# *Quantum annealers as a laboratory for Quantum Physics*

Steven Abel (Durham)

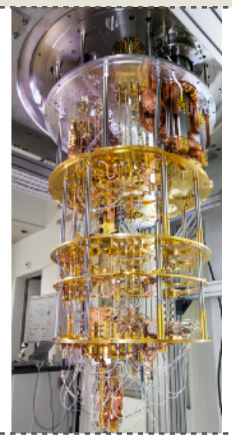
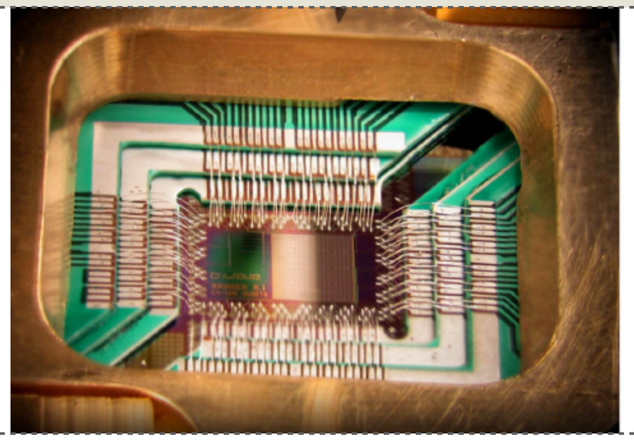
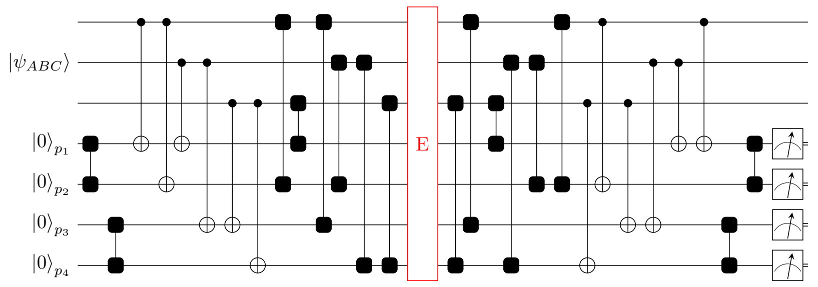
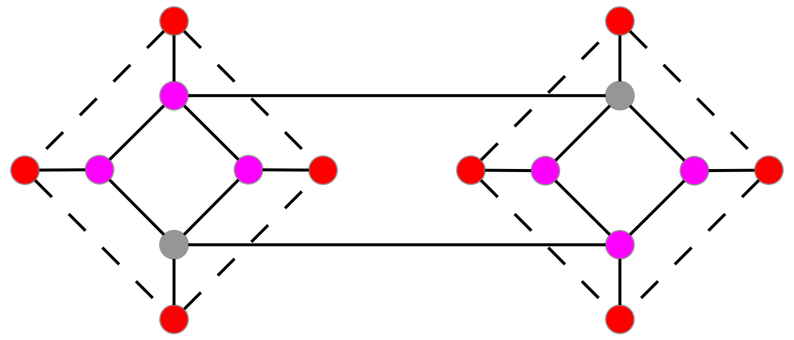
w/ Spannowsky arXiv:2006.06003
w/ Chancellor and Spannowsky, arXiv:2003.07374.

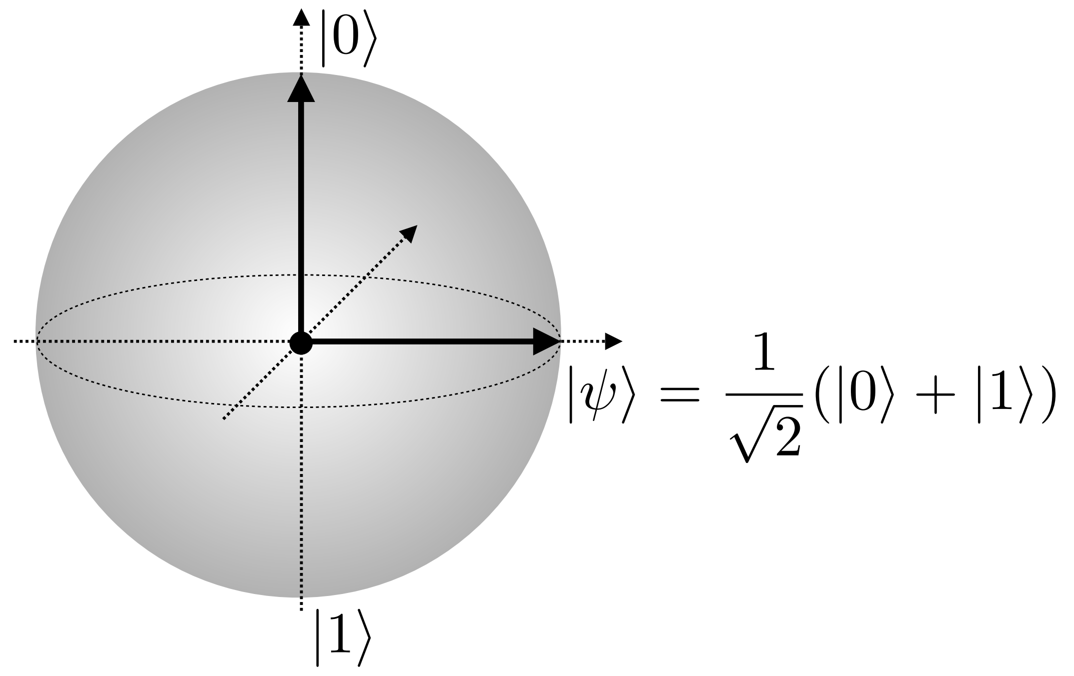
# *Overview*

- Quantum annealers background
- Ising encodings of simple problems
- Toy field-theory problem: classical tunnelling solution in QFT
- Ising encoding of QFT
- Results for thin wall limit
- Measuring quantum tunnelling in Schrodinger equation
- Thermal or quantum

# *Quantum annealers background*

Quantum computing has a long and distinguished history but is only now becoming practicable. (Feynman ’81, Zalka '96, Jordan, Lee, Preskill … see Preskill 1811.10085 for review). Two main types of Quantum Computer:

| Type | Discrete Gate | Quantum Annealer |
|---|---|---|
| Property | Universal (any quantum algorithm can be expressed) | Not universal — certain quantum systems |
| How? | IBM - Qiskit<br>~50 Qubits | DWave - LEAP<br>~5000 Qubits |
| What? |  |  |
| |  |  |




- Both types operate on the Bloch sphere: basically measuring $\sigma_i^Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ where $(\sigma^Z|0\rangle = |0\rangle, \sigma^Z|1\rangle = -|1\rangle)$ are the possible eigenvector eqns

- Each *i* represents a single qubit

- A discrete quantum gate system is good for looking at things like entanglement, Bell's inequality etc. Also discrete problems, cryptographical problems, Shor's, Grover's algorithms, etc.

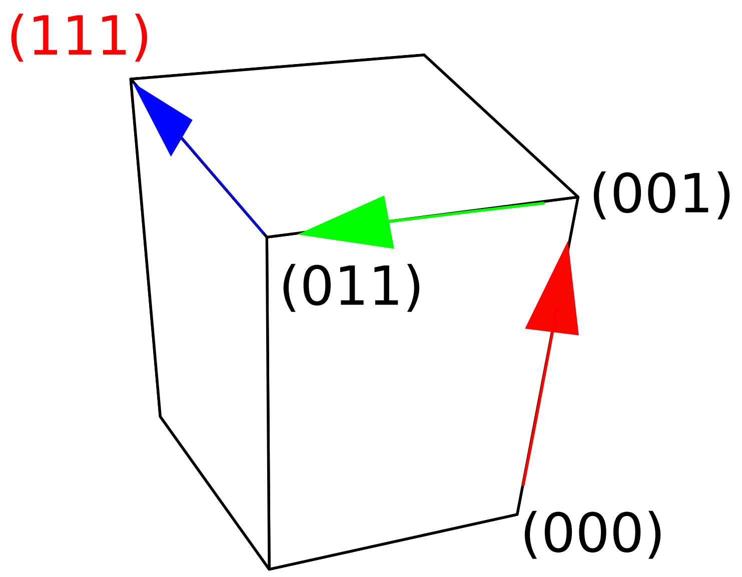
- A quantum annealer is good for looking at network optimisation problems but from our perspective it is also a more natural tool for thinking about field theory. It is based on the general transverse field Ising model (Kadowaki, Nishimori):

$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z\sigma_j^Z + \sum_i h_i\sigma_i^Z + \Delta(t)\sum_i \sigma_i^X$$

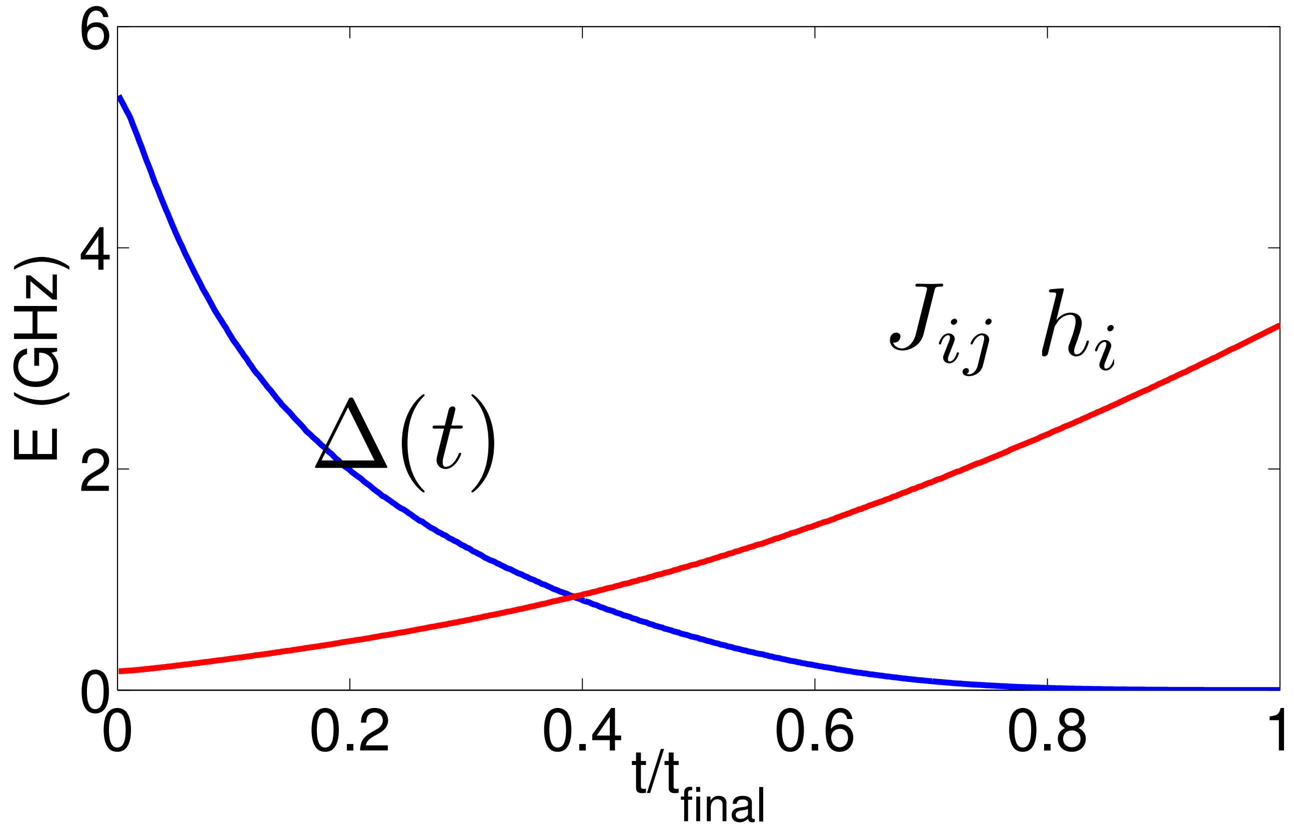
- What does “anneal” mean?

$$\mathcal{H}_{\mathrm{QA}}(t) = \sum_i \sum_j J_{ij}\sigma_i^Z\sigma_j^Z + \sum_i h_i\sigma_i^Z + \Delta(t)\sum_i \sigma_i^X$$

$\Delta(t)$ induces bit-hopping in the Hamming/Hilbert space

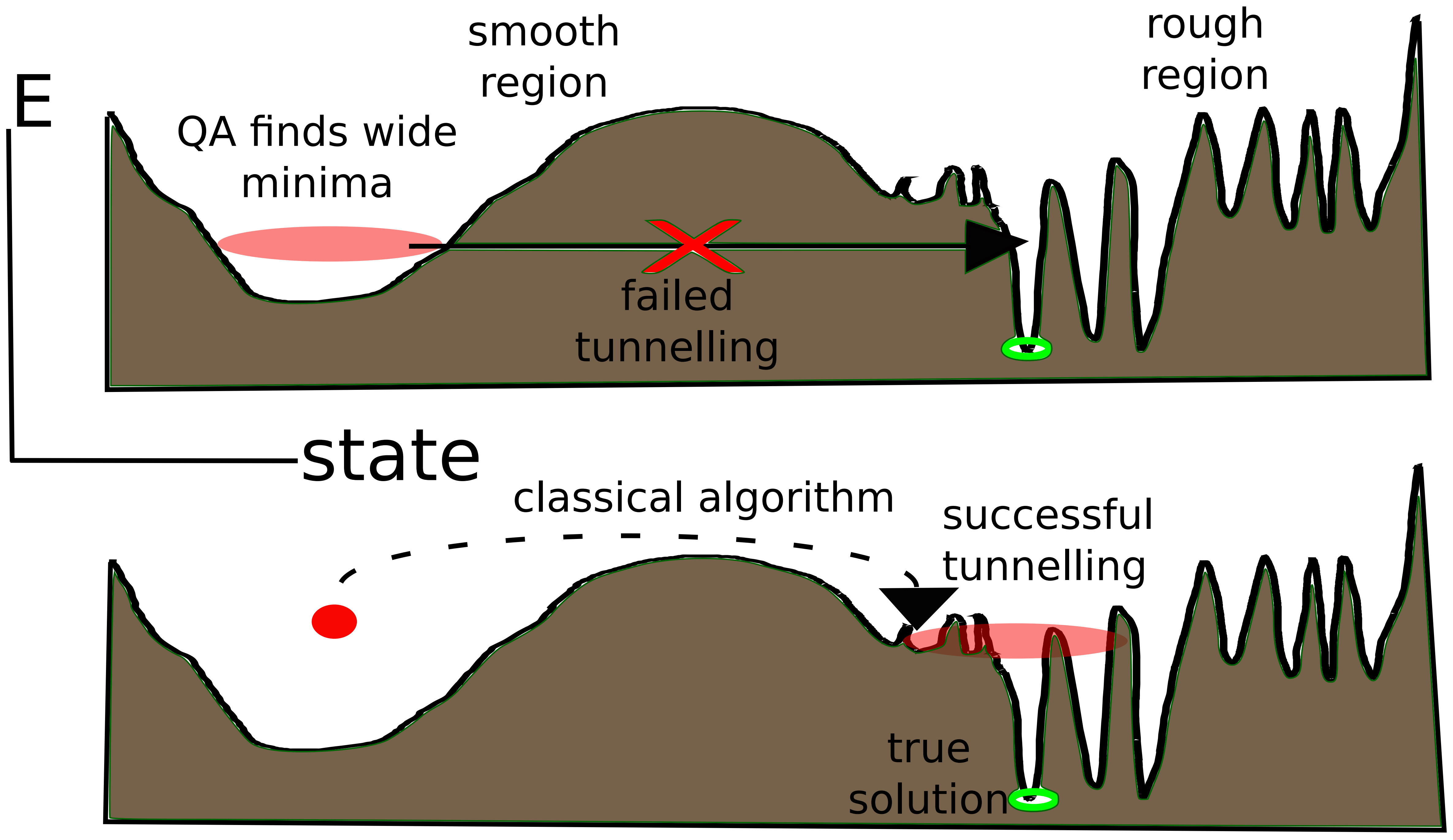



The idea is to dial this parameter to land in the global minimum (i.e. the solution) of some “problem space” described by *J,h*:

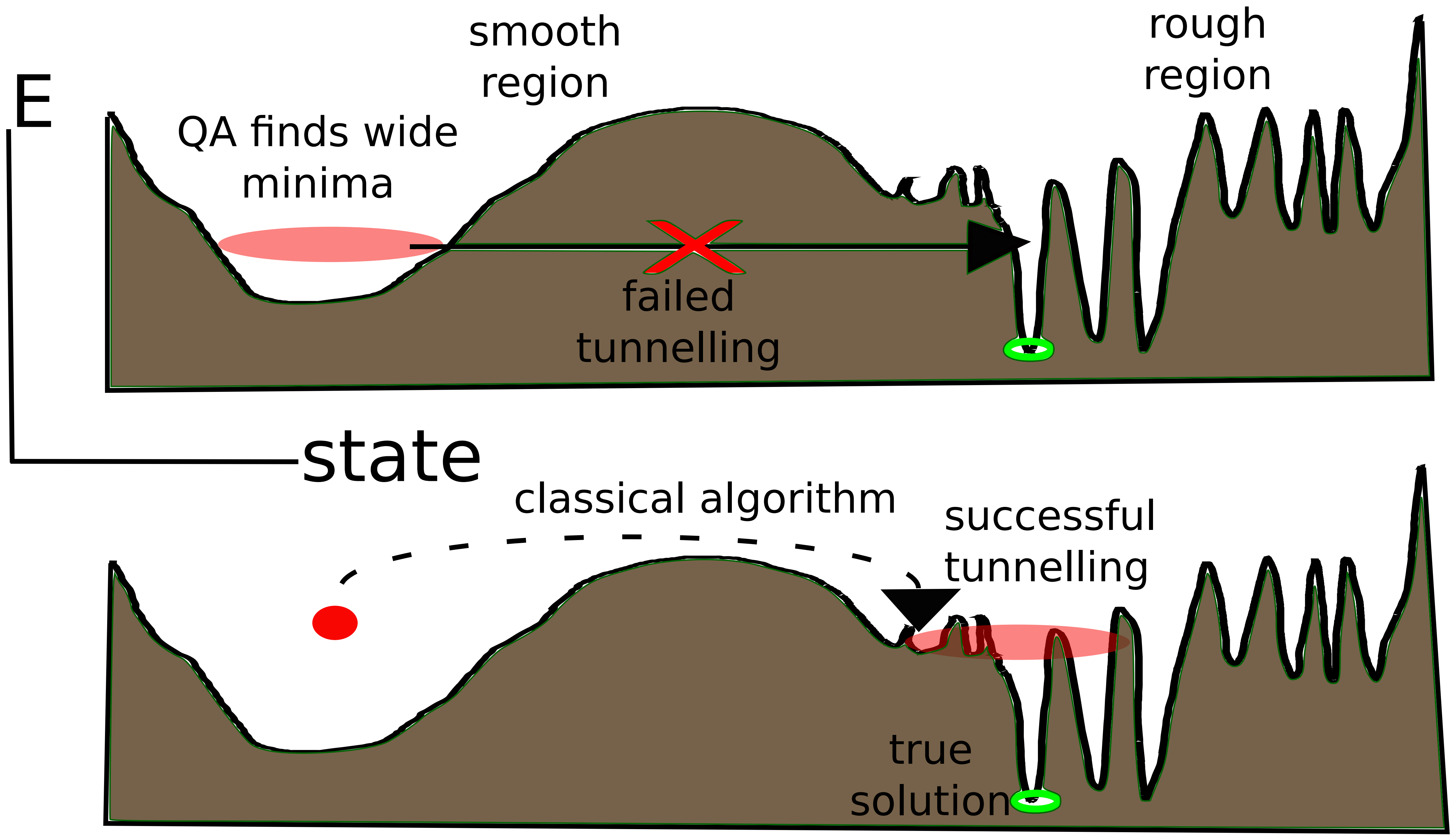


***Thermal (classical) and Quantum Annealing are complementary:***

- Thermal tunnelling is fast over broad shallow potentials (Quantum "tunnelling" is exponentially slow)
- Quantum Tunnelling is fast through tall thin potentials (Thermal "tunnelling" is exponentially slow — Boltzmann suppression)



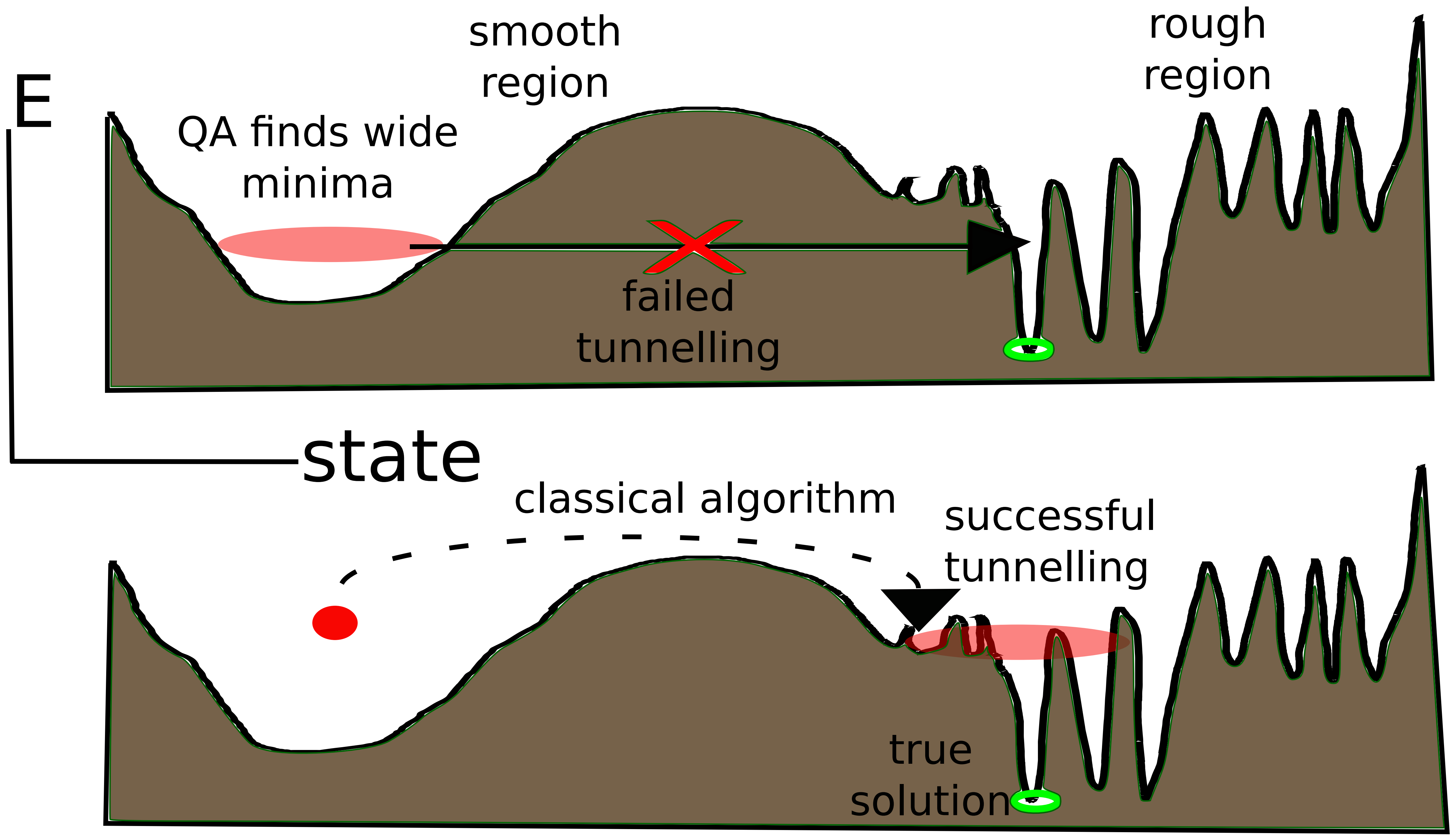
***Hence hybrid approach to Quantum Annealing can be useful depending on the solution landscape:***



***More specifically: thermal annealing uses Metropolis algorithm: accept random*** $\sigma_i^Z$ ***flips with probability***

$$P = \begin{cases} 1 & \Delta H \le 0 \\ e^{-\Delta H/KT} & \Delta H > 0 \end{cases}$$

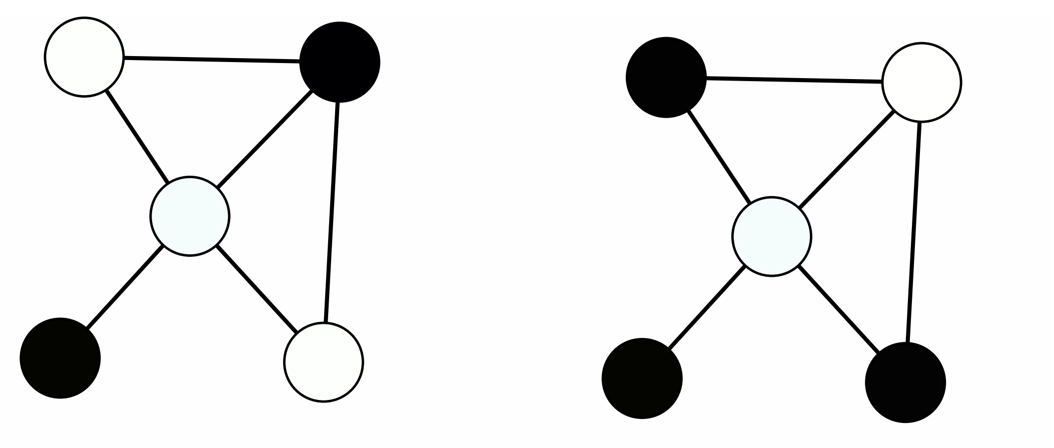
***Quantum tunnelling in QFT happens with probability*** $P \sim e^{-w\sqrt{2m\Delta H}/\hbar}$
***so by contrast it can be operative for tall barriers if they are made thin***



## *Simple examples of Ising encodings*

### *Encoding network problems in a general Ising model*

- Example 1: how many vertices on a graph can we colour so that none touch? NP-hard problem (from N.Chancellor).



