Dark matter in astrophysics/cosmology

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1. A brief introduction to cosmology

2. Observational evidence for dark matter

3. **Dark matter distribution**

4. Constraints on the properties of dark matter

5. Overspill + what would you like to hear more about?

Dark matter distribution (on galactic scales)

Theory

Numerical simulations

Observations

Small scale challenges

Will focus here on ‘vanilla’ cold dark matter,
self-interacting and warm dark matter in section 4

see also Lam Hui lectures on ‘Fuzzy’ ultralight dark matter

Recommended further reading

- ‘Dark matter haloes and subhaloes’, Zavala & Frenk, arXiv:1907.11775.
- ‘Numerical simulations of the dark Universe: state of the art and the next decade’ , Kuhlen, Vogelsberger & Angulo, arXiv:1209.5745 n.b. significant progress in hydrodynamical simulations including baryons since then.
- ‘Streams, substructures and the early history of the Milky Way’, Helmi, arXiv:2002.04340, Ann. Rev. Astro.
- ‘Small-scale challenges to the ΛCDM paradigm’, Bullock & Boylan-Kolchin, arXiv. 1707.04256, Ann. Rev. Astro.

Useful textbooks:

- ‘Galactic dynamics’, Binney & Tremaine, Princeton University Press.
- ‘Dynamics and astrophysics of galaxies’, Bovy, Princeton University Press (in preparation)

Why is the dark matter distribution important?

All the observational evidence for dark matter arises from its gravitational effects.

If we want to confirm the existence of dark matter (and the standard cosmological model) and understand its nature we need to detect it.

The signals in DM detection experiments depend on:

Lab based direct detection experiments (see Jody Cooley & Igor Irastorza lectures)
the local (i.e. at Solar radius $r = R_{\odot}$) Milky Way DM density and speed distribution.
$R_{\odot} = (8.178 \pm 0.013 \pm 0.022)\,\mathrm{kpc}$ GRAVITY Collaboration, using orbit of star S2 around Sgr A* (massive BH at MW centre).

Indirect detection via annihilation products (see Tracy Slatyer lectures)
the DM density distribution: density profile of individual (sub)halos & subhalo mass function, in particular Milky Way and dwarf galaxies.

Theory

'standard halo model' (isothermal sphere)

density profile: $\rho(r) \propto r^{-2}$

('Maxwell-Boltzmann' or 'Maxwellian') velocity distribution:

$$f(\mathbf{v}) = \frac{1}{N}\left(\frac{3}{2\pi\sigma_v^2}\right)^{3/2}\exp\left(-3\mathbf{v}^2/2\sigma_v^2\right)$$

N normalisation constant

$\sigma_v = \sqrt{3/2}\,v_c$ r.m.s. velocity dispersion

(v_c circular speed, $v_c^2 = r(\mathrm{d}\Phi/\mathrm{d}r)$, $\Phi(r)$ potential)

Formally infinite, velocity distribution usually truncated by hand at the escape speed: $v_e = \sqrt{2\,|\,\Phi(r)\,|}$

Standard values of constants:

$$\rho(R_\odot) = 0.3\,\mathrm{GeV\,cm^{-3}}, \quad v_c = 220\,\mathrm{km\,s^{-1}}, \quad v_{esc} \approx (550 - 600)\,\mathrm{km\,s^{-1}}$$

(will cover latest values, and errors, in 'Observations' section)

What's the physical origin of this model?

Phase space distribution function: $f(\mathbf{x}, \mathbf{v}, t)$

Number of particles with phase space co-ordinates in $\mathbf{x} \rightarrow \mathbf{x} + \mathrm{d}\mathbf{x}$ and $\mathbf{v} \rightarrow \mathbf{v} + \mathrm{d}\mathbf{v}$ at time t: $f(\mathbf{x}, \mathbf{v}, t)\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{v}$

Steady-state phase space distribution of a collection of collisionless particles is given by the solution of the collisionless Boltzmann equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = 0$$

in Cartesian co-ordinates:

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} + \frac{\partial \Phi}{\partial \mathbf{x}}.\frac{\partial f}{\partial \mathbf{v}} = 0$$

Poisson's equation for a self-consistent system (where density distribution generates potential):

$$\nabla^2\Phi = 4\pi G\rho = 4\pi G \int f \mathrm{d}^3\mathbf{v}$$

A solution is:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

$$f \propto \exp(-v^2/2\sigma^2)$$

c.f. the phase-space distribution of a self-gravitating isothermal sphere with

$$\sigma^2 = k_{\mathrm{B}}T/m$$

Collisionless particles can change their energy & reach the steady-state configuration if they experience a fluctuating gravitational potential (violent relaxation). However real DM halos haven't reached a steady state and contain substructure (subhalos and streams).

Numerical simulations

see lectures by Annika Peter

In CDM cosmologies structure forms hierarchically: small halos (on average) form first and then larger halos form via mergers and accretion.

Dark matter only simulation of a Milky Way like halo

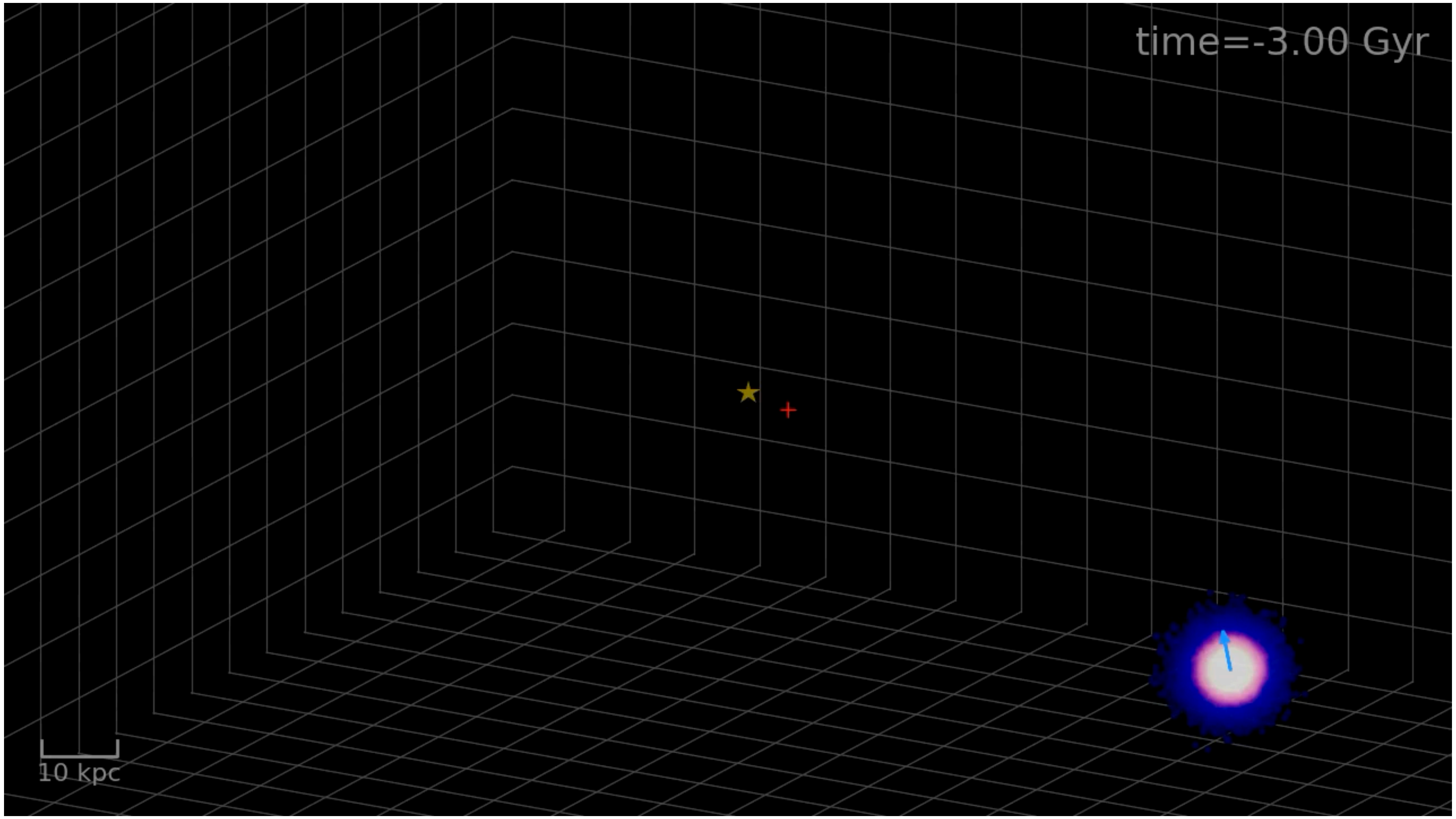
Aquarius

DM distribution today has not reached a steady state!

(c.f. one of the assumptions behind the standard halo model)

Subhalos reduced in mass, or destroyed, by tidal stripping (tidal force across sub halo larger than self-gravity):

Simulation of the tidal stripping of the Sagittarius dwarf galaxy, Erkal/Vasiliev
red/yellow stars, blue dark matter, LMC



Simulating Milky Way like halos (a ‘non-expert’ summary):

Chose input cosmological parameters (e.g. $h,\ \Omega_{\rm m},\ \Omega_{\Lambda}, n_{\rm s}$) and calculate input power spectrum.

Carry out large volume simulation.

Select Milky Way like halo ($M \sim 10^{12} M_{\odot}$, no massive close neighbours/a M31-like neighbour, no recent major mergers).

Resimulate, using lower mass particles in region that forms halo of interest (“zoom technique”).

Carry out convergence tests (do properties change when you change the particle mass or gravitational softening?) and study halo-to-halo scatter. (mass of simulation ‘particles’ is >> mass of DM particles).

N-body simulations (e.g. Aquarius): dark matter only

Hydrodynamical simulations (e.g APOSTLE, Auriga, FIRE): include baryons (i.e. stars and gas) using prescriptions for 'sub-grid' physics.

