Manchester, January 11 2022

Standard Model prediction of the B_c lifetime

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Outline

based on: 2105.02988, 2108.10285 in collaboration with Benjamín Grinstein

Outline

Status

Experimental value

$\tau_{B_c} = 0.510(9)\text{ps}$

LHCb: 1401.6932, 1411.6899
CMS:1710.08949

Theoretical predictions

Operator Product Expansion (OPE)

Beneke/Buchalla(BB): hep-ph/9601249
Bigi: hep-ph/9510325
Chang/Chen/Feng/Li: hep-ph/0007162

QCD sum rules

Kiselev/Kovalsky/Likhoded: hep-ph/0002127

Potential Models

Gershtein/Kiselev/Likhoded/Tkabladze: hep-ph/9504319

OPE result from BB

$\tau_{B_c} = 0.52\,\text{ps}, \quad 0.4\,\text{ps} < \tau_{B_c} < 0.7\,\text{ps}$

Beneke/Buchalla(BB): hep-ph/9601249

Outline

EFT approach

Effective Hamiltonian

At μ_W, RGE running

OPE

At μ_{low}

Non-Relativistic QCD (NRQCD)

Integrate out (anti-)quark fields

Outline

Results

Massles strange quark

$$\begin{aligned}
\Gamma_{B_c}^{\overline{\text{MS}}} &= \left(1.58 \pm 0.40\right|^{\mu} \pm 0.08\left|^{\text{n.p.}} \pm 0.02\right|^{\overline{m}} \pm 0.01\left|^{V_{cb}}\right) \text{ ps}^{-1} \\
\Gamma_{B_c}^{\text{meson}} &= \left(1.77 \pm 0.25\right|^{\mu} \pm 0.20\left|^{\text{n.p.}} \pm 0.01\right|^{V_{cb}}\Big) \text{ ps}^{-1} \\
\Gamma_{B_c}^{\text{Upsilon}} &= \left(2.51 \pm 0.19\right|^{\mu} \pm 0.21\left|^{\text{n.p.}} \pm 0.01\right|^{V_{cb}}\Big) \text{ ps}^{-1}
\end{aligned}$$

Massive strange quark

$$\begin{aligned}
\Gamma_{B_c}^{\overline{\text{MS}}} &= \left(1.51 \pm 0.38\right|^{\mu} \pm 0.08\left|^{\text{n.p.}} \pm 0.02\right|^{\overline{m}} \pm 0.01\left|^{m_s} \pm 0.01\right|^{V_{cb}}\Big) \text{ ps}^{-1} \\
\Gamma_{B_c}^{\text{meson}} &= \left(1.70 \pm 0.24\right|^{\mu} \pm 0.20\left|^{\text{n.p.}} \pm 0.01\right|^{m_s} \pm 0.01\left|^{V_{cb}}\right) \text{ ps}^{-1} \\
\Gamma_{B_c}^{\text{Upsilon}} &= \left(2.40 \pm 0.19\right|^{\mu} \pm 0.21\left|^{\text{n.p.}} \pm 0.01\right|^{m_s} \pm 0.01\left|^{V_{cb}}\right) \text{ ps}^{-1}
\end{aligned}$$

$(\Gamma_{B_c}^{exp} = 1.961 \pm 35 \text{ ps}^{-1})$

Outline

Summary

Three different mass schemes

$\overline{\text{MS}}$, meson, Upsilon

Results

Agreement with experiment: large scheme dependence

Improvements

NNLO, n.p. corrections, lattice results

Backup

Motivation

B-anomalies in charged currents

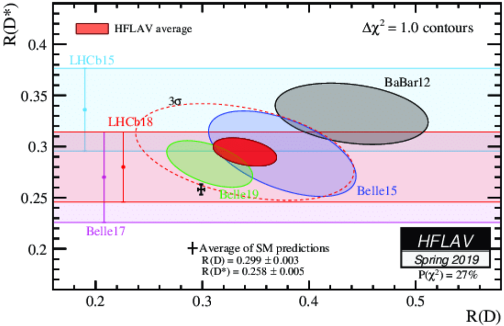
Measurement

R_D and R_{D^*}

BaBar: 1205.5442, 1303.0571, LHCb: 1506.08614, 1708.08856
Belle: 1507.03233, 1607.07923, 1612.00529

