

# Electroweak physics - Experimental overview

Oldrich Kepka  
Institute of Physics, Prague

LeptonPhoton, Manchester, 10. 1. 2022

On behalf of ATLAS, CDF, CMS, D0, LHCb  
Collaborations



# Outline

## Single V-boson production

- Drell-Yan production
- Weak mixing angle
- W-mass
- Branching ratios to leptons

## Diboson measurements

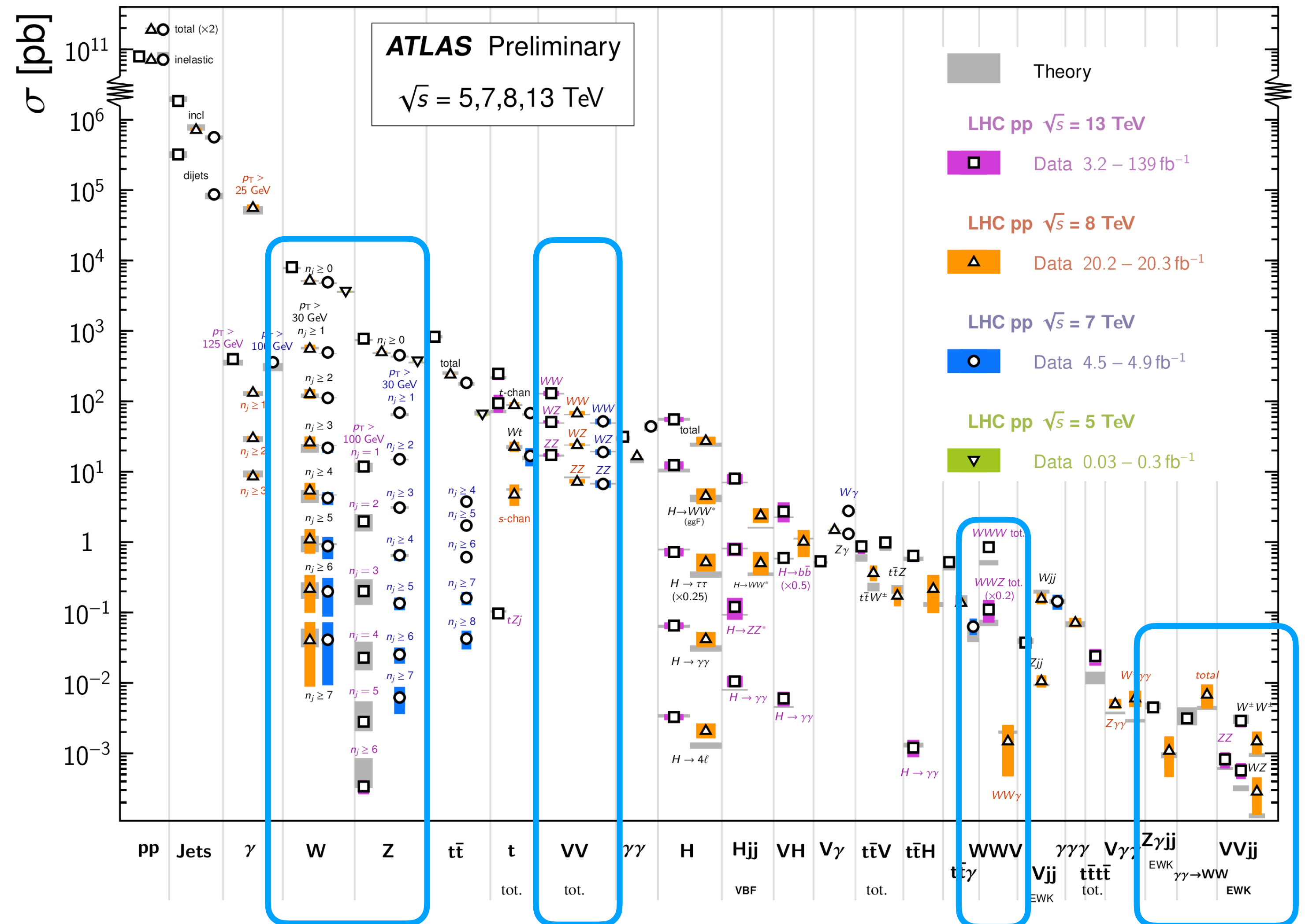
## Electroweak production of bosons

- VBF/VBS processes

## Triboson production

Standard Model Production Cross Section Measurements

Status: July 2021



## Drell-Yan process $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ is very important

- Many results from LHC experiments at different energies

## Determination of electron/muon/tau/jet ... performance

- Calibrating against  $m_Z$  precisely determined at LEP

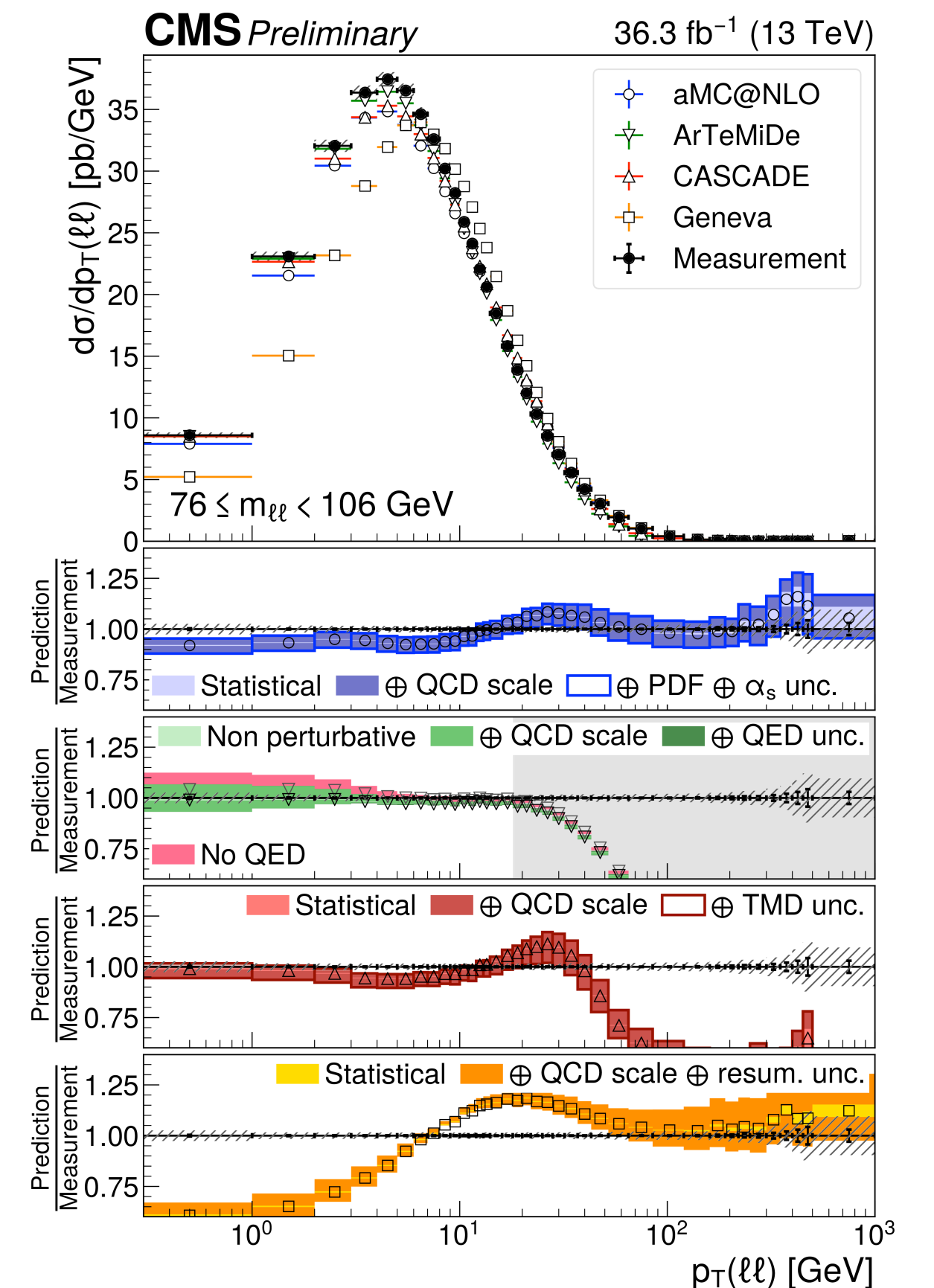
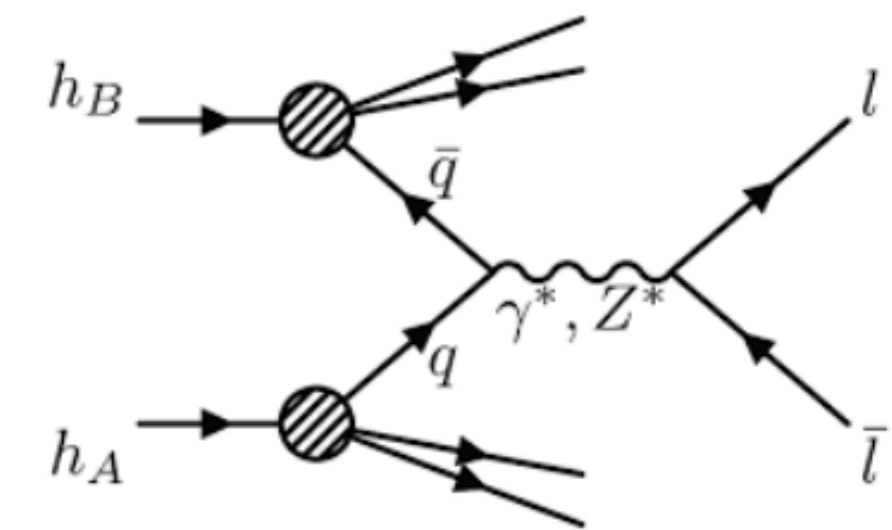
## Testing theoretical predictions

- State-of-the art NLO/NNLO QCD calculations and parton showers
- Soft-gluon resummation, PDF, extraction of EW parameters

## CMS measurement of transverse lepton-pair momentum

- On-shell as well as off-shell productions for  $m_{\ell\ell} > 50$  GeV
- Inclusive and  $n_{\text{jet}} \geq 1$ , ratios normalised to Z-peak
- sensitivity to soft QCD initial state radiation at low- $p_T$ , compared to TMD parton showers and hadronization predictions

