Next-to soft virtual resummed Drell-Yan cross section beyond Leading-logarithm

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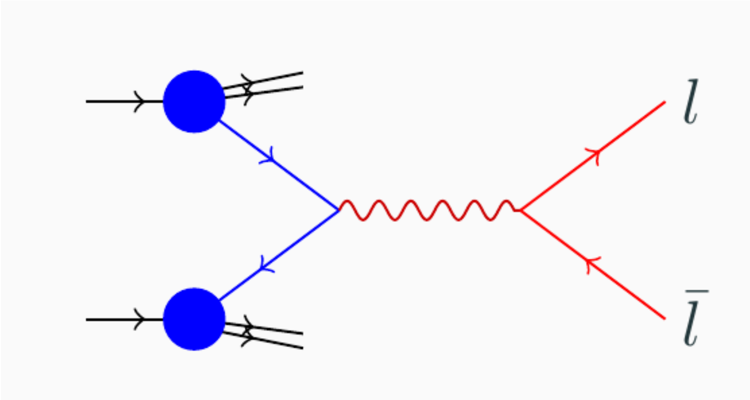
(Joint work with Ajjath A.H, P. Mukherjee, V. Ravindran and Surabhi Tiwari)

The Institute of Mathematical Sciences, India

30th International Symposium on Lepton Photon Interactions at High Energies

Overview & Background

Drell-Yan



- **One of the standard candle processes**
  Large cross section and clean experimental signature - important for detector calibration and constraining parton distribution functions

- Experimentally, one has a very clean environment for precise measurements

- Well-understood theoretically - known to N^3LO accuracy in QCD
  Duhr, Dulat et.al ('20)

- DY serves as an important process in collider experiments

- Higher order perturbative QCD corrections to DY provides ample opportunity to explore the structure of the perturbation series

Overview & Background

- Large logarithms at kinematic threshold region spoil the reliability of fixed-order pertutrbative sries

- Resolution: Thresold resummation

  Sterman-Catani-Trentedue

- Resummation is necessary to provide reliable theoretical predictions

- Threshold resummation : known to N^3LL accuracy

Resummed Drell-Yan cross-section at N^3LL

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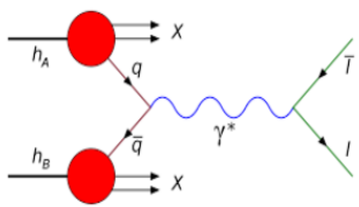
Incusive Reactions – QCD Improved Parton Model

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2) \int \frac{dz}{z} \Phi_{ab}\left(\frac{\tau}{z}, \mu_F^2\right) \Delta_{ab}(q^2, \mu_F^2, z)$$

Partonic Coeff. function

perturbative

Partonic flux



$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

Parton distribution fns
(PDFs - non-perturbative)

τ	Hadronic scaling variable
q^2	Invariant mass sq
z	Partonic scaling variable
μ_R^2	Renormalisation scale
μ_F^2	Factorisation scale

$$\tau = \frac{q^2}{S}$$

Hadronic cm
energy squared

$$z = \frac{q^2}{\hat{s}}$$

Partonic cm
energy squared

Threshold Expansion

The partonic CF near threshold (z->1) take this form:

$$\Delta_c^{\mathrm{SV+NSV,i}}(z,q^2) = \sum_{k=0}^{2i-1} c_{ik}^{\mathcal{D}}\ \mathcal{D}_k + c_i^{\delta}\ \delta(1-z) + \sum_{k=0}^{2i-1} c_{ik}^{L}\ \log^k(1-z)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{(1-z)}\right)_+$$

Plus distribution

Soft-virtual [SV] corrections

Leading power (LP) logarithms
Most Singular when z→ 1
Corrections from diagonal Channels
Resummation to N3LL accuracy
Well-understood

Next-to SV [NSV] corrections

Next-to LP (NLP) logarithms
Next-to-dominant singular
Corrections from both diagonal & off-diagonal
Resummation to LL accuracy
Not much studied

NSV in History

* The earliest evidence that IR effects can be studied at NLP
  [Low, Burnett, Kroll]

* Early attempts :
  [Kraemer, Laenen, Spira (98)]
  [Akhoury, Sotiropoulos & Sterman (98)]

* Important Results & Predictions using Physical Kernel Approach & explicit computation:
  [Moch , Vogt et al. (09-20)]
  [Anastasiou, Duhr, Dulat et al.(14)]

* Universality of NLP effects and LL Resummation:
  [Laenen, Magnea, et al. (08-19)]
  [Grunberg & Ravindran (09)]
  [Ball, Bonvini, Forte, Marzani, Ridolfi (13)]
  [Del Duca et al. (17)]

* Subleading Factorisation and LL Resummation at NLP using SCET:
  [Larkoski, Nelli , Stewart et al. (14)]
  [Kolodrubetz, Moult, Neill ,Stewart et al. (17)]
  [Beneke et al. (19-20)]