Baryons affect the dark matter distribution by, e.g. :

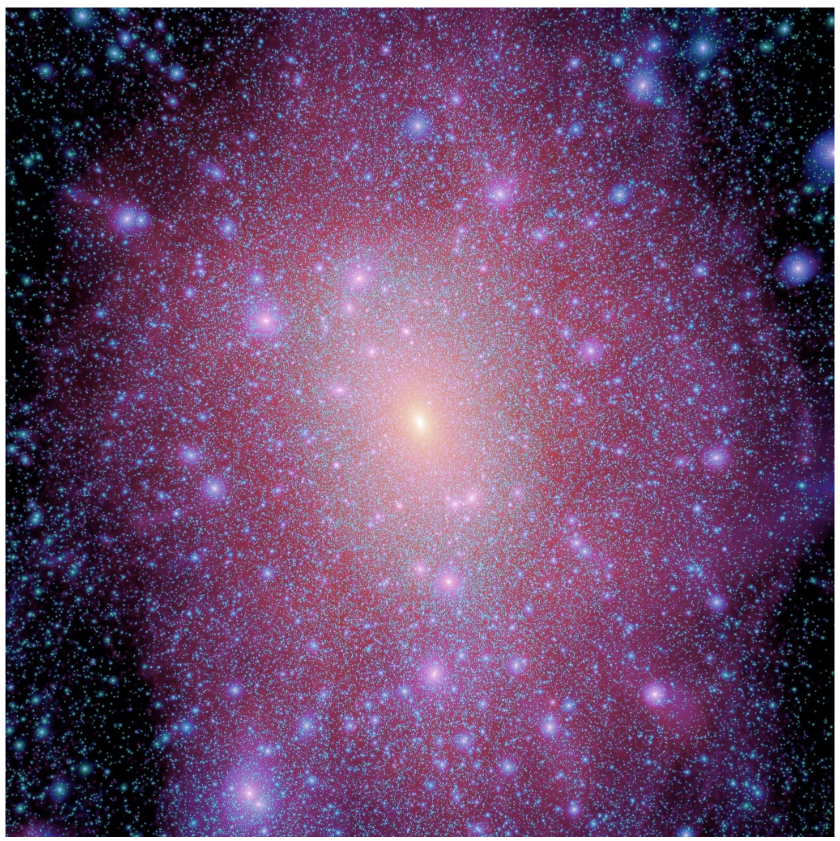
baryonic contraction: infall of baryons pulls in DM, steepening DM density profile Blumenthal et al.

stellar feedback: can reduce density in inner regions and form a core

disk shocking: tidal shock (rapid gravitational perturbation), increases internal energy

Subhalo mass function

e.g. Aquarius (DM only):

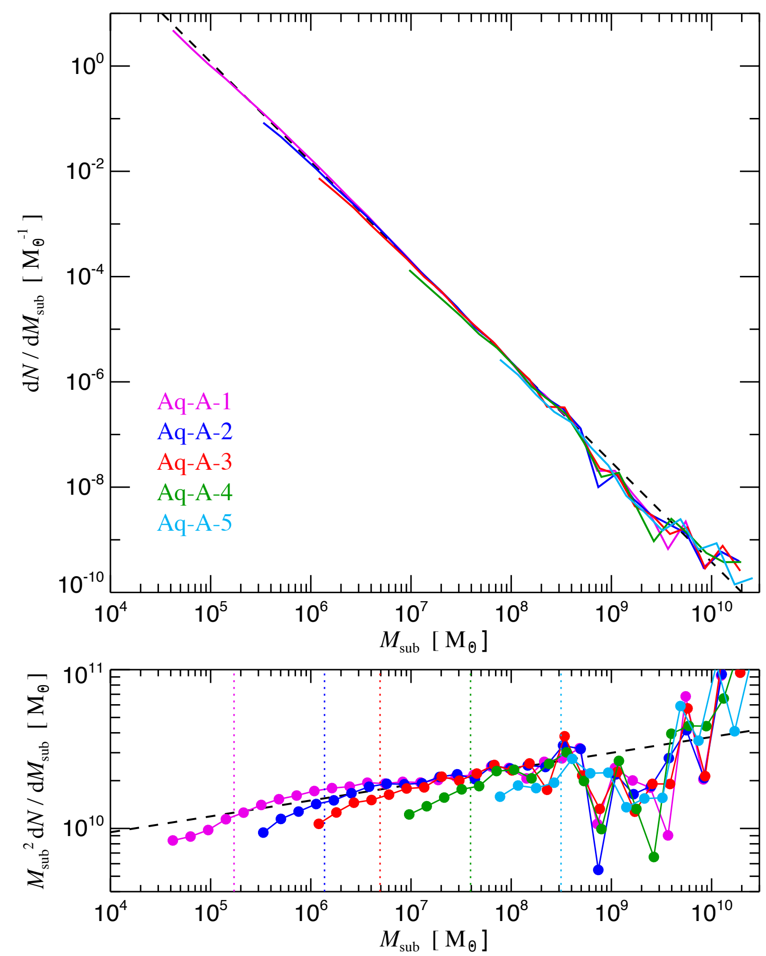


$M/M_\odot$

10^4 10^{10}

$\frac{\mathrm{d}n}{\mathrm{d}M}$

$M^2 \frac{\mathrm{d}n}{\mathrm{d}M}$



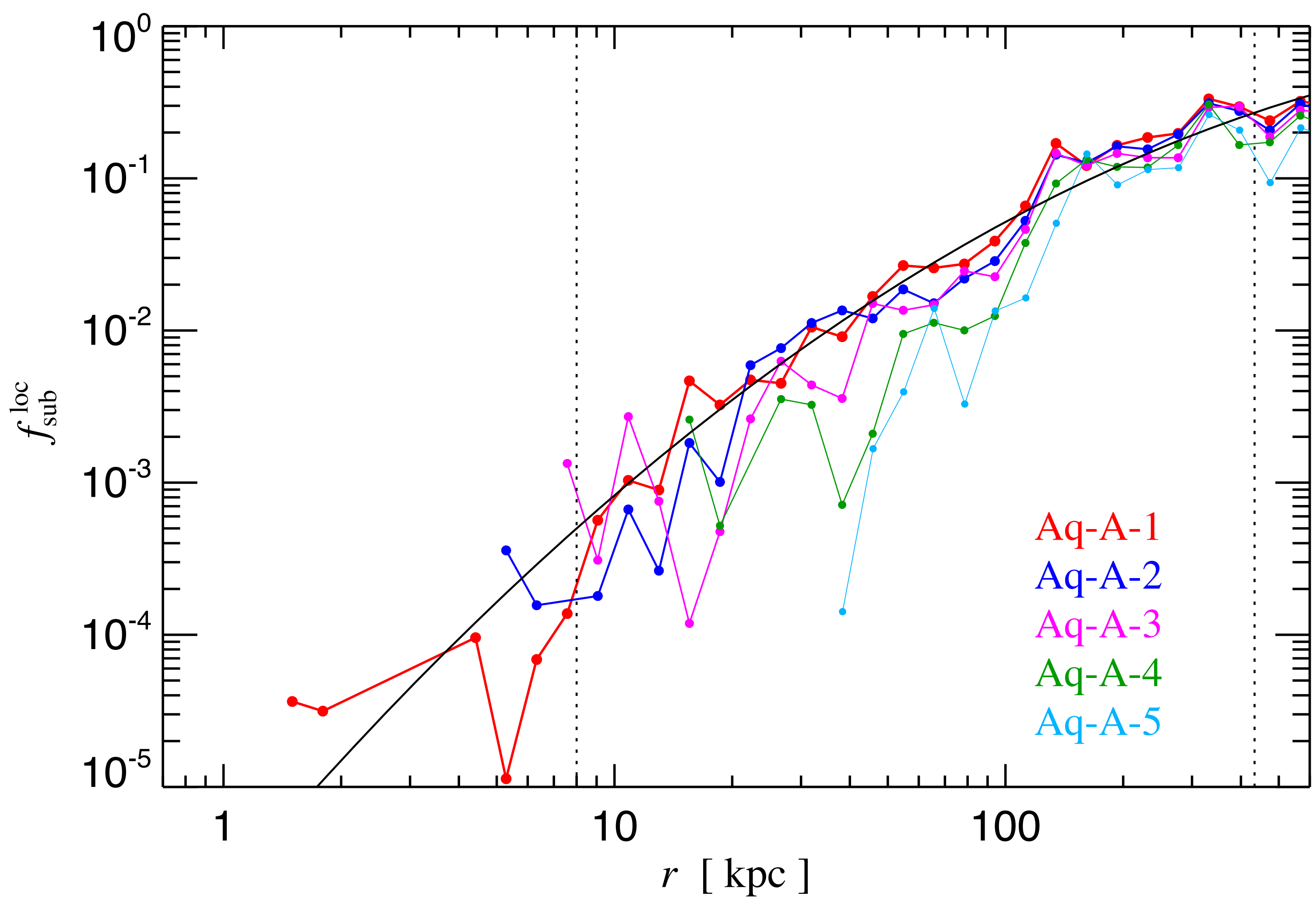
subhalo mass function for 5 varying resolution simulations of same halo

mass function: $\frac{\mathrm{d}n}{\mathrm{d}M} \propto \left(\frac{M}{M_\odot}\right)^{-\alpha}$ $\alpha = 1.90 \pm 0.03$

Subhalo radial distribution

e.g. Aquarius (DM only):