Drell-Yan process

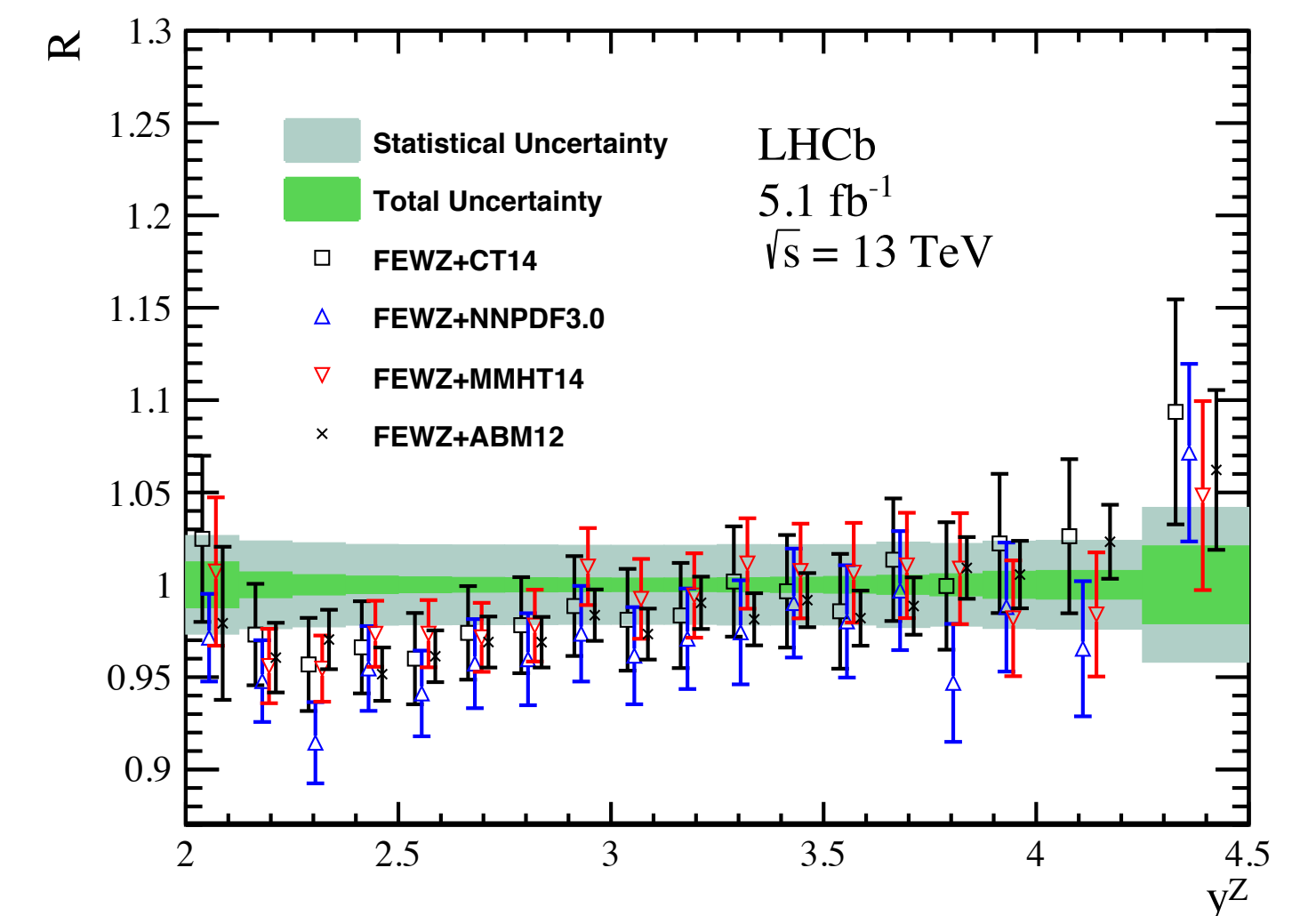
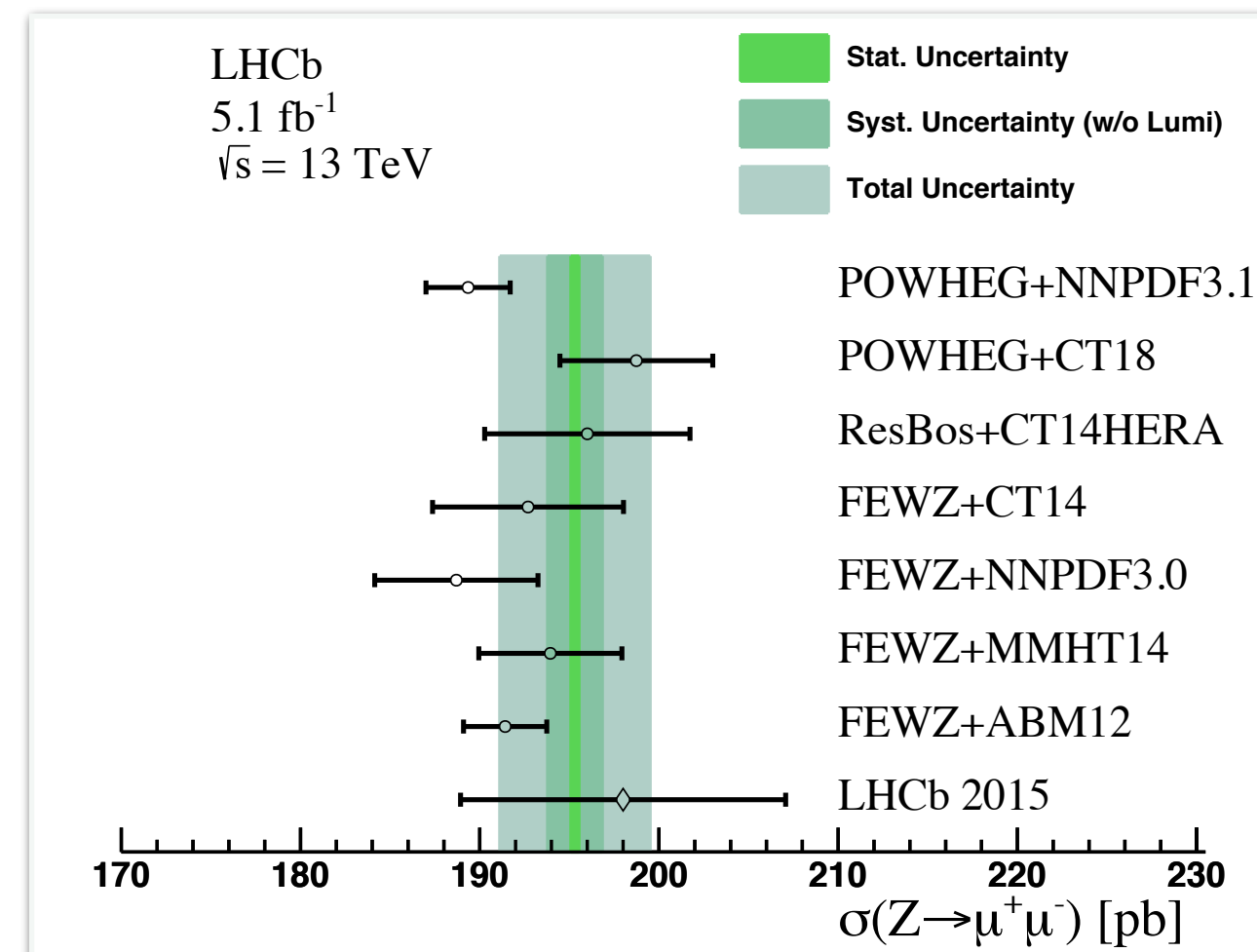
