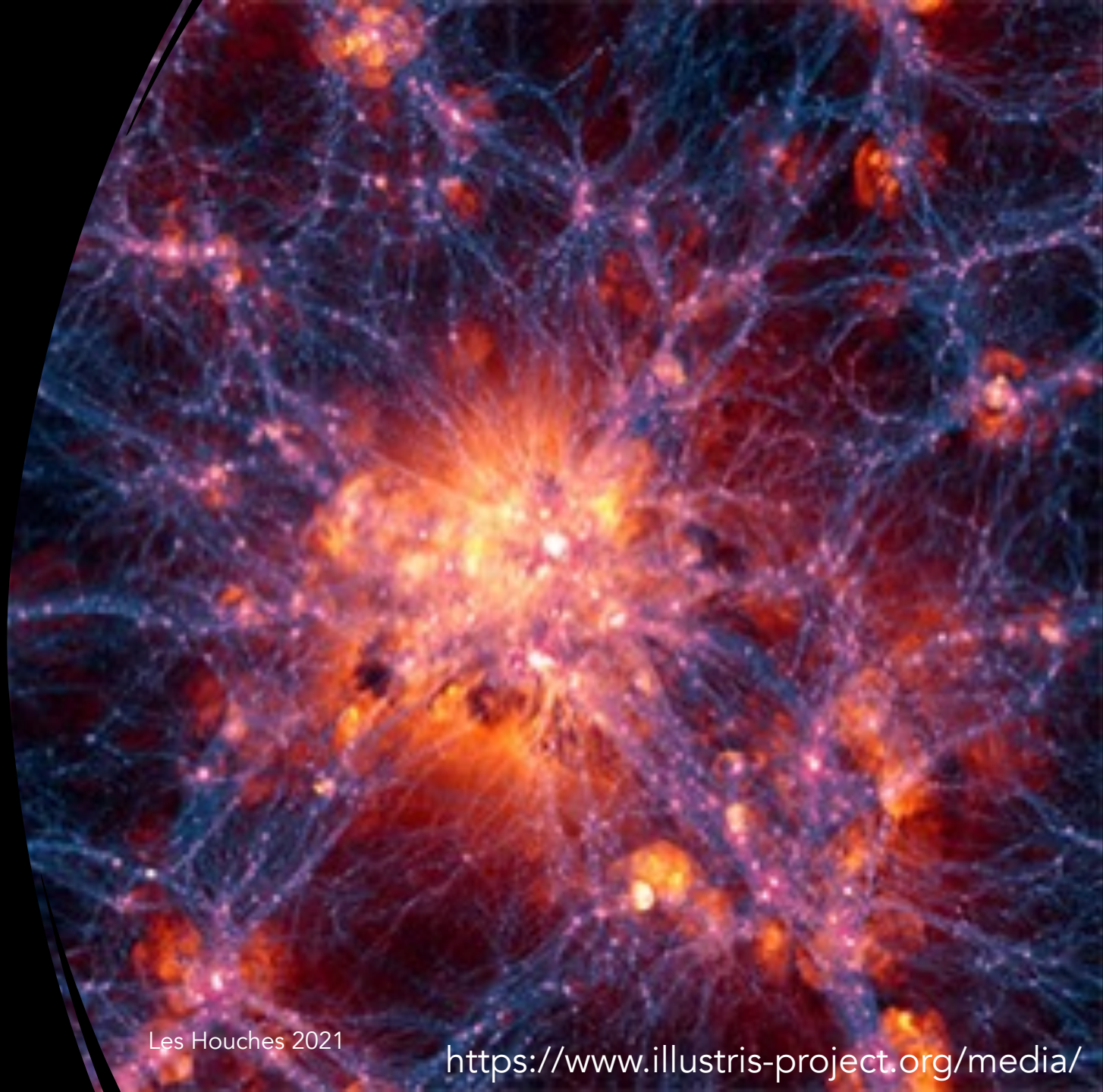
So you want to run a simulation…

Annika Peter
The Ohio State University



Les Houches 2021

https://www.illustris-project.org/media/

How to use these notes

- I have added some notes with additional references in the PowerPoint slides.
- HOWEVER, the bulk of the references are in the separate reading list document in the Google Drive. Those references are meant as starting points for your own literature safari.
- There are some ideas for ways to get started in the exercises directory.

These lectures in a meme

The fabulous Drake
Ruth Durrer lecture notes:
https://arxiv.org/pdf/astro-ph/0402129.pdf
Ohio Supercomputer Center

Perturbation theory

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon a^2 h_{\mu\nu} \tag{12}$$

$\bar{g}_{\mu\nu}$ being the unperturbed Friedmann metric. We conventionally set (absorbing the "smallness" parameter ε into $h_{\mu\nu}$)

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu} + a^2 h_{\mu\nu}, \quad \bar{g}_{00} = -a^2, \quad \bar{g}_{ij} = a^2 \gamma_{ij} \quad |h_{\mu\nu}| \ll 1 \\ T^\mu_\nu &= \overline{T}^\mu_\nu + \theta^\mu_\nu, \qquad \overline{T}^0_{\ 0} = -\bar{\rho}, \quad \overline{T}^i_{\ j} = \bar{p}\delta^i_j \qquad |\theta^\mu_\nu|/\bar{\rho} \ll 1. \end{aligned} \tag{13}$$

But τ^i_j contains in general perturbations. We set

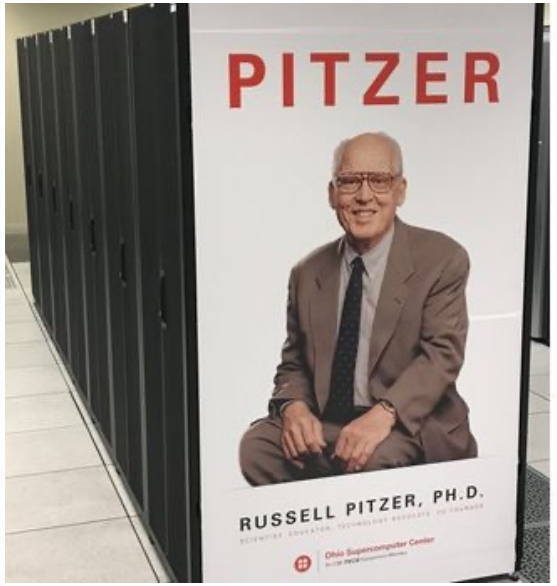
$$\tau^i_j = \bar{p}\left[(1+\pi_L)\,\delta^i_j + \Pi^i_j\right], \quad \text{with} \quad \Pi^i_i = 0. \tag{53}$$

We decompose Π^i_j as

$$\Pi^i_j = \Pi^{(S)} Y^{(S)\,i}_j + \Pi^{(V)} Y^{(V)\,i}_j + \Pi^{(T)} Y^{(T)\,i}_j. \tag{54}$$

We shall not derive the gauge transformation properties of these perturbation variables in detail, but just state some results which can be obtained as an exercise (see also [5]):

Simulation



When to go to a computer?

Your system is non-linear.

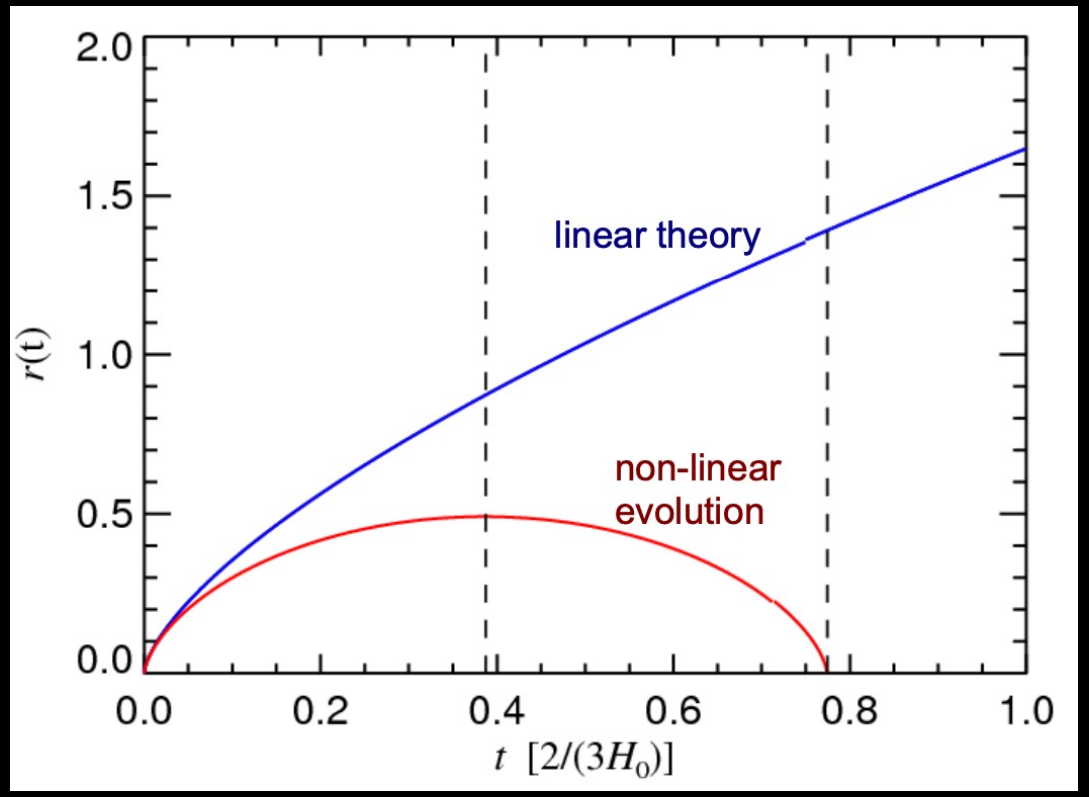
Your system is chaotic.

Perturbation theory breaks down.

See Springel video

- https://wwwmpa.mpa-garching.mpg.de/galform/data_vis/index.shtml

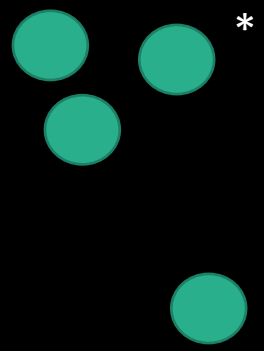
Springel lecture notes (see notes for link)



Goal

1.

Initial conditions



*

$\{q_i(t=0), p_i(t=0)\}$

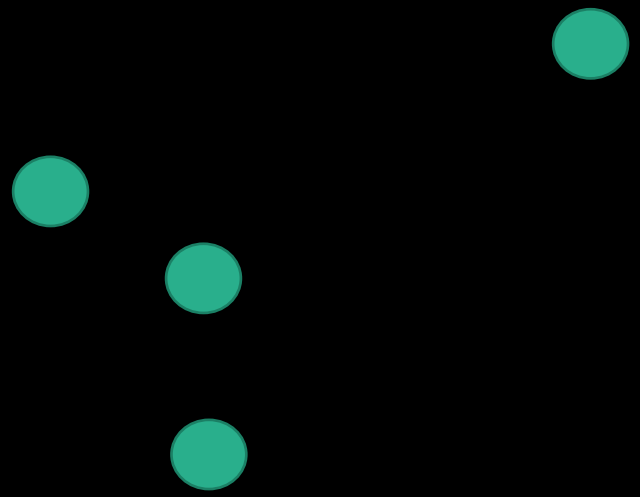
Position and momentum coordinates

2.

Some forces

3.

Conditions at some other time

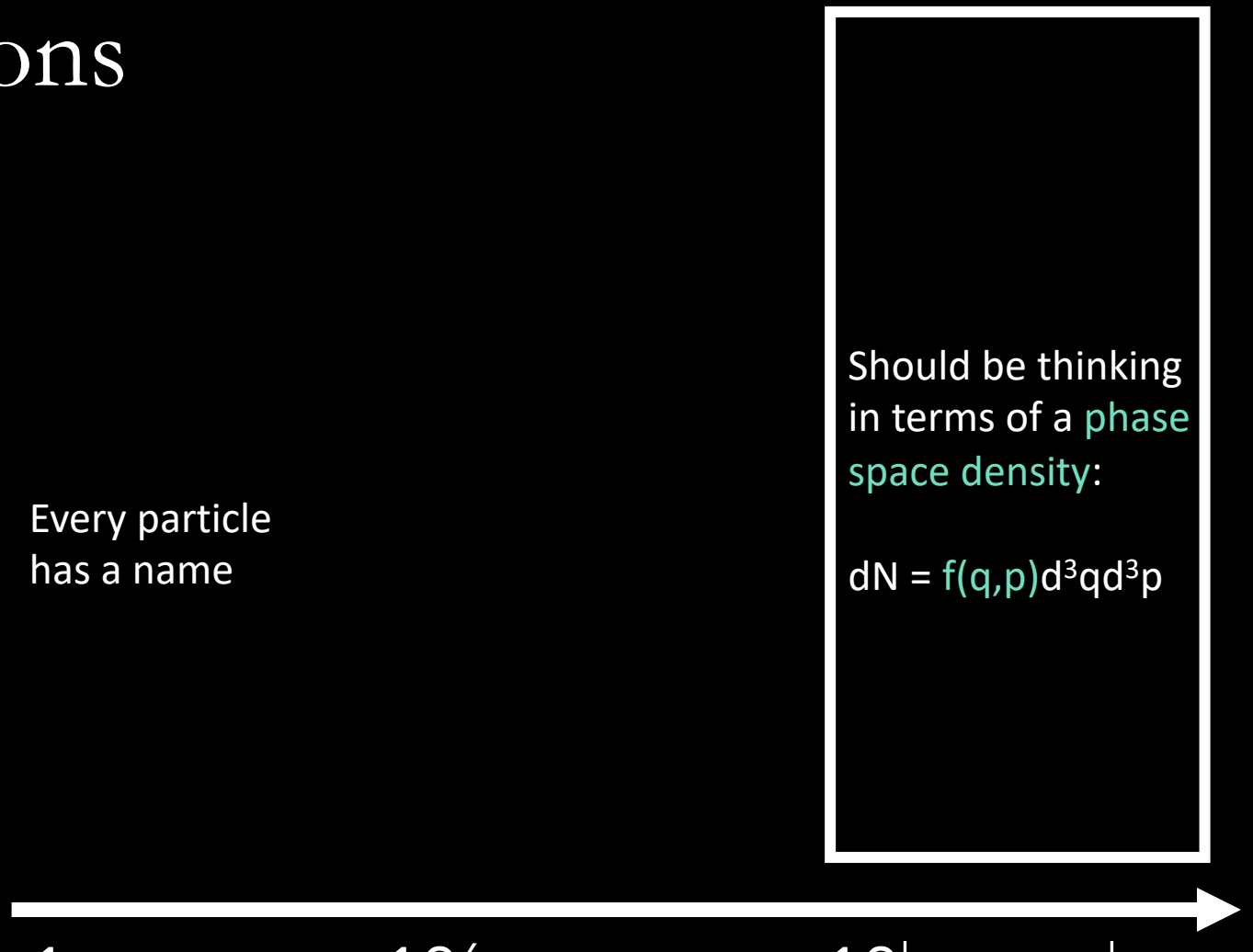


$\{q_i(t=whatever), p_i(t=whatever)\}$

*Assumes particles here, can be waves

How do we figure this out?

Considerations

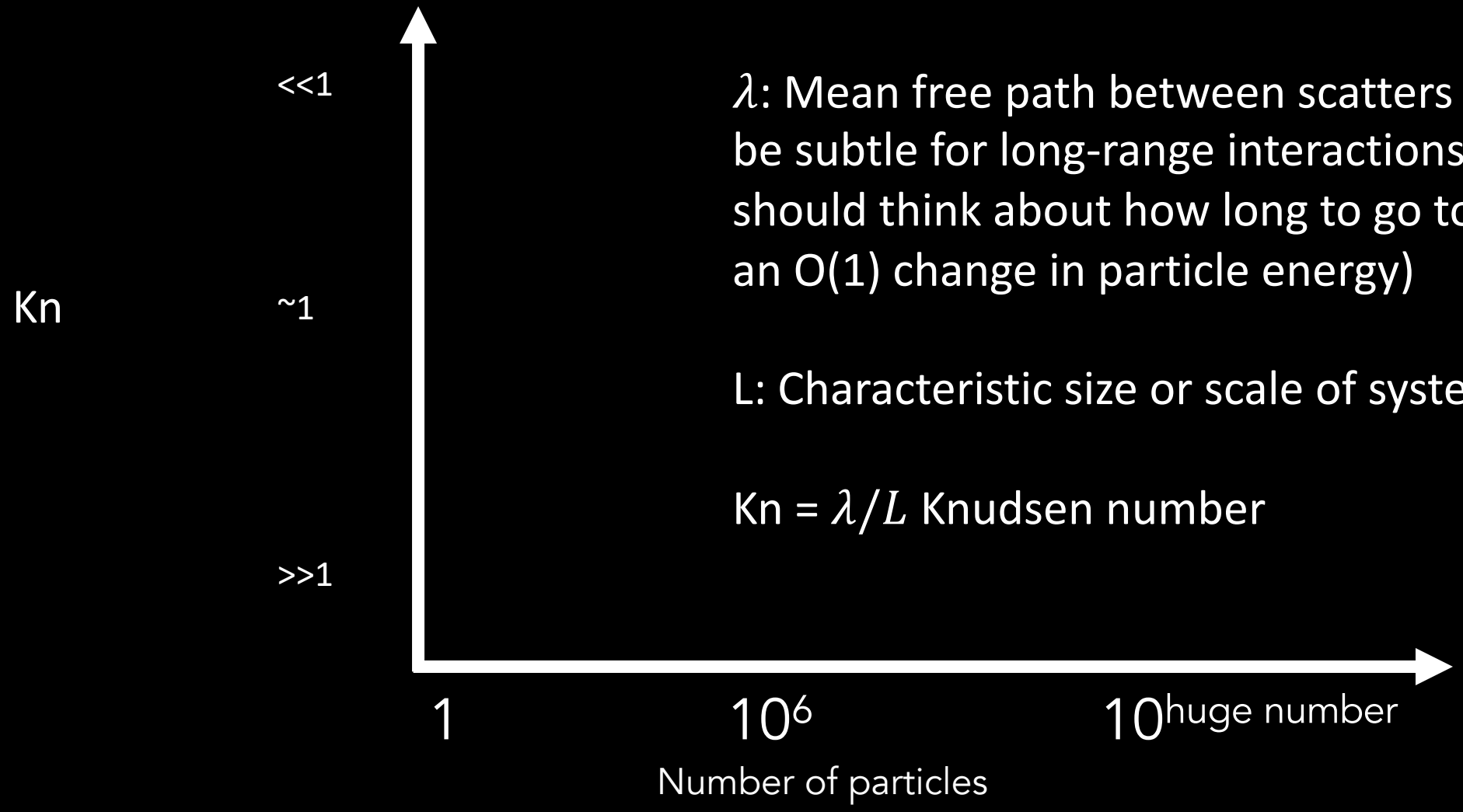


Advanced-level consideration: high occupancy in states or not?

Low occupancy: WIMPs, treat as particles

High occupancy: Axion-like particles, treat as waves

Considerations

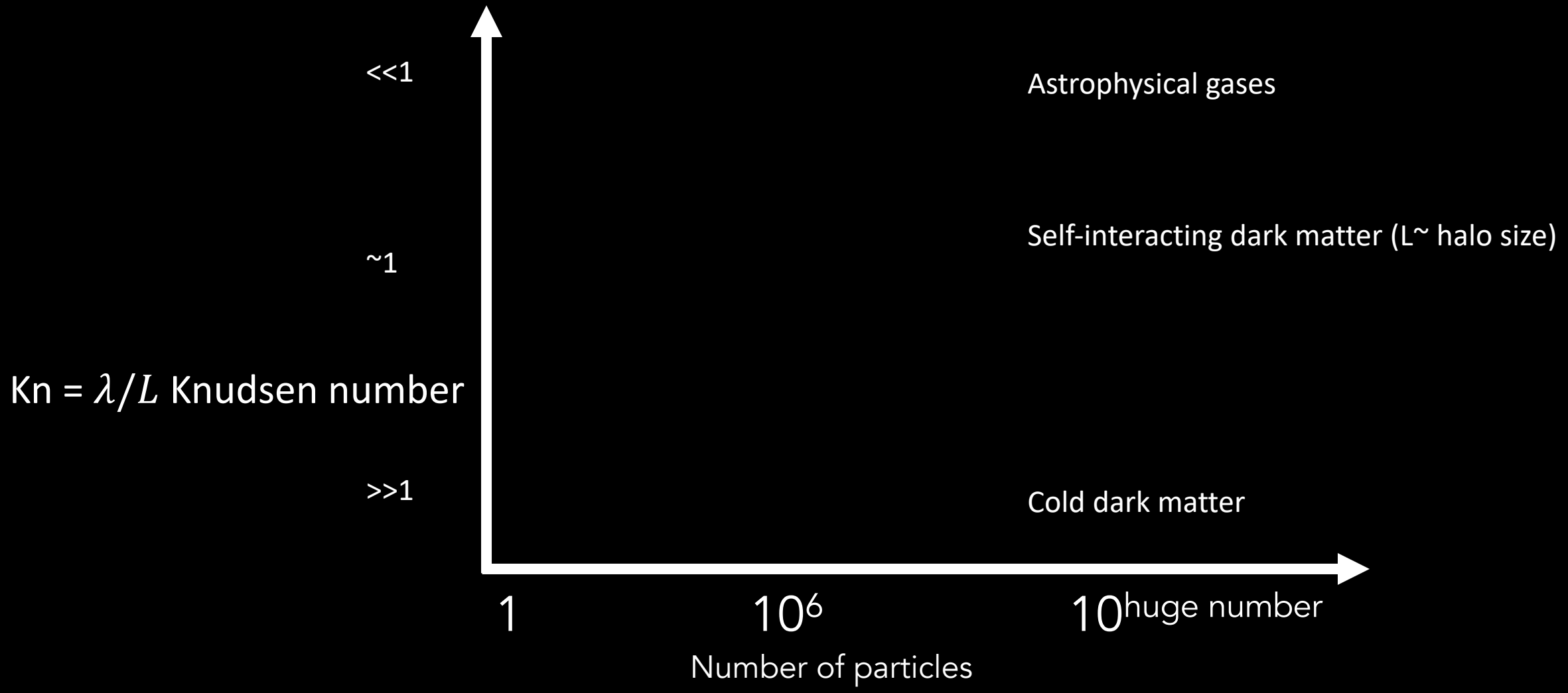


λ: Mean free path between scatters (can be subtle for long-range interactions, should think about how long to go to get an O(1) change in particle energy)

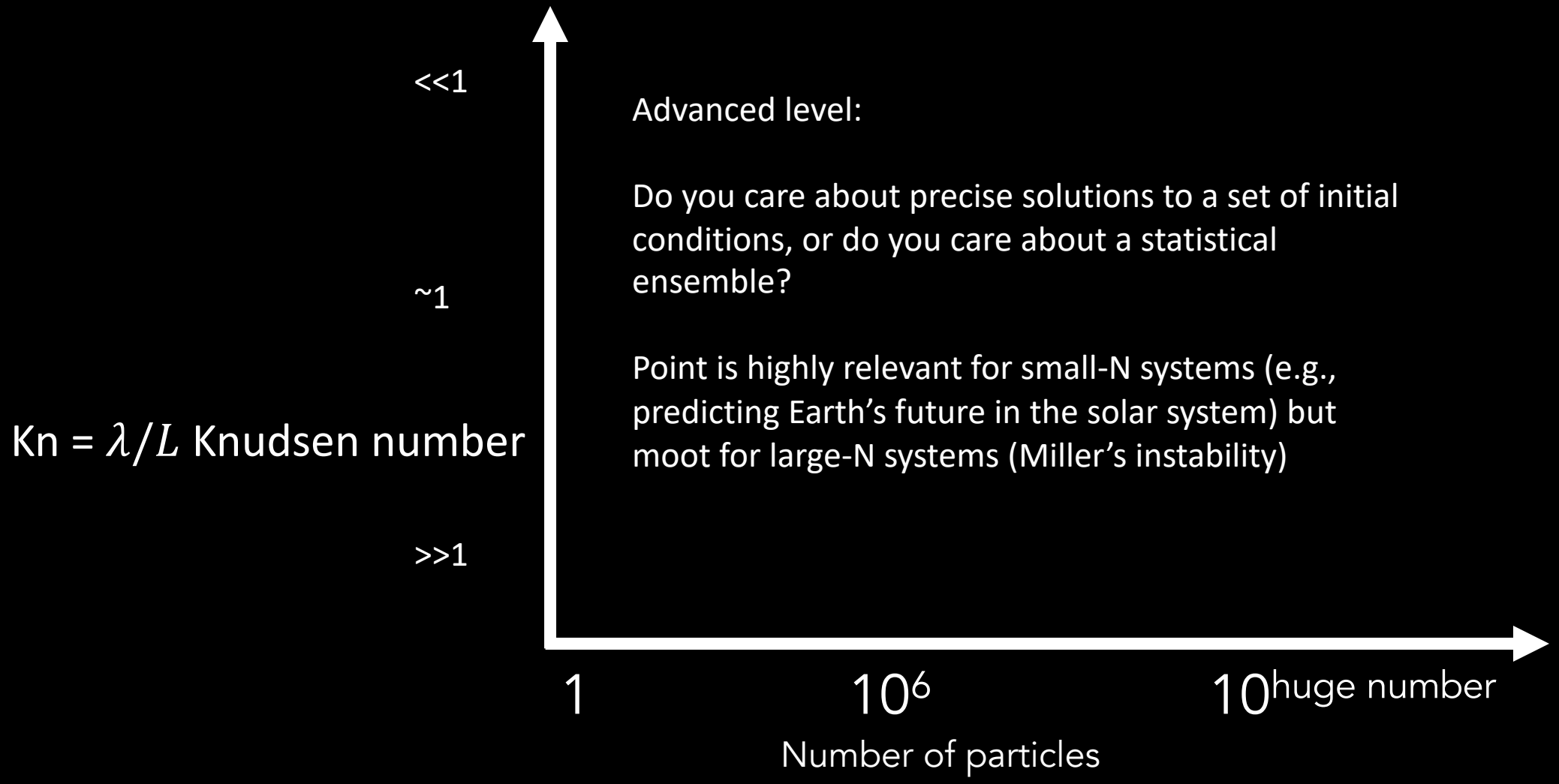
L: Characteristic size or scale of system

Kn = λ/L Knudsen number

Considerations



Considerations

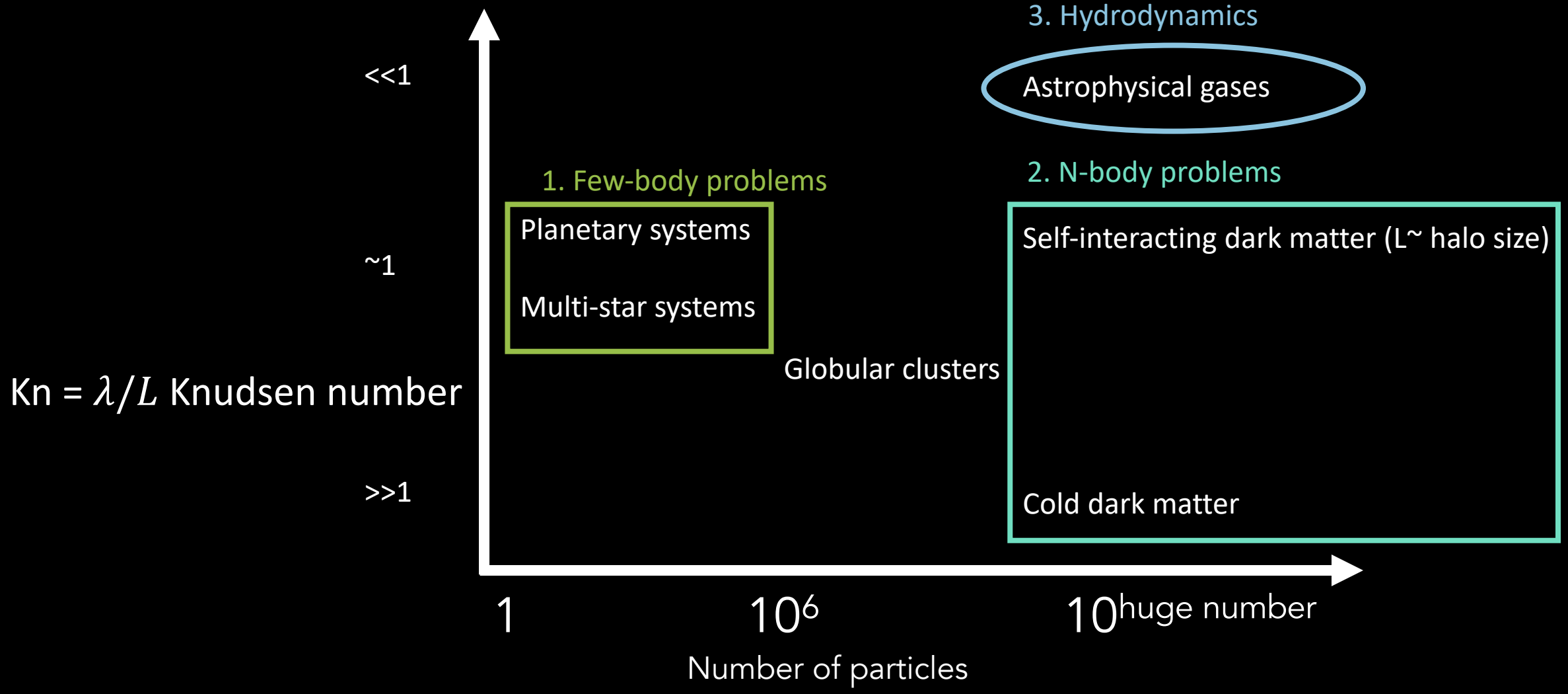


Advanced level:

Do you care about precise solutions to a set of initial conditions, or do you care about a statistical ensemble?

Point is highly relevant for small-N systems (e.g., predicting Earth's future in the solar system) but moot for large-N systems (Miller's instability)

Considerations



Problems and approaches

Few-body gravitational problems

N-body problems

Hydrodynamics

Problems and approaches

Few-body gravitational problems

N-body problems

Hydrodynamics

Gravity is mean

2-body problem

Closed orbits only for the inverse square law and for orbits in constant-density systems (question for the reader: what potential/force law does this correspond to?)

https://en.wikipedia.org/wiki/Two-body_problem_in_general_relativity#/media/File:Elliptic_orbit.gif

Beautiful, classical, closed solution for non-relativistic gravity.

3+body problem

https://twitter.com/ThreeBodyBot

Literally chaos: when you have initial conditions extremely close together that end up in wildly different places at later times.

Dramatic example

Laskar + Gastineau 2009

Note: can only make predictions for ~O(10 Myr) that are reliable.

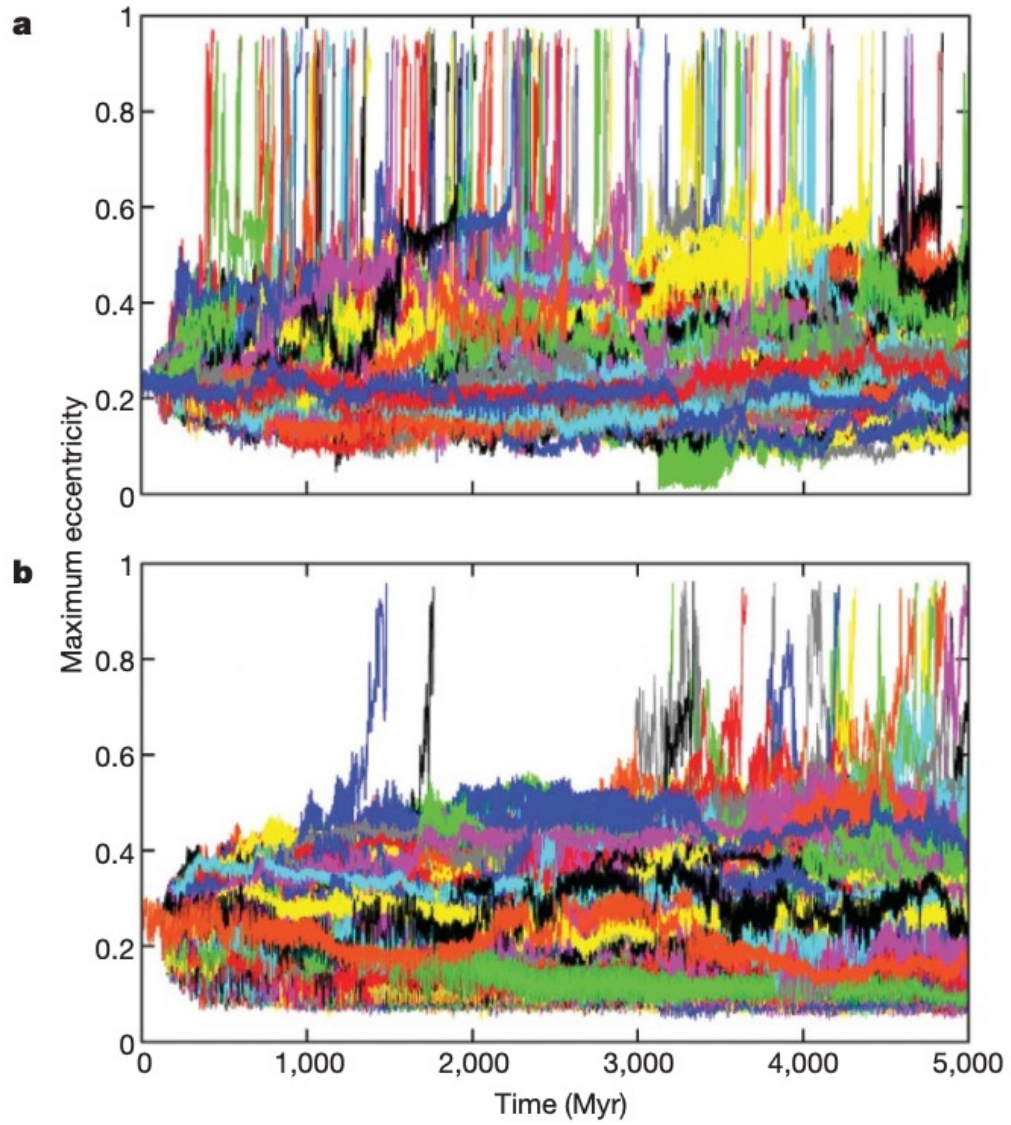


Figure 1 | Mercury's eccentricity over 5 Gyr. Evolution of the maximum eccentricity of Mercury (computed over 1-Myr intervals) over 5 Gyr. **a**, Pure Newtonian model without the contribution of the Moon, for 201 solutions with initial conditions that differ by only 3.8 cm in the semi-major axis of Mercury. **b**, Full Solar System model with relativistic and lunar contributions, for 2,501 solutions with initial conditions that differ by only 0.38 mm in the semi-major axis of Mercury.

Fundamental physics applications

Not dark matter but GR and compact objects

Joe Antognini: https://www.youtube.com/watch?v=Y3nPOQXl6co

See also papers like this; also by Fang, Thompson & Hirata

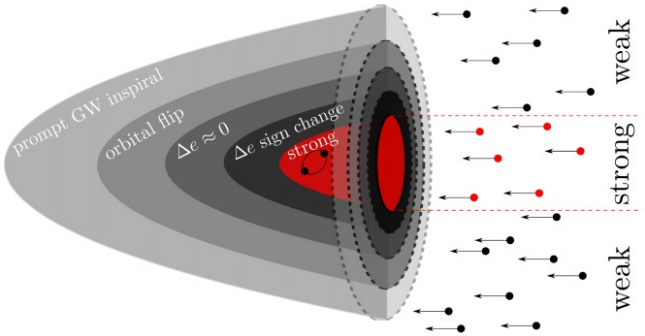


FIG. 2. Illustration of a binary interacting with an incoming population of single objects, where the strong interaction regime is highlighted in *red*, and the weak interaction regime in *grey*. The weak regime is further divided into different subregimes, each giving rise to a distinct type of outcome. The range of outcome types depends on the ICs, the shown here are the ones relevant for our fiducial parameters outlined in the end of Section II A. As described in Section II, the outcomes of a binary-single interaction with our chosen ICs can, from high to low values of $r_{\rm p}/a_0$, broadly be divided into the following regimes: (*i*) Distant weak interaction regime. Here the change in binary eccentricity is relatively small, and can be described using FO theory. For our ICs $\Delta e > 0$. (*ii*) Prompt GW inspiral regime. Here the final eccentricity is $e_{\rm fin} \approx 0$, which results in a prompt GW inspiral merger during the interaction. This also marks the limit below which SO theory has to be used. (*iii*) Orbital flip regime. Below

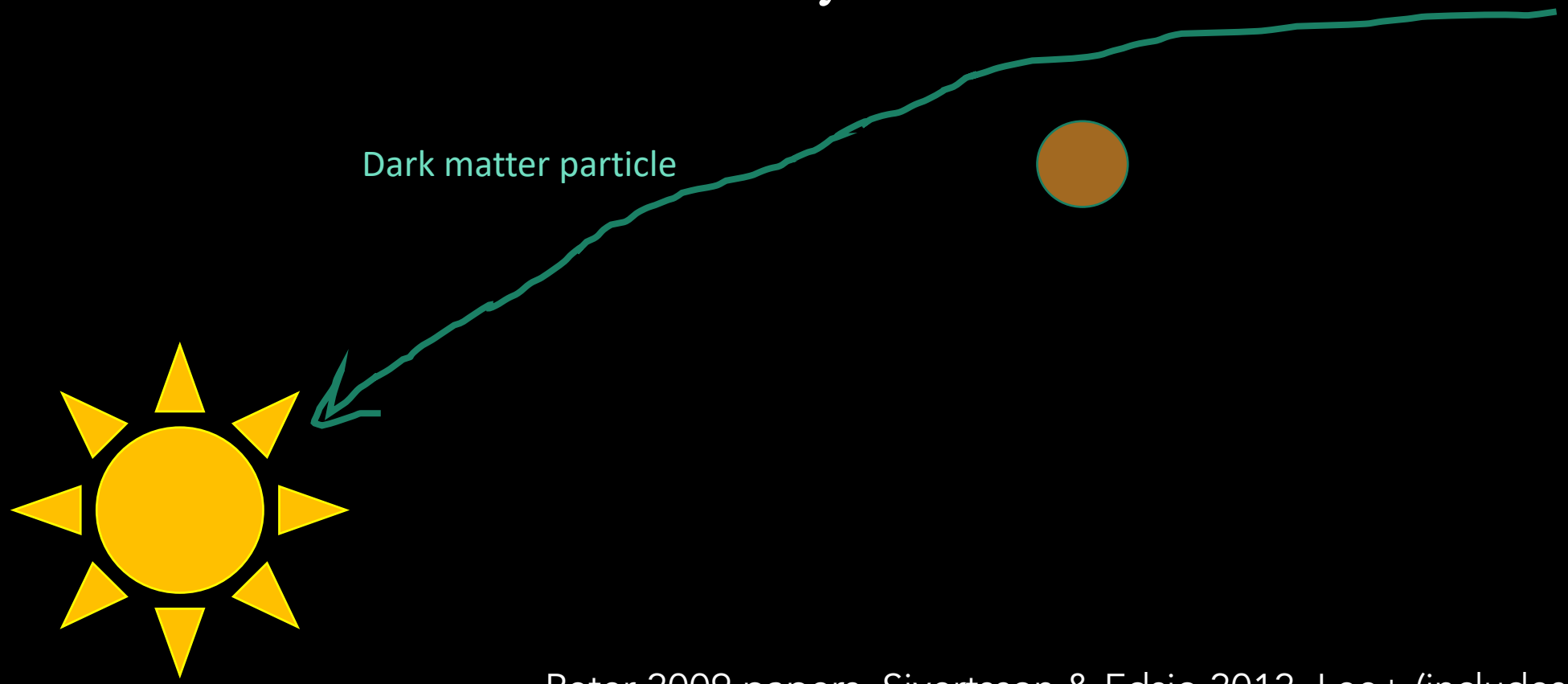
The Effect of Distant Encounters on Black Hole Binaries in Globular Clusters: Systematic Increase of In-cluster Mergers in the LISA band

Johan Samsing[1], Adrian S. Hamers[2], Jacob G. Tyles[1]

[1] *Department of Astrophysical Sciences, Princeton University, Peyton Hall, 4 Ivy Lane, Princeton, NJ 08544, USA.*
[2] *Institute for Advanced Study, School of Natural Sciences, Einstein Drive, Princeton, NJ 08540, USA.*

We present the first systematic study on how distant weak interactions impact the dynamical evolution of merging binary black holes (BBHs) in dense stellar clusters. Recent studies indicate that dense clusters are likely to significantly contribute to the rate of merging BBHs observable through gravitational waves (GWs), and that many of these mergers will appear with notable eccentricities measurable in the LISA and LIGO sensitivity bands. This is highly interesting, as eccentricity can be used to distinguish between different astrophysical merger channels. However, all of these recent studies are based on various Monte Carlo (MC) techniques that only include strong interactions for the dynamical evolution of BBHs, whereas any binary generally undergoes orders-of-magnitude more weak interactions than strong. It is well known that weak interactions primarily lead to a change in the binary's eccentricity, which for BBHs implies that weak interactions can change their GW inspiral time and thereby their merger probability. With this motivation, we perform MC simulations of BBHs evolving in dense clusters under the influence of both weak and strong interactions. We find that including weak interactions leads to a notable increase in the number of BBHs that merge inside their cluster, which correspondingly leads to a higher number of eccentric LISA sources. These preliminary results illustrate the importance of including weak interactions for accurately modeling how BBHs merge in clusters, and how to link their emitted GW signals to their astrophysical environment.