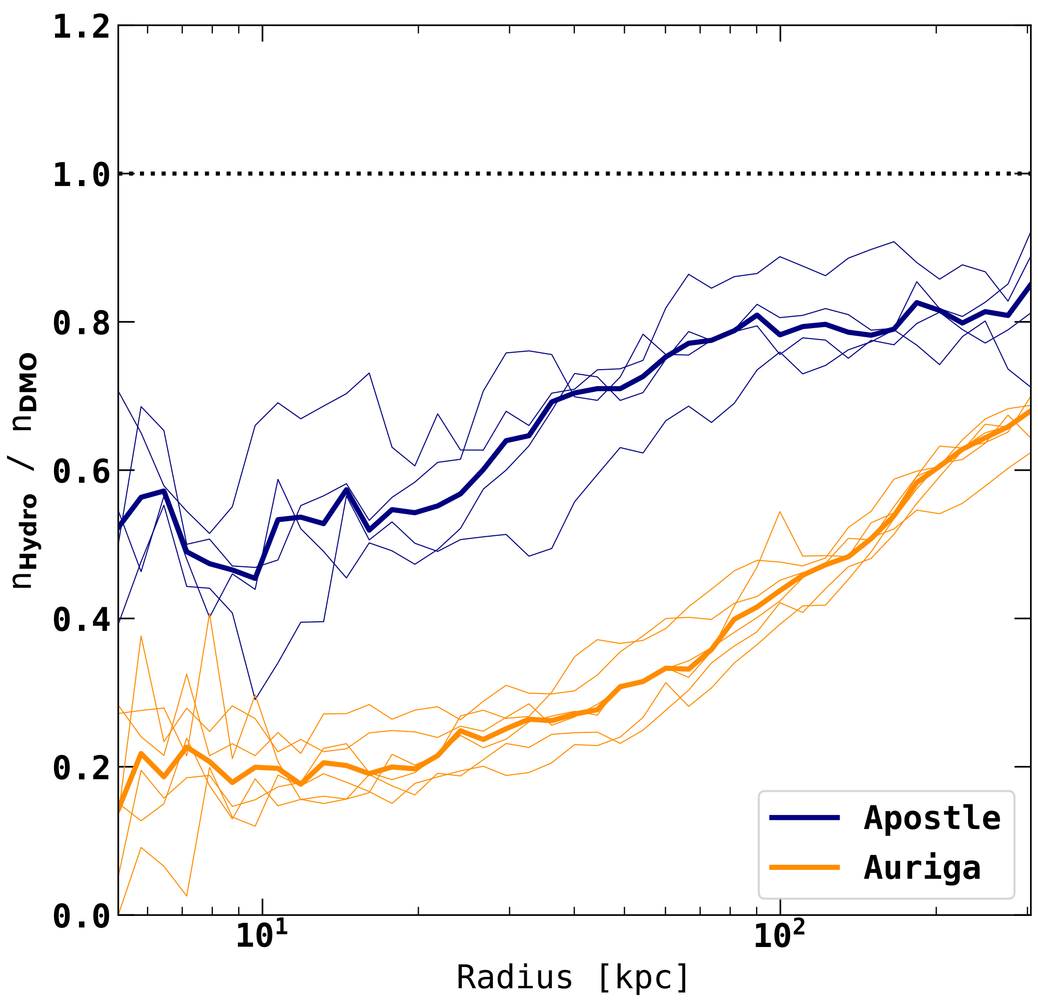
Fraction of local mass in subhalos as a function of radius



~10% of the total mass in DM only simulations is in (resolved) subhalos

< 0.1% of the mass at the solar radius is in (resolved) subhalos
(less subhalos in inner regions as they're more effectively disrupted there)

Ratio of the number density of halos with mass $(10^{6.5} - 10^{8.5})M_\odot$ in hydrodynamical (APOSTLE & Auriga) and dark matter only simulations, as a function of radius



Richings et al.

Fraction of halo in subhalos is smaller in hydrodynamical simulations, and is decreased more at small radii.

Size of reduction depends on how baryonic component is modelled.

Density profile

- Halo definition:

  virial radius, $r_{\rm vir}$: radius within which density is Δ (viral overdensity-see spherical collapse in Section 1) times background density $\bar{\rho}$

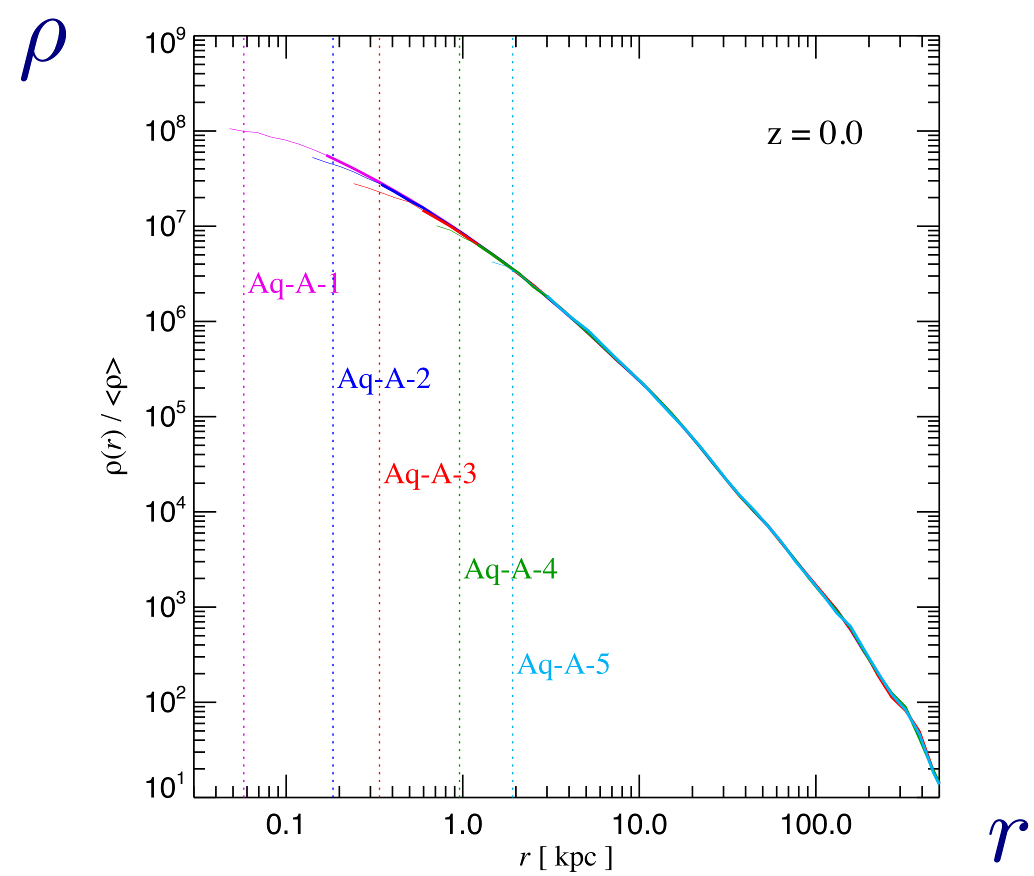
  virial mass: mass within virial radius $$M_{\rm vir} = \frac{4\pi}{3} r_{\rm vir}^3 \Delta\, \bar{\rho}$$

- Different papers use different conventions (see e.g. 'Bullock & Boylan-Kolchin):
  - value for virial overdensity, Δ = 200 or Bryan & Norman red-shift dependent fitting function for ΛCDM, which has $\Delta(z=0) = 333$),
  - background density = critical density or matter density.

- Halo size often parameterised using maximum circular speed rather than viral mass

$$V_{\rm max} = \sqrt{\frac{GM(<r)}{r}}\Bigg|_{\rm max}$$

Aquarius
(dark matter only)



Navarro, Frenk & White (NFW) profile

$$\rho(r) = \frac{\rho_0}{(r/r_\mathrm{s})[1+(r/r_\mathrm{s})]^2}$$

r_s scale radius $\left(\dfrac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}\right)_{r=r_\mathrm{s}} = -2$

$c = r_\mathrm{vir}/r_\mathrm{s}$ concentration

$$\rho(r) \propto r^{-1}\,, \ \text{as } r \to 0\,, \ \rho(r) \propto r^{-3}\,, \ \text{for } r \gg r_\mathrm{s}$$

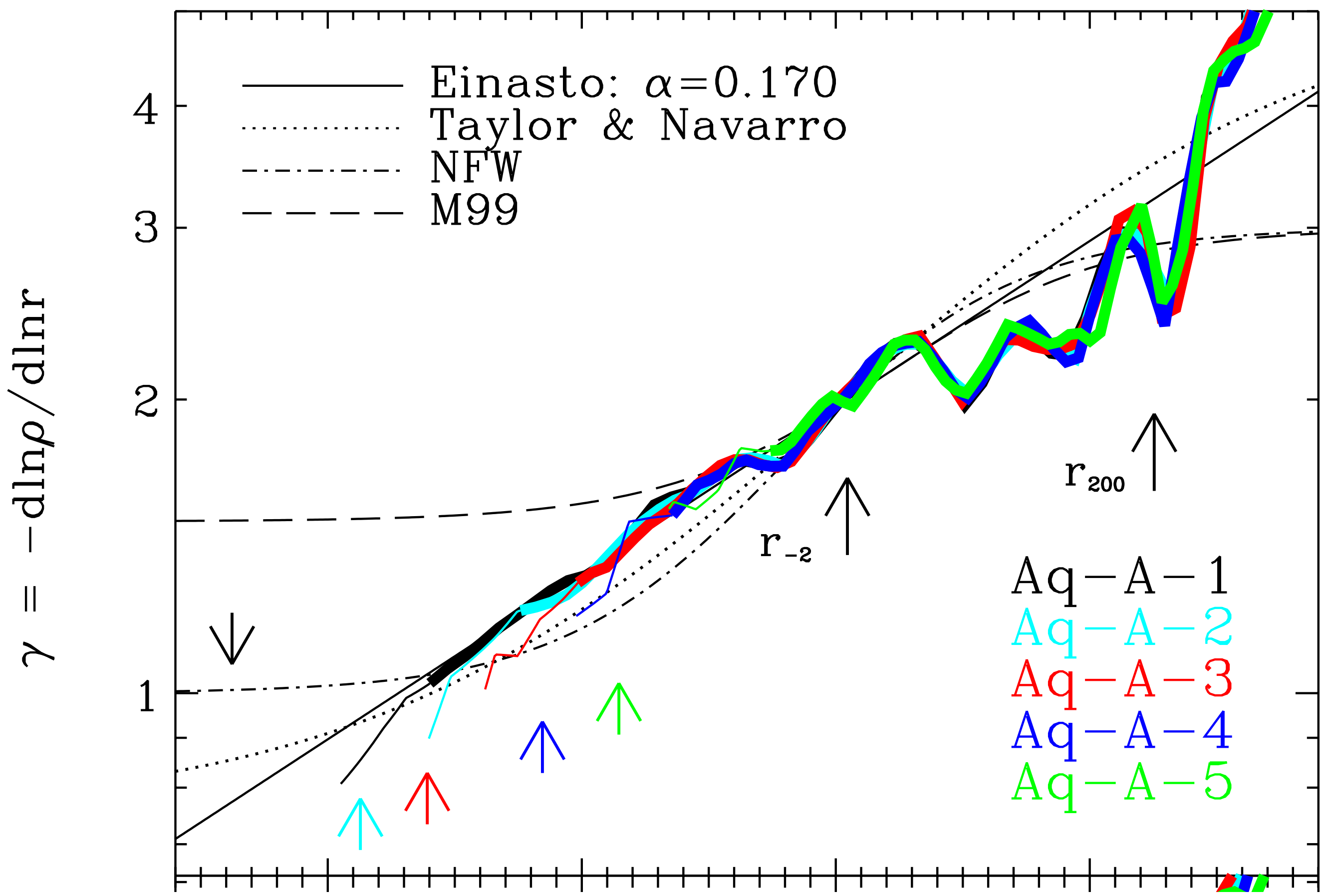
c.f. (singular) isothermal sphere: $\rho(r) \propto r^{-2}$ for all r.