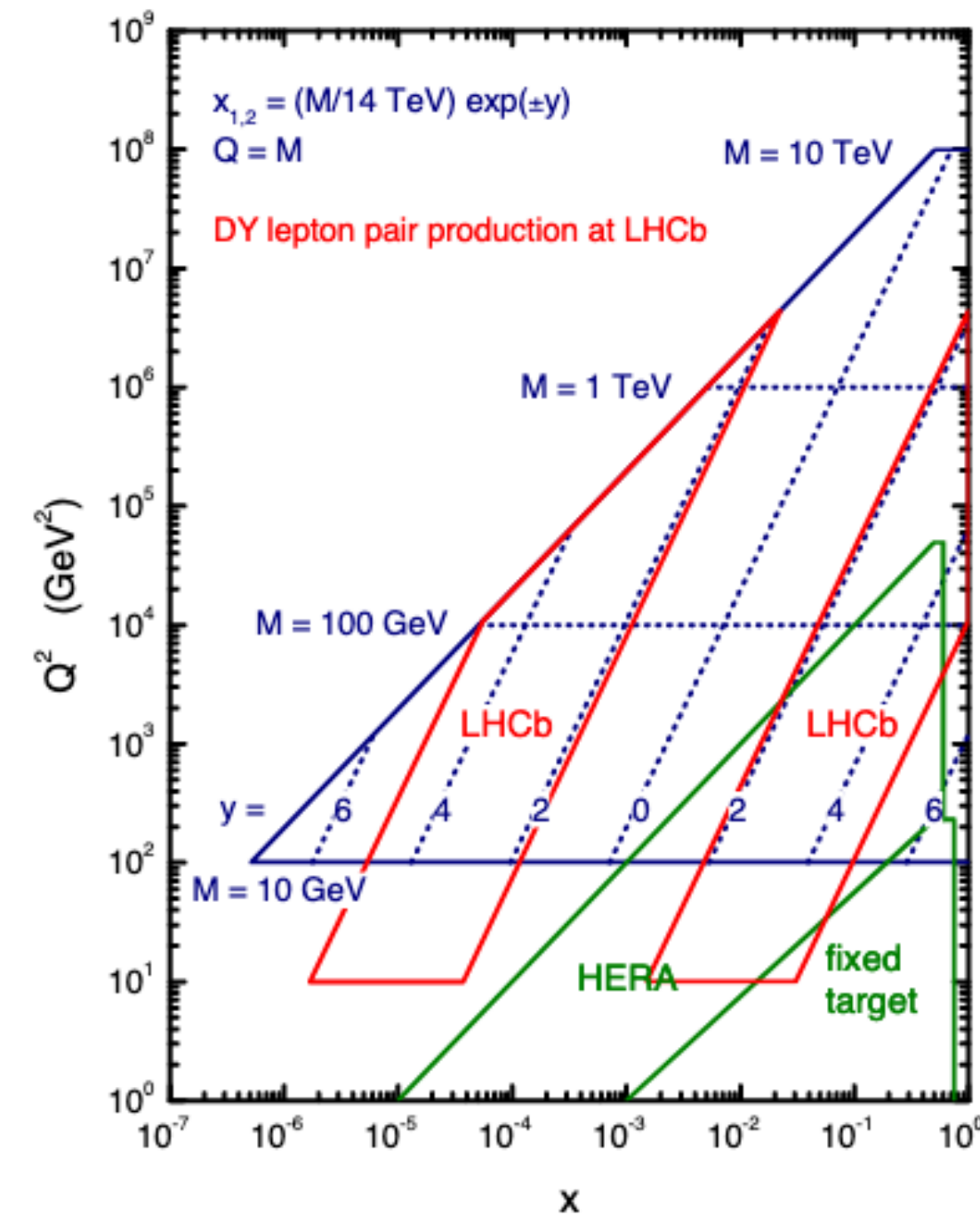
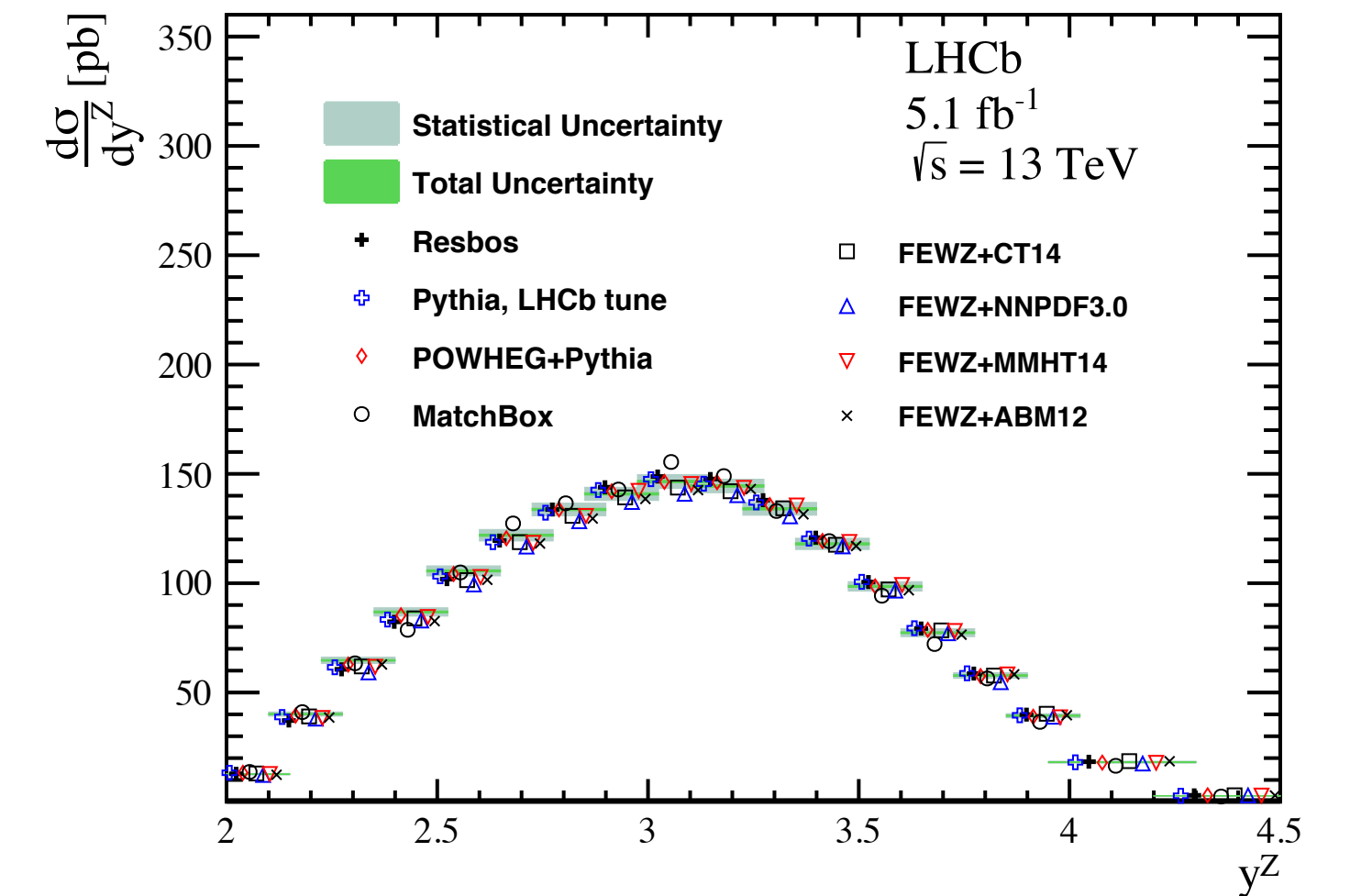