Dark matter in the solar system

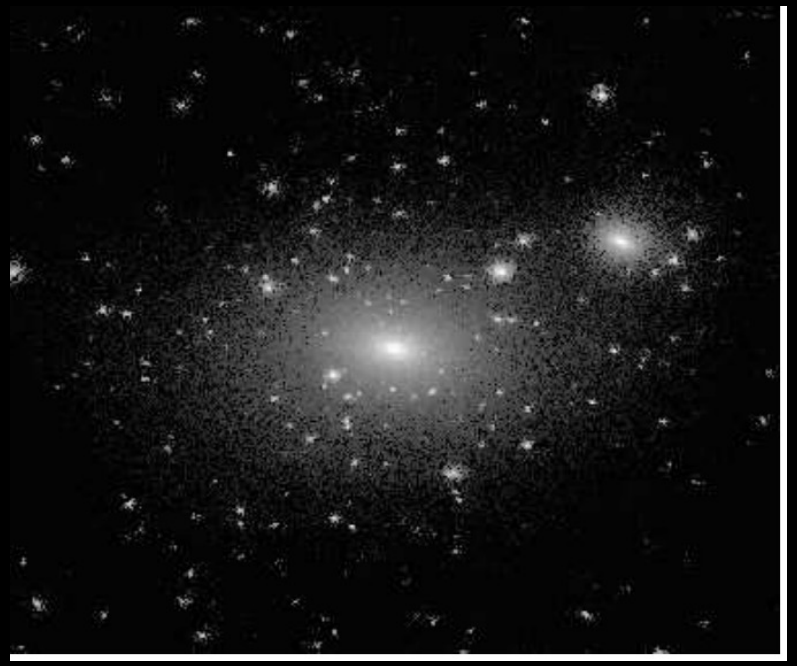


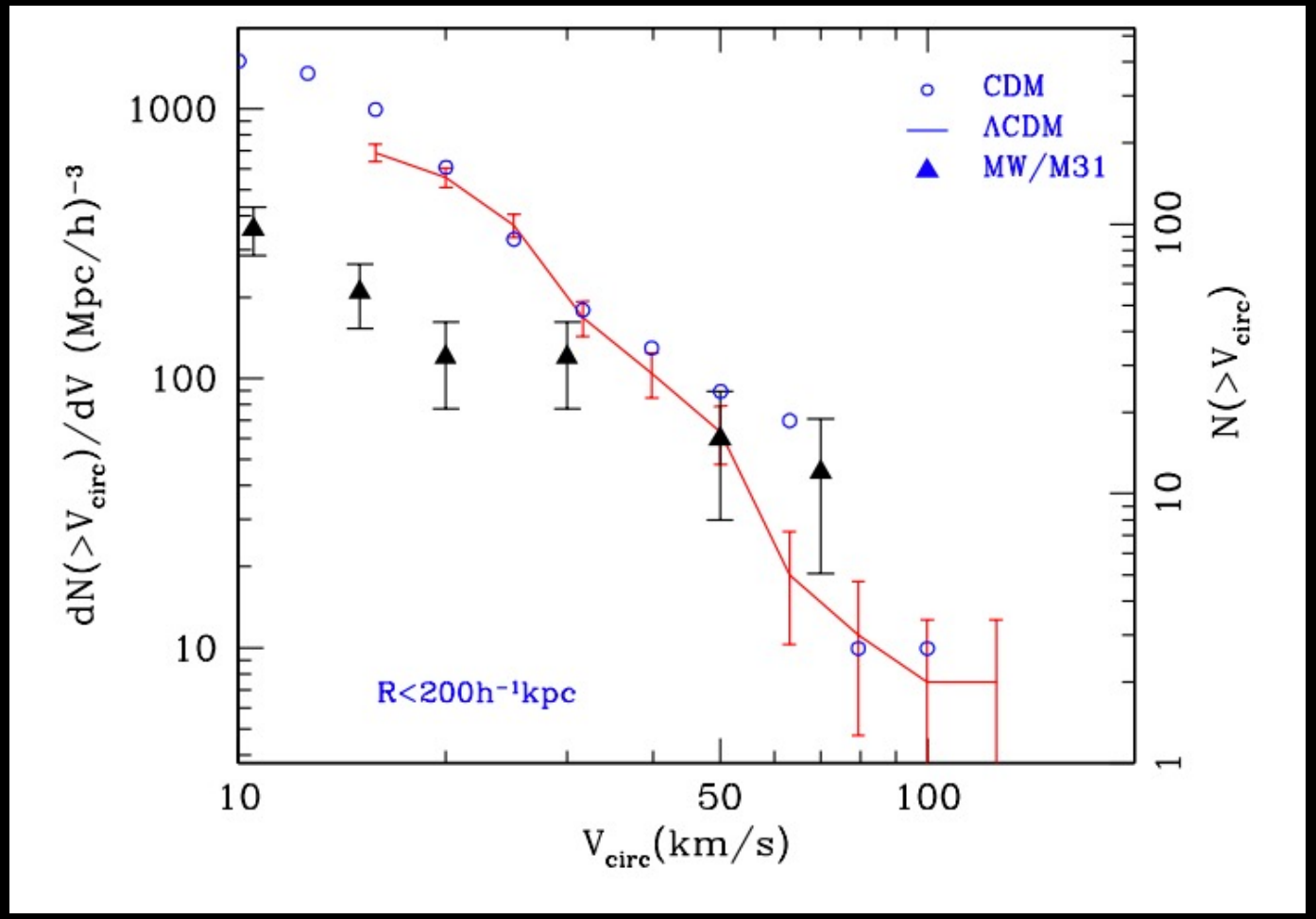
Peter 2009 papers, Sivertsson & Edsjo 2012, Lee+ (includes me!) on gravitational focusing from 2014

Original “missing satellites problem”

Klypin+ 9901240 →

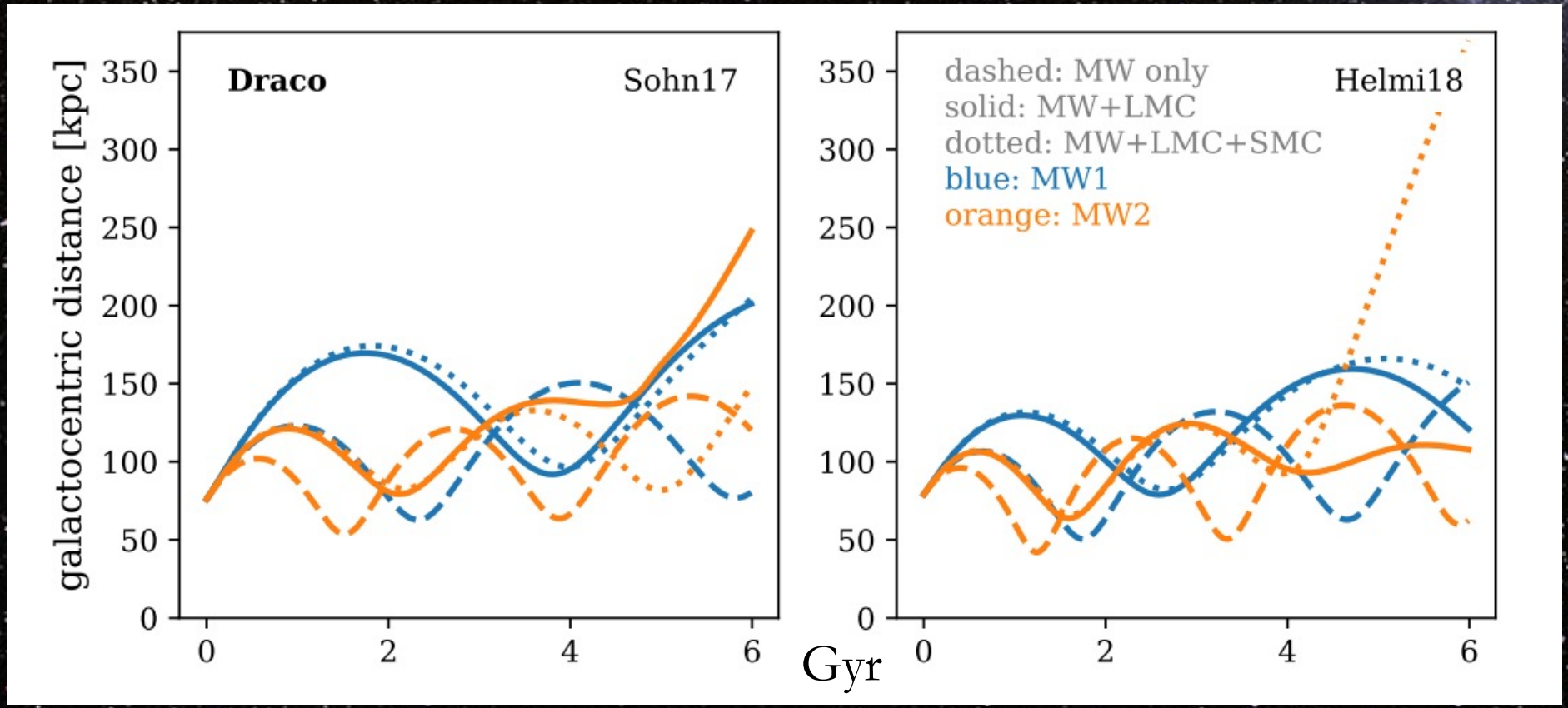
Moore+ 9907411





Milky Way (and Magellanic) satellites

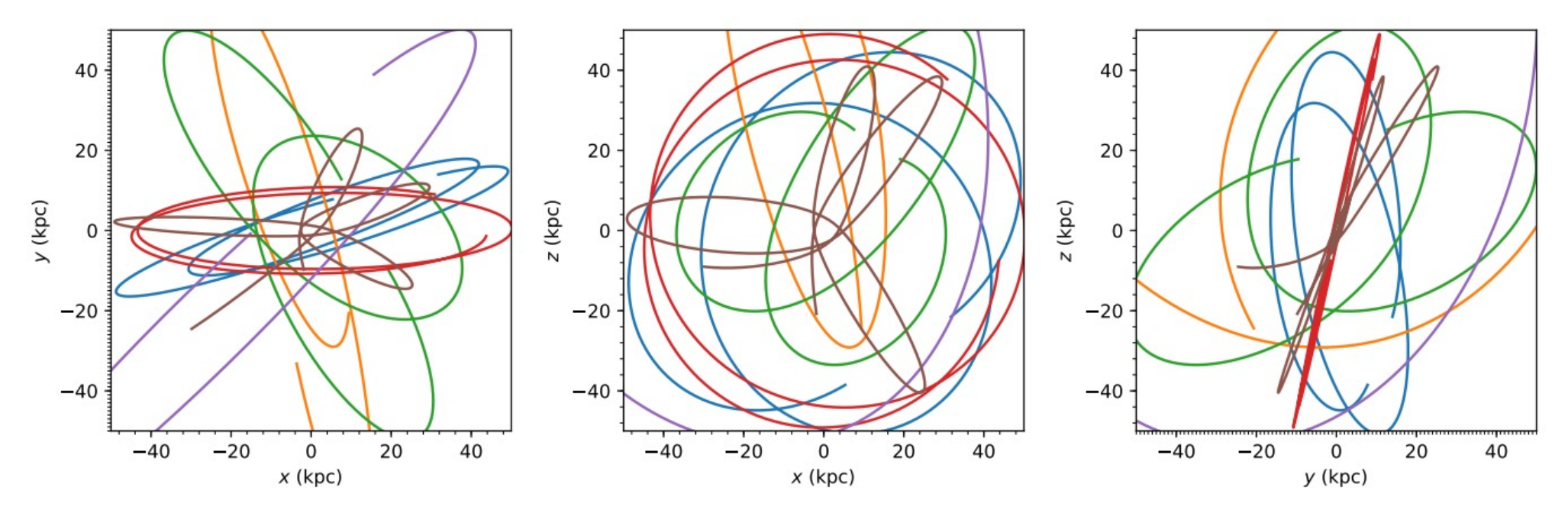
"Missing satellites problem"–but see Kim, Peter & Hargis 1711.06267; Kim & Peter 2106.09050, papers led Ethan Nadler



Patel et al. 2001.01746

Satellite orbits & missing satellites problem

Orbits of Milky Way satellites based on Gaia DR2 proper motions and galpy orbit simulations
Simon 1804.10230

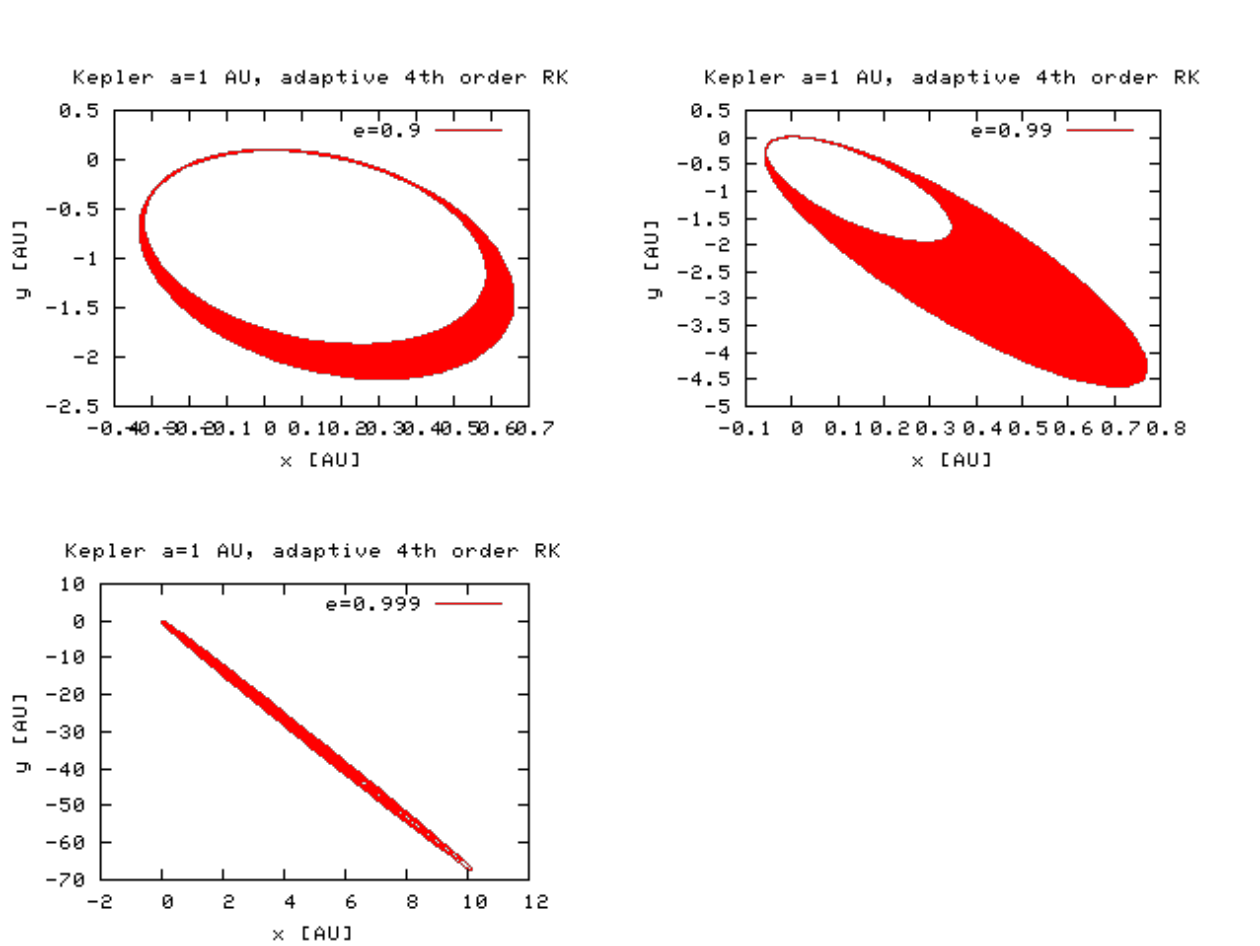


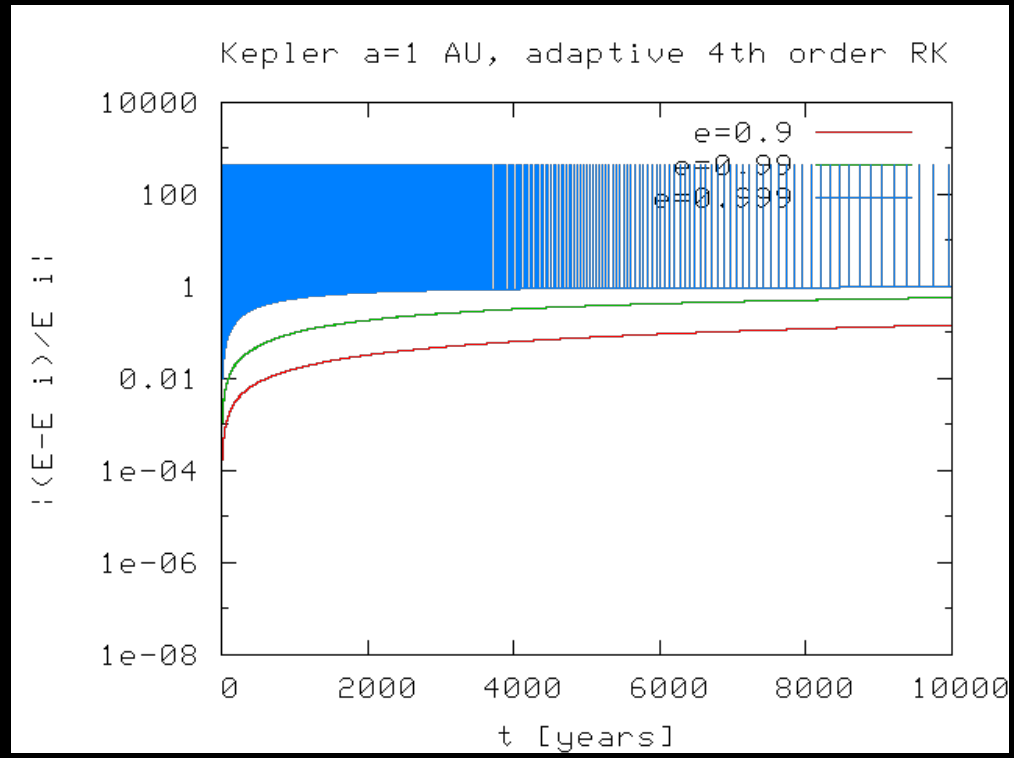
Jo Bovy (Toronto) wrote galpy, a python software package to model galaxies and the orbits of things in galaxies
https://docs.galpy.org/en/v1.6.0/ (see a tutorial in the Google Drive)

How do we solve few-N problems?

Seems simple enough to write down some equations of motion. Coupled ODEs, what is so bad about that?

Kepler problem with Runge-Kutta integration





From my first talk on my thesis work, in 2005

Instead: geometric integration

- Preserves some geometric property of the differential equations.
- Energy-conserving algorithms for energy-conserving systems.
- Time-reversable integration.
- Symplectic integration: each step in time to new coordinates is a canonical transformation (for an approximate Hamiltonian).
- Note: sometimes to get good behavior, you have to make a clever change of variables to make the equations of motion be better behaved numerically (e.g., time-transformed Hamiltonian I used in my thesis, Kustaanheimo-Stiefel coordinates used in MERCURY).

Symplectic: Leapfrog algorithm

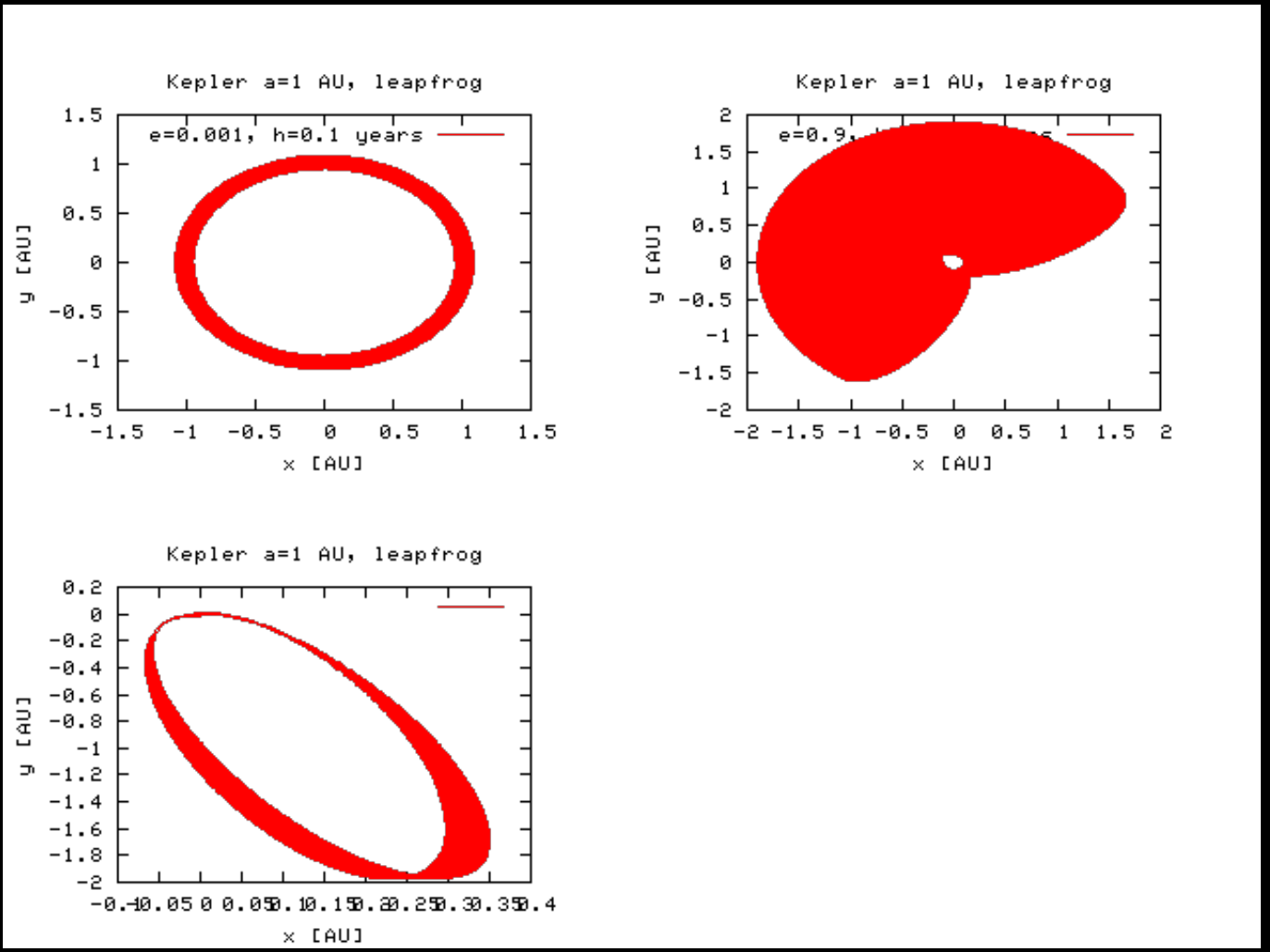
$$q_{1/2} = q_0 + \frac{h}{2} p_0$$

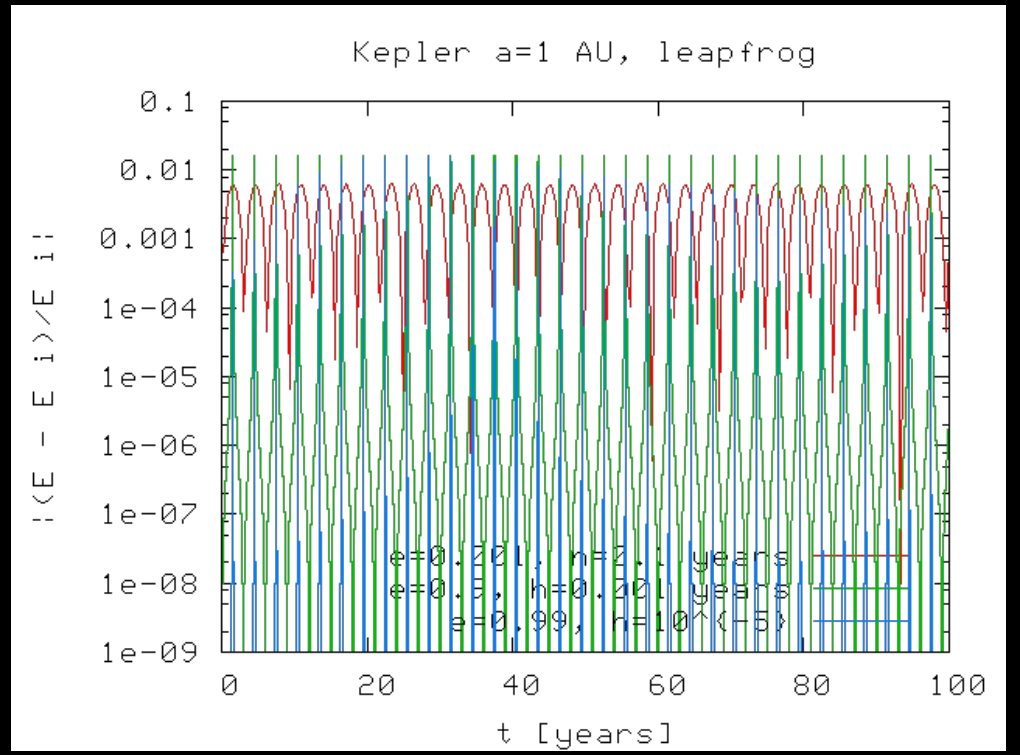
$$t_{1/2} = t_0 + \frac{h}{2}$$

$$p_1 = p_0 - h \left. \frac{\partial U}{\partial q} \right|_{q=q_{1/2}}$$

$$q_1 = q_0 + \frac{h}{2} p_1$$

$$t_1 = t_{1/2} + \frac{h}{2}$$





Higher order accuracy (example)

1409.4779

IAS15: A fast, adaptive, high-order integrator for gravitational dynamics, accurate to machine precision over a billion orbits

Hanno Rein[1,2], David S. Spiegel[2]
[1]*University of Toronto, Department of Environmental and Physical Sciences, Scarborough*
[2]*School of Natural Sciences, Institute for Advanced Study, Princeton*

Accepted 2014 October 15. Received 2014 October 14; in original form 2014 September 7

ABSTRACT

We present IAS15, a 15th-order integrator to simulate gravitational dynamics. The integrator is based on a Gauß-Radau quadrature and can handle conservative as well as non-conservative forces. We develop a step-size control that can automatically choose an optimal timestep. The algorithm can handle close encounters and high-eccentricity orbits. The systematic errors are kept well below machine precision and long-term orbit integrations over 10^9 orbits show that IAS15 is *optimal* in the sense that it follows Brouwer's law, i.e. the energy error behaves like a random walk. Our tests show that IAS15 is superior to a mixed-variable symplectic integrator (MVS) and other popular integrators, including high-order ones, in both speed and accuracy. In fact, IAS15 preserves the symplecticity of Hamiltonian systems better than the commonly-used nominally symplectic integrators to which we compared it.

We provide an open-source implementation of IAS15. The package comes with several easy-to-extend examples involving resonant planetary systems, Kozai-Lidov cycles, close encounters, radiation pressure, quadrupole moment, and generic damping functions that can, among other things, be used to simulate planet-disc interactions. Other non-conservative forces can be added easily.

Key words: methods: numerical — gravitation — planets and satellites: dynamical evolution and stability

Leapfrog is the workhorse for cosmological simulations, though.

- As long as you make the timestep adaptive, you'll break the symplectic nature a bit. See Tremaine lecture notes and Springel's 2005 paper on the Gadget-2 code for details.

Things you should know
(and know how to do if you're interested in this field)

- Write your own leapfrog integrator (there is an exercise on this in Drive folder).
- Learn how to test for convergence.
- Learn more about geometric integration.
- If you are interested in anything related to galactic dynamics, you should learn to use Jo Bovy's galpy python package (https://docs.galpy.org/en/v1.6.0/) AND he's about to publish and interactive galactic dynamics textbook. Alternatively, check out Adrian Price-Whelan's gala python package, especially if you are interested in stellar streams (http://gala.adrian.pw/en/latest/index.html).

Problems and approaches

Few-body gravitational problems

N-body problems

Hydrodynamics

What's so interesting about this for dark matter?

Expansion history of the Universe REQUIRES dark matter

Cosmology

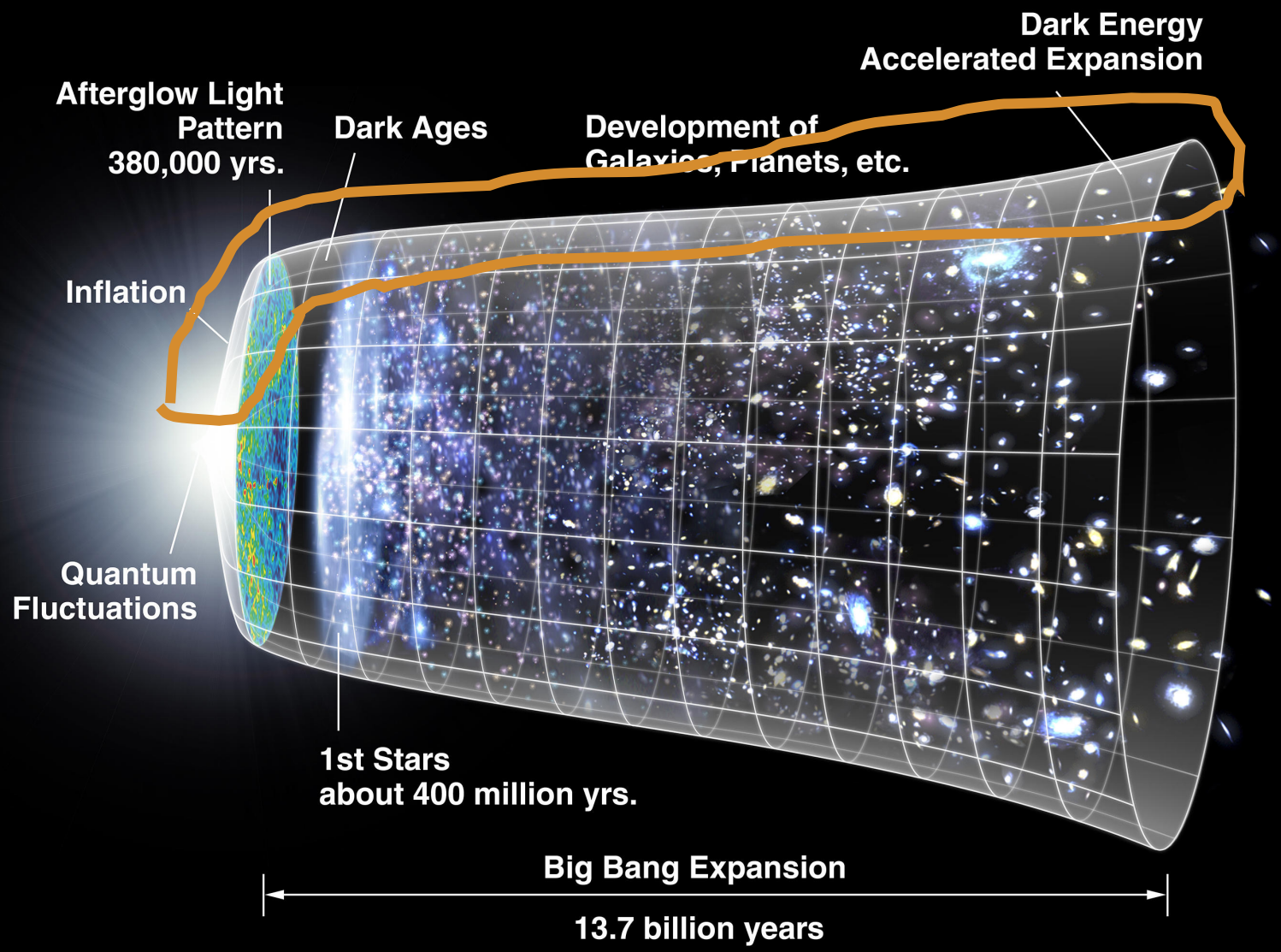


Image credit: NASA

Growth and clustering of galaxies REQUIRES dark matter

Cosmology

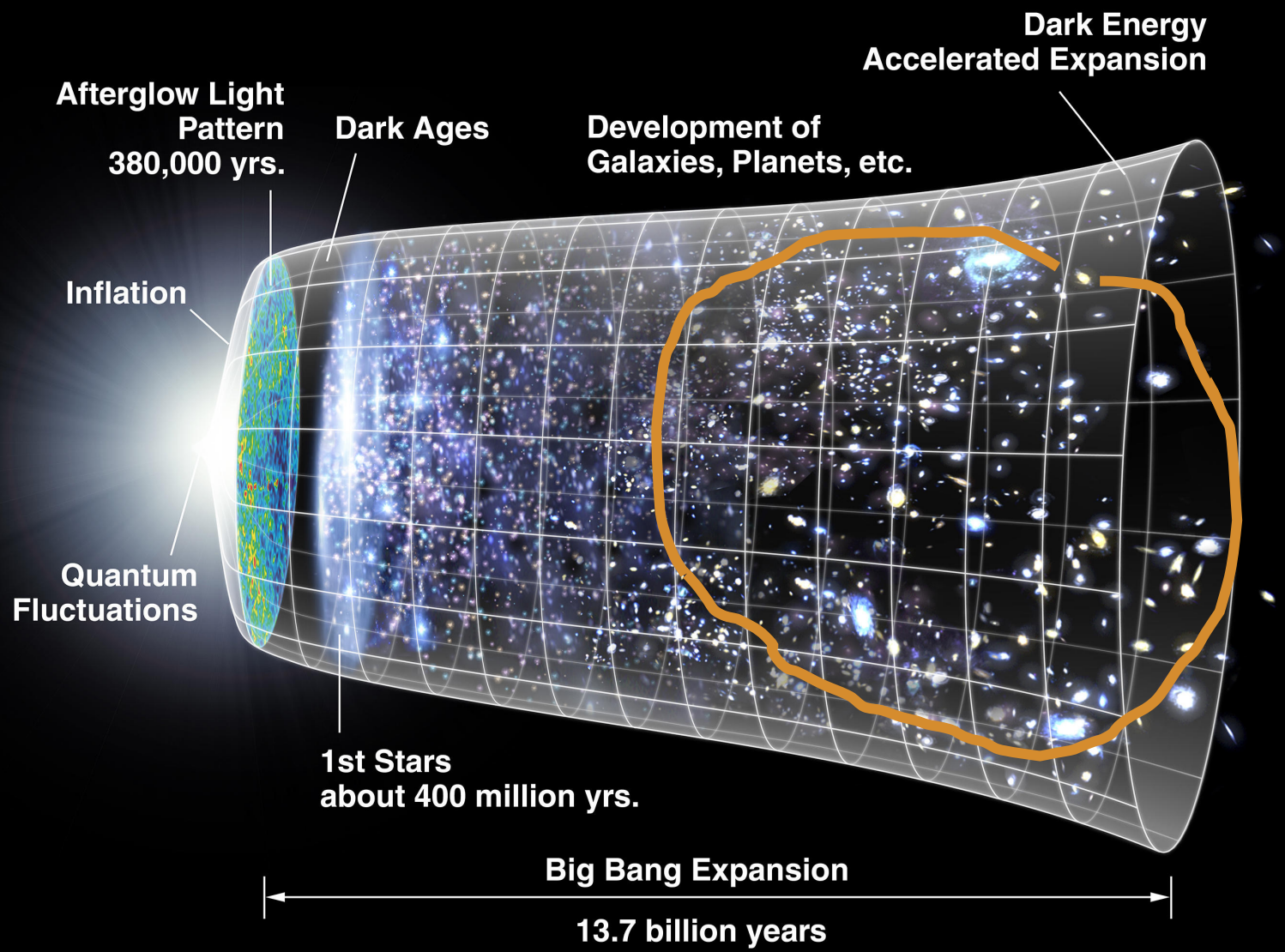
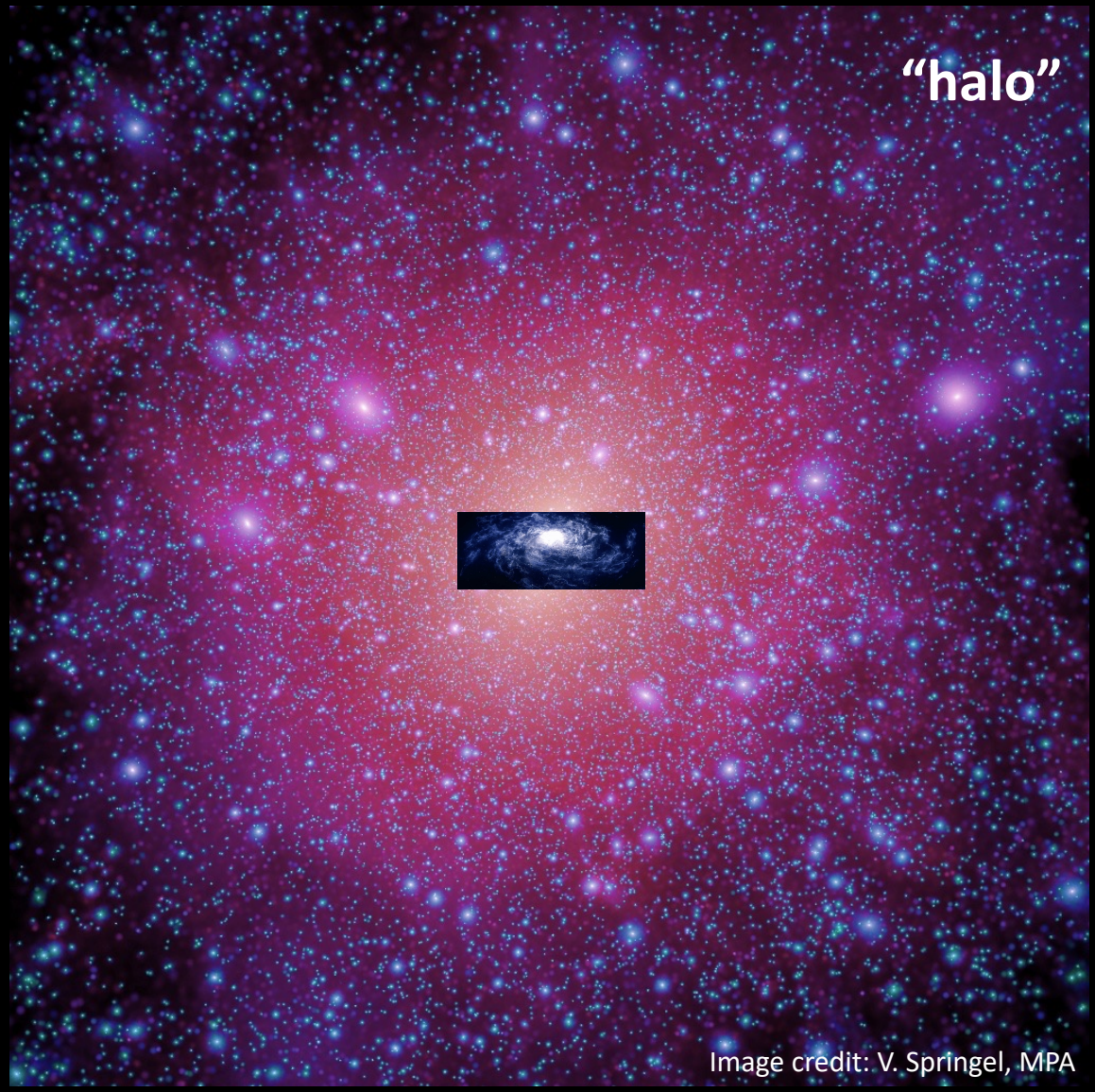


