Dark matter in astrophysics/cosmology

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1. **A brief introduction to cosmology**
2. Observational evidence for dark matter
3. Dark matter distribution
4. Constraints on the properties of dark matter
5. Overspill + what would you like to hear more about?

nb 1 section ≠ 1 lecture

About these lectures

Aim is to provide you with the background knowledge of cosmology/astrophysics required for the other courses during the School, and also for carrying out research in the field of dark matter.

Introduction to each section contains a list of books and web-pages recommended for further reading.

For background material I've referenced books/reviews (more accessible than original papers). For more specific topics I've attempted to cite reviews, and/or the first and/or most recent papers. These references are intended to serve as 'jumping-off points' to the literature.

Depth of coverage is deliberately varied: 'key concepts' in detail, 'useful to be aware ofs' much more briefly.

In some cases topics will be covered in more detail later by experts (in the case of over-simplifications/discrepancies, trust the expert…).

Please ask questions, if you don't understand something or would like to know more.

I wish we could all be together in Les Houches in person (and I hope that those of you that are there get to enjoy the mountains…).

Questions

Questions on things which aren't clear:

during the pauses in the lectures
or (if it's urgent) raise your (real or virtual) hand to interrupt

Questions on technical details or extensions after/between lectures:

by email: anne.green@nottingham.ac.uk
or padlet: https://padlet.com/annegreen3/k33gx7aeoyggy61j

if I think the answer is of broad interest I'll share it on padlet or in a lecture.

A brief introduction to cosmology

A rapid ‘crash course’ for people who haven't already studied this material or a recap for those who have.

Evolution of the Universe

Thermal history

- Nucleosynthesis
- Cosmic microwave background

Structure formation

Gravitational lensing

Inflation

Type 1a supernovae (and the accelerated expansion of the Universe)

Recommended further reading

- Particle Data Group Review of Particle Physics
  - Big Bang cosmology, Olive & Peacock
  - Nucleosynthesis, Fields, Molaro & Sarkar
  - Cosmic microwave background, Scott & Smoot

Useful textbooks/lecture notes:

- ‘Theoretical astrophysics volume III: galaxies and cosmology’, Padmanabhan, Cambridge University Press
- ‘Introduction to cosmology’, Ryden, Cambridge University Press.
- ‘Lectures on cosmology’, Tong, University of Cambridge

Evolution of the Universe

Key equations and quantities

Friedmann equation: how the expansion of the Universe (Hubble parameter H) depends on its contents (energy density ε and cosmological constant Λ) and geometry (curvature parameter k).

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a = scale factor, value today, a_0, sometimes set to 1
. = d/dt

Here, and throughout, set c=1.

Subscript ‘0’ often used to denote value of quantity at the present day.

Hubble constant H_0: present day value of the Hubble parameter.

$$H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

From the Planck (and other ‘early’ measurements): h ≈ 0.68±0.004
From distance ladder measurements: h ≈ 0.73 ± 0.02

‘Hubble tension’ see e.g. Perivolaropoulos & Skara

Fluid equation: how density varies due to the expansion of the universe.

$$\dot{\epsilon} = -3H\left(\epsilon + p\right)$$

Equation of state: relationship between pressure and energy density $p = w\epsilon$

radiation, relativistic particles (photons, light neutrinos): w=1/3
matter, non-relativistic particles (baryons, dark matter): w=0
dark energy: w<-1/3
[cosmological constant: w=-1]

Critical density: density for which geometry of universe is flat (k=0)

$$\epsilon_{\mathrm{c}} = \frac{3H^2}{8\pi G}$$

Density parameter: density relative to the critical density $\Omega = \frac{\epsilon}{\epsilon_{\mathrm{c}}}$

Comoving coordinates: carried along with the expansion of the universe

$$\mathbf{R}(t) = a(t)\mathbf{x}$$

physical coordinate

comoving coordinate

Friedmann-Lemaitre-Robertson-Walker metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right]$$

Photons have $\mathrm{d}s^2 = 0$ → in a flat (k=0) universe $\mathrm{d}r = \frac{\mathrm{d}t}{a(t)}$

Redshift: wavelength of light grows with scale factor: $\lambda \propto a$

$$a = \frac{1}{1+z}$$

Horizons: how far light/relativistic particles can travel in a given time

$$d_{\mathrm{hor}} = a(t)\int_0^t \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})} \sim H^{-1}$$

For significant periods of time, the evolution of the Universe is dominated by a single component.

Matter dominated universe

Baryons, dark matter $p = 0$

From the fluid equation: $\epsilon \propto a^{-3}$

physically: number density ∝ a^{-3}

From the Friedmann equation: $a \propto t^{2/3}$

Cosmological constant (Λ) dominated universe

From the Friedmann equation: $a(t) \propto \exp\left(\sqrt{\frac{\Lambda}{3}}t\right)$

The cosmological constant can also be described as a fluid with $p = -\epsilon$
i.e. $w = -1$.

Dark energy

More generally accelerated expansion can be caused by a fluid with
equation of state: $p = w\epsilon$ $\quad w < -1/3$

Radiation dominated universe

Photons, neutrinos $p = \frac{1}{3}\epsilon$

From the fluid equation: $\epsilon \propto a^{-4}$

physically: number density $\propto$ a^{-3}

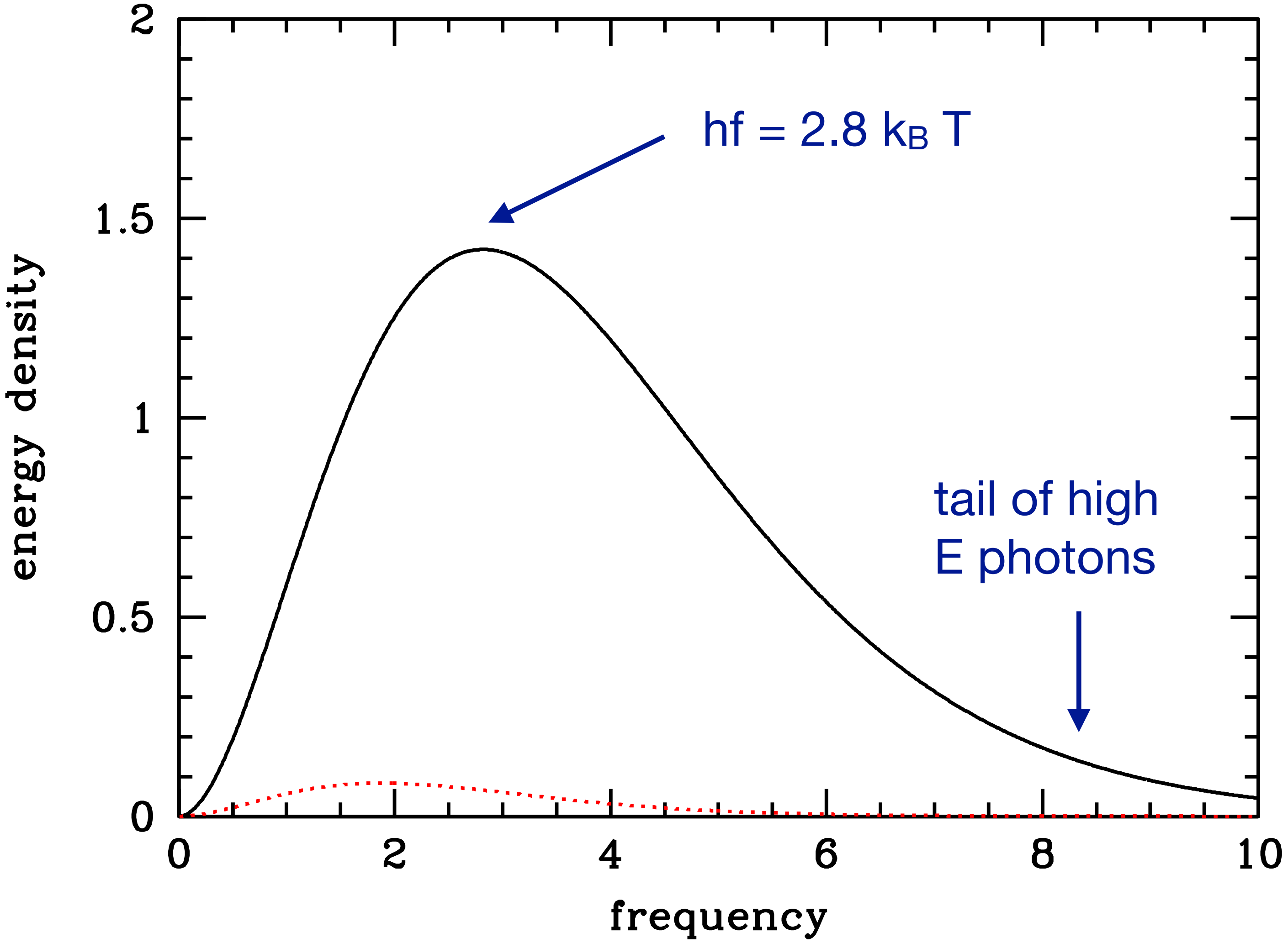
energy of individual photons $\propto$ a^{-1}, due to redshift

From the Friedmann equation: $a \propto t^{1/2}$

Photons have a blackbody distribution:

Planck constant

$$\epsilon(f)\,\mathrm{d}f = 8\pi h\frac{f^3\,\mathrm{d}f}{\exp\left(\frac{hf}{k_\mathrm{B}T}\right)-1}$$



total energy density: $\epsilon_\gamma = \alpha T^4$ α = radiation constant

present day number density: $n_{\gamma,0} = 4.1 \times 10^8\,\mathrm{m}^{-3}$ $\left(\frac{n_\gamma}{n_\mathrm{b}} \gg 1\right)$

Neutrino energy density: $$\epsilon_\nu = \left[3 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right] \epsilon_\gamma = 0.68\epsilon_\gamma$$

(3 species of light neutrino, are fermions, electron-position annihilation produces photons but not neutrinos)

In general: $$\epsilon_{\rm r} = \frac{\pi^2}{30} g_\star T^4$$

$g_\star$ = total number of effectively massless degrees of freedom

$$a \propto g_\star^{-1/4} T^{-1}$$

time-temperature relationship: $$\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2} \approx \left(\frac{k_{\mathrm{B}}T}{1\,\mathrm{MeV}}\right)$$

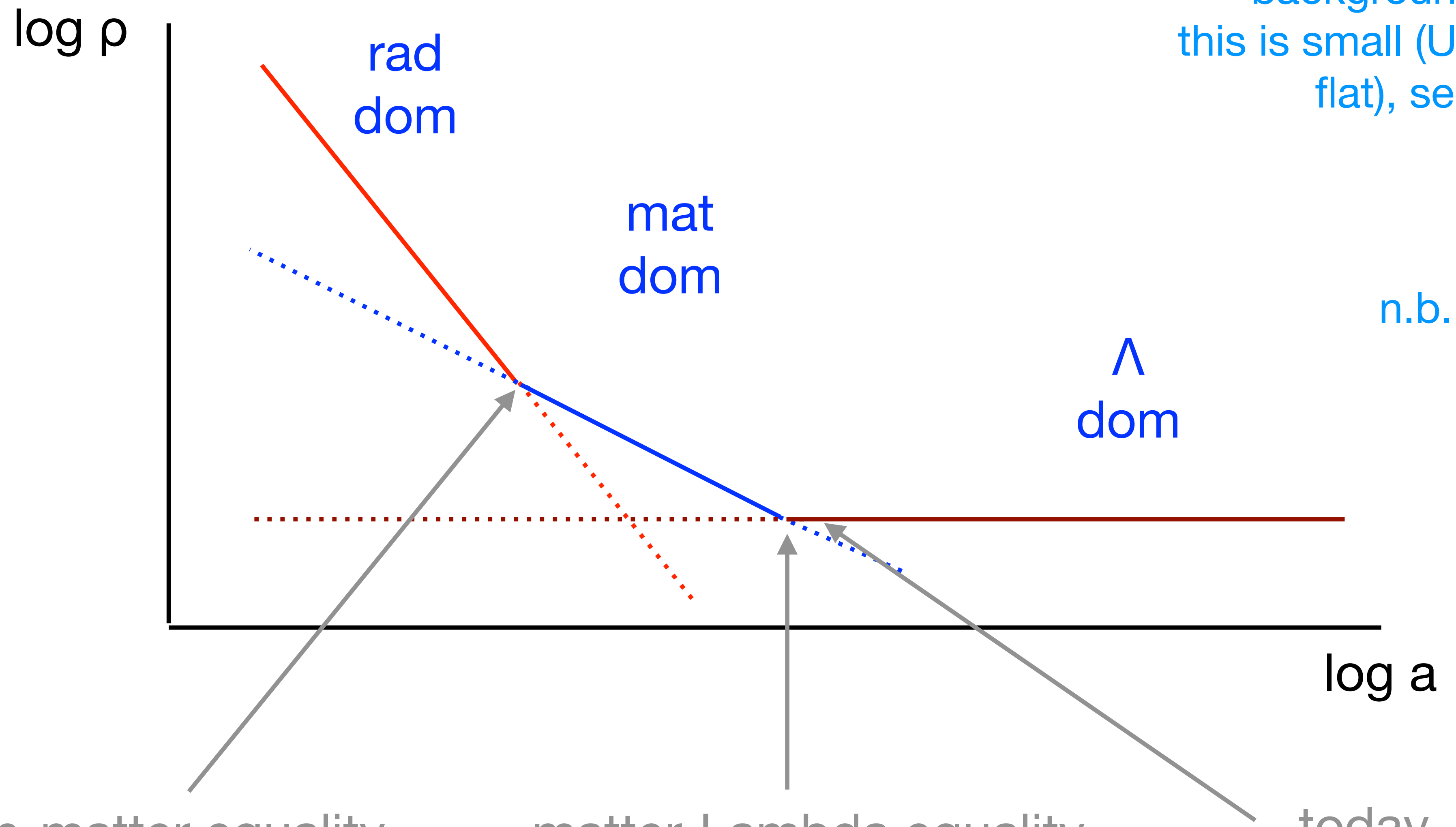
In general

$$H^2 = H_0^2 \left[\Omega_{\mathrm{r},0}(1+z)^4 + \Omega_{\mathrm{m},0}(1+z)^3 + \Omega_{\mathrm{k},0}(1+z)^2 + \Omega_{\Lambda,0}\right]$$