# Forward Z boson production

arXiv:2112.07458

## LHCb forward muons $2.0 < \eta < 4.5$

- Unique complementary phase-space to other LHC experiments
- Transverse momentum and  $\phi^*$
- Good modelling in general, but potential to constrain PDF at low and high- $x$



# FB asymmetry and effective weak mixing angle

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**Forward-backward asymmetry in  $q\bar{q} \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$  is sensitive to weak mixing angle**

- $\sin^2 \theta_{\text{eff}}^f$  determines relative strengths of vector and axial-vector couplings of Z to fermions,  $M_W^2 = M_Z^2 \cos^2 \theta_{\text{eff}}^f$
- Drell-Yan differential cross section,  $\theta^*$  - polar angle of  $\ell^-$  in Collins-Soper frame

$$\frac{d\sigma}{d \cos \theta^*} = A((1 + \cos^2 \theta^*) + A_4 \cos \theta^*)$$

- Experimentally measured  $A_{\text{FB}} \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8} A_4$

**Different approaches by ATLAS and CMS to extract  $\sin^2 \theta_{\text{eff}}^f$**

- ATLAS: from  $A_4$  coefficients through effective linear dependence
- CMS: fit of  $A_{\text{FB}}(m_{\text{II}}, y)$  using predictions obtained for different  $\sin^2 \theta_{\text{eff}}^f$