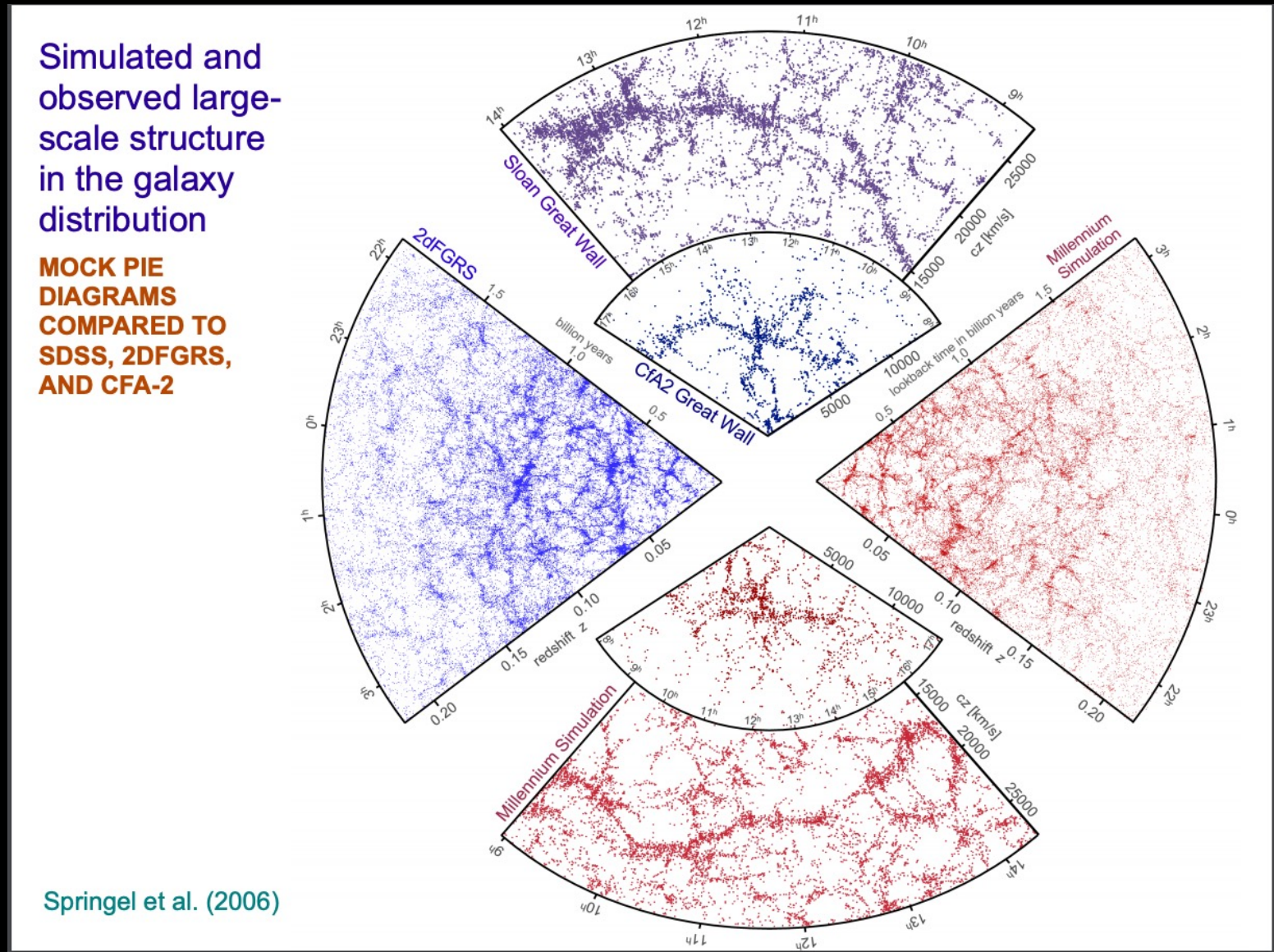
Image credit: NASA

Cosmologies & galaxies



Galaxies trace halo clustering on large scales

Slide from Springel (link in notes)

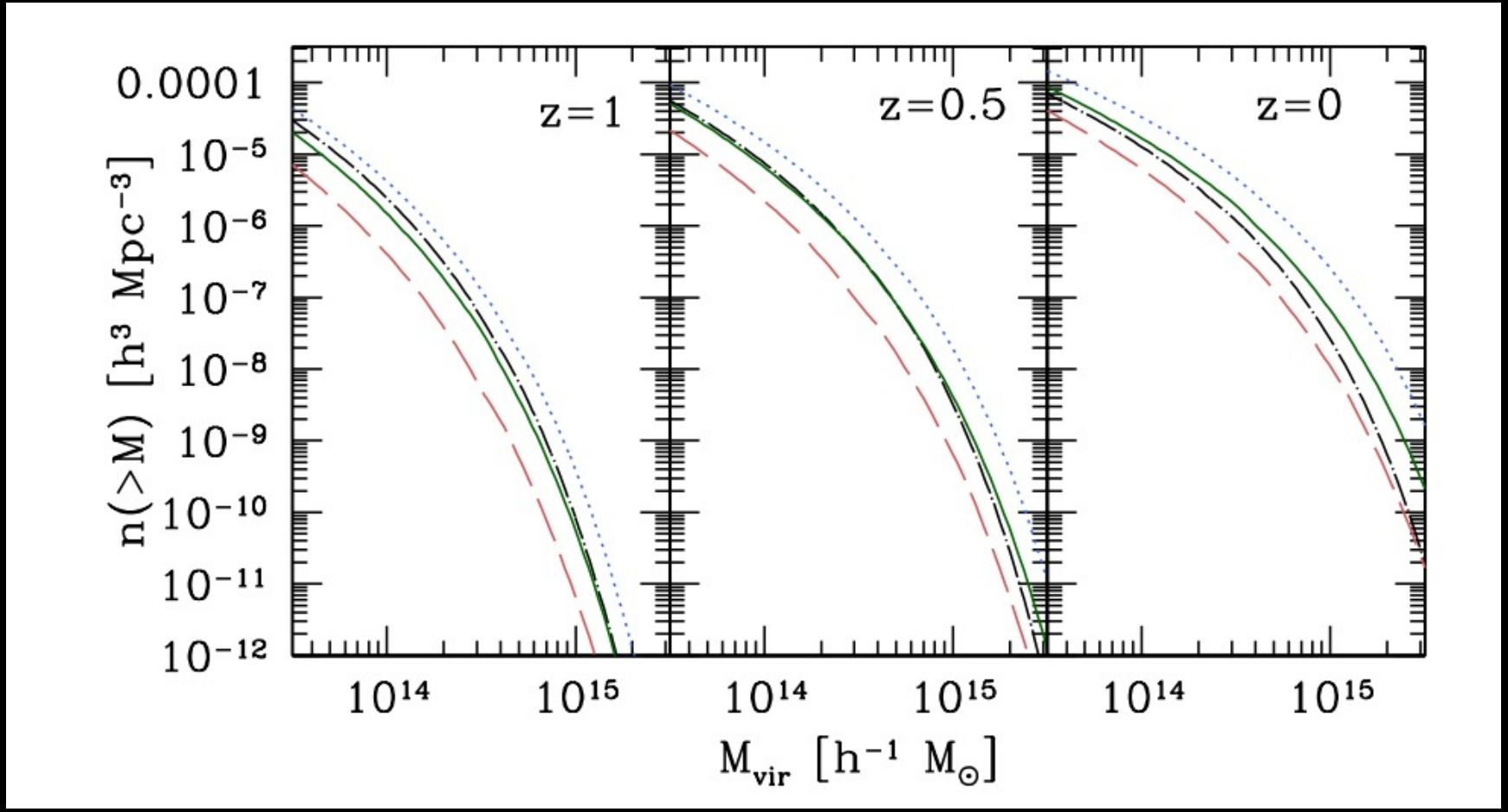


Constraining dark matter physics on large scales

Colored lines: CDM cosmologies from CMB

Black dashed line: non-relativistic decaying dark matter with a half life of 4x the Hubble time

AP 1001.3870



Dark matter and small scales

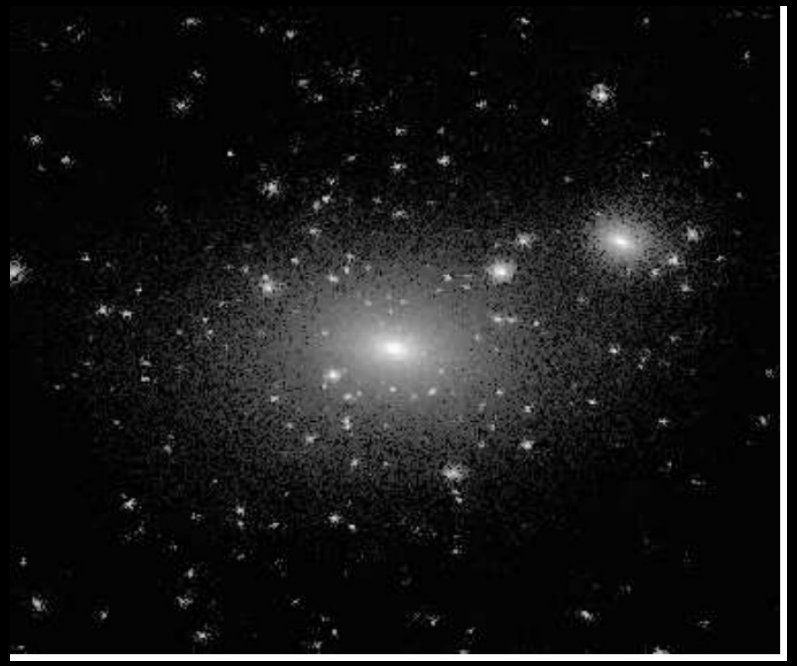
Uncloaking dark matter with galaxies

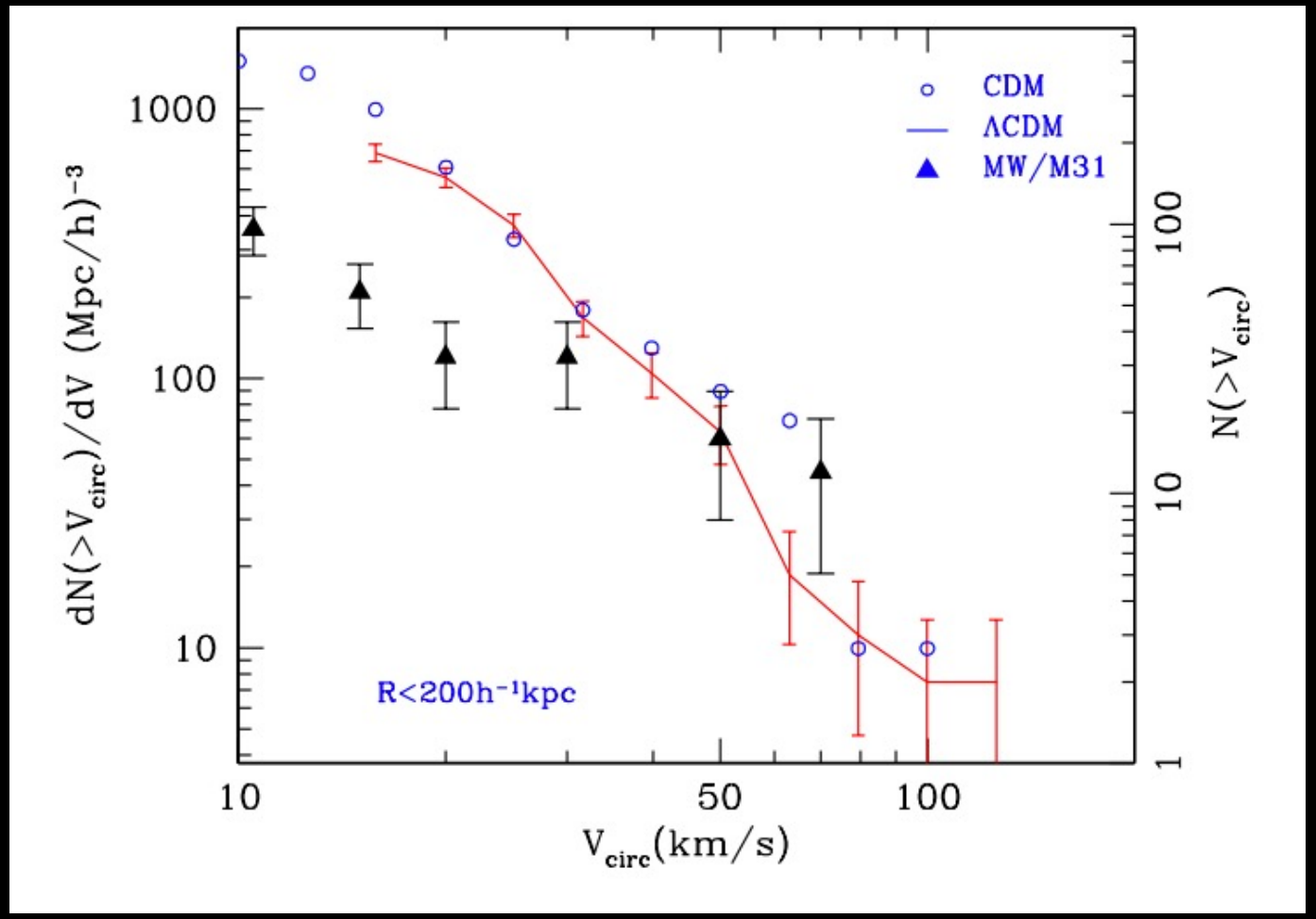
Fornax as seen by the ASAS-SN survey; Segue I, courtesy of Marla Geha

Original “missing satellites problem”

Klypin+ 9901240 →

Moore+ 9907411





2021 view

Kim & AP 2106.09050

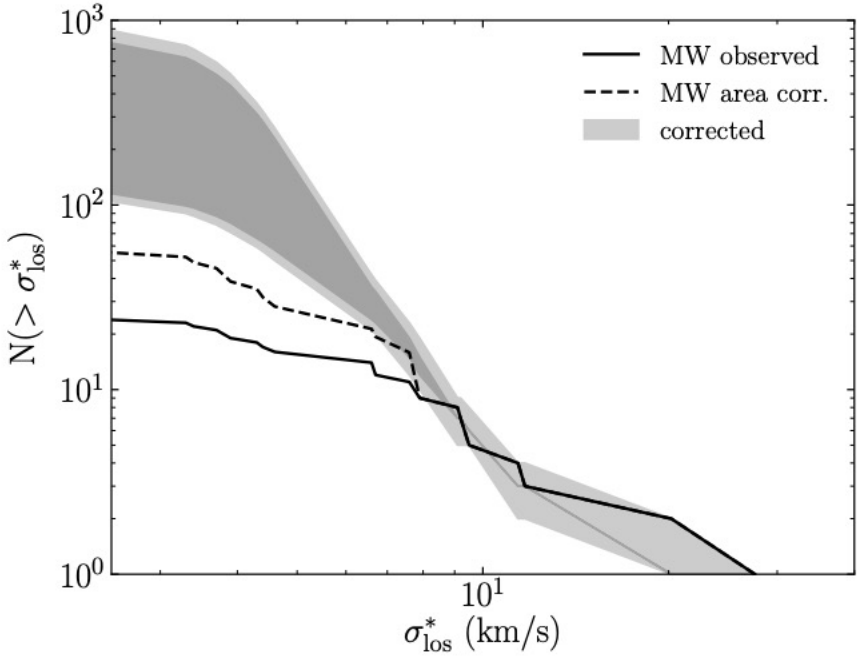
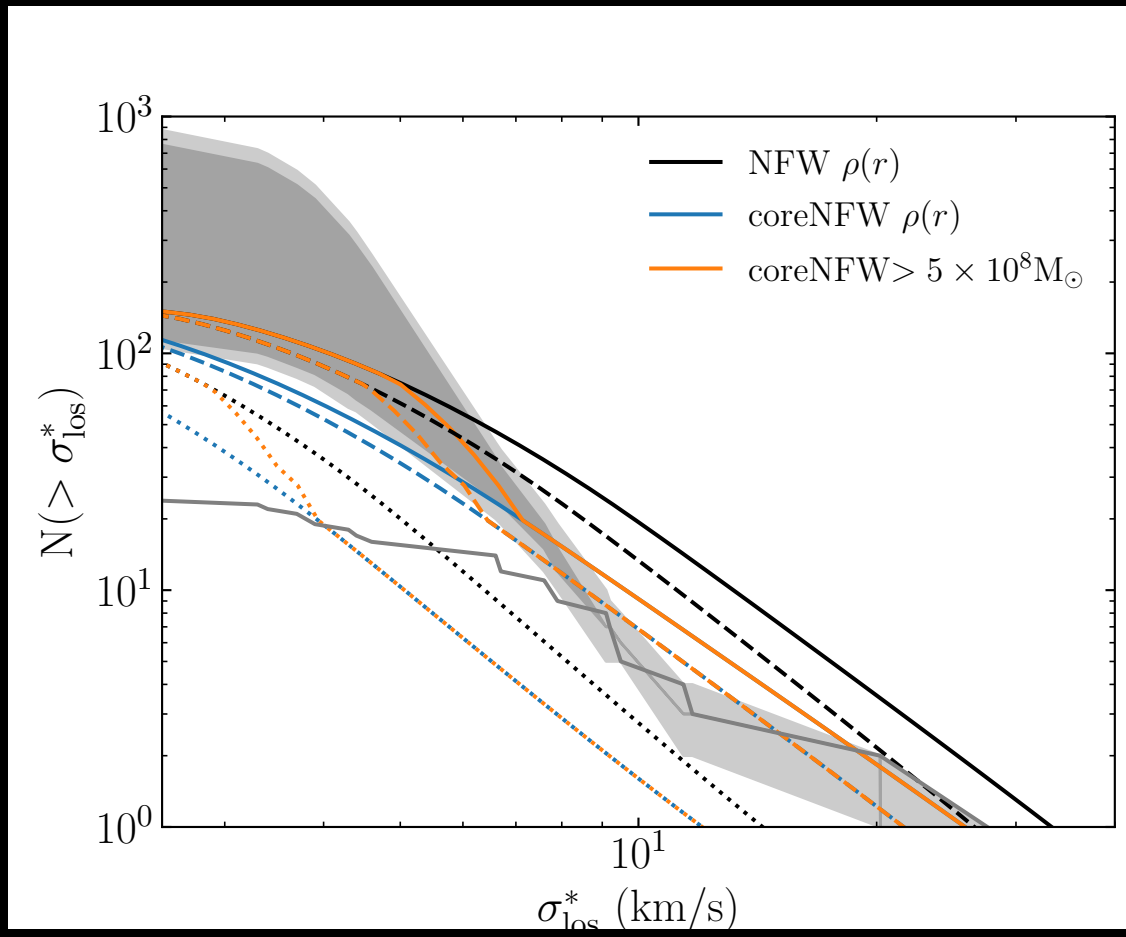


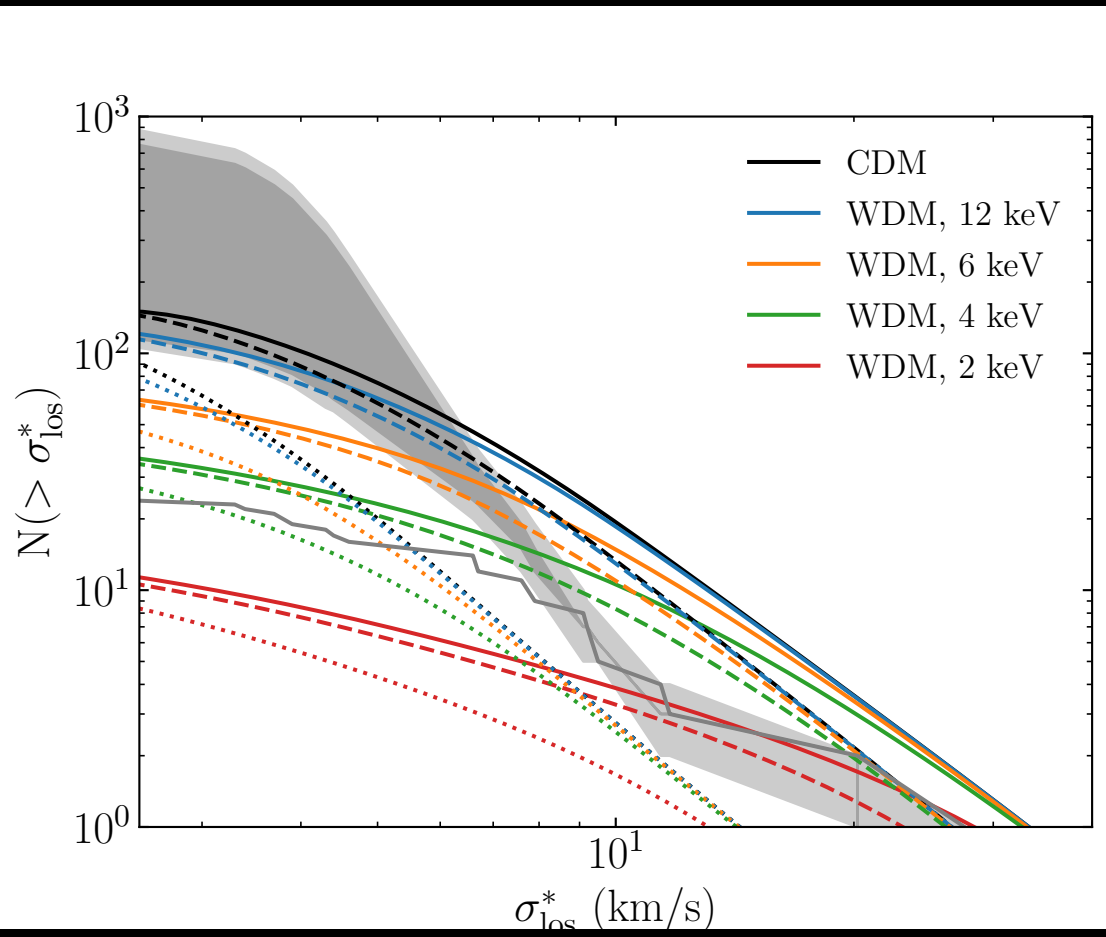
Figure 1. The completeness-corrected velocity function (gray band) based on satellites discovered in the SDSS survey. The dark gray band encompasses the uncertainty due to the unknown radial distribution of satellites. The bottom edge of the band assumes the distribution mirrors the NFW profile of the underlying MW halo, while the upper edge includes the effect of satellite destruction by the MW's disk. The light gray bands denotes the measurement uncertainties on σ^*_{los} as well as the scatter due to the anisotropy of the MW satellite distribution. In comparison, the velocity function for the observed classical and SDSS dwarfs is shown with a black line. Including a simple area correction (i.e. no radial correction) for the SDSS dwarfs would produce the black dashed line.

See also papers by Tollerud, Jethwa, Koposov, Newton, Hargis

2021 view

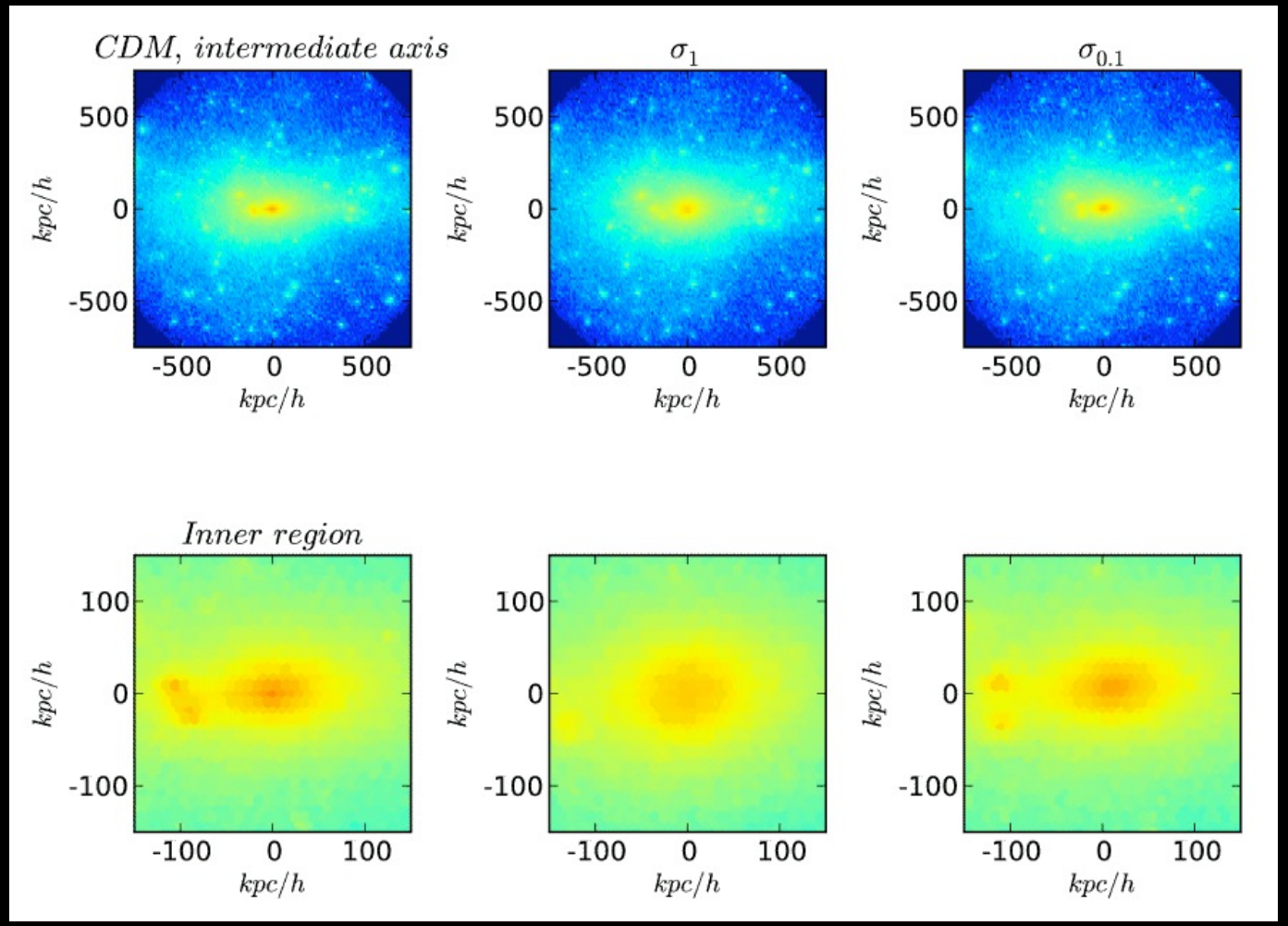
Kim & AP 2106.09050





Self-Interacting Dark Matter only affects the inner part of halos…

AP+ 1208.3026



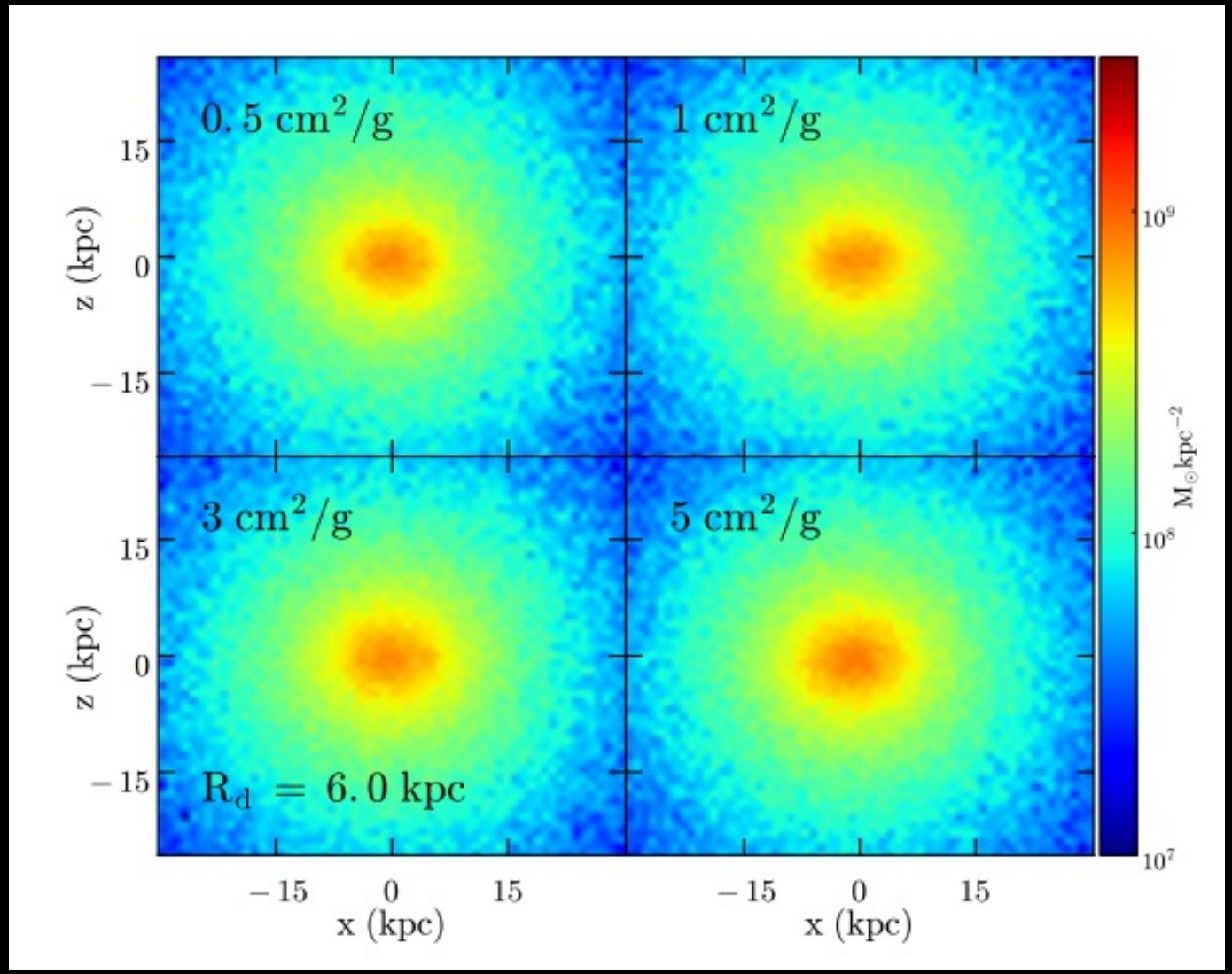
…Where they must Follow Baryons' lead

Sameie+ 1801.09682

Original idea: Kaplinghat+ 1311.6524

See other work by Hai-Bo Yu and collaborators.

DM halo of a disk galaxy!