$$R_{D^{(*)}} = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\ell\nu)}$$

$$\ell \in \{e, \mu\}$$



HFLAV: 1909.12524

Constrain NP from τ_{B_c}

$B_c \to \tau\nu_\tau$

Not exceed τ_{B_c}

$Br(B_c \to \tau\nu_\tau)$

Pseudoscalar scenarios constrained

Alonso/Grinstein/Camalich: 1611.06676

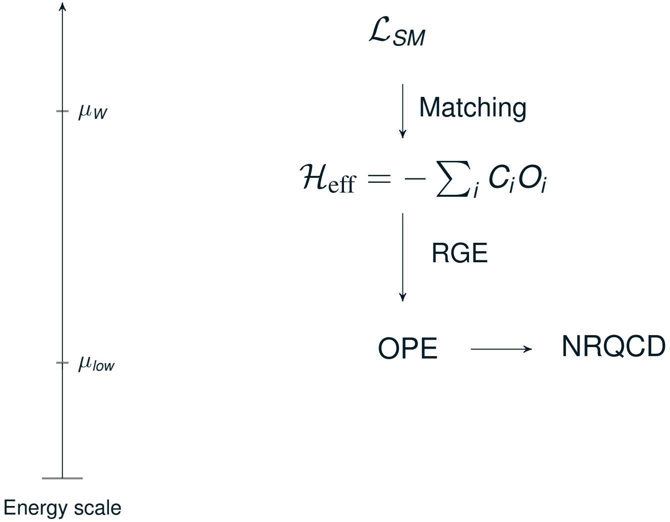
Polarization observables

$F_L(D^*)$, τ-polarization

Blanke/Crivellin/de Boer/Kitahara/Moscati/Nierste/Nisandzic: 1811.09603
Blanke/Crivellin//Kitahara/Moscati/Nierste/Nisandzic: 1905.08253

Procedure

EFT approach



Overview of BB

Beneke/Buchalla(BB): hep-ph/9601249

OS scheme

m_b^{OS}, m_c^{OS}

Error estimate

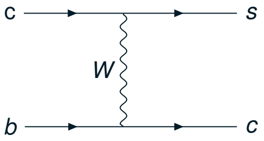
Vary $1.4\,\mathrm{GeV} < m_c < 1.6\,\mathrm{GeV}$

fix m_b by B_d lifetime

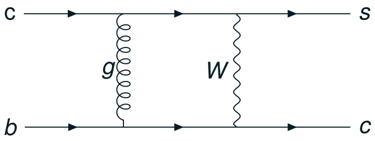
Penguin contributions

Neglected

Current-current operators



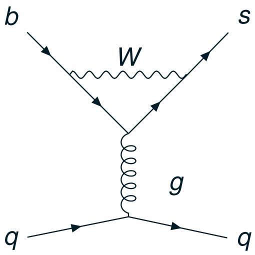
$$\longrightarrow \qquad Q_2 = (\bar{c}b)_{V-A}\,(\bar{s}c)_{V-A}$$



$$\longrightarrow \qquad Q_1 = (\bar{c}_\beta b_\alpha)_{V-A}\,(\bar{s}_\alpha c_\beta)_{V-A}$$

$$V \pm A = \gamma^\mu(1 \pm \gamma_5)$$

QCD-penguins



$$\longrightarrow$$

$$Q_3 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 = (\bar{s}b)_{V-A} \sum_q (\bar{q}q)_{V+A}$$

$$Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

Optical Theorem

Forward scattering

$\Gamma_{B_c} = \frac{1}{2M_{B_c}} \langle B_c | \mathcal{T} | B_c \rangle$

Transition Operator

$\mathcal{T} = \mathrm{Im}\, i \int d^4x\, T\, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0)$

OPE

$\mathcal{T} =$ series of local operators

OPE

Transition operator

$$\mathcal{T}_Q = C_Q^{(3)}\bar{Q}Q + C_Q^{(5)}\frac{1}{m_Q^2}g_s\bar{Q}\sigma_{\mu\nu}Q\,G^{\mu\nu} + \sum_i C_{Q,i}^{(6)}\frac{1}{m_Q^3}O_i^{(6)} + \mathcal{O}\left(\frac{1}{m_Q^4}\right)$$

Wilson coefficients

Spectator decays, WA, PI

Contributions

$$\mathcal{T}_{B_c} = \mathcal{T}_{\bar{b}} + \mathcal{T}_c + \mathcal{T}_{\text{WA}} + \mathcal{T}_{\text{PI}}$$