# Status of $\sin^2 \theta_{\text{eff}}^f$

ATLAS: [ATLAS-CONF-2018-037](#)

CMS: [EPJC 78 \(2018\) 701](#)

LHCb: [JHEP 1511\(2015\) 190](#)

Tev: [PRD 97 \(2018\) 112007](#)

## Current LHC measurements of $\sin^2 \theta_{\text{eff}}^f$ based only on Run 1 data

- Uncertainties dominated by PDFs and limited data sample
- ATLAS more precise than CMS thanks to additional category with one forward electron (only calorimeter measurement) which helps to constrain PDFs

### ATLAS (Run 1)

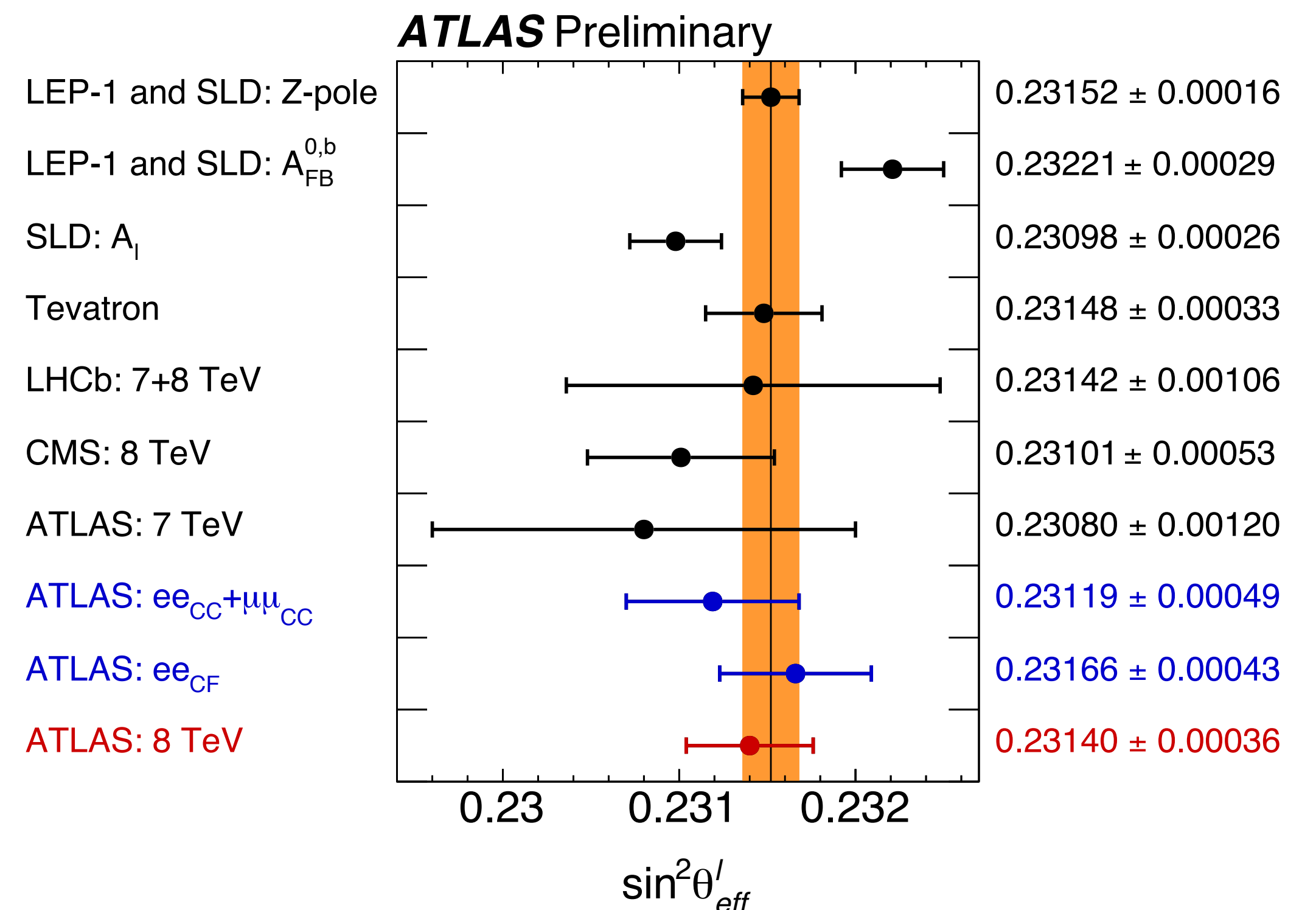
$$\sin^2 \theta_{\text{eff}}^f = 0.23140 \pm 0.00021 \text{ (stat)} \pm 0.00016 \text{ (syst)} \\ \pm 0.00024 \text{ (PDF)}$$

### CMS (Run 1)

$$\sin^2 \theta_{\text{eff}}^f = 0.23101 \pm 0.00036 \text{ (stat)} \pm 0.00018 \text{ (syst)} \\ \pm 0.00016 \text{ (theory)} \pm 0.00031 \text{ (PDF)}$$

**Tevatron** combination is the most precise result from hadron collider

**Expect improvement with Run 2 data and PDF using LHC Run 2 measurements**



# Measuring W mass at hadron colliders

## W boson mass is a fundamental parameter of SM

- Receives loop corrections (mainly higgs/top/BSM)

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_F} (1 + \Delta r)$$

Phys. Rev. D 42, 152

- Indirect global fit to EW parameters: 7 MeV uncertainty
- PDG average has precision of 12 MeV (twice larger than global fit)

