

Weighing the top with Energy Correlators

Aditya Pathak

Jack Holguin, Ian Mould, Massimiliano Procura

The top quark mass and fate of the universe

Right now: An exciting era of precision collider physics!

The top and the higgs masses determine the nature of the stability of the electroweak vacuum

[Buttazzo, et al., 2013][Andreassen, et al. 2014] [See Andrea Knue's talk]

→ Direct top mass the most precise:

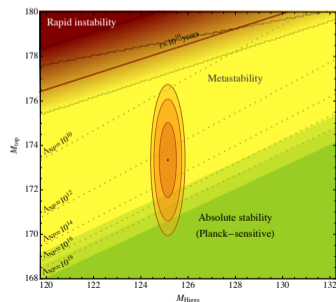
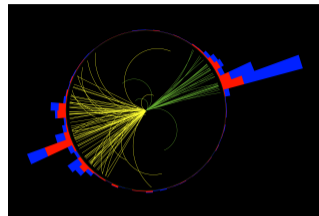
$m_t^{\text{MC}} = 173.34 \pm 0.76 \text{ GeV}$ [World combination 1403.4427],
but ...

- *MC top mass parameter: No field theoretic mass scheme, $\mathcal{O}(1)$ GeV theory uncertainty argued*
[Hoang 2004.12915]
- Sensitive to the parton shower, hadronization model
[Hoang et al. 1807.06617]

→ Indirect cross section measurements not precise enough

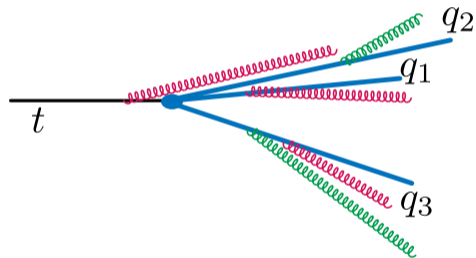
$m_t^{\text{pole}} = 172.9^{+2.5}_{-2.6} \text{ GeV}$ [ATLAS, 1406.5375]

$m_t^{\text{pole}} = 172.7^{+2.4}_{-2.7} \text{ GeV}$ [CMS, 1701.06228]



Kinematic top mass measurements

Knowing kinematics of the top decay improves precision on m_t

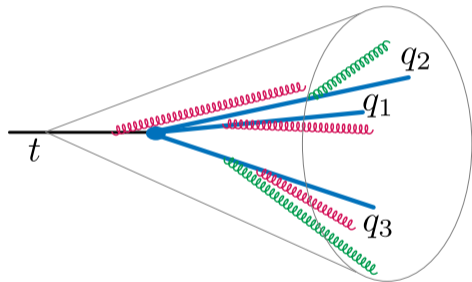


Consider the jet mass:

$$M_J^2 = \left(\sum_{i \in J} p_i^\mu \right)^2 \simeq m_t^2 + \Gamma_t m_t + \dots$$

[Fleming et al. hep-ph/0703207, 0711.2079]

[Bachu, Hoang, AP, Mateu, Stewart 2012.12304]



Kinematic top mass measurements

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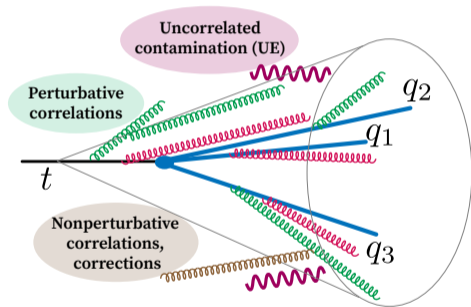
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[Fleming et al. hep-ph/0703207, 0711.2079]

[Bachu, Hoang, AP, Mateu, Stewart 2012.12304]

Challenging to calculate:

- Strongly correlated with the outside radiation
- Precision spoiled by uncorrelated contamination (UE)



Kinematic top mass measurements

Consider **Soft drop jet mass**

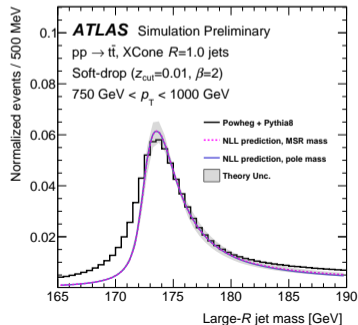
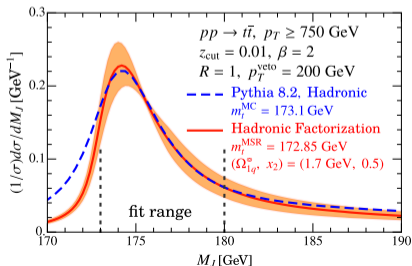
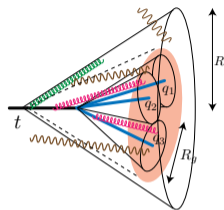
$$M_{J,\text{sd}}^2 = \left(\sum_{i \in J^{\text{groomed}}} p_i^\mu \right)^2 \simeq m_t^2 + \Gamma_t m_t + \dots$$

