COMPLEMENTARY APPROACHES

$$\mathcal{L} = -\partial_\mu \psi \, \partial^\mu \psi^* - \underline{m^2 |\psi|}^2 - \frac{g}{2} |\psi|^4 .$$

TAKE NR LIMIT: $\quad \psi(\vec{x},t) = \frac{1}{\sqrt{2m}} \Psi(\vec{x},t) \, e^{-imt}$

$$\partial_0 \psi \, \partial_0 \psi^* = \frac{1}{2m} \Big(\cancel{\dot{\Psi} \dot{\Psi}^*} - i\underline{m} \Psi \dot{\Psi}^* + i m \Psi^* \dot{\Psi} + \underline{\underline{m^2 |\Psi|^2}} \Big)$$

$$\therefore \; \mathcal{L} = \frac{i}{2} \left(\Psi^* \dot{\Psi} - \Psi \dot{\Psi}^* \right) - \frac{|\vec{\nabla} \Psi|^2}{2m} - \frac{g}{8m^2} |\Psi|^4 .$$

VARY WRT Ψ^*:

$$\boxed{i \dot{\Psi} = \left(-\frac{1}{2m} \vec{\nabla}^2 + \frac{g}{4m^2} |\Psi|^2 \right) \Psi}$$

GROSS-PITAEVSKII.

MADELUNG DECOMPOSITION:

$$\underline{\Psi}(\vec{x},t) = \sqrt{n} \, e^{i\theta} = \sqrt{\frac{\rho(\vec{x},t)}{m}} \, e^{i\theta(\vec{x},t)} .$$

DEFINE $\quad \boxed{\vec{v}(\vec{x},t) = \frac{1}{m} \vec{\nabla} \theta} \leftarrow$

SUBSTITUTE:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{v}) = 0 \qquad \text{(CONTINUITY)}$$

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \frac{n}{2m^2}\frac{\vec{\nabla}^2\sqrt{n}}{\sqrt{n}} \qquad \text{(EULER)}$$

FUZZY DM.

WHERE $\boxed{P = \frac{g}{8m^4}\rho^2}$.

- EQU'NS INVISCID
- POTENTIAL FLOW (IRROTATIONAL)

RECALL MOND LAGRANGIAN:

$$\mathcal{L} = -\frac{2}{3}\frac{M_{Pl}^2}{a_0}(\vec{\nabla}\phi)^2\sqrt{(\vec{\nabla}\phi)^2} + \underline{\phi\rho_b}.$$

$$a_0 = \frac{1}{6}cH_0 \simeq 10^{-33}\,\text{eV}.$$

IDENTIFY ϕ WITH PHONONS:

$$\mathcal{L}_\pi = \frac{2}{3}\Lambda(2m)^{3/2} X\sqrt{|X|} + \frac{\Lambda}{M_{Pl}}\pi\rho_b$$

BREAKS U(1) AT $1/M_{Pl}$.

$$X = \mu + \dot{\pi} - \frac{(\vec{\nabla}\pi)^2}{2m}.$$

FIELD REDEF'N: $\phi = \frac{\Lambda}{M_{Pl}}\pi$.

FIND: $\boxed{\Lambda \simeq \sqrt{a_0 M_{Pl}} \simeq \text{meV}.}$

- $m \sim \text{eV}$
- $\Lambda \sim \text{meV}$

DM PHONONS MEDIATE LONG-RANGE FORCE:

$$\boxed{a_\pi = \sqrt{a_0\, a_b}}$$

ONLY OCCURS IN REGIONS OF SUPERFLUIDITY.

NO SUPERFLUID $\Longrightarrow$ NO MOND.

$$\frac{\sigma}{m} \gtrsim \underline{\left(\frac{m}{eV}\right)}^{4} \frac{cm^2}{g} \,.$$

$$\boxed{\text{SIDM:}} \; \frac{\sigma}{m} \lesssim \frac{cm^2}{g}$$