How does the density profile behave in $r \to 0$ limit?

Aquarius:

logarithmic slope
of density profile

$$\gamma = -\frac{d\ln\rho}{d\ln r}$$



log r [kpc/h]

_._._._ NFW
$(\gamma \to 1 \text{ as } r \to 0)$

______ Einasto

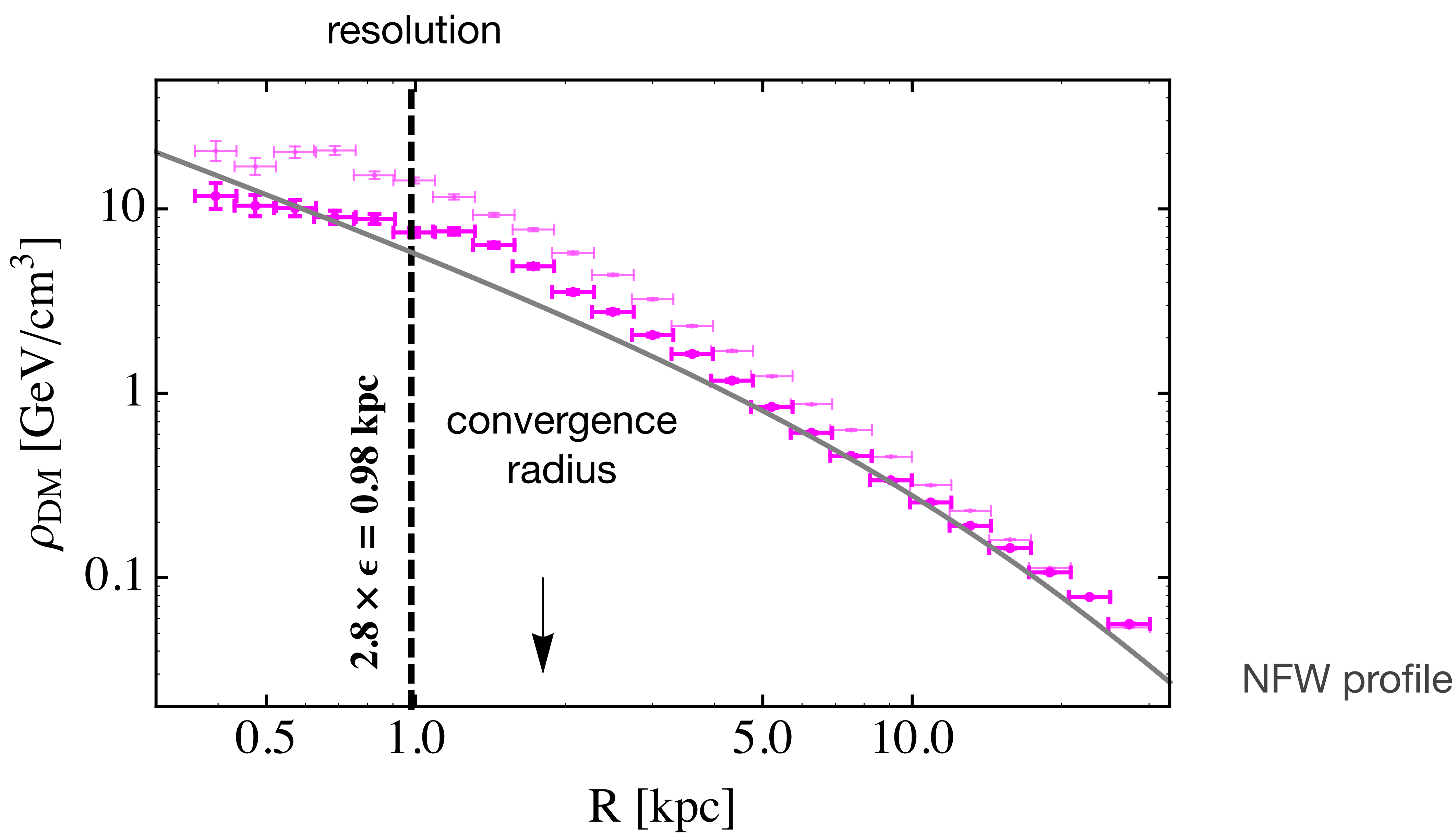
Einasto profile:

$$\rho(r) = \rho_s \exp\left\{-\frac{2}{\alpha}[(r/r_s)^\alpha - 1]\right\}$$

$$\gamma = 2\left(\frac{r}{r_s}\right)^\alpha$$

EAGLE (hydrodynamical simulations of MW like galaxies, with baryons)

Calore et. al:

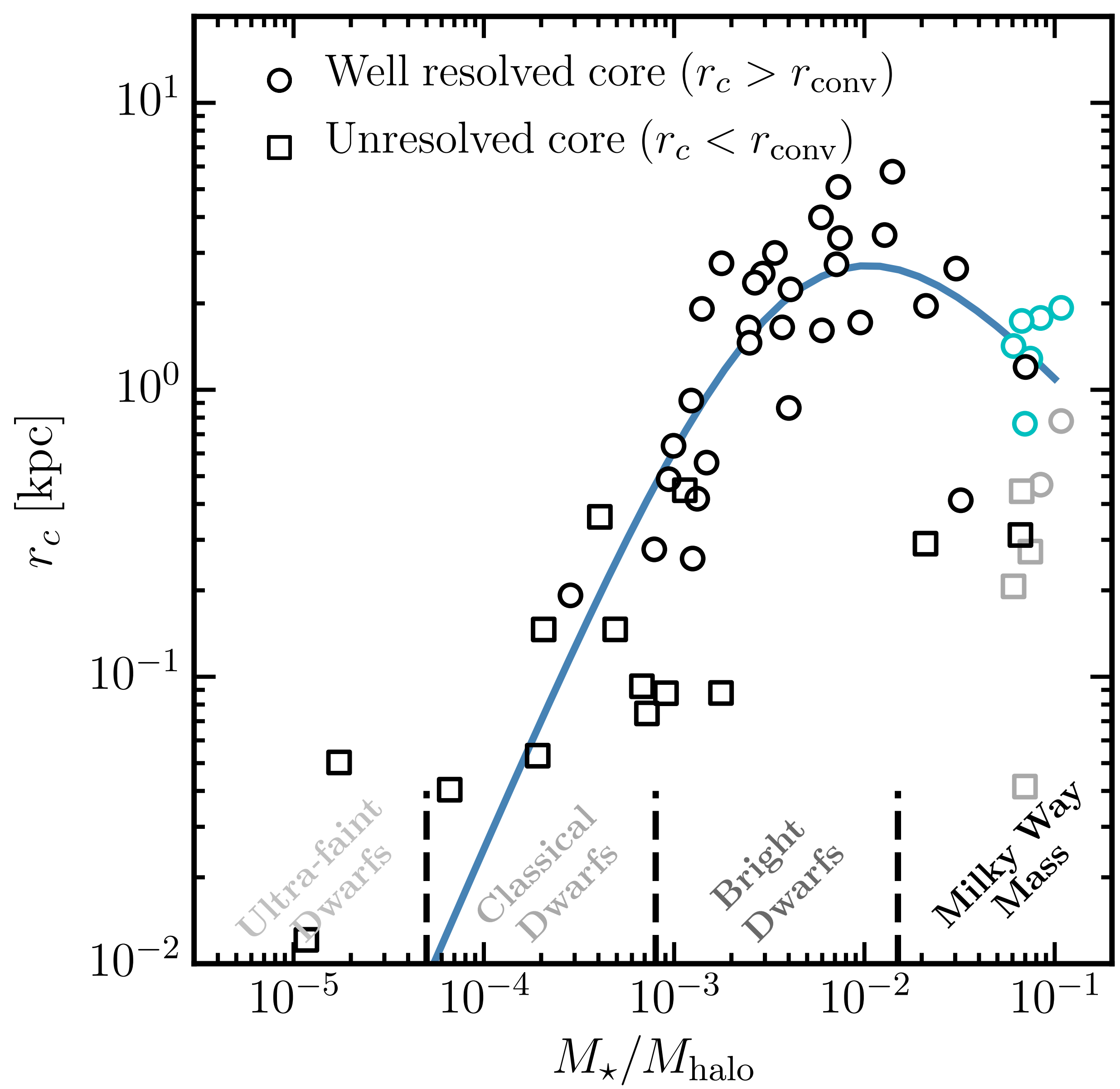


Density profile steeper than NFW for r~(1.5-6) kpc due to baryonic contraction.

Stellar feedback is most efficient at producing cores (constant density inner regions) in bright dwarf galaxies with radius r_{core} ~ (1-5) kpc (see Lazar et al. and references therein).

Can get cores in Milky Way sized galaxies with r_{core} ~ (0.5-2) kpc (e.g. Lazar et al. using FIRE-2)

Core formation depends on gas density threshold for star formation.



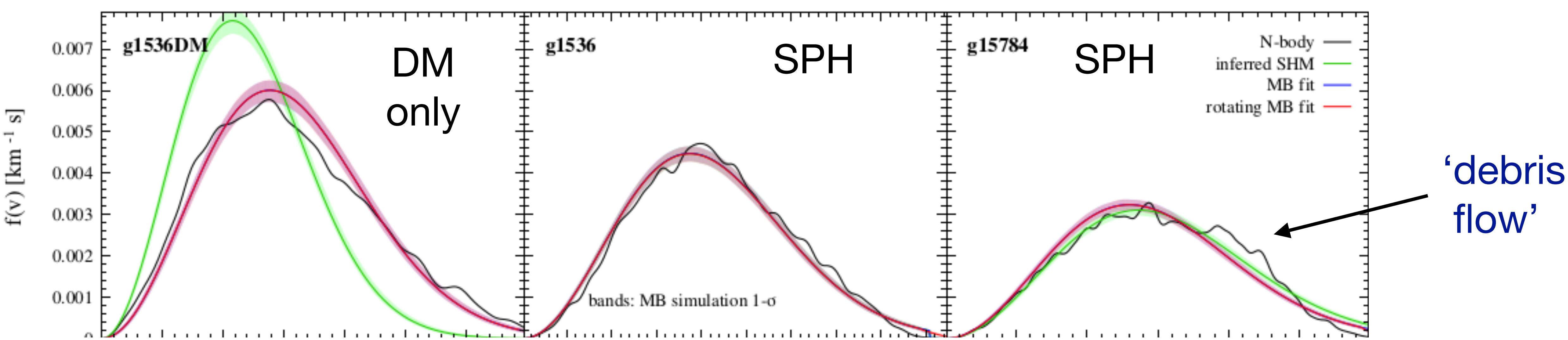
Lazar et al.