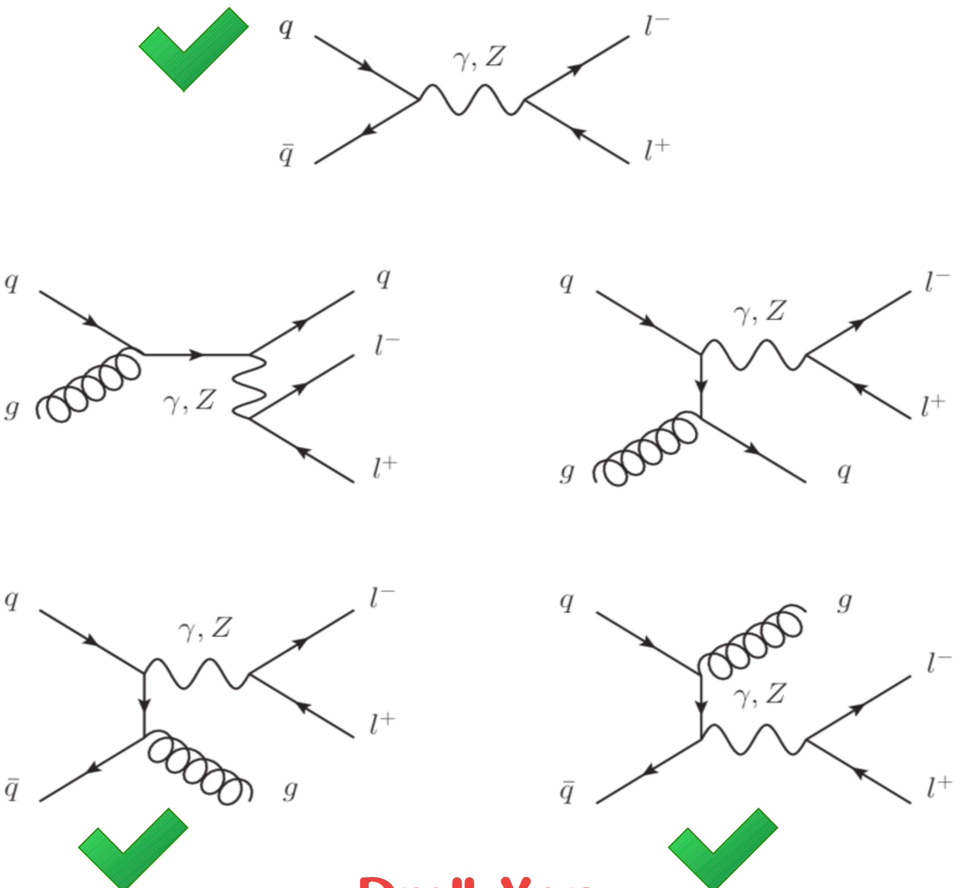
And many other works…

Our Works

- Factorisation and RG invariance approach to study NSV resummation effects
  [Ajjath, Pooja, Ravindran , hep-ph/ 2006.06726]
- On next to soft threshold corrections to DIS and SIA processes
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, JHEP 04 (2021) 131]
- Next-to SV resummed Drell-Yan cross section beyond Leading-logarithm ✔
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, hep-ph/2107.09717]
  Today's talk
- Resummed Higgs boson cross section at next-to SV to NNLO + $\overline{\text{NNLL}}$
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, hep-ph/2109.12657]
- Rapidity distribution at soft-virtual and beyond for n-colorless particles to N4LO in QCD
  [Taushif, Ajjath, Pooja, Ravindran, A.Sankar, Eur. Phys. J. C 81, 943 (2021)]
- Next-to-soft corrections for Drell-Yan and Higgs boson rapidity distributions beyond N3LO
  [Ajjath, Pooja, Ravindran, A.Sankar, S.Tiwari, Phys.Rev.D 103 (2021) L111502]

Our Approach

Considered only diagonal channels :



Drell-Yan

Keypoints

- Collinear Factorisation
- Renormalisation Group (RG) Invariance
- Logarithmic structure of higher order perturbative results

The Theory - Formalism

- Factoring out the pure virtual contributions

Soft+Next-to soft corrections

$$\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \left(Z_{c,UV} \right)^2 |\hat{F}_c(\epsilon)|^2 S_c(z,\epsilon)$$

Partonic cross-section

UV Renormalisation constant

Unrenormalised Form Factor (FF)
(pure virtual corrections)

- <u>Mass Factorisation</u>

Altarelli-Parisi (AP) kernel

$$\frac{1}{z}\hat{\sigma}_{ab}(z,\epsilon) = \sigma_0 \sum_{a'b'} \Gamma_{aa'}(\mu_F^2,z,\epsilon) \otimes \left(\frac{1}{z}\Delta_{a'b'}(\mu_F^2,z,\epsilon) \right) \otimes \Gamma_{b'b}(\mu_F^2,z,\epsilon)$$

Partonic cross-section containing only
Initial state collinear singularities

Collinear Finite

Collinear Singular

Coefficient function – Diagonal channel

UV finite mass-factorised partonic coefficient function for the diagonal channels:

$$\Delta_{c\bar{c}}(z, \epsilon, q^2 \mu_R^2, \mu_F^2) = \left(\Gamma^T\right)^{-1} \otimes \left\{ \left(Z_{c,UV}\right)^2 |\hat{F}_c(Q^2, \epsilon)|^2 S_c(q^2, z, \epsilon) \right\} \otimes \left(\Gamma\right)^{-1}$$

Set of governing differential eqns

$$Q^2 \frac{d}{dQ^2} \log \hat{F}^c = \frac{1}{2} \left[K^c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^c\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) \right]$$

K+G

$$\mu_R^2 \frac{d}{d\mu_R^2} \log Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \quad \gamma_{i-1}^c$$

RGE

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z, \mu_F^2, \epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}\left(z, a_s(\mu_F^2)\right) \otimes \Gamma_{a'b}(z, \mu_F^2, \epsilon)$$

AP evolution eqn

Altarelli-Parisi kernels

[Moch,Vogt,Vermaseren]

Required to remove the initial state collinear singularities

AP kernels which satisfy renormalisation group equations

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma_{ab}(z,\mu_F^2,\epsilon) = \frac{1}{2} \sum_{a'=q,\bar{q},g} P_{aa'}(z, a_s(\mu_F^2)) \otimes \Gamma_{a'b}(z,\mu_F^2,\epsilon)\,, \qquad a,b = q,\bar{q},g$$

AP Splitting function

Collinear ano dim
(contributes to NSV)

$$P_{cc}(z,\mu_F^2) = 2\left[\frac{A^c}{(1-z)_+} + B^c\,\delta(1-z) + C^c\,\log(1-z) + D^c + \mathcal{O}(1-z)\right]$$

SV **NSV** **beyond NSV**

We consider only diagonal parts of splitting functions

Differential Eqn – Soft + Next-to Soft

Using finiteness of the partonic coefficient function and Sudakov differential eqn of FFs (K+G eqn) :

$$q^2 \frac{d}{dq^2} \mathcal{S}_c = \frac{1}{2}\left[\overline{K}_c\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon, z\right) + \overline{G}_c\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon, z\right)\right] \otimes \mathcal{S}_c$$

IR singular — IR finite

Soft-collinear contributions exhibits exponential behaviour:

$$\mathcal{S}_c = \mathcal{C} \exp\left(2\Phi_c\right)$$

Soft- collinear function

$$\mathcal{C} \exp\left(2\Phi_c(z)\right) = \frac{\hat{\sigma}_{c\bar{c}}(z)}{Z_{c,UV}^2 \left|\hat{F}_c\right|^2}, \qquad c = q, b, g$$

No pure virtual , Only Real-Virtual (RV), Real-Real (RR) etc

[Ravindran]

Soft-Collinear Function -- Solution

$$\Phi_c(\hat{a}_s, q^2, \mu^2, \epsilon, z) = \sum_i \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left(\frac{i\epsilon}{1-z}\right) \left[\hat{\phi}_{\rm SV}^{c,(i)}(\epsilon) + (1-z)\,\hat{\phi}_{\rm NSV}^{c,(i)}(z,\epsilon)\right]$$