$$\Omega_k = -\frac{k}{H^2} \qquad \Omega_\Lambda = -\frac{\Lambda}{H^2}$$

from the cosmic microwave background anisotropies this is small (Universe is close to flat), see section 2

n.b. slopes not to scale



radiation-matter equality
t ~ 0.05 Myr

matter-Lambda equality
t ~ 10 Gyr

today
t_0 ~ 13.7 Gyr
(for best fit ΛCDM model)

Thermal history

(Big Bang) Nucleosynthesis

The synthesis of the nuclei of the light elements (D, ^{3}He, ^{4}He and Li) seconds to minutes after the Big Bang.

Before t~ 1 s ($k_B T$ ~ 1 MeV) protons and neutrons are kept in thermal equilibrium by weak interactions:

$$\mathrm{n} + \nu_\mathrm{e} \leftrightarrow \mathrm{p} + \mathrm{e}^-$$

$$\mathrm{n} + \mathrm{e}^+ \leftrightarrow \mathrm{p} + \bar{\nu}_\mathrm{e}$$

Relative number densities:

$$\frac{N_\mathrm{n}}{N_\mathrm{p}} \approx \exp\left(-\frac{(m_\mathrm{n} - m_\mathrm{p})}{k_\mathrm{B} T}\right)$$

$$k_\mathrm{B} T \gg (m_\mathrm{n} - m_\mathrm{p}) = 1.3\,\mathrm{MeV}\,, \quad N_\mathrm{n} \sim N_\mathrm{p}$$

$$k_\mathrm{B} T \not\gg 1.3\,\mathrm{MeV}, \qquad N_\mathrm{n} < N_\mathrm{p}$$

Once $k_B T_{fo}$ ~ 0.8 MeV the timescale on which these reactions occurs becomes longer than the age of the Universe, so they effectively cease ('freeze-out') when:

$$\frac{N_\mathrm{n}}{N_\mathrm{p}} \approx 0.20$$

Production of the nuclei of the light elements starts through a chain of reactions:

$$
\begin{gathered}
\mathrm{p} + \mathrm{n} \to \mathrm{D} \\
\mathrm{D} + \mathrm{p} \to {}^{3}\mathrm{He} \\
\mathrm{D} + \mathrm{D} \to {}^{4}\mathrm{He} \\
\vdots
\end{gathered}
$$

Until T drops to $k_B T_{nuc} \sim 0.1$ MeV high E tail of photon distribution destroy nuclei (and free neutrons decay into protons, so the neutron to proton ratio drops to $N_n/N_p \sim 0.18$).

The majority of the remaining neutrons form ^{4}He, with trace amounts of other light nuclei. Remaining p are H.

mass fractions:

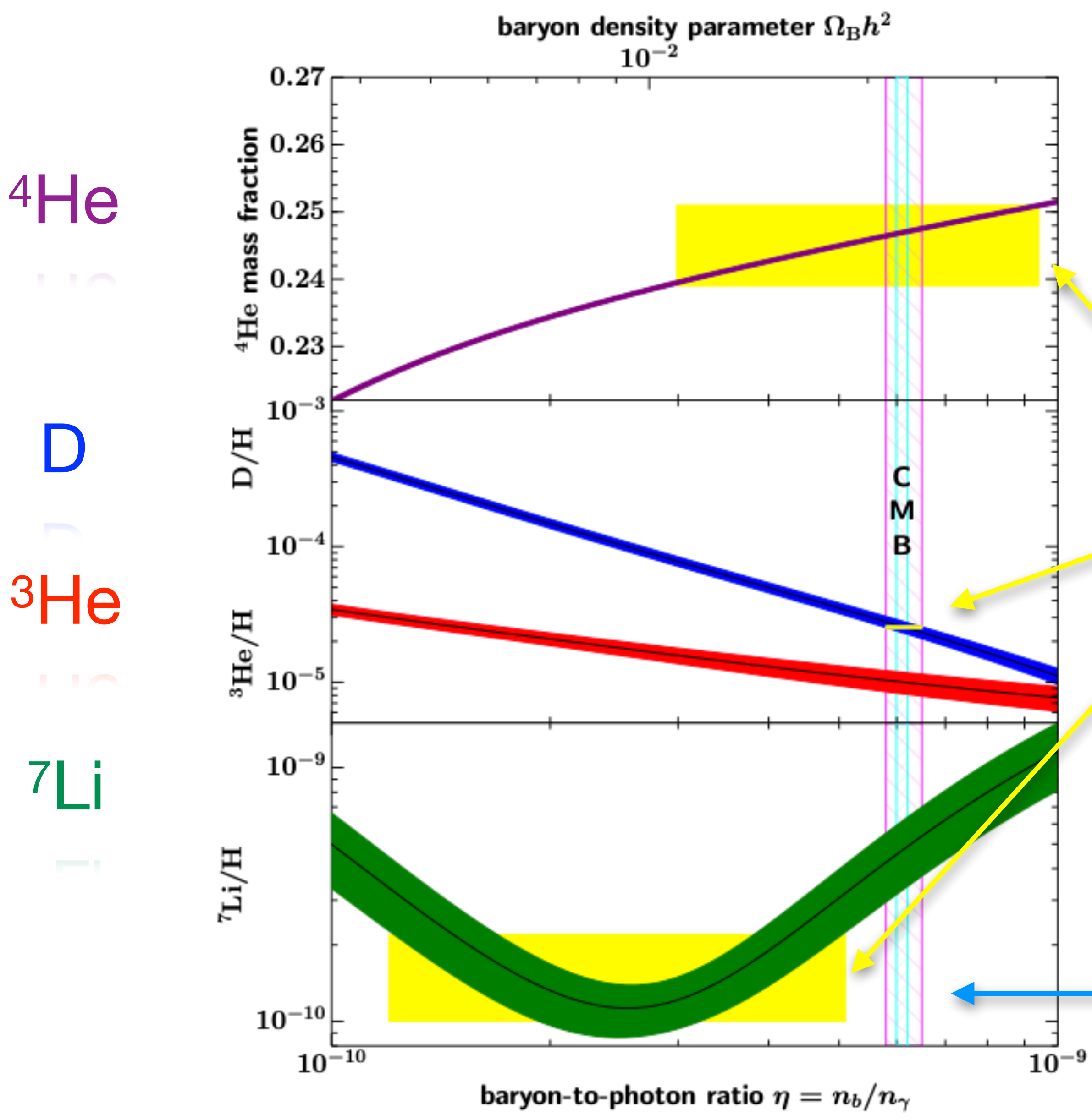
$$Y_{^4\mathrm{He}} = 0.23 - 0.24$$

$$Y_\mathrm{D} \approx 10^{-4} \qquad Y_{^3\mathrm{He}} \approx 10^{-5} \qquad Y_{^7\mathrm{Li}} \approx 10^{-10}$$

Exact abundances depends on the photon to baryon ratio (or equivalently, since the photon number density is known from the CMB temperature) the abundance of baryons.

abundances of the light elements

baryon density



baryon-to-photon ratio

Lines: theoretical predictions (thickness of lines denotes nuclear physics uncertainties)

Yellow boxes: observations.

^{4}He emission lines of metal poor galaxies

D absorption of quasar light by primordial gas clouds.

^{7}Li metal poor stars (“cosmological lithium problem”)

Blue vertical band: baryon density determined by CMB observations.

Fields, Molaro & Sarkar.

$$0.021 \le \Omega_b h^2 \le 0.024$$

Cosmic Microwave Background (CMB)

At high T photons have energy greater than the ionisation energy of hydrogen (13.6 eV) and Universe composed of nuclei and electrons.

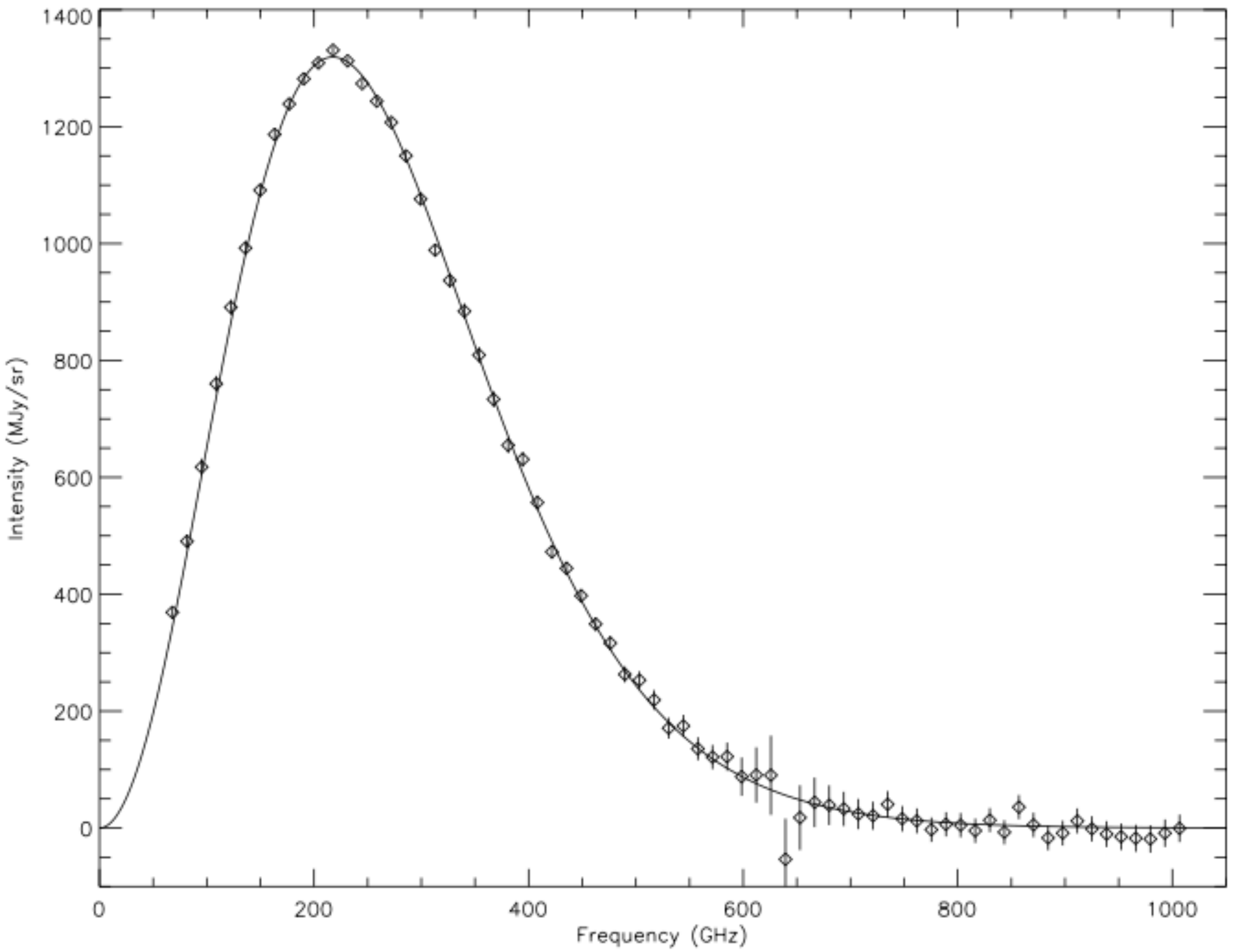
Mean free path of photons is very short, & Universe contains an ionised plasma of frequently colliding particles.