Contributions

$\bar{b}$-decays

$\bar{b} \to \bar{c}u(\bar{s}+\bar{d}), \bar{c}c(\bar{s}+\bar{d}), \bar{c}\ell\nu$

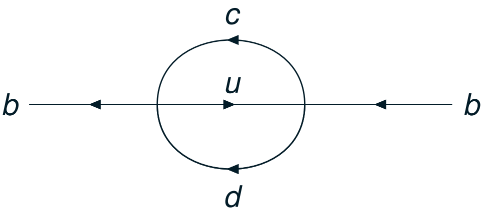
c-decays

$c \to (s+d)u(\bar{s}+\bar{d}), (s+d)\ell\nu$

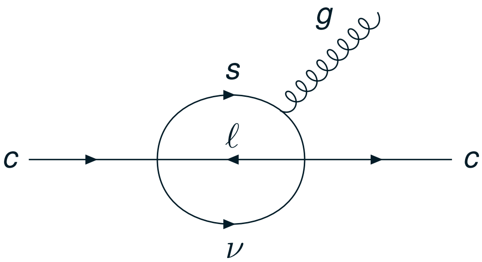
Weak Annihilation (WA), Pauli Interference (PI)

1-loop graphs

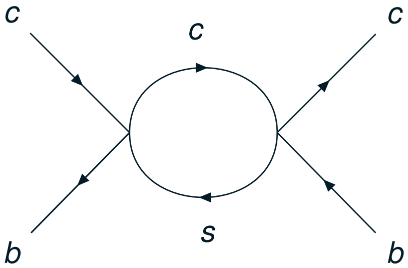
$\bar{b}$-decay



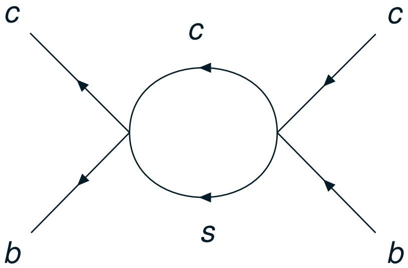
c-decay



WA



PI



NRQCD

Heavy quark Q

$$Q(x) = e^{-imu\cdot x}(\Psi_+(x) + \Psi_-(x))\,, \qquad \text{where} \qquad \Psi_\pm = \left(\frac{1\pm\not{u}}{2}\right)\Psi_\pm\,,$$

$u =$ Meson velocity

Equation of motion

$$(i\not{D} - m)Q = 0 \quad \Rightarrow \quad \Psi_- = \frac{1}{2m+iu\cdot D} i\not{D}_\perp \Psi_+\,, \qquad D^\mu_\perp = D^\mu - (u\cdot D)u^\mu$$

Lagrangian

$$\mathcal{L} = \overline{\Psi}_+ \left(iu\cdot D + i\not{D}_\perp \frac{1}{2m+iu\cdot D} i\not{D}_\perp\right)\Psi_+$$

Velocity expansion

Small quark velocity

$v \ll 1$

Velocity counting

$[\Psi] = v^{3/2}$, $[D] = v$, etc.

Expanded Lagrangian

$\mathcal{L}_0 = \overline{\Psi}_+ \left(iu \cdot D - \frac{1}{2m} D_\perp^2 \right) \Psi_+$

Two-component notation

Field

$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$

Dirac basis

$\gamma^0 = \sigma^3 \otimes \mathbb{1}, \vec{\gamma} = i\sigma^2 \otimes \vec{\sigma}$

Expanded Lagrangian

$\mathcal{L}_0^{\text{2-com}} = \psi^\dagger \left(iD_t - \frac{1}{2m}(i\vec{D})^2 \right)\psi$

Expansion of OPE operators

dim 3

$$\bar{Q}Q \to \psi_q^\dagger \left(1 + \frac{1}{2m^2}(\vec{D})^2 + \frac{g_s}{2m^2}\vec{\sigma}\cdot\vec{B} + \frac{g_s}{4m^3}[\vec{D}\cdot\vec{E}] + \frac{25}{64m^4}(\vec{D})^4 + \cdots\right)\psi_q$$

dim 5

$$\bar{Q}\sigma_{\mu\nu}g_s G^{\mu\nu}Q \to \psi_q^\dagger\left(-2g_s\vec{\sigma}\cdot\vec{B} - \frac{g_s}{m}[\vec{D}\cdot\vec{E}] + \cdots\right)\psi_q$$

dim 6

$$(\bar{b}\gamma_\mu P_L b)(\bar{c}\gamma^\mu P_L c) = (\overline{X}_-^{(b)}\gamma_\mu P_L X_-^{(b)})(\overline{\Psi}_+^{(c)}\gamma^\mu P_L \Psi_+^{(c)}) + \cdots$$