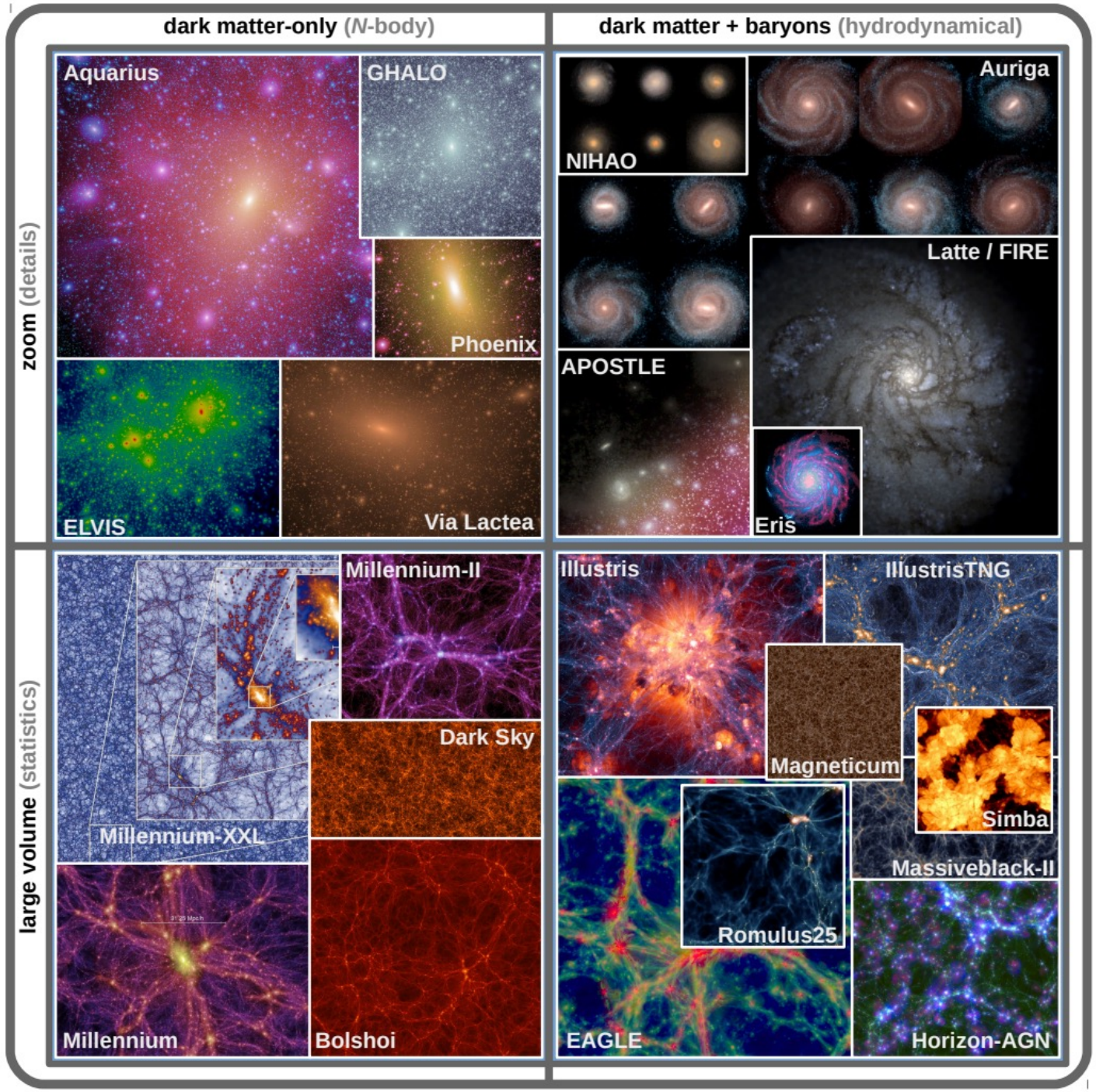


You can’t understand dark matter without understanding its connection to galaxies

Approaches

From Vogelsberger+ 1909.07976

YOU SHOULD READ THIS PAPER



Problems and approaches

Few-body gravitational problems

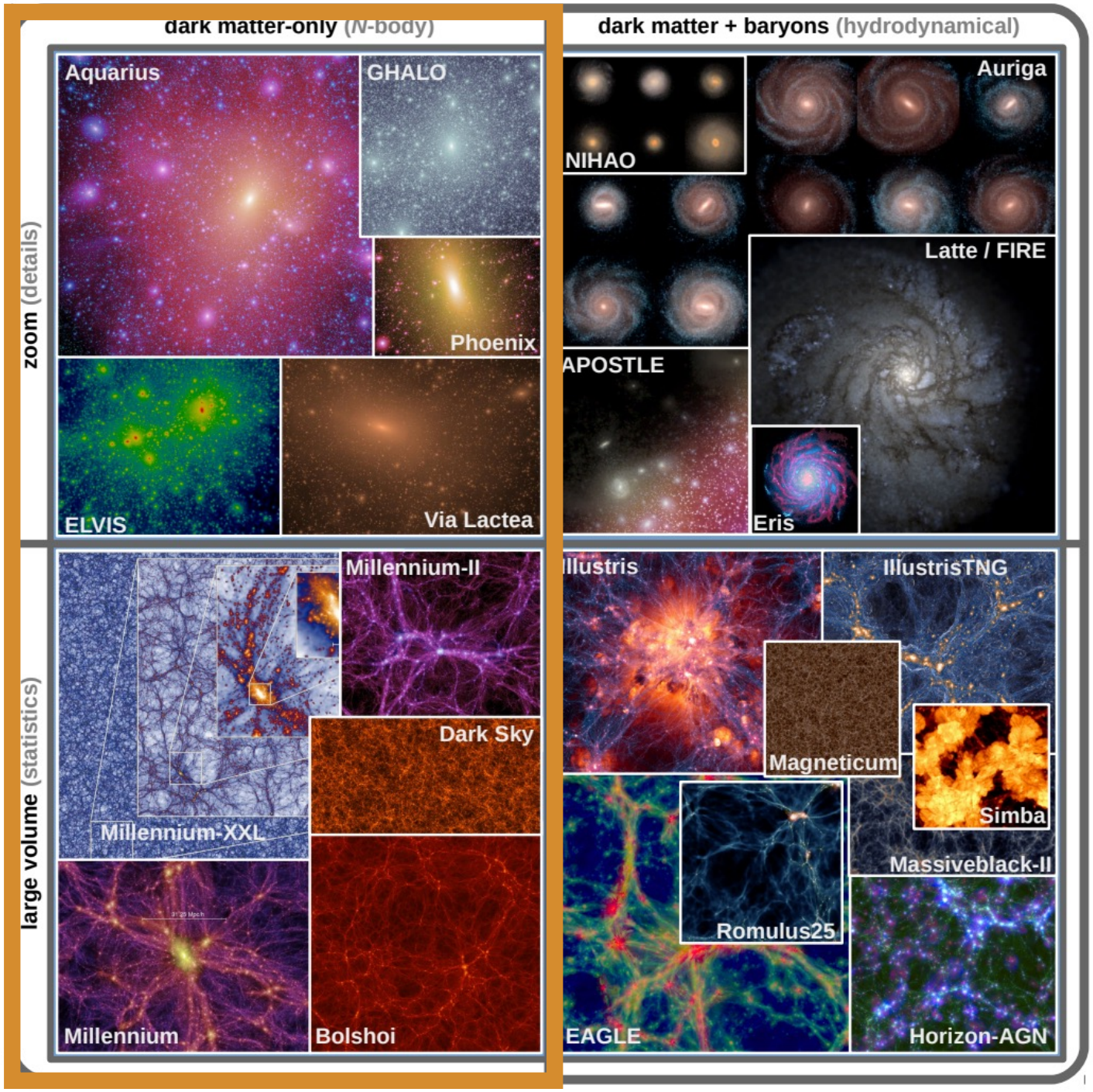
N-body problems

Hydrodynamics

Approaches

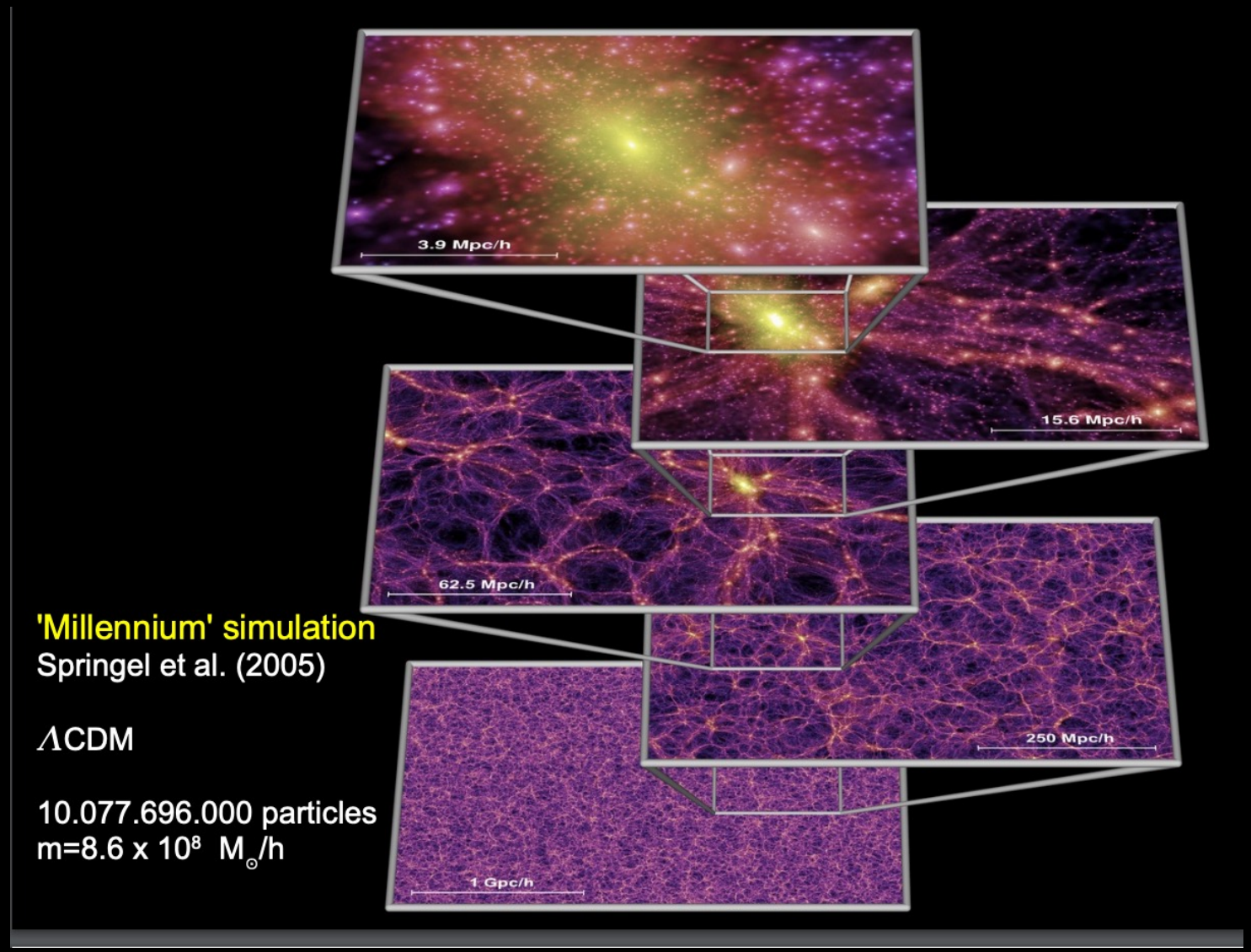
From Vogelsberger+ 1909.07976

YOU SHOULD READ THIS PAPER



Big boxes

Slide from Springel (link in notes)



Zoom-in simulations

Onorbe+ "How to zoom"
1305.6923

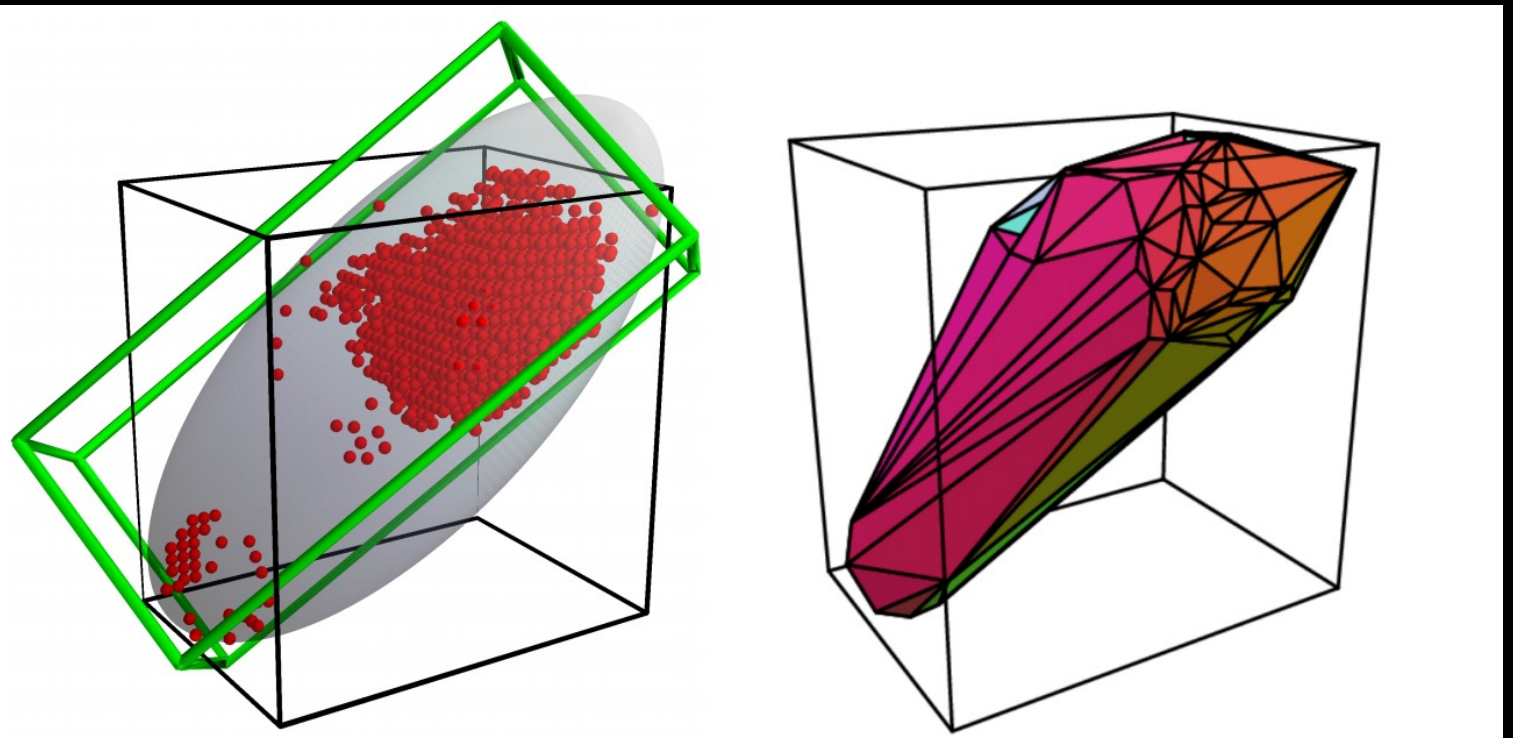
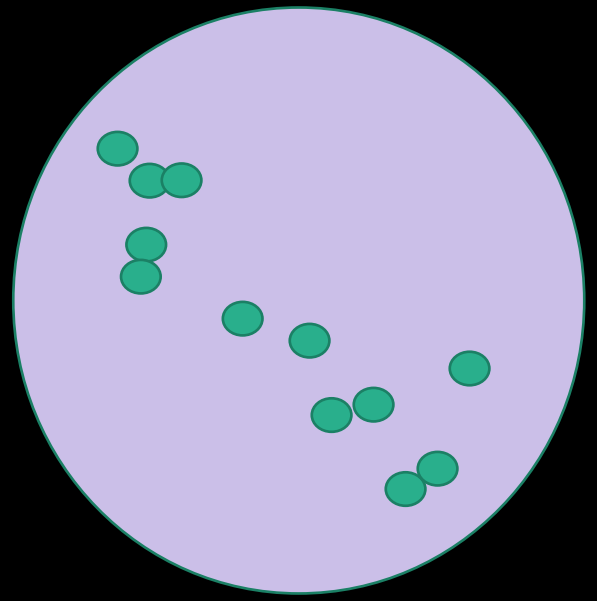


Figure 1. Lagrange volume definitions. In the left panel, the red points indicate the $z = z_{ini}$ positions of all the particles (1510) within R_v at $z = 0$ of a halo with $M_v = 1.03 \times 10^{11} h^{-1} M_\odot$ (from L50n512). The black cube stands for the cuboid Lagrange volume, $\mathcal{V}$, i.e., the cuboid that is aligned with the simulation axis and that encloses all the particles. The green cuboid shows the minimum cuboid that encloses the selected particles ($\mathcal{V}_{MC}$). The transparent grey ellipsoid shows the minimum ellipsoid that encloses the selected particles ($\mathcal{V}_{ME}$). Note the majority of the particles at $z = z_{ini}$ are nearly planar; we find that this is a generic feature of the Lagrange volumes we explore. The right panel again shows $\mathcal{V}$ (black cube), along with the convex hull Lagrange volume, $\mathcal{V}_{CH}$, for the same halo. As expected, the convex hull volume is much closer to the actual particle distribution.

Why these different simulation methods?



"Simulation particle"—typically many many solar masses.
In other words, we are COARSE-GRAINING the phase space.

Individual WIMP: 100 GeV = 10^{-55} $M_\odot$.

Milky Way halo: 10^{12} $M_\odot$

What do I mean by this/how to think about N-body simulations

1. When you have so many particles, we should really be thinking about phase-space density $dN = f(q,p)d^3qd^3p$
2. We are coarse-graining this phase-space density to make solutions tractable.
3. Fundamentally, an N-body simulation is a set of initial conditions evolved with the Boltzmann and Poisson equations (either cosmological or Newtonian simulations).

Boltzmann-Poisson equations

- Boltzmann equation:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Phi \cdot \nabla_v f = Q + \Gamma$$

Lagrangian derivative

Eulerian derivative

Source/sink

Scattering

- Poisson equation:

$$\nabla^2 \Phi = 4\pi G \, \Sigma_i m_i \int f \, d^3 v$$

i for each species of particle, to get total mass density

https://www.youtube.com/watch?v=UoJxBEQRLd0

Liouville theorem of phase-space conservation

Cold dark matter (CDM)

Non-relativistic-ish during birth epoch

Stable on long time scales

Hardly interacts with us beyond gravity

(nearly synonymous with "WIMP"
-- Weakly Interacting Massive Particle,
but not quite, and other types of dark
matter can be CDM, like axions)

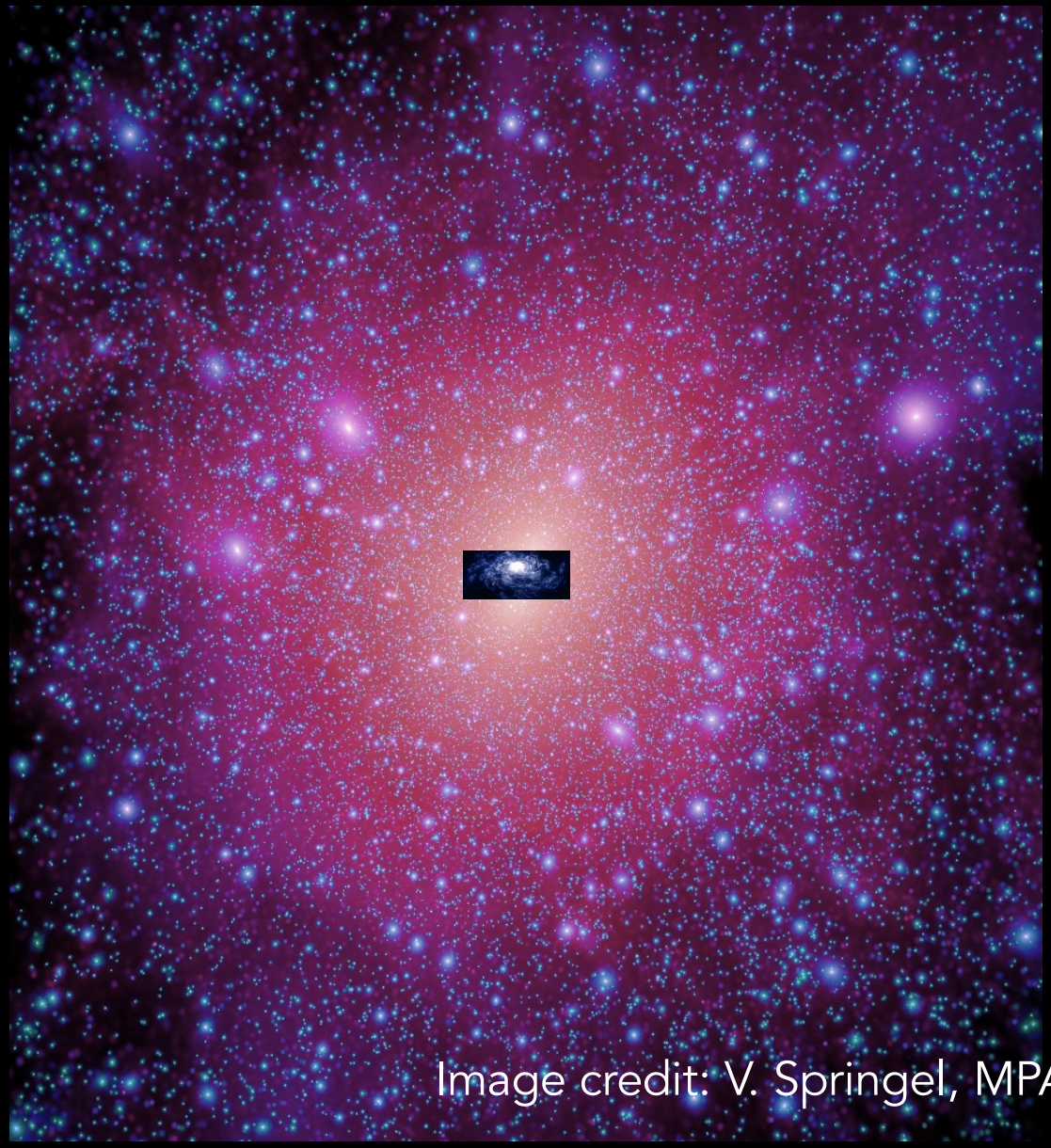
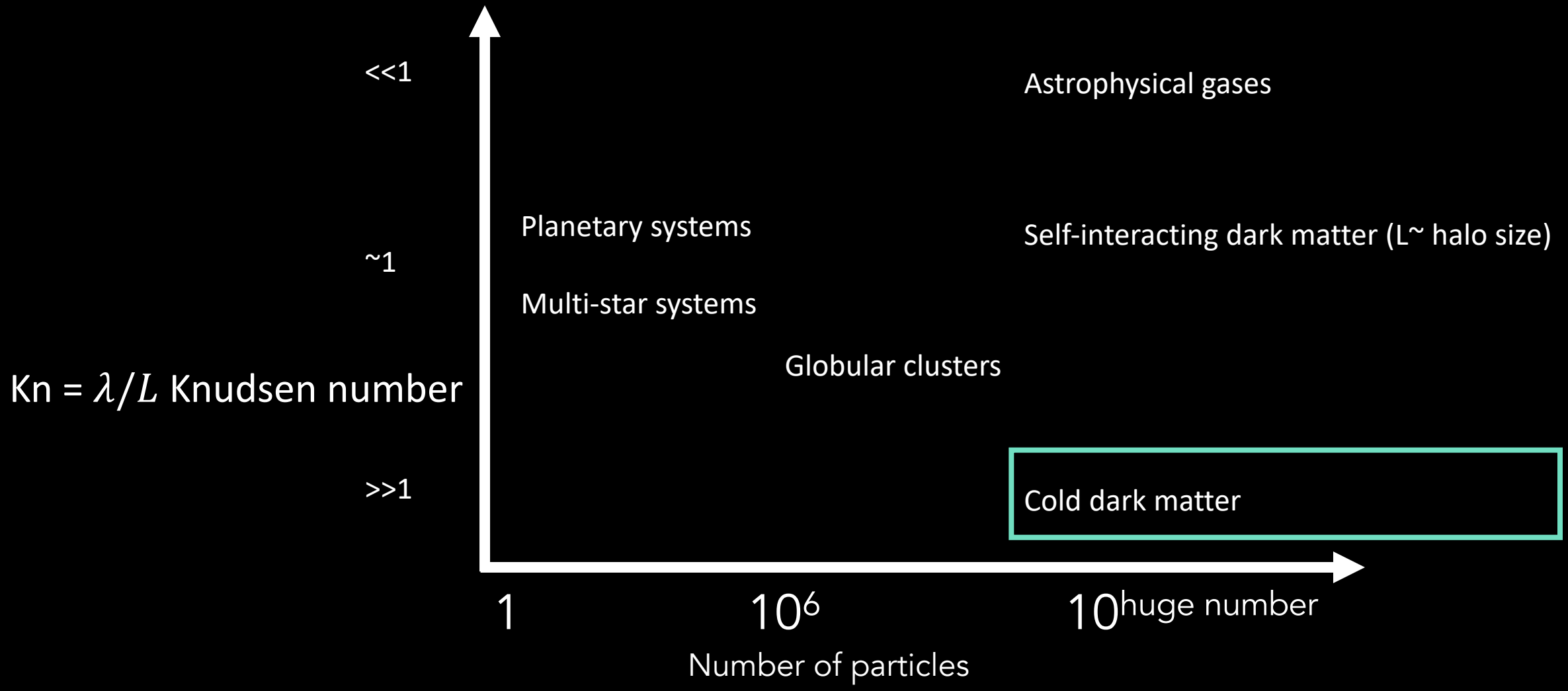


Image credit: V. Springel, MPA

Considerations



Boltzmann-Poisson equations

- (collisionless) Boltzmann equation:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - \nabla \Phi \cdot \nabla_v f = 0$$

Lagrangian derivative

Eulerian derivative

No source or sink, no scattering

- Poisson equation:

$$\nabla^2 \Phi = 4\pi G \, \Sigma_i m_i \int f \, d^3 v$$

i for each species of particle, to get total mass density

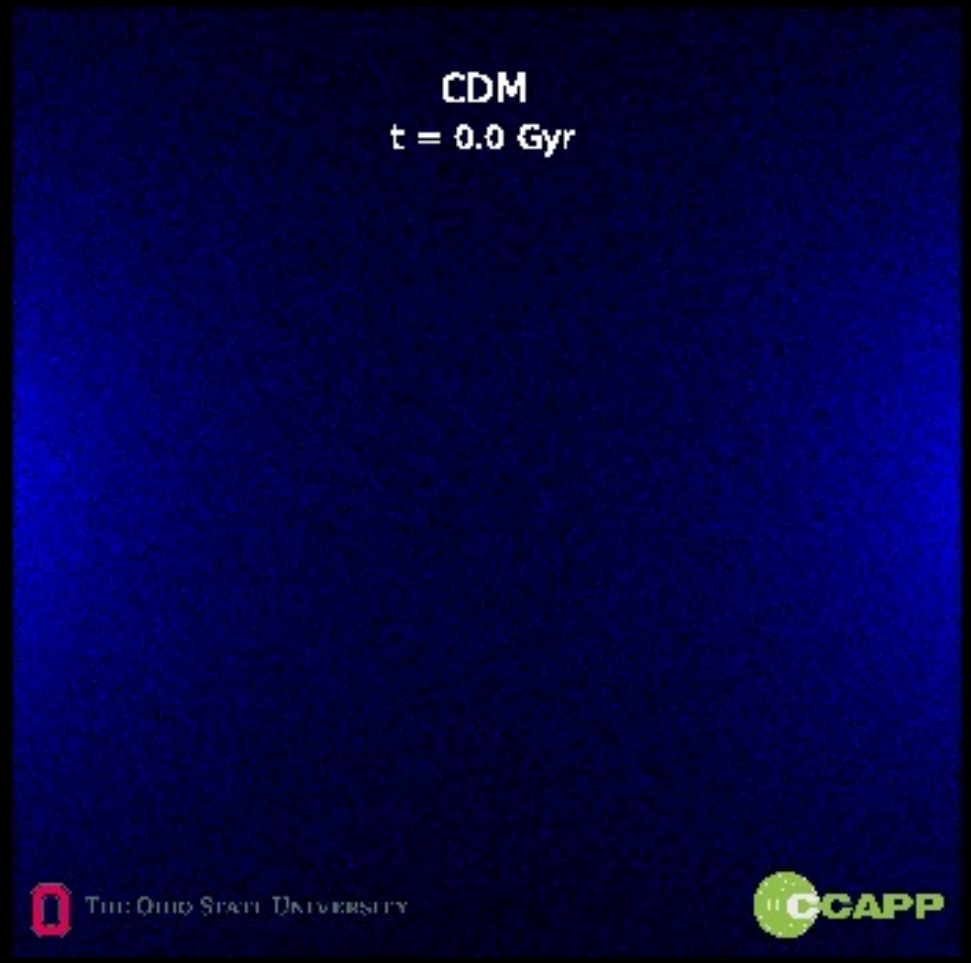
And then, we coarse-grain it. Each simulation particle represents a patch of phase space. Follow each particle on its Lagrangian trajectory through space—integrate its equation of motion (this is where the leapfrog integrator comes in).

What do I mean by this/how to think about N-body simulations

1. When you have so many particles, we should really be thinking about phase-space density $dN = f(q,p)d^3qd^3p$
2. We are coarse-graining this phase-space density to make solutions tractable.
3. Fundamentally, an N-body simulation is a set of **initial conditions** evolved with the Boltzmann and Poisson equations (either cosmological or Newtonian simulations).

Initial conditions: Newtonian

Merging cluster simulation
Kim, AP, Wittman
1608.08630



Initial conditions: cosmological

Start at a high enough redshift that perturbation theory still works!

We are creating realizations of density fields.

See also Springel's 2005 Gadget-2.

Mon. Not. R. Astron. Soc. **000**, 1–22 (2011) Printed 10 June 2011 (MN LATEX style file v2.2)

Multi-scale initial conditions for cosmological simulations

Oliver Hahn[1⋆] and Tom Abel[1,2,3]
[1] *Kavli Institute for Particle Astrophysics and Cosmology, SLAC/Stanford University, 2575 Sand Hill Road, Menlo Park, CA 94025, USA*
[2] *Zentrum für Astronomie der Universität Heidelberg, Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, 69120 Heidelberg, Germany*
[3] *Heidelberg Institut für Theoretische Studien, Schloss-Wolfsbrunnenweg 35, 69118 Heidelberg, Germany*

MNRAS in press

2 [astro-ph.CO] 9 Jun 2011

ABSTRACT
We discuss a new algorithm to generate multi-scale initial conditions with multiple levels of refinements for cosmological "zoom-in" simulations. The method uses an adaptive convolution of Gaussian white noise with a real space transfer function kernel together with an adaptive multi-grid Poisson solver to generate displacements and velocities following first (1LPT) or second order Lagrangian perturbation theory (2LPT). The new algorithm achieves RMS relative errors of order 10^{-4} for displacements and velocities in the refinement region and thus improves in terms of errors by about two orders of magnitude over previous approaches. In addition, errors are localized at coarse-fine boundaries and do not suffer from Fourier-space induced interference ringing. An optional hybrid multi-grid and Fast Fourier Transform (FFT) based scheme is introduced which has identical Fourier space behaviour as traditional approaches. Using a suite of re-simulations of a galaxy cluster halo our real space based approach is found to reproduce correlation functions, density profiles, key halo properties and subhalo abundances with per cent level accuracy. Finally, we generalize our approach for two-component baryon and dark-matter simulations and demonstrate that the power spectrum evolution is in excellent agreement with linear perturbation theory. For initial baryon density fields, it is suggested to use the local Lagrangian approximation in order to generate a density field for mesh based codes that is consistent with Lagrangian perturbation theory instead of the current practice of using the Eulerian linearly scaled densities.

Key words: cosmology: theory, large-scale structure of Universe – galaxies: formation – methods: numerical

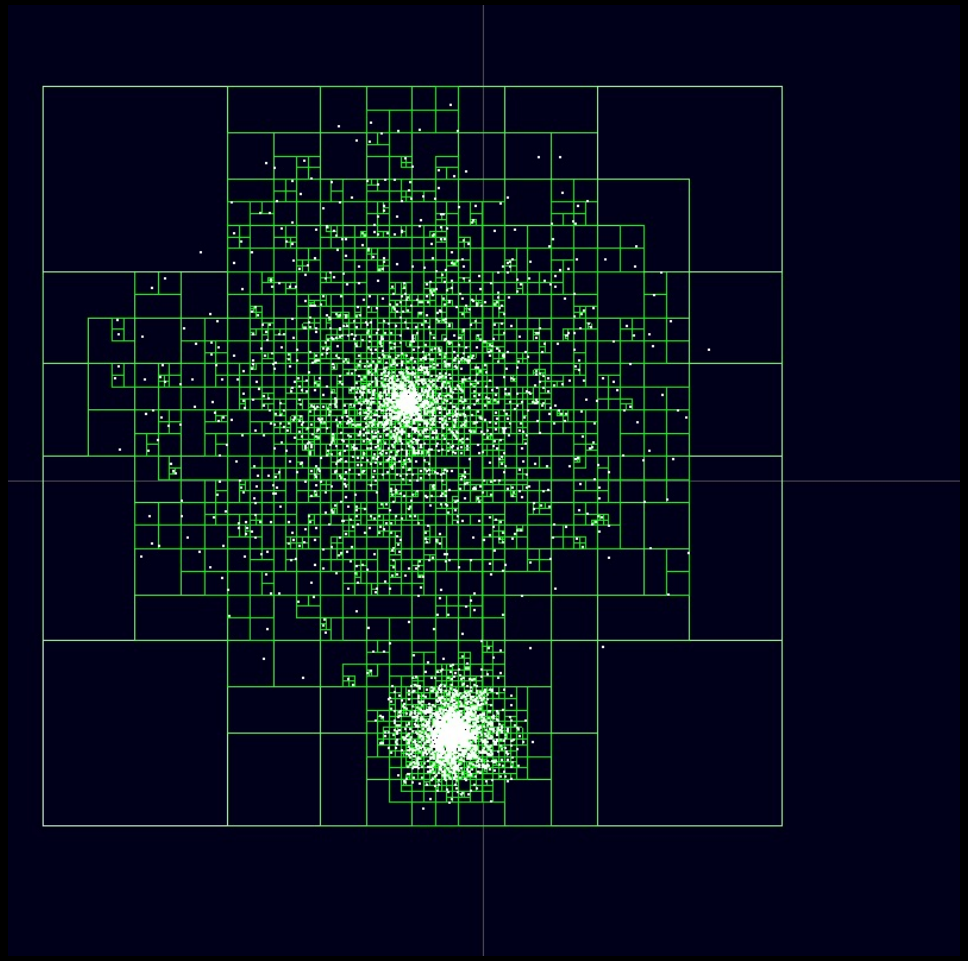
What do I mean by this/how to think about N-body simulations

1. When you have so many particles, we should really be thinking about phase-space density $dN = f(q,p)d^3qd^3p$
2. We are coarse-graining this phase-space density to make solutions tractable.
3. Fundamentally, an N-body simulation is a set of initial conditions **evolved with the Boltzmann and Poisson equations** (either cosmological or Newtonian simulations).

Gridless codes (tree)

N^2 operations to NlogN

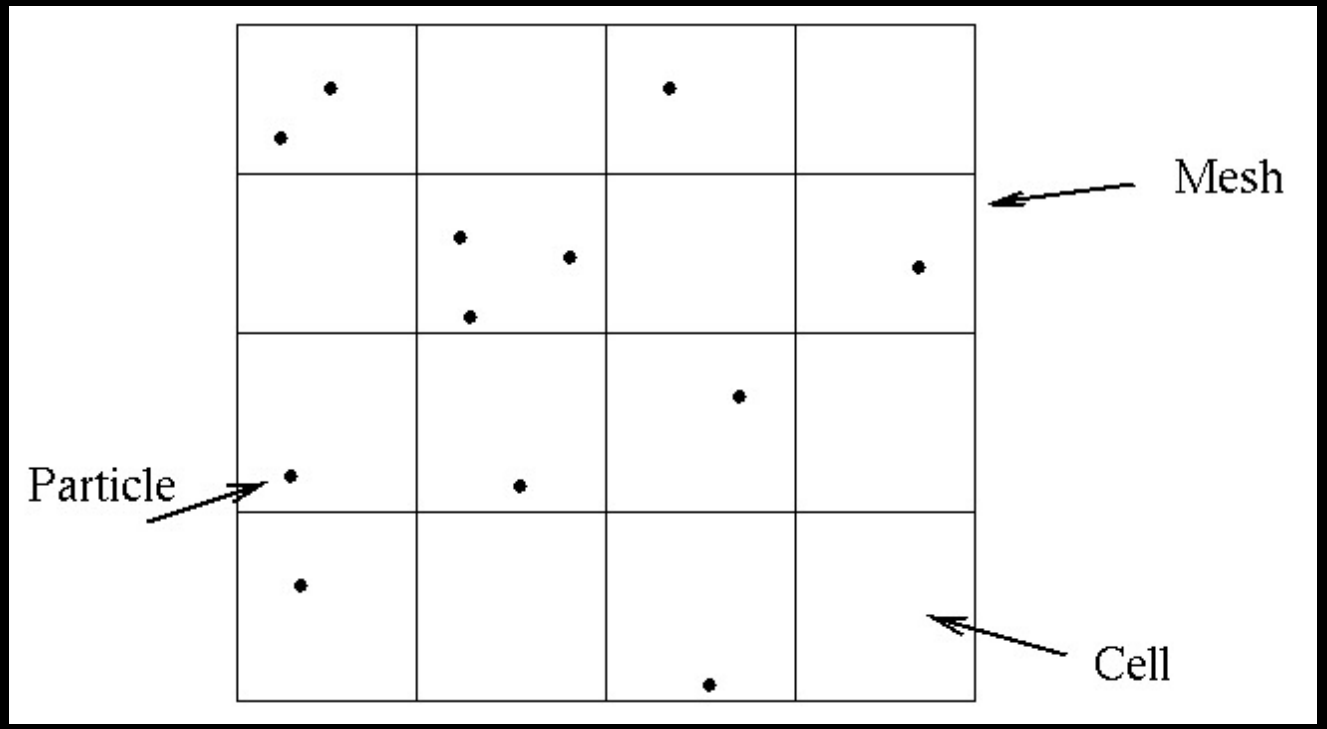
Calculate gravitation force directly (bypass the potential)



Barnes & Hut 1986

https://commons.wikimedia.org/wiki/File:Barnes_hut_tree.png

Grid codes



http://www.new-npac.org/projects/cdroms/cewes-1999-06-vol1/cps615course/nbody-materials/nbody-simulations.html

Gravity: collisionless Boltzmann

- Grid codes
  - Particle-mesh codes; FFT of density field; solve Poisson equation; FFT back
  - $\nabla^2 \Phi = 4\pi G\ \rho$
  - $\rightarrow k^2 \tilde{\Phi} = -4\pi G\ \tilde{\rho}$

Then, take a derivative to find the force for the leapfrong integrator

What the major codes actually use

From Vogelsberger+ 1909.07976

Table 1: Major galaxy formation simulation codes

code name	gravity treatment[a]	hydrodynamics treatment[b]	parallelization technique[c]	code availability[d]	primary reference
ART	PM/ML	AMR	data-based	public	Kravtsov (1997)[27]
RAMSES	PM/ML	AMR	data-based	public	Teyssier (2002)[38]
GADGET-2/3	TreePM	SPH	data-based	public	Springel (2005)[39]
Arepo	TreePM	MMFV	data-based	public	Springel (2010)[40]
Enzo	PM/MG	AMR	data-based	public	Bryan et al. (2014)[41]
ChaNGa[e]	Tree/FM	SPH	task-based	public	Menon et al. (2015)[42–44]
GIZMO[f]	TreePM	MLFM/MLFV	data-based	public	Hopkins et al. (2015)[45]
HACC	TreePM/P^3M	CRK-SPH	data-based	private	Habib et al. (2016)[46]
PKDGRAV3	Tree/FM	–	data-based	public	Potter et al. (2017)[47]
Gasoline2	Tree	SPH	task-based	public	Wadsley et al. (2017)[48]
SWIFT	TreePM/FM	SPH	task-based	public	Schaller et al. (2018)[49]

[a] PM: particle-mesh; TreePM: tree + PM, FM: fast multipole, P^3M: particle-particle-particle-mesh; ML: multilevel; MG: multigrid

[b] SPH: smoothed particle hydrodynamics, CRK-SPH: conservative reproducing kernel smoothed particle hydrodynamics , AMR: adaptive-mesh-refinement, MMFV: moving-mesh finite volume, MLFM/MLFV: mesh-free finite mass / finite volume

[c] data-based: data parallelism focuses on distributing data across different nodes, which operate on the data in parallel; task-based: task parallelism focuses on distributing tasks concurrently performed

[d] private: private code; public: publicly available code (in some cases with limited functionality)

[e] gravity solver is based on PKDGRAV3

[f] based on the GADGET-3 code

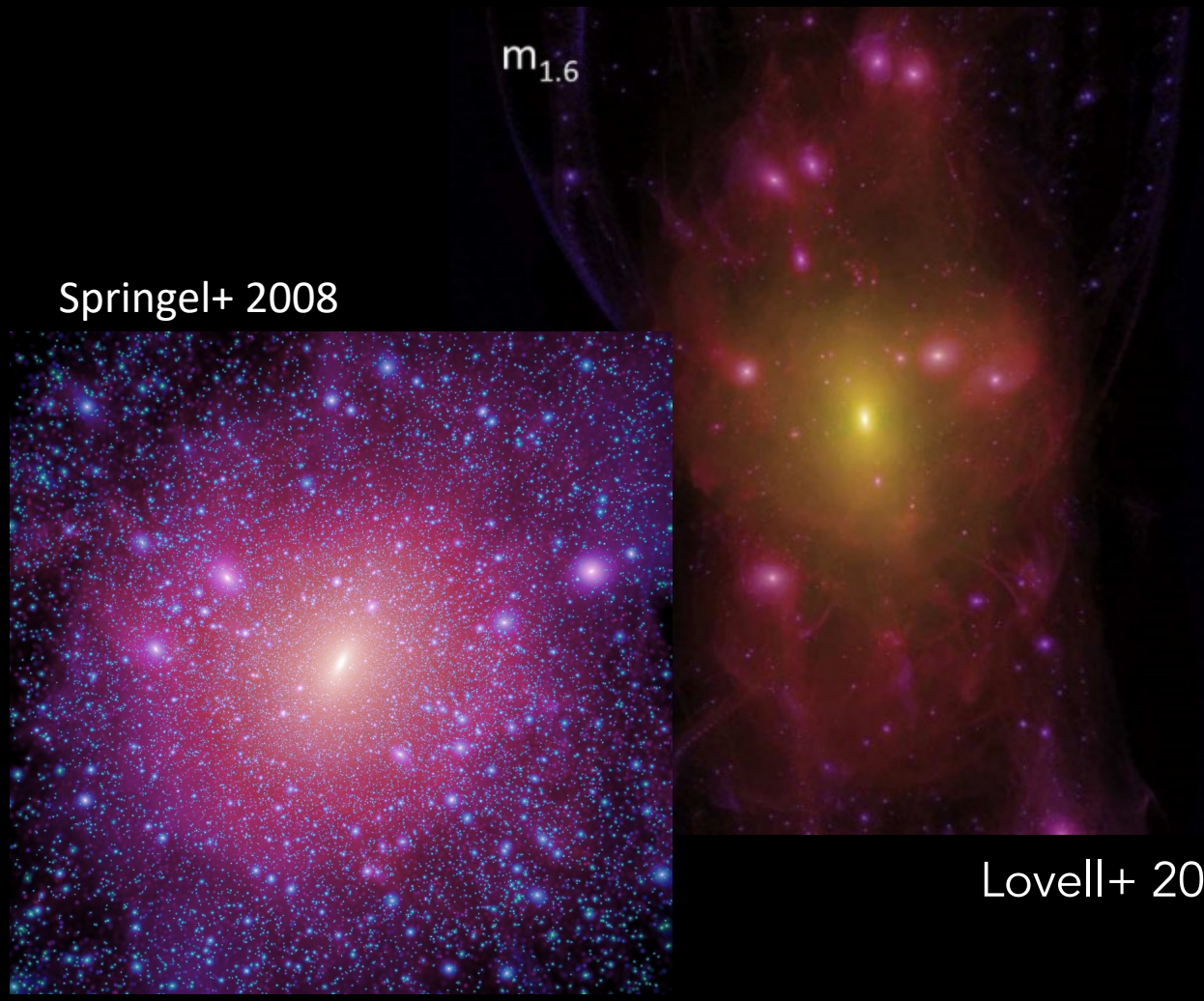
A good check

- Make an equilibrium halo, and evolve it in isolation. If the halo moves, check to make sure that you've set the free parameters in your gravitational force calculator finely enough.
- Expect chaos—even with identical initial conditions, you will get numerical chaos if you change resolution/time-stepping. Expect good statistical convergence (halo mass functions, etc), but not exactly replicating xyz positions of every particle.