As Universe expands and cools energy of photons drops and atoms form.
Recombination, t ~ 0.25 Myr, $k_B T$ ~ 0.32 eV (< (13.6/2.8) eV since $n_\gamma \gg n_b$).

Photons stop scattering and free-stream.
Decoupling/last scattering, t ~ 0.37 Myr, $k_B T$ ~ 0.26 eV

Spectrum of CMB is a blackbody with: $T_0 = (2.7255 \pm 0.0006)\,\mathrm{K}$

CMB intensity as a function of frequency

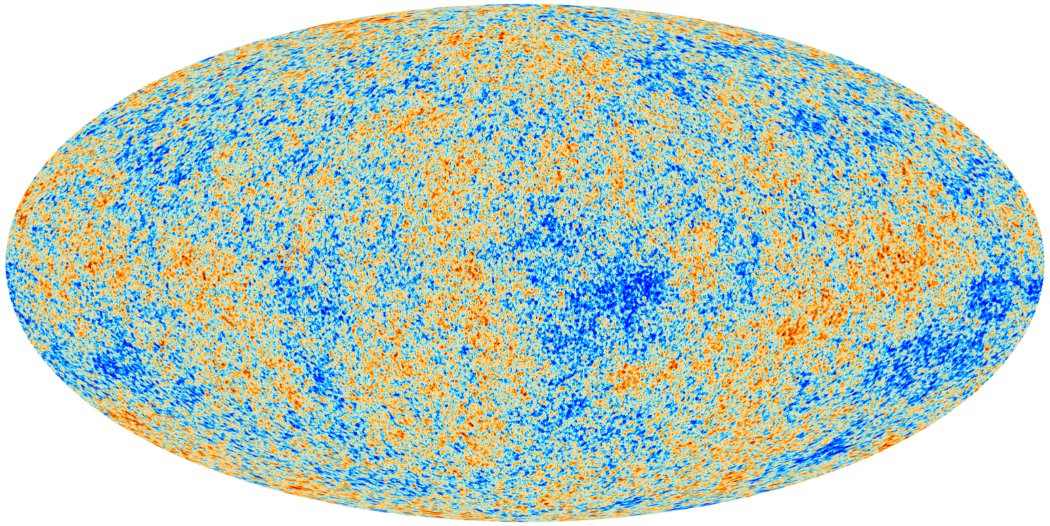


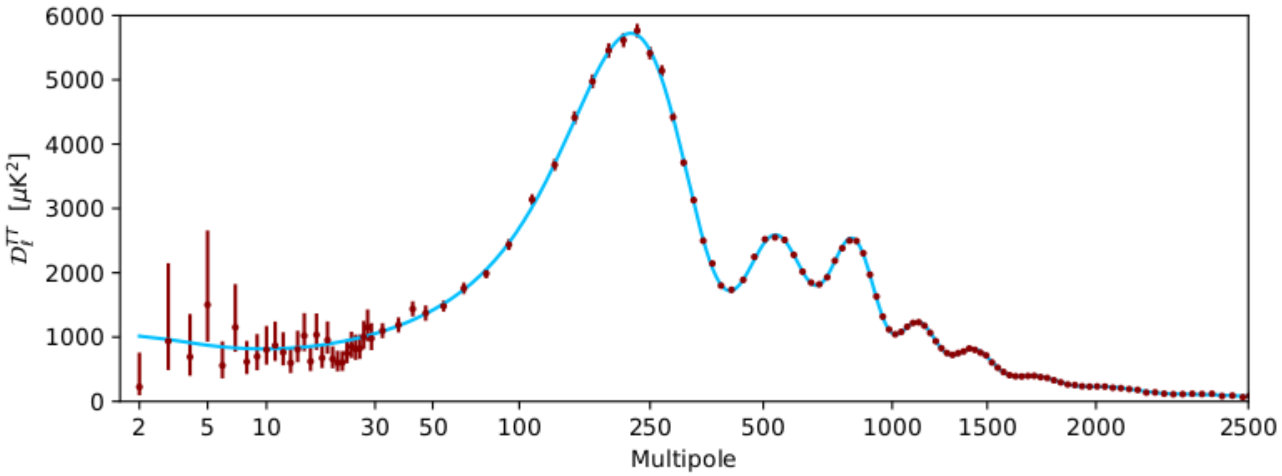
Fixen

CMB temperature anisotropies (more in section 2)

Structure formation occurs due to the growth of initially small density perturbations, these perturbations also lead to anisotropies in the temperature of the CMB.

Fluctuation distribution depends on spectrum of primordial perturbations and also contents of Universe (because of growth of fluctuations and also projection of length scales onto sky).





Planck 2018

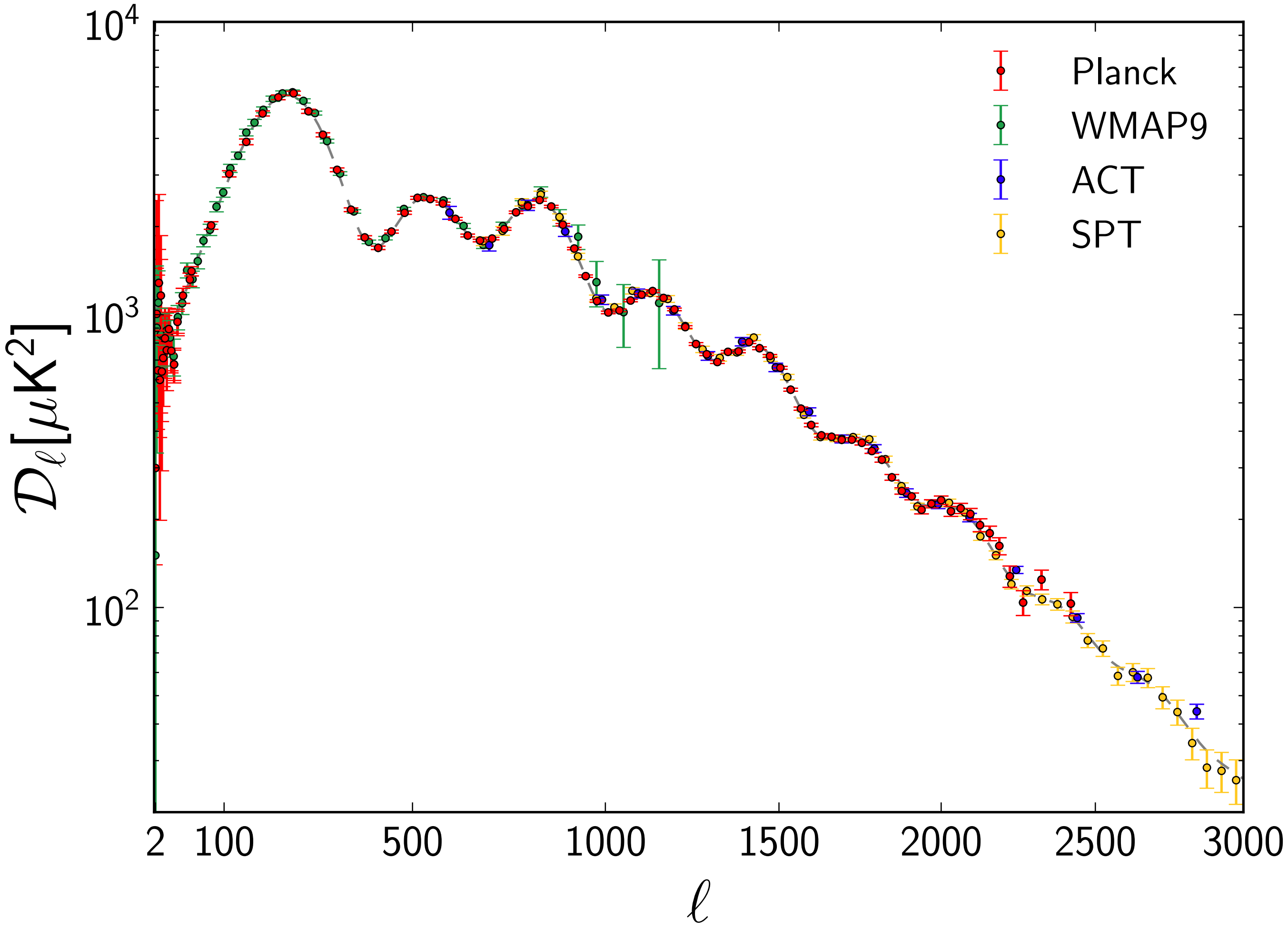
Expand temperature fluctuations in spherical harmonics:

$$\frac{\Delta T(\theta,\phi)}{\bar{T}} \equiv \frac{T(\theta,\phi)-\bar{T}}{\bar{T}} = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\phi)$$

Angular power spectrum:

$$C_l = \langle |a_{lm}|^2 \rangle$$

Temperature power spectrum (TT)



n.b. log scale

$$l \propto \frac{1}{\theta}$$

Planck 2013

Several characteristic regions

The 'Sachs-Wolfe' plateau, low l:
temperature variations arise from variations in the gravitational potential.

The acoustic (or Doppler) peaks, intermediate l:
from oscillations in photon-baryon fluid due to competition between gravity and pressure (due to interactions between photons and electrons), with potential wells.

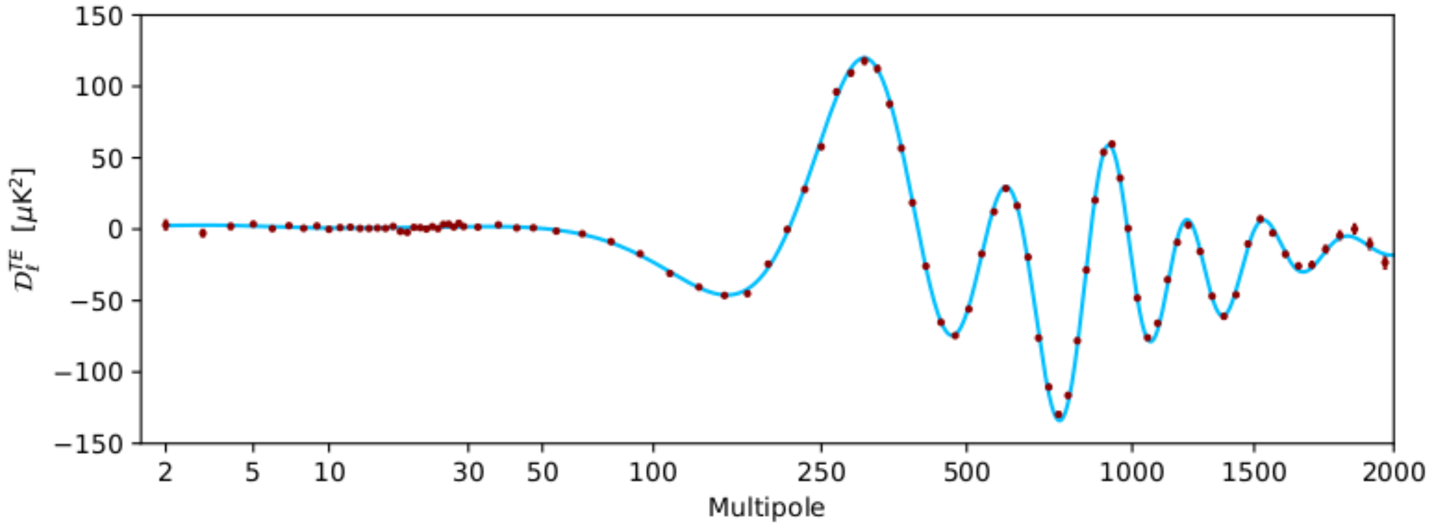
The Silk damping tail, high l:
due to diffusion of photons during the recombination process, fluctuations on small scales are damped.