- Let non-coloured vertices have $\sigma_i^Z = -1$ and coloured ones have $\sigma_i^Z = +1$ .

- Add a reward for every coloured vertex, and for each link between vertices *i,j* we add a penalty if there are two +1 eigenvalues:

$$\mathcal{H} = -\Lambda \sum_i \sigma_i^Z + \sum_{\text{linked pairs } \{i,j\}} \left[\sigma_i^Z + \sigma_j^Z + \sigma_i^Z \sigma_j^Z\right]$$

- Example 2: N^2 students are to sit an exam in a square room with NxN desks 1.5m apart. half the students (A) have a virus while half of them (B) do not. How can they be arranged to minimise the number of ill students that are less than 2m from healthy students?

- Call the eigenvalue of A == +1 and that of B == -1. That is if I measure $\sigma^Z$ at a point to have value +1 then I conclude that I should put an ill person there, and vice-versa.

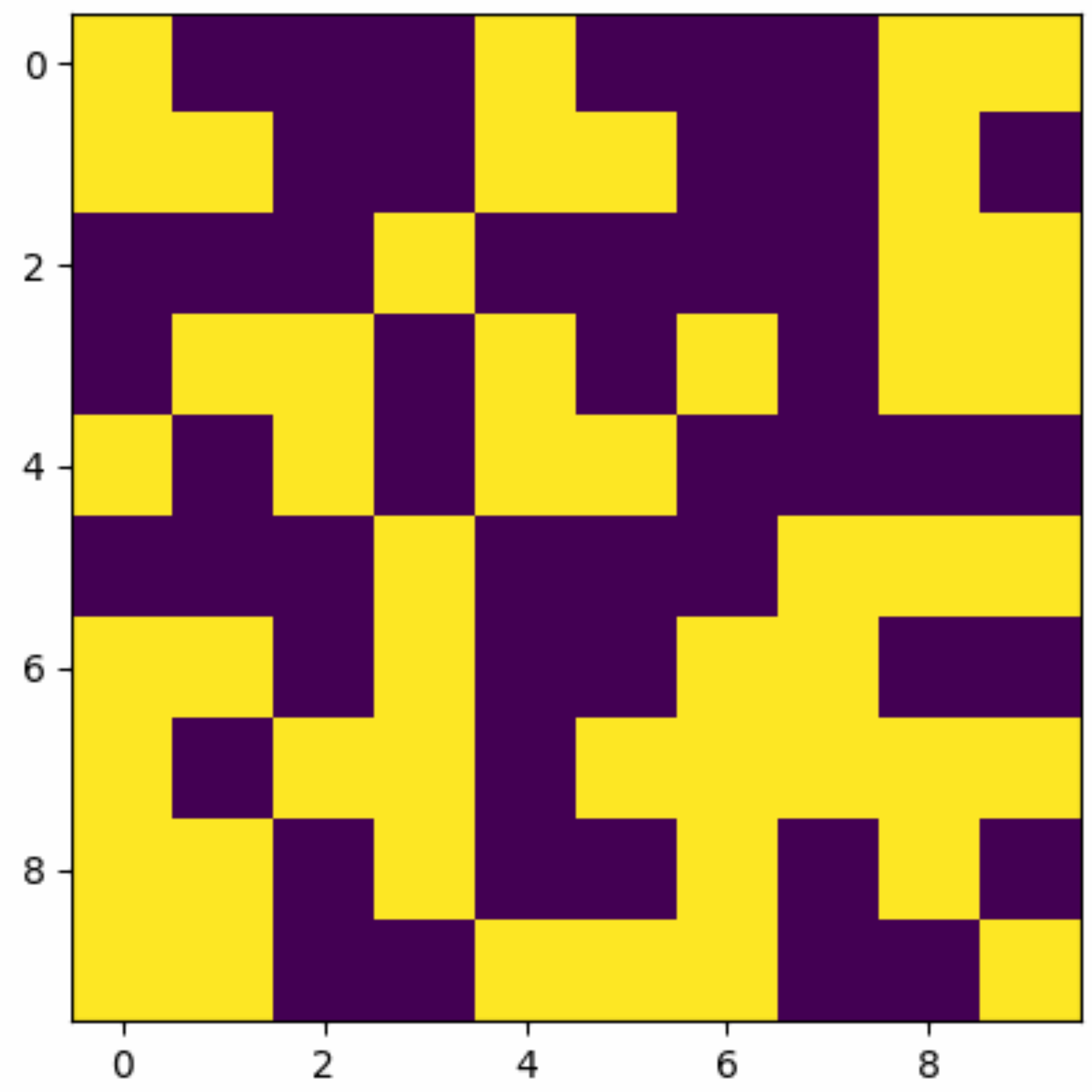
- There are N^2 spins $\sigma^Z_{\ell N+j}$ arranged in rows and columns. I do not care if A>=<A or B>=<B, but if A>=<B then I put a penalty of +2 on the Hamiltonian (ferromagnetic coupling). So …

$$\mathcal{H} = \sum_{\ell m=1}^{N} \sum_{ij=1}^{N} \left(\delta_{\ell m}(\delta_{(i+1)j} + \delta_{(i-1)j}) + \delta_{ij}(\delta_{(\ell+1)m} + \delta_{(\ell-1)m})\right) \left[1 - \sigma^Z_{\ell N+i}\sigma^Z_{mN+j}\right]$$

- Finally I need to apply the constraint that #A = #B:

$$\begin{aligned} \mathcal{H}^{(\mathrm{constr})} &= \Lambda\,(\#A - \#B)^2 \\ &= \Lambda \left(\sum_{\ell,i}^{N} \sigma^Z_{\ell N+i}\right)^2 \\ &= \Lambda \sum_{\ell m=1}^{N} \sum_{ij=1}^{N} \sigma^Z_{\ell N+i}\sigma^Z_{mN+j} \end{aligned}$$

- Example 2 done with classical thermal annealing using the Metropolis algorithm. Note this represents a search over ${}_{100}C_{50} \sim\ 2^{100}$ configurations:




- Importantly the constraint hamiltonian cannot be too big otherwise the hills are too high and it freezes too early. This makes the process require a (polynomial sized) bit of "thermal tuning".

- In principle this could be done more easily on a quantum annealer as the constraints could be high and it would still work.

- To do this we would simply fill h and J and call the quantum annealer from python as follows:

```
response = sampler.sample_ising(h,J,seed=1234+i,num_reads=3000000, num_sweeps=1)
```

- “response” is a list of [+1,-1,+1,+1 …..] spins ordered by energy

- However the architecture (connectivity of J,h) is limited. (Later)

## *A toy field-theory problem: find classical tunnelling solutions in QFT*

- We think of the general Ising model as a “universal QFT computer”

- Simple problem to demonstrate encoding QFT — quantum tunnelling in a scalar theory

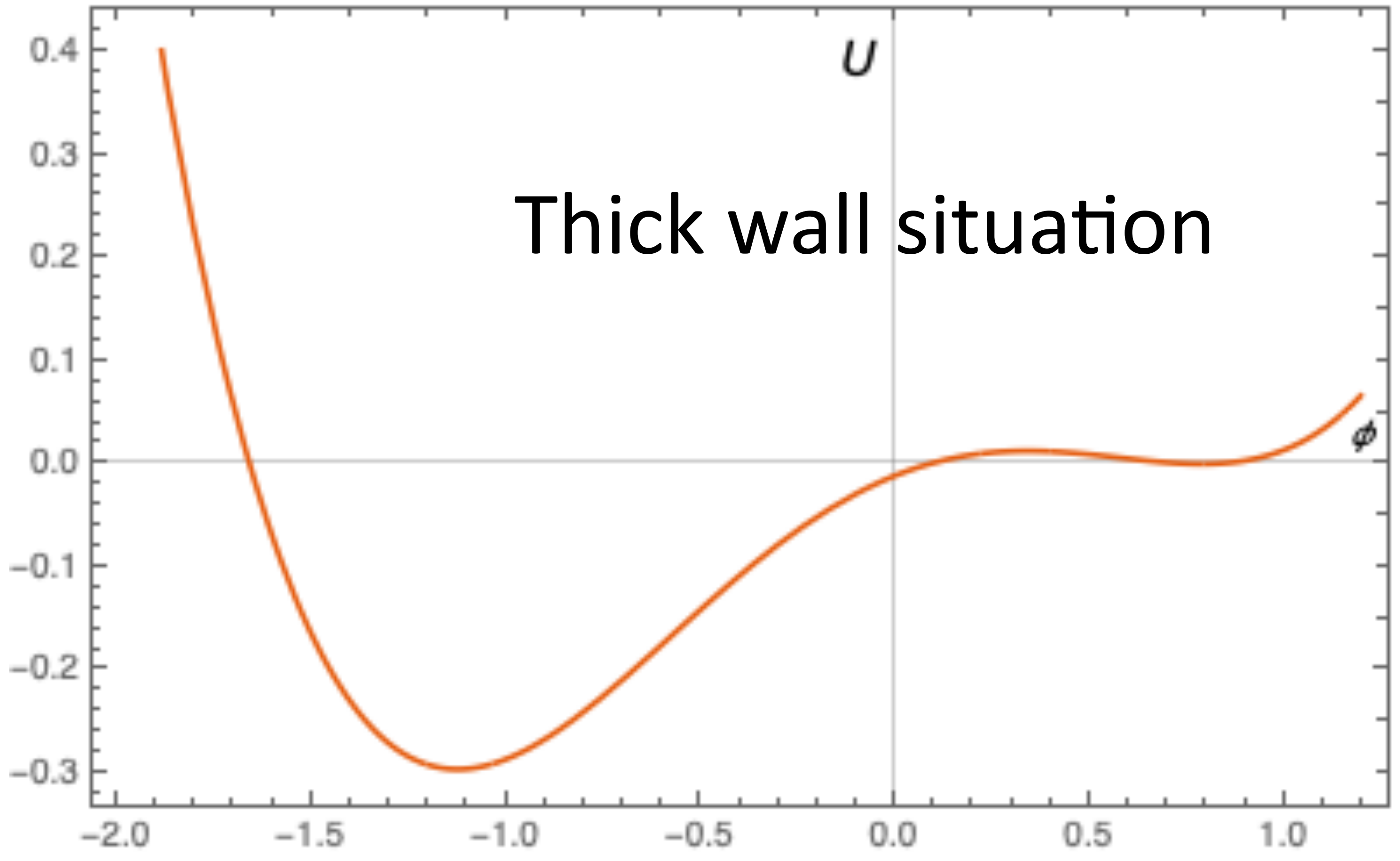
- Advantage 1: easy to prepare the initial state (this non-perturbative process is much easier than preparing scattering states).

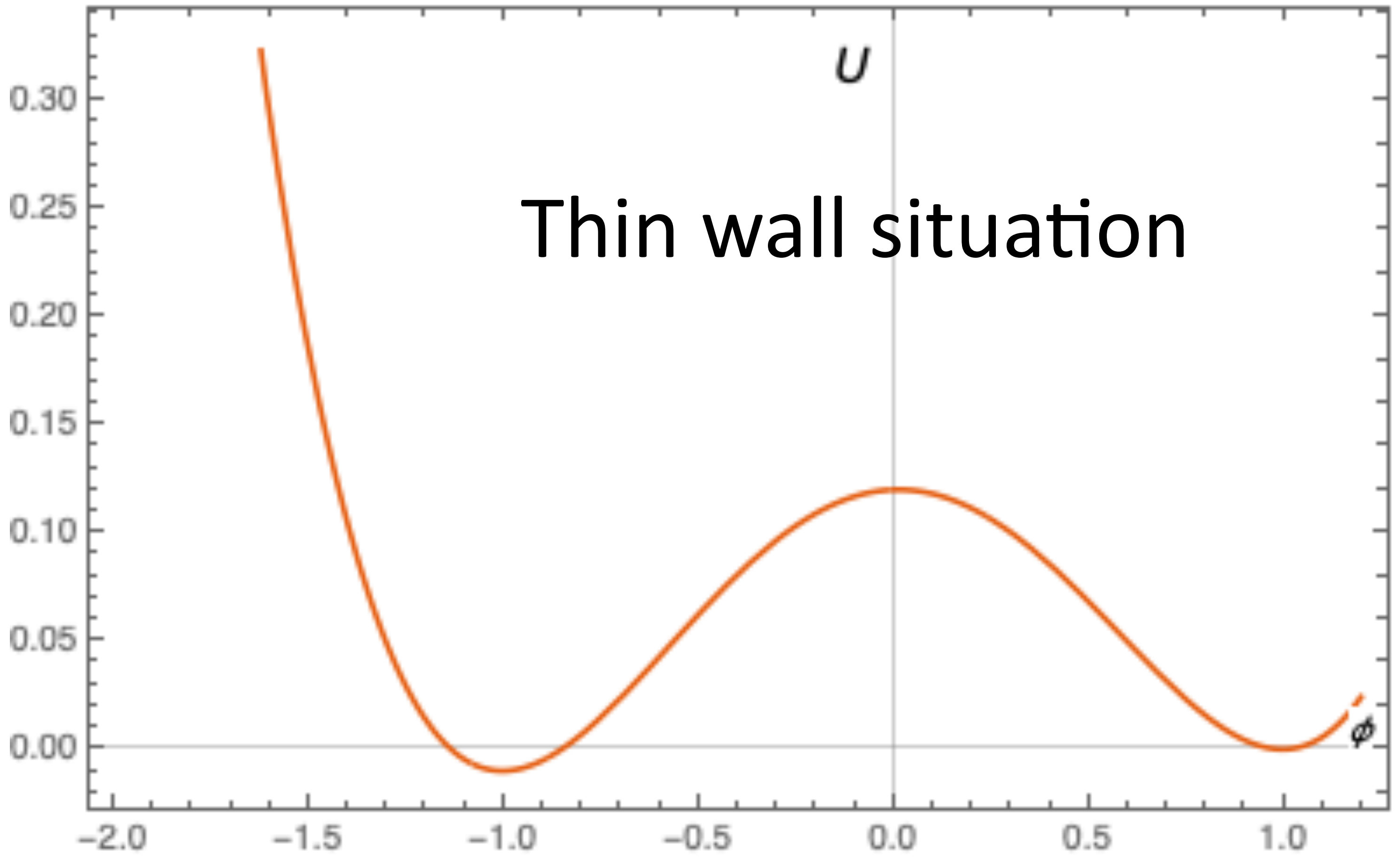
- Advantage 2: we could in principle observe genuine tunnelling in the annealer rather than just simulate it.

- Advantage 3: the system is dissipative (reaches a ground state and then tunnels: we do not need very short nano-sec times to preserve coherence)

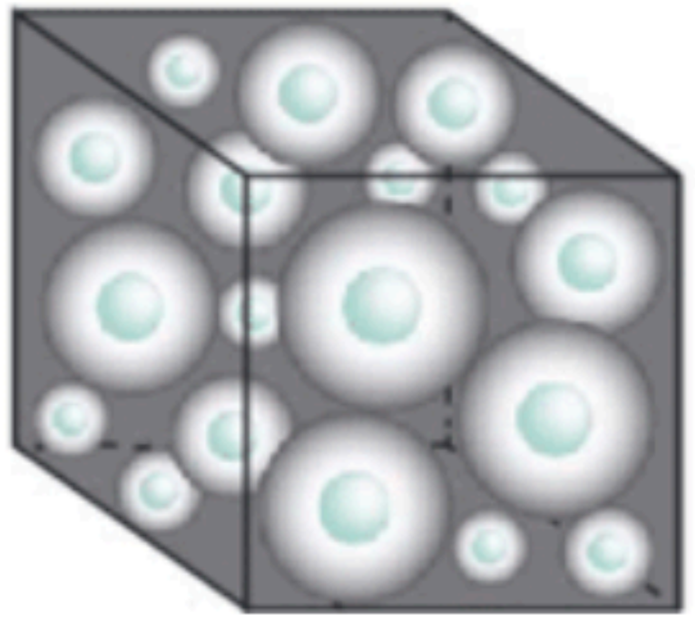
$$V(\phi) = \frac{\lambda}{8}(\phi^2 - v^2)^2 + \frac{\epsilon}{2v}(\phi - v)$$

$$U(\phi) = V(\phi) - V(\phi_+)$$




- A system trapped in the false vacuum will decay by forming bubbles …

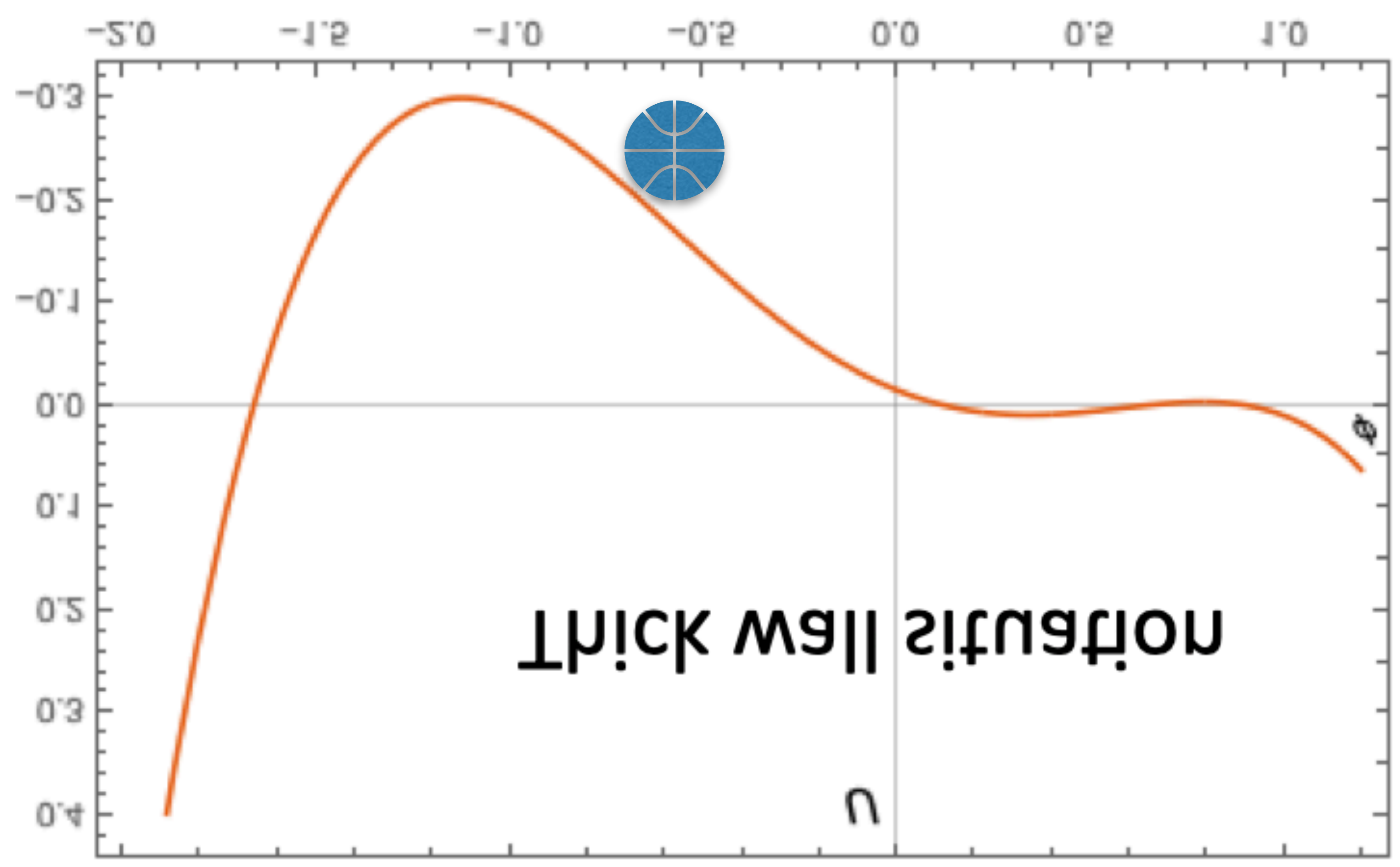


- The analytic result for the tunnelling rate was worked out in several famous papers by Callan, Coleman, de Luccia and Linde

- Decay rate per unit volume is given by the Euclidean actions of the O(4) or O(3) symmetric "bounce" solution (for instanton or thermal resp):

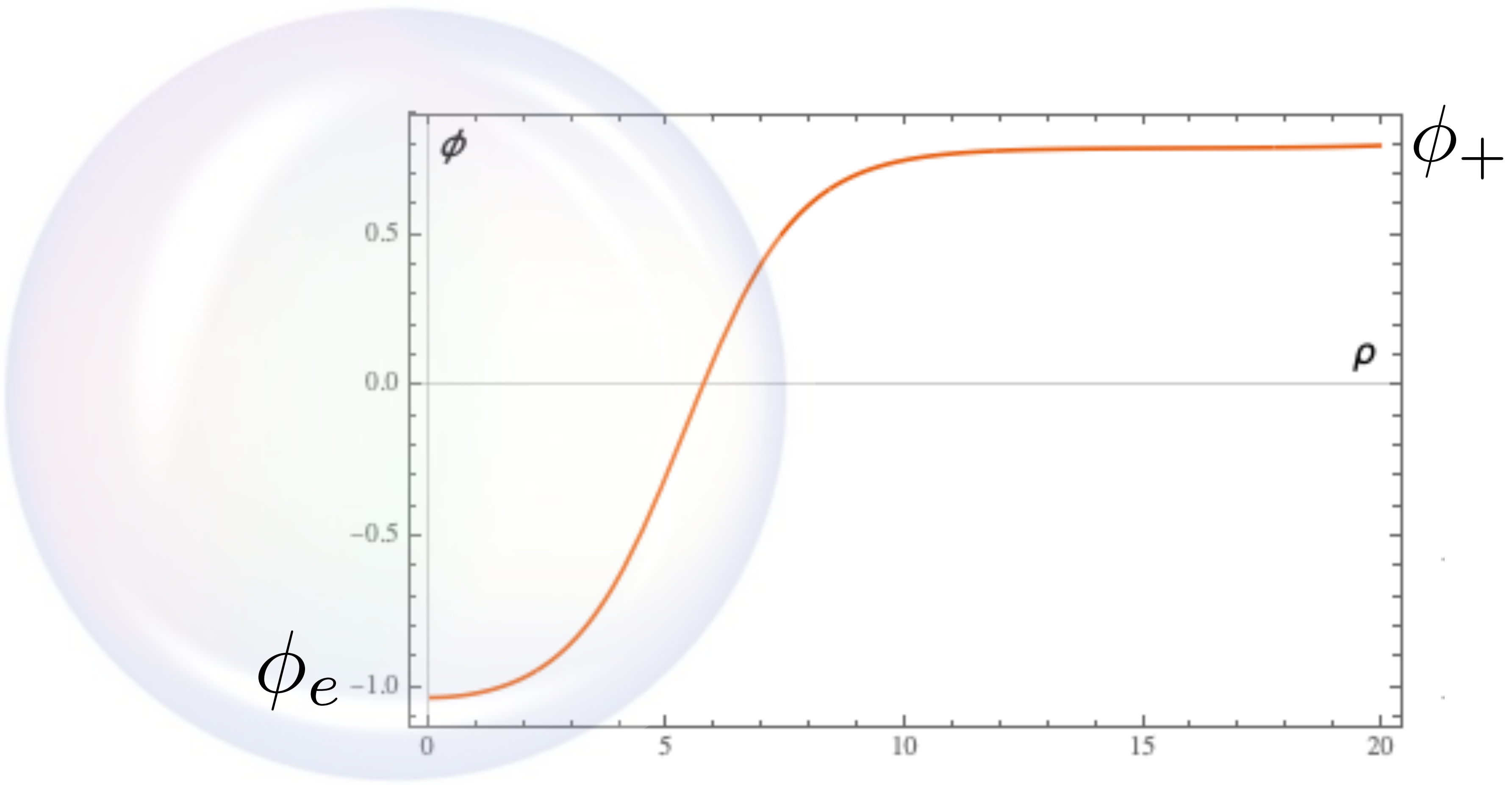
$$\Gamma_4 = A_4\, e^{-S_4[\phi]}\,,$$
$$\Gamma_3 = A_3\, T e^{-S_3[\phi]/T}\,,$$

where $S_{c+1} = \int_0^\infty d\rho \rho^c \left( \frac{\dot{\phi}^2}{2} + U(\phi) \right)$



- Normally solution found by solving Euler-Lagrange equations with boundary conditions:

$$\frac{d^2\phi}{d\rho^2} + \frac{c}{\rho}\frac{d\phi}{d\rho} = U' \ , \quad d\phi/d\rho = 0 \quad \text{as} \quad \rho \to 0, \infty$$




- "Escape point" found with overshoot/undershoot method.

