Weighing the top with Energy Correlators

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The top quark mass and fate of the universe

Right now: An exciting era of precision collider physics!

The top and the higgs masses determine the nature of the stability of the electroweak vacuum

[Buttazzo, et al., 2013][Andreassen, et al. 2014] [See Andrea Knue’s talk]

→ Direct top mass the most precise:
  \[ m^\text{MC}_t = 173.34 \pm 0.76 \text{ GeV} \] [World combination 1403.4427],
  but . . .
  • \textit{MC top mass parameter:} No field theoretic mass scheme,
    \( O(1) \) GeV theory uncertainty argued
    [Hoang 2004.12915]
  • Sensitive to the parton shower, hadronization model
    [Hoang et al. 1807.06617]

→ Indirect cross section measurements not precise enough
  \[ m^\text{pole}_t = 172.9^{+2.5}_{-2.6} \text{ GeV} \] [ATLAS, 1406.5375]
  \[ m^\text{pole}_t = 172.7^{+2.4}_{-2.7} \text{ GeV} \] [CMS, 1701.06228]
Knowing kinematics of the top decay improves precision on $m_t$
Consider the jet mass:

\[ M_J^2 = \left( \sum_{i \in J} p_i^\mu \right)^2 \simeq m_t^2 + \Gamma t m_t + \ldots \]

[Fleming et al. hep-ph/0703207, 0711.2079]
[Bachu, Hoang, AP, Mateu, Stewart 2012.12304]
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Challenging to calculate:

→ Strongly correlated with the outside radiation
→ Precision spoiled by uncorrelated contamination (UE)
Kinematic top mass measurements

Consider **Soft drop jet mass**

\[ M_{J,\text{sd}}^2 = \left( \sum_{i \in J^{\text{groomed}}} p_i^\mu \right)^2 \approx m_t^2 + \Gamma_t m_t + \ldots \]

→ Removes wide-angle soft radiation
→ Improves robustness significantly for the LHC

[Hoang, Mantry, AP, Stewart 1708.02586] [Hoang, Mantry, Michel, AP, Stewart]

[ATL-PHYS-PUB-2021-034]
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Still quite a challenging task to quantify the remaining corrections in ‘…’!

\[ M_J^2 \simeq m_t^2 + \Gamma_t m_t + \ldots \]
Lessons from other sciences

Correlation functions are extremely powerful!

[Arkani-Hamed, Maldacena 1503.08043]
[Arkani-Hamed, Baumann, Lee, Pimentel;1811.00024]

[Schweigler et al. 1505.03126]

Correlators for collider physics

For a precise theoretical formulation:

→ Consider correlation functions of energy flow operators: \( \langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots\mathcal{E}(\vec{n}_N) \rangle \) [Hofman, Maldacena; 0803.1467]

→ Cannot work with local operators: need light ray operators

[Hofman, Maldacena; 0803.1467][Sveshnikov, Tkachov hep-ph/9512370][Kravchuk, Simmons-Duffin 1805.00098]

→ Borrow a wealth of insights from CFT and string theory

[Hofman, Maldacena; 0803.1467][Belitsky et al. 1311.6800][Korchemsky, Sokatchev 1504.07904][Kologlu et al. 1904.05905, 1905.01311]
Why have *we* not already been using correlators?

Cosmic variance is not an excuse for collider physics!

Polchinski: There is a lot of QCD data... can they see this scaling?

Maldacena: People do not do this, I haven’t figured out why they don’t. I think they just haven’t thought about this.
Why have *we* not already been using correlators?

Since the beginning of the LHC we have exclusively focused on jet-based observables \((m_J, \tau_a, e_n^{(\alpha)}, \ldots)\)
Why have \textit{we} not already been using correlators?

Energy-correlators only require angles to be measured precisely!

→ Can be computed on tracks!  

[Chang, Procura, Thaler, Waalewijn, 1303.6637][Moult, van Velzen, Waalewijn, Zhu 2108.01674]
Progress in recent years

→ Resummation in the small angle limit in SCET
→ EEC: mapping transition from perturbative to free hadron phase
→ EEEC: Interference in the squeezed limit

[Dixon, Moult, Zhu, 1904.01310]

[Komiske, Moult, Thaler, Zhu]

[Chen, Luo, Moult, Yang, Zhang, Zhu 1912.11050]

[Chen, Moult, Zhu 2104.00009]

[Karlberg, Salam, Scyboz, Verheyen 2103.16526]
Probing the top with the three-point correlator

\[ \langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3) \rangle_t \equiv \frac{\langle \psi_t | \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3) | \psi_t \rangle}{\langle \psi_t | \psi_t \rangle} \]
Three-point correlator:

\[ G^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \int d\sigma \widehat{M}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \]

\[ \widehat{M}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k} \frac{E^i_n E^j_n E^k_n}{Q^{3n}} \delta (\zeta_{12} - \hat{\zeta}_{ij}) \delta (\zeta_{23} - \hat{\zeta}_{ik}) \delta (\zeta_{31} - \hat{\zeta}_{jk}) \]