What happens for not-CDM?

Warm dark matter

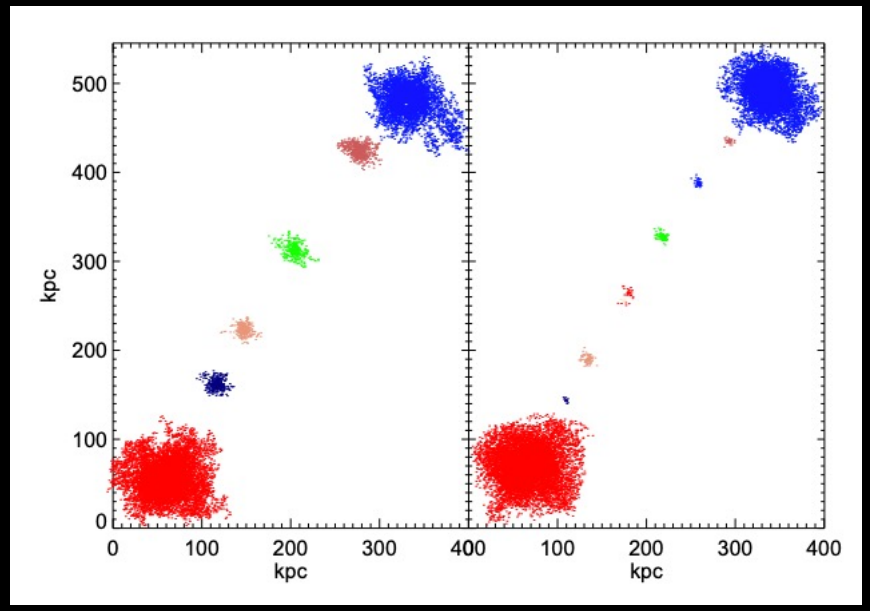
- Suppression of power spectrum
- Particle velocities



Springel+ 2008

Lovell+ 2014

Beware of artifacts



Lovell+
1308.1399

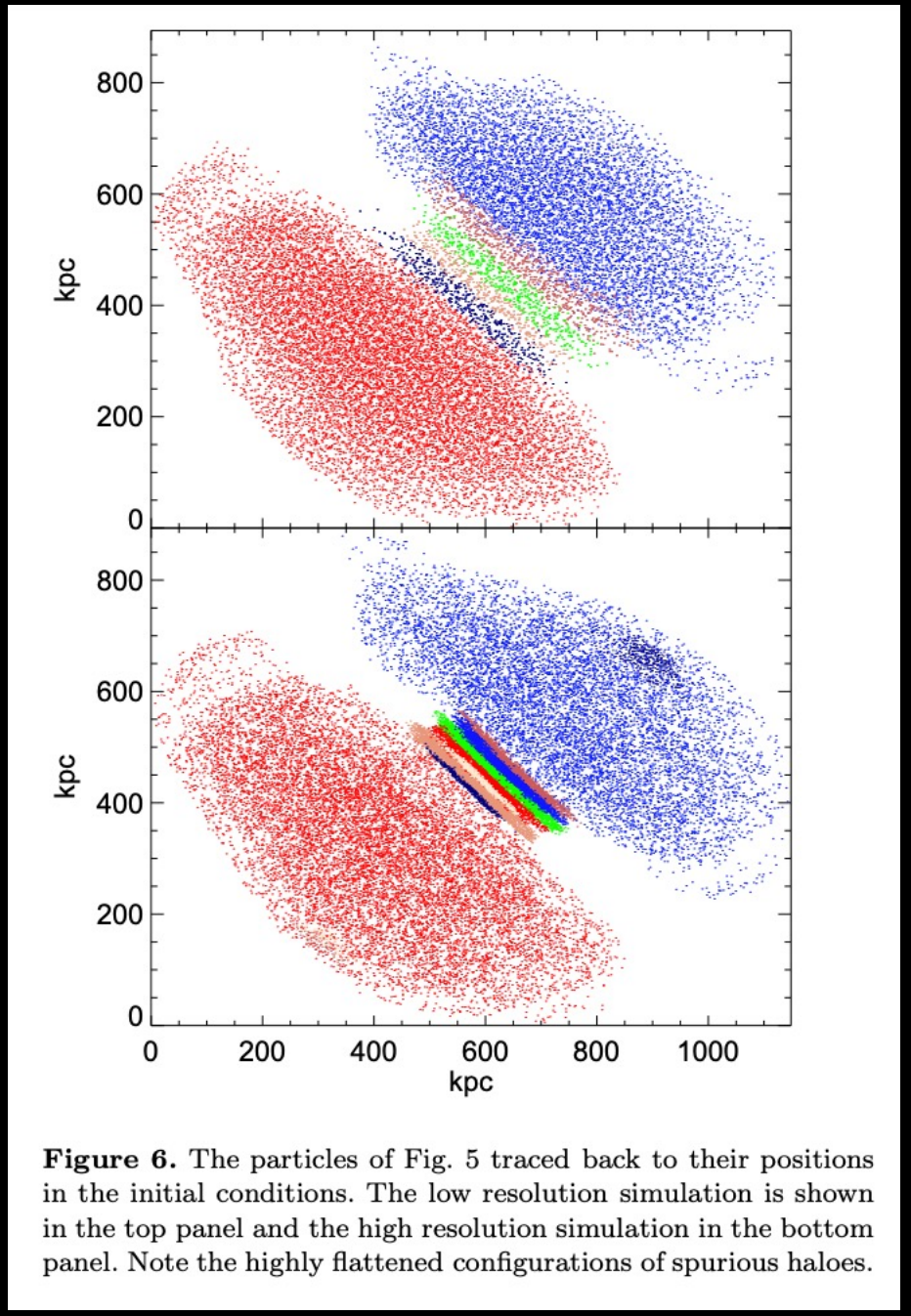
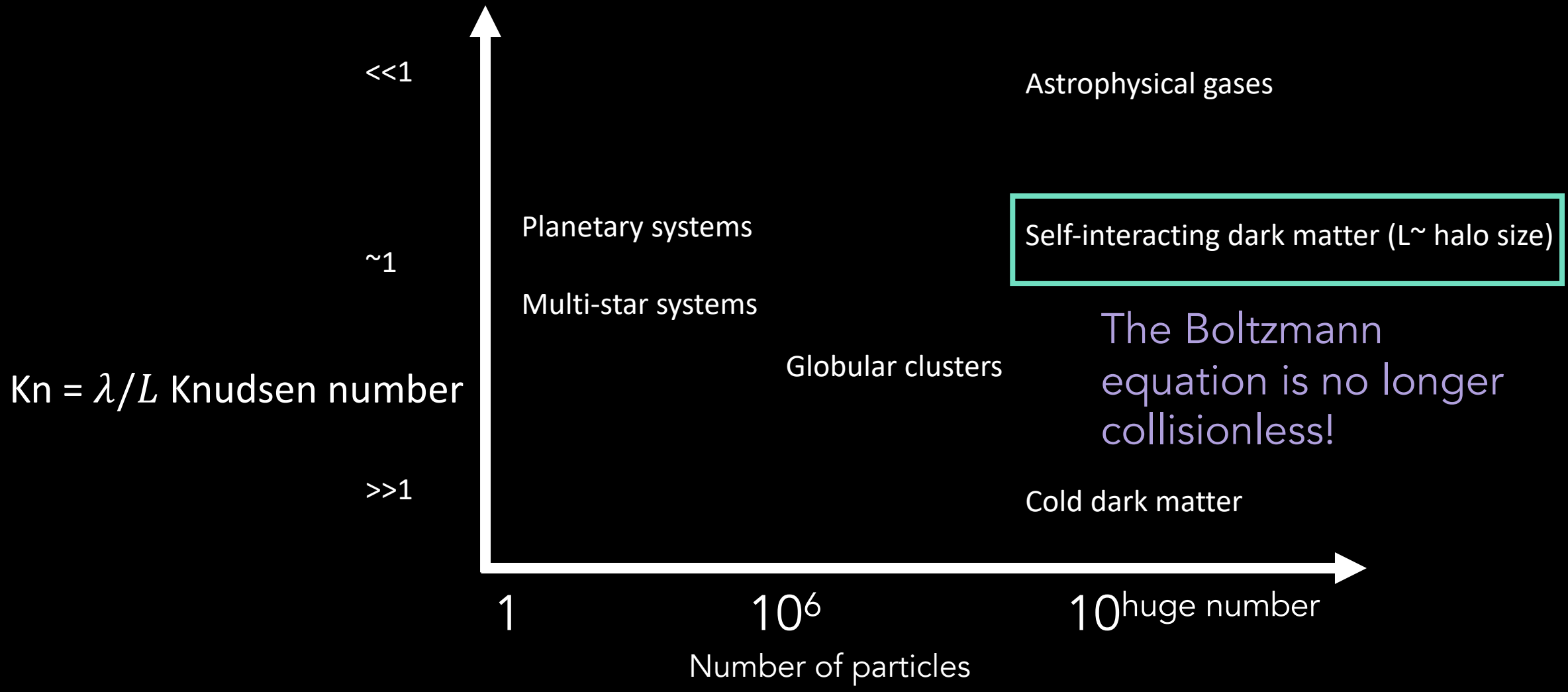


Figure 6. The particles of Fig. 5 traced back to their positions in the initial conditions. The low resolution simulation is shown in the top panel and the high resolution simulation in the bottom panel. Note the highly flattened configurations of spurious haloes.

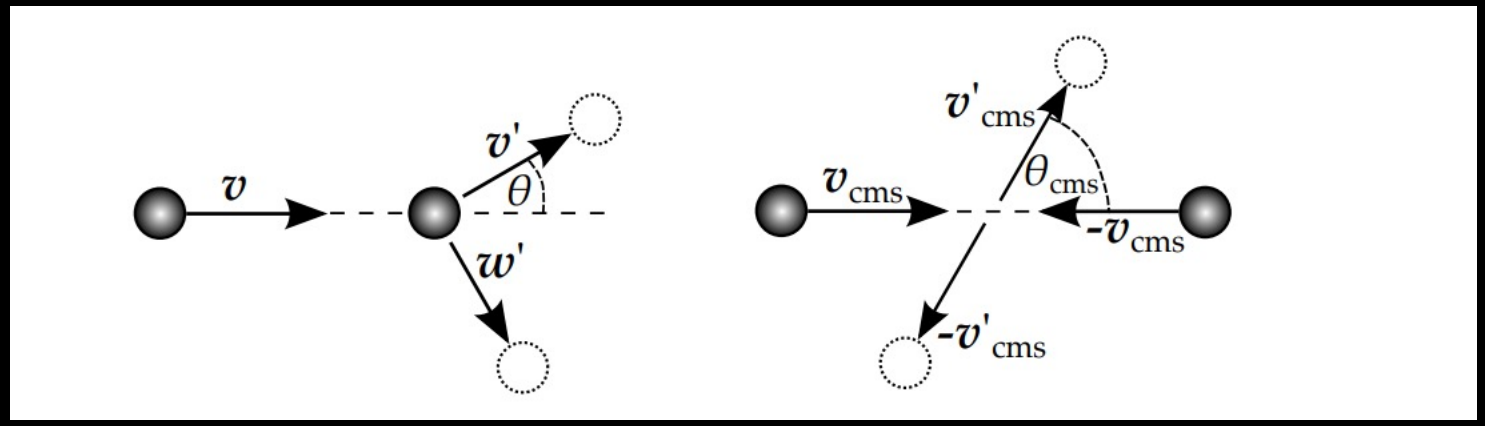
See Hahn+ 1210.6652 for a different simulation approach

Self-interacting dark matter

Considerations



Simulation approach depends on nature of scatter



Kahlhoefer+ 1308.3419
Has a very nice explanation of why large-angle scattering is different from small-angle scattering.

Velocity-dependence matters, too.

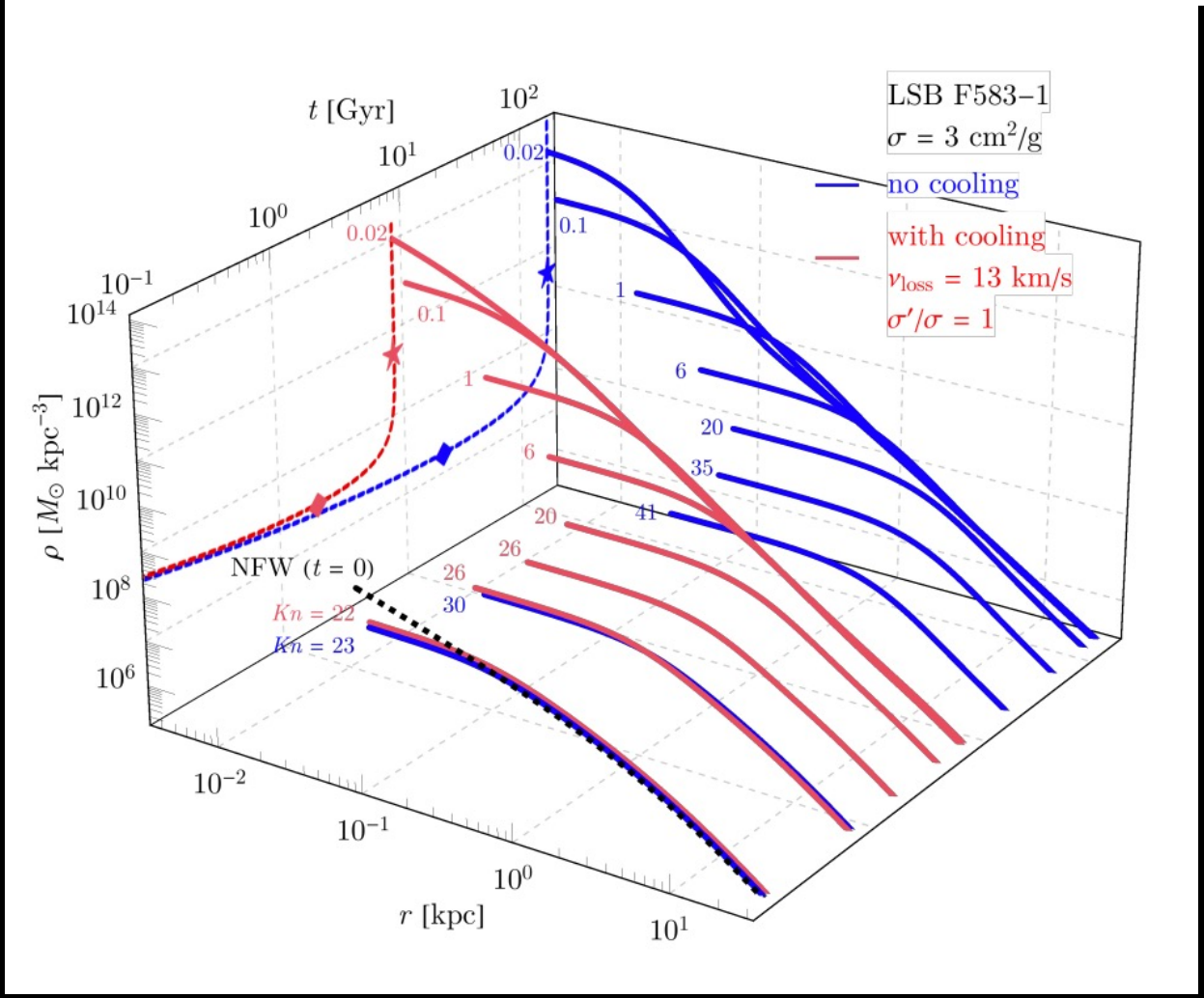
How is momentum moving around the system?

See the references document for different algorithms for different situations.

Usually need to consider a transfer or viscosity cross section.

Do you have a system where anisotropy in the scatter matters? (e.g., mergers) See papers by Robertson, Banerjee, Kahlhoefer.

(Gravothermal) catastrophe!



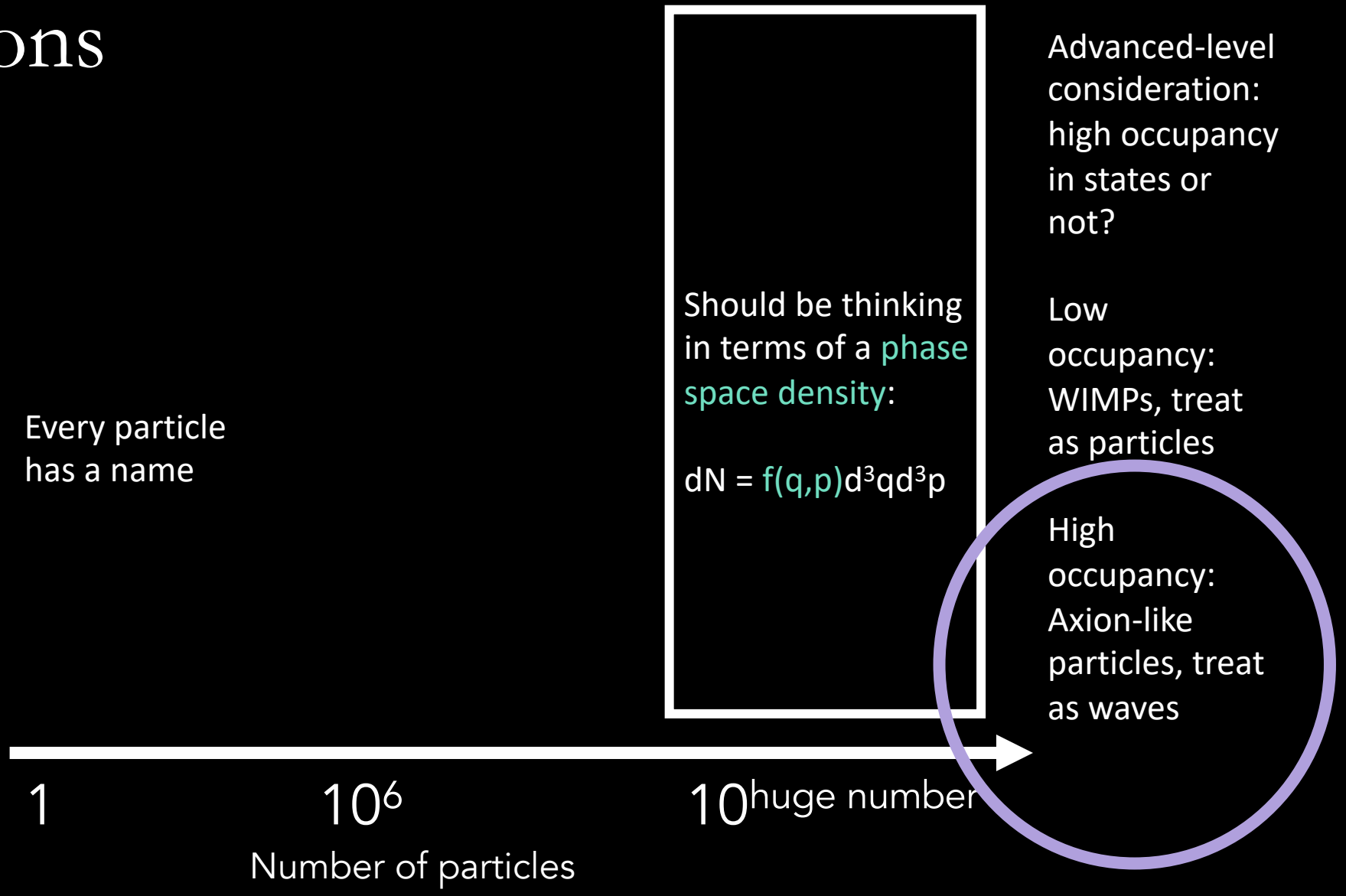
Essig+ 1809.01144

See also 2002-era papers by Colin, Shapiro(s), new papers by Kaplinghat and collaborators

Stay tuned for a forthcoming paper by Carton Zeng (OSU!)

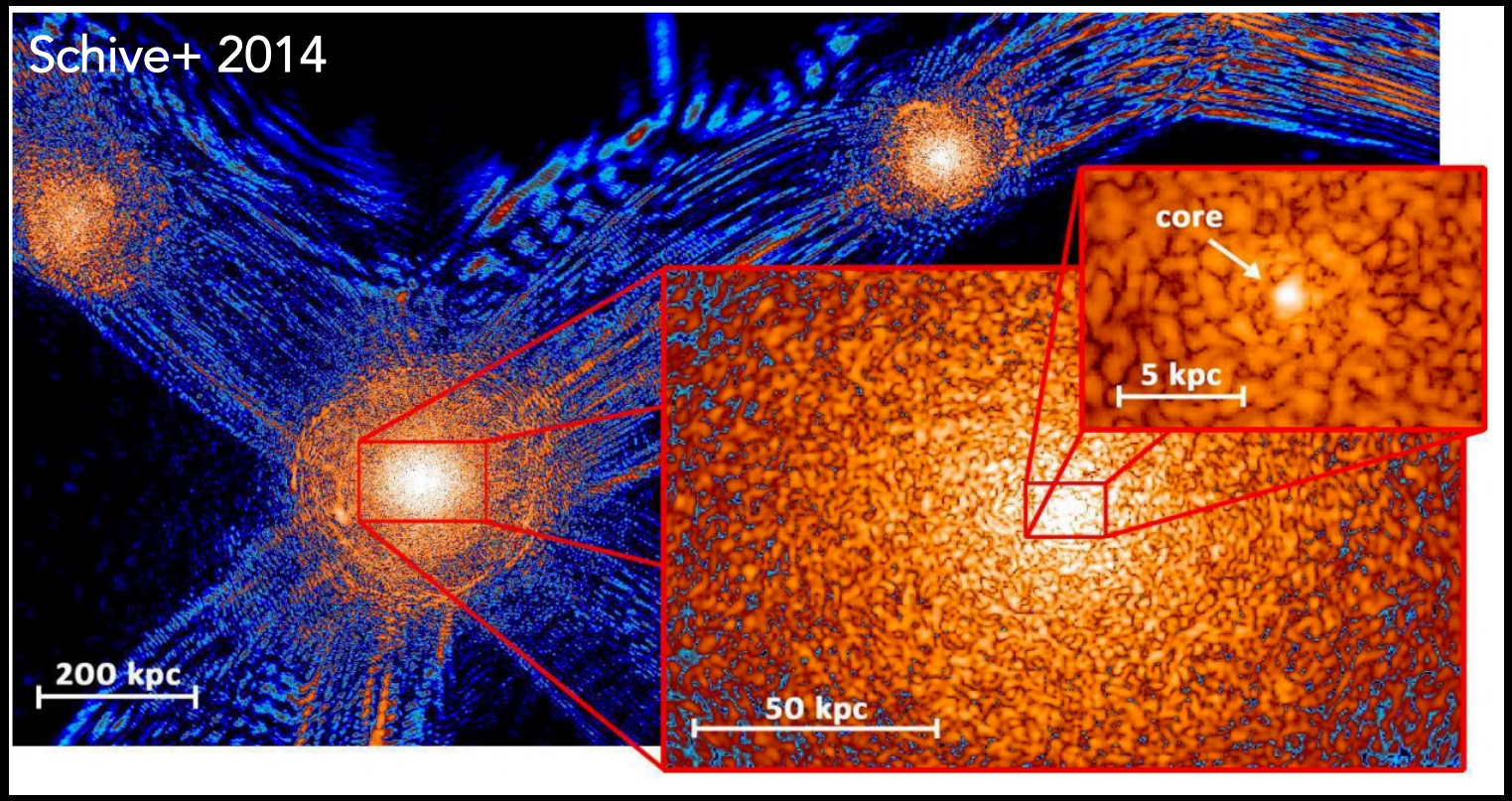
Fuzzy dark matter/wave dark matter

Considerations



COMPLETELY different phenomenology & methods

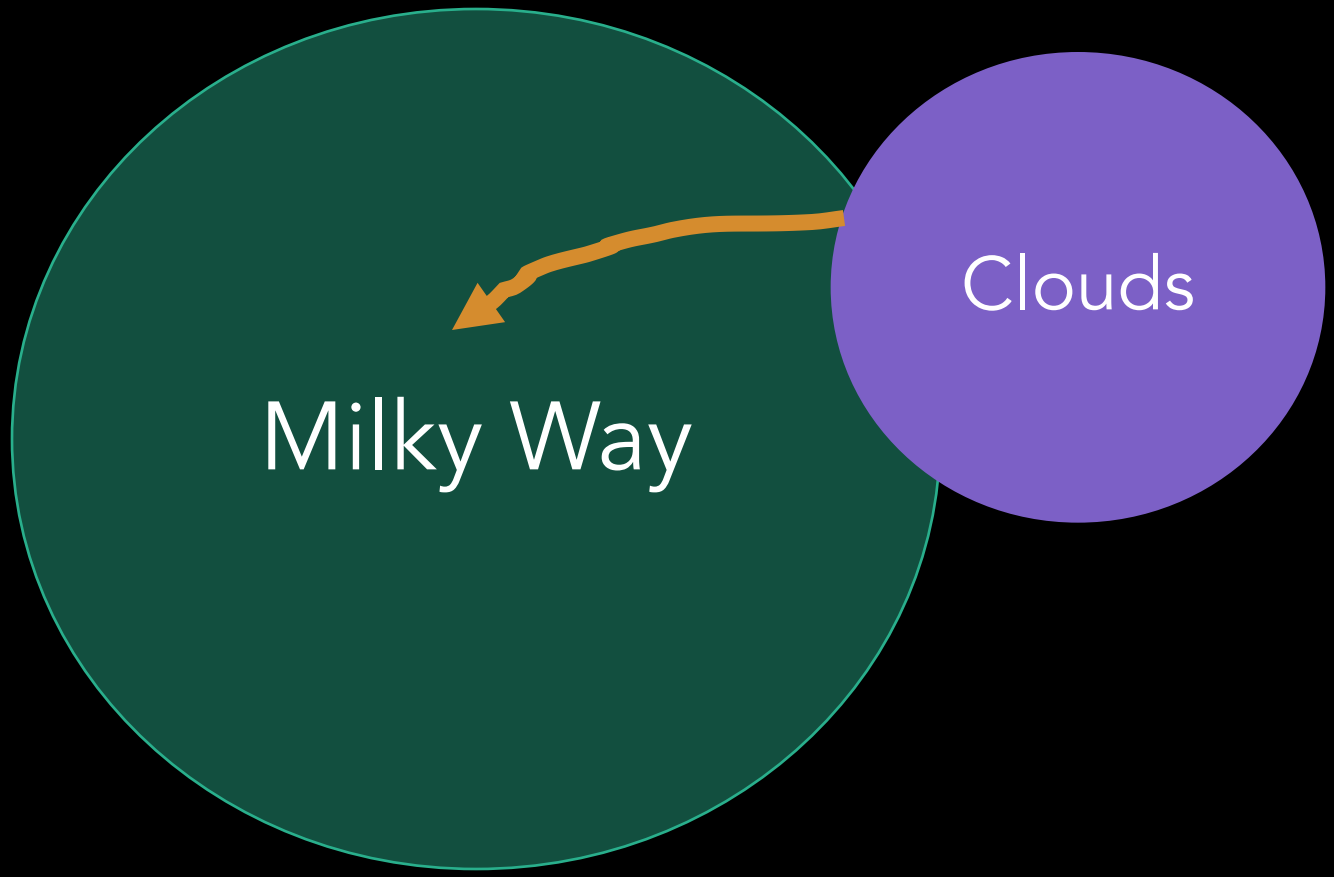
"Fuzzy" dark matter



Something that's just plain cool

And can ONLY be done in simulation

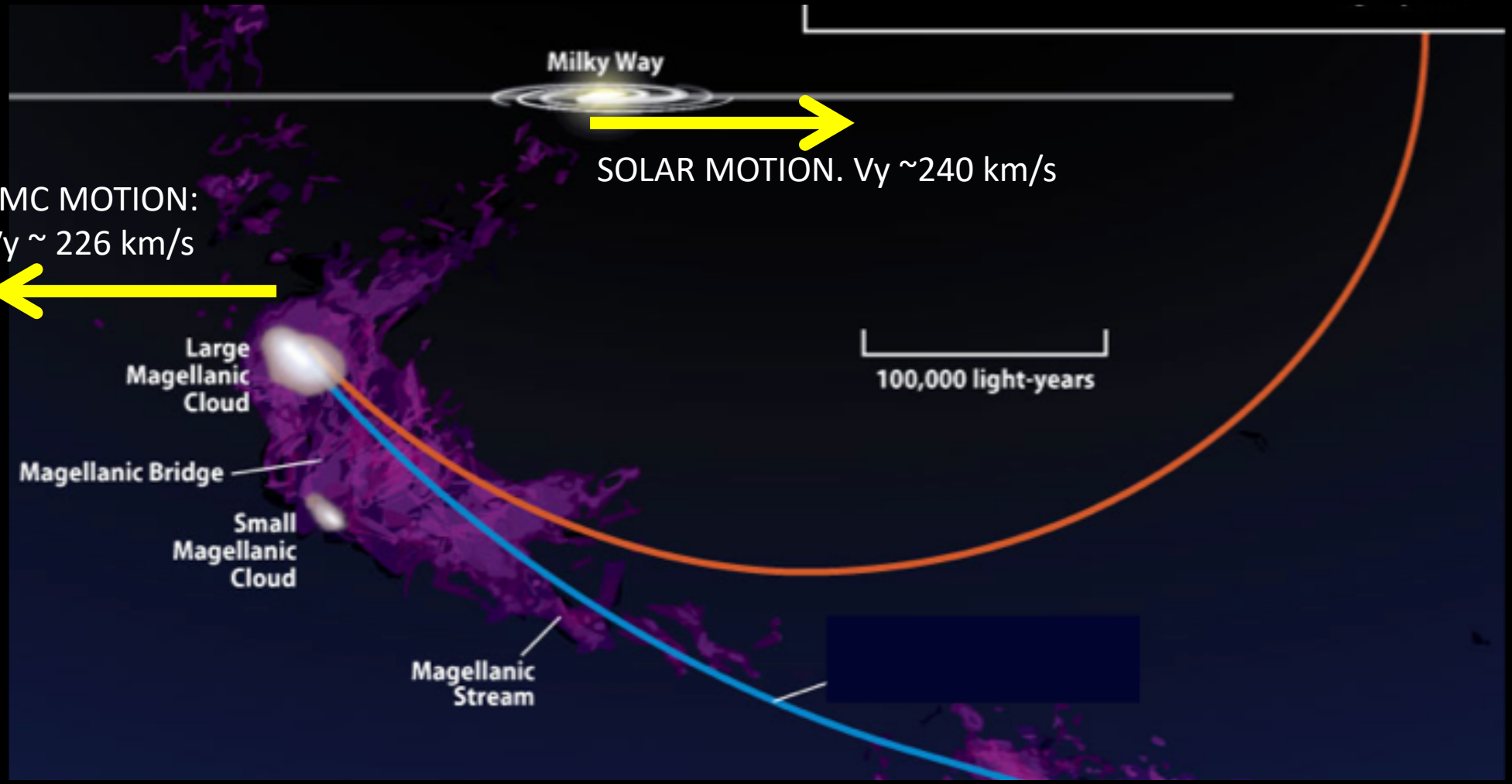
The LMC as a Massive Satellite



Only recently captured by the Milky Way

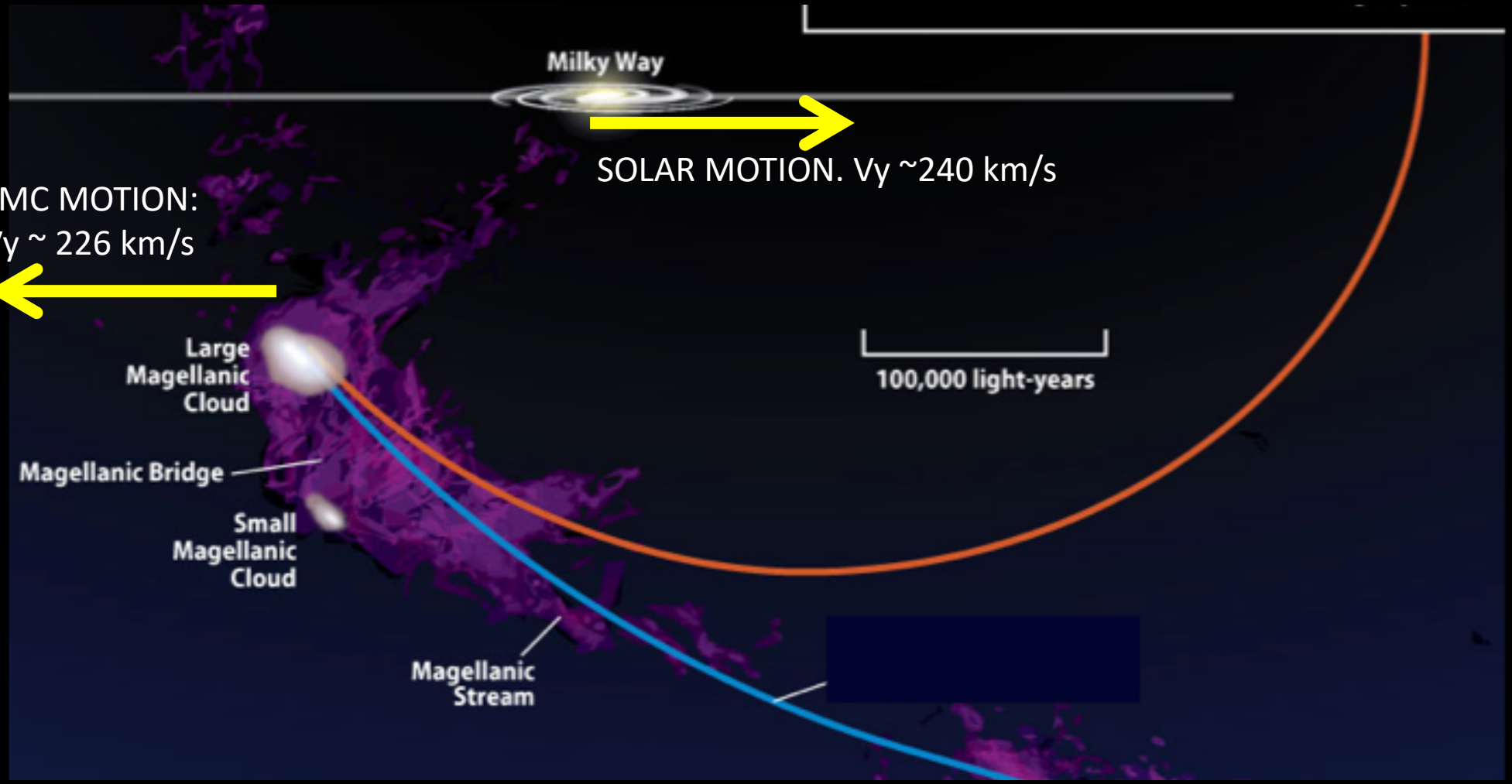
The LMC has made its closest approach to the Milky Way ~ 50 Myr ago at a distance of ~49 kpc.