- Thick-wall approximation: rescaling arguments give answer in terms of “standard action”

$$S_4 = \frac{3\xi}{\lambda} S_4^0 \quad ; \quad S_4^0 = 91$$

$$S_3 = \frac{3v\xi^{3/2}}{\lambda^{1/2}} S_3^0 \quad ; \quad S_3^0 = 19.4$$

where

$$\xi = \sqrt{2/3(1-\epsilon/\epsilon_0)}$$

$$\epsilon_0 = 2\lambda v^4/3\sqrt{3}$$

- Thin-wall approximation: action written in terms of c=0 action (Z2 domain wall)

$$S_4 = \frac{27\pi^2 S_1^4}{2\epsilon^3} \quad ; \quad S_3 = \frac{16\pi^3 S_1^3}{3\epsilon^2} \quad .$$

*In principle if we can encode this field theory on a quantum annealer, we will be able to vary the parameters and perform a tunnelling experiment. As a first step, we will determine S1: finding the extremum of the action is a quasi-convex problem (convex in a finite box).*

This means for the $c = 0$ action we will attempt to minimise the Euclidean action holding the endpoints fixed at +/- v:

$$S_1 = 2\pi^2 \int_0^\infty d\rho \; \frac{1}{2}\dot{\phi}^2 + U(\phi)$$

## *Encoding a scalar QFT on an Ising model*

- Chancellor
- SAA, Chancellor and Spannowsky, arXiv:2003.07374.

First encode $\phi$ by discretising its value using *N* qubits:

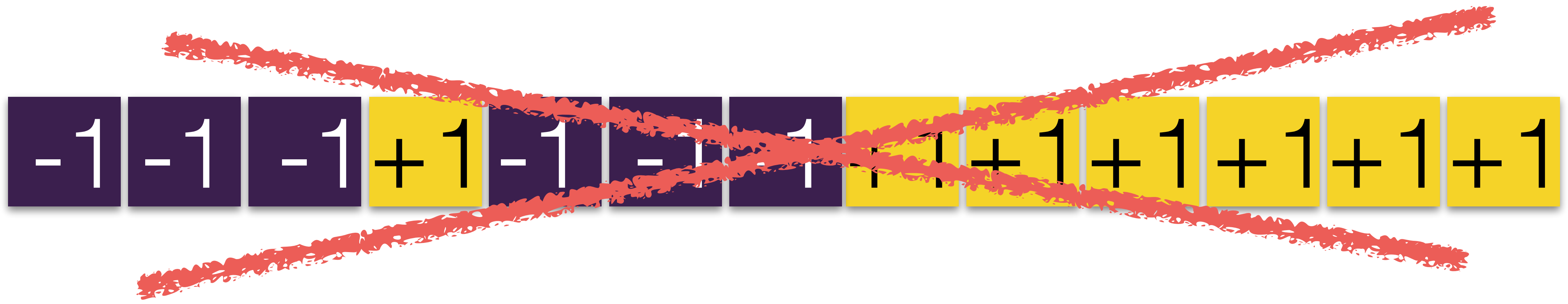
$$\phi \;=\; \phi_0 + \; j\,\xi \quad = \quad \phi_0 + \xi \; \ldots \; \phi_0 + N\xi$$

Represent it as a point on a spin chain == domain wall encoding (Chancellor):


We can translate any spin chain to a field value using

$$\phi = \phi_0 + \frac{\xi}{2} \sum_{i=1}^{N} (1 - \sigma_i^Z)$$

For this to work as a consistent encoding we have to avoid e.g.




This is the domain-wall encoding. Begin in the Ising model with a ferromagnetic interaction that favours as few flips as possible, but frustrate at least one by having the endpoints pinned at -1 … +1.

$$\mathcal{H}^{(\mathrm{chain})} = \Lambda \left( \sigma_1^Z - \sigma_N^Z - \sum_i^{N-1} \sigma_i^Z \sigma_{i+1}^Z \right)$$

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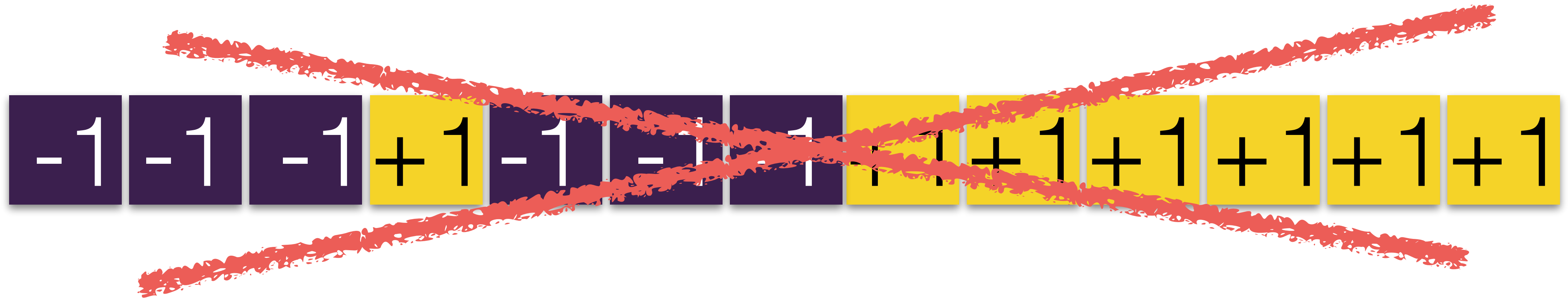


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$$\mathcal{H}^{(\text{chain})} = \Lambda \left( \sigma_1^Z - \sigma_N^Z - \sum_i^{N-1} \sigma_i^Z \sigma_{i+1}^Z \right)$$

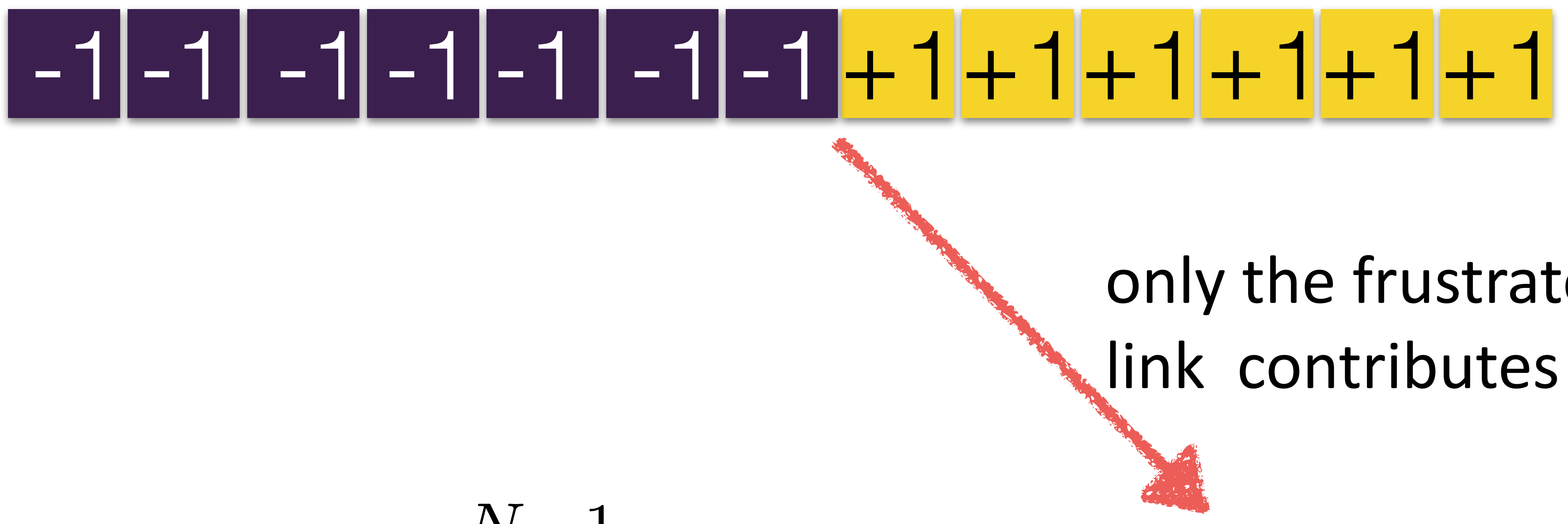
Pins the end spins at opposing values

$$h_j^{(\text{chain})} = \Lambda \left( \delta_{j1} - \delta_{jN} \right)$$

penalty for different adjacent spin

$$J_{ij}^{(\text{chain})} = -\frac{\Lambda}{2} \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & 1 & 0 & & \\ & & & \ddots & & \\ & & & & 0 & 1 \\ & & & & 1 & 0 \end{pmatrix}_{ij}$$

To add a potential we can add a contribution to the linear *h* couplings

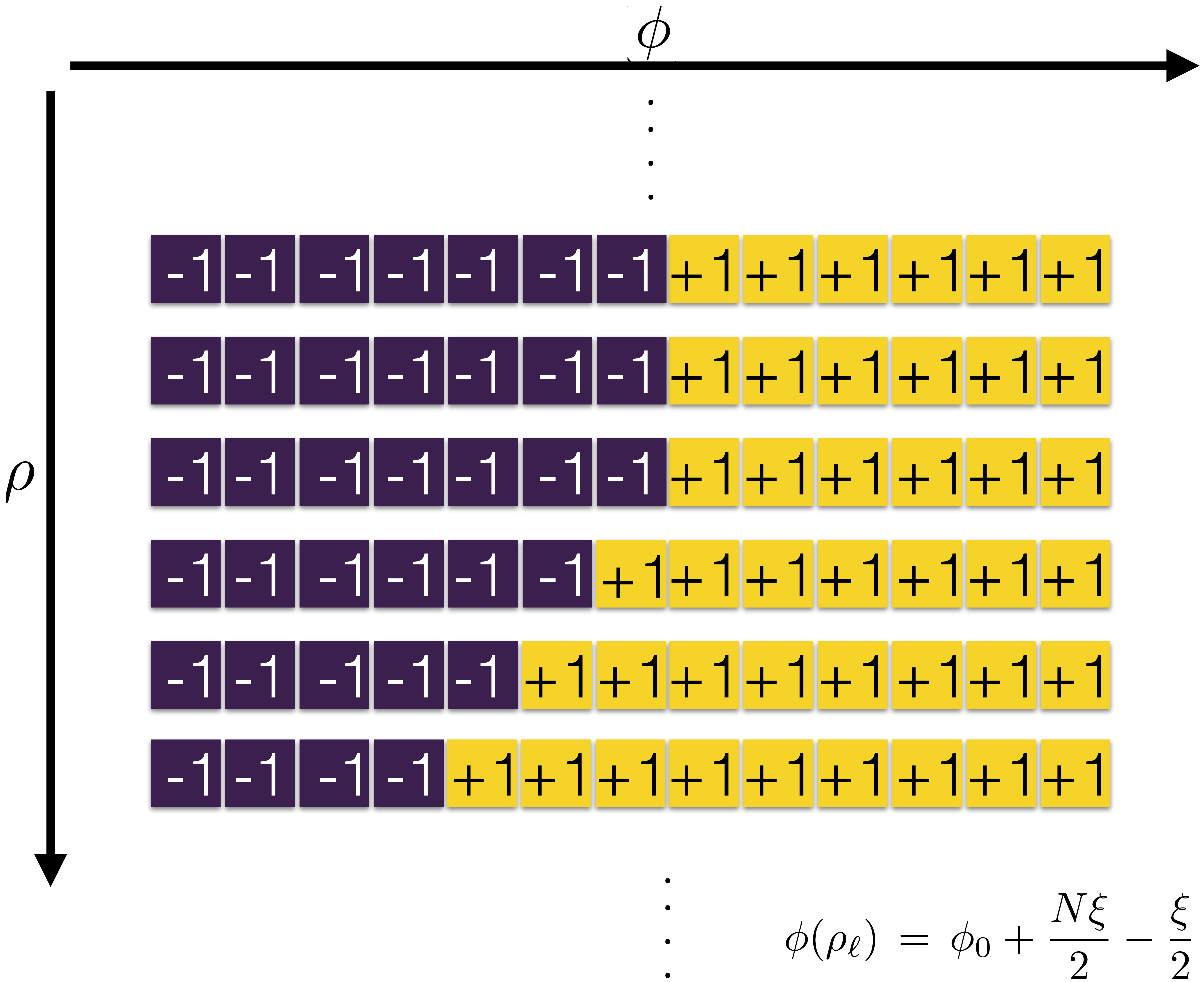



only the frustrated link contributes

$$U(\phi) = \frac{1}{2}\sum_{j}^{N-1} U(\phi_0 + j\xi)\left(\sigma^Z_{j+1} - \sigma^Z_j\right)$$

$$\equiv -\frac{1}{2}\sum_{j}^{N-1} U'(\phi_0 + j\xi)\sigma^Z_j$$

Next add the discretised radial spacetime coordinate: $\rho_\ell \;=\; \ell\nu \;=\; \nu \ldots M\nu$




$$\phi(\rho_\ell) \;=\; \phi_0 + \frac{N\xi}{2} - \frac{\xi}{2}\sum_{j=1}^{N}\langle\sigma^Z_{\ell N+j}\rangle$$

Everything done so far is then trivially extended in the *l* spacetime index:

$$h^{(\text{chain})}_{\ell N+j} = \Lambda\left(\delta_{j1} - \delta_{jN}\right) \qquad J^{(\text{chain})}_{\ell N+i, mN+j} = -\frac{\Lambda}{2}\,\delta_{\ell m}\begin{pmatrix} 0 & 1 & & & & \\ 1 & 0 & 1 & & & \\ & 1 & 0 & & & \\ & & & \ddots & & \\ & & & & 0 & 1 \\ & & & & 1 & 0 \end{pmatrix}_{ij}$$

$$h^{(\text{QFT})}_{N\ell+j} = \begin{cases} -\frac{\nu\xi}{2}U'(\phi_0 + j\xi)\ ; & j < N \\ \frac{\nu}{2}U(\phi_0 + (N-1)\xi)\ ; & j = N \end{cases}$$

Then kinetic terms are as follows:

$$S_{KE} \equiv \int_0^{\Delta\rho} d\rho \frac{1}{2}\dot{\phi}^2 = \lim_{M\to\infty}\sum_{\ell=1}^{M-1}\frac{1}{2\nu}\left(\phi(\rho_{\ell+1}) - \phi(\rho_\ell)\right)^2$$

$$= \sum_{\ell=1}^{M-1}\sum_{ij}^{N}\frac{\xi^2}{8\nu}\left[\sigma^Z_{(\ell+1)N+i} - \sigma^Z_{\ell N+i}\right]\times$$

$$\left[\sigma^Z_{(\ell+1)N+j} - \sigma^Z_{\ell N+j}\right]$$

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$$J^{(\text{QFT})}_{\ell N+i,mN+j} = \frac{\xi^2}{8\nu}\,\left(2\delta_{\ell m} - \delta_{\ell(m+1)} - \delta_{(\ell+1)m}\right)$$

Next we need to impose the physical boundary condition with:

$$\mathcal{H}^{(BC)} = \frac{\Lambda'}{2}(\phi(0)+v)^2 + \frac{\Lambda'}{2}(\phi(\rho_M)-v)^2$$

We can think of these as just boundary mass-term potentials in *U*:

$$h^{(\mathrm{BC})}_{N\ell+j} = \begin{cases} -\Lambda'(\phi_0 + j\xi + v) \ ; \ \ell = 1, \forall j \\ -\Lambda'(\phi_0 + j\xi - v) \ ; \ \ell = M-1, \forall j \end{cases}$$

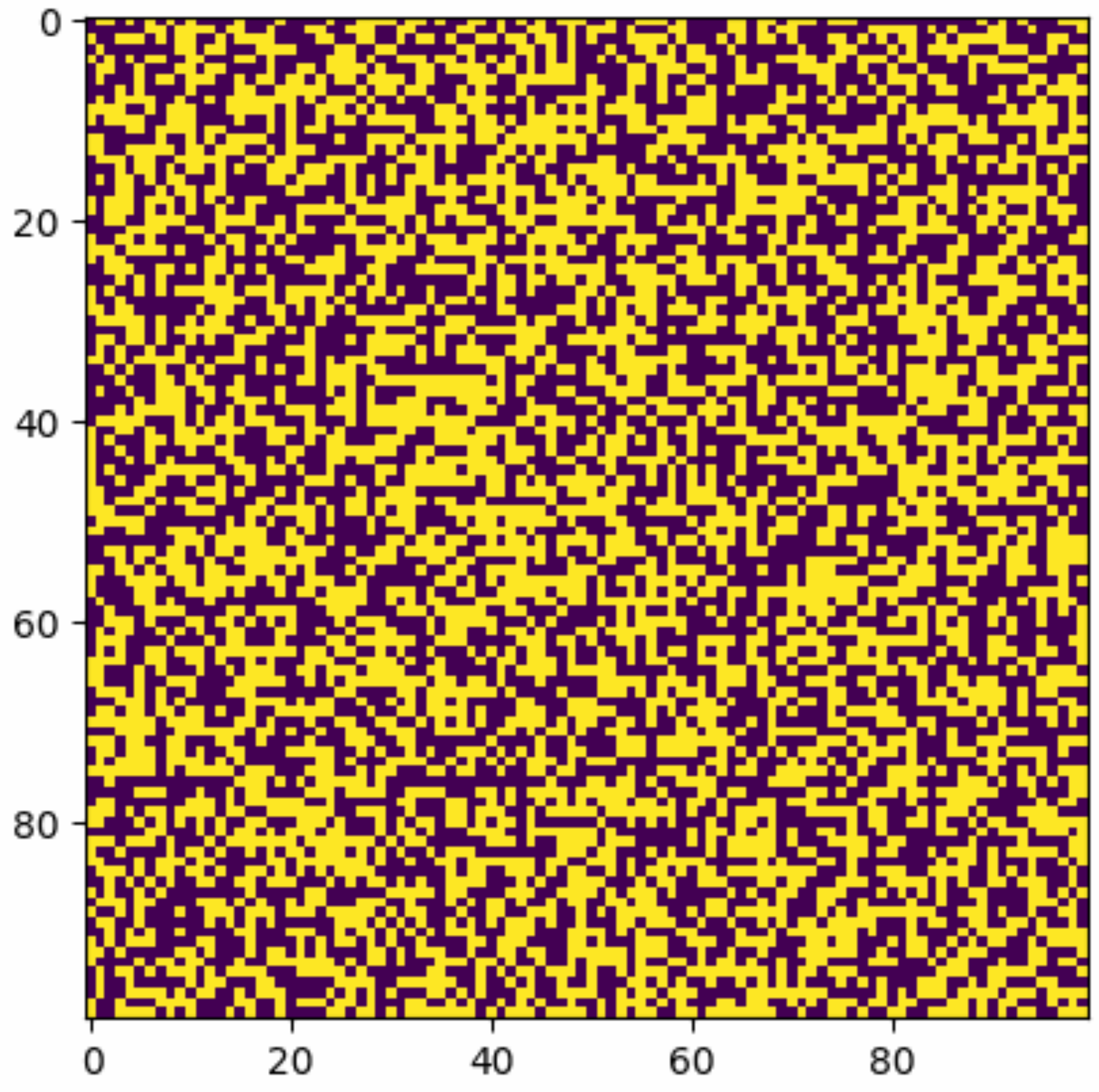
Finally add everything together!

$$\mathcal{H} = \mathcal{H}^{(\mathrm{chain})} + \mathcal{H}^{(\mathrm{QFT})} + \mathcal{H}^{(\mathrm{BC})}.$$