Phase-space factor

From matrix elements

Inspired from explicit results

Solution verified up to 3rd order

$A^c, f^c, \overline{\mathcal{G}}^c$

singularities get canceled against those in FF entirely and AP kernels partially

$C^c, D^c, \varphi_c(z)$

Process dependent

Singularities get canceled against the residual div in AP kernels

- Expanding the ansatz:

$$\frac{1}{(1-z)}\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \frac{\delta(1-z)}{i\epsilon} + \sum_{k=0}^{\infty}[i\epsilon]^k \frac{\mathcal{D}_k}{k!}$$

Contributes to SV

$$\left[(1-z)^2\right]^{i\frac{\epsilon}{2}} = \sum_{n=0}^{\infty}\frac{\left[i\epsilon\ \log(1-z)\right]^n}{n!}$$

Contributes to NSV

SV+NSV in Nutshell

Restricting to diagonal channels and around z=1

↓

Mass Factorization Formula

↓

All-order factorisation formula for $\Delta_{c\bar{c}}$

↓

Sudakov type differential Equation & RG Eq.

↓

$$\Delta_c(q^2, \mu_R^2, \mu_F^2, z) = C \exp\Big(\Psi^c\big(q^2, \mu_R^2, \mu_F^2, z, \varepsilon\big)\Big)\Big|_{\varepsilon=0}$$

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z) \otimes f(z) + \cdots .$$

The Master Formula

$$\Psi_c = \left(\ln \left(Z_{c,UV}\left(\hat{a}_s, \mu^2, \mu_R^2, \epsilon\right)\right)^2 + \ln \left|\hat{F}_c\left(\hat{a}_s, \mu^2, q^2, \epsilon\right)\right|^2 \right) \delta(1-z)$$
$$+2\Phi_c\left(\hat{a}_s, \mu^2, q^2, z, \epsilon\right) - 2\mathcal{C} \ln \Gamma_{cc}\left(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon\right)$$

- **SV Ditributions** $\rightarrow$ $\delta(1-z), \left(\frac{\ln^k(1-z)}{(1-z)}\right)_+$
- **NSV Logarithms** $\rightarrow$ $\ln^k(1-z)$

All order structure – Predictive power

$$\Delta_c(q^2,\mu_R^2,\mu_F^2,z) = C\exp\Big(\Psi^c(q^2,\mu_R^2,\mu_F^2,z,\varepsilon)\Big)\Big|_{\varepsilon=0}$$

- What do we achieve as a consequence to this exponential structure:

$L_z^k = \log^k(1-z)$

	GIVEN	PREDICTIONS					
	FO Coefficient	$\Delta^{(2)}_{c\bar{c}}$	$\Delta^{(3)}_{c\bar{c}}$	$\Delta^{(4)}_{c\bar{c}}$	$\Delta^{(5)}_{c\bar{c}}$	$\Delta^{(6)}_{c\bar{c}}$	$\Delta^{(i)}_{c\bar{c}}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
2-loop	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
3-loop	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

In general, using as$^{(n-1)}$ info:

$$\log^{\kappa}(1-z),\ \ n+1\ \le k \le 2n-1$$

at order as^{n}

Checked up to 4th order
[Claude et.al]
[Moch. Vogt et.al]

Integral representation in z-space

- **Knowing the functional form of each building blocks one can derive the integral form as:**

Integral representation:

captures the delta contribution from FF and S_c

$$\Delta_c(q^2, z) = C_0^c(q^2) \;\; \mathcal{C} \exp\left(2\Psi_{\mathcal{D}}^c(q^2, z)\right),$$

Exponent:

$$\Psi_{\mathcal{D}}^c(q^2, z) = \frac{1}{2}\int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + \mathcal{Q}^c(a_s(q^2(1-z)^2), z)$$

Finite contributions from cancellation between Γ_{cc} & S_c

$$P'_{cc} = 2\Big[A^c \mathcal{D}_0(z) + C^c \ln(1-z) + D^c\Big]$$

$$\mathcal{Q}^c(a_s(q^2(1-z)^2), z) = \left(\frac{1}{1-z}\overline{G}_{SV}^c(a_s(q^2(1-z)^2))\right)_+ + \varphi_{f,c}(a_s(q^2(1-z)^2), z).$$

Finite contribution coming from S_c

In the Mellin N space

- Mellin moment of CFs

$$\Delta_N^c = \int_0^1 dz \; z^{N-1} \Delta_c(z)$$

- Threshold limit $z \to 1$ in z-Space translates to $N \to \infty$ in N-Space

- $N \to \infty$ Taking into account SV and NSV terms

$$\left(\frac{\log(1-z)}{1-z}\right)_+ = \frac{\log^2 N}{2} - \frac{\log N}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\log^k(1-z) = \frac{\log^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

Tower of NSV logarithms

- Structure of Next to SV terms

$$\Delta^c_N = 1 + a_s\Big[c_1^2 \log^2 N + c_1^1 \log N + c_1^0 + d_1^1 \frac{\log N}{N} + \mathcal{O}(1/N)\Big]$$

$$+a_s^2\Big[c_2^4 \log^4 N + \cdots + c_2^0 + d_2^3 \frac{\log^3 N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$$+\cdots$$

$$+a_s^n\Big[c_n^{2n} \log^{2n} N + \cdots + d_n^{2n-1} \frac{\log^{2n-1} N}{N} + \cdots + \mathcal{O}(1/N)\Big]$$

$a_s \log N$ is of order \`one\` when a_s is very small at every order 1/N

NSV Resummation

- Inclusion of the NSV logarithms modifies the existing resumed expression as :

$$\omega = 2\beta_0 a_s(\mu_R^2)\log N$$

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = C_0^c(q^2,\mu_R^2,\mu_F^2)\exp\left(\Psi^c_{\mathrm{SV,N}}(q^2,\mu_F^2) + \Psi^c_{\mathrm{NSV,N}}(q^2,\mu_F^2)\right)$$

N-independent coefficient

$$\Psi^c_{\mathrm{SV,N}} = \log(g_0^c(a_s(\mu_R^2))) + g_1^c(\omega)\log N + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g^c_{i+2}(\omega)$$

Known since 1989

[Sterman et.al]

[Catani et.al]