Local velocity distribution

Simulations of ‘MW like halos’ with baryons (use different prescriptions for ‘sub-grid’ physics): Bozorgnia et al.; Kelso et al.; Sloane et al.

Maxwellian f(v) (with larger velocity dispersion than standard halo model) is a fairly good fit-better than for dark matter only simulations (since baryonic contraction means logarithmic slope of density list is closer to -2 at Solar radius??).

Kelso et al.:



Standard halo model Maxwellian

Features in tail of dist, ‘debris flows’, incompletely phased mixed material.
Lisanti & Spergel; Kuhlen, Lisanti & Spergel

Observations

Huge progress in understanding the Milky Way in recent years thanks to Gaia:

ongoing ESA space astrometry mission (2013-2022+?)

positions, parallaxes and proper motions (change in apparent position) of >1 billion stars (~1% of MW)

20 (200) million stars with distances measured to 1 (10)%

40 million stars with tangential velocities measure to $< 0.5\,\mathrm{km\,s^{-1}}$

7 million stars with full 6d phase space coordinates (x, y, z, v_x, v_y, v_z)

Often combined with info on metallicity, [Fe, H], from spectroscopic surveys e.g. APOGEE, RAVE, LAMOST.

Need modelling/simulations to interpret observations, and systematic errors are often now comparable or similar to statistical errors.

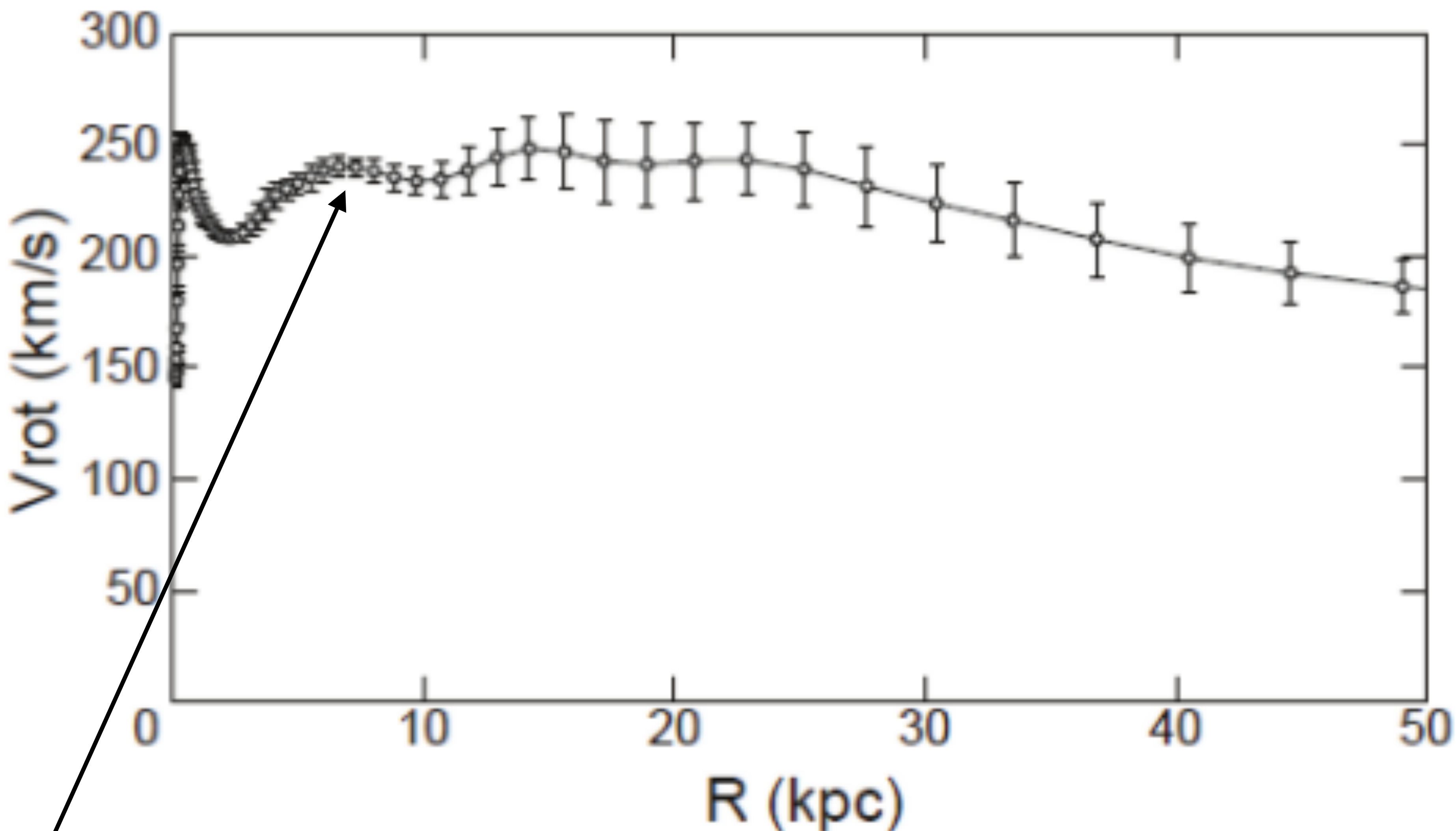
For more info see Helmi 2020 Annual Reviews article
(or, for implications for DM experiments O'Hare talk slides 1 and 2).

Local density $\rho(R_\odot)$:

Various techniques: local (using kinematics of nearby stars) and global (e.g. mass modelling) see Read's 2014 extensive review and de Salas & Widmark 2020 review for more recent measurements.

Mass modelling: use multiple data sets (e.g. rotation curve, velocity dispersions of halo stars, local surface mass density, total mass…) to constrain a model for the MW (luminous components + halo).

Sofue ‘unified rotation curve of MW’, combining a large number of data sets

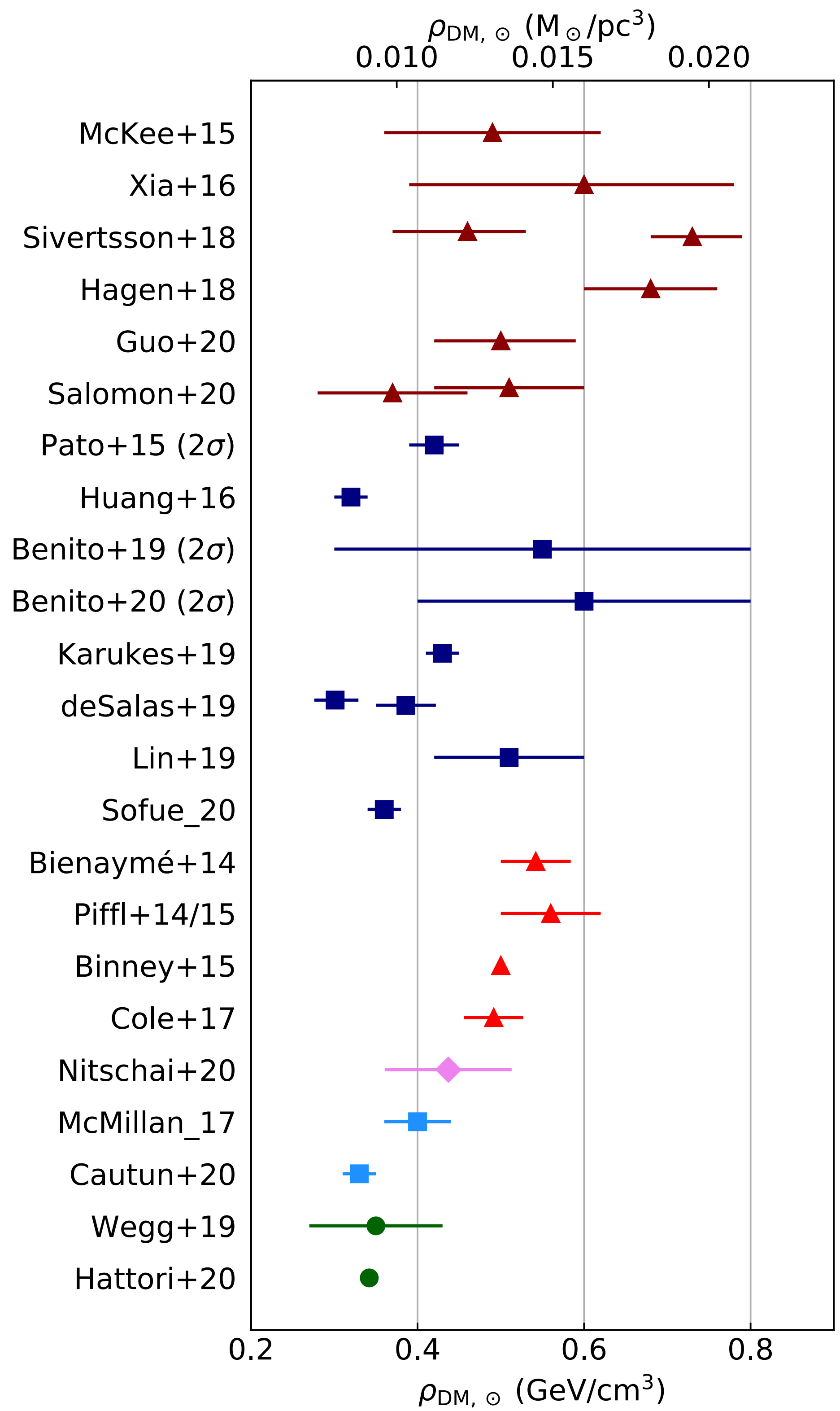


close to flat
(i.e. $\rho \propto r^{-2}$)
at Solar radius

$$\rho(R_\odot) = (0.36 \pm 0.02)\,\mathrm{GeV\,cm^{-3}}$$

for best fit, NFW, profile for DM halo

de Salas & Widmark summary of recent determinations of $\rho(R_\odot)$



local stars

rotation curve

mass modelling

Jeans modelling disc stars

circular velocity based mass modelling

halo stars

Local circular speed e.g:

Reid et al. proper motion of Sgr A*:

$$v_{\rm c}(R_\odot)/R_\odot = (30.3 \pm 0.9)\,{\rm km\,s^{-1}\,kpc^{-1}}$$

and using new precise measurement of $R_\odot$ gives

$$v_{\rm c}(R_\odot) = (248 \pm 7)\,{\rm km\,s^{-1}\,kpc^{-1}}$$

Eilers et al. Jeans analysis from taking moment of collision less Boltzmann equations (in cylindrical co-cordinates):

$$v_{\rm c}^2(R) = R\frac{\partial\Phi}{\partial R}_{z\approx 0} = \langle v_\phi^2\rangle - \langle v_R^2\rangle\left(1 + \frac{\partial \ln \nu}{\partial \ln R} + \frac{\partial \ln \langle v_R^2\rangle}{\partial \ln R}\right)$$

ν = density of tracer stars.

combing data from Gaia, APOGEE and other sources:

$$v_{\rm c}(R_\odot) = (229.0 \pm 0.2)\,{\rm km\,s^{-1}}$$

with (2-5)% systematic uncertainty (from e.g. uncertainty in distribution of tracer stars).

