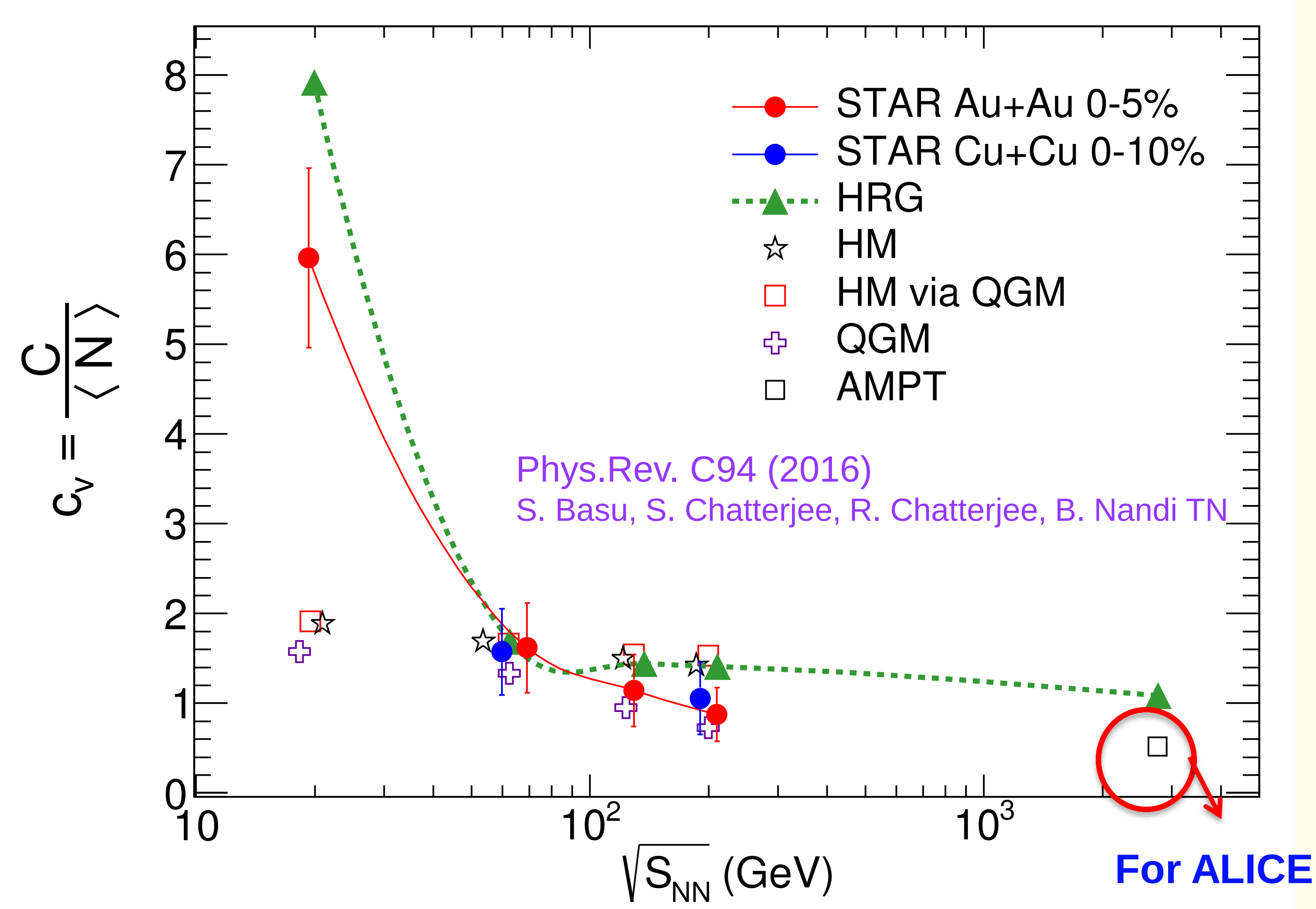
Specific heat:

$$c_V = \frac{C}{\langle N \rangle} = \frac{C}{VT^3}$$

Phys.Rev. C94 (2016)
Sumit Basu, TN et al



Specific heat (c_V): Beam Energy Dependence



For ALICE

- STAR data: central Au+Au and Cu+Cu collisions
- Hadron Resonance Gas (HRG)
- AMPT
- Three different quark gluon models