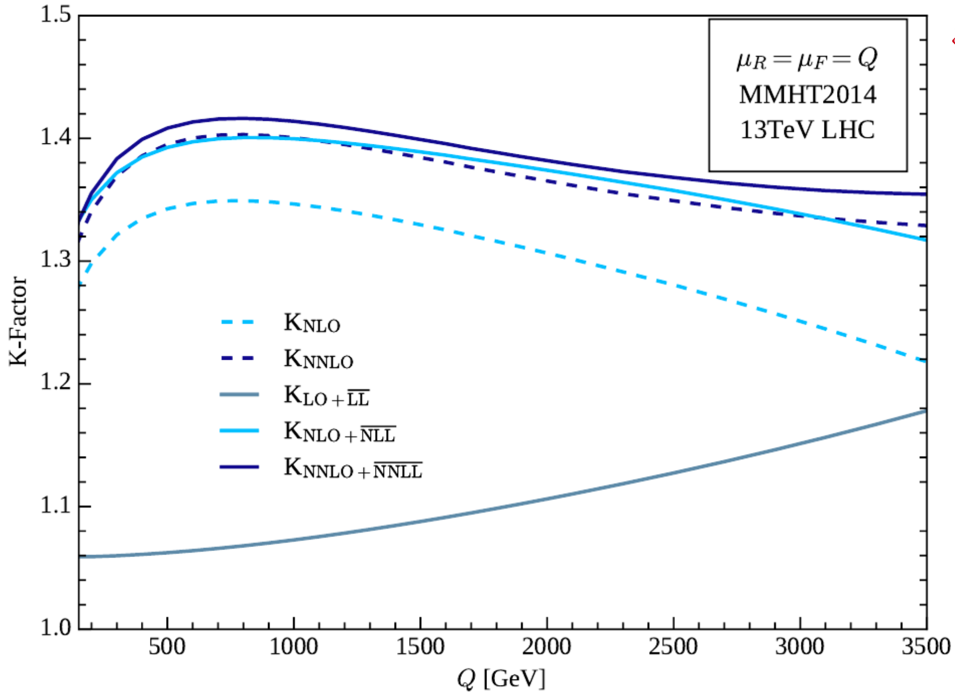
New Result!!

$$\Psi^c_{\mathrm{NSV},N} = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\left(\bar{g}^c_{i+1}(\omega) + h_i^c(\omega,N)\right)$$

$$h_i^c(\omega,N) = \sum_{k=0}^{i} h^c_{ik}(\omega)\ \log^k N.$$

Phenomenology - Drell-Yan

K-Factor Analysis



- resummed curves lie above their corresponding fixed order ones - enhancement due to the resummed corrections
[For Q= 500 GeV, LO->LO+$\overline{LL}$: 6.2 %
NLO->NLO+$\overline{NLL}$: 3.7 % , NNLO->NNLO+$\overline{NNLL}$: 0.94 %]

- resummed curves are closer
resummed correction decreases as we go for higher order resummed contributions

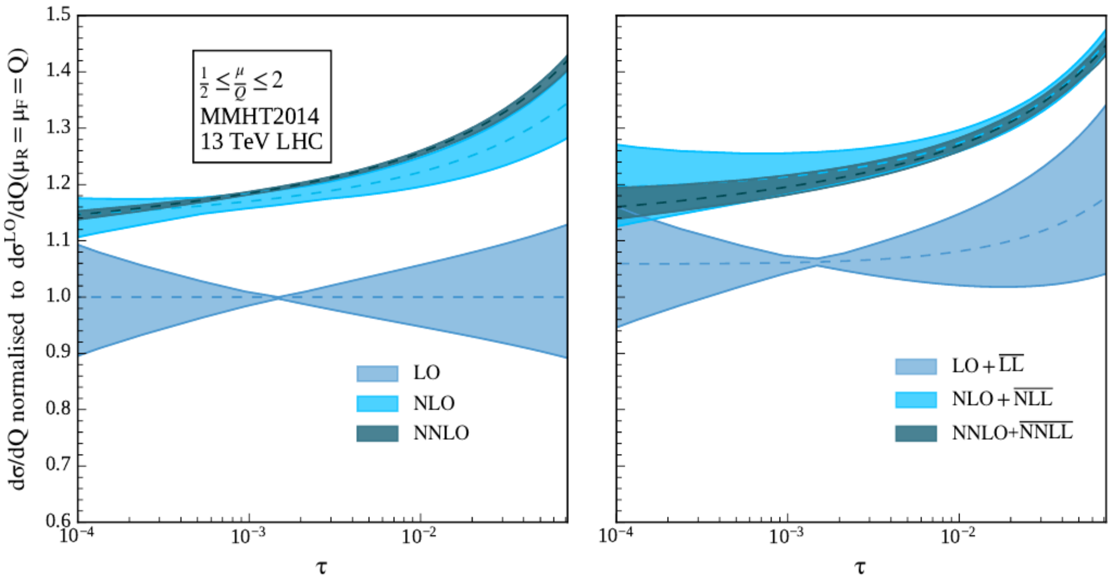
Perturbative convergence in the RES predictions
reliability of RES predictions

$\mu_R = \mu_F = Q$(GeV)	LO + $\overline{\text{LL}}$	NLO	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + $\overline{\text{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

$$\mathrm{K}(Q) = \frac{\dfrac{d\sigma}{dQ}(\mu_R = \mu_F = Q)}{\dfrac{d\sigma^{\mathrm{LO}}}{dQ}(\mu_R = \mu_F = Q)}$$

Phenomenology - Drell-Yan

7-point scale uncertainities of the resummed results



- resummed result shows a systematic reduction of the uncertainties with the inclusion of each logarithmic corrections

- improvement at the $\mathrm{NLO}+\overline{\mathrm{NLL}}$ than at the $\mathrm{NNLO}+\overline{\mathrm{NNLL}}$ in comparison to their respective F.O predictions Why ?