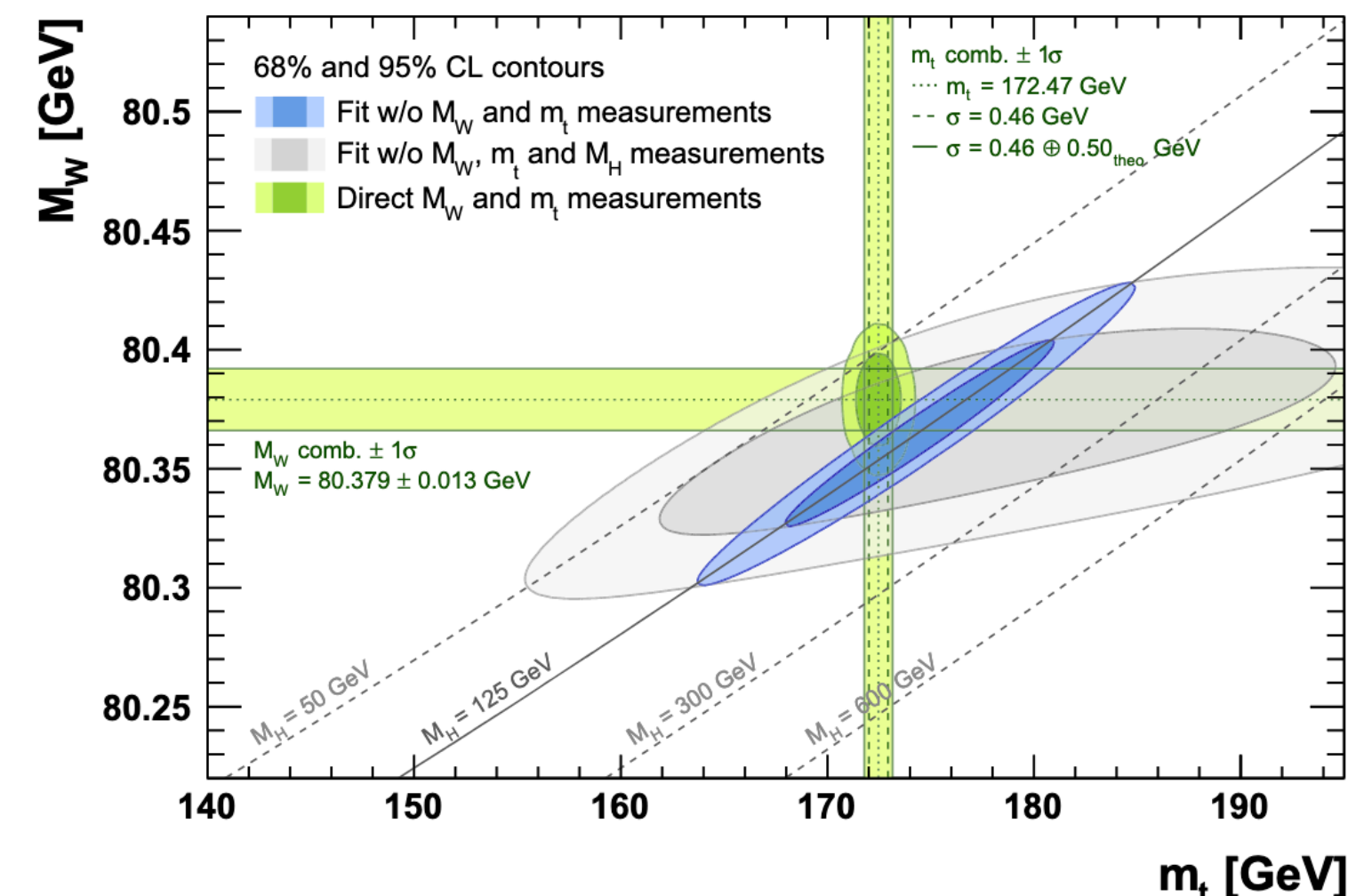
## Standard approach is to fit templates to $p_T$ and $m_T$ distributions in data (Jacobian peak $m_W/2$ or $m_W$ )

- $p_T(l)$  experimentally more precise, but sensitive to theoretical modeling: W polarisation (PDFs) and  $p_T(W)$  (QCD higher orders)

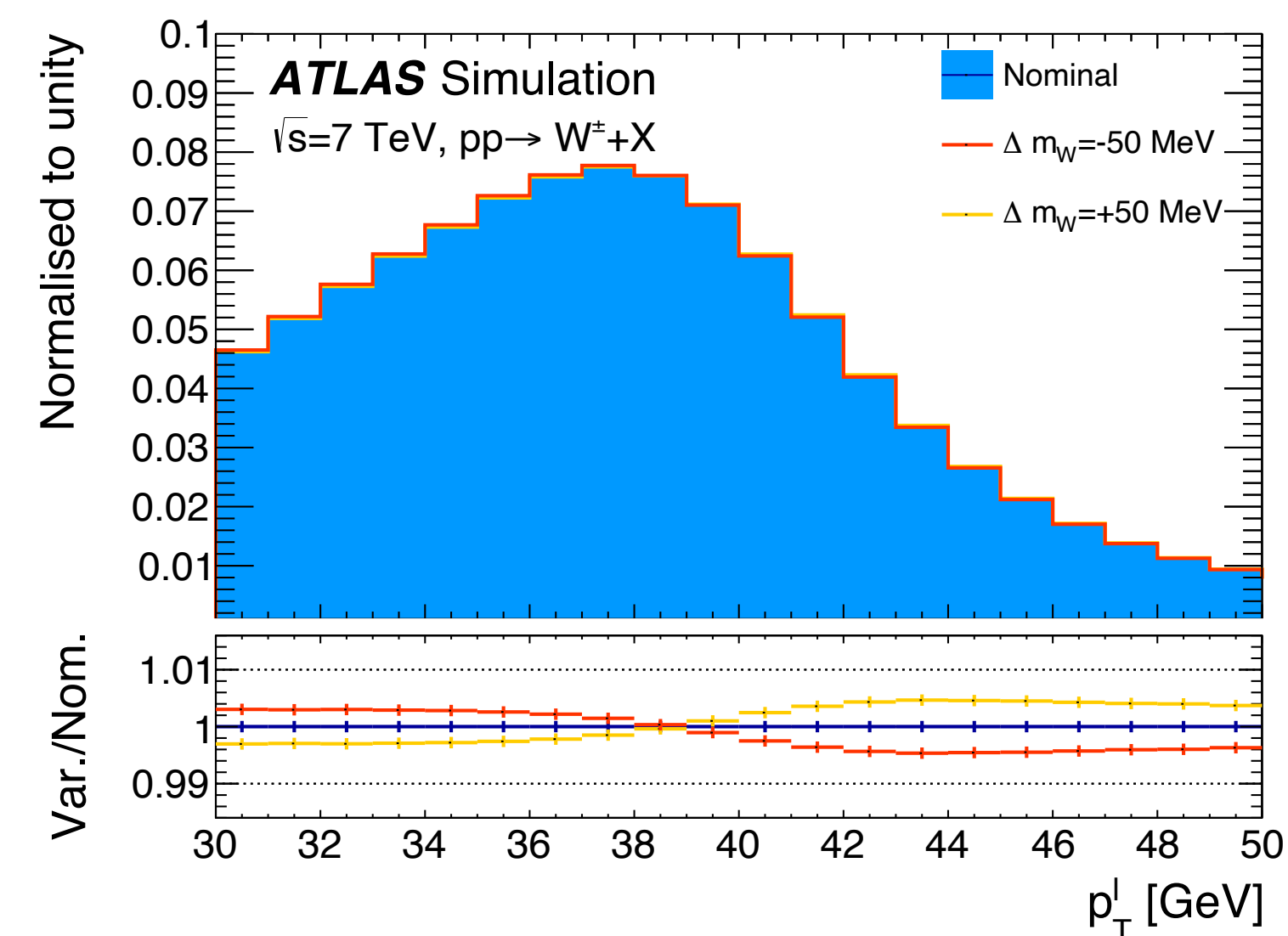
## First LHC measurement from ATLAS

- $\Delta m_W = 7_{\text{stat}} \pm 11_{\text{exp syst}} \pm 14_{\text{model}} = 19 \text{ MeV}$
- Dominated by theoretical uncertainties - 9 MeV (PDF), 8 MeV from  $p_T(W)$
- Similar precision as Tevatron

$$\left. \begin{array}{l} \text{CDF: } \Delta m_W = 12_{\text{stat}} \pm 10_{\text{exp syst}} \pm 12_{\text{model}} = 19 \text{ MeV} \\ \text{D0: } \Delta m_W = 13_{\text{stat}} \pm 18_{\text{exp syst}} \pm 13_{\text{model}} = 26 \text{ MeV} \end{array} \right\} \text{PRD 88 (2013) 052018}$$