In the CFT limit EEEC exhibits a featureless power law

[Dixon, Moult, Zhu, 1904.01310][Korchemsky 1905.01444][Kologlu et al. 1905.01311]

\[ G^{(1)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \xrightarrow{\text{CFT}} \zeta_{31}^{-1 + \gamma(4)} G(z, \bar{z}), \]

\[ z\bar{z} = \zeta_{12}/\zeta_{31}, \quad (1 - z)(1 - \bar{z}) = \zeta_{23}/\zeta_{31} \]
The top quark decay imprints $m_t$ as a characteristic scale in the 3-point correlator:

$$\zeta_{ij} \sim \frac{m_t^2}{p_{T,t}^2}$$
Study the simplest configuration of an equilateral triangle $\hat{\zeta}_{ij} = \zeta$:

→ Hadronization effects: small effect on the peak $\Delta m_t^{\text{Had.}} \sim 230$ MeV.

→ Peaked at $\zeta_{\text{peak}} \approx 3m_t^2/Q^2$: peak dominated by hard decay of the top.

→ Resilient to collinear radiation $\alpha_s \ln \zeta_{\text{peak}} < 1$: fixed order perturbation theory sufficient.
At the LHC we can measure EEEC on jets containing top decay products:

\[ p_{T,t} \]

Unlike EEEC, \( p_{T,jet} \) is susceptible to hadronization and underlying event:

→ Consider multiple \( p_{T,jet} \) bins

Assuming that \( p_{T,jet} \) can be unfolded to 1 GeV accuracy, then

→ Differences between parton, hadron and with MPI: well less than 1 GeV (likely less than 500 MeV)

→ Impact of the UE: less than 300 MeV (in the PYTHIA MPI model)

The EEEC observable is fully robust against the UE: Obtain corrections to \( p_{T,jet} \) independently
1. Correlation functions are powerful tools for precision collider physics.
2. The 3-point correlation function is very sensitive to the top mass.
3. Very small hadronization corrections to the normalized 3-point correlator.
4. The only sizable corrections from hadronization and the UE enter solely into $p_{T\text{jet}}$.
5. Corrections to the jet $p_T$ are observable-independent and can be independently studied and quantified.
Thank you
Supplementary slides
More on correlation functions of energy flow operators

Energy flow operators are natural objects in QFT:

\[
\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r \to \infty} r^2 n_i T_{0i}(t, r\vec{n})
\]

that can be used to define energy weighted cross sections:

\[
\sigma_w = \int d^4x e^{i\vec{q} \cdot \vec{x}} \langle 0 | \mathcal{O}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \ldots \mathcal{E}(\vec{n}_N) \mathcal{O}^\dagger(0) | 0 \rangle
\]

such as the 2-point correlator (EEC):

\[
\frac{d\Sigma}{d \cos \chi} = \sum_{ij} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi) d\sigma
\]

Each event contributes to multiple bins, with the final distribution being an ensemble average over all events:
EEEC on tops: Compare different energy weights

Study the simplest configuration of an equilateral triangle $\hat{\zeta}_{ij} = \zeta$:

$$
\frac{d\Sigma(\delta\zeta)}{dQd\zeta} = \int d\zeta_{12} d\zeta_{23} d\zeta_{31} \int d\sigma \widehat{M}_{\Delta}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta)
$$

$n = 2$ is not IRC safe, but can be computed using the track-functions formalism.
At the LHC we can measure EEEC on jets containing top decay products:

Measurement operator for $pp$ collisions:

$$\hat{\mathbf{M}}^{(n)}_{(pp)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k \in \text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \delta \left( \zeta_{12} - \hat{\zeta}_{ij}^{(pp)} \right) \delta \left( \zeta_{23} - \hat{\zeta}_{ik}^{(pp)} \right) \delta \left( \zeta_{31} - \hat{\zeta}_{jk}^{(pp)} \right)$$

$$\hat{\zeta}_{ij}^{(pp)} = \Delta R_{ij} = \sqrt{\Delta \eta_{ij}^2 + \Delta \phi_{ij}^2}$$
EEEC on tops: *nitty-gritty* of measurements in hadron collisions

At the LHC we can measure EEEC on jets containing top decay products:

The peak the distribution is determined by $p_{T,t}$ and *not* $p_{T,\text{jet}}$. But the parton level top $p_{T,t}$ is not directly accessible, so consider

$$
\frac{d\Sigma(\delta\zeta)}{dp_{T,\text{jet}}d\zeta} = \frac{d\Sigma(\delta\zeta)}{dp_{T,t}d\zeta} \frac{dp_{T,t}}{dp_{T,\text{jet}}}
$$

Because of energy weighting and small angle limit, the basic properties of $d\Sigma(\delta\zeta)/dp_{T,t}d\zeta$ are completely insensitive to the underlying event.
Impact of asymmetry cut $\delta \zeta$