→ Removes wide-angle soft radiation

→ Improves robustness significantly for the LHC

[Hoang, Mantry, AP, Stewart 1708.02586] [Hoang, Mantry, Michel, AP, Stewart]

[ATL-PHYS-PUB-2021-034]



Kinematic top mass measurements

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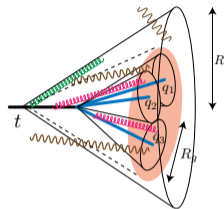
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[ATL-PHYS-PUB-2021-034]



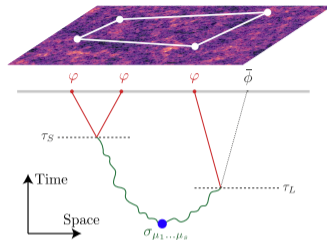
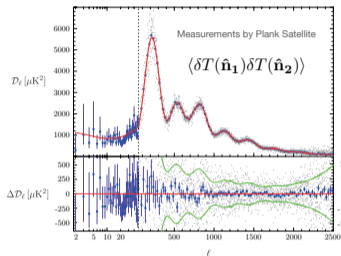
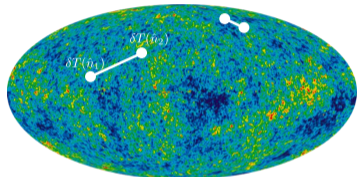
Still quite a challenging task to quantify the remaining corrections in ‘...’!

$$M_J^2 \simeq m_t^2 + \Gamma_t m_t +$$



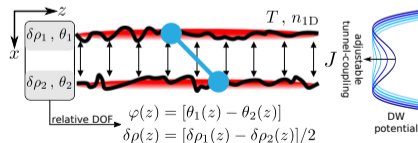
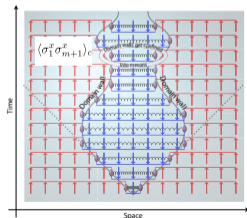
Lessons from other sciences

Correlation functions are extremely powerful!



[Arkani-Hamed, Maldacena 1503.08043]

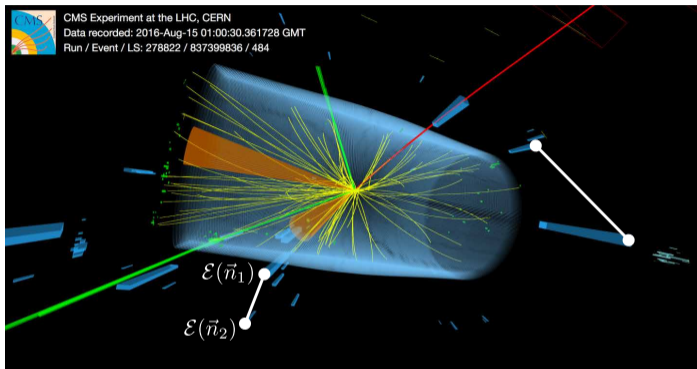
[Arkani-Hamed, Baumann, Lee, Pimentel;1811.00024]



$$G^N(\mathbf{z}, \mathbf{z}') = \langle [\varphi(z_1) - \varphi(z'_1)] \dots [\varphi(z_N) - \varphi(z'_N)] \rangle$$

[Schweigler et al. 1505.03126]

Correlators for collider physics

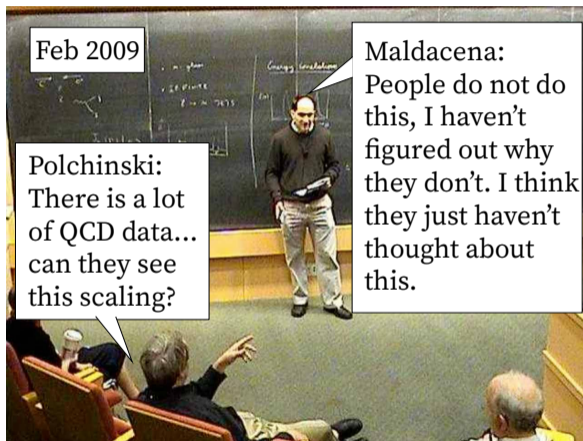


For a precise theoretical formulation:

- Consider correlation functions of energy flow operators: $\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \dots \mathcal{E}(\vec{n}_N) \rangle$ [Hofman, Maldacena; 0803.1467]
- Cannot work with local operators: need light ray operators
[Hofman, Maldacena; 0803.1467][Sveshnikov, Tkachov hep-ph/9512370][Kravchuk, Simmons-Duffin 1805.00098]
- Borrow a wealth of insights from CFT and string theory
[Hofman, Maldacena; 0803.1467][Belitsky et al. 1311.6800][Korchinsky, Sokatchev 1504.07904][Kologlu et al. 1904.05905, 1905.01311]

Why have *we* not already been using correlators?