Let us analyze the effect of each scale individually on the resummed result

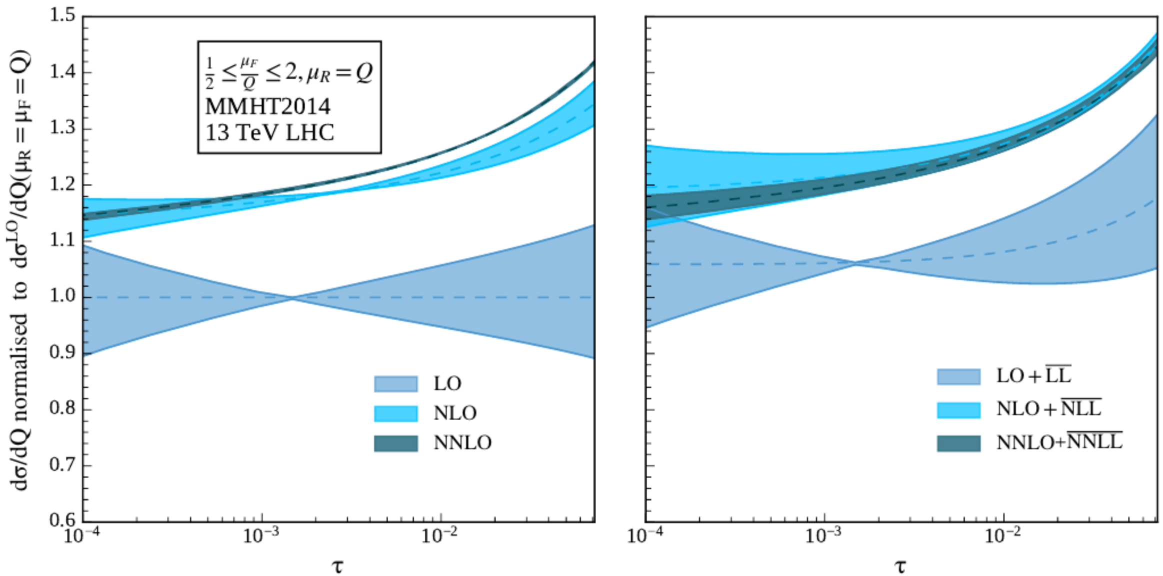
Q	LO	$\mathrm{LO}+\overline{\mathrm{LL}}$	NLO	$\mathrm{NLO}+\overline{\mathrm{NLL}}$	NNLO	$\mathrm{NNLO}+\overline{\mathrm{NNLL}}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

cross section in 10^−5 pb/GeV

7-point var: $\mu = \{\mu F , \mu R\}$ is varied in the range [1/2Q, 2Q] keeping the ratio $\mu R / \mu F$ not larger than 2 and smaller than 1/2.

Phenomenology - Drell-Yan

Uncertainities w.r.t μ_F scale variation



F.O – All the channels
dependence on μF from PDFs + $q\bar{q}$ + qg ->
compensation due to RGE w.r.t μF
[μF var mixes different channels]

RES – Only $q\bar{q}$ channel
qg ✖
Not much compensation under μF scale
Variation,

- resummed bands look similar to that of 7-point bands --width of the 7-point bands mainly comes from the μF uncertainties
- NLO band gets improved with the inclusion of $\overline{\text{NLL}}$, but NNLO band increses with the inclusion of $\overline{\text{NNLL}}$

NLO : $q\bar{q}$ -> 22 %
qg -> -5 %

$q\bar{q}$ dominating

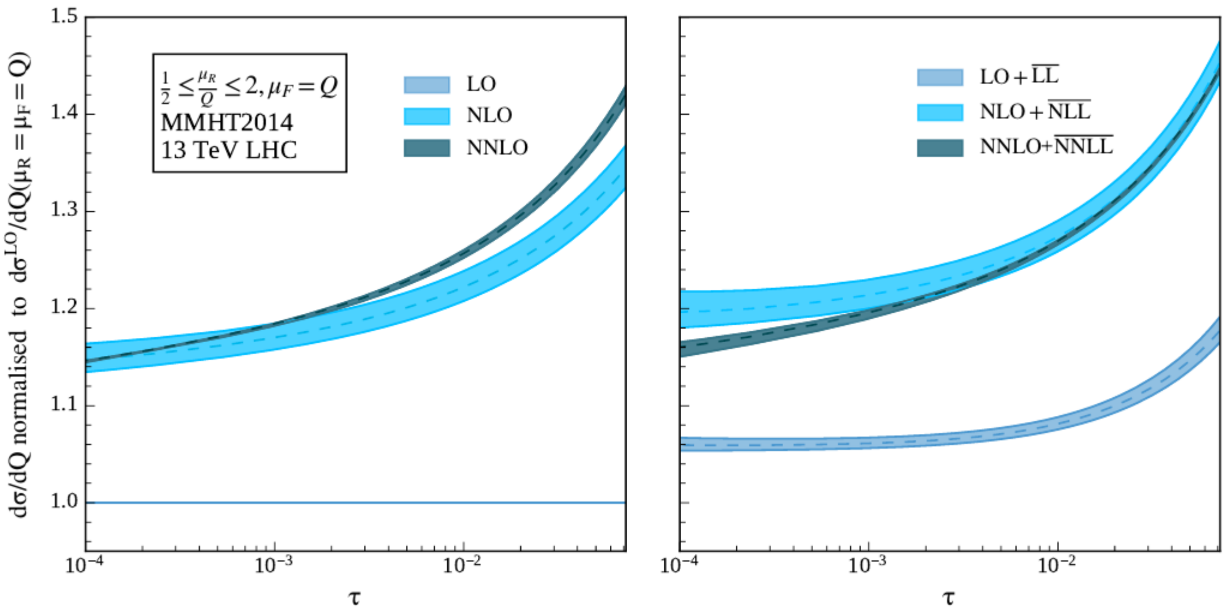
NNLO : $q\bar{q}$ -> 4.9 %
qg -> -2.8 %

Bigger cancellation b/w $q\bar{q}$ & qg

Missing qg-channel resummed contribution leads to more uncertainty at NNLO+$\overline{\text{NNLL}}$

Phenomenology - Drell-Yan

Uncertainities w.r.t μ_R scale variation



- NNLO+$\overline{\text{NNLL}}$ the error band becomes substantially thinner
- each partonic channel is invariant under μ_R variation and hence inclusion of more corrections within a channel is expected to reduce the uncertainity

Inclusion of resummed result reduces the μ_R uncertainty remarkably as compared to the fixed order ones

Summary & Outlook

- Studied Next-to Soft virtual (NSV) terms in Inclusive reactions
- Set up a formalism to compute NSV terms using factorisation & RG Invariance
- The Inclusive cross-sections takes an exponential form which allows for all order predictions for certain SV & NSV logarithms
- Integral representation in z-space leads to a framework to resum the NSV logarithms in N-space ($\overline{\text{NNLL}}$ accuracy)

Summary & Outlook

- The inclusion of resummed NSV terms improves perturbative convergence and reduces the uncertainty from the choice of renormalisation scale.