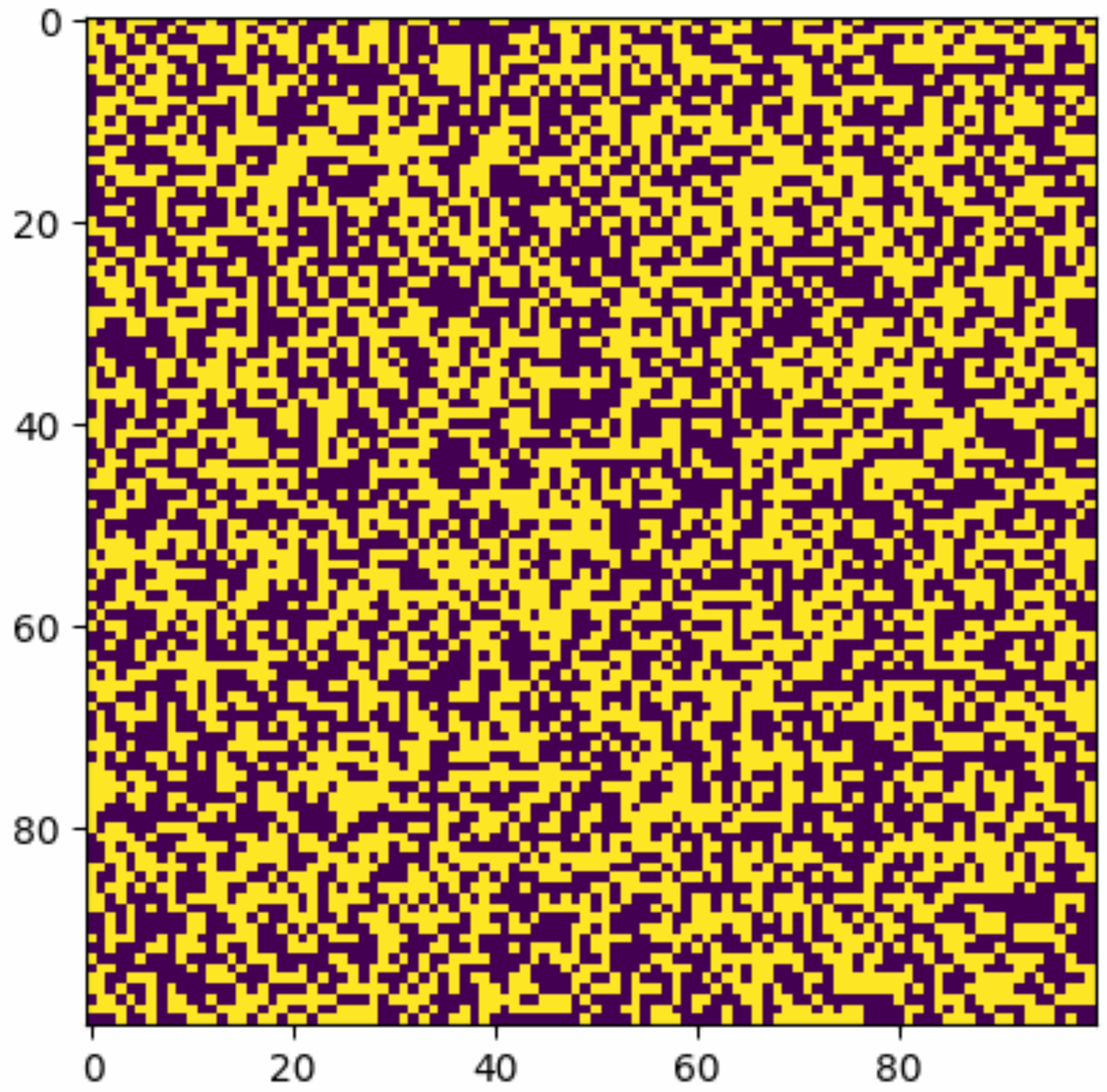
## *Results for thin wall limit*

Can solve classical simulated annealing with the Metropolis algorithm. Again have to be careful how we set the temperatures and parameters:

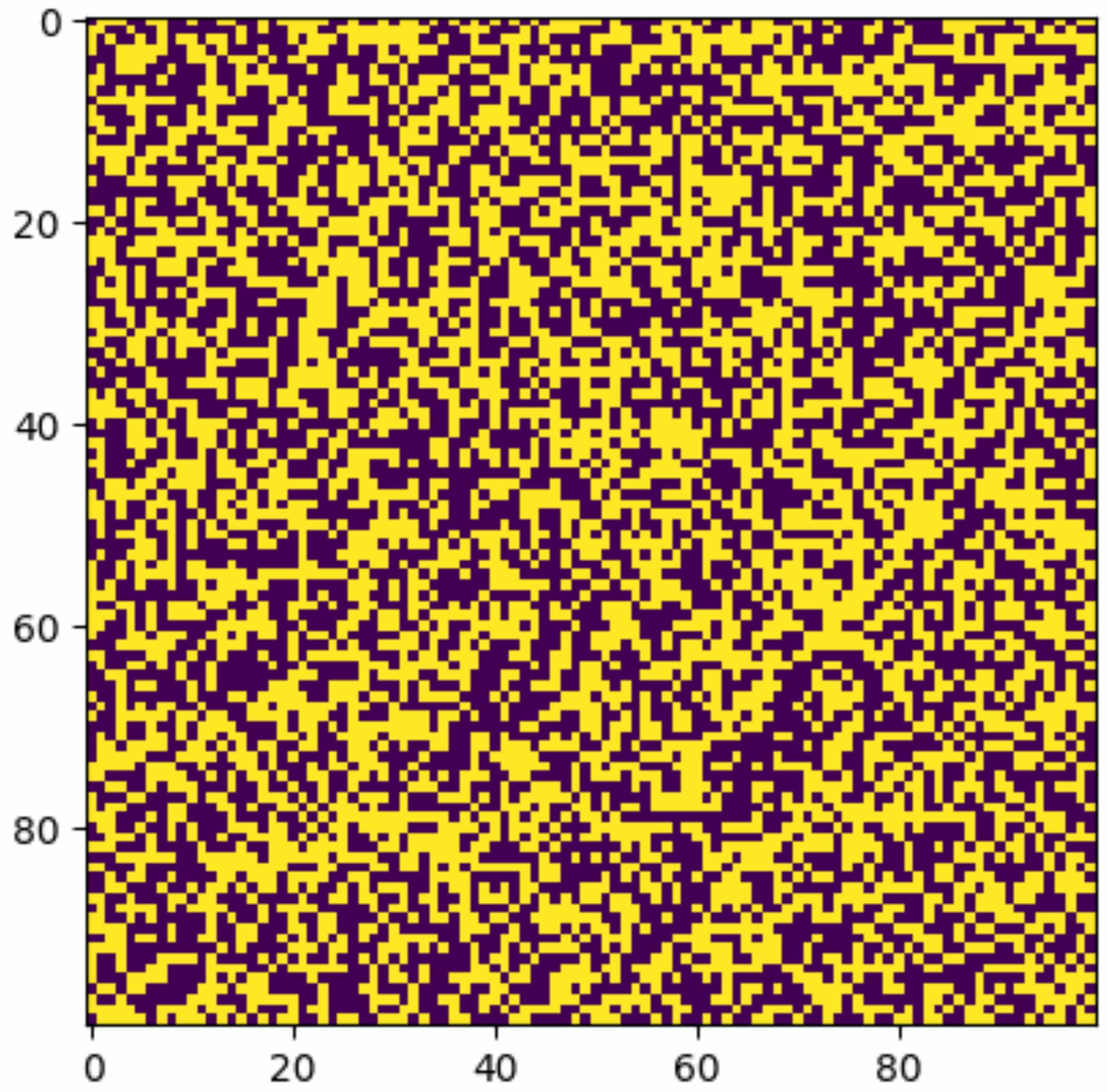
Too hot



Too cold

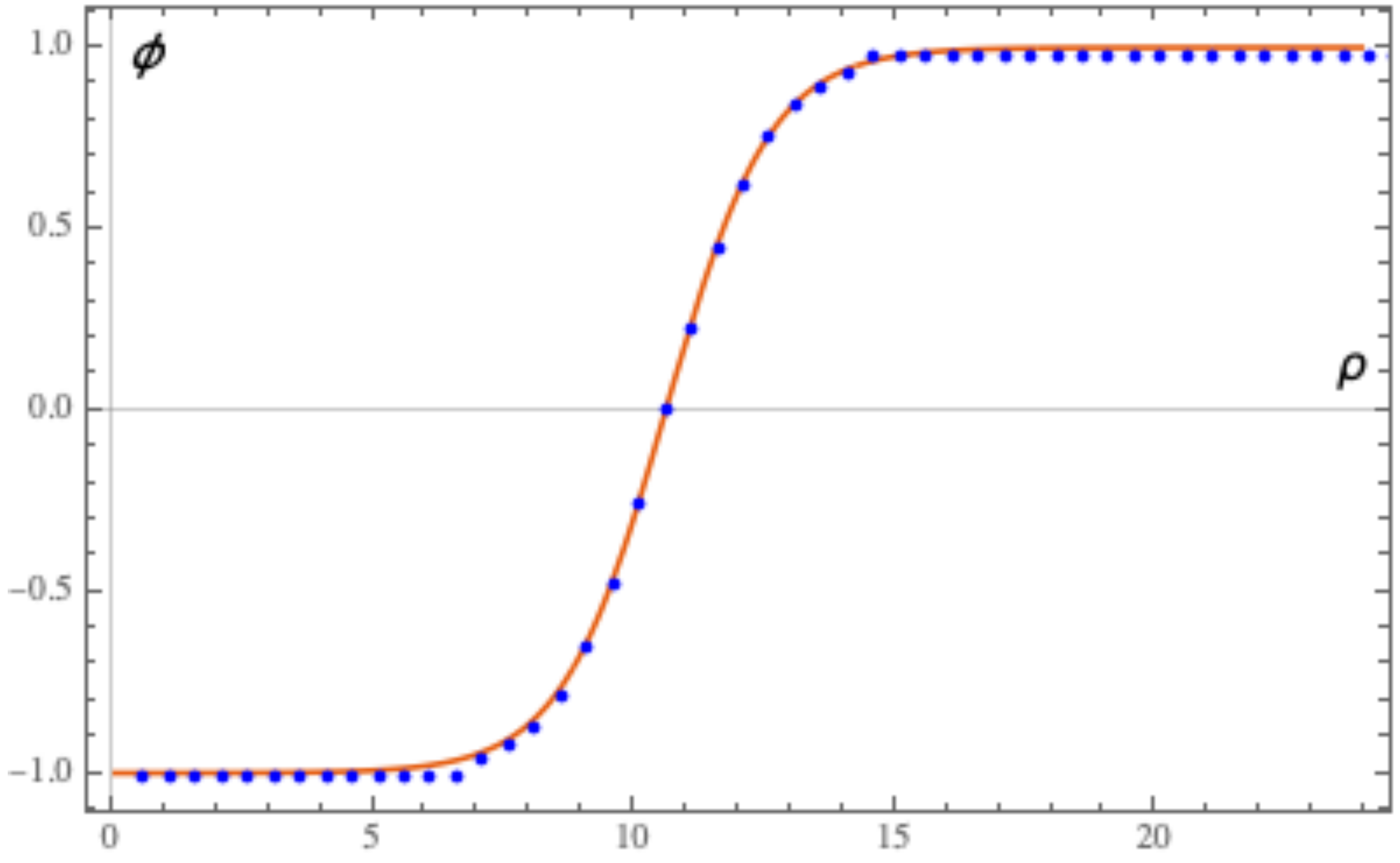


Just right (two stage annealing process)



Same result on Dwave using hybrid quantum/classical Kerberos annealer (It finds best samples of parallelised tabu search + simulated annealing + D-Wave subproblem sampling)

Notably the Kerberos sampler is much more robust than pure simulated annealing.

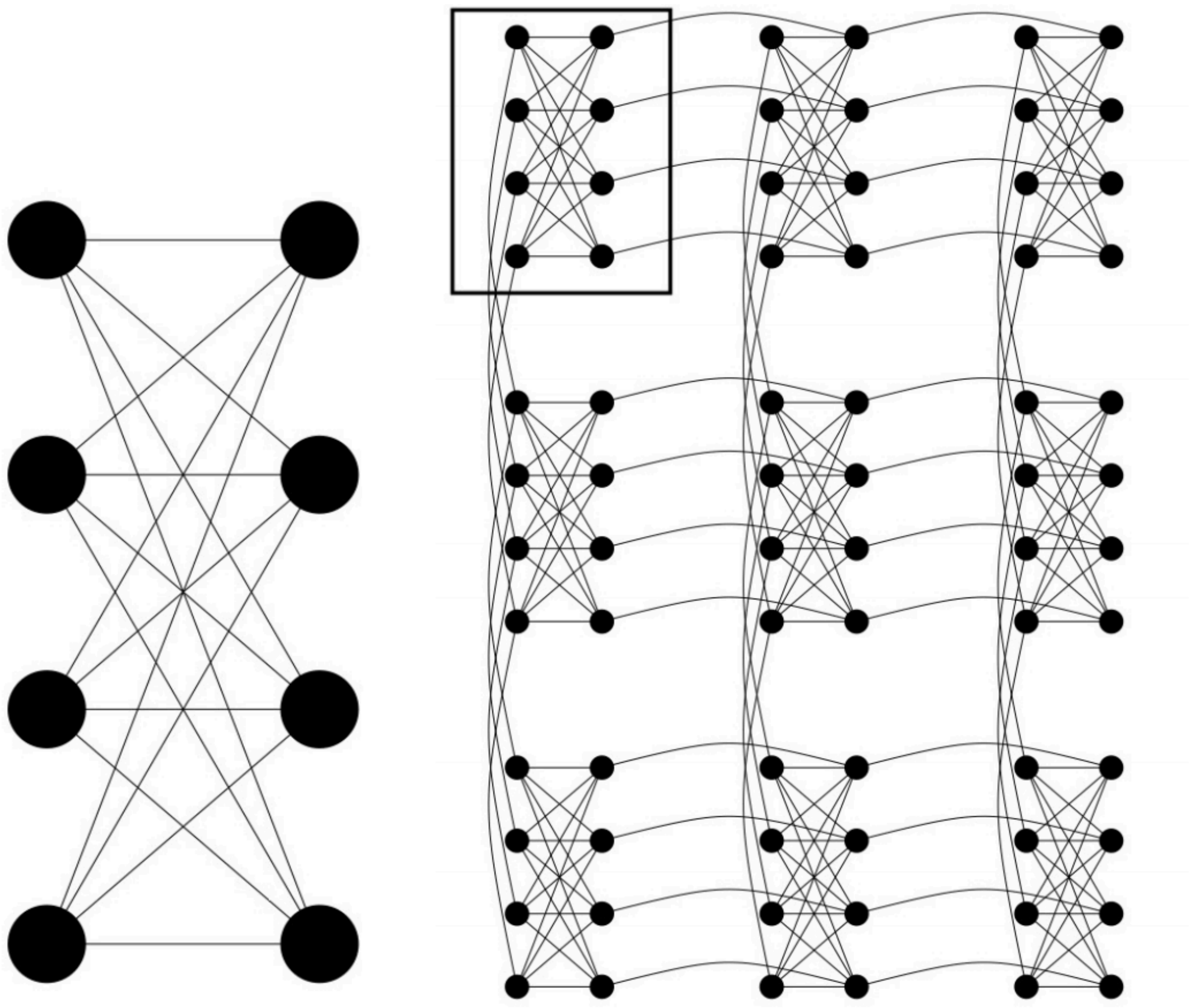



But this proves the principle: we can encode a pure field theory potential on the annealer, so we can experiment with QFT tunnelling

***Addendum to this part:*** The “instanton” solution is of course a classical object. We have not yet done any actual quantum tunnelling.

# *Quantum Tunnelling: the Schrodinger eq.*

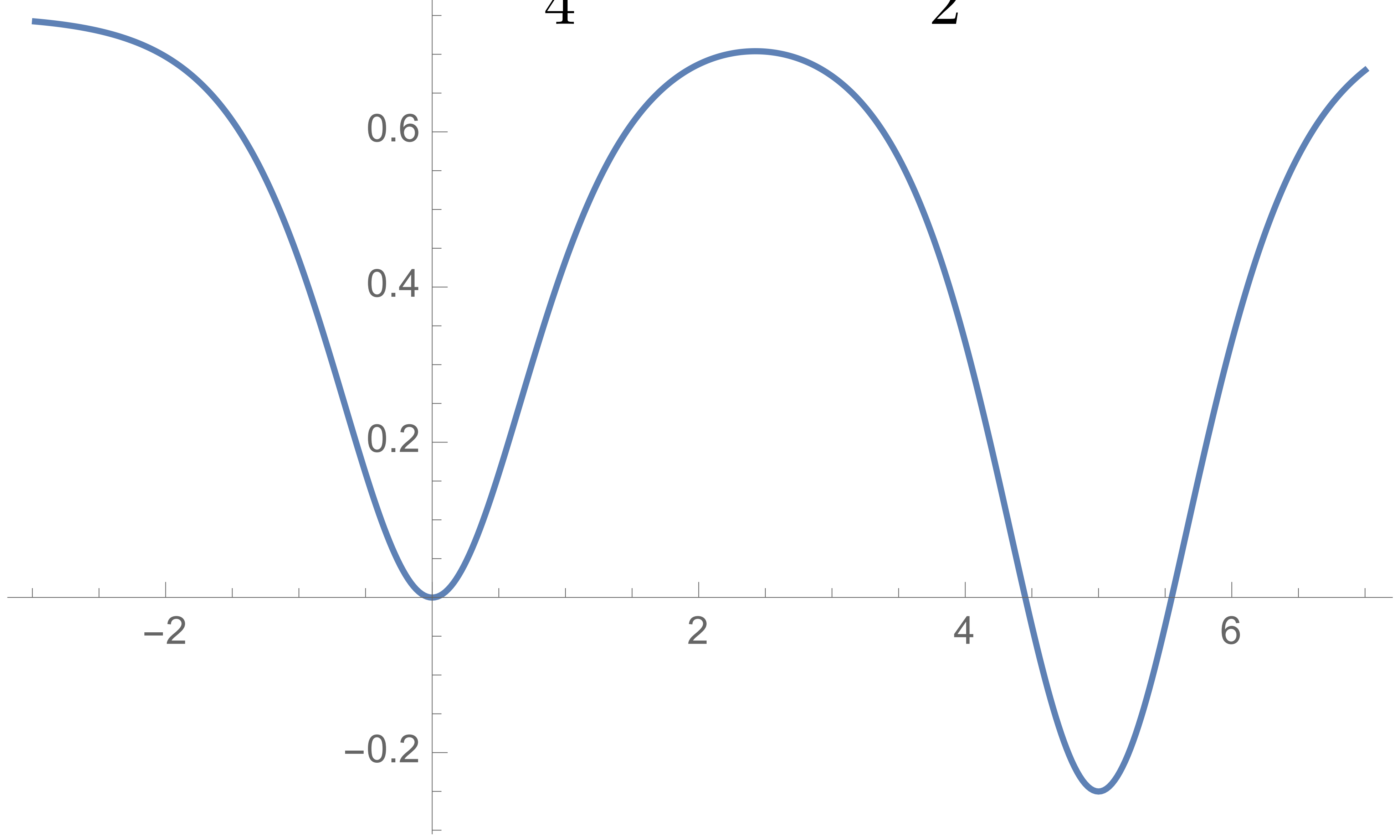
***Why did we not use a pure Quantum annealer?*** The connectivity is not general enough for this problem (in particular encoding the kinetic terms): it has a Chimera structure …



***But using a "minor embedding" we can currently achieve the equivalent of a ~ 200 qubit general Ising model.  This is enough for the zero space-dimension problem.***

***So we should be able to do d=1 field theory (aka Quantum Mechanics). That is we set up the annealer with ONLY a potential and NO dynamics at all.***

$$U(\phi) = \frac{3}{4}\tanh^2\phi - \frac{k(t)}{2}\mathrm{sech}^2(\phi - v)$$




If it is quantum then we should find the c=0 tunnelling corresponding to

$$\Delta_E = \int_{\phi_i}^{\phi_f} \mathcal{D}\phi\, e^{-\hbar^{-1}\int dt\left(\frac{m\dot{\phi}^2}{2} + U(\phi)\right)}$$

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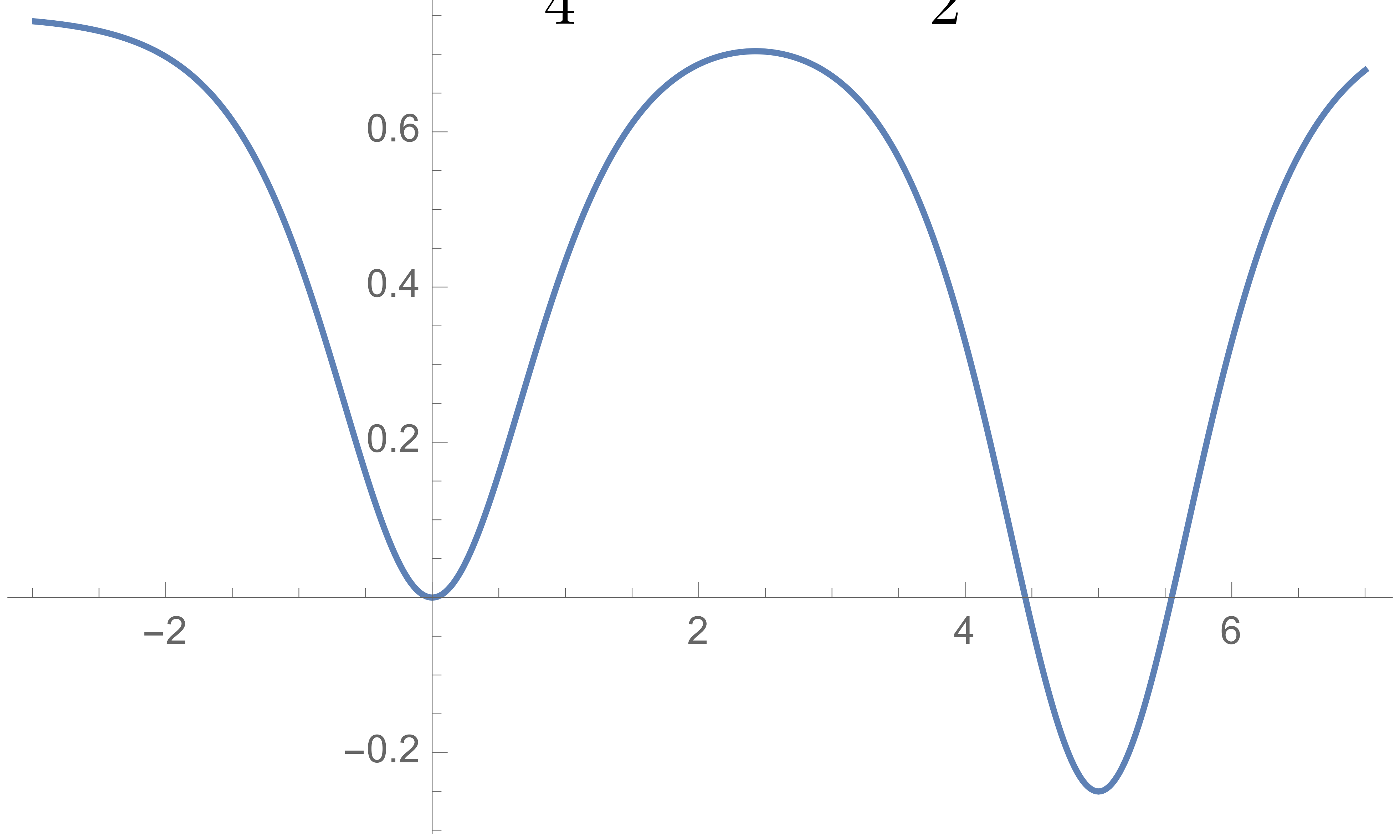


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***Dynamics, time-dependence (and also m) should all come from the annealer now***