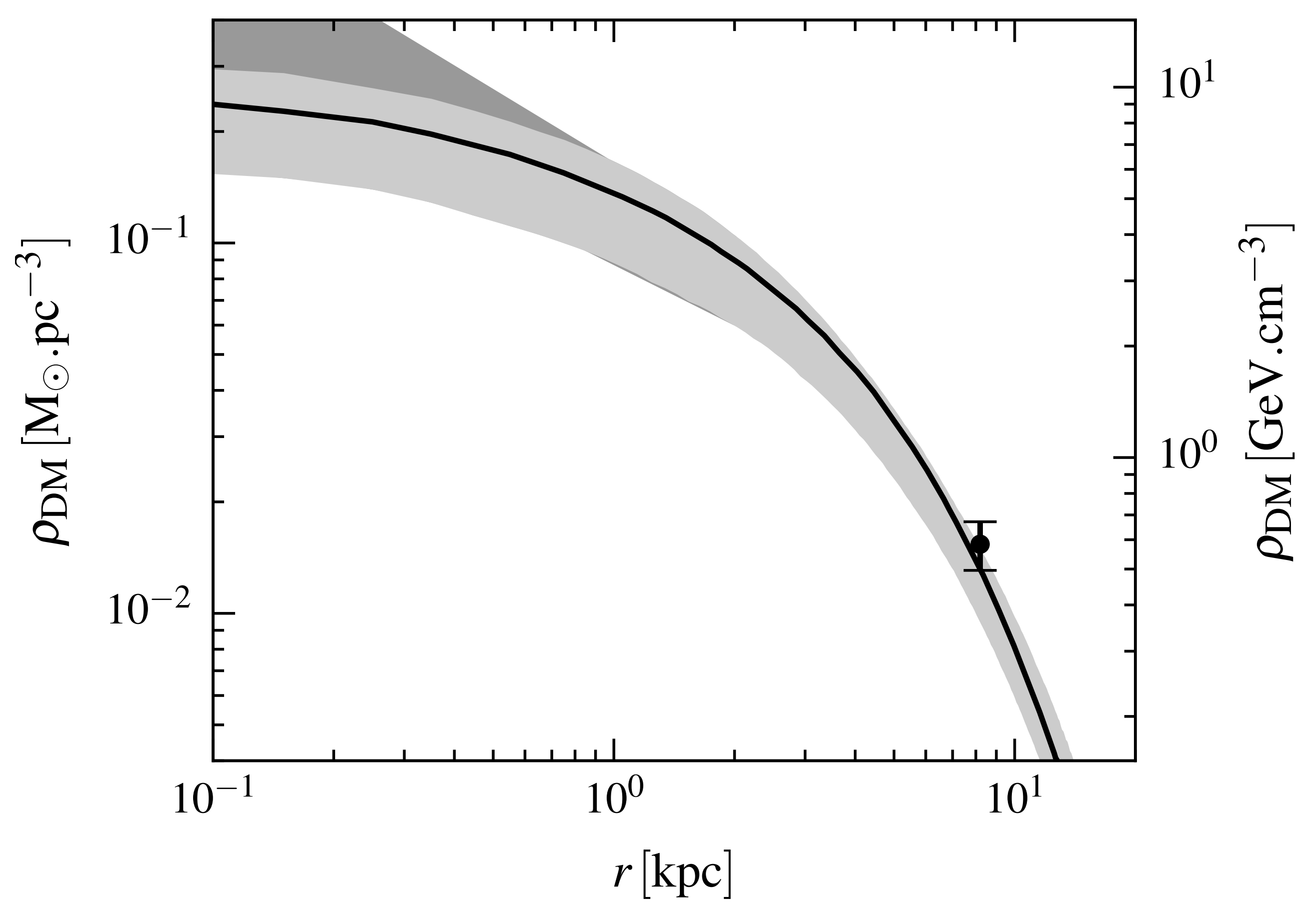
n.b. Standard halo has one-to-one relationship between circular speed and velocity dispersion & peak speed, but in general this isn't the case.

dark matter density profile: Milky Way

Hard to measure inner slope, as baryons dominate in inner regions.

Large measured microlensing optical depth towards Galactic centre implies large stellar density in inner MW, in tension with the high DM densities of ‘cuspy’ density profiles. Binney & Evans

Dynamical modelling, taking into account various observational constraints on the galactic bulge and bar find a dark matter density profile which flattens to a shallow cusp or core in inner regions: Portail et al.

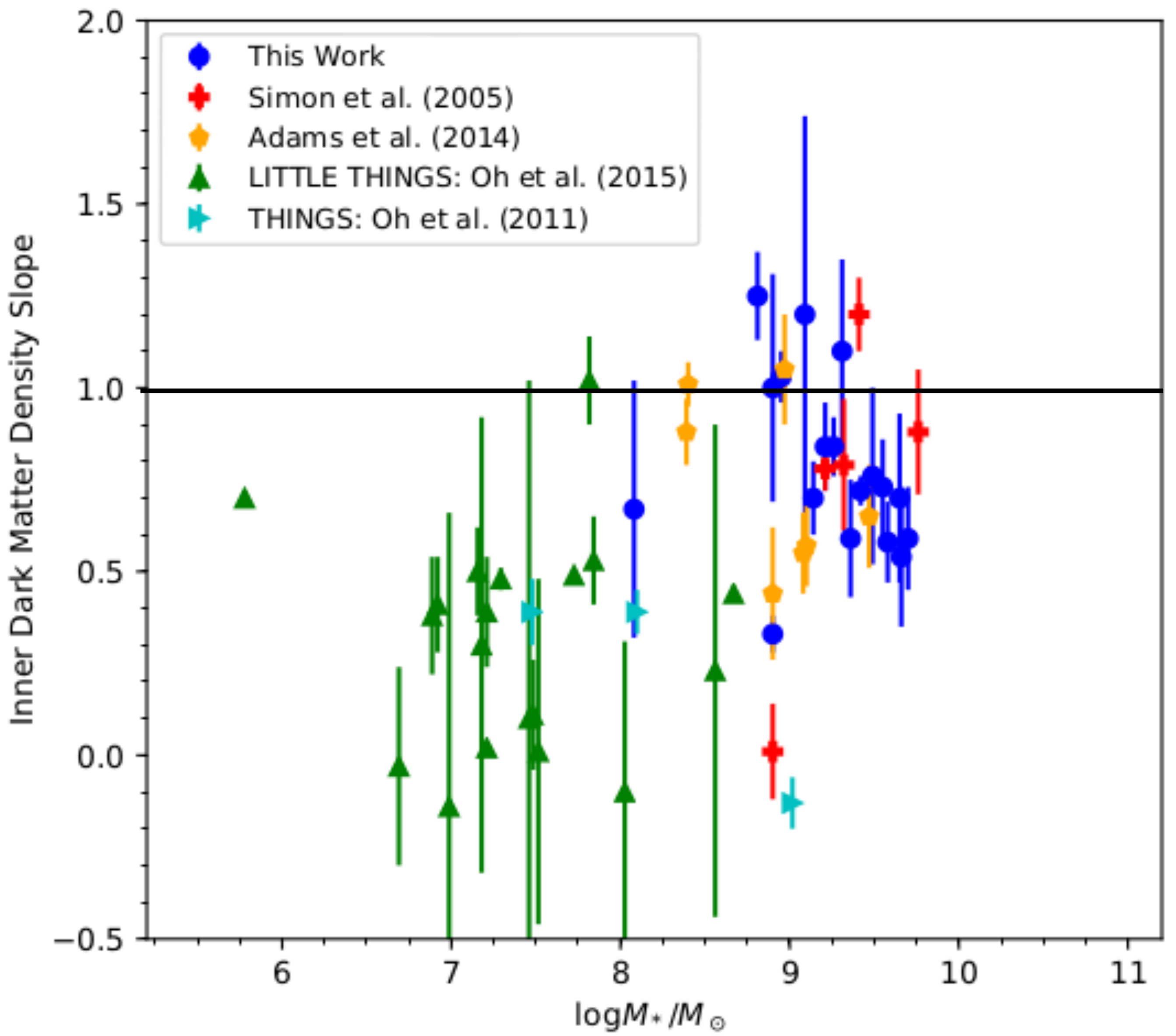


dark matter density profile: dwarf galaxies

Compilation of rotation curve measurements of inner slope:
Relatores et al.

$$\gamma = -\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}$$

at $r = 0$



NFW

stellar mass

Local escape speed:

Piffl et al: high velocity stars from the RAVE survey,

assume $f(|\mathbf{v}|) \propto (v_{\rm esc} - |\mathbf{v}|)^k$ in tail of distribution

k in range 2.3 to 3.7 (motivated by numerical simulations)