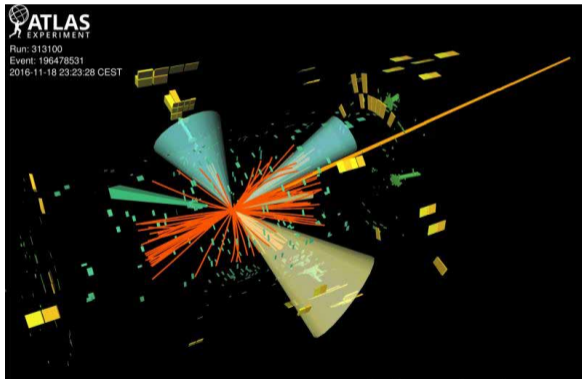
Cosmic variance is not an excuse for collider physics!



[KITP Seminar, Feb 2009; 47 min into the talk]

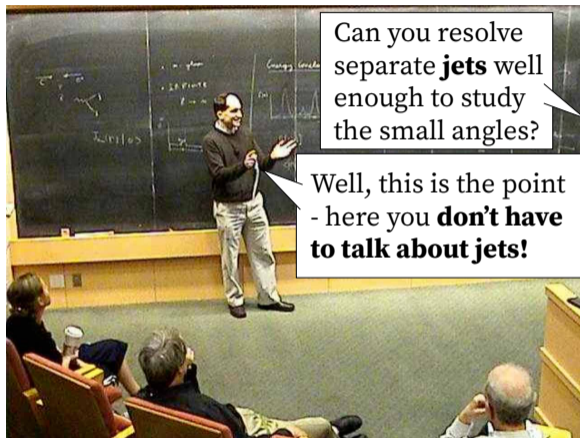
Why have *we* not already been using correlators?

Since the beginning of the LHC we have exclusively focused on jet-based observables ($m_J, \tau_a, e_n^{(\alpha)}, \dots$)



Why have *we* not already been using correlators?

Energy-correlators only require angles to be measured precisely!

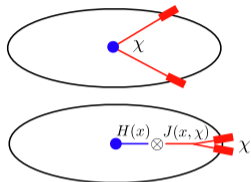


[KITP Seminar, Feb 2009; 49 min into the talk]

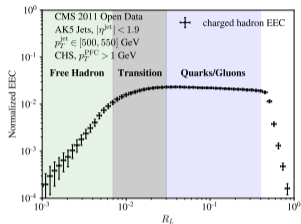
→ Can be computed on tracks! [Chang, Procura, Thaler, Waalewijn, 1303.6637][Moult, van Velzen, Waalewijn, Zhu 2108.01674]

Progress in recent years

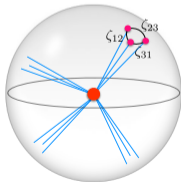
- Resummation in the small angle limit in SCET
- EEC: mapping transition from perturbative to free hadron phase
- EEEC: Interference in the squeezed limit



[Dixon, Moulton, Zhu, 1904.01310]

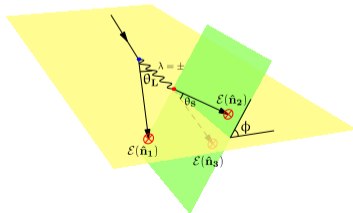


[Komiske, Moulton, Thaler, Zhu]

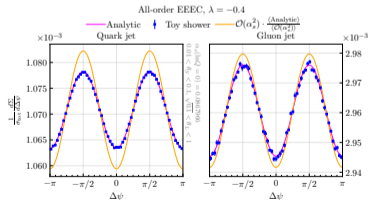


[Chen, Luo, Moulton, Yang, Zhang, Zhu

1912.11050]

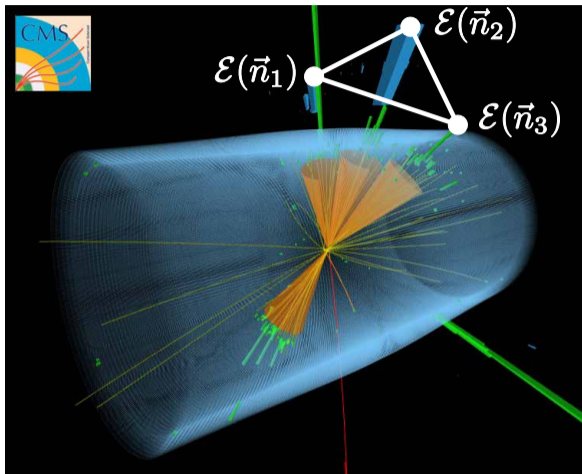


[Chen, Moulton, Zhu 2104.00009]