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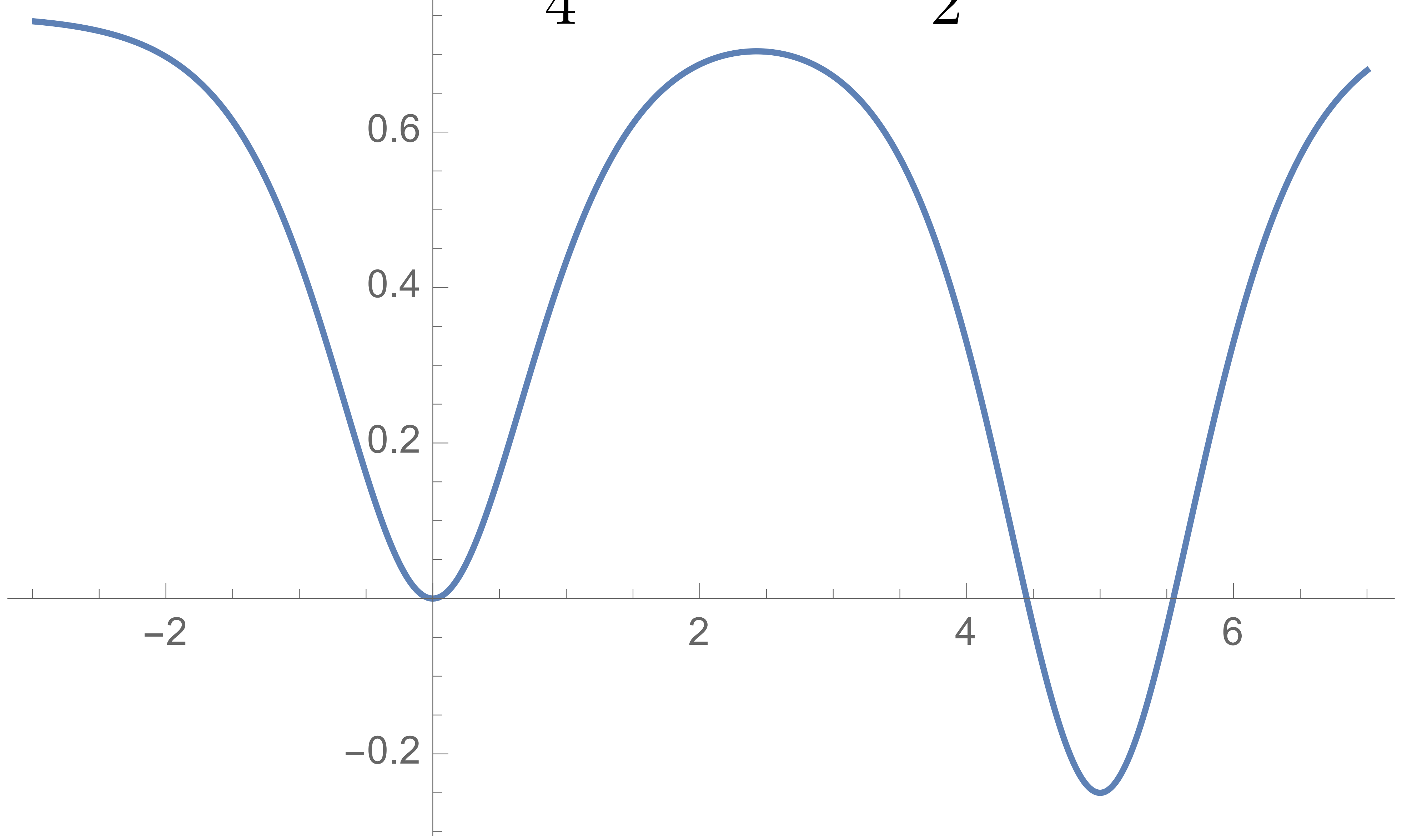
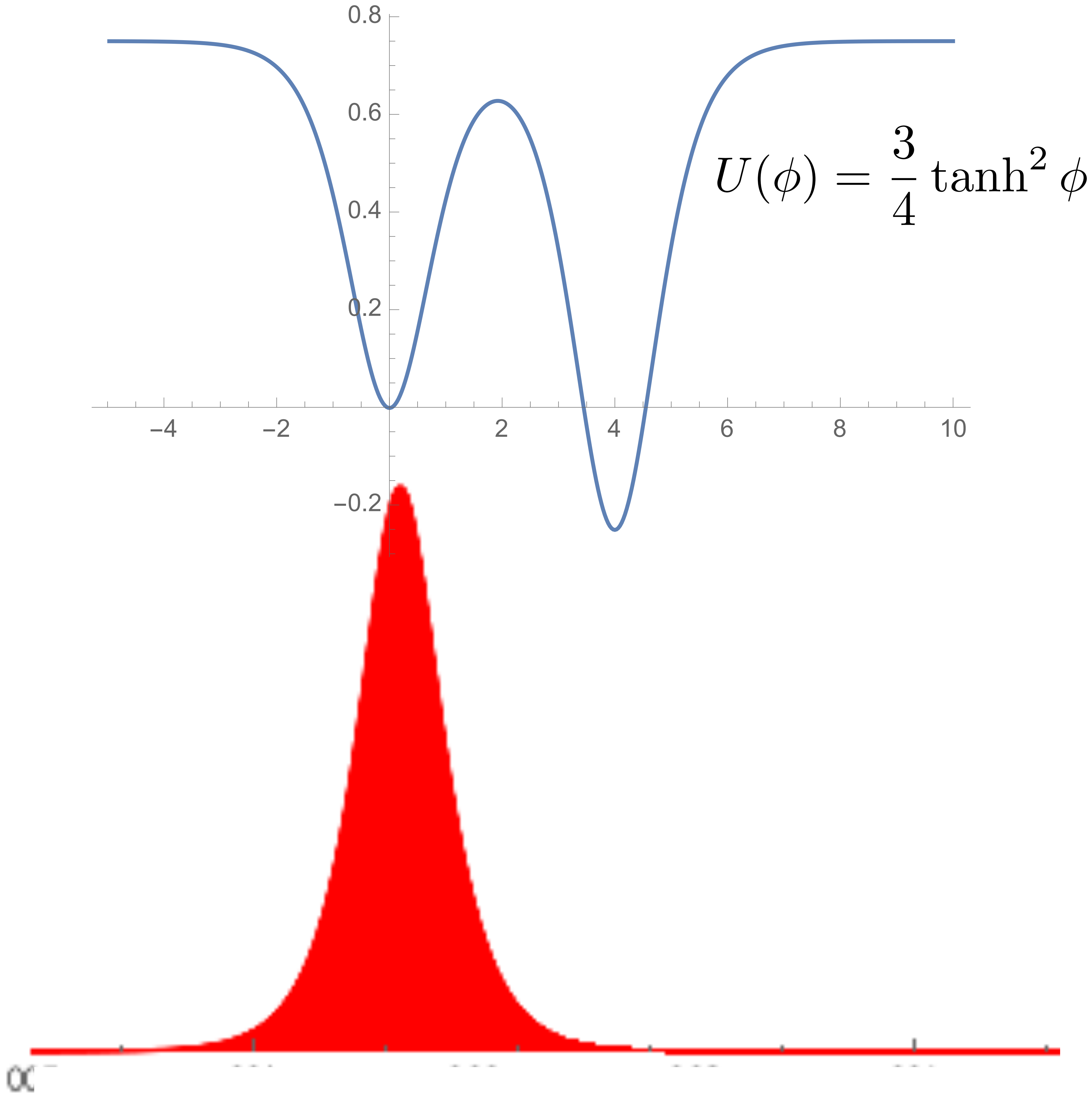



***This is equivalent to solving the one dimensional Schroedinger Equation:***

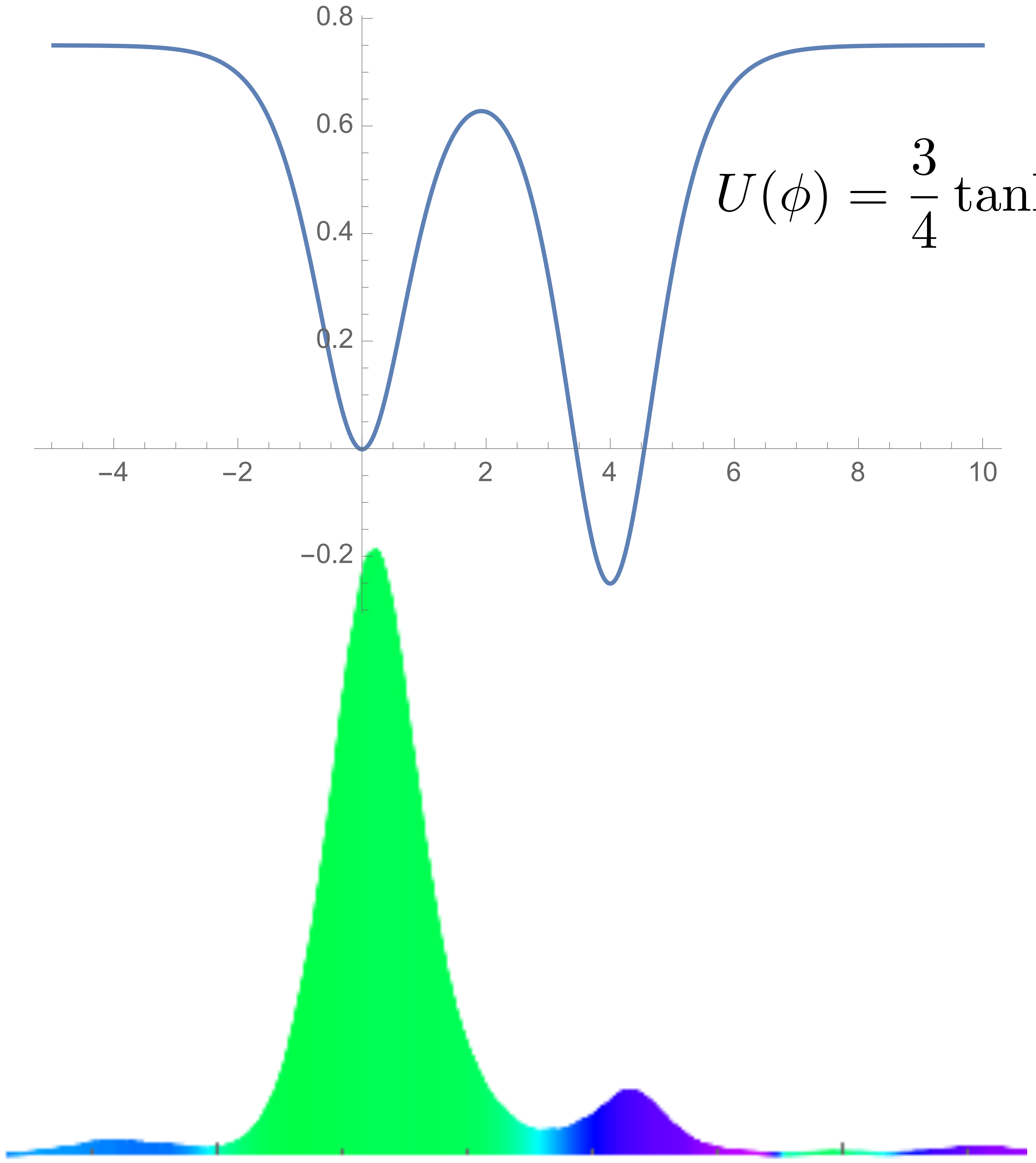
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial\phi^2} + U\psi = i\hbar\frac{\partial\psi}{\partial t}$$

## ***Begin in the false minimum and evolve numerically (takes a long time!)***




$$U(\phi) = \frac{3}{4}\tanh^2\phi + \tanh^2(\phi - v) - 1$$

***Begin in the false minimum and evolve numerically (takes a long time!)***

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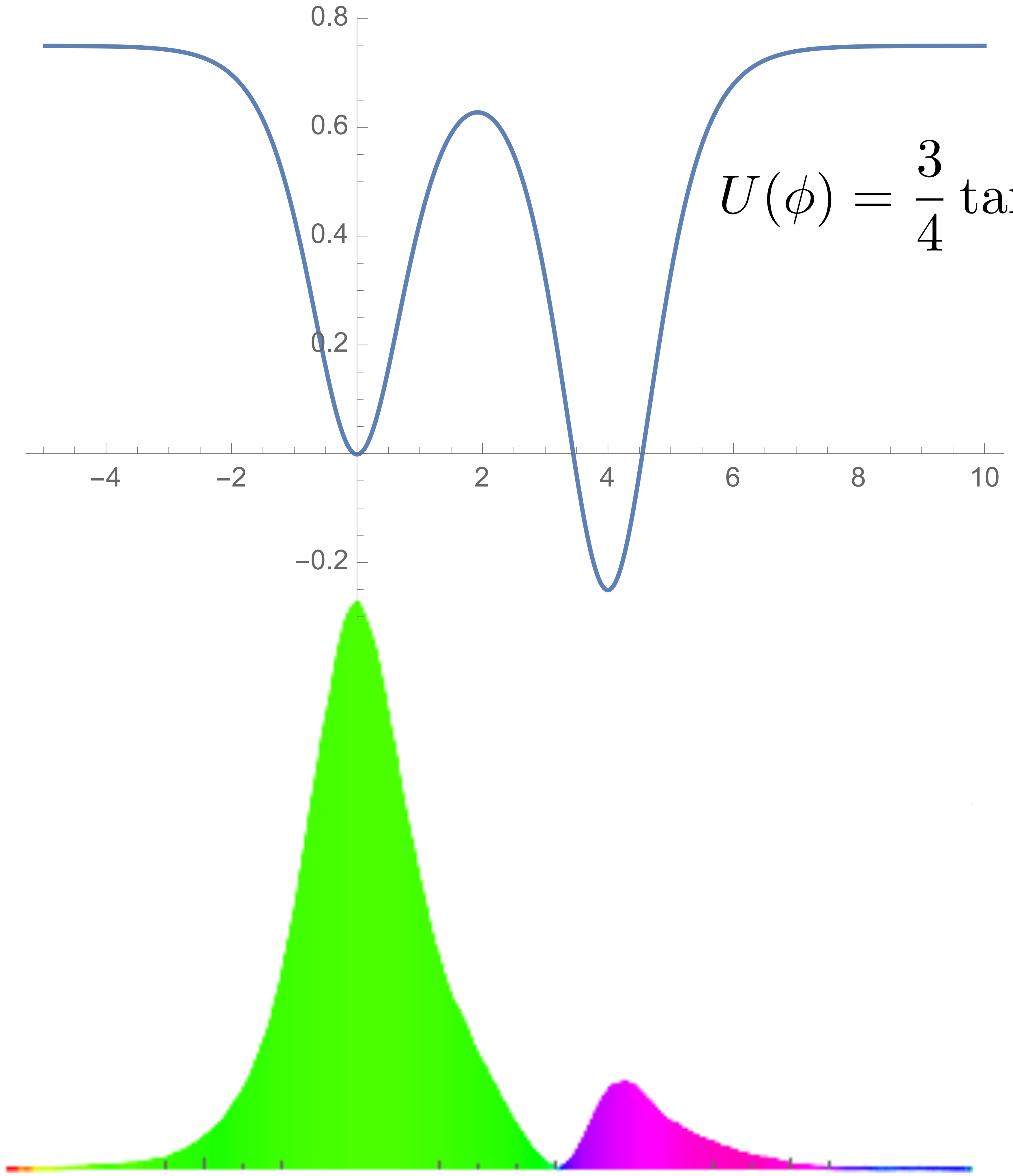


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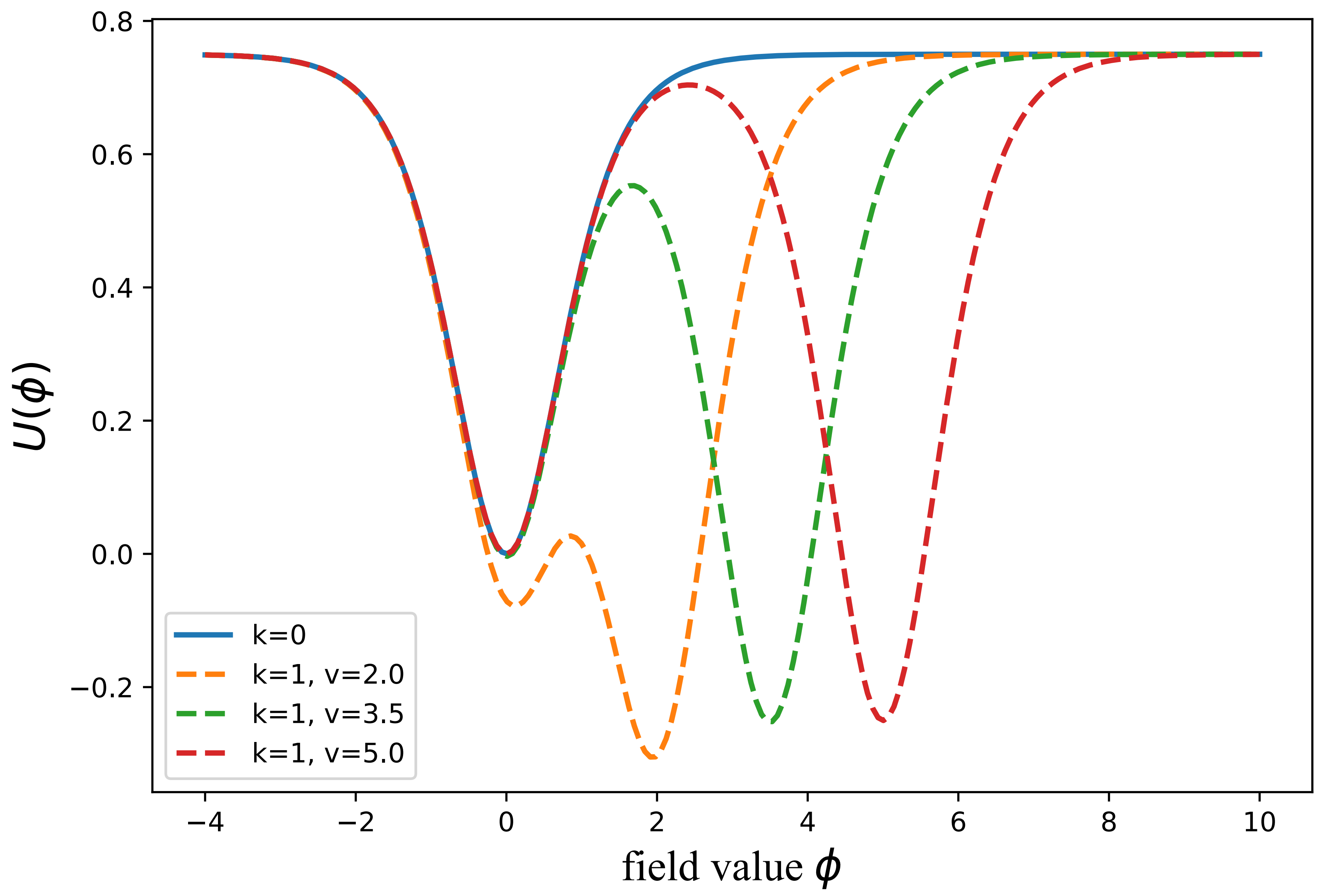
In the worldline formalism we get the usual WKB like decay rate; $\Gamma \approx e^{-2\hbar^{-1}S_E}$

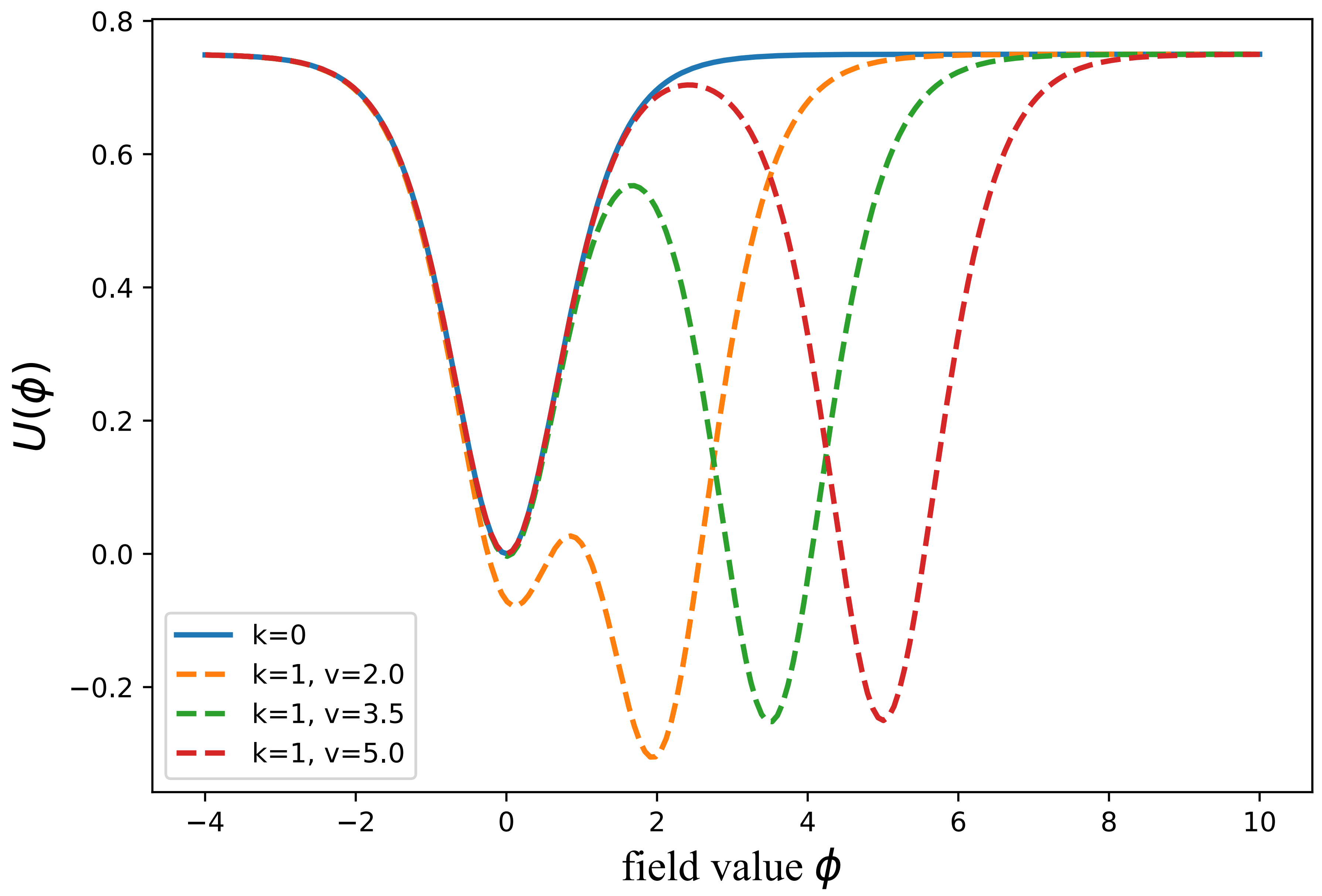
$$\hbar^{-1}S_E \approx \gamma^{-\frac{1}{2}} \int_{\phi_+}^{\phi_e} \sqrt{\frac{3}{4}\tanh^2\phi - \mathrm{sech}^2(\phi - v)}\, d\phi$$

where $\gamma = \hbar^2/2m$ is something we must measure. e.g. use the SHO groundstate



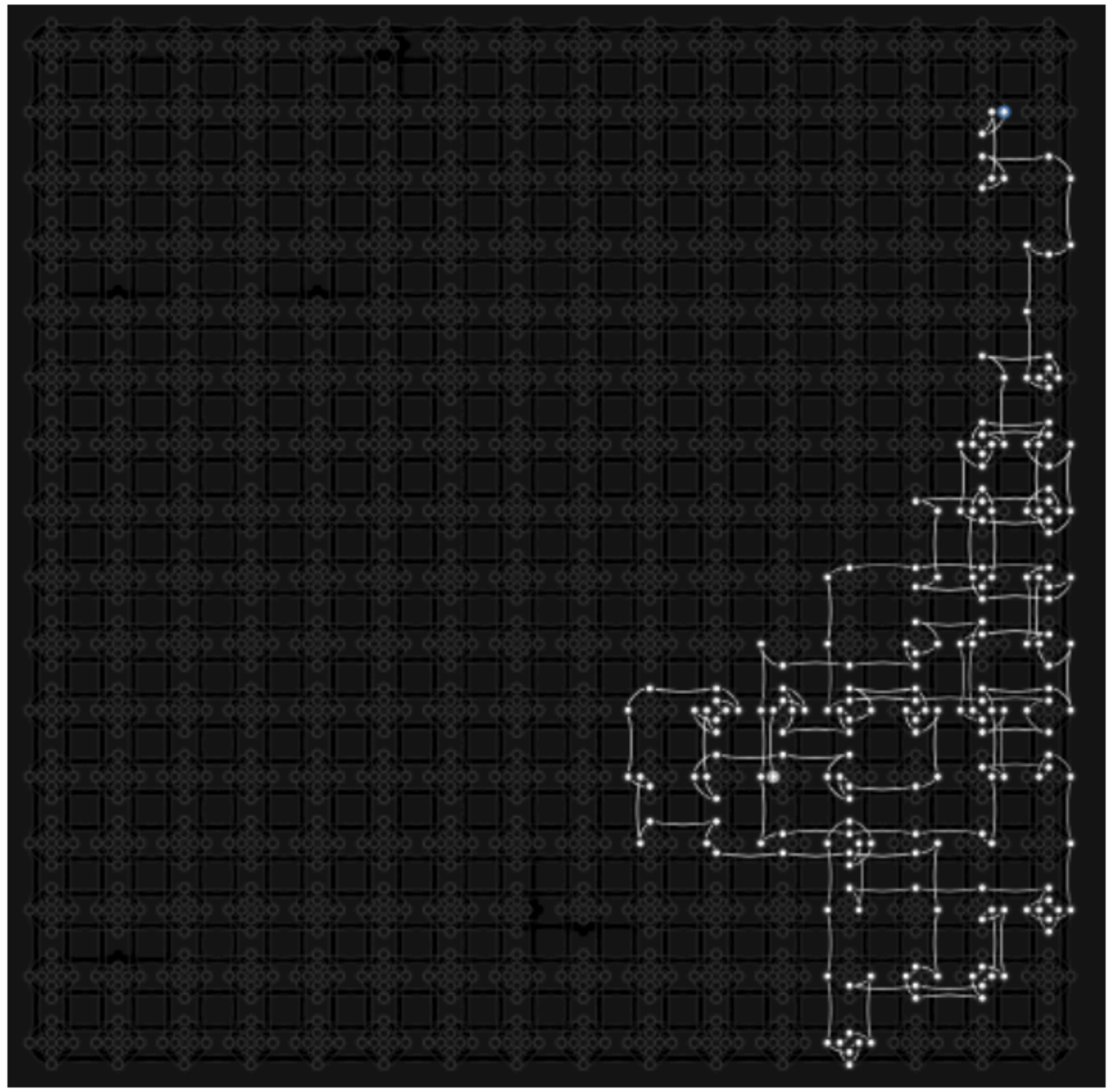
The real part of this integral is linear to a very good approximation for all v>5/3:

$$\log\Gamma \approx -2\hbar^{-1}S_E \approx \sqrt{\frac{3}{\gamma}}\left(\frac{5}{3}-v\right)$$