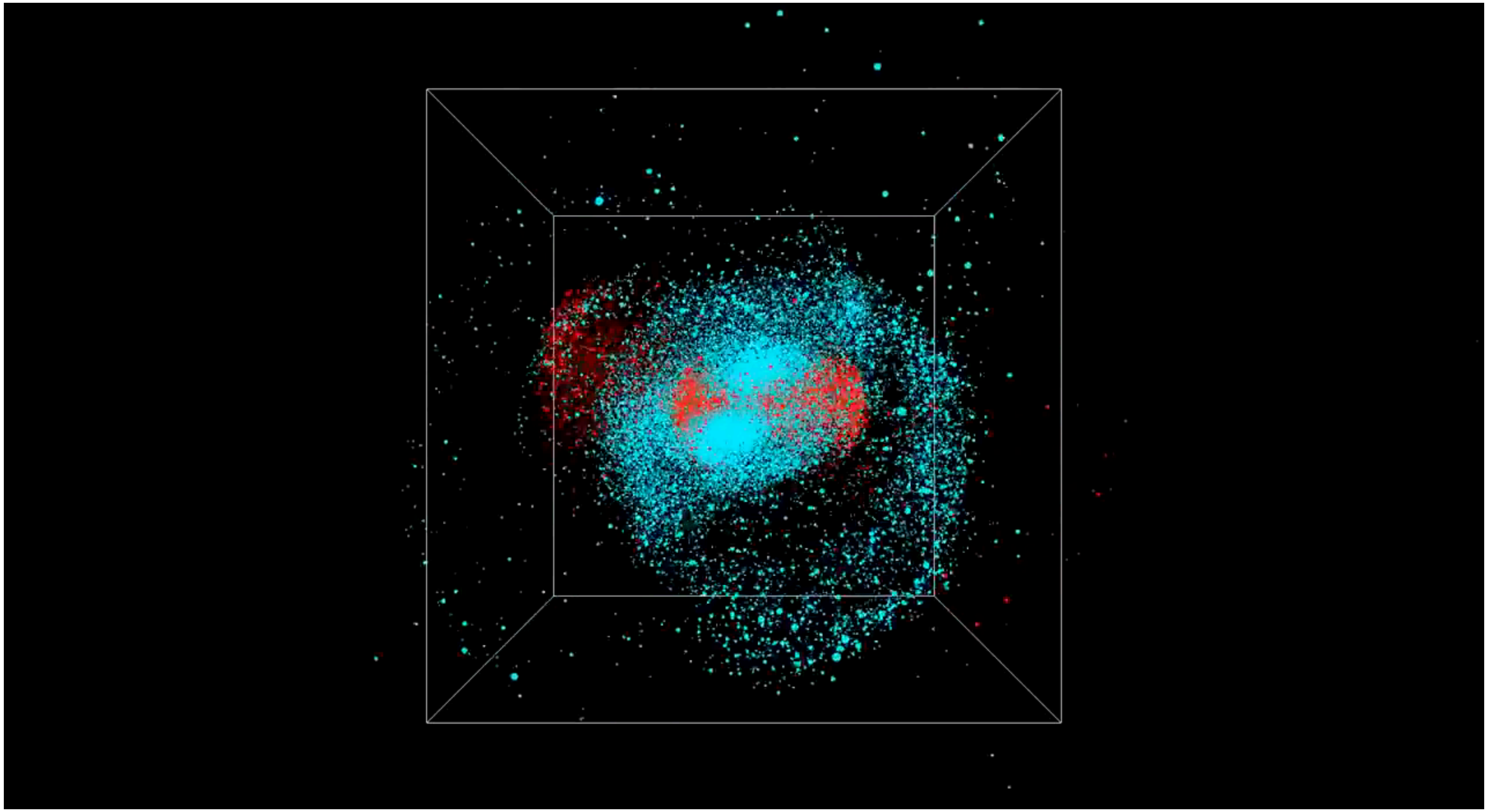
$$v_{\rm esc}(R_\odot) = 533^{+54}_{-41}\,{\rm km\,s^{-1}}$$

Monari et al. similar approach using Gaia Data Release 2, but without assuming a potential in modelling

$$v_{\rm esc}(R_\odot) = (580 \pm 63)\,{\rm km\,s^{-1}}$$

Gaia-Enceladus/Sausage

The aftermath of a major merger with a $\sim 10^{11} M_{\odot}$ dwarf galaxy (8-10) Gyr ago
Helmi et al.



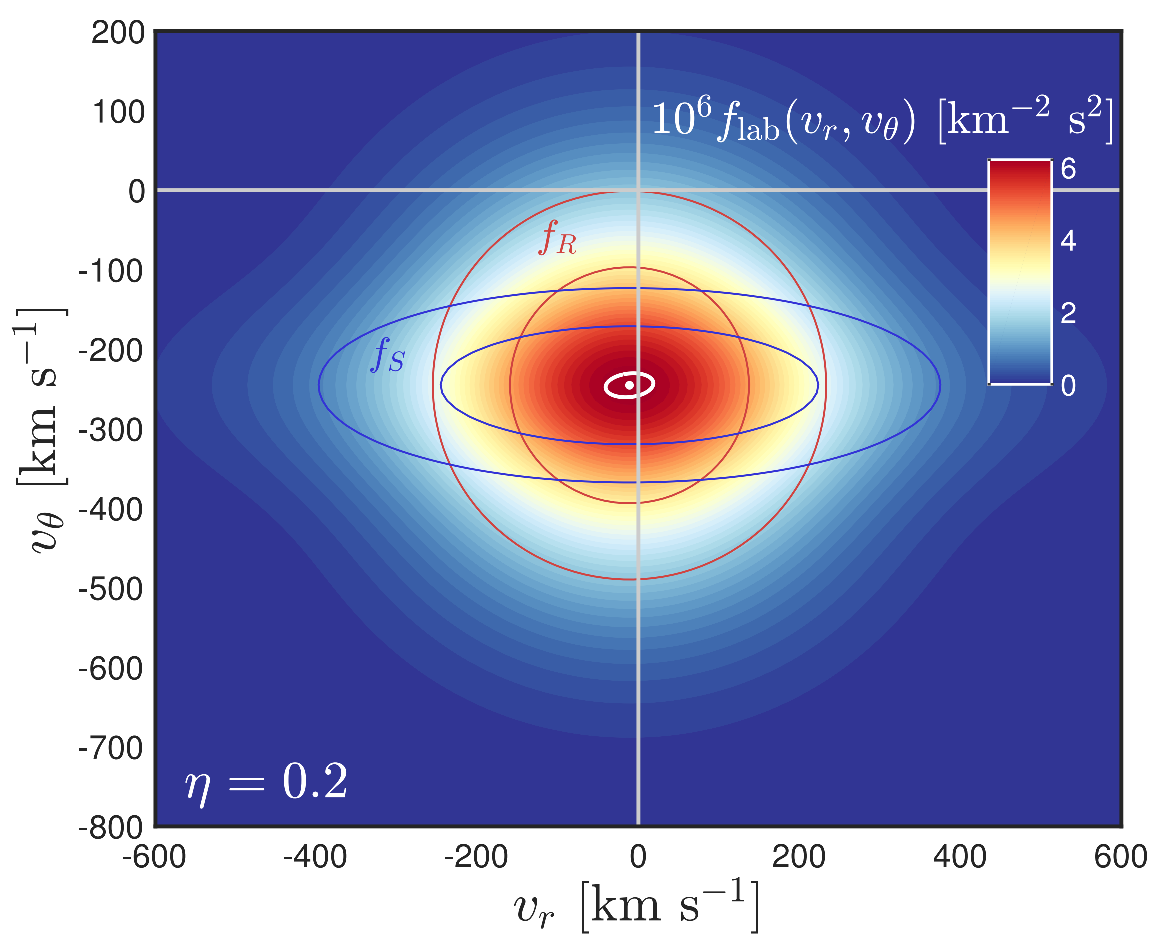
Simulation: Koppelman, Villalobos & Helmi

Gaia-Enceladus/Sausage

A significant fraction of the halo near the Sun is made up of debris from a major merger with a $\sim 10^{11} M_\odot$ dwarf galaxy (8-10) Gyr ago. Helmi et al.

Stars have radially biased orbits, distribution of v_r is 'sausage like'. Belokurov et al.

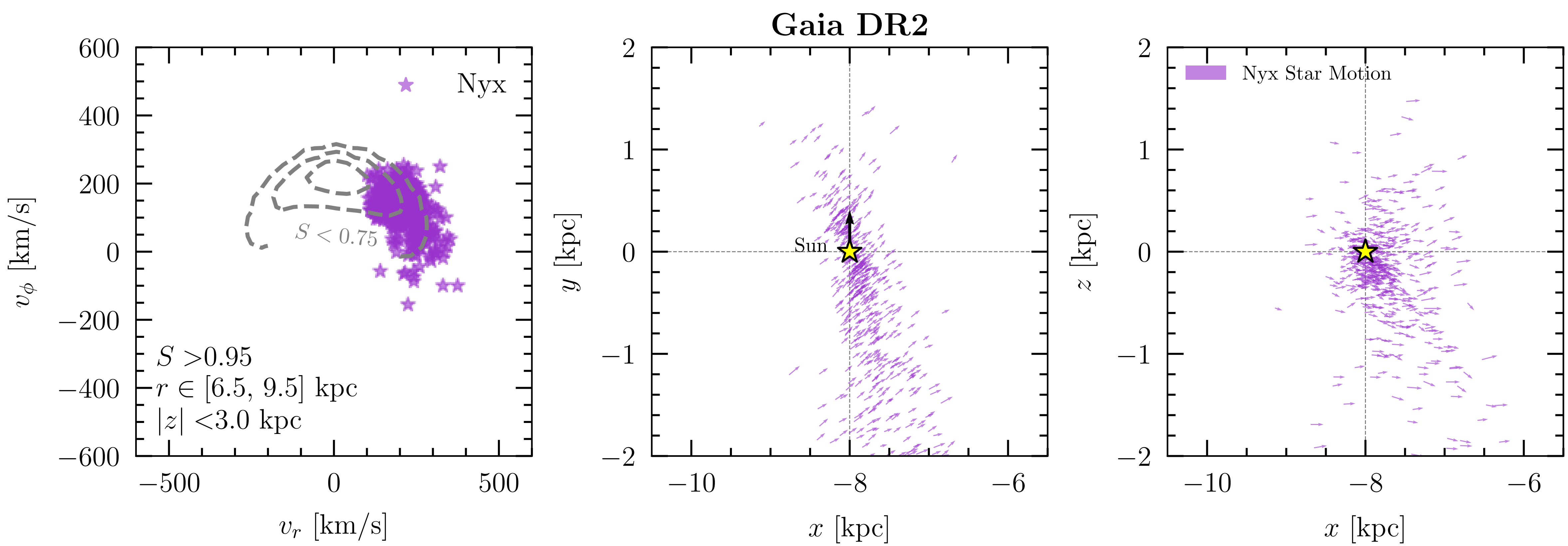
Fraction of local dark matter density it makes up is ~(10-30)% (see e.g. Evans et al., also for effects on WIMP and axion direct detection experiments).