Orbit over the past 1 Gyr is largely independent of Milky Way halo mass or density profile.



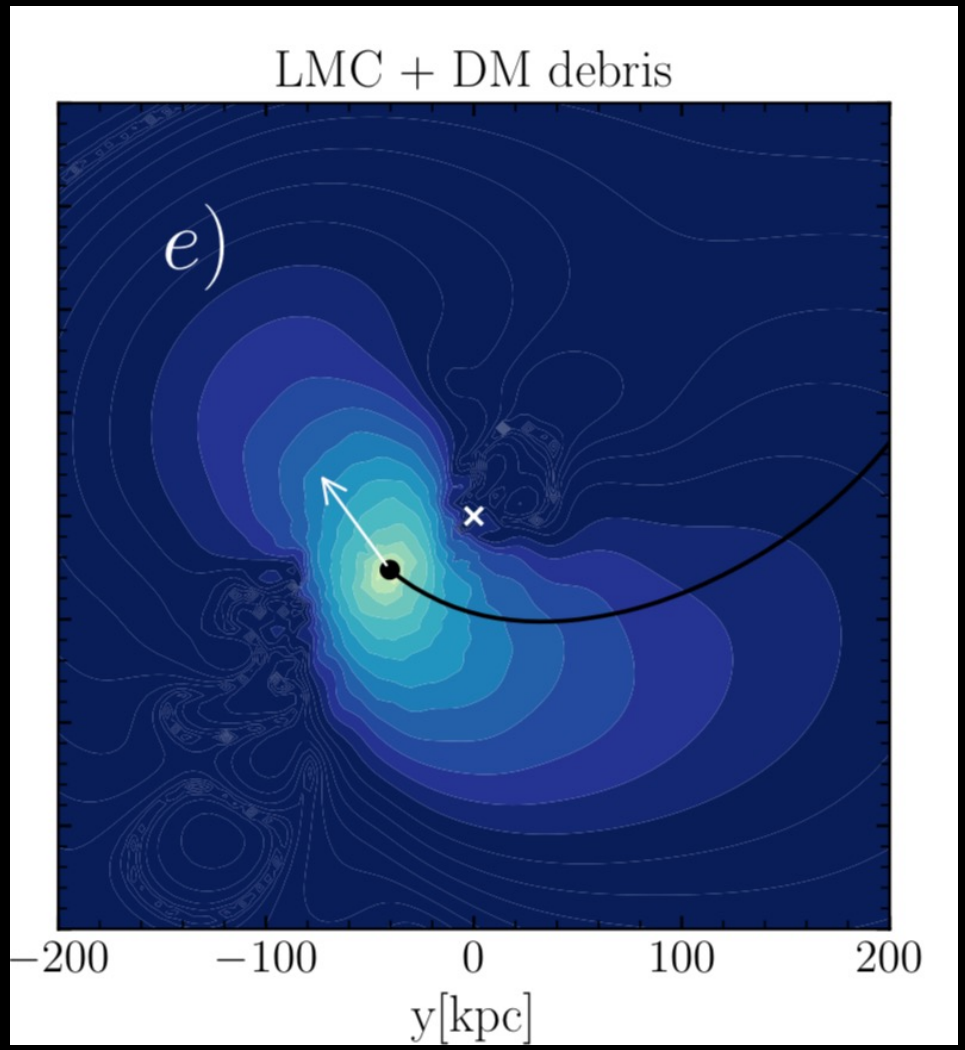
Only recently captured by the Milky Way

Dark matter from the Clouds is tidally stripped by the Milky Way—but stars only start being stripped when >90% of the dark matter is gone.



Dark matter effects of the Clouds

1. Tidal debris from the Clouds.



Garavito-Camargo+ 2020

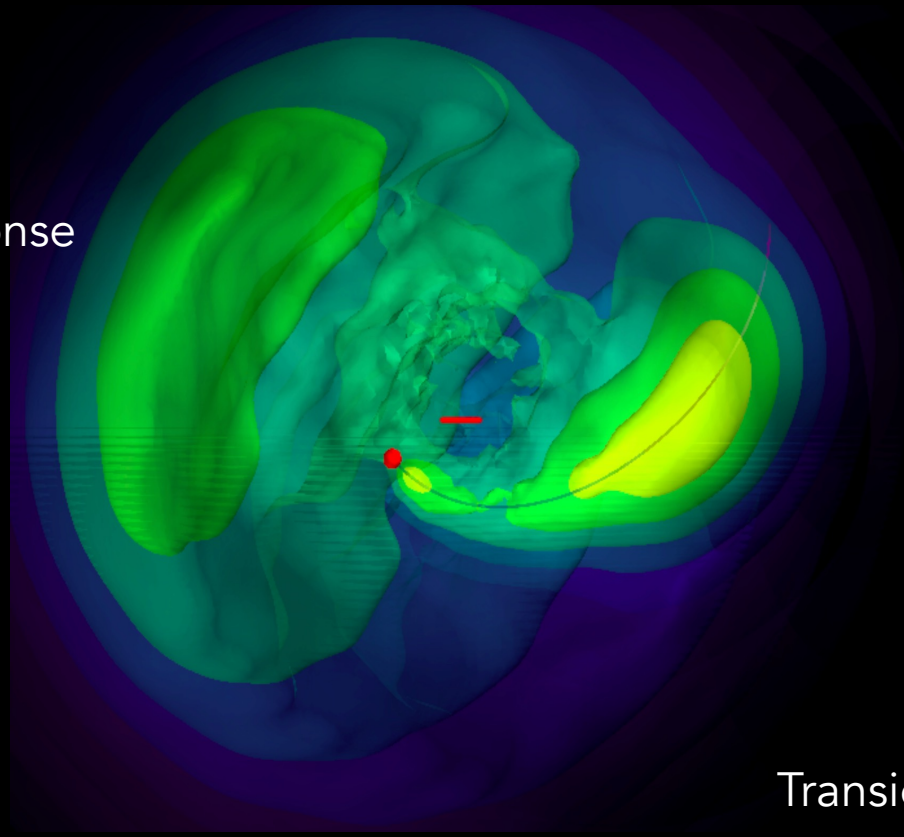
Dark matter effects of the Clouds

1. Tidal debris from the Clouds.
2. Dynamical imprint of the Clouds on the Milky Way dark matter halo.

Dynamical Friction

(details depend on nature of dark matter particle)

Collective response



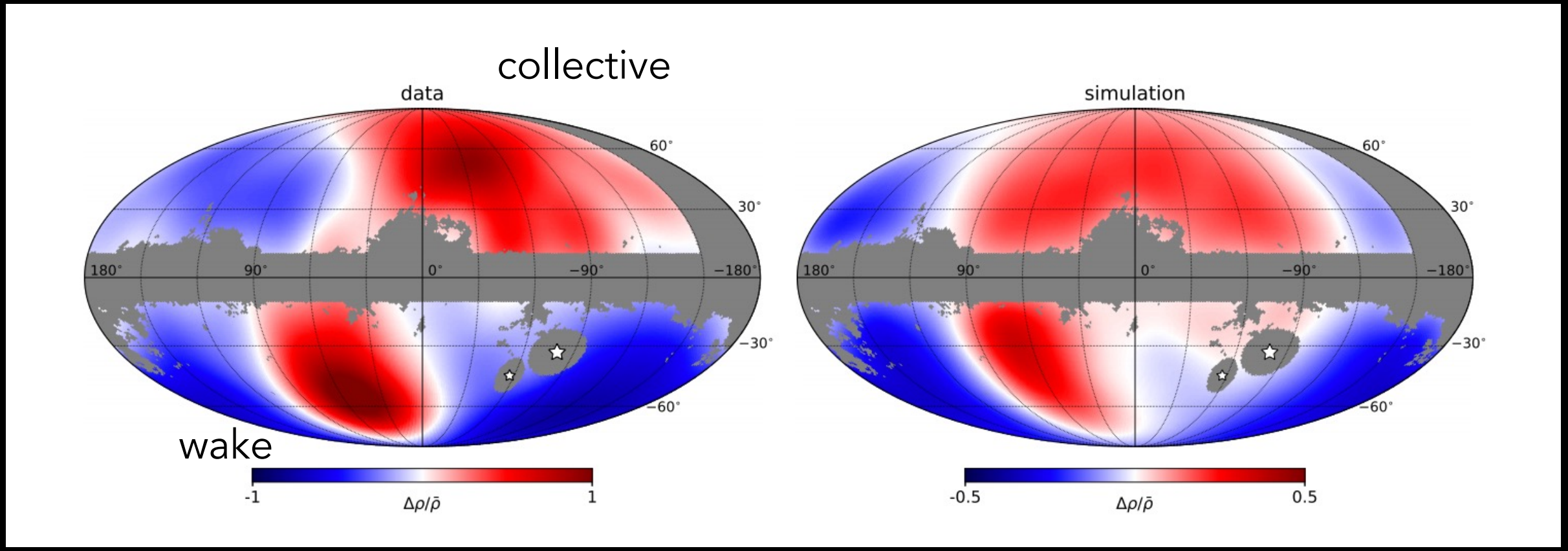
Transient wake

Garavito-Camargo, Besla+ 2019, 2020

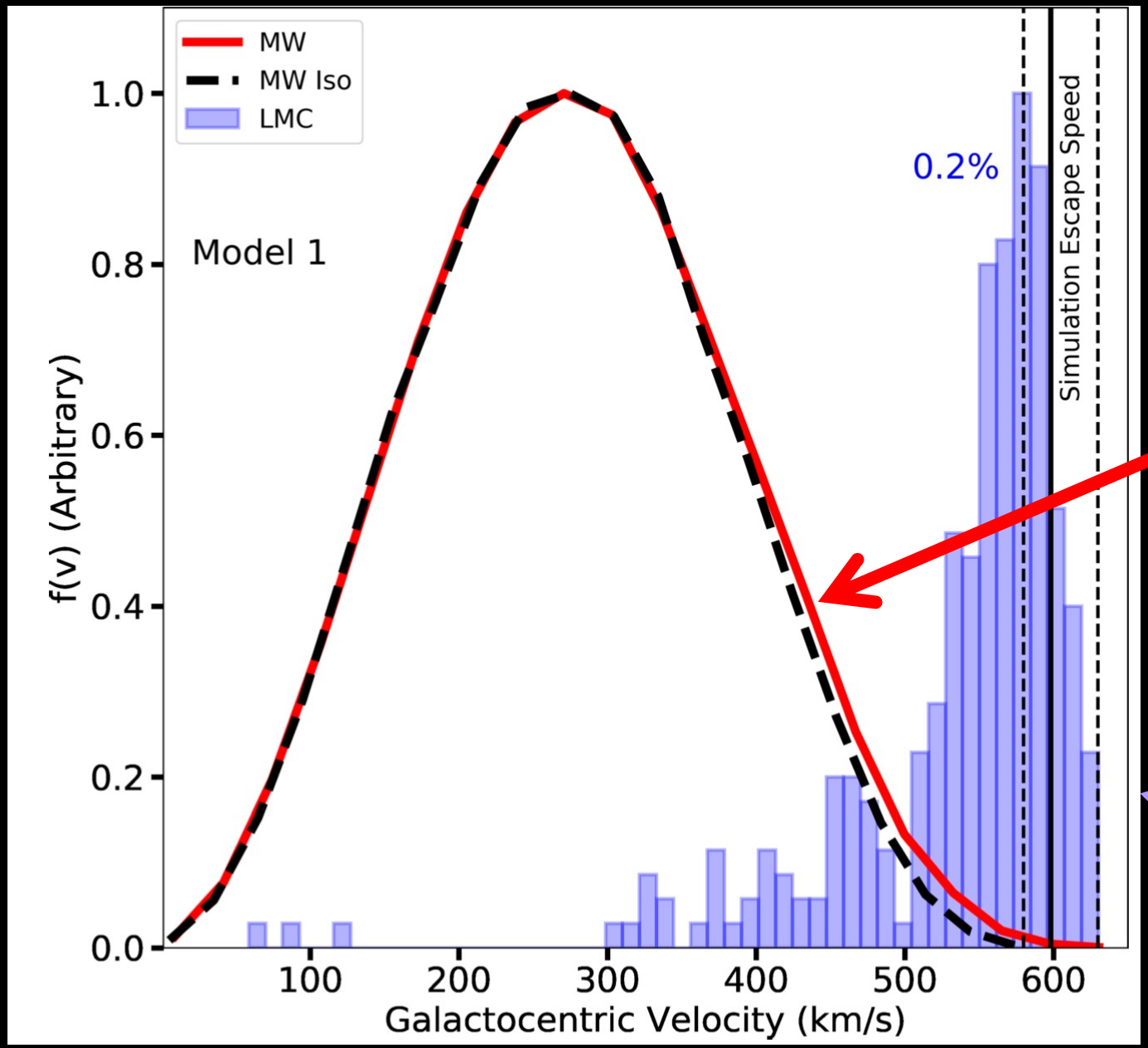
See also
Vesperini & Weinberg 2000

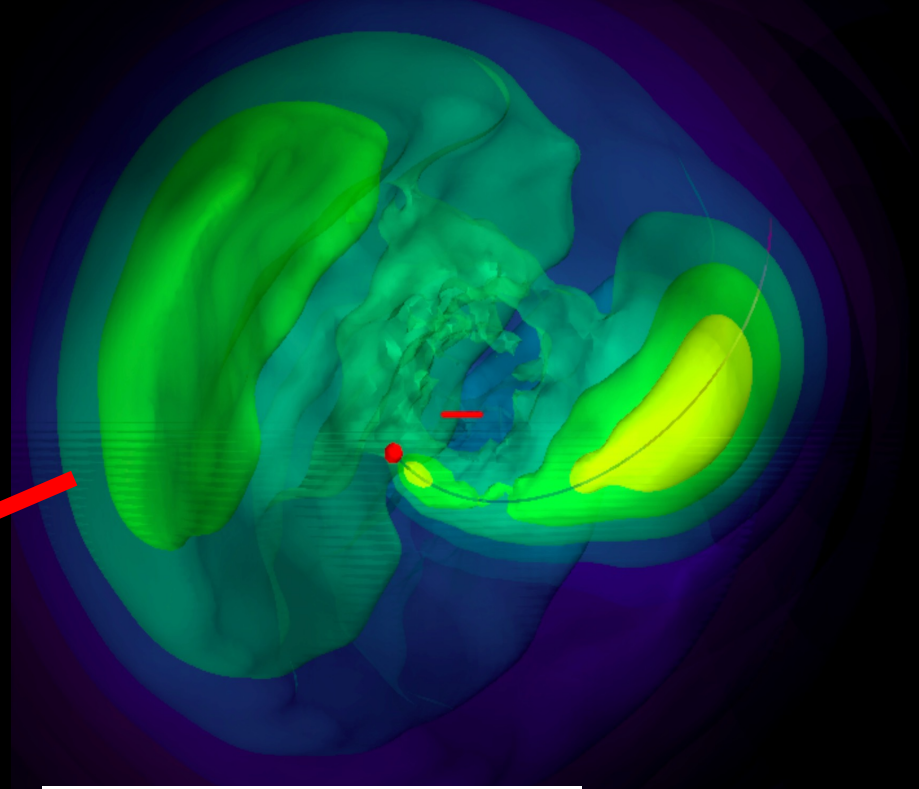
Video courtesy of Gurtina Besla

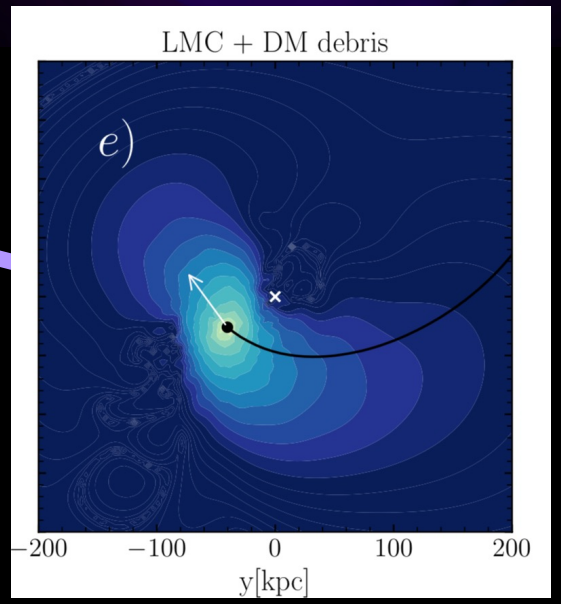
Observed in stars in the Milky Way!



Conroy+ 2021

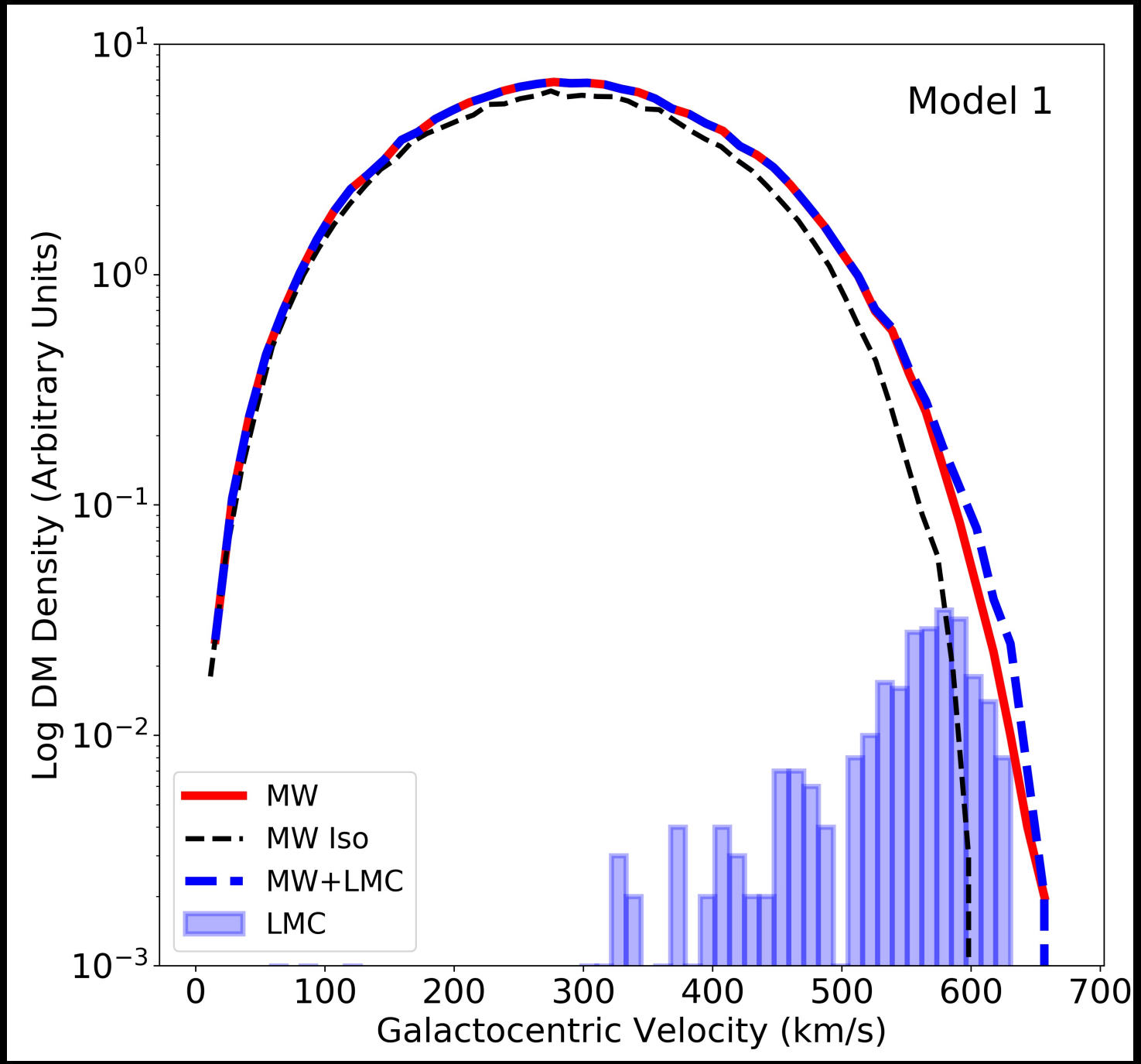






76% of DM particles with Galactocentric speeds > 600 km/s originate from the LMC

Besla, Peter & Garavito-Camargo + 2019



The combined effect of LMC particles and accelerated MW halo particles together augment the local DM density distribution by ~5%.

Besla, AP & Garavito-Camargo 1909.04140

Nico & Gurtina are working on similar sims with other types of dark matter.

Problems and approaches

Few-body gravitational problems

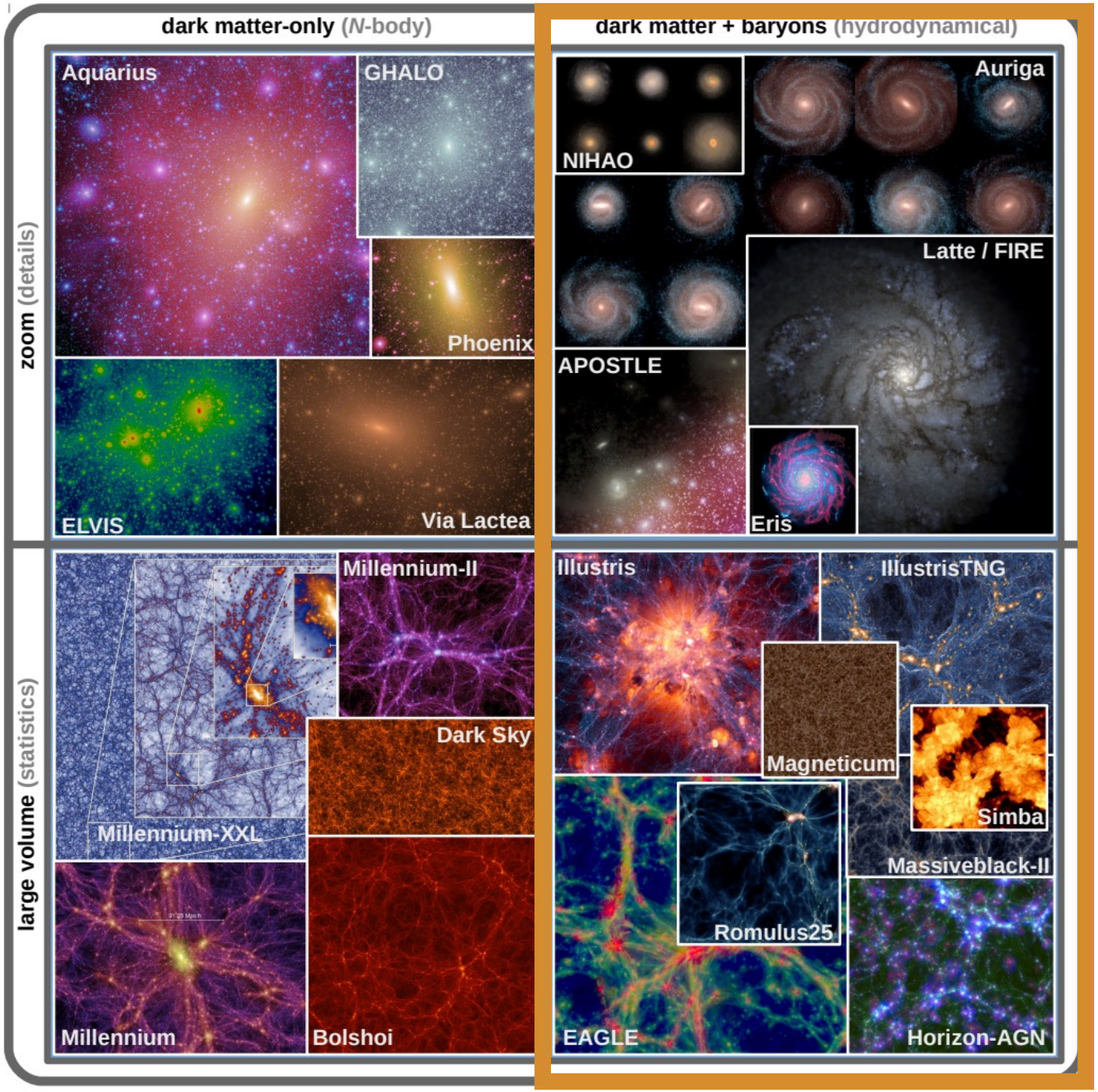
N-body problems

Hydrodynamics

Approaches

From Vogelsberger+ 1909.07976

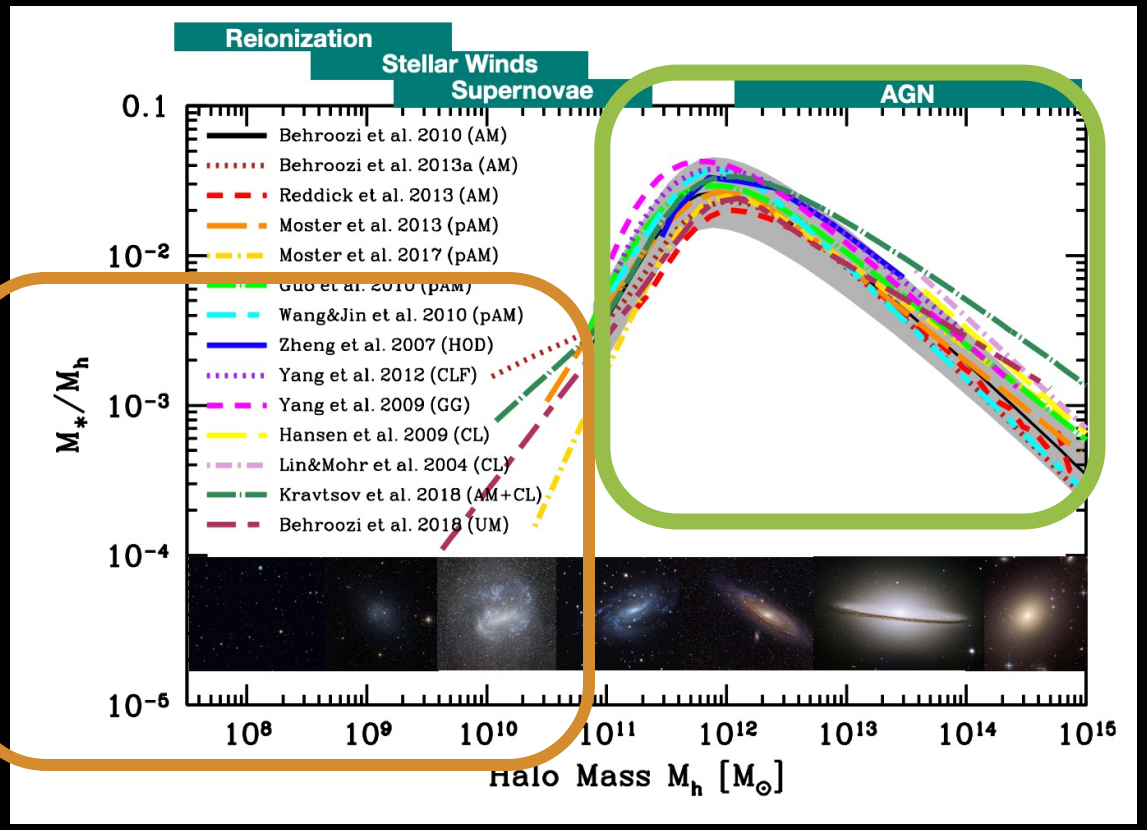
YOU SHOULD READ THIS PAPER



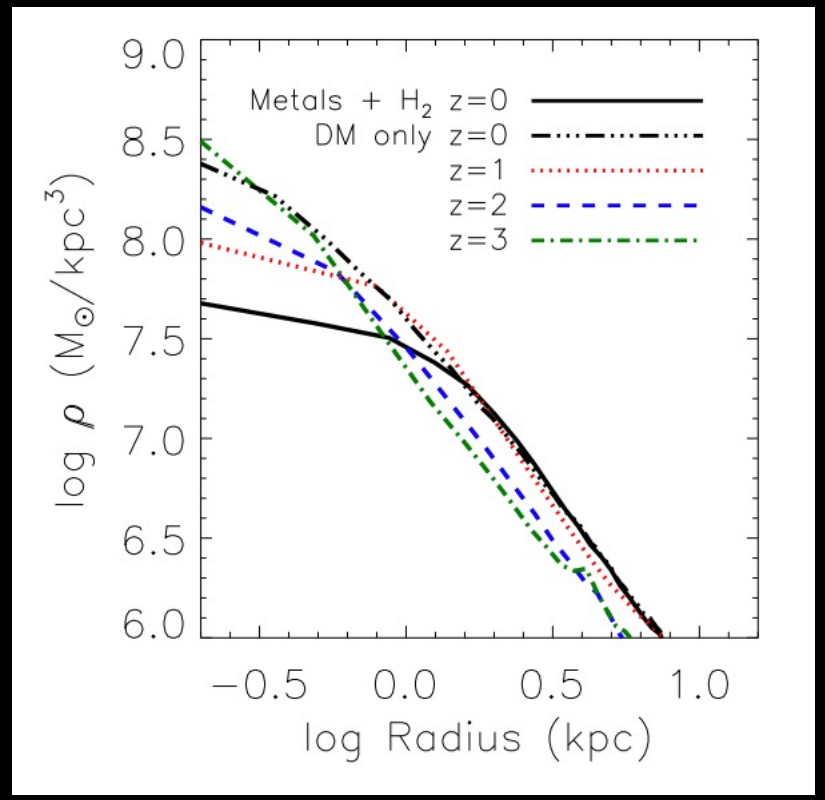
Why you should care

(even if you are a dark matter fanatic)

Baryons are the David to dark matter's Goliath



Wechsler & Tinker 1804.03097



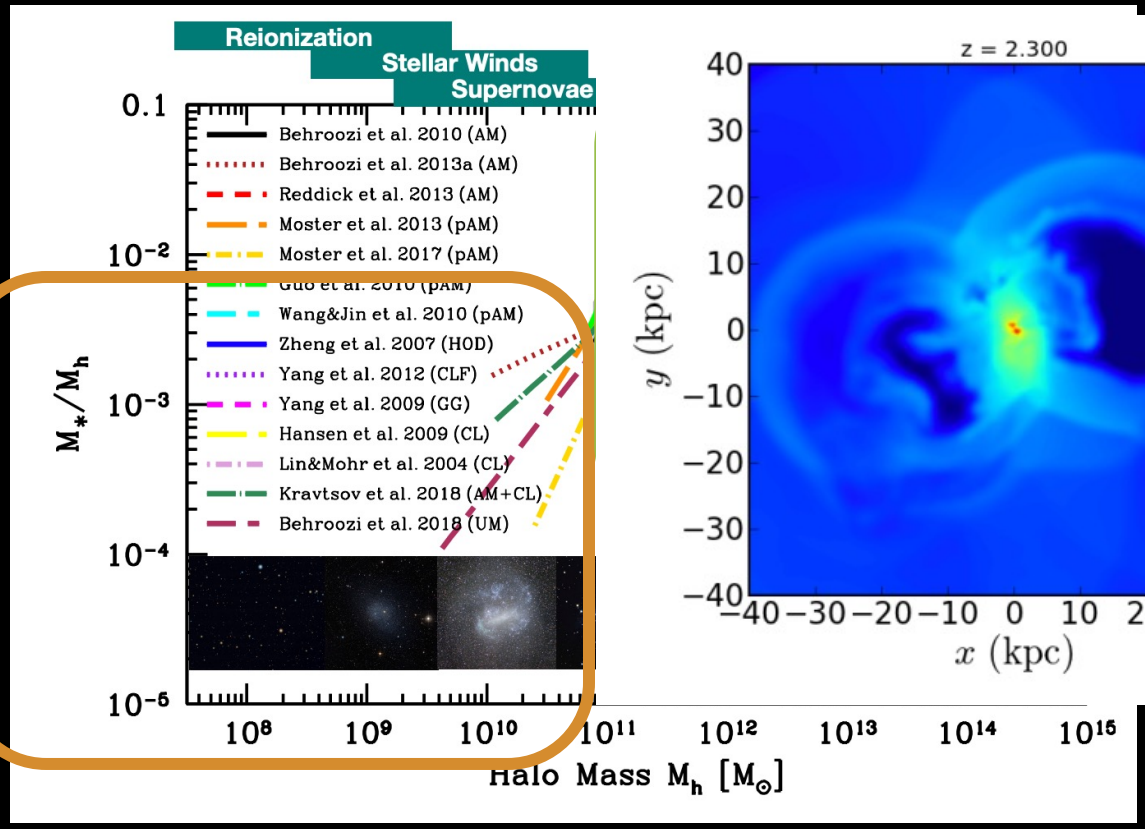
Governato+ 1202.0554

Why you should care

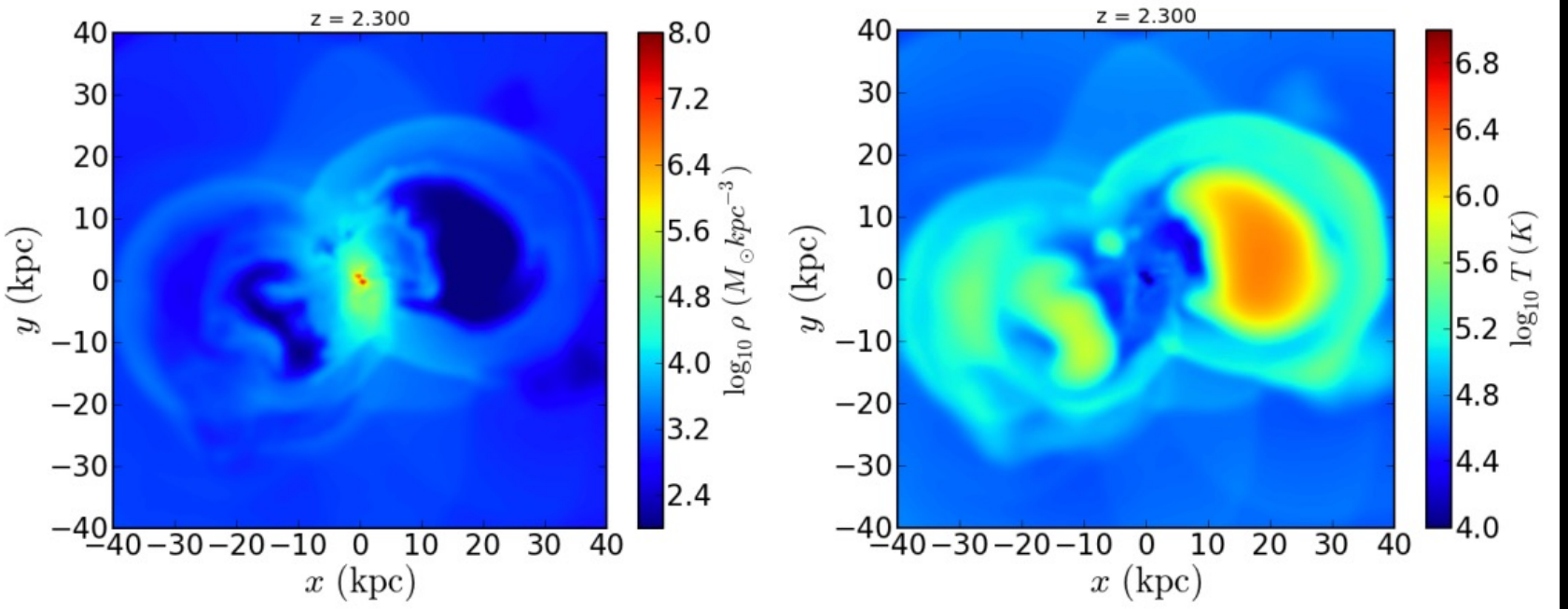
(even if you are a dark matter fanatic)

Changes the interpretation of small-scale structure problems related to density and galaxy abundance.