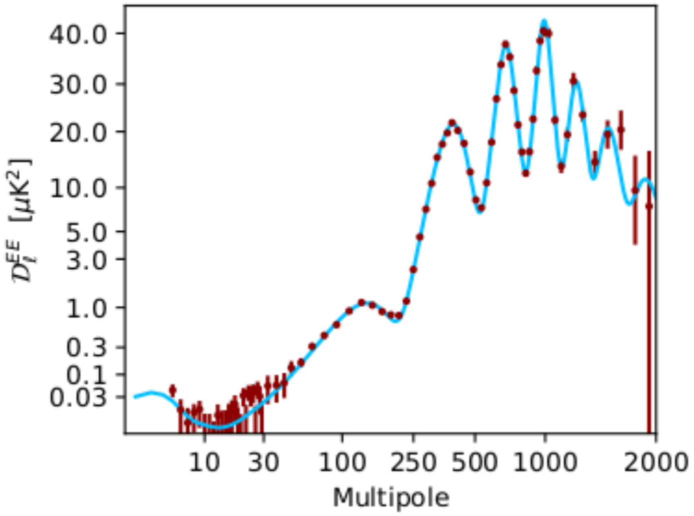
In addition to temperature anisotropies:

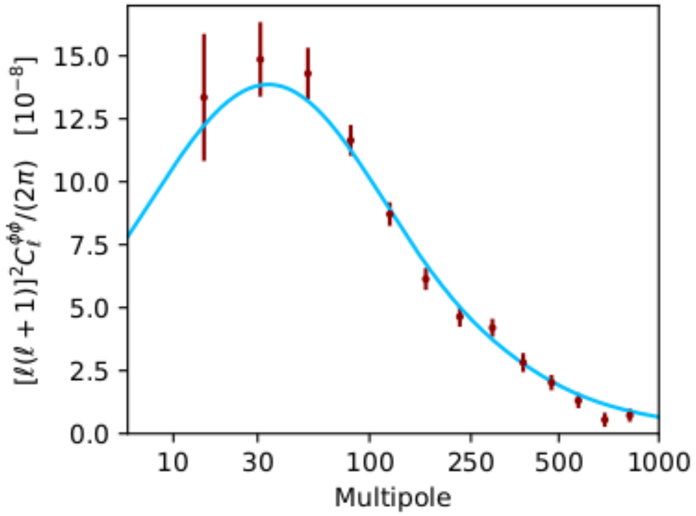
Polarization: E mode (Thomson scattering of photons off free charged particles) B mode (lensing of E modes, dust, tensor modes)

Lensing: photons deflected by gravitational potentials, smooths out acoustic peaks

Temperature-E mode polarisation cross correlation (TE)







E mode polarisation power spectrum (EE)

Lensing

Planck 2018

Other events

Electroweak phase transition

$k_B\,T \approx 100$ GeV

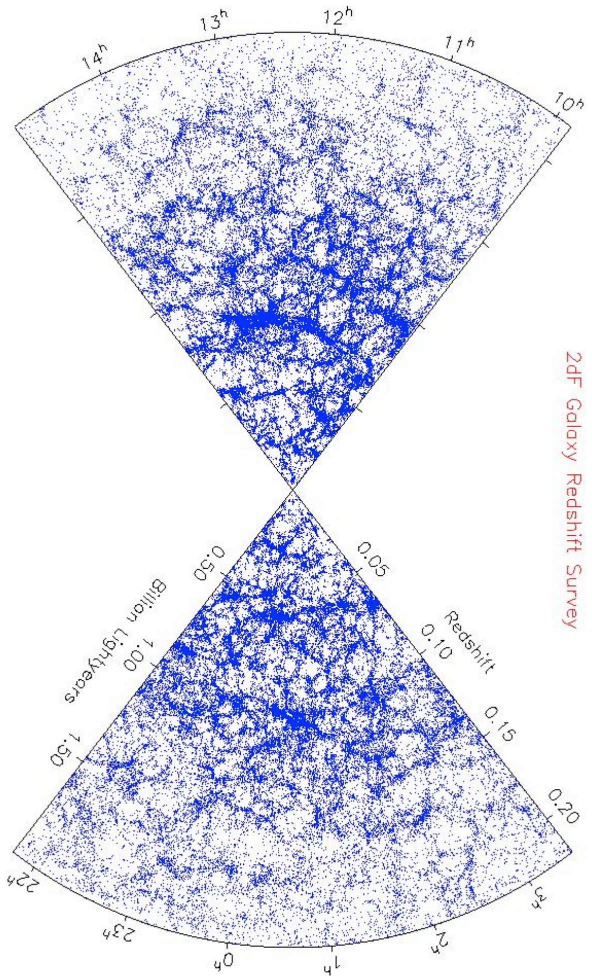
QCD phase transition

$k_B\,T \approx 150$ MeV, quark-gluon plasma → protons and neutrons

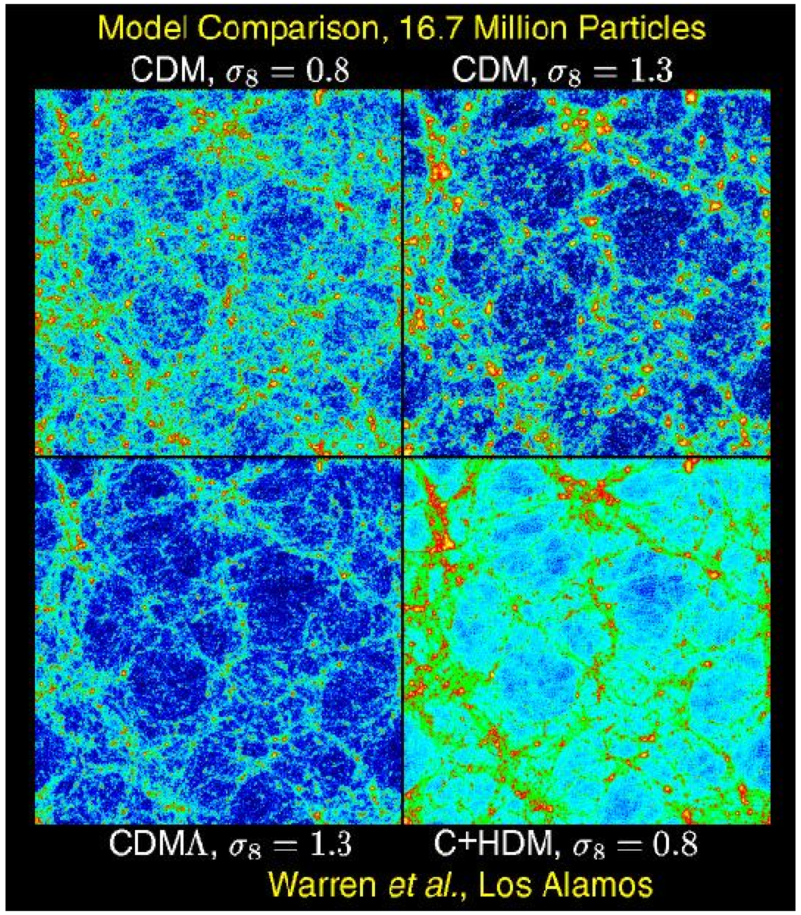
Structure formation

Structures (galaxies, galaxy clusters) in the Universe form from gravitational instability: small perturbations in the density grow with time.

Growth of perturbations, and hence clustering of galaxies, depends on contents of Universe.



2dF galaxy red-shift survey



Simulations

n.b. current state of the art simulations have much, much better resolution (more particles) but aren't carried out for 'wrong' cosmologies

Horizon entry:

Perturbations can only grow once a scale is smaller than the horizon scale.

A given scale with comoving wavenumber k ‘enters the horizon’ when

$$k^{-1} = \frac{H^{-1}}{a}$$

$$k = aH$$

Warning! this is a somewhat qualitative outline c.f. Ryden textbook, for a more rigorous analysis see e.g. Tong lecture notes

Jeans instability

first consider over-dense sphere in pressure-less, static universe: $\rho = \bar{\rho}(1+\delta)$

density perturbation $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

by considering gravitational acceleration at sphere's surface plus mass conservation:

$$\ddot{\delta} = 4\pi G\bar{\rho}\delta \longrightarrow \delta(t) = A_1 \exp\left(t/t_{\rm dyn}\right) + A_2 \exp\left(-t/t_{\rm dyn}\right)$$

$$t_{\rm dyn} = \frac{1}{(4\pi G\bar{\rho})^{1/2}}$$

pressure will resist collapse, but takes time $t_{\rm pre} \sim \dfrac{R}{c_{\rm s}}$

c_s = sound speed $c_{\rm s} = (\mathrm{d}p/\mathrm{d}\epsilon)^{1/2} = \sqrt{w}$

perturbations will grow if $t_{\rm pre} > t_{\rm dyn}$

$$R > \lambda_{\rm J} \sim c_{\rm s} t_{\rm dyn} \sim c_{\rm s}\left(\frac{1}{G\bar{\rho}}\right)^{1/2}$$

Jeans length

expanding universe

on sub-horizon scales:

[horizon= maximum distance light/particles can have travelled since Big Bang, perturbations on super-horizon scales can't grow due to causality]

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}\Omega_{\rm m} H^2 \delta = 0$$

during radiation domination ($\Omega_{\rm m} \ll 1, a \propto t^{1/2}$):

$$\delta(t) = B_1 + B_2 \ln t$$

during matter domination ($\Omega_{\rm m} \approx 1, a \propto t^{2/3}$):

$$\delta(t) = D_1 t^{2/3} + D_2 t^{-1}$$

Dark matter perturbations can grow from radiation-matter equality.

Before decoupling baryons are tightly coupled to photons and have $c_{\rm s} = 1/\sqrt{3}$, and hence baryonic perturbations can't grow.

After decoupling, baryons 'fall into potential wells' created by dark matter.

→ evidence for DM from amplitude of CMB anisotropies (see section 2)

Cold dark matter (CDM): decouples when non-relativistic (and v ≈ 0 today).

Hot dark matter (HDM): decouples when relativistic (and v ≉ 0 today).

effect of hot dark matter on density perturbations

HDM becomes non-relativistic when $3k_{\rm B}T_{\rm HDM} \approx m_{\rm HDM}$, prior to this it free streams and erases density fluctuations on length scales smaller than $d_{\rm min,HDM} = t_{\rm HDM}$.

For $m_{\rm HDM} > 2.4\,{\rm eV}$ (so that this happens during rad dom), this corresponds to a mass

$$M_{\rm min,HDM} \approx 10^{16}\,M_{\odot}\left(\frac{m_{\rm HDM}}{3\,{\rm eV}}\right)^{-3}$$

aside: a similar thing happens for (thermal relic) CDM but, because of its much smaller velocity dispersion, on much smaller scales

$$M_{\rm min,CDM} \sim 10^{-6}\,M_{\odot}$$

Hofmann, Schwarz & Stocker

power spectrum

Fourier transform density perturbations: $\delta_k = \frac{1}{V} \int \delta(\mathbf{r}) \exp(i\mathbf{k}.\mathbf{r})\, \mathrm{d}^3\mathbf{r}$

Power spectrum: $P(k) \equiv \langle |\delta_k|^2 \rangle$

$$P(k,t) = \frac{2\pi^2}{k^3} T^2(k,t) \mathscr{P}(k)$$

$T(k,t)$ transfer function, describes evolution of density perturbations

$\mathscr{P}(k)$ primordial power spectrum, usually assumed to have form

$$\mathscr{P}(k) = A_\mathrm{s} \left(\frac{k}{k_\star} \right)^{n_\mathrm{s}-1}$$

n_s = scalar spectral index [n_s =1 scale invariant, Harrison-Zeldovich]

A_s = amplitude

From Planck: $n_\mathrm{s} = 0.959 \pm 0.006\,, \quad A_\mathrm{s} = 2.2 \times 10^{-9}$