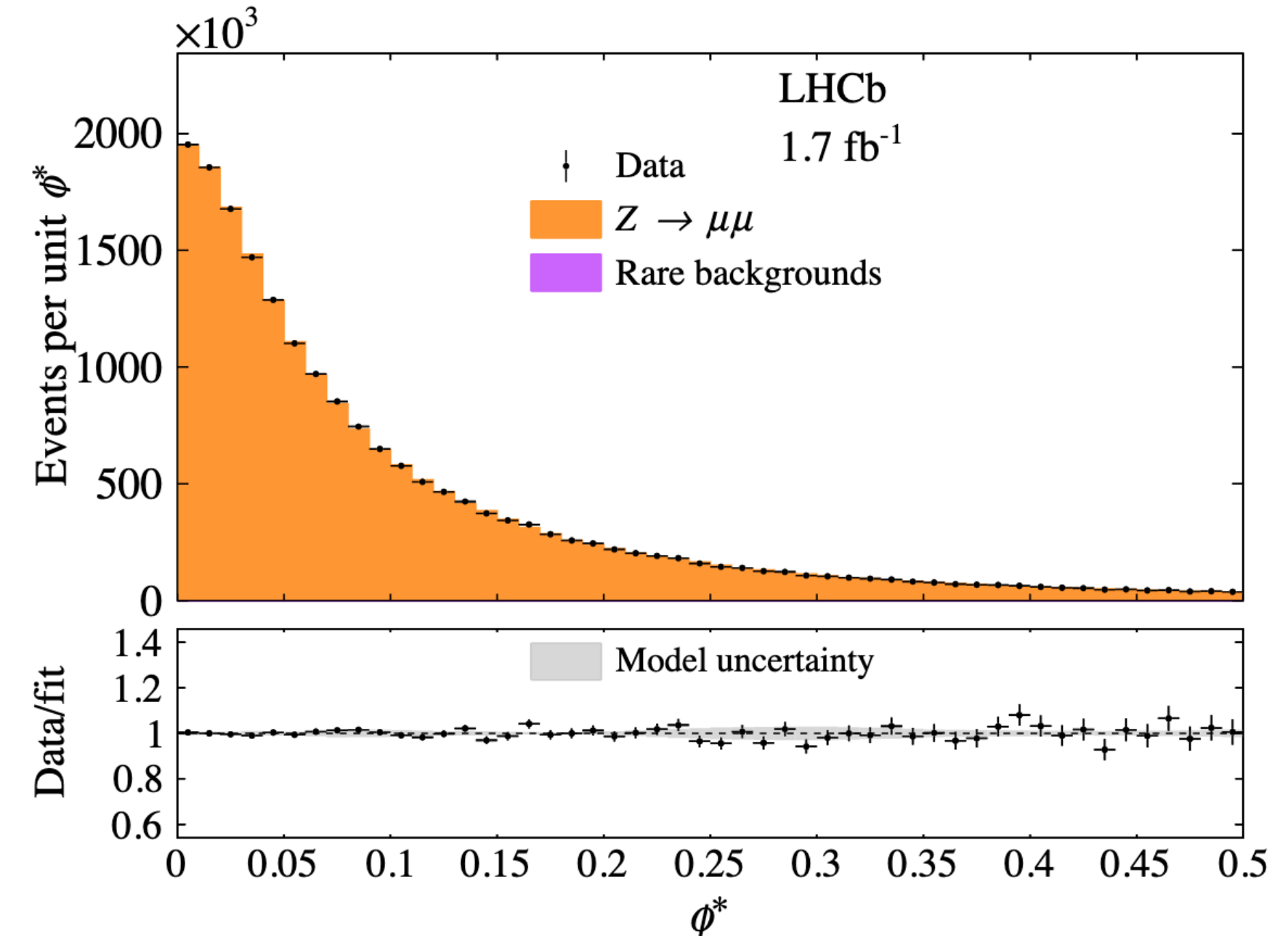
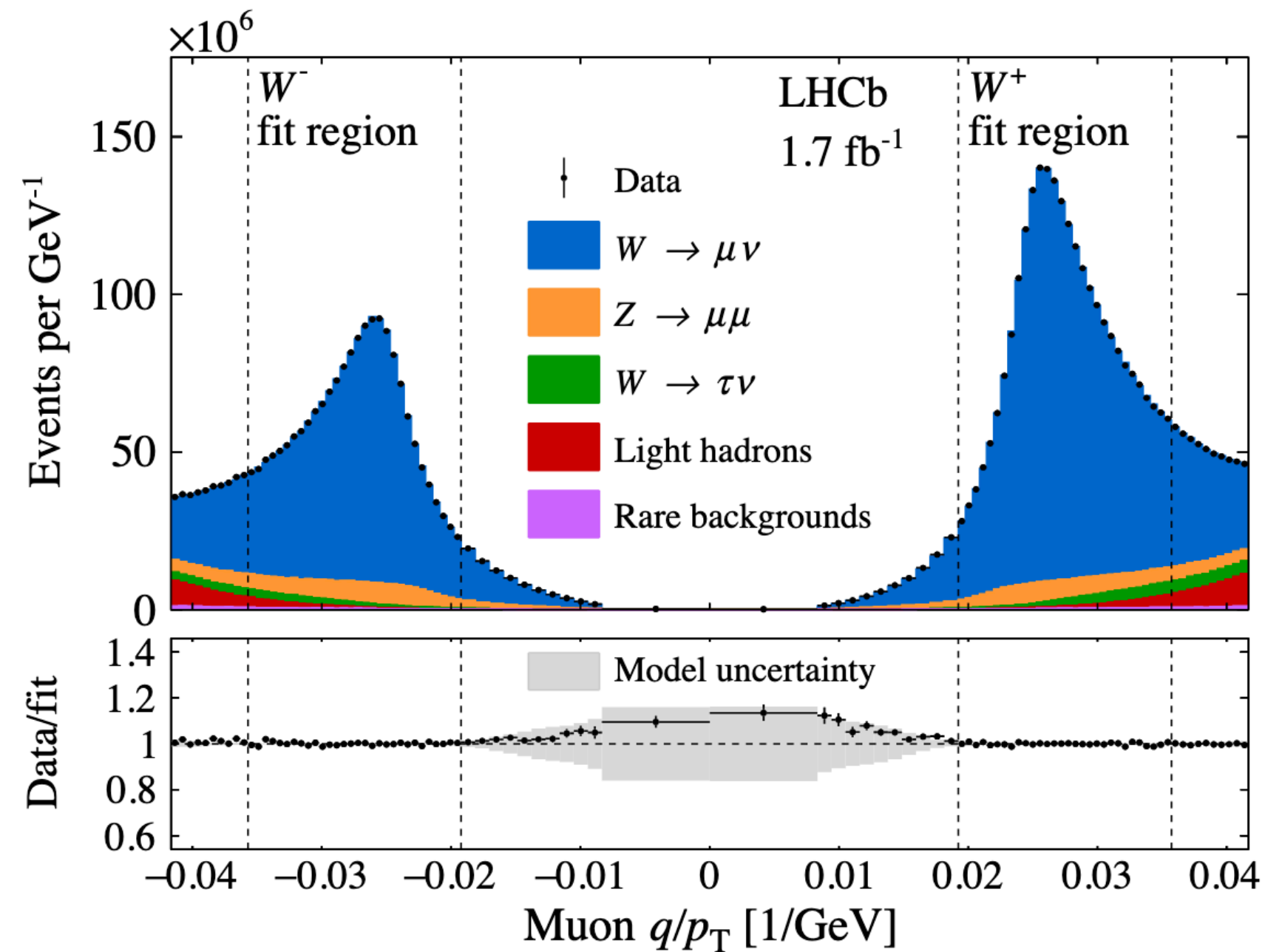
EPJC 78 (2018) 675