- The absence of quark gluon initiated contributions to NSV part in the resummed terms leaves large factorisation scale dependence indicating their importance at NSV level for DY.

Summary & Outlook

What more to do ?

Modify the existing formalism for off-Diagonal Channels.

THANK YOU

Additional Slides ...

Soft-Virtual (SV)

$$\Delta_{ab}^{SV,(i)}(z) = \delta_{a\bar{a}}\left(\Delta_{\bar{a}b,\delta}\ \delta(1-z) + \sum_{k=0}^{2i-1}\Delta_{\bar{a}b,\mathcal{D}_k}^{(i)}\mathcal{D}_k(z)\right)$$

$$\mathcal{D}_k = \left(\frac{\log^k(1-z)}{(1-z)}\right)_+$$

Plus distribution

Regular part

$$\Delta_{ab}^{reg,(i)}(z) = \sum_{k=0}^{2i-1}\sum_{l=0}^{\infty}\Delta_{ab,l,k}^{reg,(i)}(1-z)^l\log^k(1-z)$$

Next-to Soft-Virtual (NSV)

$$\Delta_{ab}^{NSV,(i)}(z) = \sum_{k=0}^{2i-1}\Delta_{ab,0,k}^{reg,(i)}\log^k(1-z)$$

Form Factor – K+G Eqn

IR singularities factorise

[Sen,sterman,Magnea]

[Moch,Vogt,Vermasern; Ravindran]

$$\hat{F}^c(Q^2,\mu^2,\epsilon) = Z_{IR}(Q^2,\mu^2,\mu_R^2,\epsilon)\hat{F}_c^{fin}(Q^2,\mu^2,\mu_R^2,\epsilon)$$

universal IR counter term contains poles

Finite part

Differentiating both sides with respect to Q^2, we obtain **K+G equation** for the FFs

$$Q^2\frac{d}{dQ^2}\log\hat{F}^c = \frac{1}{2}\left[K^c\left(\hat{a}_s,\frac{\mu_R^2}{\mu^2},\varepsilon\right) + G^c\left(\hat{a}_s,\frac{Q^2}{\mu_R^2},\frac{\mu_R^2}{\mu^2},\varepsilon\right)\right]$$

Poles

No Poles

RG Invariance

$$\mu_R^2\frac{d}{d\mu_R^2}K^c(a_s(\mu_R^2)) = -\mu_R^2\frac{d}{d\mu_R^2}G^c(a_s(\mu_R^2)) = -\overline{A}^c(a_s(\mu_R^2))$$

$$A_q = \frac{C_F}{C_A}A_g$$

Maximally non-abelian, verified up to 4 loops

Factorisation – Diagonal channel

For Drell-Yan process:

Diagonal Channel:
$$\frac{\hat{\sigma}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \frac{\Delta_{qq}}{z} \otimes \Gamma_{q\bar{q}} + \Gamma^T_{qq} \otimes \frac{\Delta_{qg}}{z} \otimes \Gamma_{g\bar{q}} + \cdots$$

In the threshold limit z -> 1 , keeping only $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+ \quad \delta(1-z_i)$ SV

$\log^k(1-z_i), \quad k = 0, \cdots \infty$ next to SV

dropping $(1-z_i)^k, \quad k = 1, \cdots \infty$

$$\boxed{\frac{\hat{\sigma}^{\rm sv+nsv}_{q\bar{q}}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\rm sv+nsv}_{q\bar{q}} \otimes \Gamma_{\bar{q}\bar{q}} \,.}$$

Remarkably Simple form !

Factorisation – off-diagonal channel

Off-diagonal Channel: $$\frac{\hat{\sigma}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta_{qq} \otimes \Gamma_{qg} + \Gamma^T_{qq} \otimes \Delta_{qg} \otimes \Gamma_{gg} + \cdots$$

In the threshold limit z -> 1 , keeping only $\log^k(1 - z_i), \quad k = 0, \cdots \infty$ next to SV

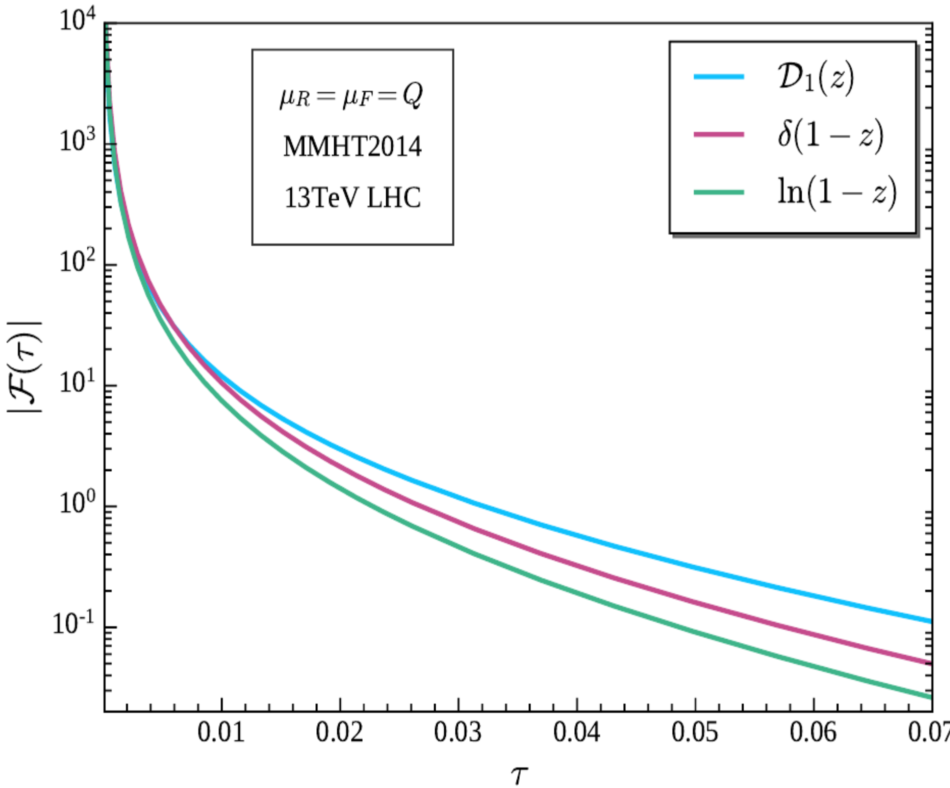
$$\boxed{\frac{\hat{\sigma}^{\mathrm{sv+nsv}}_{qg}}{z\sigma_0} = \Gamma^T_{qq} \otimes \Delta^{\mathrm{sv+nsv}}_{q\bar{q}} \otimes \Gamma_{\bar{q}g} + \Gamma^T_{qq} \otimes \Delta^{\mathrm{nsv}}_{qg} \otimes \Gamma_{gg}\,.}$$