[Karlberg, Salam, Scyboz, Verheyen 2103.16526]

Probing the top with the three-point correlator



$$\langle \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) \rangle_t \equiv \frac{\langle \psi_t | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) | \psi_t \rangle}{\langle \psi_t | \psi_t \rangle}$$

EEEC on top: A kinematic top mass sensitive observable

Three-point correlator:

$$G^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \int d\sigma \widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31})$$

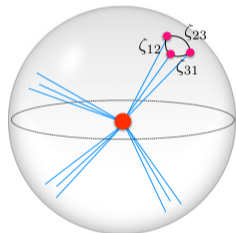
$$\widehat{\mathcal{M}}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k} \frac{E_i^n E_j^n E_k^n}{Q^{3n}} \delta(\zeta_{12} - \hat{\zeta}_{ij}) \delta(\zeta_{23} - \hat{\zeta}_{ik}) \delta(\zeta_{31} - \hat{\zeta}_{jk})$$

In the CFT limit EEEC exhibits a featureless power law

[Dixon, Moulton, Zhu, 1904.01310][Korchemsky 1905.01444][Kologlu et al. 1905.01311]

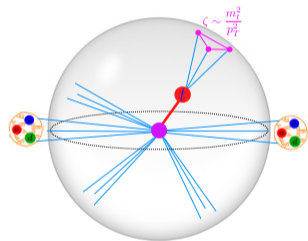
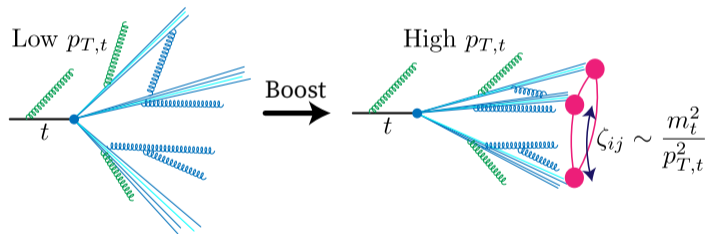
$$G^{(1)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) \xrightarrow{\text{CFT}} \zeta_{31}^{-1+\gamma(4)} G(z, \bar{z}),$$

$$z\bar{z} = \zeta_{12}/\zeta_{31}, \quad (1-z)(1-\bar{z}) = \zeta_{23}/\zeta_{31}$$



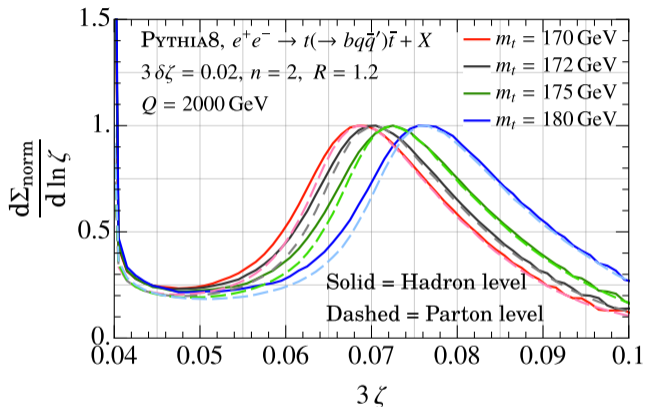
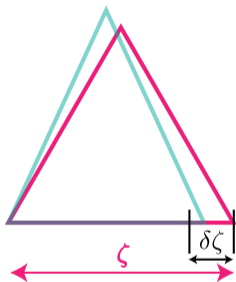
EEEC on top: A kinematic top mass sensitive observable

The top quark decay imprints m_t as a characteristic scale in the 3-point correlator:



EEEC on top: Excellent mass sensitivity

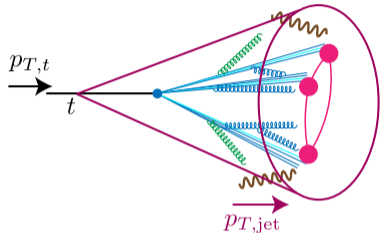
Study the simplest configuration of an equilateral triangle $\hat{\zeta}_{ij} = \zeta$:



- Hadronization effects: **small effect on the peak $\Delta m_t^{\text{Had.}} \sim 230$ MeV.**
- Peaked at $\zeta_{\text{peak}} \approx 3m_t^2/Q^2$: **peak dominated by hard decay of the top.**
- Resilient to collinear radiation $\alpha_s \ln \zeta_{\text{peak}} < 1$: **fixed order perturbation theory sufficient.**

EEEC on top: measurement at the LHC

At the LHC we can measure EEEEC on jets containing top decay products:



Unlike EEEEC, $p_{T,jet}$ is susceptible to hadronization and underlying event:

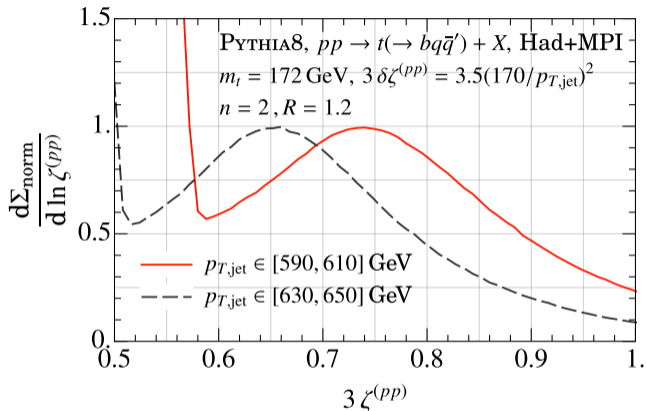
→ Consider multiple $p_{T,jet}$ bins

Assuming that $p_{T,jet}$ can be unfolded to 1 GeV accuracy, then

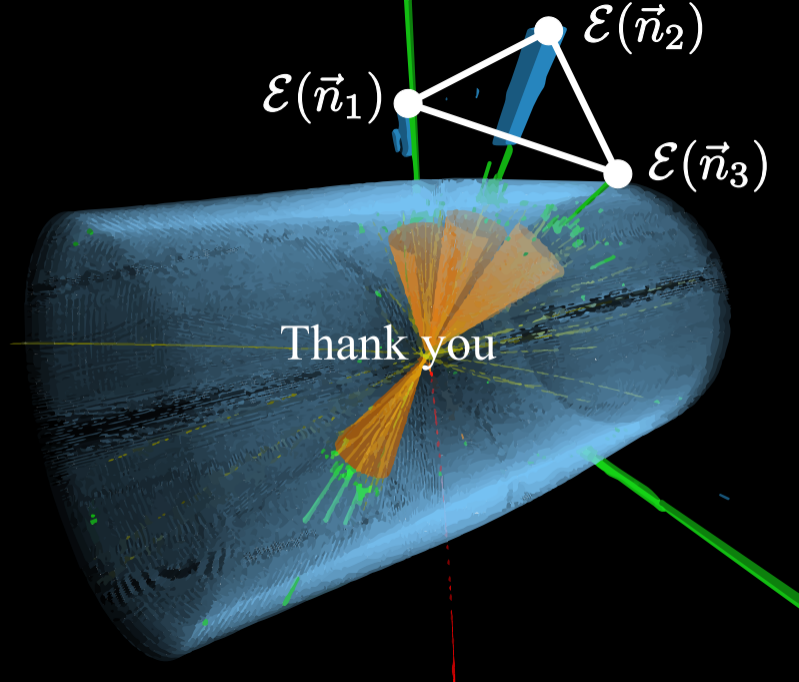
→ Differences between parton, hadron and with MPI: well less than 1 GeV (likely less than 500 MeV)

→ Impact of the UE: less than 300 MeV (in the PYTHIA MPI model)

The EEEEC observable is fully robust against the UE: Obtain corrections to $p_{T,jet}$ independently



1. Correlation functions are powerful tools for precision collider physics.
2. The 3-point correlation function is very sensitive to the top mass.
3. Very small hadronization corrections to the normalized 3-point correlator.
4. The only sizable corrections from hadronization and the UE enter solely into $p_{T\text{jet}}$.
5. Corrections to the jet p_T are observable-independent and can be independently studied and quantified.