Matrix Elements

Gershtein/Kiselev/Likhoded/Tkabladze: hep-ph/9406339

Kinetic Energy

$$\frac{\langle B_c|\psi^\dagger(i\vec{D})^2\psi|B_c\rangle}{2M_{B_c}} = \frac{2m_c m_b}{(m_c+m_b)}T\,, \qquad T = \text{Kinetic energy}$$

Fermi and Darwin

$$\frac{\langle B_c|\psi_{\bar{b}}^\dagger g_s\vec{\sigma}\cdot\vec{B}\psi_{\bar{b}}|B_c\rangle}{2M_{B_c}} = -\frac{4}{3}g_s^2\frac{|\Psi^{\mathrm{WF}}(0)|^2}{m_c}\,, \qquad M_{B_c^*} - M_{B_c} = \frac{8}{9}g_s^2\frac{|\Psi^{\mathrm{WF}}(0)|^2}{m_b m_c}$$

$$\frac{\langle B_c|\psi_{\bar{b}}^\dagger g_s[\vec{D}\cdot\vec{E}]\psi_{\bar{b}}|B_c\rangle}{2M_{B_c}} = \frac{4}{3}g_s^2|\Psi^{\mathrm{WF}}(0)|^2\,, \qquad f_{B_c}^2 = \frac{12|\Psi^{\mathrm{WF}}(0)|^2}{M_{B_c}}$$

D^4

$$\frac{\langle B_c|\psi^\dagger(i\vec{D})^4\psi|B_c\rangle}{2M_{B_c}} = \frac{4m_c^2 m_b^2}{(m_c+m_b)^2}T^2$$

Setup

Improvements over BB

Mass schemes

$\overline{\text{MS}}$, meson, Upsilon

Spin symmetry

Relates matrix elements

Typos in literature

Bagan/Ball/Fiol/Gosdzinsky: hep-ph/9502338

Corrected by KLR

Krinner/Lenz/Rauh: 1305.5390

Penguin contributions

Included

Better input values

α_s, f_{B_c}, CKM parameters

Mass schemes

$\overline{\text{MS}}$ scheme

m_b^{OS} and m_c^{OS} in terms of $\overline{m}_b(\mu_b)$ and $\overline{m}_c(\mu_c)$

Meson scheme

m_b^{OS} in terms of m_Υ
m_c^{OS} in terms of m_b^{OS} and $\overline{m}_B - \overline{m}_D$

Upsilon scheme

Like meson scheme
For c decays: m_c^{OS} in terms of Upsilon expansion of $m_{J/\psi}$

Upsilon scheme

Hoang/Ligeti/Monahar: hep-ph/9809423

$\bar{b}$ decays, WA, PI

$$\frac{1}{2} m_\Upsilon = m_b^{OS} \left[1 - \frac{(\alpha_s C_F)^2}{8} \left\{ 1 + \frac{\alpha_s}{\pi} \left[\left(\ln \left(\frac{\mu}{\alpha_s C_F m_b^{OS}} \right) + \frac{11}{6} \right) \beta_0 - 4 \right] + \cdots \right\} \right]$$

$$m_b^{OS} - m_c^{OS} = \overline{m}_B - \overline{m}_D + \frac{1}{2} \lambda_1 \left(\frac{1}{m_b^{OS}} - \frac{1}{m_c^{OS}} \right)$$

$\overline{m}_B, \overline{m}_D =$ (iso)spin-averaged masses

c decays

$$\frac{1}{2} m_{J/\Psi} = m_c^{OS} \left[1 - \frac{(\alpha_s C_F)^2}{8} \left\{ 1 + \frac{\alpha_s}{\pi} \left[\left(\ln \left(\frac{\mu}{\alpha_s C_F m_c^{OS}} \right) + \frac{11}{6} \right) \beta_0 - 4 \right] + \cdots \right\} \right]$$

strange mass

$m_s = 0$ or $\overline{\text{MS}}$

Convergence of expansion

Example: For $\Gamma\big(B \to X_c e\nu\big)$ one finds

$$1 - 0.20\epsilon - 0.20\epsilon^2 + \cdots \qquad \text{(pole mass)}$$

$$1 + 0.27\epsilon + 0.09\epsilon^2 + \cdots \qquad (\overline{\text{MS}} \text{ scheme})$$

$$1 - 0.10\epsilon - 0.03\epsilon^2 + \cdots \qquad \text{(meson/Upsilon scheme)}$$