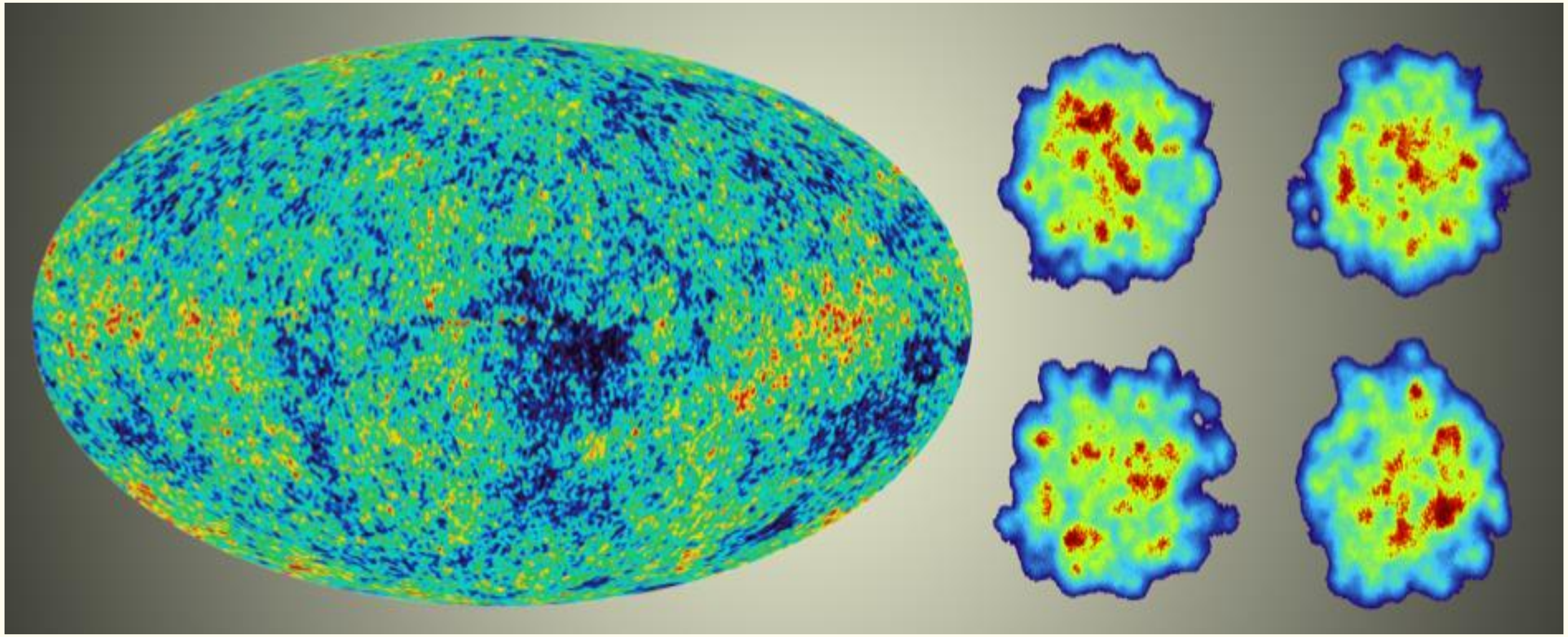
⇒ **Sudden change at low energies**

⇒ **Corresponds to RHIC BES, NICA & FAIR energies**

Next: can we use Temperature fluctuation to map the Little Bang?