Asymmetry cut $\delta \zeta$ only constrains triangles with $\zeta > 2 \delta \zeta$

\[
\frac{d\Sigma}{d\zeta} \approx 4(\delta \zeta)^2 G^{(n)}(\zeta, \zeta, \zeta; m_t), \quad \delta \zeta \ll \zeta
\]
Effects of hadronization and underlying event in $pp$ collisions

Hadronization and MPI will impact the jet $p_T$:

$$p_{T,\text{jet}}^{\text{Had.}} = p_{T,\text{jet}}^{\text{part.}} + \delta p_{T,\text{jet}}^{\text{Had.}}, \quad p_{T,\text{jet}}^{\text{UE}} = p_{T,\text{jet}}^{\text{Had.}} + \delta p_{T,\text{jet}}^{\text{UE}}$$

[Dasgupta, Magnea, Salam; 0712.3014]

$p_{T,\text{jet}}$ appears in two places:

1. $p_{T,\text{jet}}$ appears in the measurement operator

$$\hat{M}_{(pp)}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k \in \text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \delta (\zeta_{12} - \zeta_{ij}^{(pp)}) \delta (\zeta_{23} - \zeta_{ik}^{(pp)}) \delta (\zeta_{31} - \zeta_{jk}^{(pp)})$$

2. The state $|\psi_t\rangle$ in hadron colliders in $\langle \psi_t | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) | \psi_t \rangle$ cannot be described by a local operator and needs to be constrained via $p_{T,\text{jet}}$. 
If the state $|\psi_t\rangle$ is fully specified (through unphysical $p_{T,t}^{\text{hard}}$) one achieves complete insensitivity to the UE (despite $p_{T,\text{jet}}$ in $\mathcal{M}^{(n)}_{(pp)}$)

Dependence on $p_{T,\text{jet}}$ in $\mathcal{M}^{(n)}_{(pp)}$ drops out when normalized
Hadronization and UE effects directly related to the $p_{T,\text{jet}}$

The state $|\psi_t\rangle$ must be specified through $p_{T,\text{jet}}$, but the corrections to the $p_{T,\text{jet}}$ can be independently specified (something like PDFs).
Obtaining the top mass from multiple $p_{T,\text{jet}}$ bins

1. Parameterize the all orders peak position:

$$\zeta^{(pp)}_{\text{peak}} = 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{f(p_{T,\text{jet}}, m_t, \Lambda_{\text{QCD}})^2} \equiv 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{(p_{T,\text{jet}} + \Delta(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}}))^2}$$

2. Work with

$$y(\zeta^{(pp)}_{\text{peak}}, p_{T,\text{jet}}^v) = \left(\zeta^{(pp)}_{\text{peak}} - \zeta^{(pp)}_{\text{ref}}\right) \left(\frac{3(1 + \mathcal{O}(\alpha_s))}{p_{T,\text{jet}}^v} - \frac{3(1 + \mathcal{O}(\alpha_s))}{p_{T,\text{jet}}^\text{ref}}\right)^{-1},$$

3. Define

$$\Delta(p_{T,\text{jet}}^\text{ref}, m_t, \alpha_s, \Lambda_{\text{QCD}}) = \Delta^\text{ref}, \quad \Delta(p_{T,\text{jet}}^v, m_t, \Lambda_{\text{QCD}}) = \Delta^\text{ref} + \Delta^v(p_{T,\text{jet}}^v - p_{T,\text{jet}}^\text{ref}, m_t, \alpha_s, \Lambda_{\text{QCD}})$$

4. Solve for $y$:

$$y(p_{T,\text{jet}}^v, \Delta^\text{ref}, \Delta^v) = \frac{m_t}{p_{T,\text{jet}}^\text{ref}} \frac{p_{T,\text{jet}}^\text{ref}}{p_{T,\text{jet}}^v + \Delta^\text{ref}} \left(1 - \frac{2p_{T,\text{jet}}^\text{ref} \Delta^\text{ref} + (\Delta^\text{ref})^2}{2(p_{T,\text{jet}}^v)^2} + \frac{(p_{T,\text{jet}}^\text{ref} + \Delta^\text{ref})^2 (\Delta^\text{ref} + \Delta^v)}{8(p_{T,\text{jet}}^v)^3} + \mathcal{O}\left(\frac{1}{(p_{T,\text{jet}}^v)^4}\right)\right)$$

5. The asymptotic value for $p_{T,\text{jet}}^v$ depends only on $m_t$ and $\Delta^\text{ref}$.
Obtaining the top mass from multiple $p_{T,\text{jet}}$ bins

$y$

$m_t$ fit: $(172.4 \pm 0.6)(1+O(\alpha_s)) \text{ GeV}$

$\xi_{\text{peak}}$

$m_t$ fit: $(173.2 \pm 0.5 \pm 0.3)(1+O(\alpha_s)) \text{ GeV}$