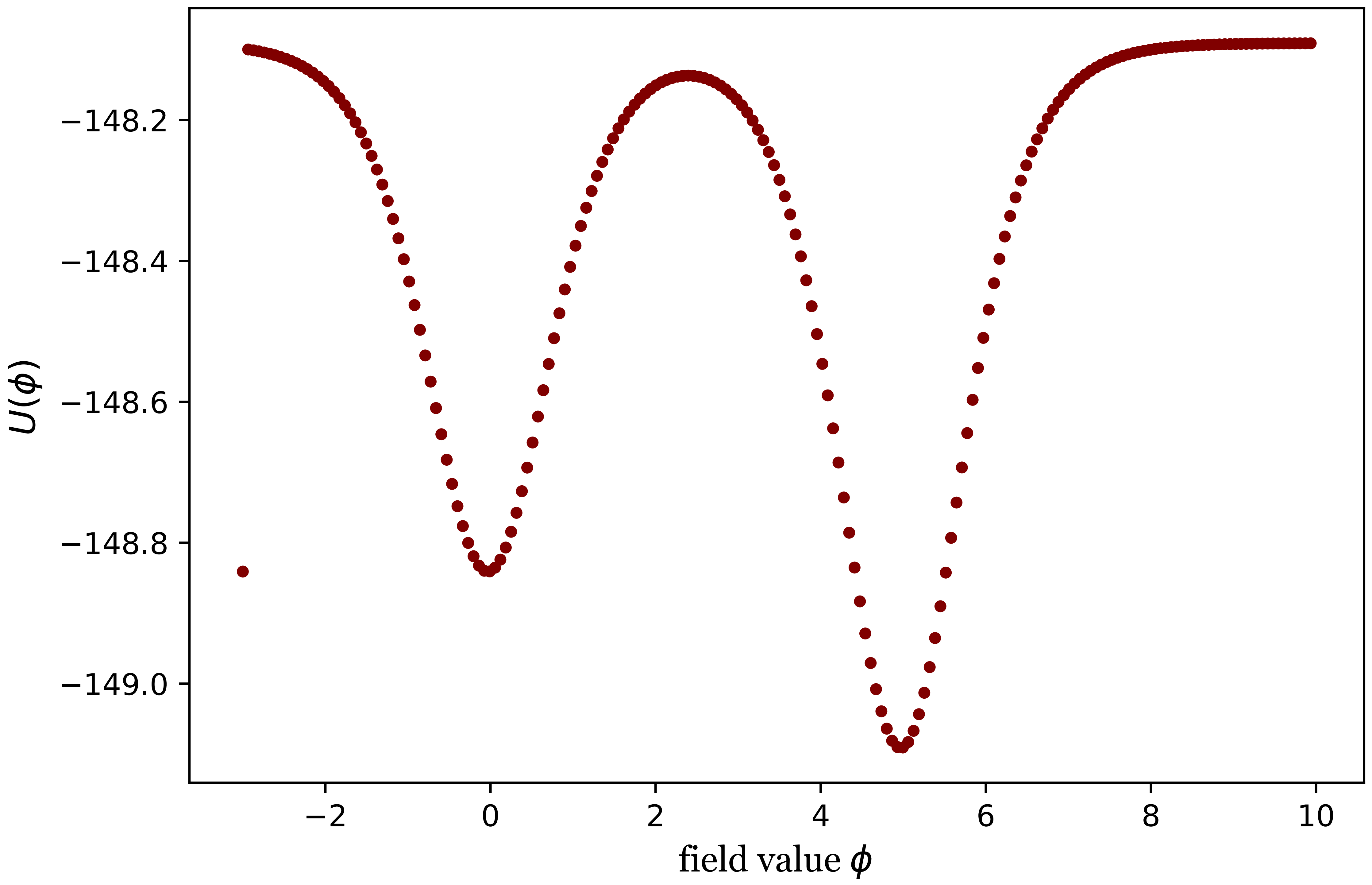


***Set up on the annealer:***

Typical “minor” embedding of the Ising model



This is what the Ising model sees when we encode the simple zero space dimension case — i.e. this is $U$ taken from what we pass to the annealer:



***We will do a reverse anneal as follows:***

a) begin with it in a classical state (choose the sigmas) with a single well potential

b) bring it to a quantum state and wait 50 microseconds for it to become stable

c) change the potential to introduce the second well

d) wait *t* microseconds and bring it back to a classical state to measure the sigmas

e) Rinse and repeat 10K times

f) work out the tunnelling fraction.

Potential is split into two parts (one for each well), and we adjust the coupling *k* using the h_gain_schedule. (i.e. well 1 is U0 and is in *J*, well 2 is U1 and is in *h*).

$$U_0 = \frac{3}{4}\tanh^2\phi, \;\; ; \;\; U_1 = -k(t)\mathrm{sech}^2\left(\phi - v\right),$$

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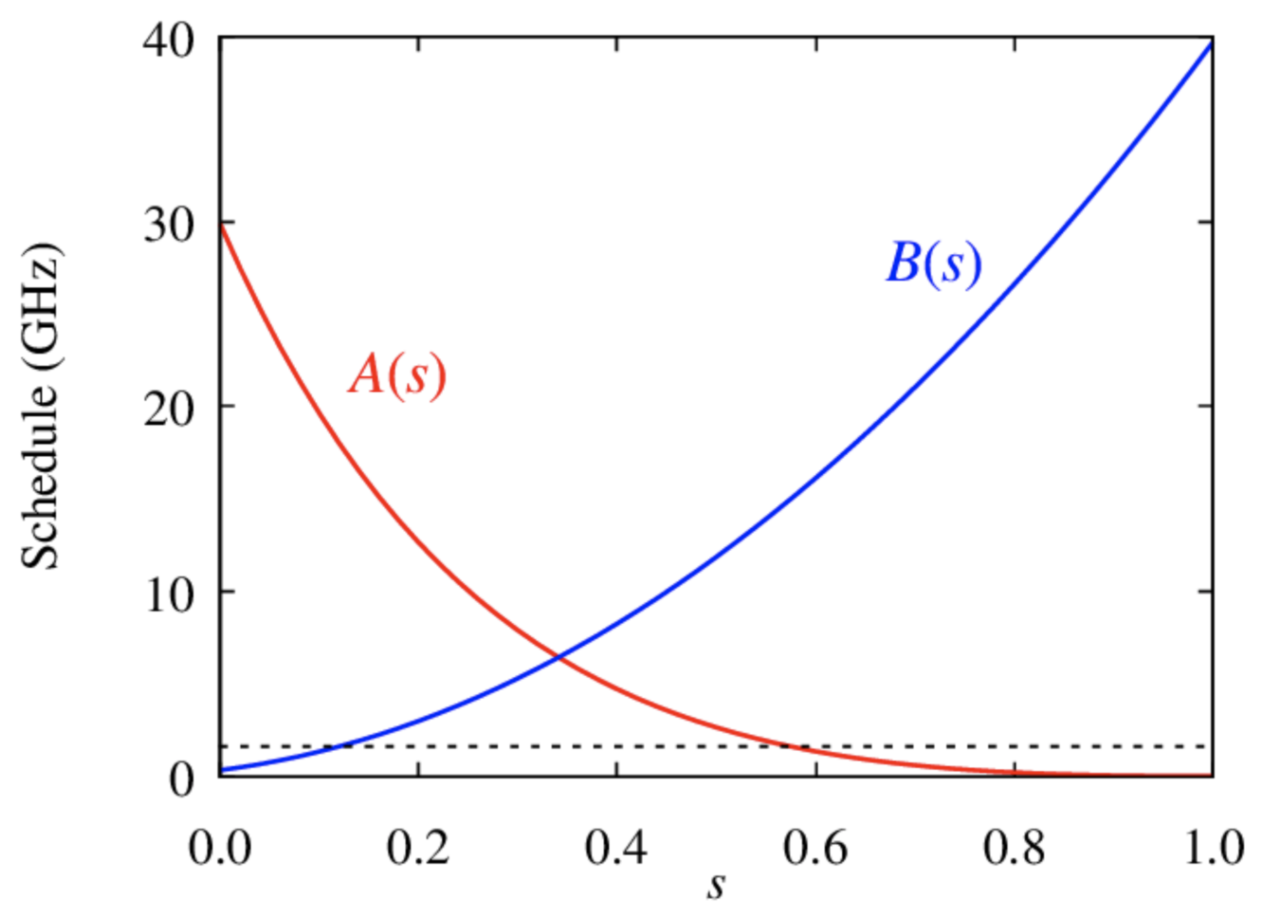


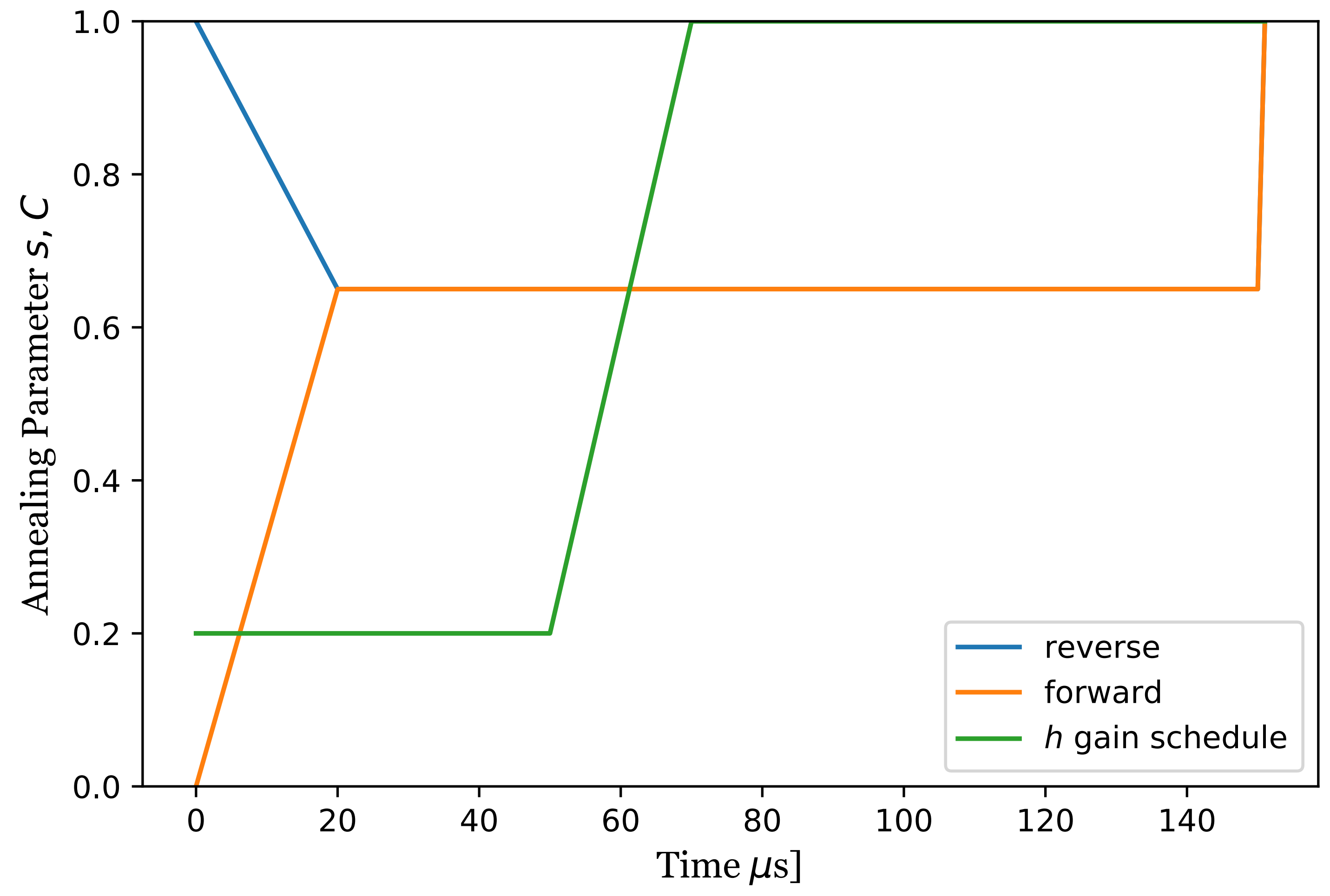

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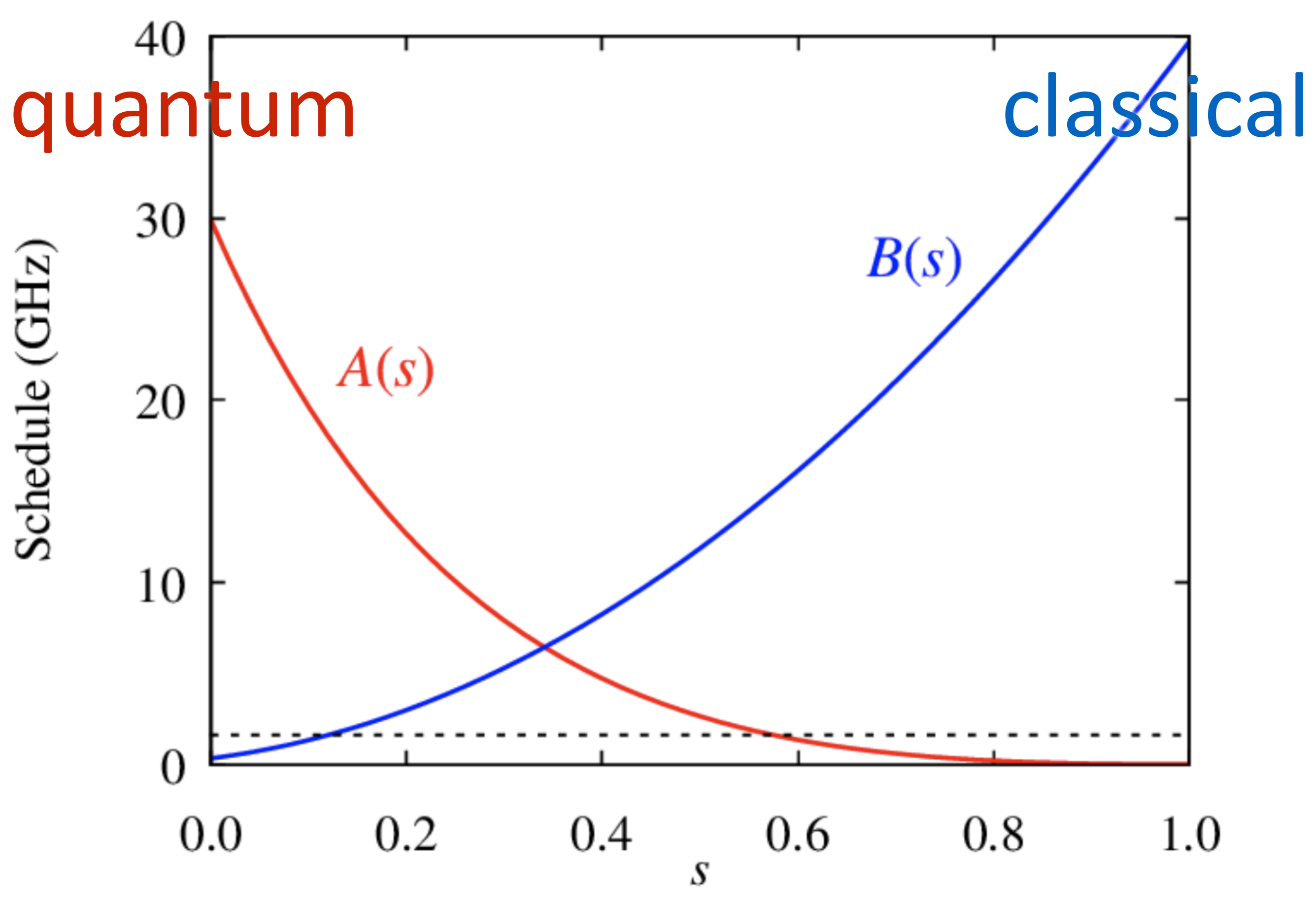
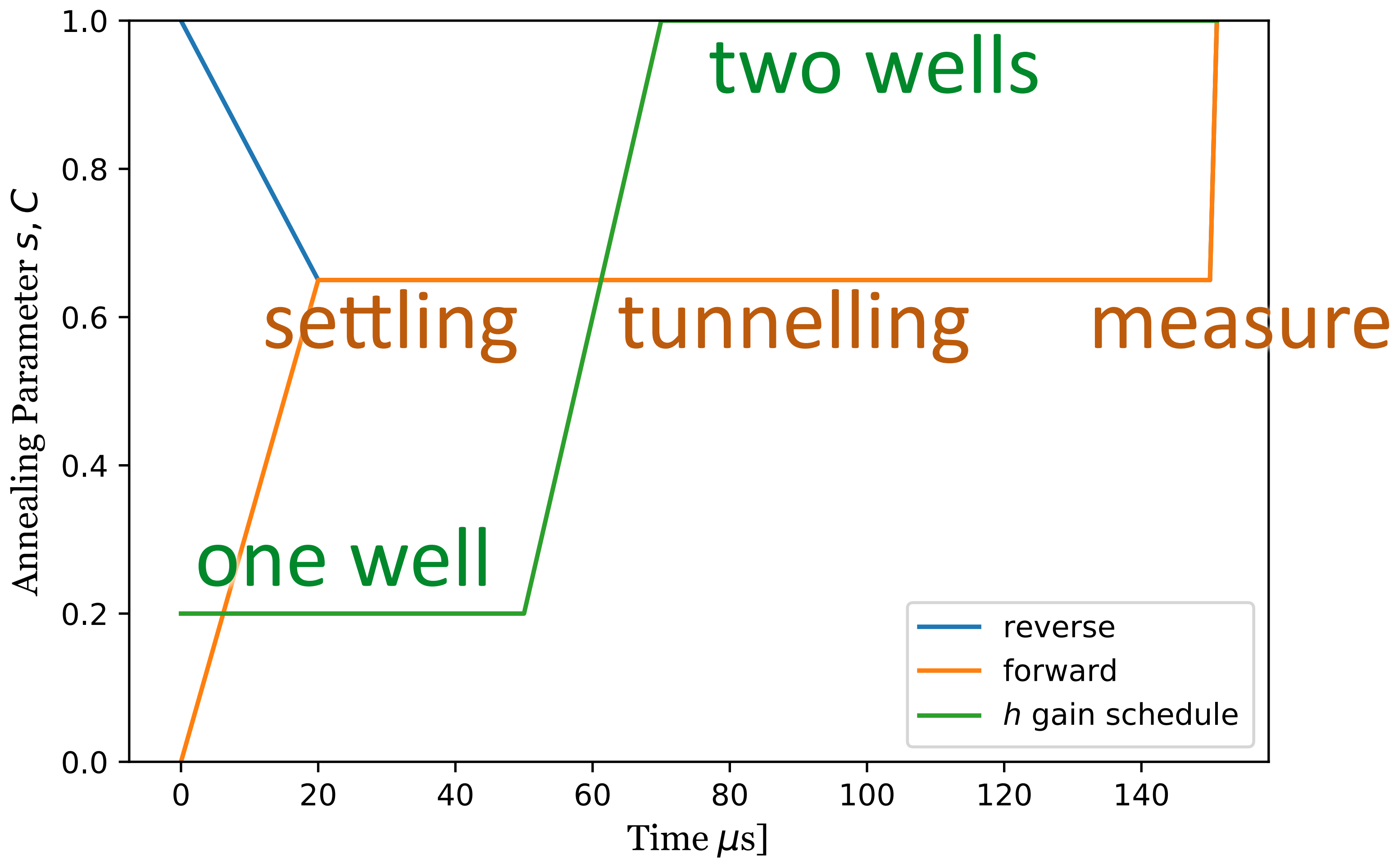
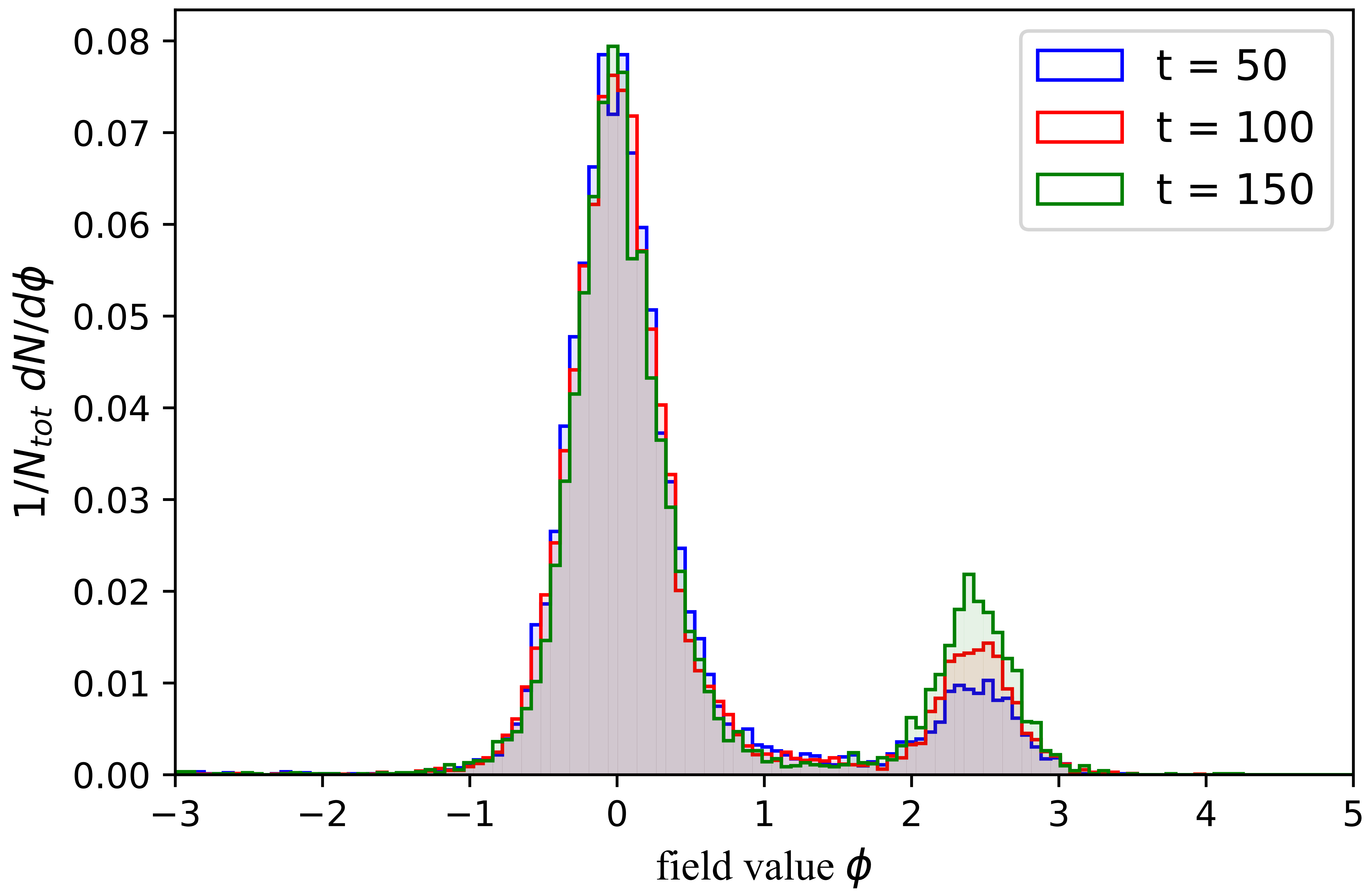
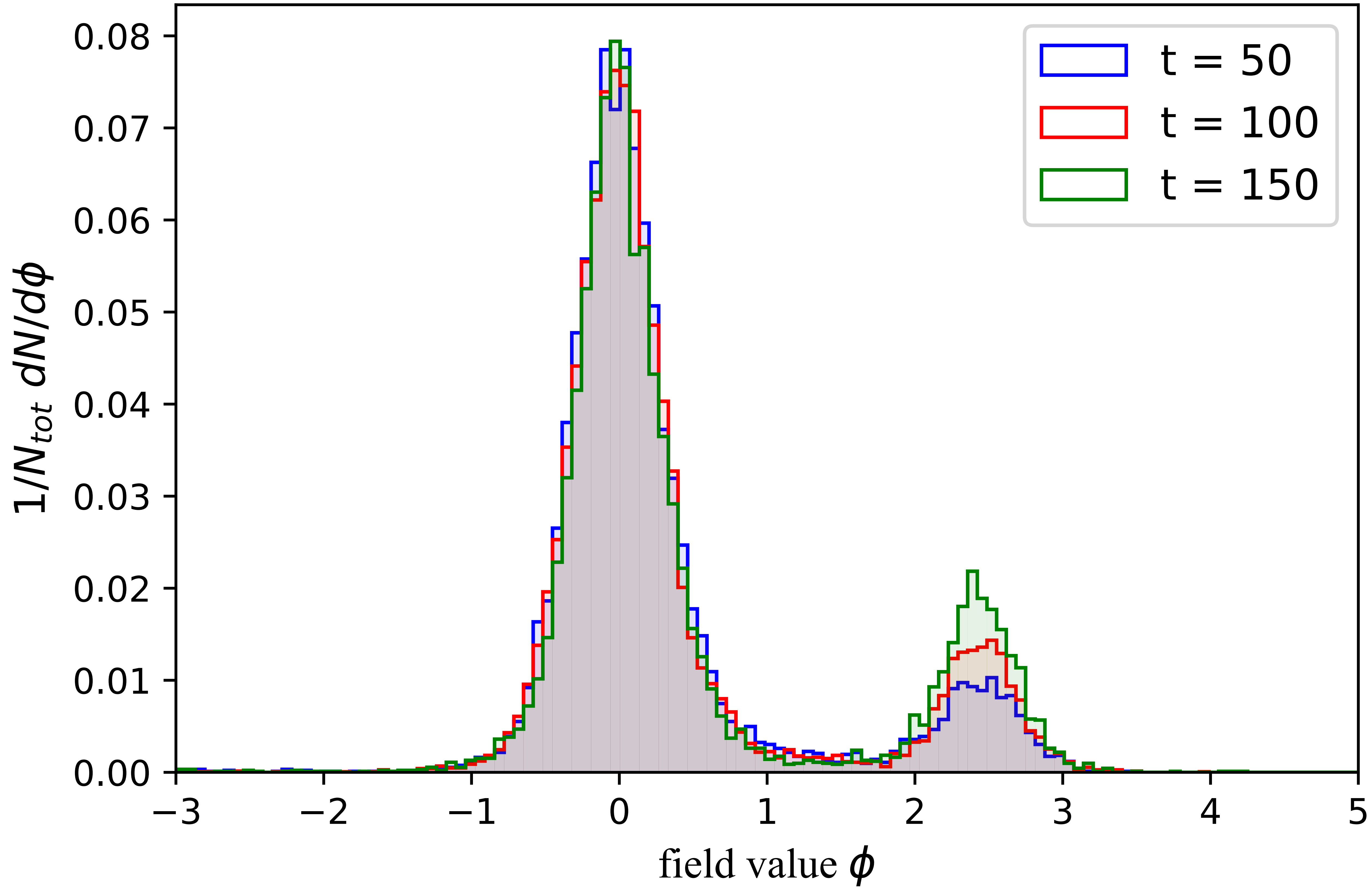