Fluctuations in the Little Bang

Uli Heinz, arXiv:1304.3634v1 [nucl-th] 11 Apr 2013

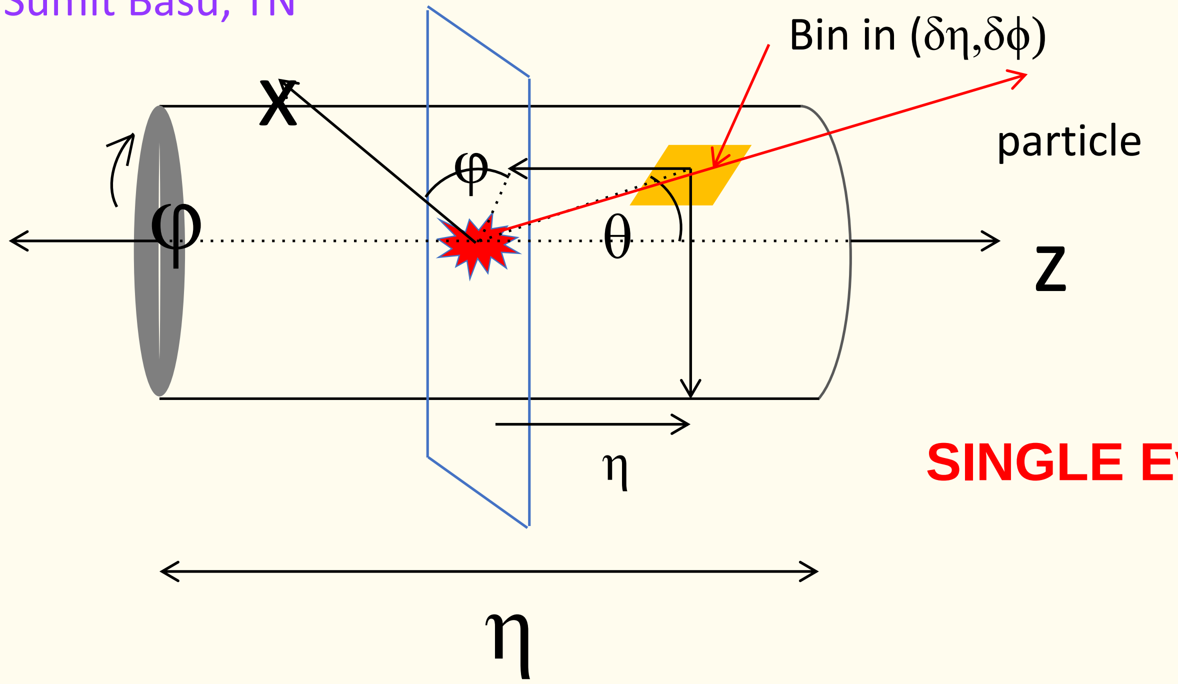


WMAP **Heavy-ion Collisions**

- Hadrons detected by the experiment are mostly emitted at the freeze-out
- Similar to the CMBR which carry information at the surface of last scattering in the Universe, these hadrons may provide information about the earlier stages (hadronization) of the reaction in heavy-ion collision =>