$k_\star = 0.05\,\mathrm{Mpc}^{-1}$

mass variance (typical amplitude of perturbations on scale R):

$$\sigma^2(R,t) = \frac{1}{2\pi^2}\int W_R^2(k)P(k,t)k^2\mathrm{d}k$$

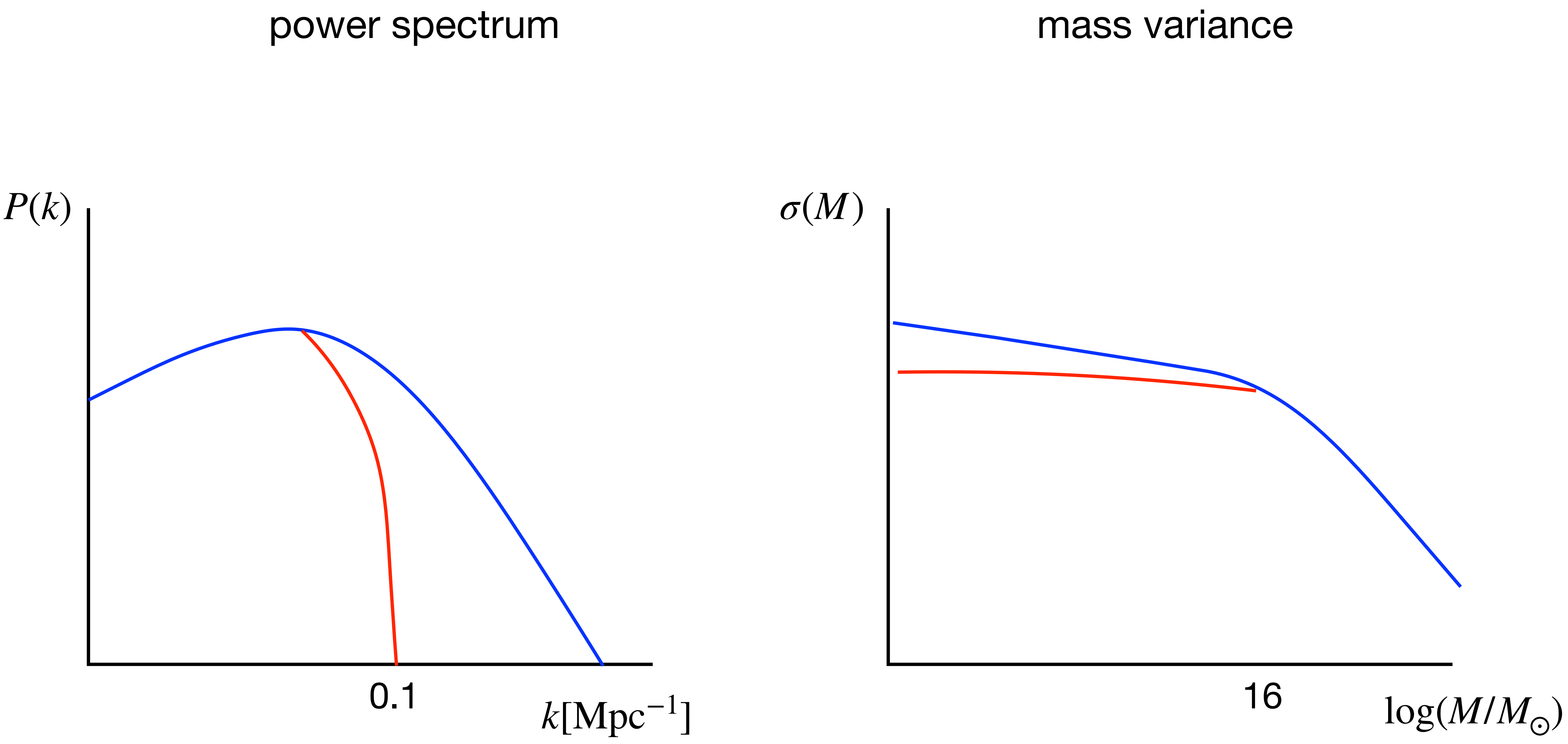
Fourier transform of top hat window function, radius R

constraints on amplitude of perturbations on large scales (from weak lensing, cluster counts and redshift space distortions) often quoted in terms of

$$S_8 \equiv \sigma_8\left(\frac{\Omega_\mathrm{m}}{0.3}\right)^{0.5}$$

where σ_8 is mass variance at $R = 8h^{-1}\,\mathrm{Mpc}$ i.e. cluster scales.

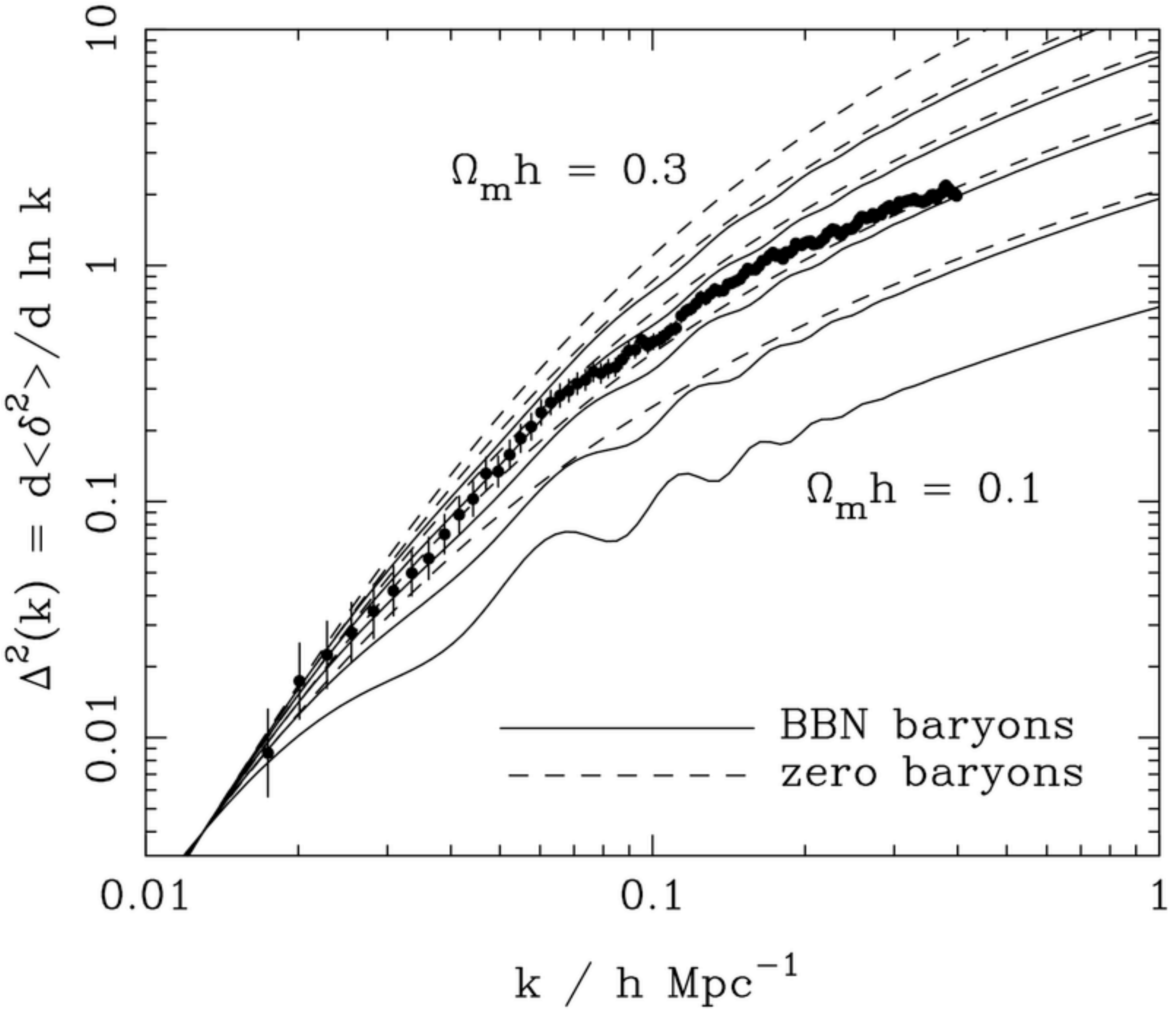
scale R goes non-linear (and structure formation starts) at when $\sigma(R,t) \approx 1$



cold dark matter: small DM halos form first (‘bottom up’)

hot dark matter: large DM halos form first (‘top down’)

Shape of power spectrum depends mainly on $\Omega_m h$ (cold dark) matter which determines horizon scale at radiation-matter equality.



2dFGRS [Cole et al.]

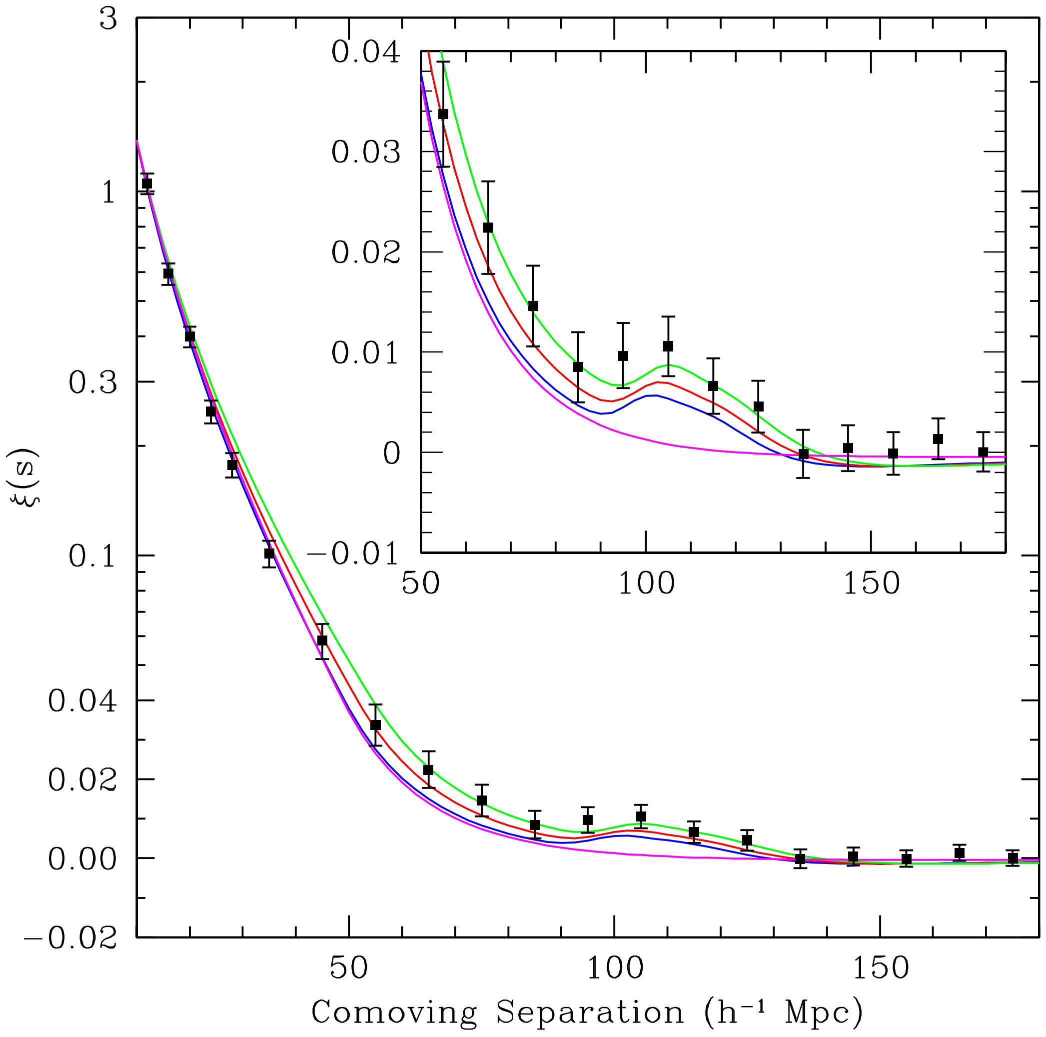
(n.b. old data c.~2005, current constraints covered in section 2)

Baryon acoustic oscillations (BAOs)

The sound horizon at decoupling (which sets the scale of the acoustic peaks in the CMB, see lecture 2) also leads to a ‘standard ruler’ feature in the clustering of galaxies:

$$\mathrm{d}N = n_{\mathrm{gal}}[1+\xi(r)]\mathrm{d}V$$

Correlation function: $$\xi(r) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle = \frac{V}{(2\pi)^3}\int P(k)\exp\left(-i\mathbf{k}.\mathbf{r}\right)\mathrm{d}^3\mathbf{k}$$



$$\Omega_{\mathrm{m}}h^2 = 0.12\,,\;\; \Omega_{\mathrm{b}}h^2 = 0.024$$

$$\Omega_{\mathrm{m}}h^2 = 0.13\,,\;\; \Omega_{\mathrm{b}}h^2 = 0.024$$

$$\Omega_{\mathrm{m}}h^2 = 0.14\,,\;\; \Omega_{\mathrm{b}}h^2 = 0.024$$

$$\Omega_{\mathrm{m}}h^2 = 0.105\,,\;\; \Omega_{\mathrm{b}} = 0$$

SDSS [Eisenstein et al.] (n.b. old data c.~2005 again)

Spherical collapse

See e.g. Tong, Padmanabhan

A spherical overdense region evolves according to the parametric solutions to the Friedmann equation for a closed universe:

$$r = A(1 - \cos\theta)\,, \quad t = B(\theta - \sin\theta)$$

At early times: $r \propto t^{2/3}$, $\delta \propto r$ (as for a flat matter dominated universe).

Expansion then slows down, and at $\theta = \pi$ region 'turns around' and starts collapsing.