$\epsilon =$ expansion parameter

Spin symmetry

Noether current

$$J^\mu = u^\mu \overline{\Psi}_+ \Psi_+ + \frac{1}{2m} \overline{\Psi}_+ i D_\perp^\mu \Psi_+$$

Relation

$$\frac{1}{2M_{B_c}} \langle B_c(p) | \overline{\Psi}_+ \Gamma \Psi_+ | B_c(p) \rangle = \frac{1}{2} \mathrm{Tr} \left[\Gamma \left(\frac{1+\not{v}}{2} \right) \right]$$

4quark MEs

$$\langle B_c(q) | (\overline{X}_-^{(b)\alpha} \gamma_\mu P_L X_-^{(b)\beta})(\overline{\Psi}_+^{(c)\beta} \gamma_\nu P_L \Psi_+^{(c)\alpha}) | B_c(q) \rangle = \frac{f_{B_c}^2 B_{B_c}}{4} \left(\frac{1}{2} q^2 g_{\mu\nu} - q_\mu q_\nu \right)$$

Uncertainties

Uncertainties

Non-Perturbative (n.p.)

velocity expansion

scale uncertainty

μ dependence

Parametric

V_{cb} etc

Strange quark mass

$m_s \neq 0$

Error estimate

velocity expansion

v^4 terms

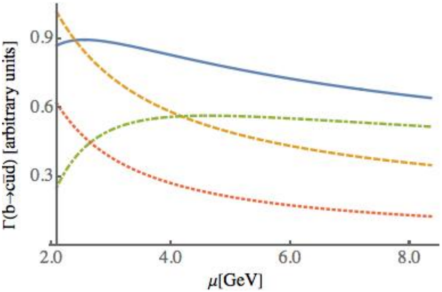
parametric, μ depdendence

vary parameter p, compute $\frac{\Delta\Gamma}{\Gamma}\frac{p}{\Delta p}$

Strange quark mass

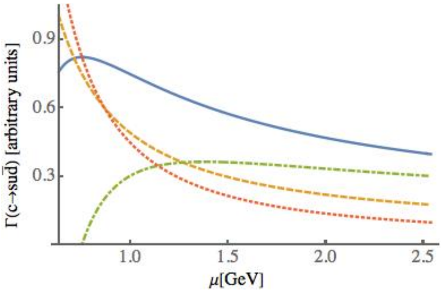
keep m_s in charm decays

$\mu-$dependence: $b \to cud$



blue: LO+NLO
orange: LO
green: LO + $\alpha_s \ln(\mu)$
red: $\alpha_s \ln(\mu)$

$\mu-$dependence: *c* $\longrightarrow$ *sud*



blue: LO+NLO
orange: LO
green: LO + $\alpha_s$ln(μ)
red: $\alpha_s$ln(μ)

Error estimate: $\frac{\Delta\Gamma}{\Gamma}\frac{p}{\Delta p}$

parameter p	$\Delta p/p$	$\overline{\text{MS}}$	meson	Upsilon
$\overline{m}_b(\overline{m}_b)$	0.2%	1.815	–	–
$\overline{m}_c(\overline{m}_c)$	0.3%	2.798	–	–
μ	10%	-0.359	-0.204	-0.112
T	10%	-0.029	-0.034	-0.057
$M_{B_c^*} - M_{B_c}$	6%	0.012	0.015	0.016
B_{B_c}	30%	-0.004	0.021	0.042
B'_{B_c}	30%	0.065	0.060	0.030
λ_1	50%	–	-0.011	0.017
f_{B_c}	1%	0.122	0.164	0.147
V_{cb}	1%	0.644	0.769	0.575