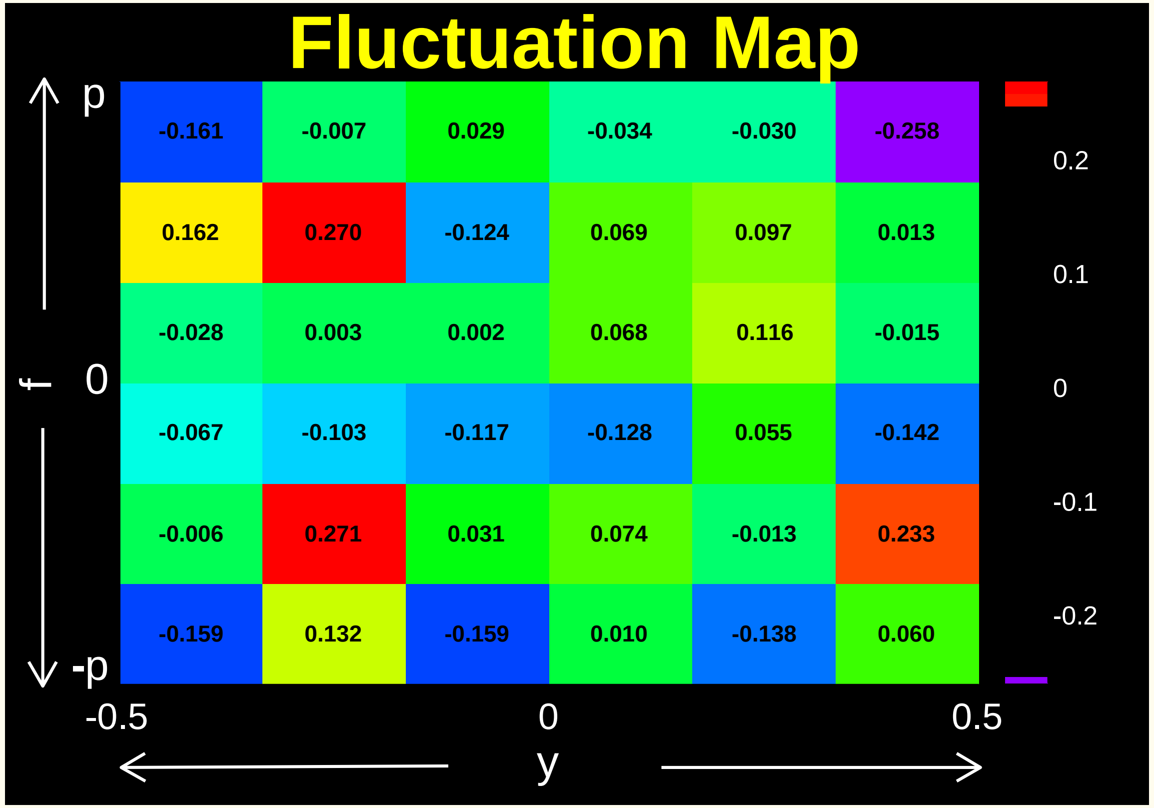
Local Fluctuation

Sumit Basu, TN



Map of $\langle m_T \rangle$

SINGLE Event

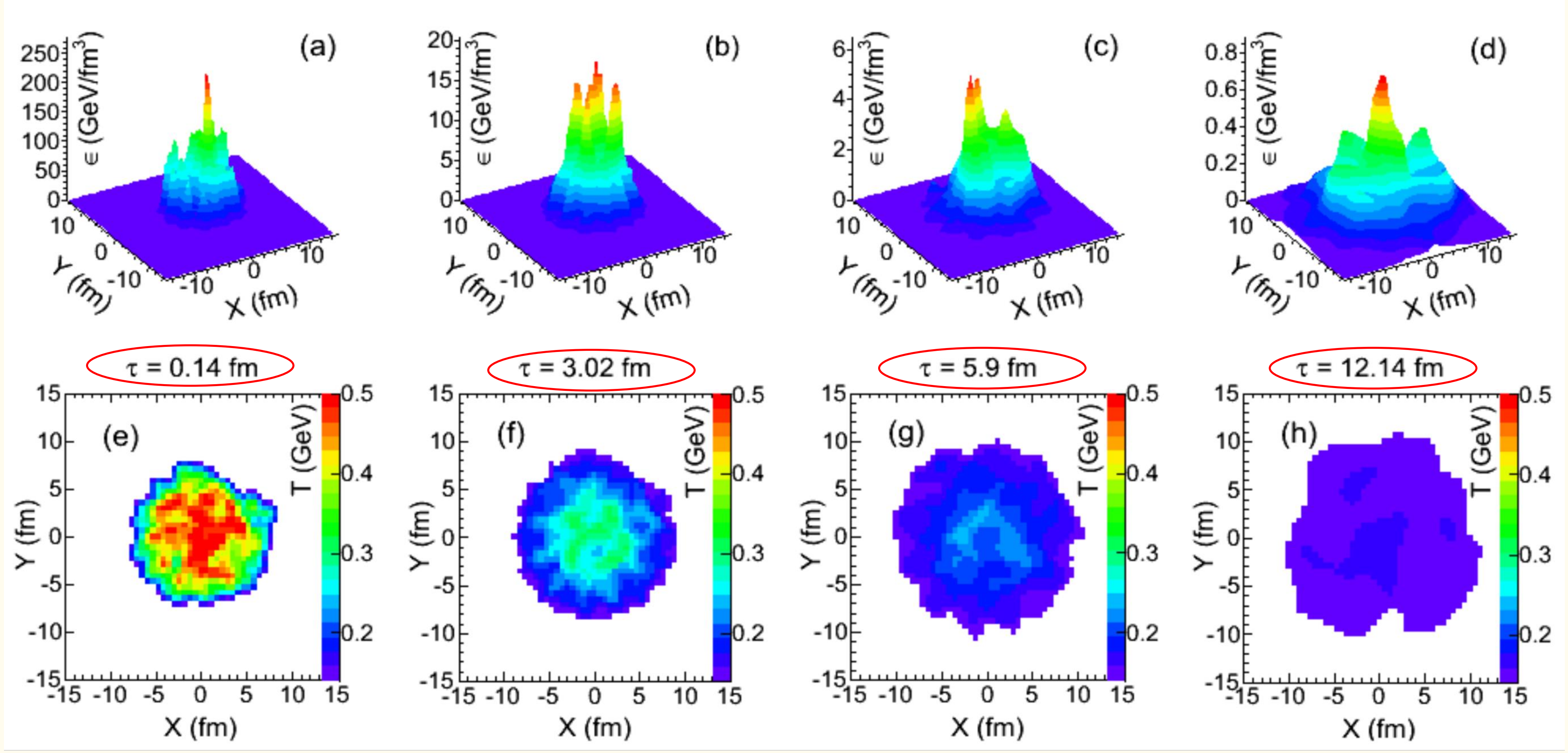


- **Need large acceptance detectors**
- **Novel method to MAP the Heavy-ion collisions**
- **Need a connection to theory for early stage fluctuations:**

Evolution of Energy Density and temperature