Formally $r = 0$ at $\theta = 2\pi$, however assumptions behind spherical collapse (matter is in spherical shells with small random velocities) breakdown, and region 'virialises' ($2T + V = 0$, T= kinetic energy, V = potential energy) with

$$\rho(t_{\rm col}) = \Delta\, \rho_{\rm c,0}(1 + z_{\rm col})^3$$

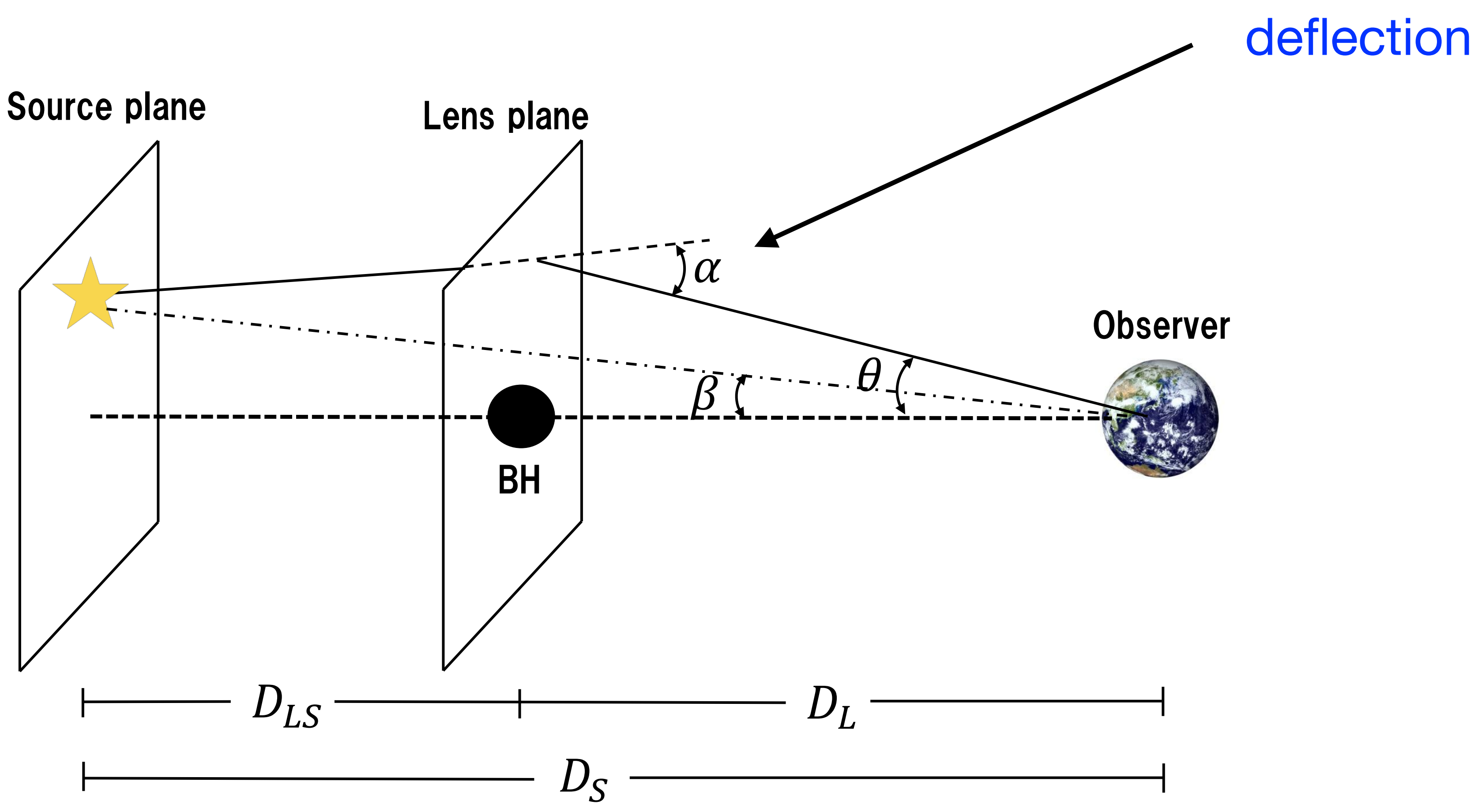
$\rho_{\rm c,0}$ = present day critical density

'col' = when $\theta = 2\pi$

For a flat matter dominated universe, viral overdensity: $\Delta = 18\pi^2 = 178 \approx 200$

Bryan & Norman have fitting functions for (red-shift dependent) Δ for non-zero cosmological constant, or curvature. But 200 sometimes used for ΛCDM (see Section 3).

Gravitational lensing



Sasaki et al.

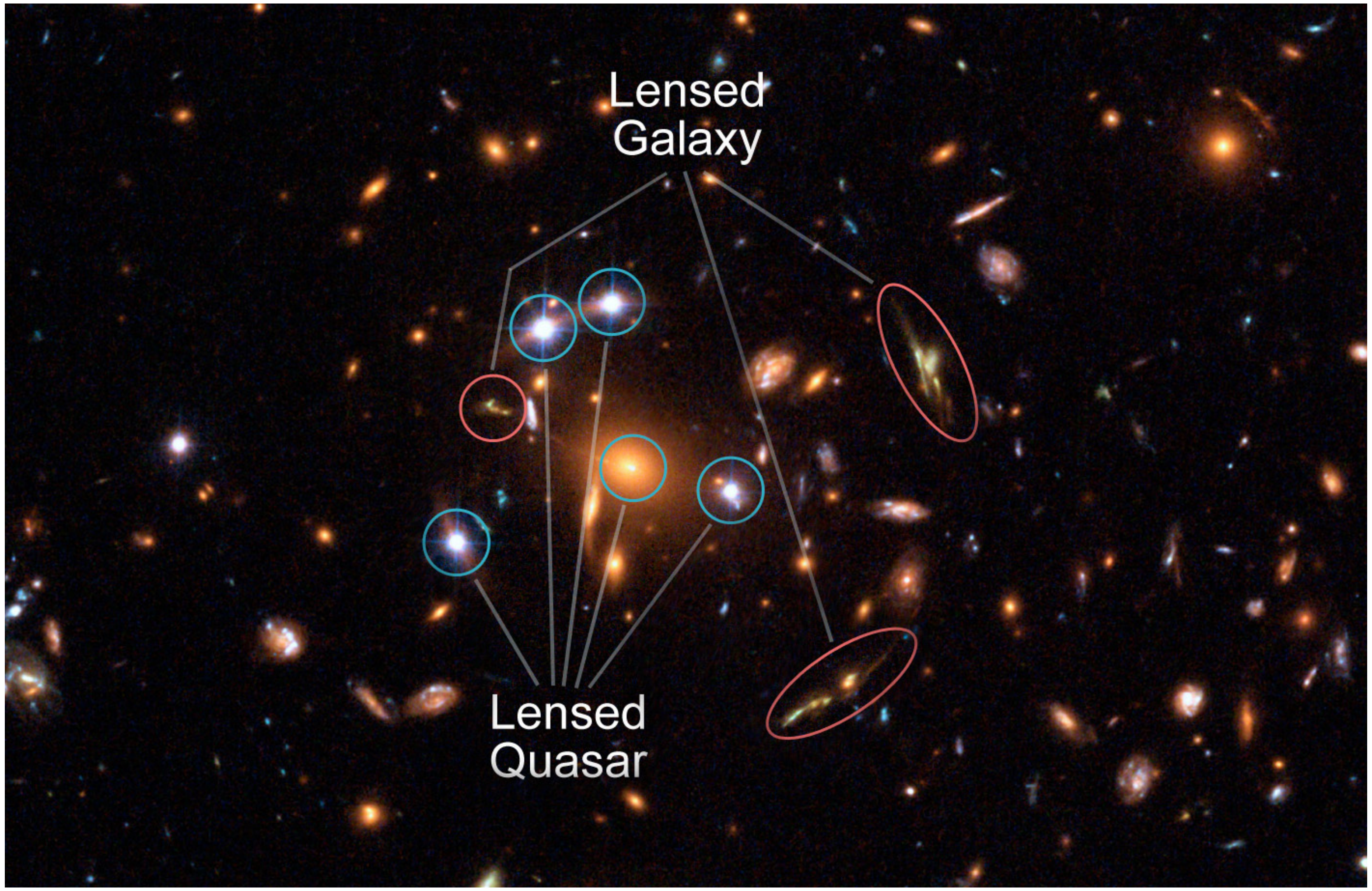
Strong lensing

See e.g. Treu for a review as of 2010.

Deflection α large $\longrightarrow$ multiple images (or Einstein ring if source, lens & observer are aligned.

Image properties (e.g. number and positions) depend on the overall mass distribution.

Substructure (i.e. subhalos, more in sections 2 & 4) can be probed via e.g. flux ratios and gravitational imaging.

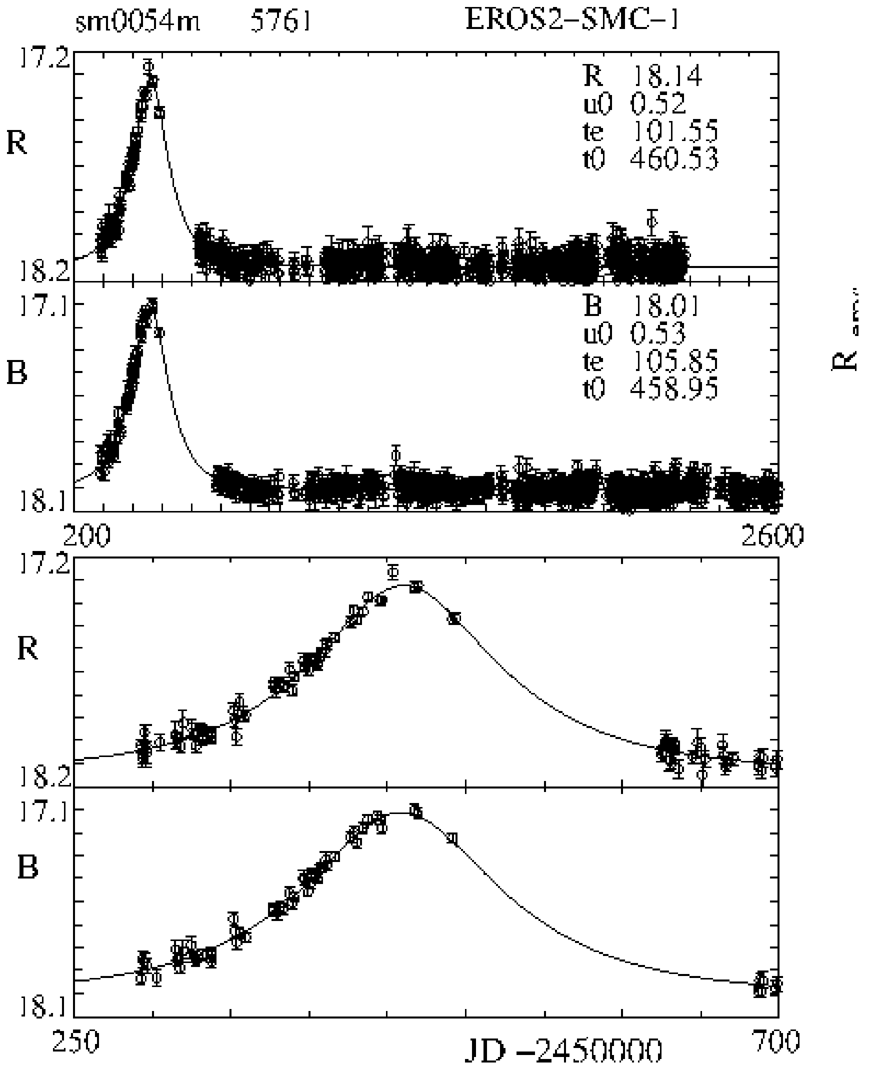


ESA/NASA, Sharon & Ofek

Microlensing

Paczynski, for a review see Mao

Angular separation of images is too small to be resolved (~micro arc seconds) so instead see temporary brightening of source, e.g. stars in Small & Large Magellanic Clouds:



EROS

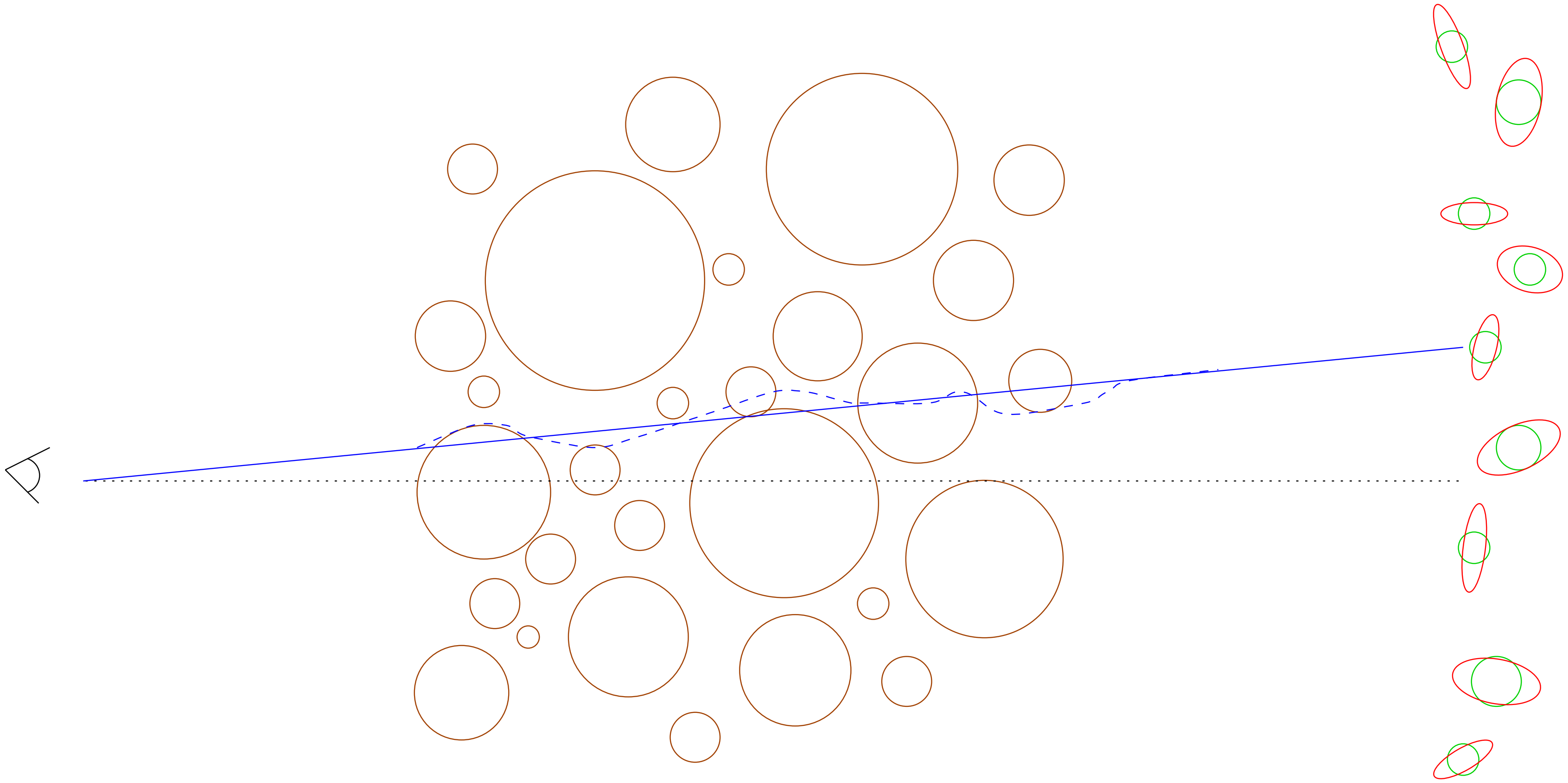
Constrains compact object dark matter, see Bernard Carr and Florian Khunel lectures on primordial black holes.

Weak lensing

See e.g. Kilbinger for a review as of 2015.

Deflection α small.

Cosmic shear: distortion of images of distant galaxies due to weak lensing, allows the matter distribution to be mapped.



Refregier

Inflation

Inflation: A period of accelerated expansion ($\ddot{a} > 0$) in the early Universe.

Problems with the Big Bang:

Flatness: if universe isn't exactly flat, density evolves away from critical density (for which geometry is flat), to be so close to critical density today requires fine tuning of initial conditions.

Horizon: regions that have never been in causal contact have the same Cosmic Microwave Background temperature and anisotropy distribution.

Monopoles/massive relics: formed when symmetry breaks, would dominate the density of the Universe.

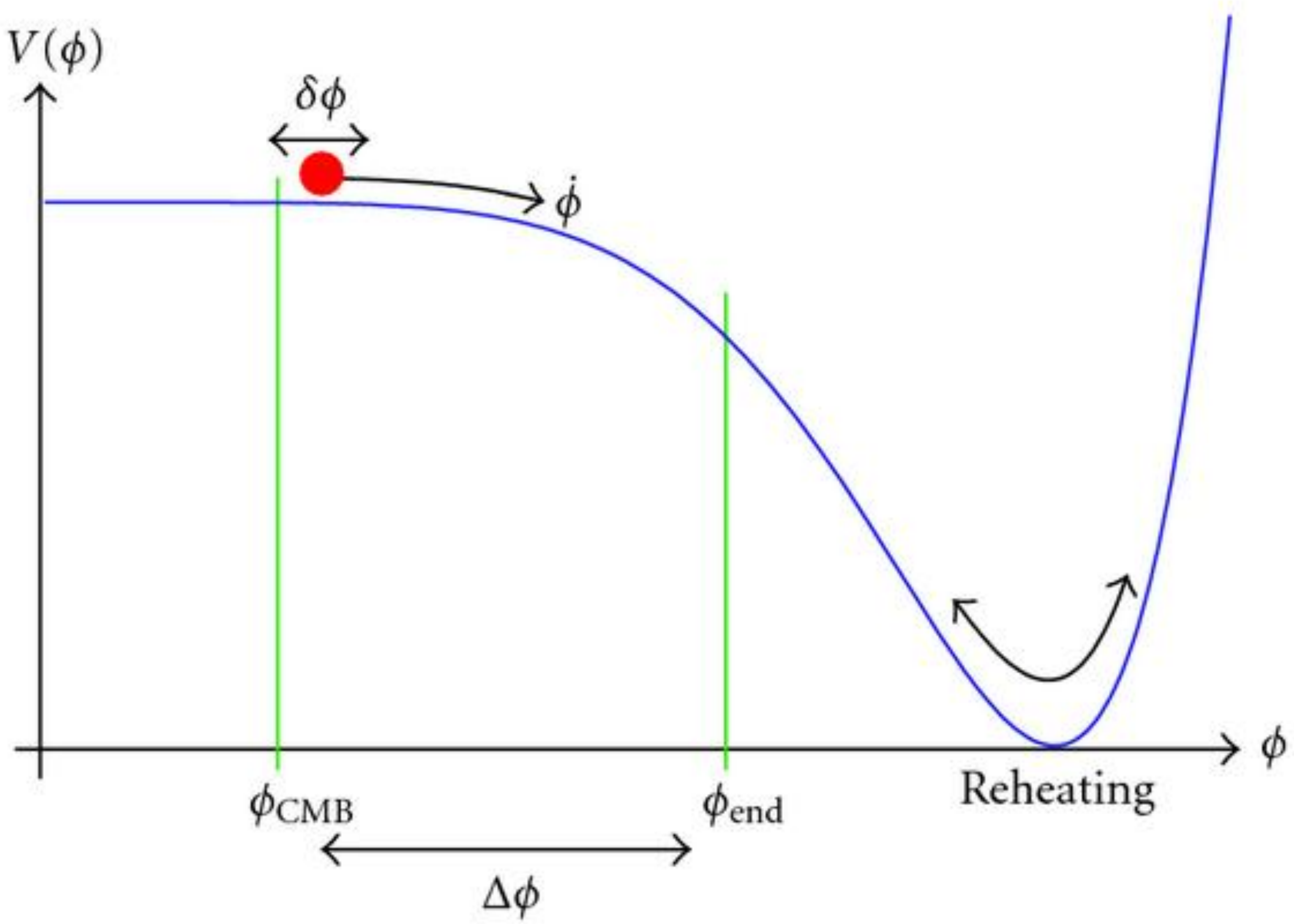
Inflation solves these problems by:

driving 'initial' density extremely close to critical density

allowing currently observable universe to originate from small region (originally in causal contact)

diluting monopoles

Driven by slowly rolling scalar field:



Yadav & Wandelt

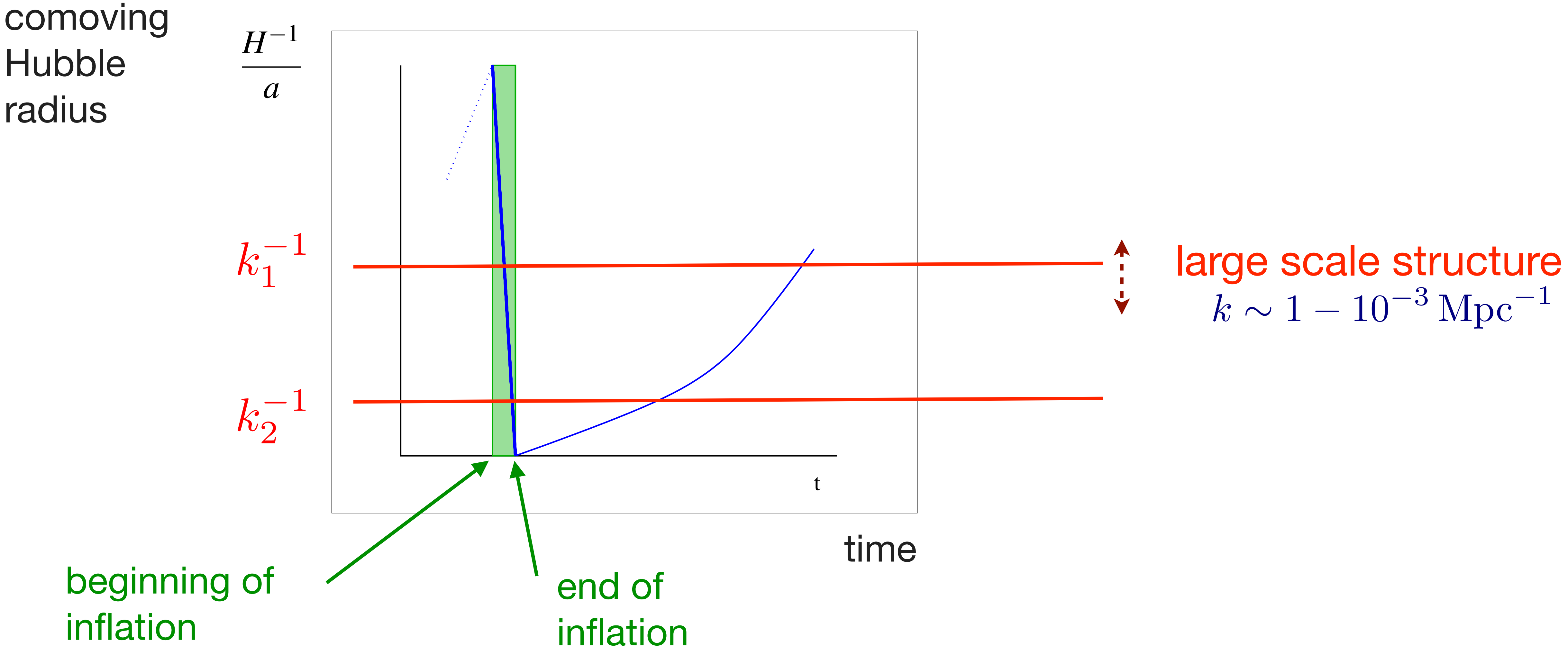
Inflation ends when potential becomes too steep:

Field oscillates around minimum of potential & decays creating radiation dominated Universe (reheating).

Quantum fluctuations in scalar field lead to density perturbations, which are close to scale invariant (same amplitude on all scales) and consistent with measurements of the temperature anisotropies in the cosmic microwave background radiation.