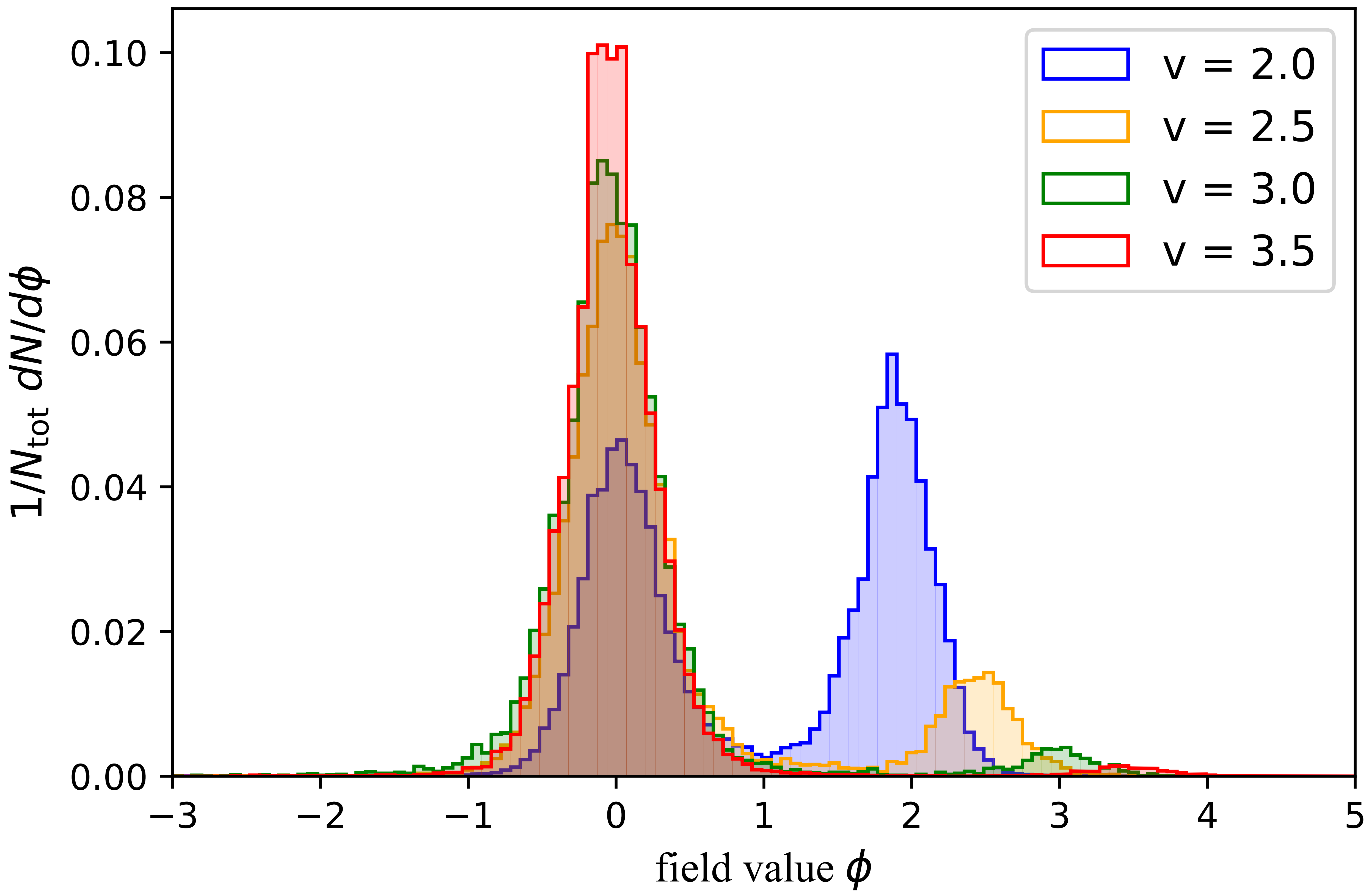
***Results: (reverse anneal with 200 qubits) we see tunnelling — e.g. at v=2.5***



***Note that this is literally an experimental measurement of the wave-function squared*** $|\psi(\phi)|^2$***!***



***Results: it appears to drop exponentially with v as predicted by WKB:***



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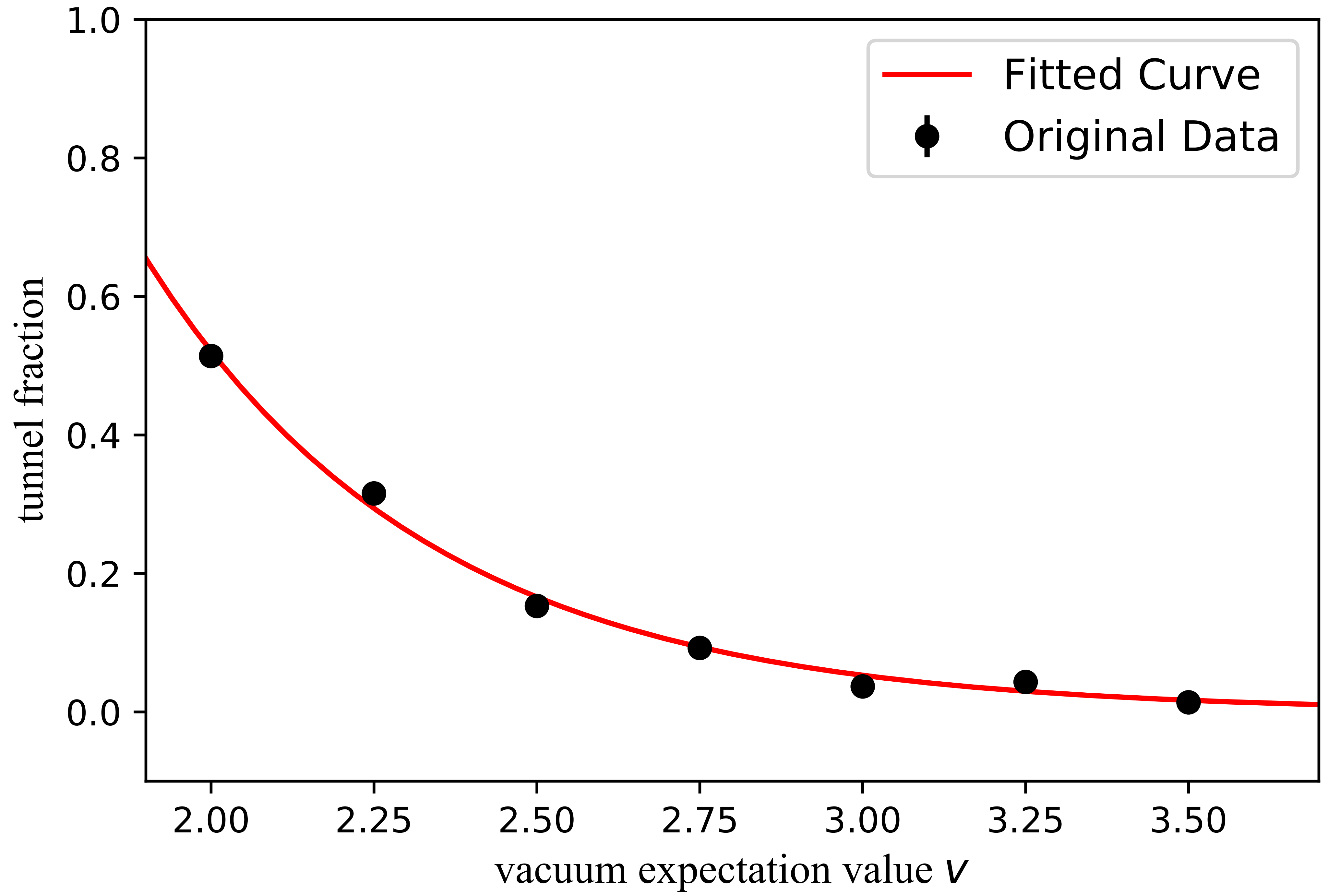
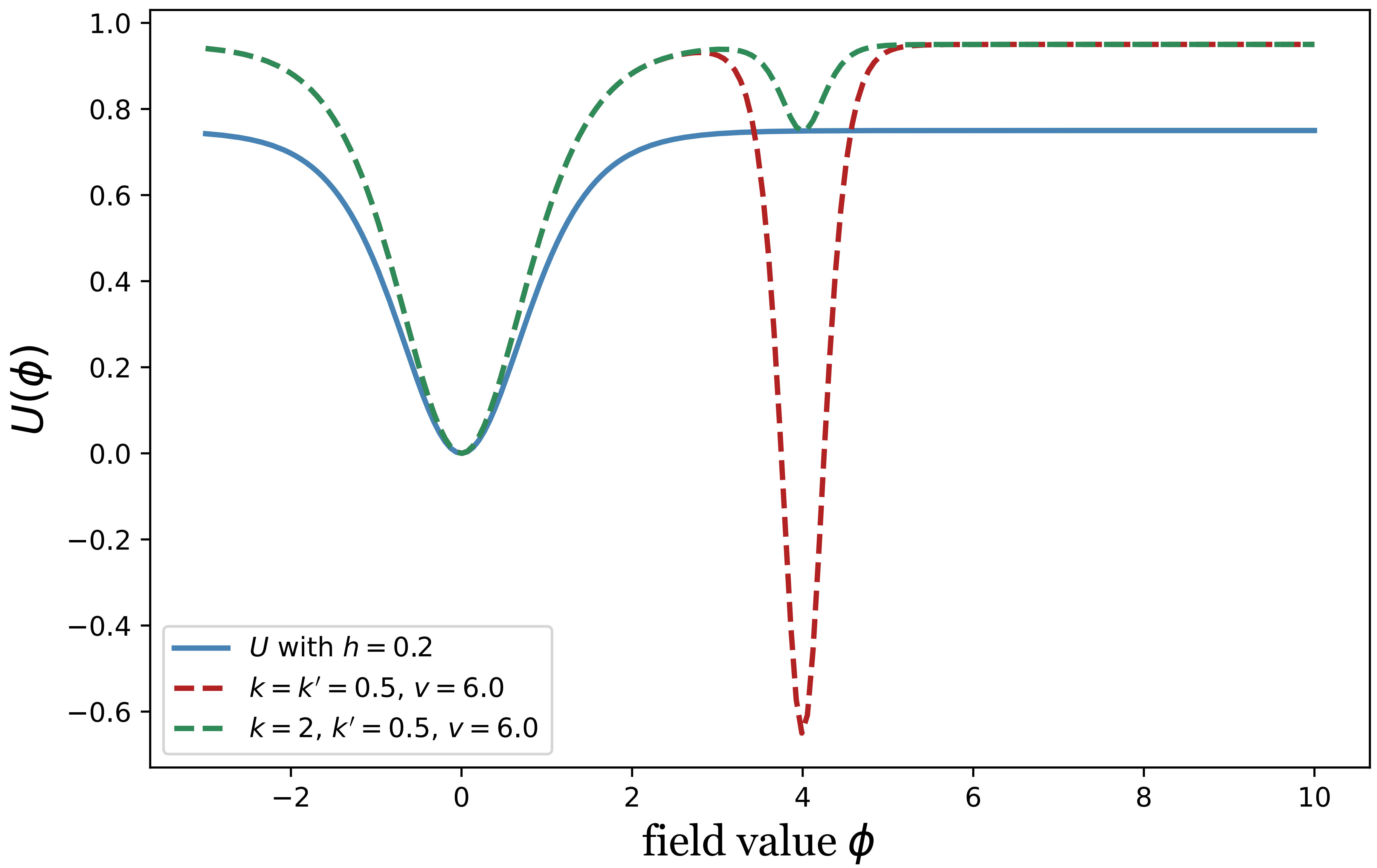



***Theory:*** $$\log\Gamma = 3.0 \times (1.66 - v)$$

***Exp:*** $$\log\Gamma = 2.29 \times (1.71 - v)$$

***Classical or Quantum?***

Could this be thermal excitation? Test with a maximally Thermal =\= Quantum set-up:




Begin in solid blue line, and turn on either the raised green potential or the deep red one.

Thermal tunnelling should give similar results. Quantum should be very different.

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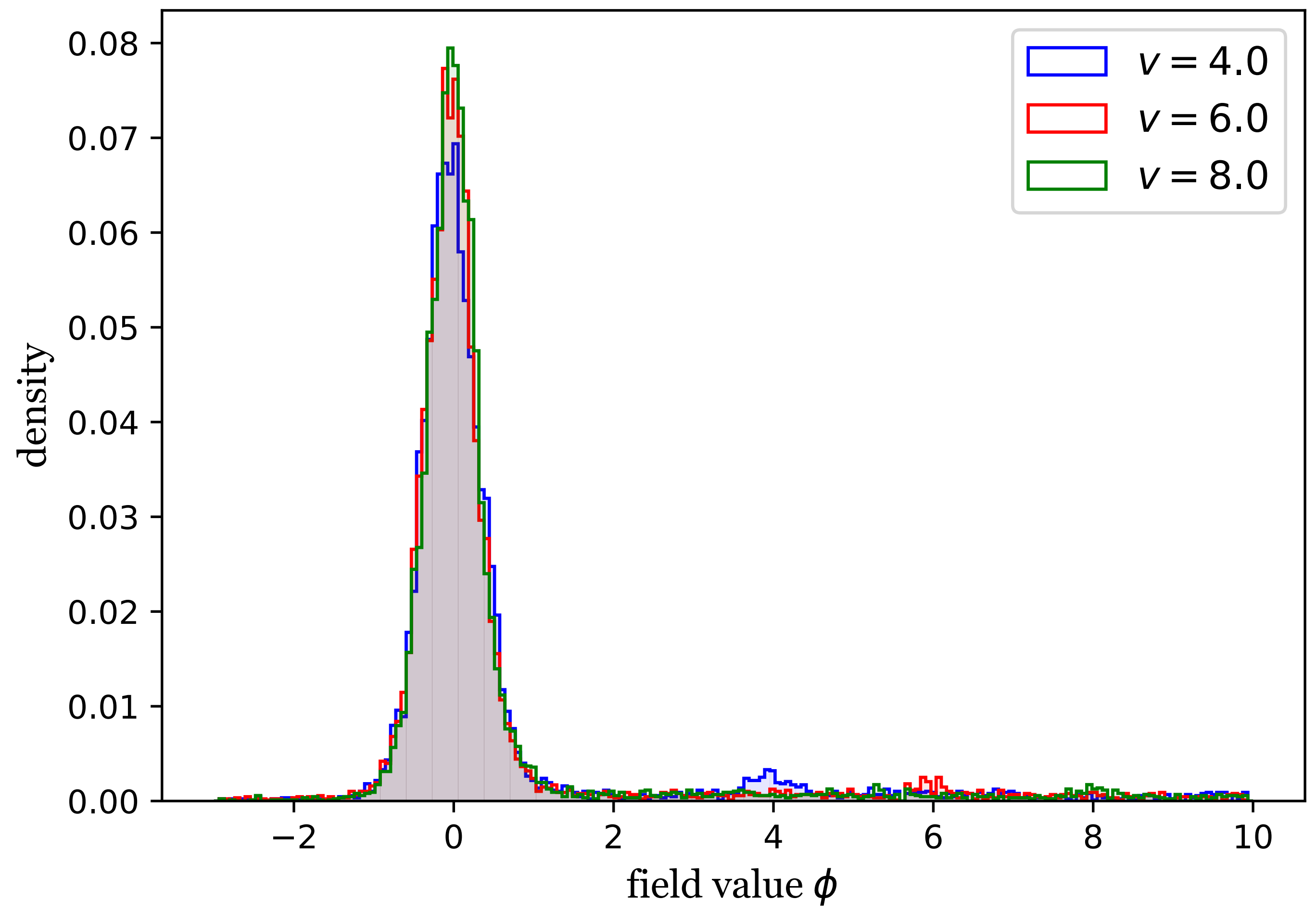
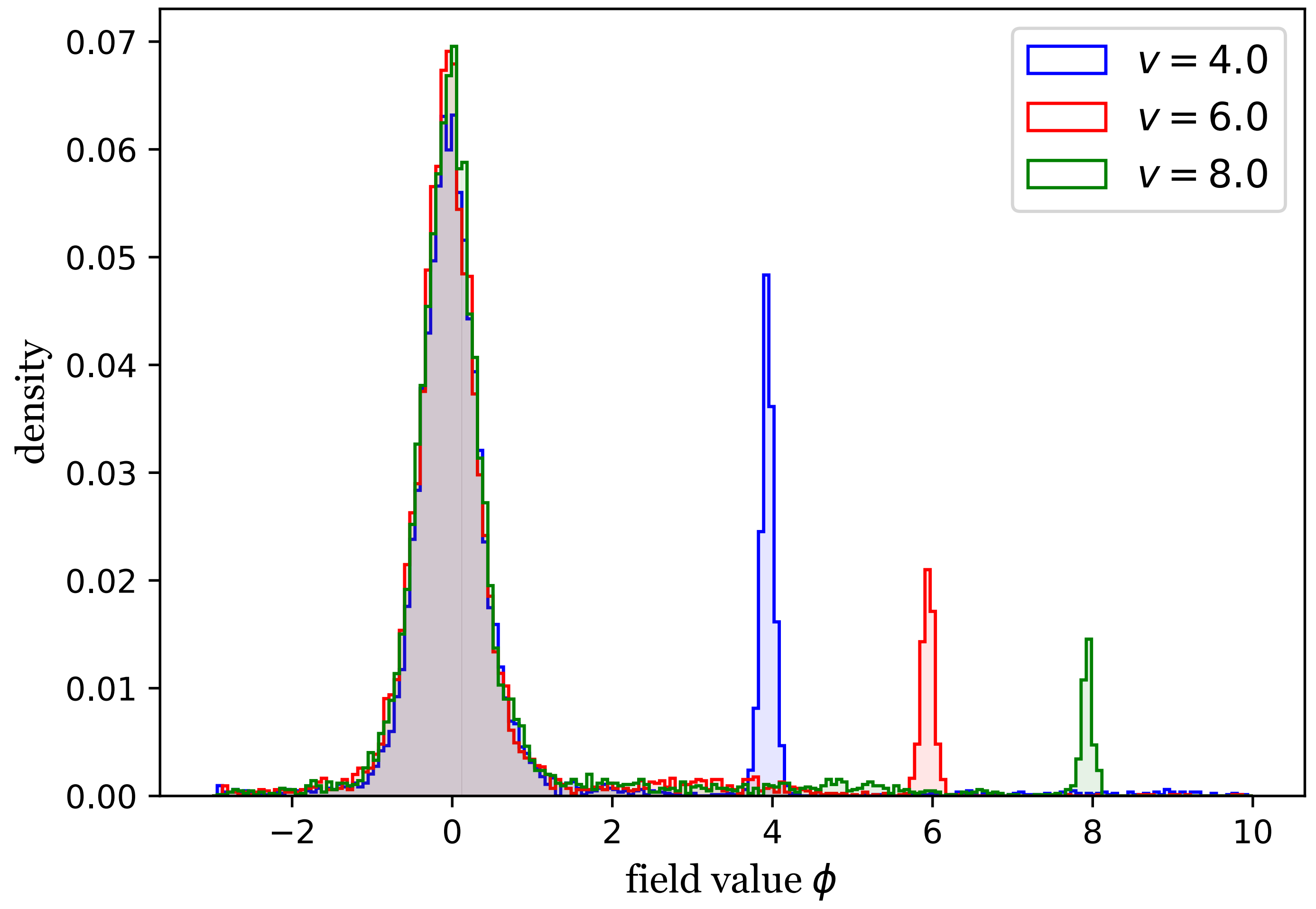
Raised minimum