Baryons are the David to dark matter's Goliath

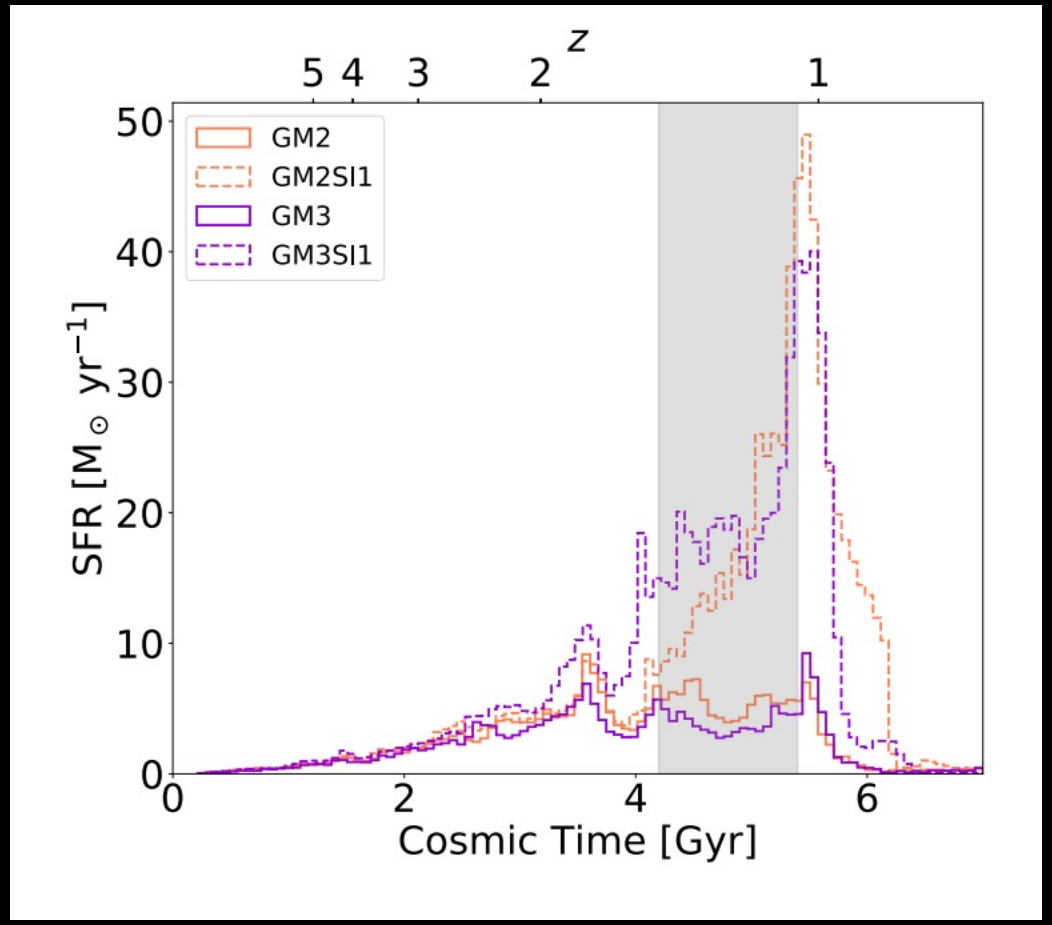


Wechsler & Tinker 1804.03097



Onorbe+ 1502.02036

Can reveal unexpected signatures of dark matter via its effects on baryonic evolution



Cruz+ 2004.08477 SIDM 1 cm^2/g

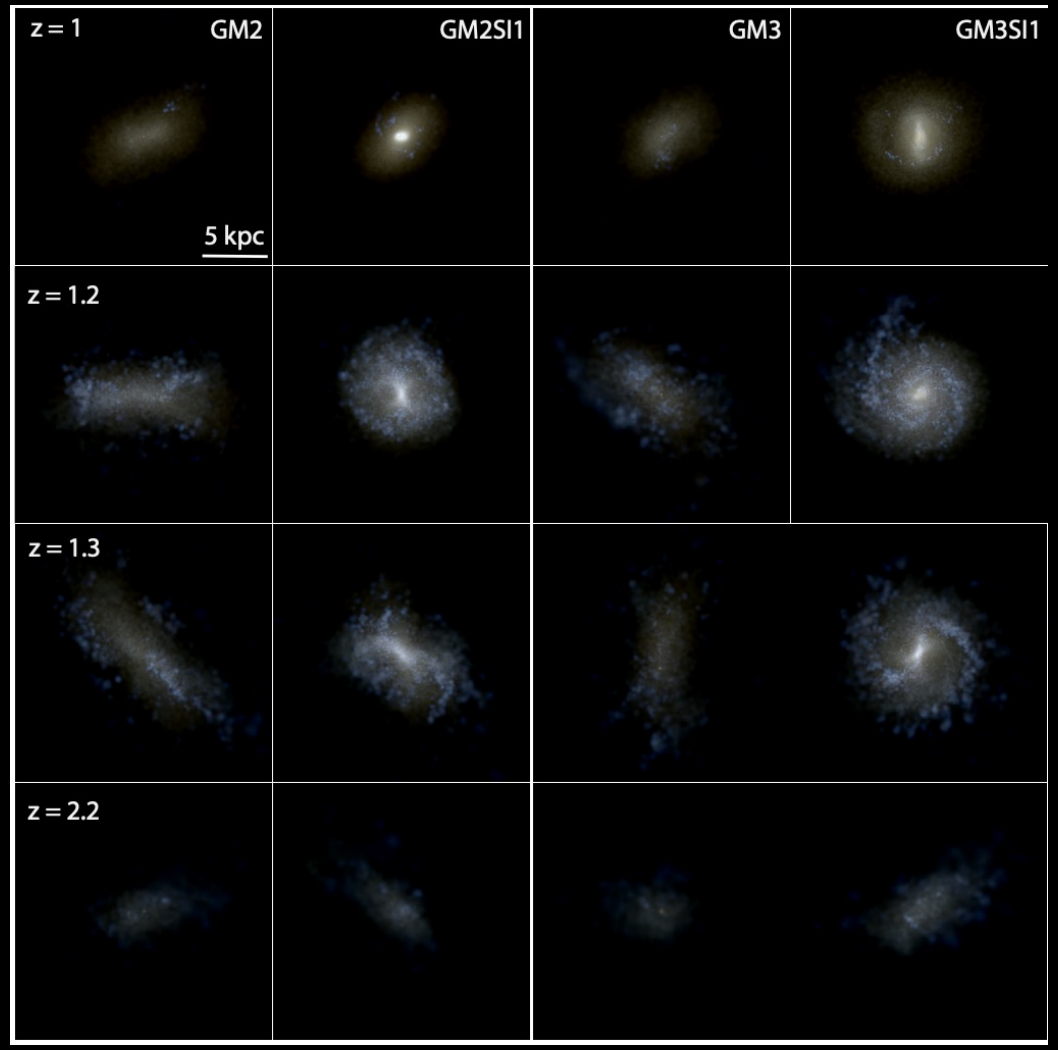
Table 1. GALAXY PROPERTIES AT $z = 0.8$

Sim	M_{vir}[a] $M_\odot$	M_*[b] $M_\odot$	M_g[c] $M_\odot$	M_{BH}[d] $M_\odot$	R_{vir}[e] kpc
GM2	4.6×10^{11}	1.3×10^{10}	3.9×10^{10}	5.1×10^{7}	140.8
GM2SI1	5.1×10^{11}	3.6×10^{10}	6.2×10^{10}	4.6×10^{7}	148.6
GM3	4.5×10^{11}	9.8×10^{9}	3.5×10^{10}	6.2×10^{7}	139.2
GM3SI1	5.0×10^{11}	3.6×10^{10}	5.2×10^{10}	4.0×10^{7}	147.5

[a] Halo virial mass as defined in Equation (1).
[b] Total stellar mass within R_{vir}.
[c] Total gas mass within R_{vir}.
[d] Mass of central SMBH in major progenitor of MW-mass galaxies.
[e] Halo virial radius, R_{vir}.

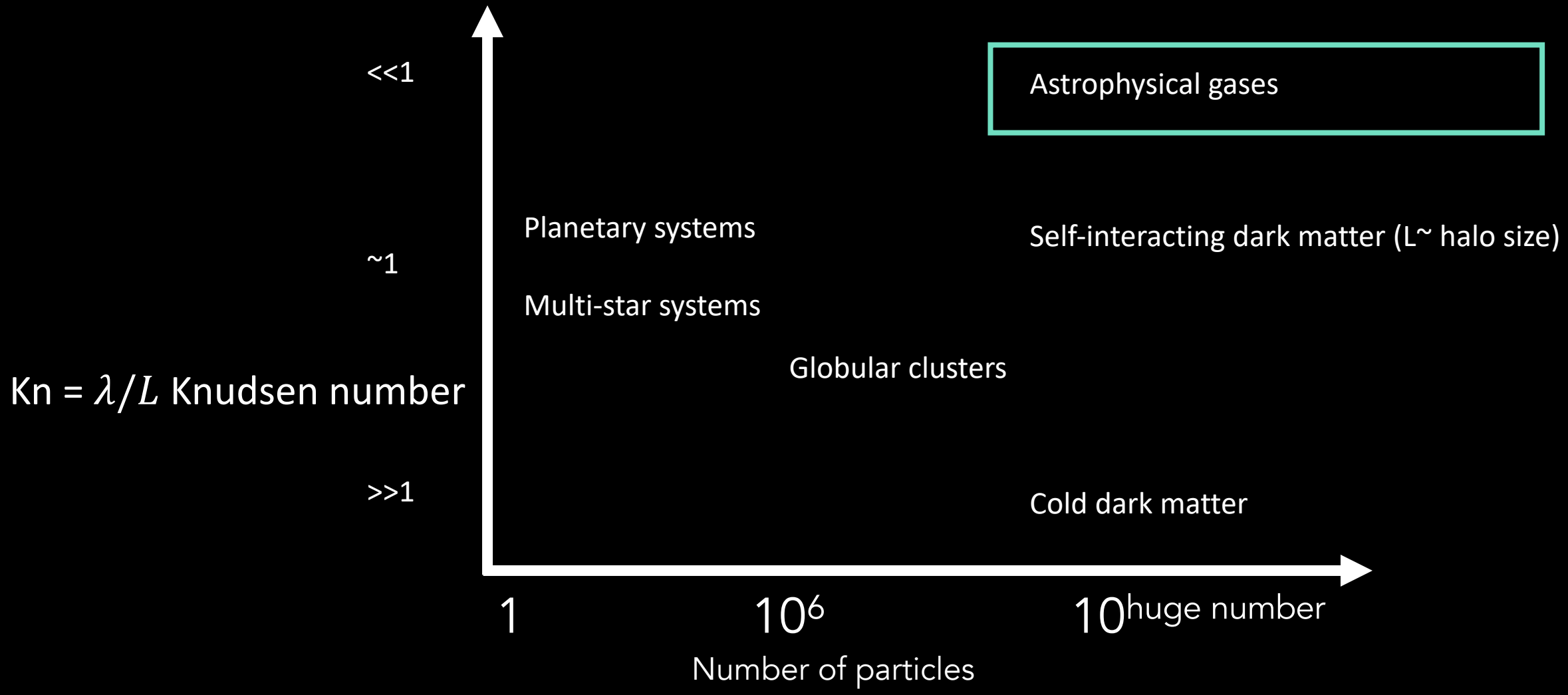
See also papers by di Cintio, Stacy Kim, Robertson, Kaplinghat

Can reveal unexpected signatures of dark matter via its effects on baryonic evolution



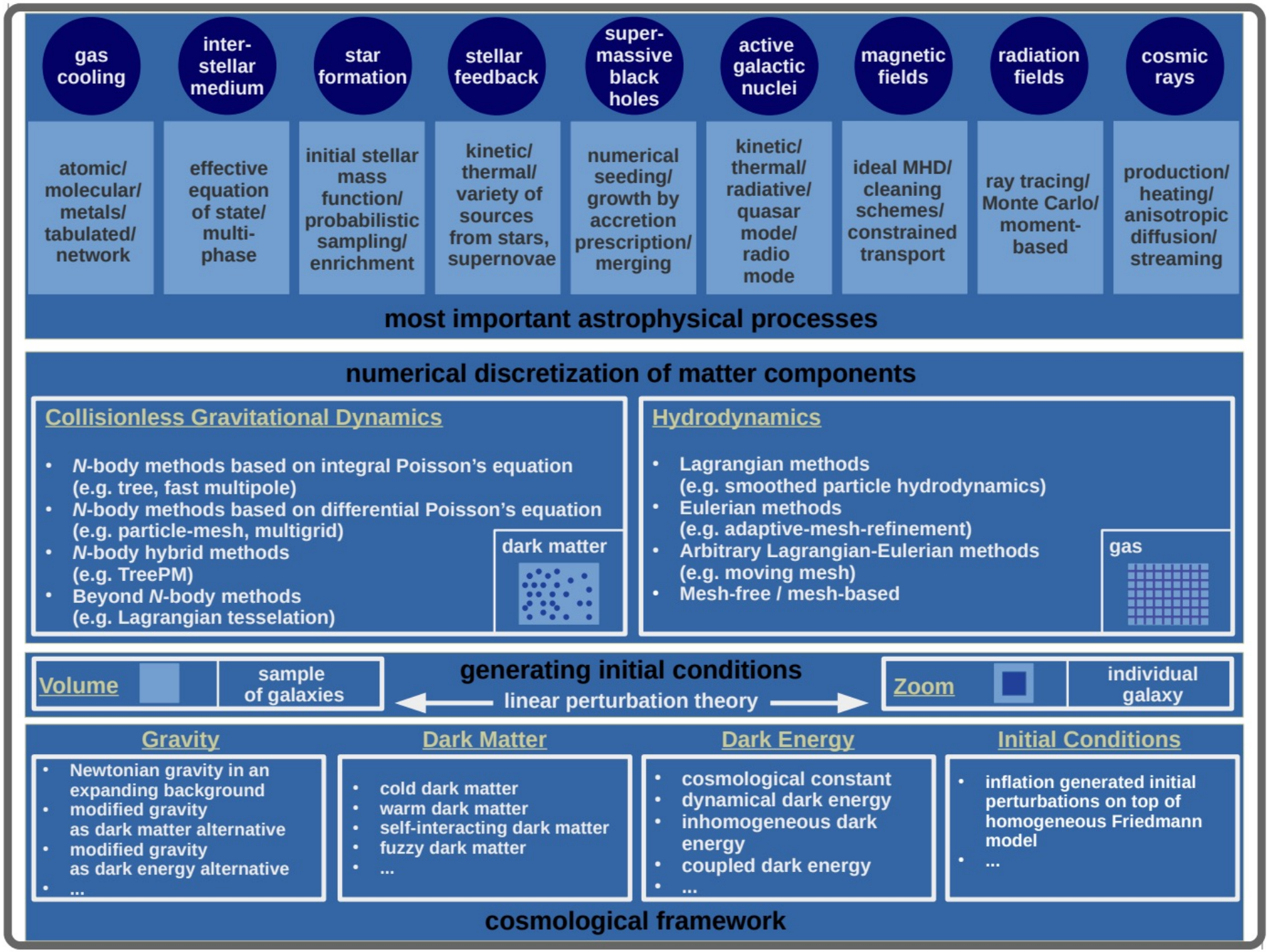
Cruz+ 2004.08477 SIDM 1 cm^2/g

Considerations



Physics

From Vogelsberger+ 1909.07976



Hydrodynamic equations

(tiny Knudsen number limit)*

Large densities = tiny timesteps = very costly

Modeling cosmic gas

Eulerian formulation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + P\mathbb{1}) = 0$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho e + P)\mathbf{v} = 0$$

Lagrangian formulation:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{v}$$

$$\frac{\mathrm{D}\mathbf{v}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla P$$

$$\frac{\mathrm{D}e}{\mathrm{D}t} = -\frac{1}{\rho}\nabla \cdot P\mathbf{v}$$

Arbitrary Lagrangian-Eulerian formulation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho \mathrm{d}V = -\int_S \rho(\mathbf{v}-\mathbf{w}) \cdot \mathbf{n}\mathrm{d}S$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho \mathbf{v}\mathrm{d}V = -\int_S \rho\mathbf{v}(\mathbf{v}-\mathbf{w}) \cdot \mathbf{n}\mathrm{d}S - \int_S P\mathbf{n}\mathrm{d}S$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho e \mathrm{d}V = -\int_S \rho e(\mathbf{v}-\mathbf{w}) \cdot \mathbf{n}\mathrm{d}S - \int_S P\mathbf{v} \cdot \mathbf{n}\mathrm{d}S$$

Different forms of the hydrodynamical equations. $\mathrm{D}/\mathrm{d}t \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$ denotes the Lagrangian derivative and $e = u + \mathbf{v}^2/2$ the total energy per unit mass. The equations are closed through $P = (\gamma - 1)\rho u$ with $\gamma = 5/3$. For the arbitrary Lagrangian-Eulerian formulation the grid moves with velocity $\mathbf{w}$ and cell volumes evolve as $\mathrm{d}V/\mathrm{d}t = \int_V (\nabla \cdot \mathbf{w})\,\mathrm{d}V$.

From Vogelsberger+ 1909.07976

See this paper for MHD, CR, radiation treatment too

*With the exception of cosmic rays

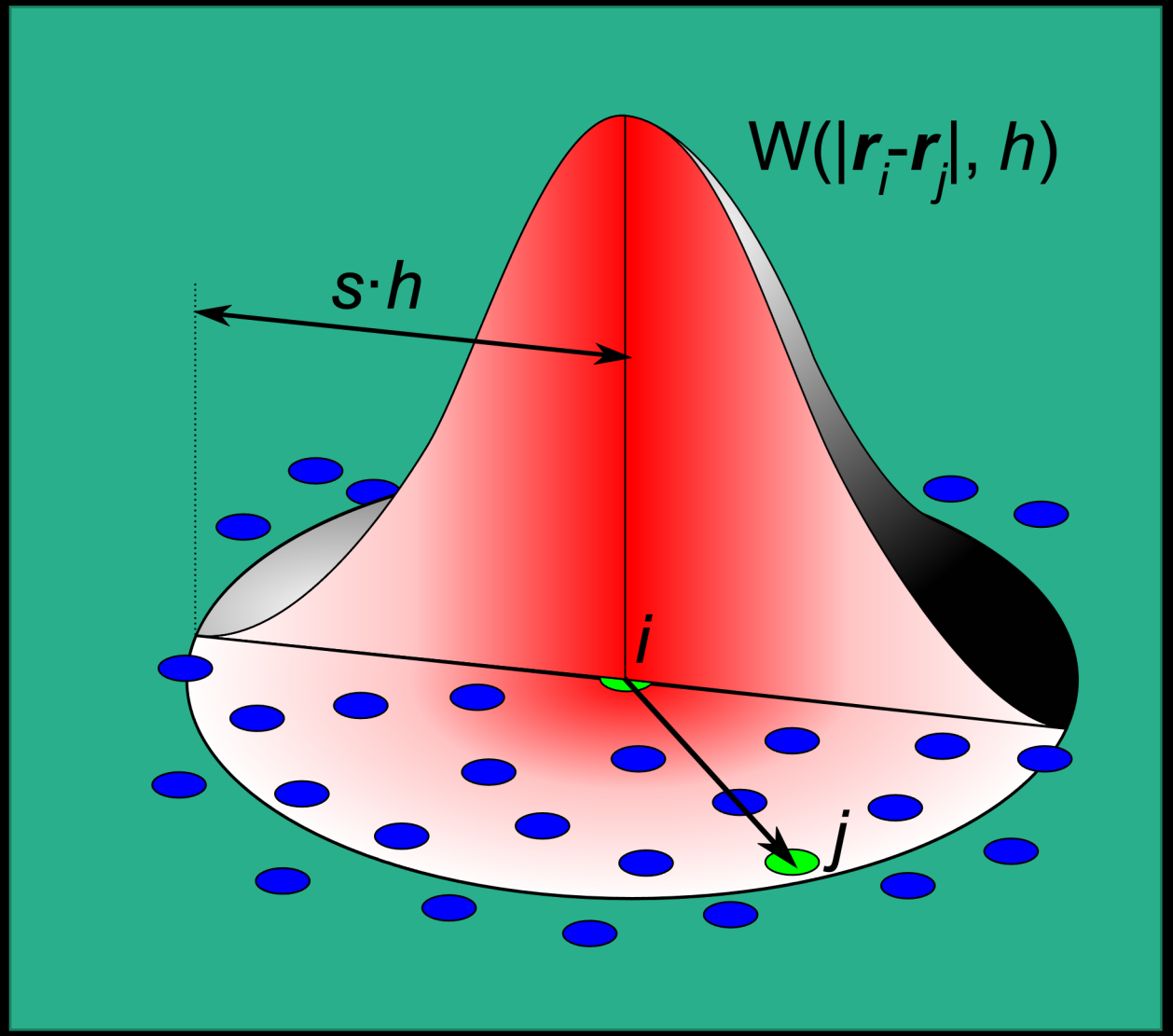
Gridless methods

Smoothed particle hydrodynamics (SPH)

Fully Lagrangian approach

$$\rho_s(\mathbf{r}) \simeq \sum_j m_j W(\mathbf{r}-\mathbf{r}_j, h),$$

Need to discretize the Lagrangian hydro equations (see references doc for refs)

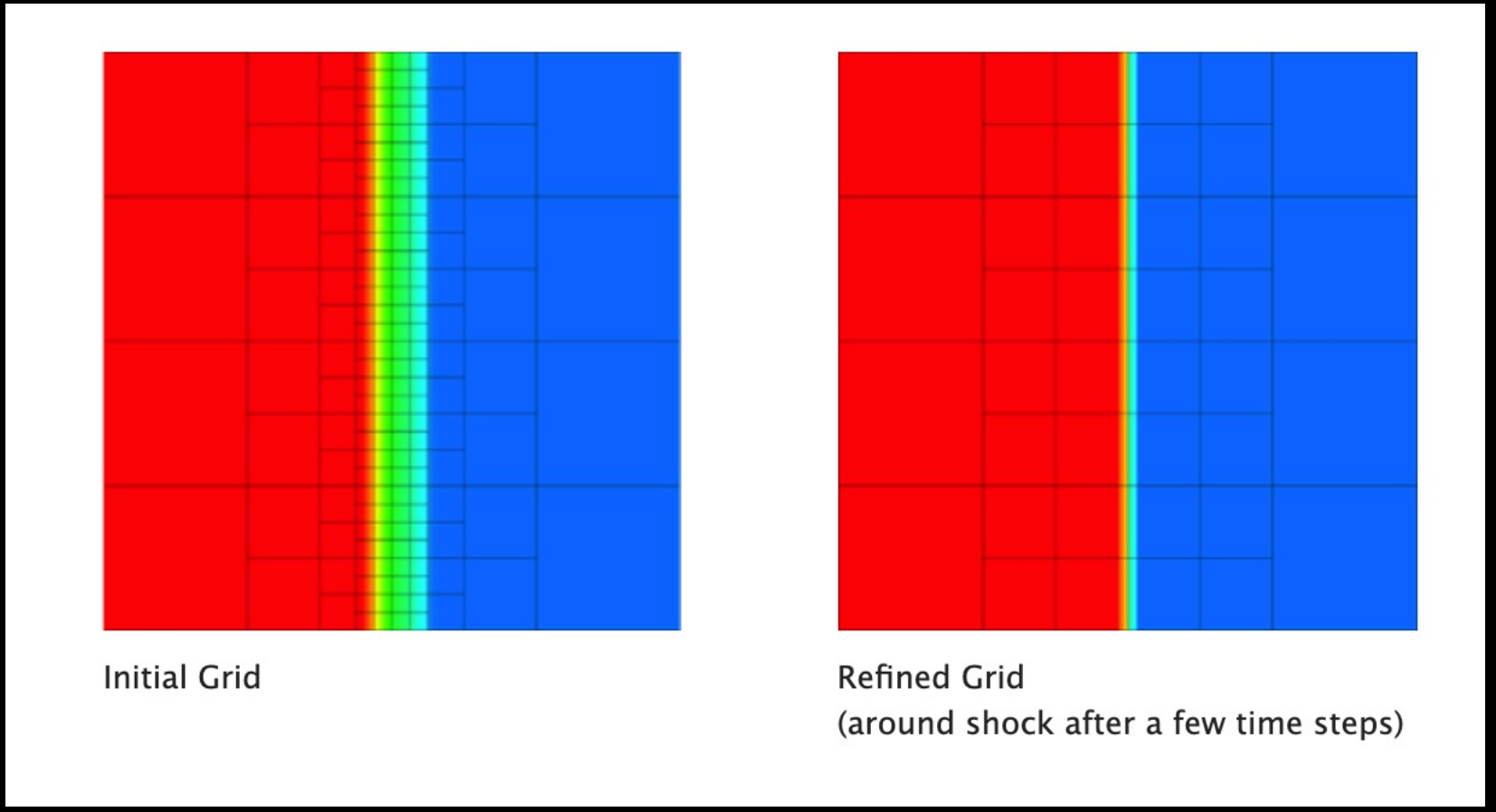


https://commons.wikimedia.org/wiki/File:SPHInterpolationColorsVerbose.svg

Grid methods

Eulerian approach

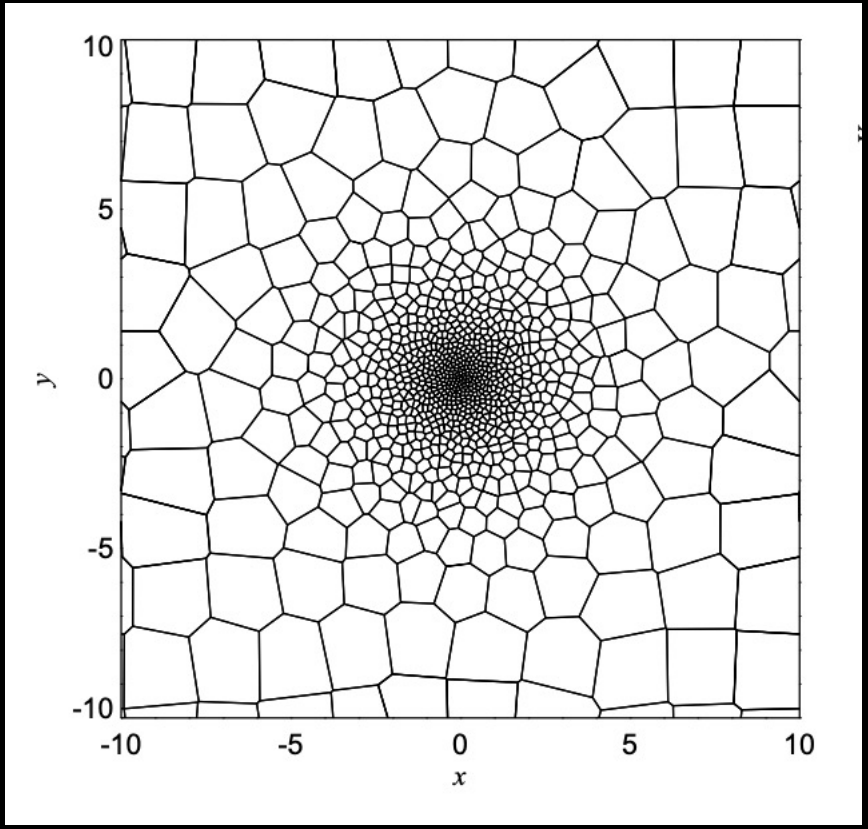
If you want to learn more, check out Mike Zingale's pyro code package and the accompanying text book:
https://python-hydro.github.io/pyro2/
http://bender.astro.sunysb.edu/hydro_by_example/CompHydroTutorial.pdf



https://www.lanl.gov/projects/codesign/codesign-summer-school/research-areas/adaptive-mesh-refinement.php

Hybrid

"Hydrodynamic cosmological simulations at present usually employ either the Lagrangian smoothed particle hydrodynamics (SPH) technique, or Eulerian hydrodynamics on a Cartesian mesh with (optional) adaptive mesh refinement (AMR). Both of these methods have disadvantages that negatively impact their accuracy in certain situations, for example the suppression of fluid instabilities in the case of SPH, and the lack of Galilean-invariance and the presence of overmixing in the case of AMR."—Springel, 0901.4107



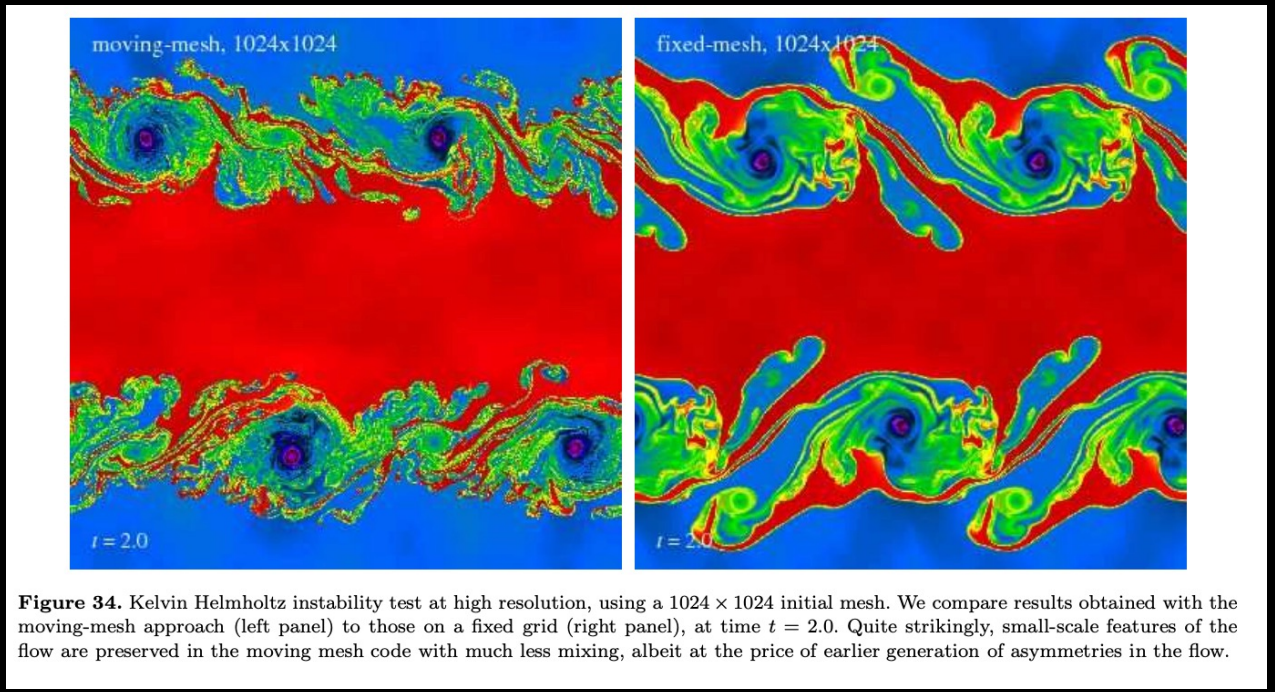


Figure 34. Kelvin Helmholtz instability test at high resolution, using a 1024×1024 initial mesh. We compare results obtained with the moving-mesh approach (left panel) to those on a fixed grid (right panel), at time $t = 2.0$. Quite strikingly, small-scale features of the flow are preserved in the moving mesh code with much less mixing, albeit at the price of earlier generation of asymmetries in the flow.

What the major codes actually use

From Vogelsberger+ 1909.07976

Table 1: Major galaxy formation simulation codes

code name	gravity treatment[a]	hydrodynamics treatment[b]	parallelization technique[c]	code availability[d]	primary reference
ART	PM/ML	AMR	data-based	public	Kravtsov (1997)[27]
RAMSES	PM/ML	AMR	data-based	public	Teyssier (2002)[38]
GADGET-2/3	TreePM	SPH	data-based	public	Springel (2005)[39]
Arepo	TreePM	MMFV	data-based	public	Springel (2010)[40]
Enzo	PM/MG	AMR	data-based	public	Bryan et al. (2014)[41]
ChaNGa[e]	Tree/FM	SPH	task-based	public	Menon et al. (2015)[42–44]
GIZMO[f]	TreePM	MLFM/MLFV	data-based	public	Hopkins et al. (2015)[45]
HACC	TreePM/P^3M	CRK-SPH	data-based	private	Habib et al. (2016)[46]
PKDGRAV3	Tree/FM	–	data-based	public	Potter et al. (2017)[47]
Gasoline2	Tree	SPH	task-based	public	Wadsley et al. (2017)[48]
SWIFT	TreePM/FM	SPH	task-based	public	Schaller et al. (2018)[49]

[a] PM: particle-mesh; TreePM: tree + PM, FM: fast multipole, P^3M: particle-particle-particle-mesh; ML: multilevel; MG: multigrid

[b] SPH: smoothed particle hydrodynamics, CRK-SPH: conservative reproducing kernel smoothed particle hydrodynamics , AMR: adaptive-mesh-refinement, MMFV: moving-mesh finite volume, MLFM/MLFV: mesh-free finite mass / finite volume

[c] data-based: data parallelism focuses on distributing data across different nodes, which operate on the data in parallel; task-based: task parallelism focuses on distributing tasks concurrently performed

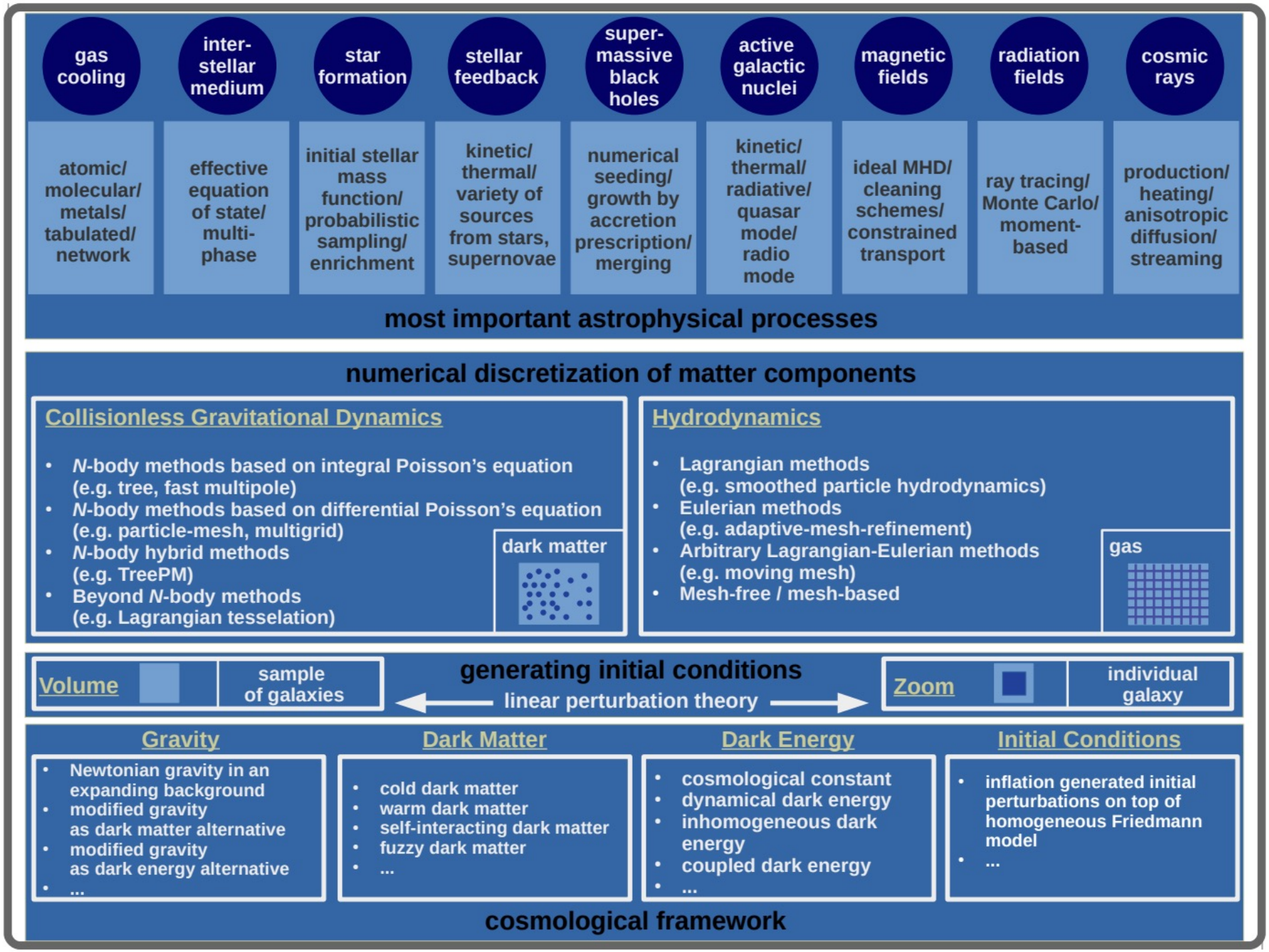
[d] private: private code; public: publicly available code (in some cases with limited functionality)

[e] gravity solver is based on PKDGRAV3

[f] based on the GADGET-3 code

Physics

From Vogelsberger+ 1909.07976



Putting it together

Few-body gravitational problems

N-body problems

Hydrodynamics

Cosmic structure

Things to know/where to start

- The first thing you do should not be to write your own code from scratch!
- Instead, start with one of the public simulation packages. You can also "phone a friend"—find someone working with a code and apprentice with them.
- Get familiar with one code and one analysis package.
- OR, you can start with public simulation data products.

Simulation codes

From Vogelsberger+ 1909.07976

Gadget-4 is now pubic (supercedes Gadget-2)

Beware that not all the bells and whistles are in the public versions of the codes.