NNSV terms

dropping $(1 - z_i)^k, \quad k = 1, \cdots \infty$

Getting complicated due to Mixing of channels

NSV contributions



$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\mathcal{D}_3$	6.13%	$\ln^3(1-z)$	12.4%
	$\mathcal{D}_2$	1.49%	$\ln^2(1-z)$	7.83%
	$\mathcal{D}_1$	-3.24%	$\ln^1(1-z)$	-2.82%
	$\mathcal{D}_0$	-4.74%	$\ln^0(1-z)$	-6.57%
	$\delta(1-z)$	0.003%		
TOTAL		-0.035%		10.8%

$Q = \mu_R = \mu_F$ (GeV)	SV		NSV	
200	$\mathcal{D}_5$	5.44%	$\ln^5(1-z)$	8.60%
	$\mathcal{D}_4$	2.62%	$\ln^4(1-z)$	9.82%
	$\mathcal{D}_3$	-2.73%	$\ln^3(1-z)$	-1.54%
	$\mathcal{D}_2$	-4.25%	$\ln^2(1-z)$	-8.98%
	$\mathcal{D}_1$	-1.94%	$\ln^1(1-z)$	-6.14%
	$\mathcal{D}_0$	-0.146%	$\ln^0(1-z)$	-1.28%
	$\delta(1-z)$	1.03%		
TOTAL		0.026%		0.47%

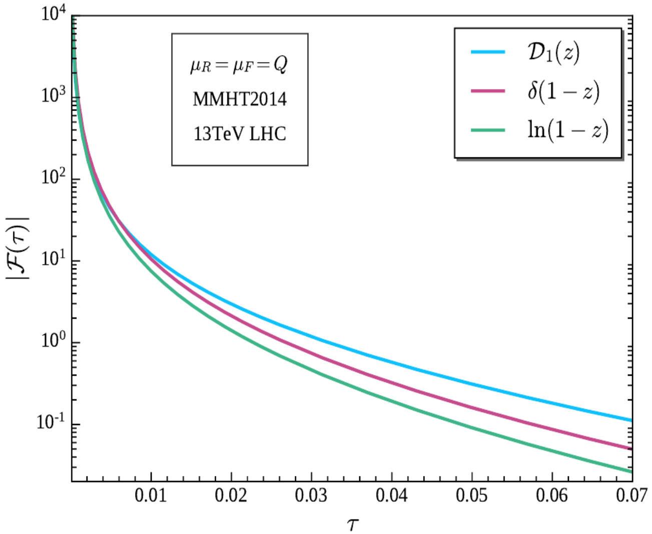
$$\mathcal{F}(\tau) = \int_\tau^1 \frac{dz}{z} \tilde{\Phi}_{q\bar{q}}\left(\frac{\tau}{z}\right) \mathcal{G}(z)$$

$$\mathcal{G}(z) = \{\delta(1-z), \mathcal{D}_1(z), \ln(1-z)\}$$

[AAH,PM,VR,AS,ST, hep-ph/2107.09717]

Size of NSV contributions - DY

[AAH,PM,VR,AS,ST, hep-ph/2107.09717]



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TOTAL		0.026%		0.47%

a_s^2 -- SV : -0.035 %

NSV : 10.8 %

a_s^3-- SV : 0.026 %

NSV : 0.47 %

- NSV gives significant contribution becuase of the large coeffts. Hence, including higher terms in the threshold expansion is essential

The Matched Result

Now we perform Mellin Inversion of the resummed result to study the numerical impact.

$$\sigma_N^{\mathrm{N^nLO}+\overline{\mathrm{N^nLL}}} = \sigma_N^{\mathrm{N^nLO}} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2) \times \left(\Delta_{q,N}\Big|_{\overline{\mathrm{N^nLL}}} - \Delta_{q,N}\Big|_{tr\ \mathrm{N^nLO}} \right) .$$

The resummed results are matched to the fixed order result in order to avoid any double counting of threshold logarithms

The contour c in the Mellin inversion is chosen according to Minimal prescription

Used for phenomenological studies

Resummed exponents & Logarithmic accuracy

Logarithmic Accuracy	Resummed Exponents
$\overline{\mathrm{LL}}$	$\tilde{g}^q_{0,0}, g^q_1, \overline{g}^q_1, h^q_0$
$\overline{\mathrm{NLL}}$	$\tilde{g}^q_{0,1}, g^q_2, \overline{g}^q_2, h^q_1$
$\overline{\mathrm{NNLL}}$	$\tilde{g}^q_{0,2}, g^q_3, \overline{g}^q_3, h^q_2$

Nomenclature :
$\overline{\mathrm{N^nLL}}$ -> NSV included resummed results at $\overline{\mathrm{N^nLL}}$ accuracy

$a_s \frac{1}{N} \log N$

$a_s^2 \frac{1}{N} \log^3 N$

$a_s^3 \frac{1}{N} \log^5 N$

⋮

$a_s^i \frac{1}{N} \log^{2i-1} N$

Only 1-loop info

$a_s^2 \frac{1}{N} \log^2 N$

$a_s^3 \frac{1}{N} \log^4 N$

$a_s^4 \frac{1}{N} \log^6 N$

⋮

$a_s^i \frac{1}{N} \log^{2i-2} N$

Only 2-loop info

. . .