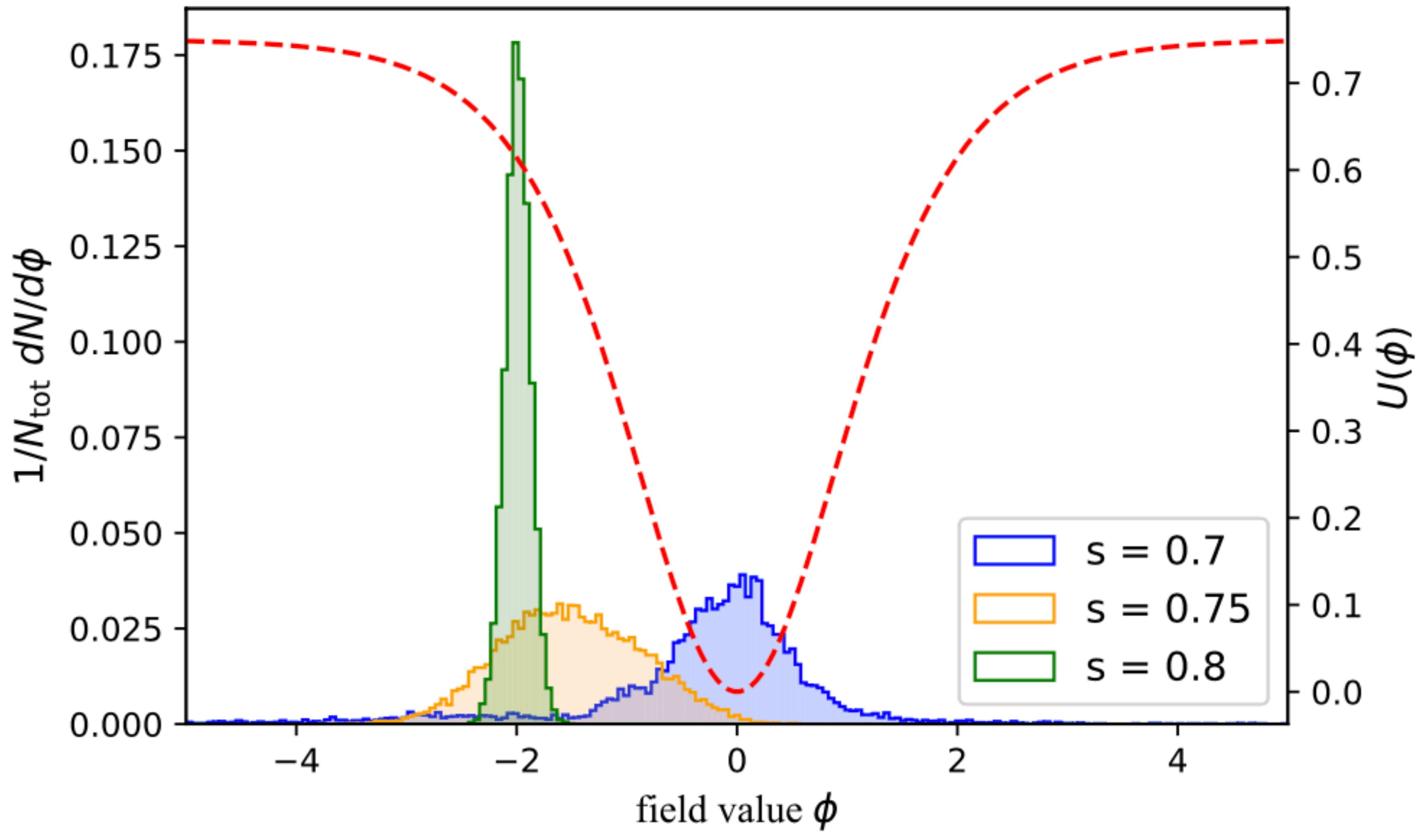



Deep minimum



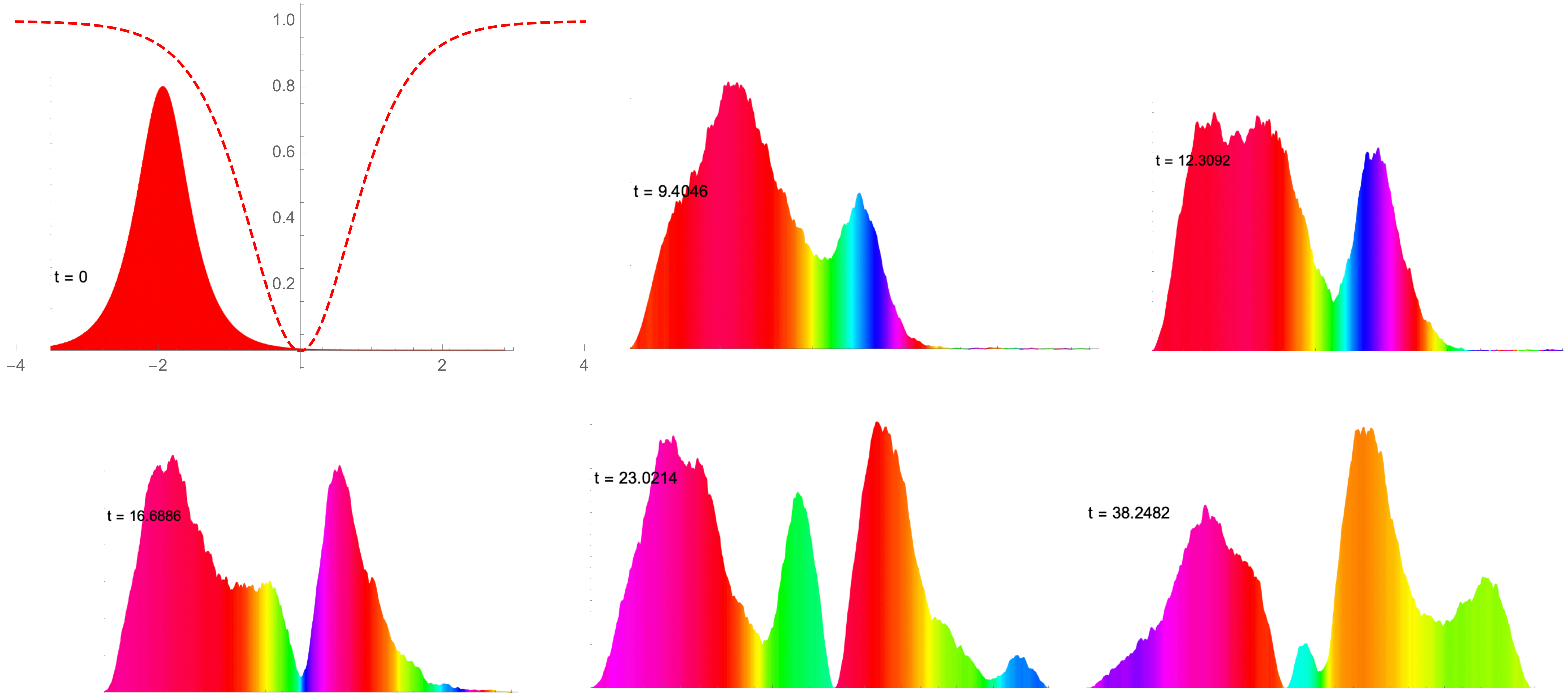
Several other checks this is genuinely quantum tunnelling and not thermal excitation. Simplest is to examine the dynamics: e.g. when we turn off the transverse field component the system won't even roll down a hill!

e.g. after t=180 $\mu$s we find the following if we start at -2:



Also dynamics has characteristic behaviour. For example it still "tunnels" to the bottom of a potential even if there is no barrier: i.e. the wave function leaks across, rather than rolling as a lump —

Numerically solving S.E. we find (this takes an hour!)



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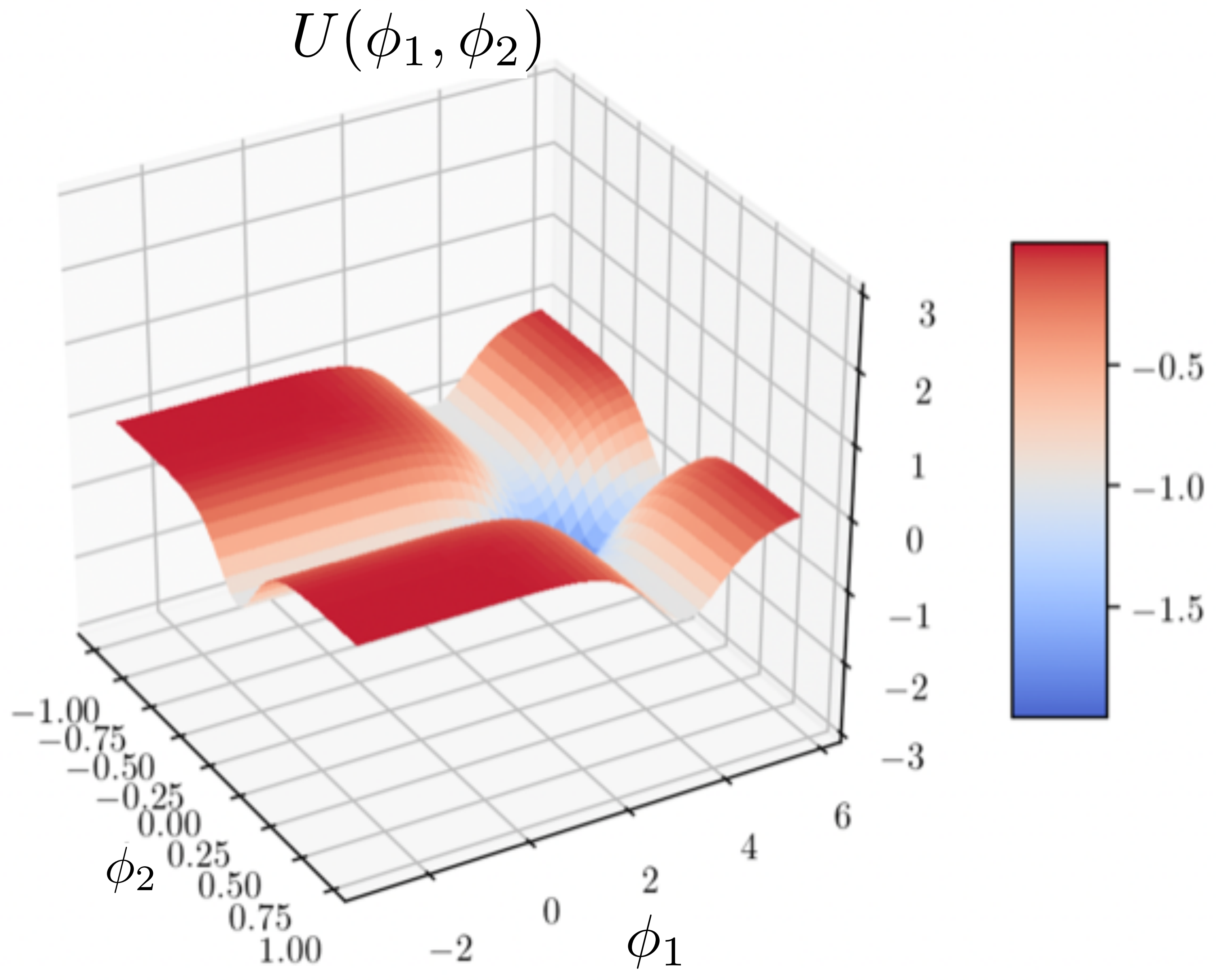
Multiple measurements on the quantum annealer:



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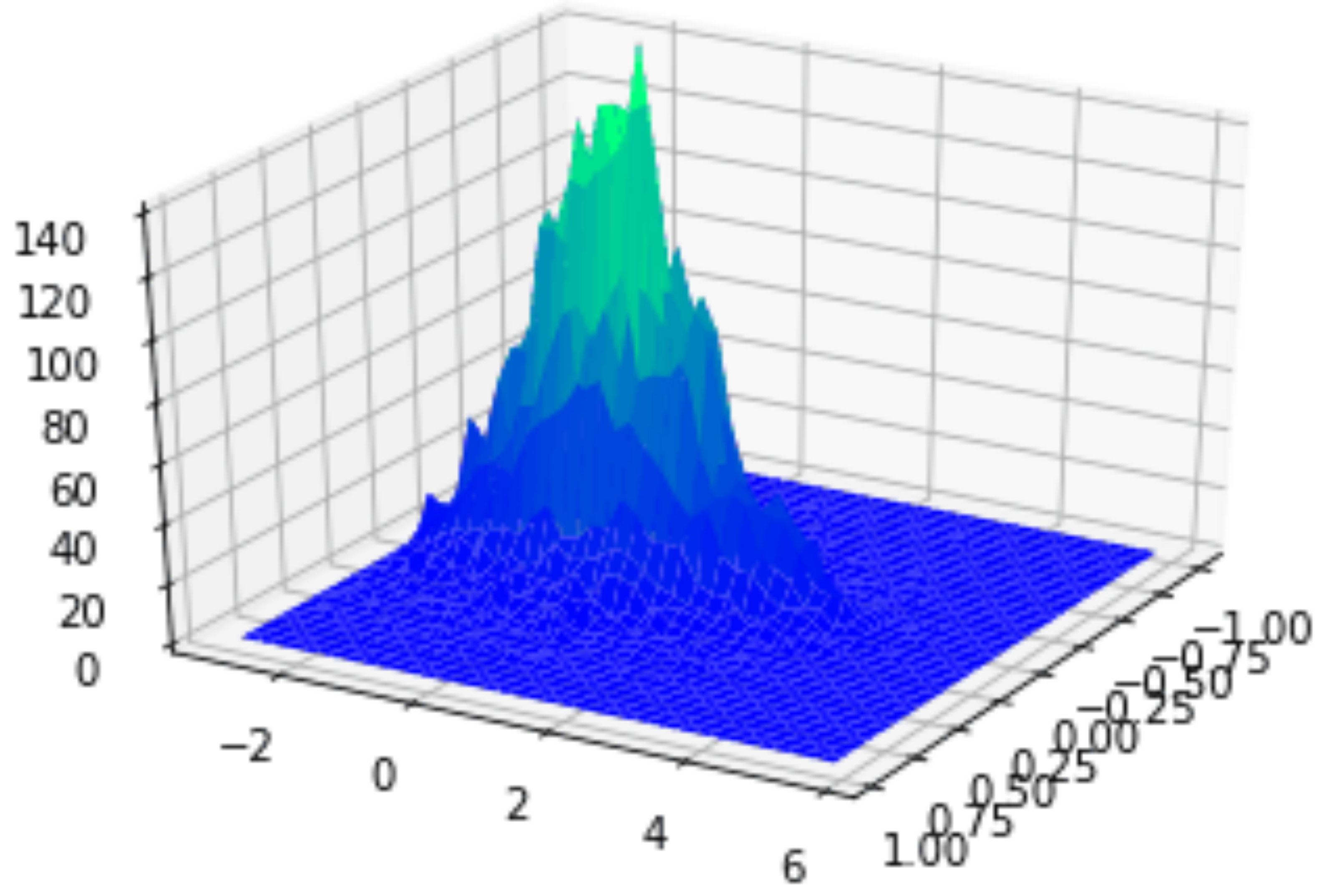

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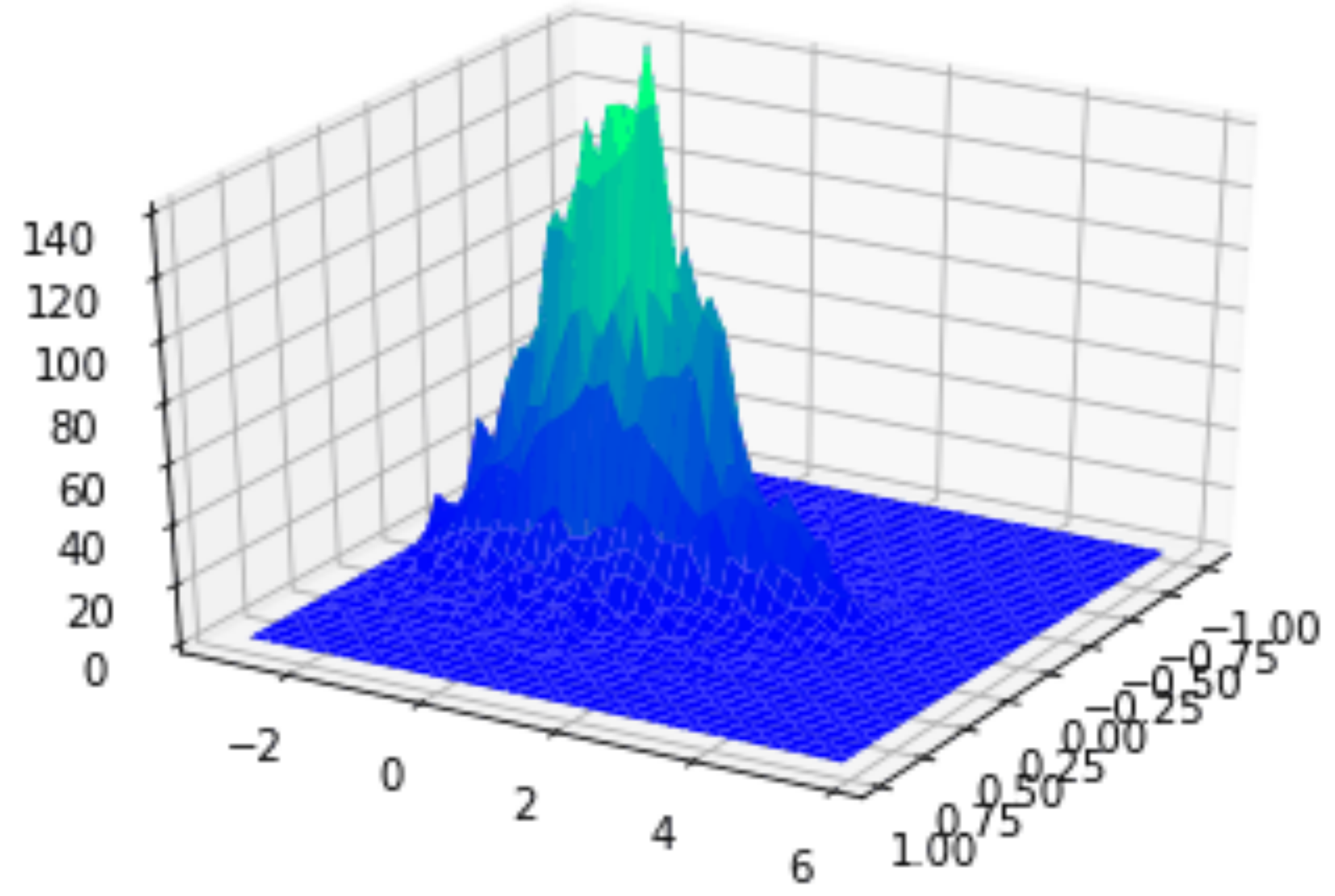


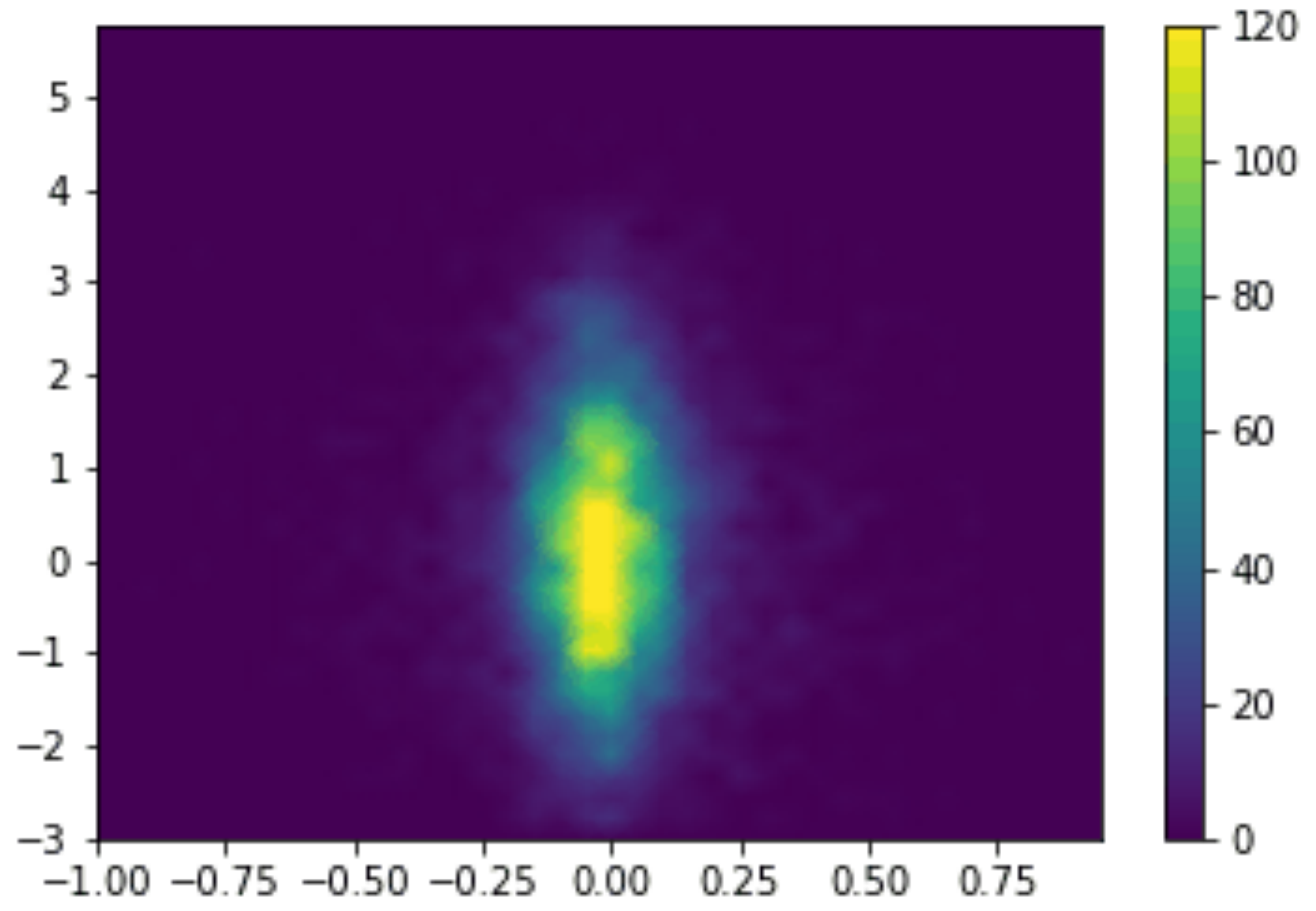

# *Conclusions*

- We have seen how the general Ising model can be used to encode QFT
- First instance of being able to build a QFT by hand in order to experimentally measure instanton and other processes in it
- Observed and measure genuine tunnelling out of false vacua (d=1 QFT)
- Behaviour is non-thermal (we are able to perform several easy tests by adjusting the potential)
- Provides a quantum lab for future tests of e.g. non-WKB situations, strongly coupled systems
- Gauge theories, more dimensions etc etc etc.