Table 1: Major galaxy formation simulation codes

code name	gravity treatment[a]	hydrodynamics treatment[b]	parallelization technique[c]	code availability[d]	primary reference
ART	PM/ML	AMR	data-based	public	Kravtsov (1997)[27]
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Enzo	PM/MG	AMR	data-based	public	Bryan et al. (2014)[41]
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HACC	TreePM/P^3M	CRK-SPH	data-based	private	Habib et al. (2016)[46]
PKDGRAV3	Tree/FM	–	data-based	public	Potter et al. (2017)[47]
Gasoline2	Tree	SPH	task-based	public	Wadsley et al. (2017)[48]
SWIFT	TreePM/FM	SPH	task-based	public	Schaller et al. (2018)[49]

[a] PM: particle-mesh; TreePM: tree + PM, FM: fast multipole, P^3M: particle-particle-particle-mesh; ML: multilevel; MG: multigrid

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[d] private: private code; public: publicly available code (in some cases with limited functionality)

[e] gravity solver is based on PKDGRAV3

[f] based on the GADGET-3 code

Analysis software

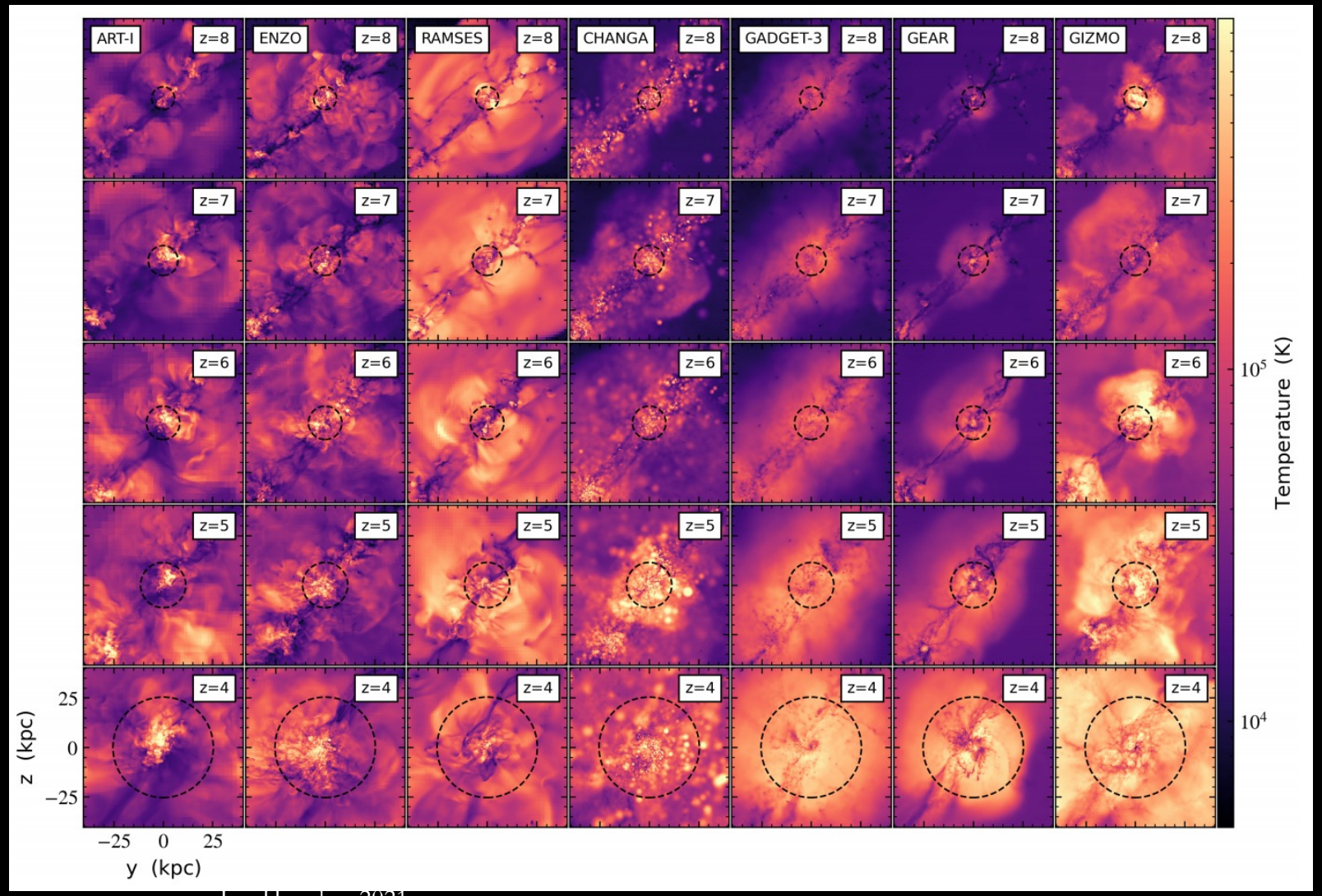
- Halo finders: https://www.cosmosim.org/cms/simulations/halo-finders/
- Check out this paper and the "Notts" papers that cite it for code comparisons: https://www.cosmosim.org/cms/simulations/halo-finders/
- Note: halo finders are usually optimized for finding cuspy CDM halos, and in not too massively dense environments. They can fail for cored halos, simulations with very fine time resolution (losing subhalos at pericenter).
- Visualization & analysis: https://yt-project.org/
- TIPSY visualization: https://github.com/N-BodyShop/tipsy
- Pynbody analysis: https://pynbody.github.io/
- Genetically modified initial conditions: https://github.com/pynbody/genetic/

Code comparisons

https://ui.adsabs.harvard.edu/abs/2021arXiv210609738R/abstract

- Check out this paper and the "Notts" papers that cite it for code comparisons: https://www.cosmosim.org/cms/simulations/halo-finders/
- AGORA project

Need something like this for dark matter physics



Big simulations

Table 2: Recent structure and galaxy formation simulations

simulation	volume	method[a]	mass resolution[b]	spatial resolution[c]	primary reference
	[Mpc^3]		[$M_\odot$]	[kpc]	
dark matter-only					
Millennium	685^3	TreePM	$1.2\times10^9/-$	6.85/–	Springel et al. (2005b)[368]
Millennium-2	137^3	TreePM	$9.4\times10^6/-$	1.37/–	Boylan-Kolchin et al. (2009)[369]
Horizon 4π	2740^3	PM/ML	$7.7\times10^9/-$	10.41/–	Teyssier et al. (2009)[370]
Bolshoi	357^3	PM/ML	$1.9\times10^8/-$	1.43/–	Klypin et al. (2011)[371]
Full Universe Run	29167^3	PM/ML	$1.4\times10^{12}/-$	55.6/–	Alimi et al. (2012)[372]
Millennium-XXL	4110^3	TreePM	$8.5\times10^9/-$	13.7/–	Angulo et al. (2012)[82]
MultiDark	1429^3	PM/ML	$1.2\times10^{10}/-$	10/–	Prada et al. (2012)[373]
Dark Sky	11628^3	Tree/FM	$5.7\times10^{10}/-$	53.49/–	Skillman et al. (2014)[54]
ν^2GC	1647^3	TreePM	$3.2\times10^8/-$	6.28/–	Ishiyama et al. (2015)[374]
Q Continuum	1300^3	TreePM/P^3M	$1.5\times10^8/-$	2.82/–	Heitmann et al. (2015)[84]
OuterRim	4225^3	TreePM/P^3M	$2.6\times10^9/-$	6.0/–	Habib et al. (2016)[46]
EuclidFlagship	20000^3	Tree/FM	$10^9/-$	5/–	Potter et al. (2017)[47]
Aquarius	zoom	TreePM	$1.7\times10^3/-$	0.02/–	Springel et al. (2008)[104]
Via Lactea II	zoom	Tree	$4.1\times10^3/-$	0.04/–	Diemand et al. (2008)[375]
GHALO	zoom	Tree	$1.0\times10^3/-$	0.06/–	Stadel et al. (2009)[376]
CLUES	zoom	TreePM	$3.4\times10^5/-$	0.21/–	Libeskind et al. (2010)[377]
Phoenix	zoom	TreePM	$8.7\times10^5/-$	0.21/–	Gao et al. (2012)[105]
ELVIS	zoom	TreePM	$1.9\times10^5/-$	0.14/–	Garrison-Kimmel et al. (2014)[378]
COCO	zoom	TreePM	$1.6\times10^5/-$	0.33/–	Hellwing et al. (2016)[379]
+ baryons					
Illustris	107^3	TreePM+MMFV	$6.7\times10^6/1.3\times10^6$	1.42/0.71	Vogelsberger et al. (2014)[149]
Horizon-AGN	142^3	PM/ML+AMR	$8.0\times10^7/1.0\times10^7$	1.0/1.0	Dubois et al. (2014)[380]
EAGLE	100^3	TreePM+SPH	$9.7\times10^6/1.8\times10^6$	0.7/0.7	Schaye et al. (2015)[124]
MassiveBlack-2	143^3	TreePM+SPH	$1.6\times10^7/3.2\times10^6$	2.64/2.64	Khandai et al. (2015)[381]
Bluetides[d]	574^3	TreePM+SPH	$1.7\times10^7/3.4\times10^6$	0.24/0.24	Feng et al. (2016)[382]
Magneticum	68^3	TreePM+SPH	$5.3\times10^7/1.1\times10^7$	1.4/0.7-1.4	Bocquet et al. (2016)[85]
MUFASA	74^3	TreePM+MLFM	$9.6\times10^7/1.8\times10^7$	0.74/0.74	Daveé et al. (2016)[383]
BAHAMAS	571^3	TreePM+SPH	$5.5\times10^9/1.1\times10^9$	0.25/0.25	McCarthy et al. (2017)[384]
Romulus25	25^3	Tree/FM+SPH	$3.4\times10^5/2.1\times10^5$	0.25/0.25	Tremmel et al. (2017)[385]
IllustrisTNG[e]	111^3	TreePM+MMFV	$7.5\times10^6/1.4\times10^6$	0.74/0.19	Springel et al. (2018)[87]
Simba[f]	147^3	TreePM+MLFM	$1.4\times10^8/2.7\times10^7$	0.74/0.74	Davé et al. (2019)[182]
Eris	zoom	Tree+SPH	$9.8\times10^4/2\times10^4$	0.12/0.12	Guedes et al. (2011)[349]
VELA	zoom	PM/ML + AMR	$8.3\times10^4/1.9\times10^5$	0.03/0.03[g]	Ceverino et al. (2014)[386]
NIHAO	zoom	Tree+SPH	$3.4\times10^3/6.2\times10^2$	0.12/0.05	Wang et al. (2015)[125]
APOSTLE	zoom	TreePM+SPH	$5.0\times10^4/1.0\times10^4$	0.13/0.13	Sawala et al. (2016)[387]
Latte/FIRE	zoom	TreePM+MLFM	$3.5\times10^4/7.1\times10^3$	0.02/0.001	Wetzel et al. (2016)[352]
Auriga	zoom	TreePM+MMFV	$4.0\times10^4/6.0\times10^3$	0.18/0.18[h]	Grand et al. (2017)[297]
MACSIS	zoom	TreePM+SPH	$6.4\times10^9/1.2\times10^9$	5.77/5.77	Barnes et al. (2017)[388]
Cluster-EAGLE	zoom	TreePM+SPH	$9.7\times10^6/1.8\times10^6$	0.7/0.7	Barnes et al. (2017)[126]
The Three Hundred Project	zoom	TreePM+SPH	$1.9\times10^9/3.5\times10^8$	9.59/9.59	Cui et al. (2018)[389]
FABLE	zoom	TreePM+MMFV	$8.1\times10^7/1.5\times10^7$	4.15/4.15	Henden et al. (2018)[390]
RomulusC	zoom	Tree/FM+SPH	$3.4\times10^5/2.1\times10^5$	0.25/0.25	Tremmel et al. (2019)[391]

[a] PM: particle-mesh; TreePM: tree + PM; FM: fast multipole; P^3M: particle-particle-particle-mesh; ML: multilevel; SPH: smoothed particle hydrodynamics; AMR: adaptive-mesh-refinement; MMFV: moving-mesh finite volume; MLFM: mesh-free finite mass

[b] highest resolution quoted (dark matter/gas)

[c] for particle based codes, the minimum softening length is reported; for mesh codes, the minimum cell size is quoted (dark matter/gas)

[d] final redshift $z = 8$; spatial resolution is in physical units at that redshift

[e] IllustrisTNG consists of three main simulations: TNG50, TNG100, TNG300; numbers are quoted for TNG100

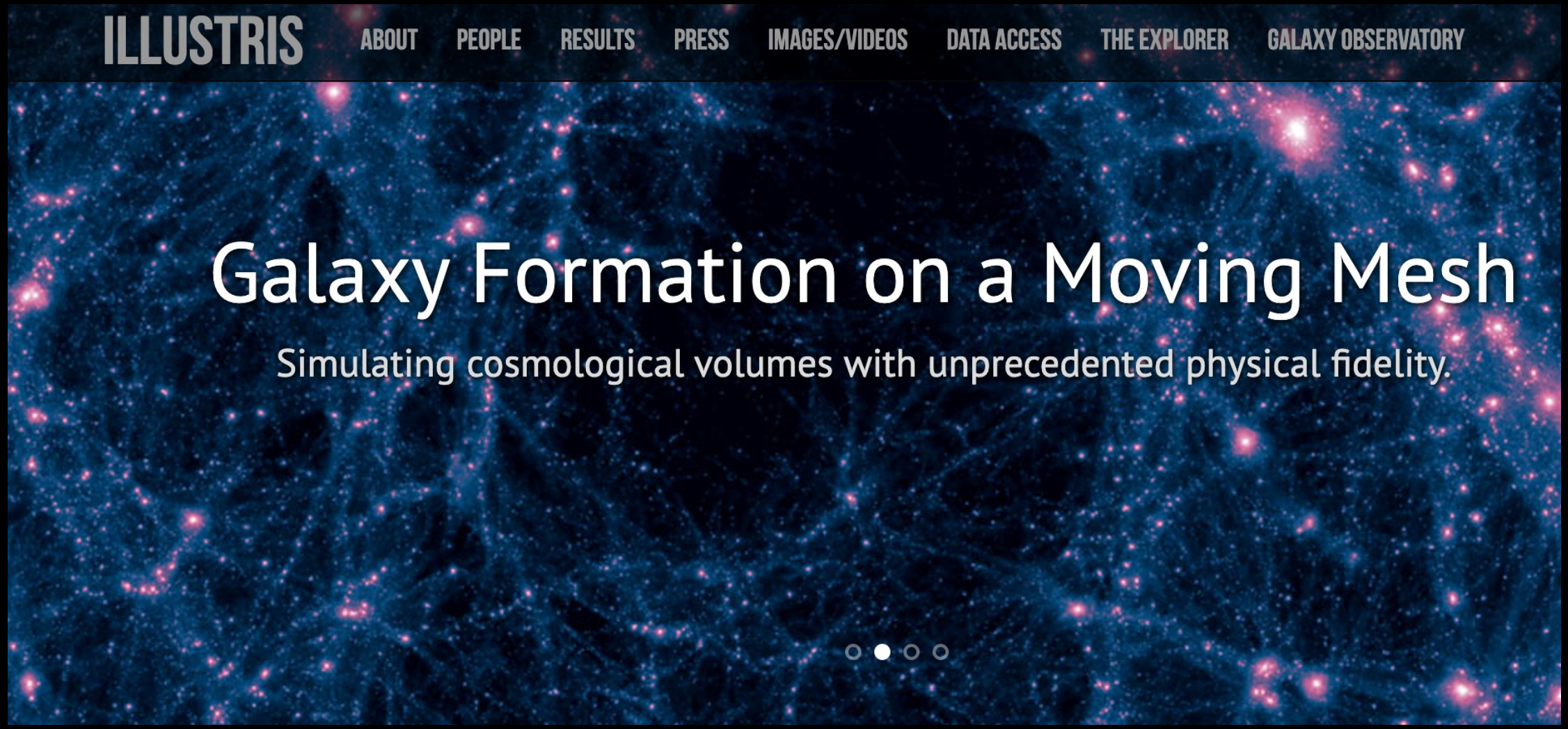
[f] numbers for largest volume simulation quoted

[g] in physical units at $z = 3$

[h] for baryons the minimum physical softening is reported

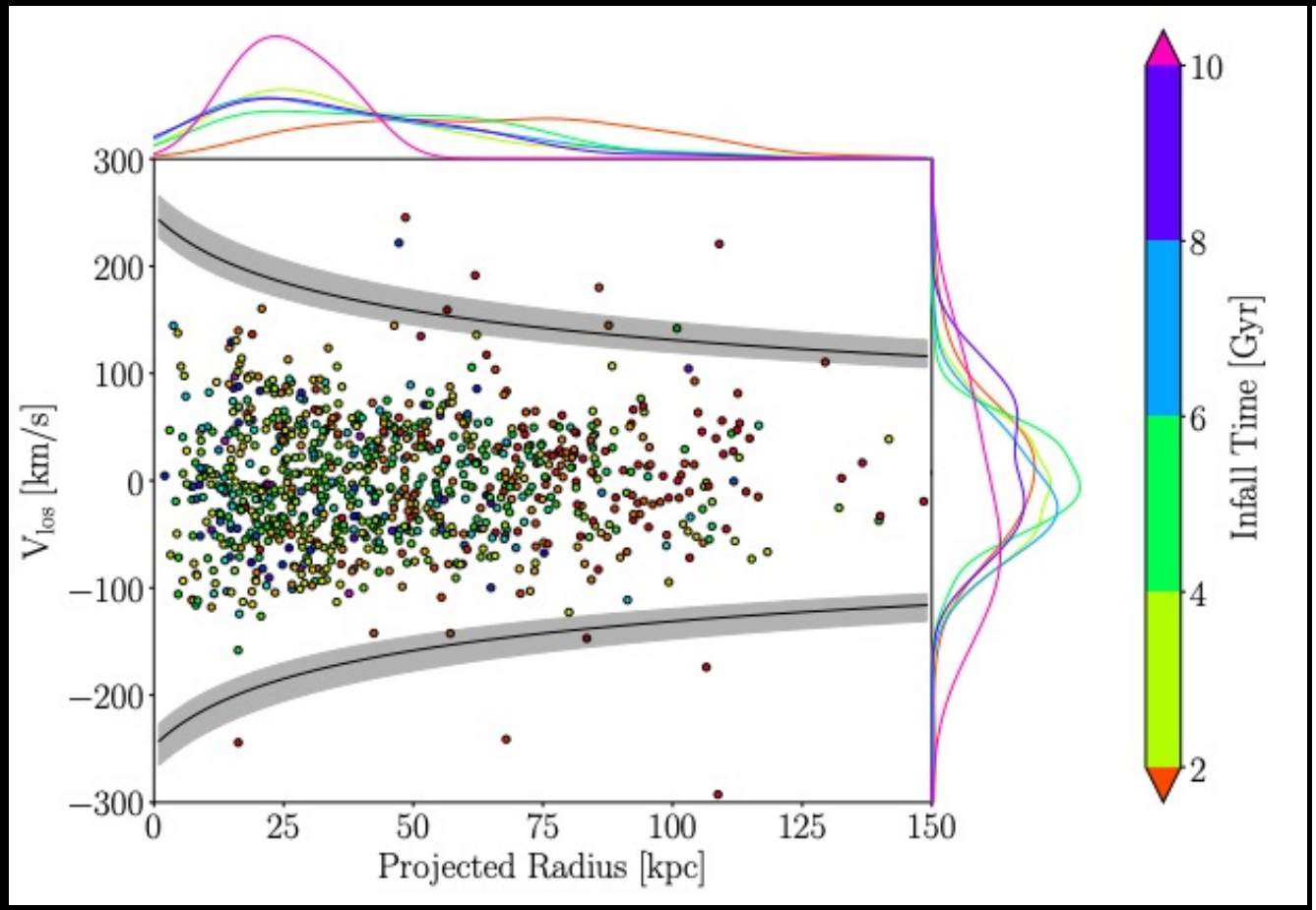
Illustris/IllustrisTNG

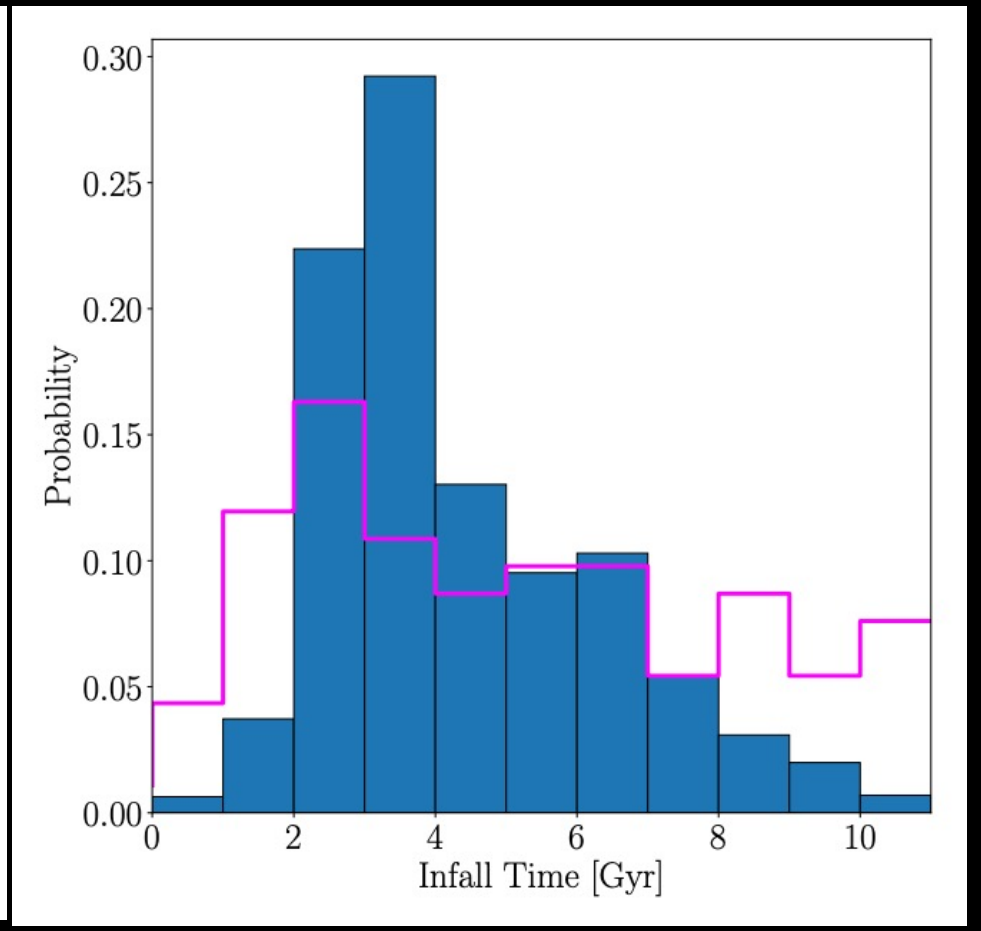
https://www.illustris-project.org/



Example of use

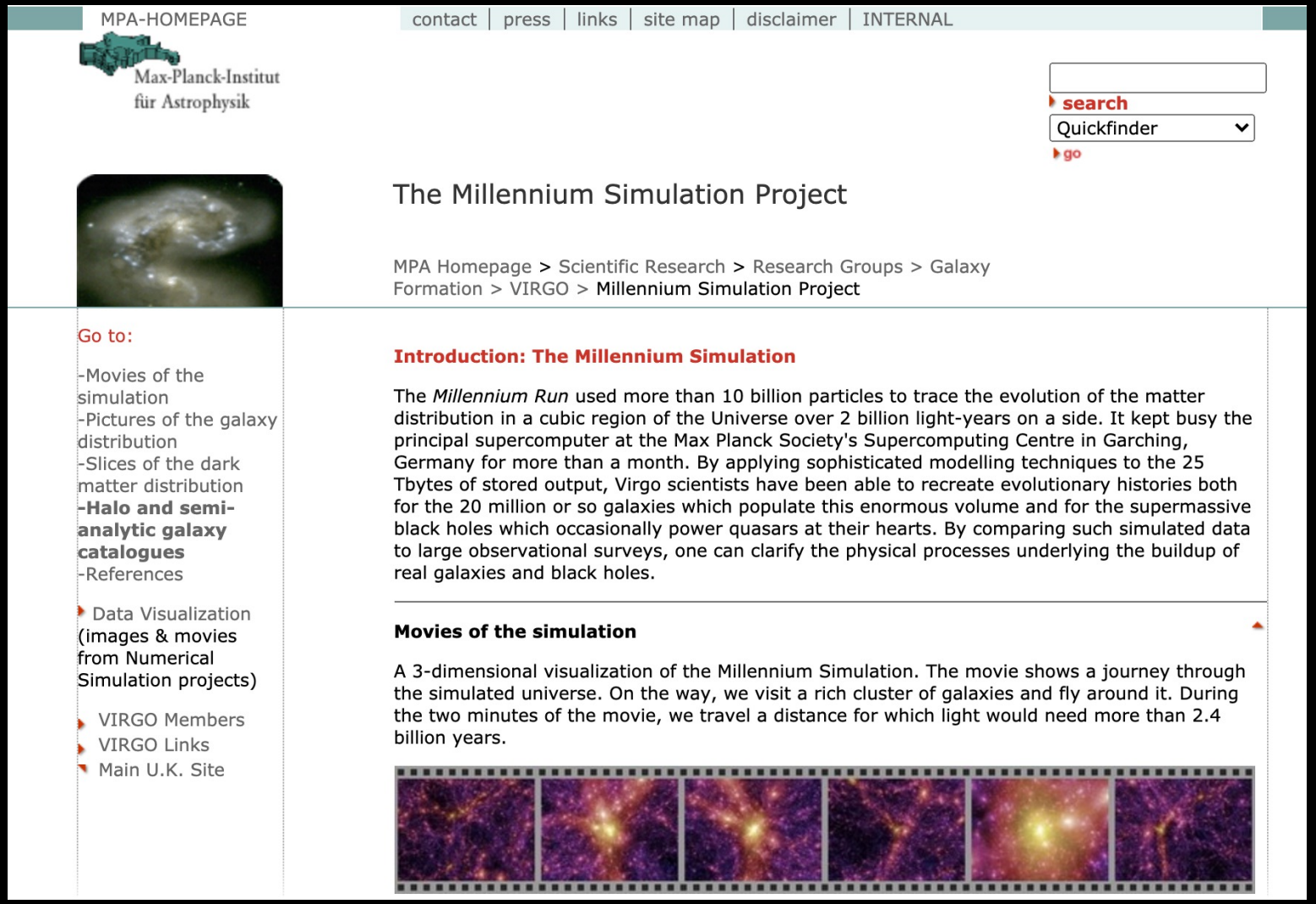
Garling, AP+ 1908.11367
Quenching mechanisms for dwarf satellite DDO 113





Millennium

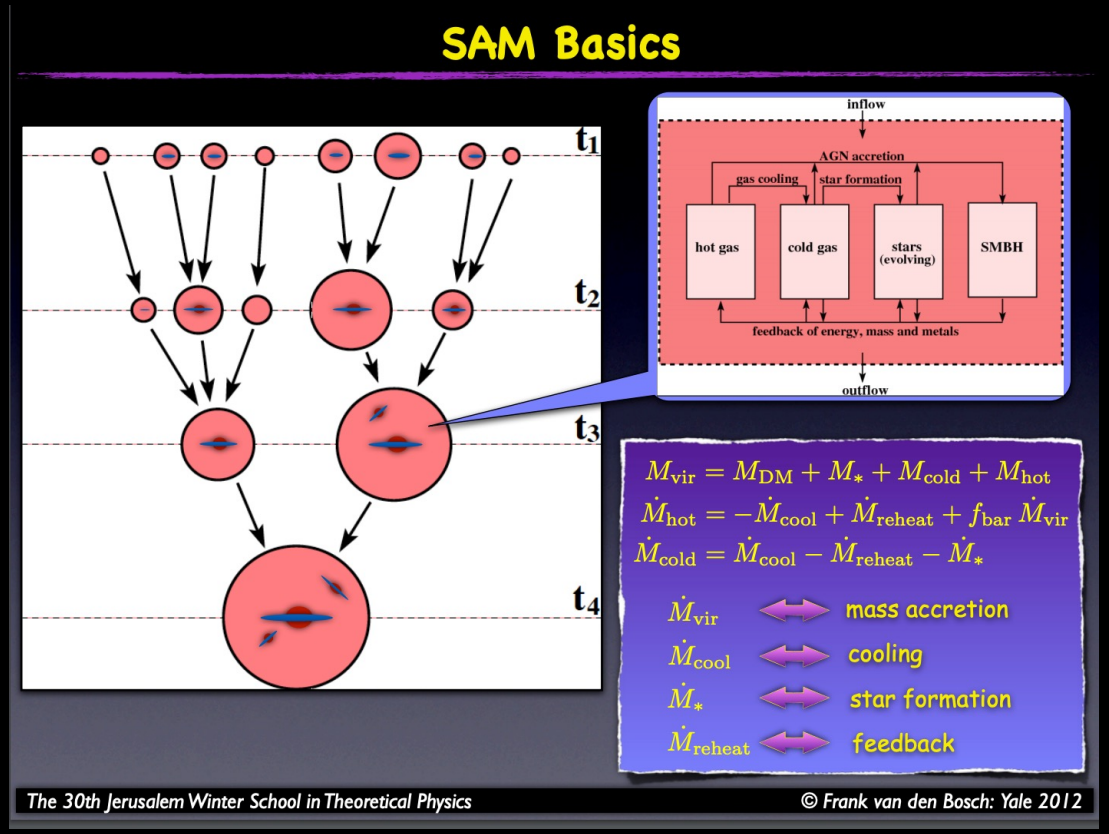
https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/#semi



Has an SQL interface!

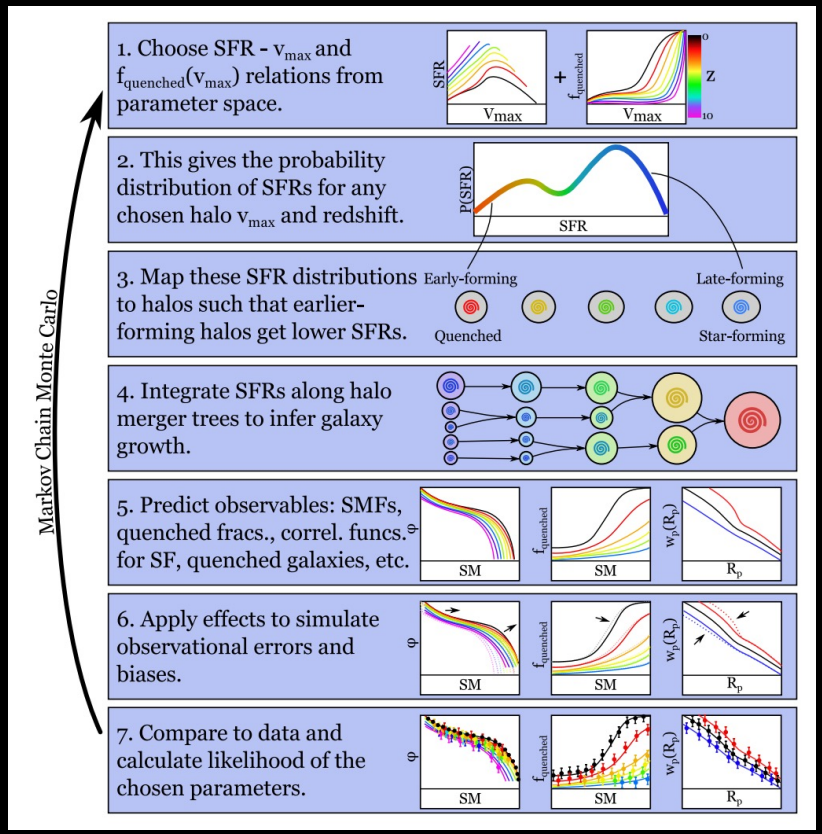
Things that I don't have time for but that you should know about

Semi-analytic modeling



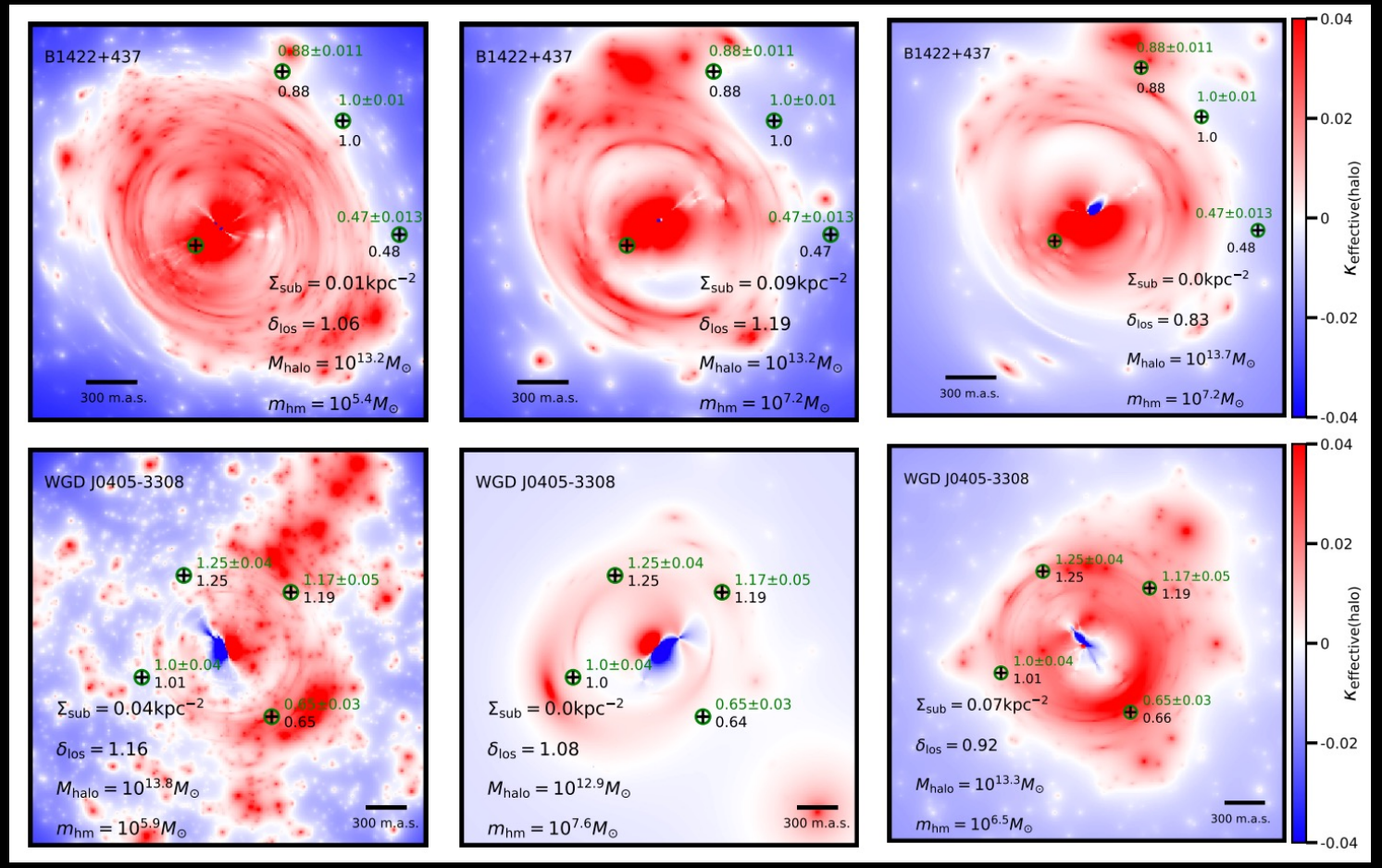
Les Houches 2021

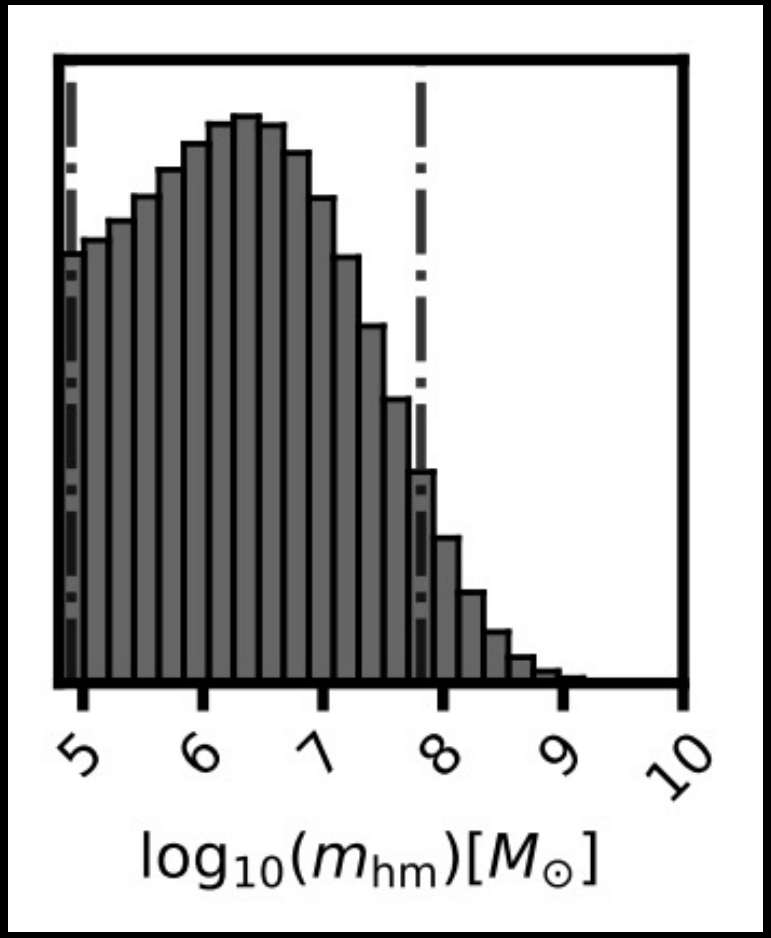
Semi-empirical modeling



Behroozi+ 1806.07893

Dark matter use case





Gilman+ 1908.06983 using galacticus by Benson 1008.1786

Every new beginning comes from some other beginning's end

- Simulations are powerful tools to reveal complex physics on small scales.
- Both the physics and implementation can be quite complex.
- Now, it is up to YOU to start simulating something!

Extra slides

Symplectic integration

A better method: Hamiltonian Mechanics

- If H(q,p), EOM are:

$$\dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q}$$

Symplectic form **z** = (q,p):

$$\dot{z} = J\frac{\partial H}{\partial z}$$

- One can make a canonical transformation with a generating function

$$p\dot{q} - H = P\dot{Q} - K + \frac{dF}{dt}$$

$$\dot{Q} = \frac{\partial K}{\partial P}, \dot{P} = -\frac{\partial K}{\partial Q}$$

$$Z = Z(z), M_{ij} = \frac{\partial Z_i}{\partial z_j}$$

$$\dot{Z} = J\frac{\partial H}{\partial Z}$$

$$\dot{Z} = MJM^T\frac{\partial H}{\partial Z}$$

Time Evolution as a Canonical Transformation

- For time evolution, the right generating function (can show this in Poisson formalism) is:

$$F = qP + dtH(q, P, t)$$

for infinitesimally small dt.

- And

$$Z = z(t+h) \approx z + \delta z$$

- Thus

$$\delta z = dtJ\frac{\partial H}{\partial z}$$

$$dt[z, H]$$

- Since , $\dot{z} = [z, H]$

$$\delta z = dz = \dot{z}dt = [z, H]dt$$

A New Mapping

- For the equation of motion:

$$\dot{z} = [z, H]$$

The formal solution if you step in time

$$t \to t + h$$

$$z(t+h) = e^{[\bullet, H]t} z(t)$$

$$z(t) + [z(t), H]h + H.O.T$$

- The goal is to find symplectic transformations that approximate the real solution. Taking the simplest generating function (qP+hH(q,P,t)), the equations of motion to first order are:

$$Q = q + h\frac{\partial H(q,P,t)}{\partial P}, P = p - h\frac{\partial H}{\partial q}$$

Leapfrog and Beyond

- It is often convenient to split the Hamiltonian so $H=H_A+H_B$, where these things could be T(p) and U(q) (for a separable Hamiltonian), or H_0 and H_1 (in a perturbative problem):

$$z(t+h) = e^{h([\bullet,H_A]+[\bullet,H_B])} z(t)$$

- This splitting means that one can find Hamiltonian systems that closely approximate the original Hamiltonian (use repeated applications of the Hausdorff identity):

$$e^{h([\bullet,H_A]+[\bullet,H_B])} = \prod_{k=1}^{k} e^{c_i h[\bullet,H_A]} e^{d_i h[\bullet,H_B]} + O(h^{n+1})$$

- The most popular scheme used in astrophysics is the "leapfrog" scheme with $H_A=T(p)$, $H_B=U(q)$, $c_1=c_2=1/2$, $d_1=1$.

Leapfrog and Beyond, II

- Therefore:

$$e^{\frac{1}{2}h[\bullet,H_A]}e^{h[\bullet,H_B]}e^{\frac{h}{2}[\bullet,H_A]} = e^{h([\bullet,H]+[\bullet,H_{err}])}$$

- The error Hamiltonian can be found by Hausdorff and is $O(h^2)$.

- The numerical Hamiltonian corresponds to:

$$H_{num} = T(p) + U(q)\sum_{k=1,odd}^{\infty}\delta(\tfrac{2t}{h}-k)$$

- Therefore, the leapfrog integrator follows the Hamiltonian:

$$H_{num} = H + H_{err}$$

- Leapfrog equations of motion (2nd order accurate):

$$q_{1/2} = q_0 + \tfrac{h}{2}p_0$$

$$t_{1/2} = t_0 + \tfrac{h}{2}$$

$$p_1 = p_0 - h\frac{\partial U}{\partial q}\Big|_{q=q_{1/2}}$$

$$q_1 = q_0 + \tfrac{h}{2}p_1$$

$$t_1 = t_{1/2} + \tfrac{h}{2}$$