$\ddot{a} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}(H^{-1}/a)}{\mathrm{d}t} < 0$

i.e. comoving Hubble radius decreases during inflation



comoving
Hubble
radius

large scale structure
$k \sim 1 - 10^{-3}\,\mathrm{Mpc}^{-1}$

time

beginning of
inflation

end of
inflation

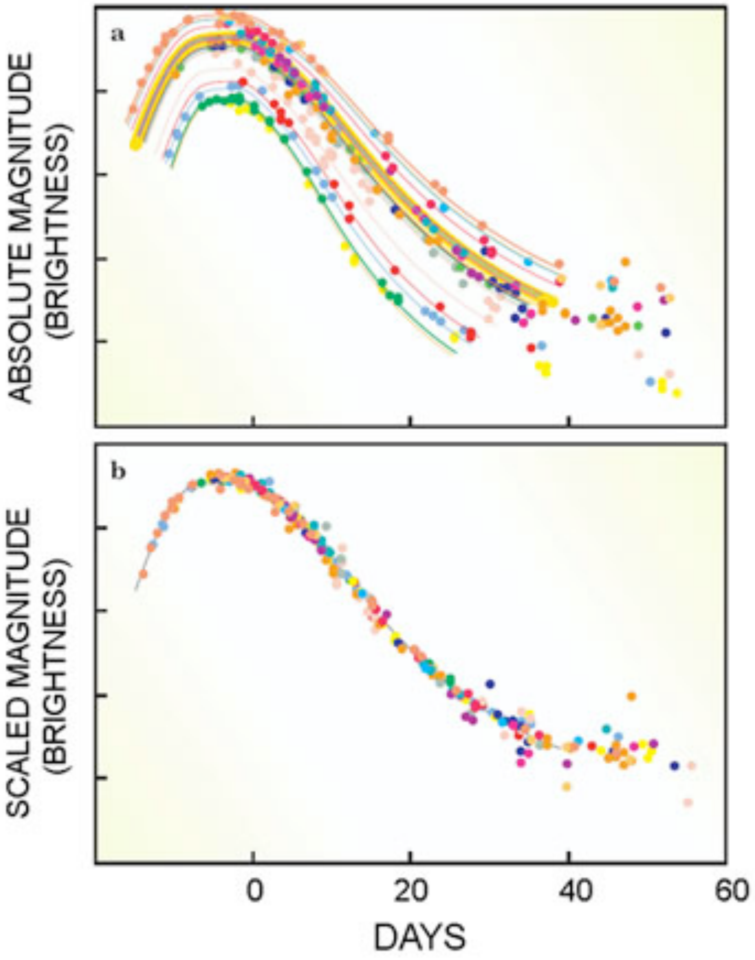
A scale exits the horizon during inflation when k = aH, re-enters when k = aH again.

Type 1a supernovae

Explosion of white dwarf exceeding Chandrasekar mass limit (maximum mass that can be supported by electron degeneracy pressure), due to accretion from binary companion.

LBL
(artists impression)

Standardisable candles: correlation between timescale and peak magnitude.



LBL

Can use type 1a SNe to measure luminosity distance:

(distance calculated assuming inverse square law for flux, f, holds)

$$d_{\mathrm{L}} \equiv \left(\frac{L}{4\pi f}\right)^{1/2}$$

expansion of universe reduces flux by $(1+z)^2$ (energy of individual photons, and arrival rate of photons both reduced by (1+z)) and area photons spread out over changed if universe isn't flat.

For a flat (k=0) universe:

$$d_{\mathrm{L}} = r(1+z) = (1+z)\int_{t_{\mathrm{e}}}^{t_0} \frac{\mathrm{d}t}{a(t)} = (1+z)\int_0^z \frac{\mathrm{d}z'}{H(z')}$$

for $z \ll 1$

$$d_{\mathrm{L}} \approx \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right]$$

q = deceleration parameter

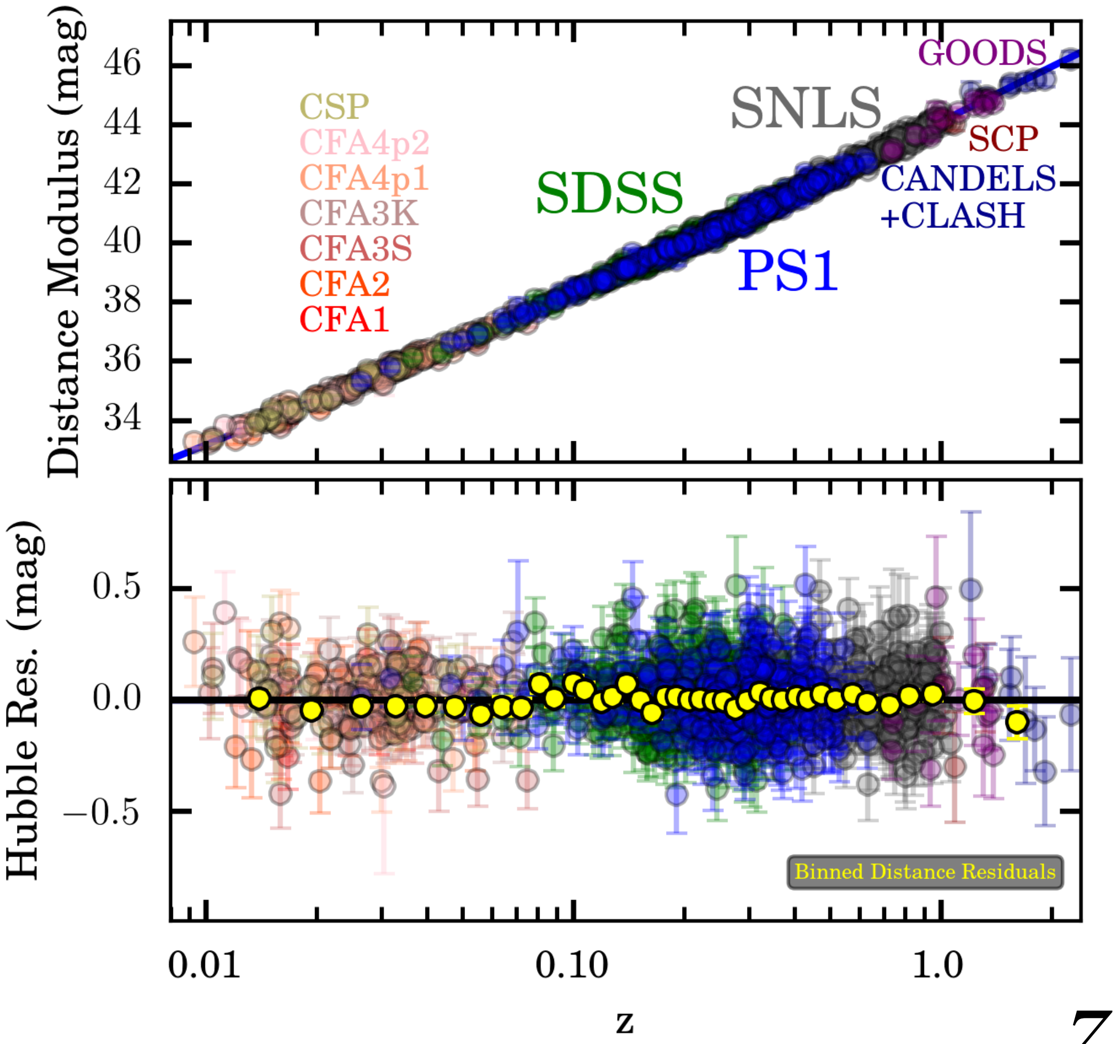
$$q = -\frac{\ddot{a}}{a}\frac{1}{H^2}$$

$$q_0 = \frac{1}{2}\Omega_{\mathrm{m},0} - \Omega_{\Lambda,0}$$

Pantheon sample: compilation of ~1000 type 1a supernovae

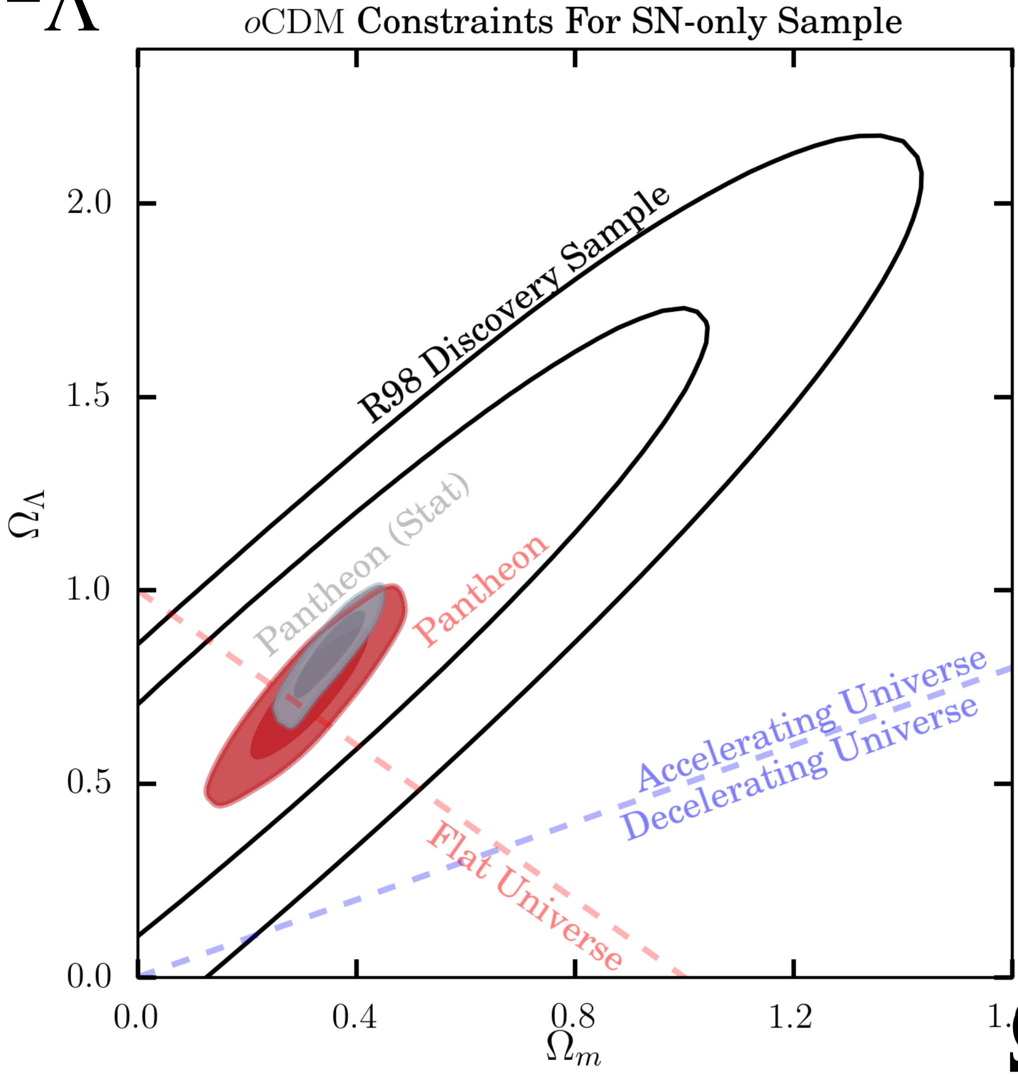
Hubble diagram

$5\log_{10}(d_{\rm L}) + 25$



z

Ω_Λ



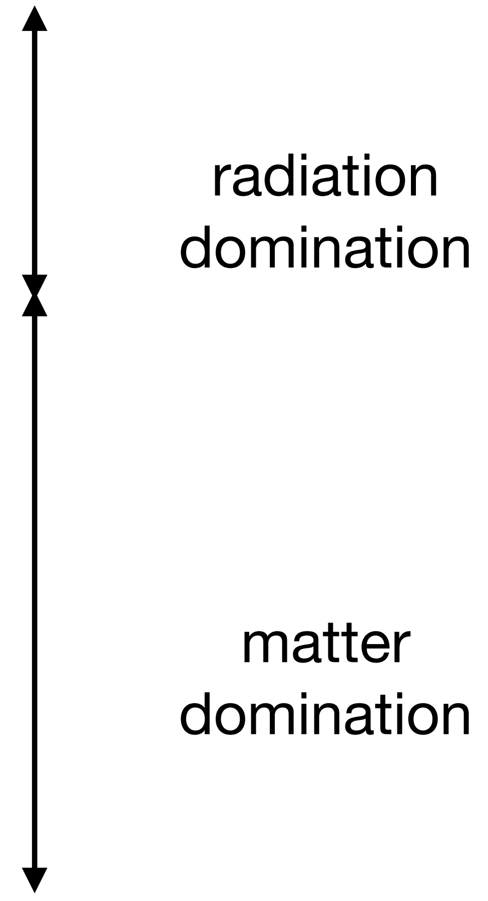
$\Omega_{\rm m}$

Scolnic et al.

Combined with CMB, BAO and H0 measurements: $w_0 = -1.01 \pm 0.09$ (i.e. consistent with a cosmological constant).

History of the Universe

What	When
nucleosynthesis (formation of the nuclei of light elements)	~1 s
radiation-matter equality	0.05 Myr
recombination, decoupling and last scattering (atoms form, CMB ‘released’)	0.3 Myr
structure formation starts	~0.1 Gyr
matter-Lambda equality	10 Gyr
today	13.7 Gyr



Next section: observational evidence for dark matter

Backup slides

scale R goes non-linear (and structure formation starts) at $z_{\rm nl}$ when

$$\sigma(R, z_{\rm nl}) = \frac{\sigma(R, 0)}{1 + z_{\rm nl}} = 1$$