- Local fluctuations in energy density arise because of the internal structures of the colliding nuclei.
- The initial fluctuations manifest into local temperature fluctuations of the fireball at different stages of the collision.

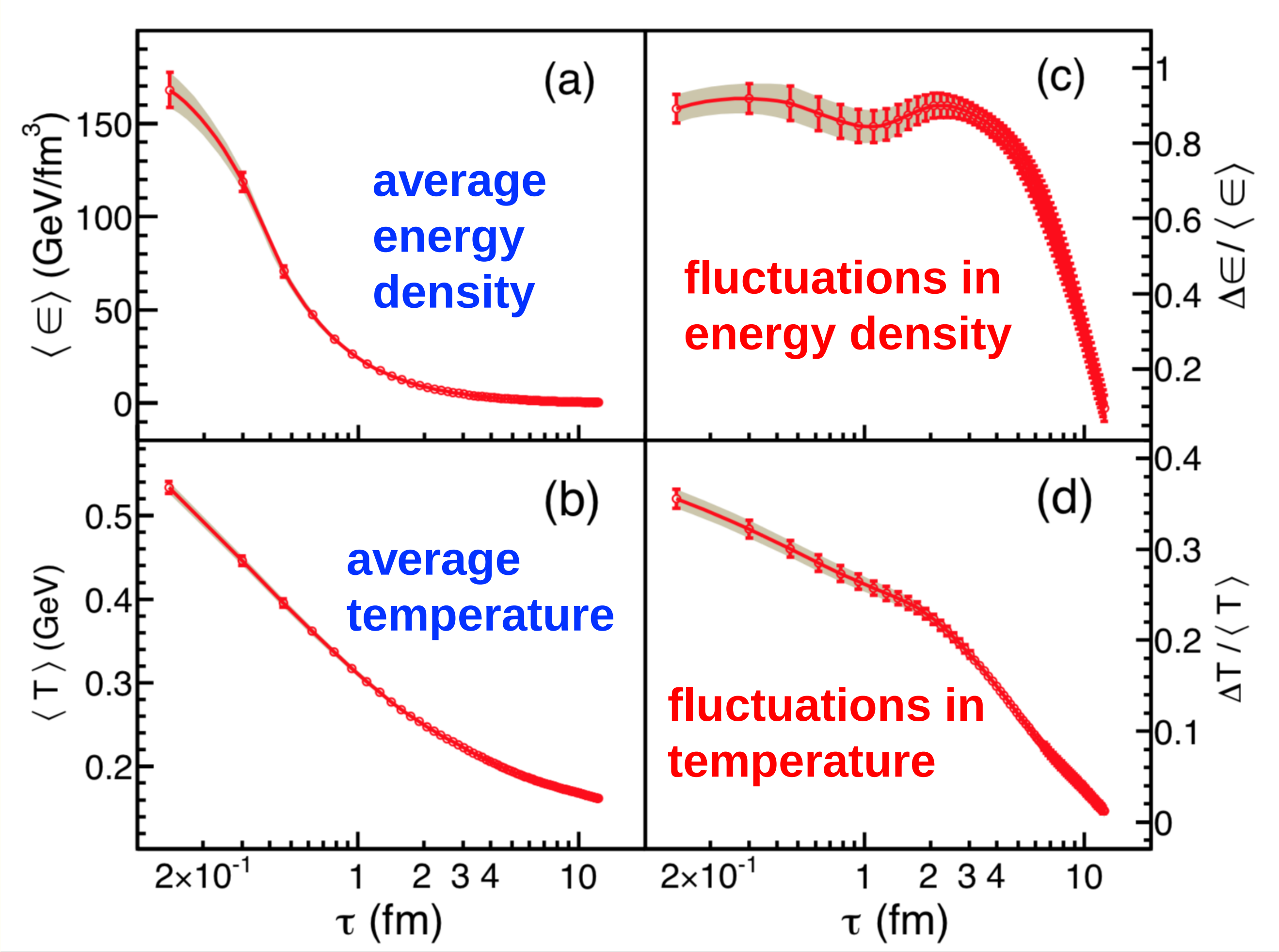
Hydrodynamic simulation of central Pb-Pb events at 2.76 TeV (LHC)