Thank you

Supplementary slides

More on correlation functions of energy flow operators

Energy flow operators are natural objects in QFT:

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r \rightarrow \infty} r^2 n^i T_{0i}(t, r\vec{n})$$

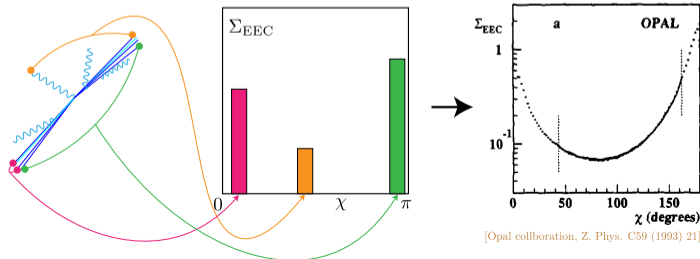
that can be used to define energy weighted cross sections:

$$\sigma_w = \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{O}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \dots \mathcal{E}(\vec{n}_N) \mathcal{O}^\dagger(0) | 0 \rangle$$

such as the 2-point correlator (EEC):

$$\frac{d\Sigma}{d \cos \chi} = \sum_{ij} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos \chi) d\sigma$$

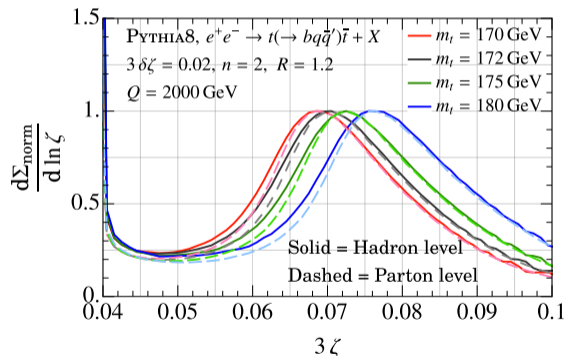
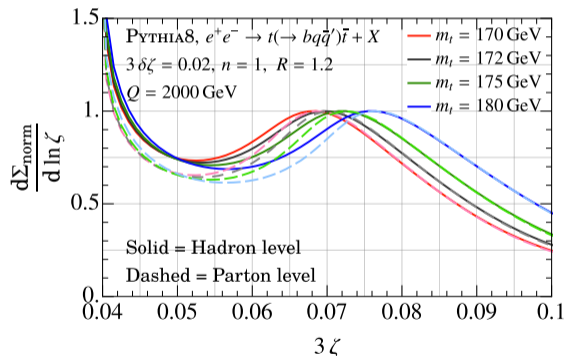
Each event contributes to multiple bins, with the final distribution being an ensemble average over all events:



EEEC on tops: Compare different energy weights

Study the simplest configuration of an equilateral triangle $\hat{\zeta}_{ij} = \zeta$:

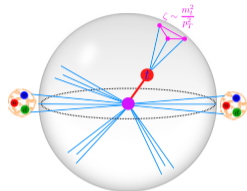
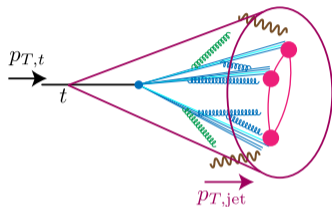
$$\frac{d\Sigma(\delta\zeta)}{dQd\zeta} = \int d\zeta_{12}d\zeta_{23}d\zeta_{31} \int d\sigma \widehat{\mathcal{M}}_{\Delta}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}, \zeta, \delta\zeta)$$



→ $n = 2$ is not IRC safe, but can be computed using the track-functions formalism

EEEC on tops: *nitty-gritty* of measurements in hadron collisions

At the LHC we can measure EEEC on jets containing top decay products:



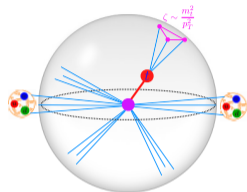
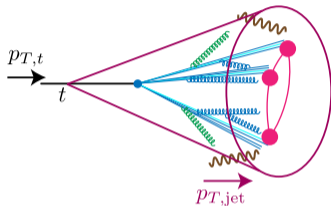
Measurement operator for pp collisions:

$$\widehat{\mathcal{M}}_{(pp)}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k \in \text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \delta\left(\zeta_{12} - \hat{\zeta}_{ij}^{(pp)}\right) \delta\left(\zeta_{23} - \hat{\zeta}_{ik}^{(pp)}\right) \delta\left(\zeta_{31} - \hat{\zeta}_{jk}^{(pp)}\right)$$

$$\hat{\zeta}_{ij}^{(pp)} = \Delta R_{ij} = \sqrt{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2}$$

EEEC on tops: *nitty-gritty* of measurements in hadron collisions

At the LHC we can measure EEEEC on jets containing top decay products:



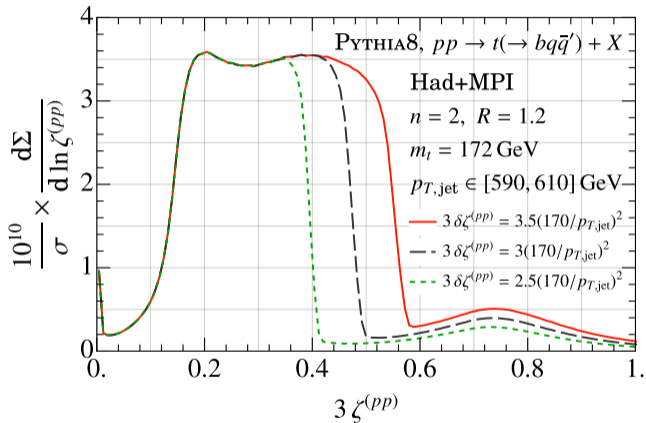
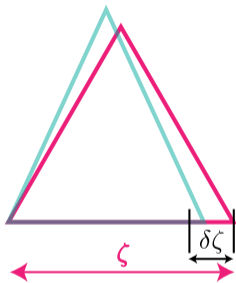
The peak the distribution is determined by $p_{T,t}$ and *not* $p_{T,jet}$. But the parton level top $p_{T,t}$ is not directly accessible, so consider

$$\frac{d\Sigma(\delta\zeta)}{dp_{T,jet}d\zeta} = \frac{d\Sigma(\delta\zeta)}{dp_{T,t}d\zeta} \frac{dp_{T,t}}{dp_{T,jet}}$$

Because of energy weighting and small angle limit, the basic properties of $d\Sigma(\delta\zeta)/dp_{T,t}d\zeta$ are **completely insensitive to the underlying event**.

Impact of asymmetry cut $\delta\zeta$

Asymmetry cut $\delta\zeta$ only constrains triangles with $\zeta > 2\delta\zeta$

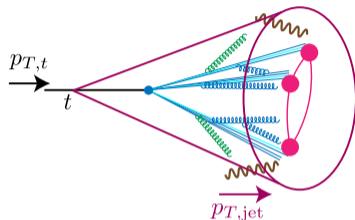


$$\frac{d\Sigma}{d\zeta} \approx 4(\delta\zeta)^2 G^{(n)}(\zeta, \zeta, \zeta; m_t), \quad \delta\zeta \ll \zeta$$

Hadronization and MPI will impact the jet p_T :

$$p_{T,\text{jet}}^{\text{Had.}} = p_{T,\text{jet}}^{\text{part.}} + \delta p_T^{\text{Had.}}, \quad p_{T,\text{jet}}^{\text{UE}} = p_{T,\text{jet}}^{\text{Had.}} + \delta p_T^{\text{UE}}$$

[Dasgupta, Magnea, Salam; 0712.3014]



$p_{T,\text{jet}}$ appears in two places:

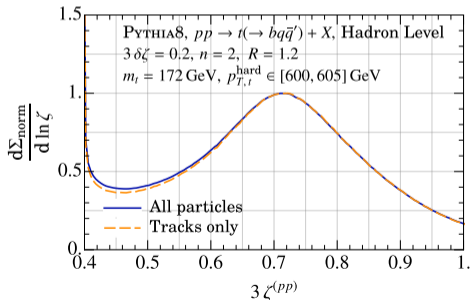
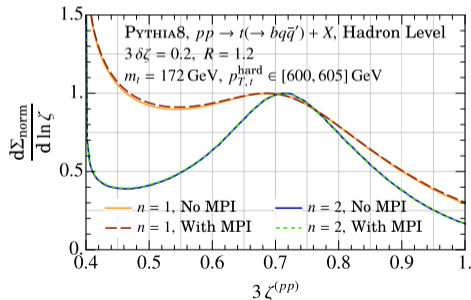
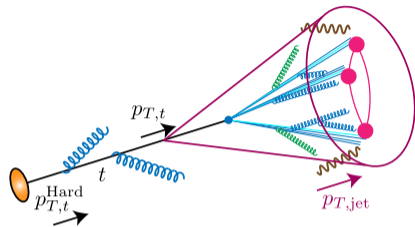
1. $p_{T,\text{jet}}$ appears in the measurement operator

$$\widehat{\mathcal{M}}_{(pp)}^{(n)}(\zeta_{12}, \zeta_{23}, \zeta_{31}) = \sum_{i,j,k \in \text{jet}} \frac{(p_{T,i})^n (p_{T,j})^n (p_{T,k})^n}{(p_{T,\text{jet}})^{3n}} \delta(\zeta_{12} - \hat{\zeta}_{ij}^{(pp)}) \delta(\zeta_{23} - \hat{\zeta}_{ik}^{(pp)}) \delta(\zeta_{31} - \hat{\zeta}_{jk}^{(pp)})$$

2. The state $|\psi_t\rangle$ in hadron colliders in $\langle \psi_t | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \mathcal{E}(\vec{n}_3) | \psi_t \rangle$ cannot be described by a local operator and needs to be constrained via $p_{T,\text{jet}}$.