Tidal streams

Also less extended tidal streams from smaller or more recent mergers.

In the Solar neighbourhood: S1, Helmi streams, Nyx,



Nyx

More on how tidal streams throughout the MW halo, and also subhalo fraction, can be used to probe the nature of dark matter in section 4

Small scale challenges

See Bullock & Boylan-Kolchin and Annika Peter's lectures.

Since 1990s, discrepancies between CDM predictions and observations

Cusp-core: DM only simulations produce halos with cuspy inner density profiles ($\rho(r) \propto r^{-\gamma}\,,\ \gamma \approx 1$) while galaxies, in particular low mass DM dominated dwarfs, have shallower profiles ($\gamma \sim 0$, core).

Missing satellites: simulated MW-size halos contain ~1000 of dwarf galaxy sized sub-halos, but 'only' ~50 dwarf galaxies have been observed (n.b. observations 'incomplete': not all of the dwarfs that exist have been observed).

'too-big-to-fail': too few medium sized ($M_{\rm dm} \sim 10^{10}\, M_{\odot}$) galaxies observed (and it's harder to explain a deficit of these larger galaxies).

These discrepancies could be resolved by

i) better understanding/modelling of baryonic physics

ii) non-vanilla DM

Summary

Simulations

Halos in dark matter only simulations have cuspy NFW/Einasto density profiles and contain lots of substructure (with more subhalos at large radii).

Hydrodynamic simulations (which include baryonic physics) find:

- density profiles are steepened, but small inner density constant cores can form,
- local velocity distribution is fairly close to Maxwellian of standard halo model, but features in high speed tail from incompletely phase-mixed material,
- fewer subhalos survive in inner region.

Observations

Recent observational determinations of the local density and circular speed have small statistical errors e.g.

$$\rho(R_\odot) = (0.36 \pm 0.02)\,\mathrm{GeV\,cm^{-3}}$$
$$v_\mathrm{c}(R_\odot) = (229.0 \pm 0.2)\,\mathrm{km\,s^{-1}}$$

but systematic errors are larger.

Historical 'standard' values ($\rho(R_\odot) = 0.3\,\mathrm{GeV\,cm^{-3}}, \;\; v_\mathrm{c} = 220\,\mathrm{km\,s^{-1}}$) are probably on the low side.

Local escape speed is harder to measure.

Substructure in spatial & velocity distribution of stellar halo: Gaia Enceladus/ sausage and streams

Inner slopes of density profiles shallower than NFW.

Next section: constraints on the nature of dark matter

Backup slides

Dark disc

Sub-halos merging at z<1 preferentially dragged towards disc, where they're destroyed leading to the formation of a co-rotating dark disc. [Read et al., Bruch et al., Ling et al.]

Detailed properties (& existence?) of dark disc are very uncertain.

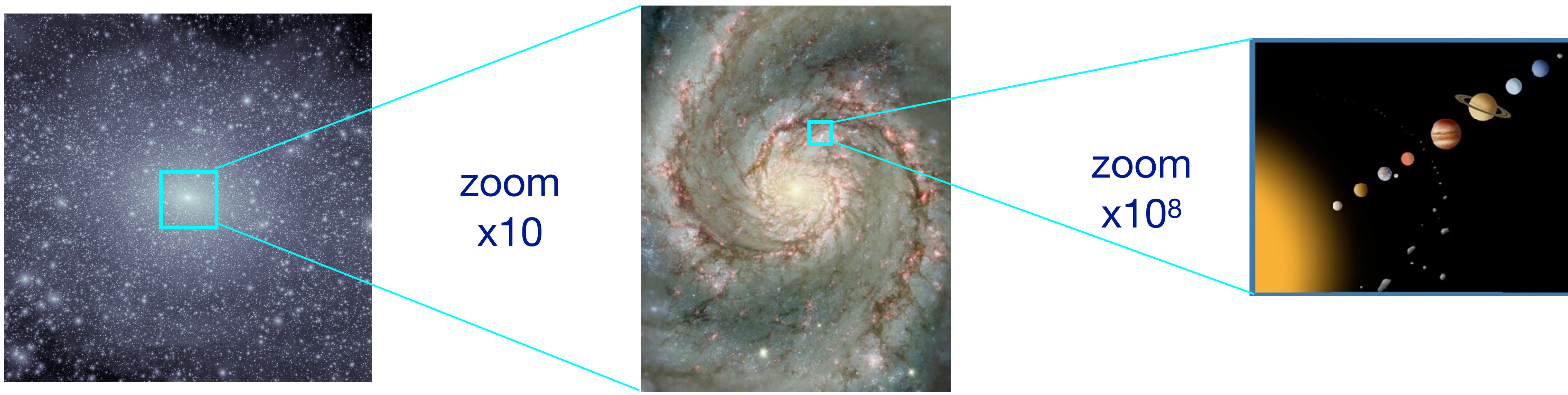
Purcell, Bullock and Kaplinghat argue that to be consistent with the observed properties of thick disc, MW's merger history must be quiescent compared with typical ΛCDM merger histories, hence the DD density must be relatively low.

Eris simulation (Guedes et al.): dark disc contributes ~10% of local density. But more recent sims no significant dark disc.

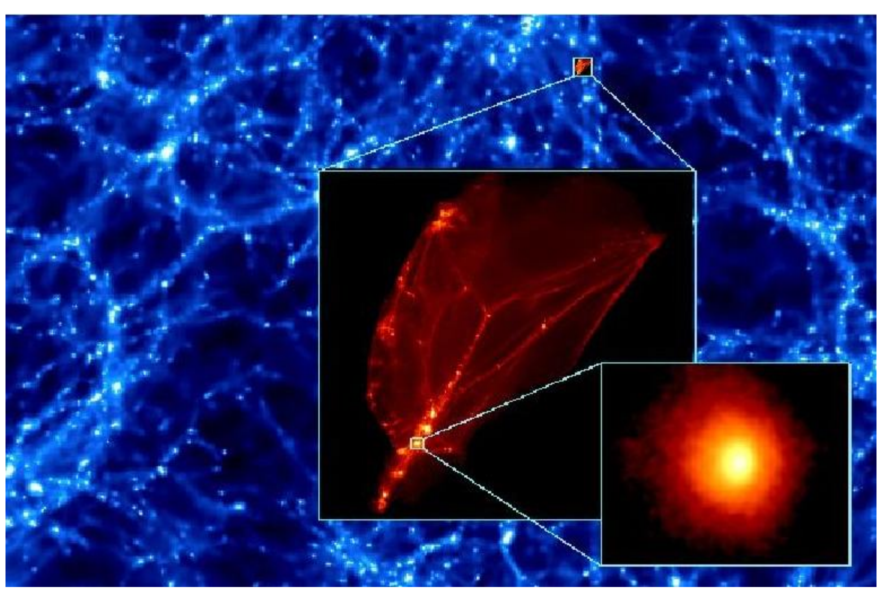
Ruchti et al.: no sign of stellar component in GAIA-ESO survey.

Caveat:

scales resolved by simulations are many orders of magnitude larger than those probed by direct detection experiments



~300 kpc ~30 kpc ~0.3 mpc



microhalo simulation
Diemand, Moore & Stadel

Resolution of best Milky Way simulations is many orders of magnitude larger than the first (~Earth mass) WIMP microhalos to form.

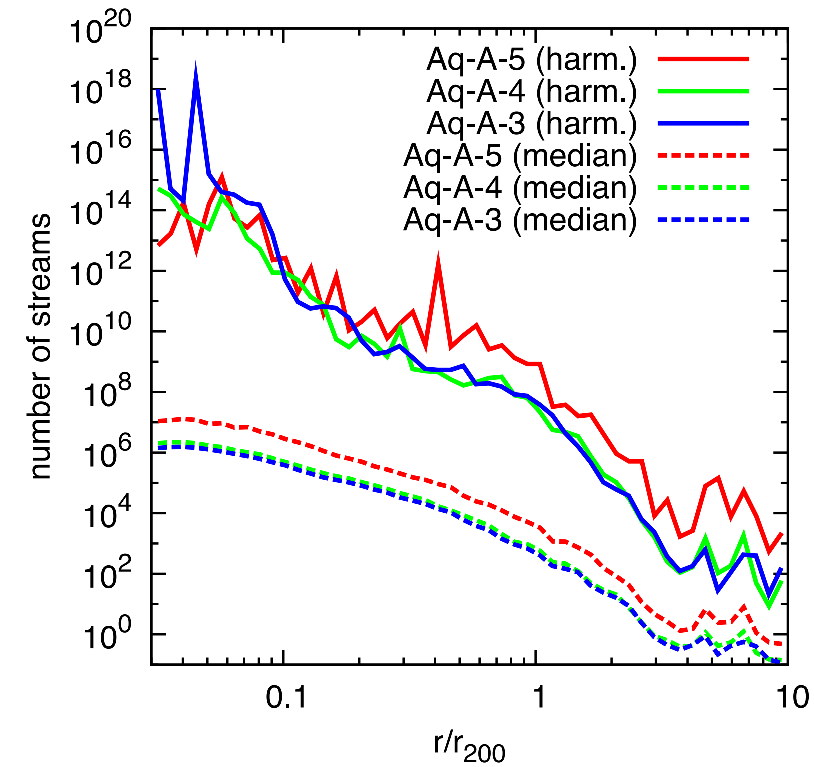
fine structure in ultra-local DM velocity distribution?

Vogelsberger & White:

Follow the fine-grained phase-space distribution, in Aquarius simulations of Milky Way like halos.

From evolution of density deduce ultra-local DM distribution consists of a huge number of streams.

At solar radius <1% of particles are in streams with $\rho > 0.01\rho_0$.



number of streams as a function of radius

calculated using harmonic mean/median stream density

Schneider, Krauss & Moore:

Simulate evolution of microhalos. Estimate tidal disruption and heating from encounters with stars, produces 10^2-10^4 streams in solar neighbourhood.