Results

Partial rates

Mode	BB	$\overline{\text{MS}}$	meson	Upsilon
$\bar{b} \to \bar{c}u(\bar{s}+\bar{d})$	0.310	0.205	0.266	
$\bar{b} \to \bar{c}c(\bar{s}+\bar{d})$	0.137	0.093	0.122	
$\bar{b} \to \bar{c}e\nu$	0.075	0.053	0.066	
$\bar{b} \to \bar{c}\tau\nu$	0.018	0.010	0.015	
$\sum \bar{b} \to \bar{c}$	0.615	0.414	0.535	
$c \to (s+d)u(\bar{d}+\bar{s})$	0.905	0.752	0.770	1.290
$c \to (s+d)e\nu$	0.162	0.161	0.162	0.250
$\sum c \to s$	1.229	1.075	1.095	1.790
WA: $\bar{b}c \to c(\bar{s}+\bar{d})$	0.138	0.079	0.126	0.157
WA: $\bar{b}c \to \tau\nu$	0.056	0.039	0.042	0.042
PI	-0.124	-0.023	-0.024	-0.017
Γ_{B_c} ($\Gamma_{B_c}^{exp} = 1.961(35)\text{ps}^{-1}$)	1.914	1.584	1.774	2.506

Branching ratios

Br(process)	$\overline{\text{MS}}$	meson	Upsilon
$b \to cu(d+s)$	13.6	15.7	11.1
$b \to cc(d+s)$	6.2	7.2	5.1
$b \to ce\nu$	3.5	3.9	2.7
$b \to c\tau\nu$	0.6	0.9	0.6
$b \to c$	27.3	31.4	22.2
$c \to su\bar{d}$	41.8	38.0	45.6
$c \to su\bar{s}$	2.1	1.9	2.4
$c \to du\bar{d}$	2.4	2.2	2.6
$c \to se\bar{\nu}$	9.4	8.4	9.2
$c \to de\bar{\nu}$	0.5	0.5	0.5
$c \to s$	66.4	60.1	70.2
$bc \to cs$	3.7	6.0	5.8
$bc \to \tau\nu$	2.6	2.5	1.8

Possible Improvements

Higher order in α_s

To reduce μ-dependence

Higher order in v

To reduce n.p. uncertainty

Matrix elements

Lattice calculation

Novel way

Novel way to determine Γ_{B_c}: Idea

JA/Grinstein: 2108.10285

Decay rates

Compute parts of $\Gamma(B)$, $\Gamma(D)$, $\Gamma(B_c)$
use $\Gamma^{exp}(B)$, $\Gamma^{exp}(D)$

Taking difference between B, D, B_c

$\Gamma(B) + \Gamma(D) - \Gamma(B_c)$

Various combinations

$(\Gamma_{B^0}, \Gamma_{D^0})$, $(\Gamma_{B^+}, \Gamma_{D^0})$, $(\Gamma_{B^0}, \Gamma_{D^+})$ and $(\Gamma_{B^+}, \Gamma_{D^+})$

Novel way to determine Γ_{B_c}

General width for meson H_Q

$$\Gamma(H_Q) = \Gamma_Q^{(0)} + \Gamma^{n.p.}(H_Q) + \Gamma^{\mathrm{WA+PI}}(H_Q) + \mathcal{O}(\frac{1}{m_Q^4})$$

Taking difference between B, D, B_c

$$\Gamma(B) + \Gamma(D) - \Gamma(B_c) = \Gamma^{n.p.}(B) + \Gamma^{n.p.}(D) - \Gamma^{n.p.}(B_c)$$
$$+ \Gamma^{\mathrm{WA+PI}}(B) + \Gamma^{\mathrm{WA+PI}}(D) - \Gamma^{\mathrm{WA+PI}}(B_c)$$

Advantage

quark decay uncertainties drop out

Results

(B^0, D^0) and (B^+, D^0)

$\Gamma_{B_c} = 3.03 \pm 0.51 \text{ ps}^{-1}$

(B^0, D^+) and (B^+, D^+)

$\Gamma_{B_c} = 3.33 \pm 1.29 \text{ ps}^{-1}$

Discrepancy with experiment

$\Gamma_{B_c}^{exp} = 1.961 \pm 35 \text{ ps}^{-1}$

Possible explanations

Underestimated uncertainties

NNLO, $1/m^4$, parametric etc.

neglected Dim7 pieces

Can have large impact King/Lenz/Piscopo/Rauh/Rusov/Vlahos: 2109.13219

Eye graph

Not included in lattice calculation, HQET sum rules King/Lenz/Rauh: to appear

Quark hadron duality

violated