Effects of hadronization and underlying event in pp collisions

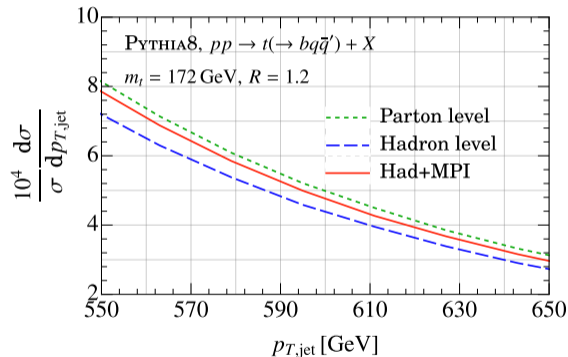
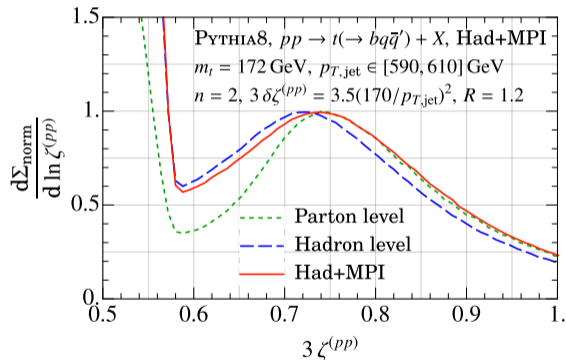
If the state $|\psi_t\rangle$ is fully specified (through *unphysical* $p_{T,t}^{\text{hard}}$) one achieves **complete insensitivity to the UE** (despite $p_{T,\text{jet}}$ in $\mathcal{M}_{(pp)}^{(n)}$)



Dependence on $p_{T,\text{jet}}$ in $\mathcal{M}_{(pp)}^{(n)}$ drops out when normalized

Hadronization and UE effects directly related to the $p_{T,\text{jet}}$

The state $|\psi_t\rangle$ must be specified through $p_{T,\text{jet}}$, but the corrections to the $p_{T,\text{jet}}$ can be *independently* specified (something like PDFs)



Obtaining the top mass from multiple $p_{T,\text{jet}}$ bins

1. Parameterize the all orders peak position:

$$\zeta_{\text{peak}}^{(pp)} = 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{f(p_{T,\text{jet}}, m_t, \Lambda_{\text{QCD}})^2} \equiv 3(1 + \mathcal{O}(\alpha_s)) \frac{m_t^2}{(p_{T,\text{jet}} + \Delta(p_{T,\text{jet}}, m_t, \alpha_s, \Lambda_{\text{QCD}}))^2}$$

2. Work with

$$y(\zeta_{\text{peak}}^{(pp)\text{v}}, p_{T,\text{jet}}^{\text{v}}) = \left(\zeta_{\text{peak}}^{(pp)\text{ref}} - \zeta_{\text{peak}}^{(pp)\text{v}} \right) \left(\frac{3(1 + \mathcal{O}(\alpha_s))}{p_{T,\text{jet}}^{\text{v}}} - \frac{3(1 + \mathcal{O}(\alpha_s))}{p_{T,\text{jet}}^{\text{ref}}} \right)^{-1},$$

3. Define

$$\Delta(p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}}) = \Delta^{\text{ref}}, \quad \Delta(p_{T,\text{jet}}^{\text{v}}, m_t, \Lambda_{\text{QCD}}) = \Delta^{\text{ref}} + \Delta^{\text{v}}(p_{T,\text{jet}}^{\text{v}} - p_{T,\text{jet}}^{\text{ref}}, m_t, \alpha_s, \Lambda_{\text{QCD}})$$

4. Solve for y :

$$y(p_{T,\text{jet}}^{\text{v}}, \Delta^{\text{ref}}, \Delta^{\text{v}}) = \frac{m_t}{p_{T,\text{jet}}^{\text{ref}}} \frac{p_{T,\text{jet}}^{\text{ref}}}{p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}}} \left(1 - \frac{2p_{T,\text{jet}}^{\text{ref}} \Delta^{\text{ref}} + (\Delta^{\text{ref}})^2}{2(p_{T,\text{jet}}^{\text{v}})^2} + \frac{(p_{T,\text{jet}}^{\text{ref}} + \Delta^{\text{ref}})^2 (\Delta^{\text{ref}} + \Delta^{\text{v}})}{8(p_{T,\text{jet}}^{\text{v}})^3} + \mathcal{O}\left(\frac{1}{(p_{T,\text{jet}}^{\text{v}})^4}\right) \right)$$

5. The asymptotic value for $p_{T,\text{jet}}^{\text{v}}$ depends only on m_t and Δ^{ref} .

Obtaining the top mass from multiple $p_{T,\text{jet}}$ bins