arXiv:1504.04502 [nucl-ex]
S. Basu, R. Chatterjee, B Nandi, TN

Let's look at the fluctuations in energy density and temperature:

Fluctuations in Energy density and Temperature



Time →

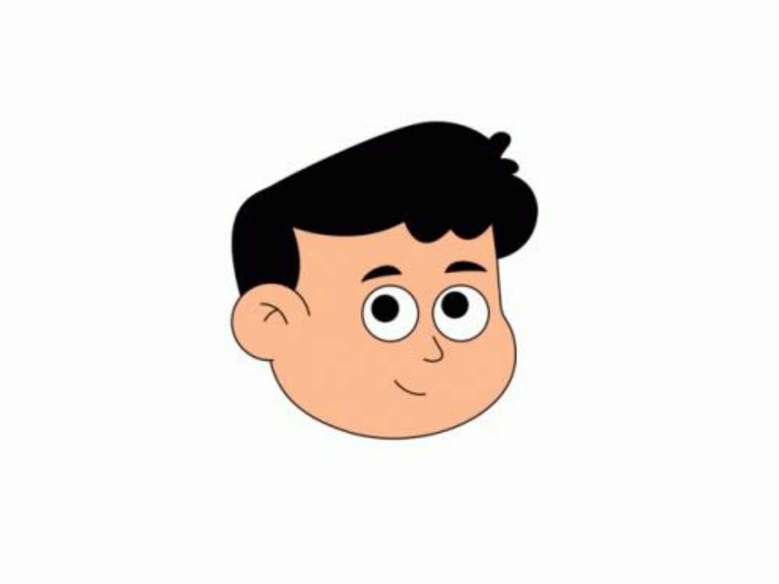
Average is taken over the X-Y bins in every event. The shaded regions represent the extent of event-by-event variations for a large number of events

What can we learn by combining measurements with hydro.. calculations?

Need theory support and hydrodynamic calculations ….

arXiv:1504.04502 [nucl-ex]
S. Basu, R. Chatterjee, B Nandi, TN

To recap:



- **Fluctuations play a major role in our lives:**
    - Understanding fluctuations help to deal with the issues at present and take proactive action for future.
    - Understanding fluctuations => understanding the nature ….

- **Fluctuations play a major role in our physics:**
    - Fluctuations/correlations tools are important for accessing critical phenomena of QCD, probing the critical point and detailed understanding the QCD phase transition. With the availability of low energy beams at RHIC, FAIR and NICA …. and with new ALICE: looking forward to a brighter future ….