$a_s^n \frac{1}{N} \log^n N$

⋮

$a_s^i \frac{1}{N} \log^{2i-n} N$

Only n-loop info

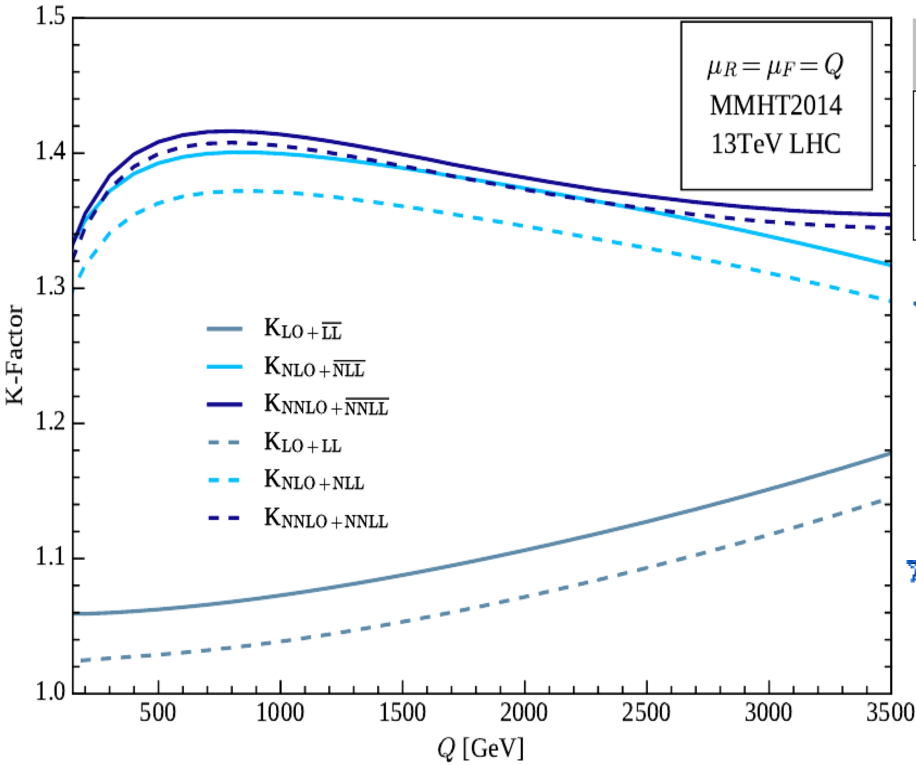
Tower of NSV logarithms that we sum over

Checked till $\overline{\mathrm{LL}}$ accuracy
[Beneke et.al]
[Laenen et.al]

Phenomenology - Drell-Yan

SV resummation vs SV+NSV resummation

K- Factor



$Q = \mu_R = \mu_F$	NLO+NLL	NLO+$\overline{\text{NLL}}$	NNLO+NNLL	NNLO+$\overline{\text{NNLL}}$
1000	1.3711	1.3995	1.4053	1.4138
2000	1.3459	1.3739	1.3729	1.3818

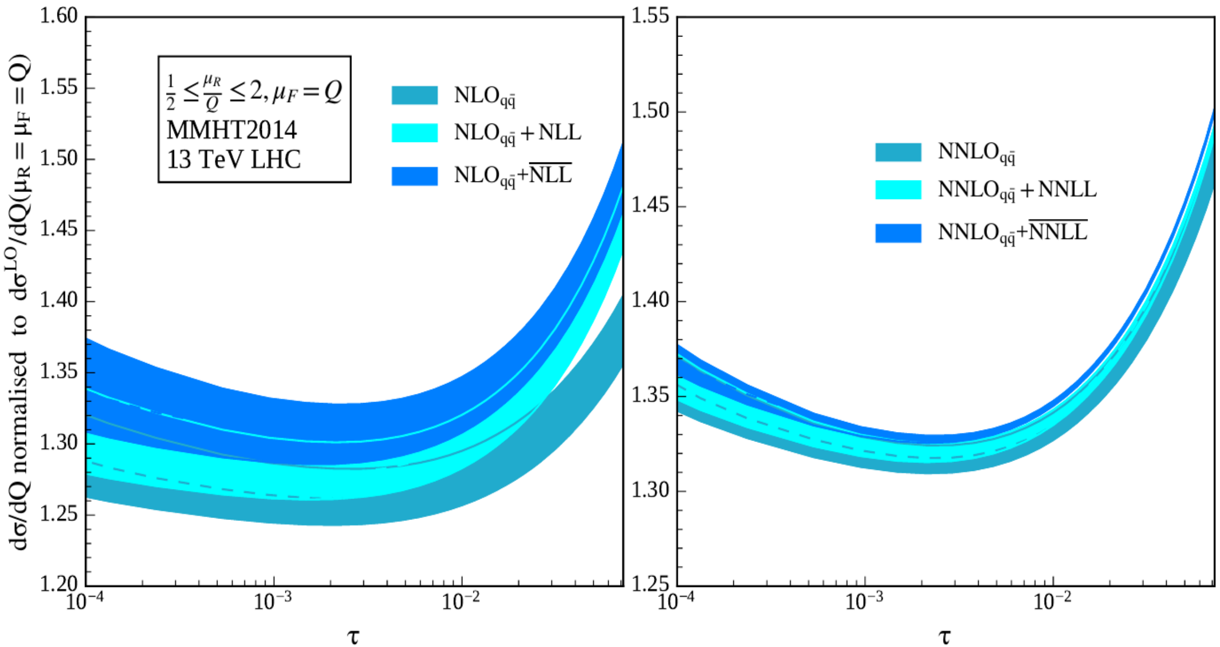
- considerable amount of increment of 2.08% when we go from NLL to $\overline{\text{NLL}}$ & 0.64% from NNLL to $\overline{\text{NNLL}}$ at Q = 2 TeV

- SV+NSV resummed results at NLO + $\overline{\text{NLL}}$ and NNLO + $\overline{\text{NNLL}}$ are closer compared to the SV counter parts -> perturbative convergence due to NSV effects

Phenomenology - Drell-Yan

- **SV resummation vs SV+NSV resummation**

  μ_R scale variation within $q\bar{q}$- channel



Behaviour of $\mathrm{NNLO}_{q\bar{q}} + \overline{\mathrm{NNLL}}$ is significantly improved from the corresponding SV $\mathrm{NNLO}_{q\bar{q}} + \mathrm{NNLL}$

Considerable improvement when adding the NSV resummation over the SV one leading to more reliable predictions

$Q = \mu_R = \mu_F$	$\mathrm{NNLO}_{q\bar{q}}$	$\mathrm{NNLO}_{q\bar{q}} + \mathrm{NNLL}$	$\mathrm{NNLO}_{q\bar{q}} + \overline{\mathrm{NNLL}}$
1000	$3.5260^{+0.49\%}_{-0.58\%}$	$3.5376^{+0.25\%}_{-0.39\%}$	$3.5576^{+0.006\%}_{-0.20\%}$
2000	$0.0717^{+0.54\%}_{-0.62\%}$	$0.0721^{+0.19\%}_{-0.33\%}$	$0.0725^{+0.0\%}_{-0.15\%}$

cross section in 10^−5 pb/GeV for $q\bar{q}$-channel

Phenomenology – Drell-Yan Process

$q\bar{q}$ & qg contributions under μ_F variation keeping μ_R fixed