EPJC 78 (2018) 110

## First W mass measurement at LHCb

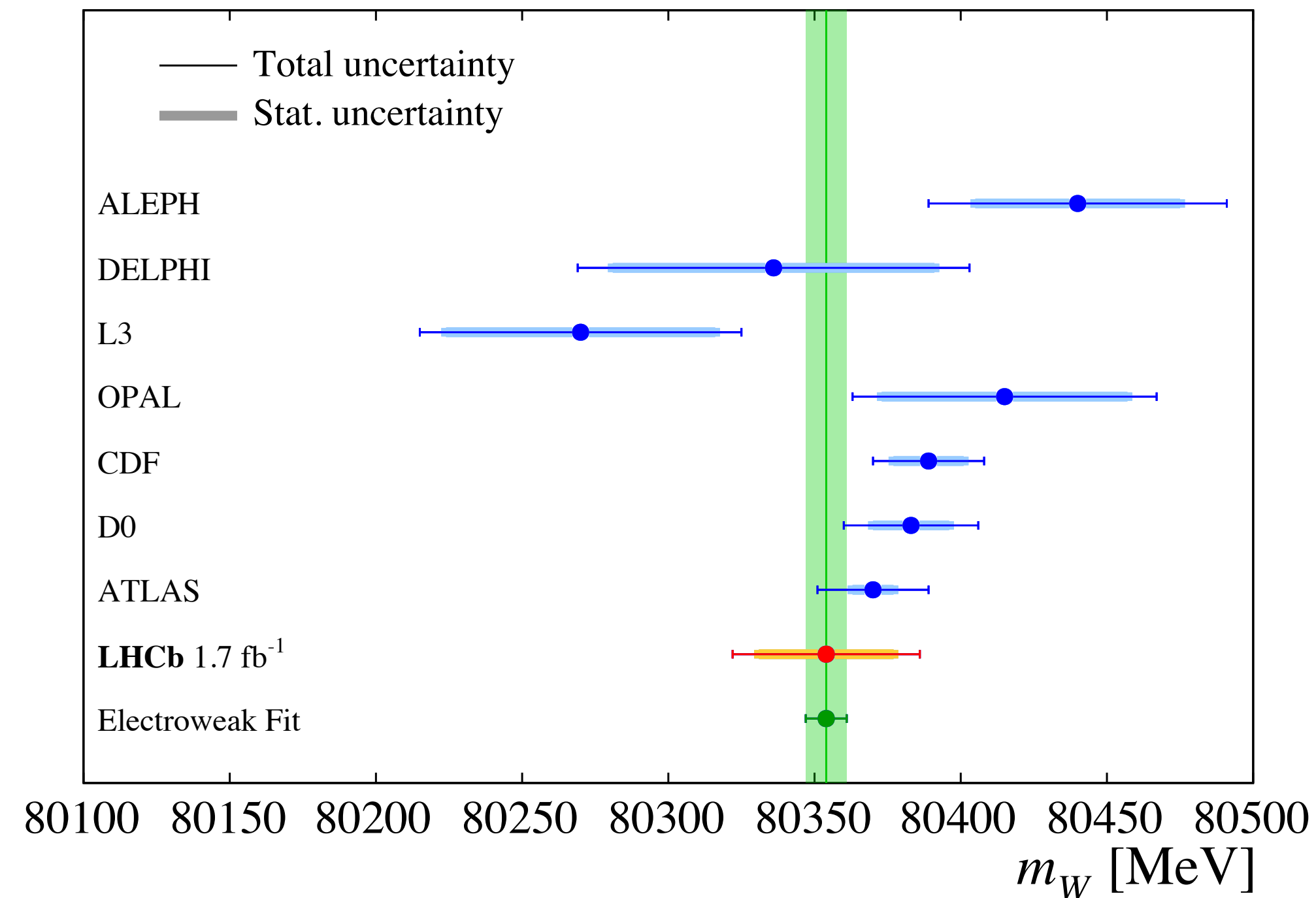
- $W \rightarrow \mu\nu$  events from  $1.7 \text{ fb}^{-1}$ , only 1/3 of full Run2
- Simultaneous fit of  $q/p_T$  and  $\phi^*$  of W and Z  
( $\phi^*$  proportional to Z  $p_T$ , but is angular measurement and so more precise)
- Constrain of uncertainties by in-situ  $Z \rightarrow \mu\mu$  measurement





$$m_W = 80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$

$\Delta m_W \approx 32 \text{ MeV}$



## Pivotal measurement

- Larger uncertainties than ATLAS/Tevatron
- Systematic uncertainties from PDF, theory and experiment
- Expected LHCb precision < 20 MeV with full Run 1 and Run 2 datasets
  - Will employ 2D fits of  $q/p_T$  and  $\eta$  to reduce PDF uncertainties

## Future combination

- Explores complementary region and can constrain PDF in future LHC+Tevatron combination

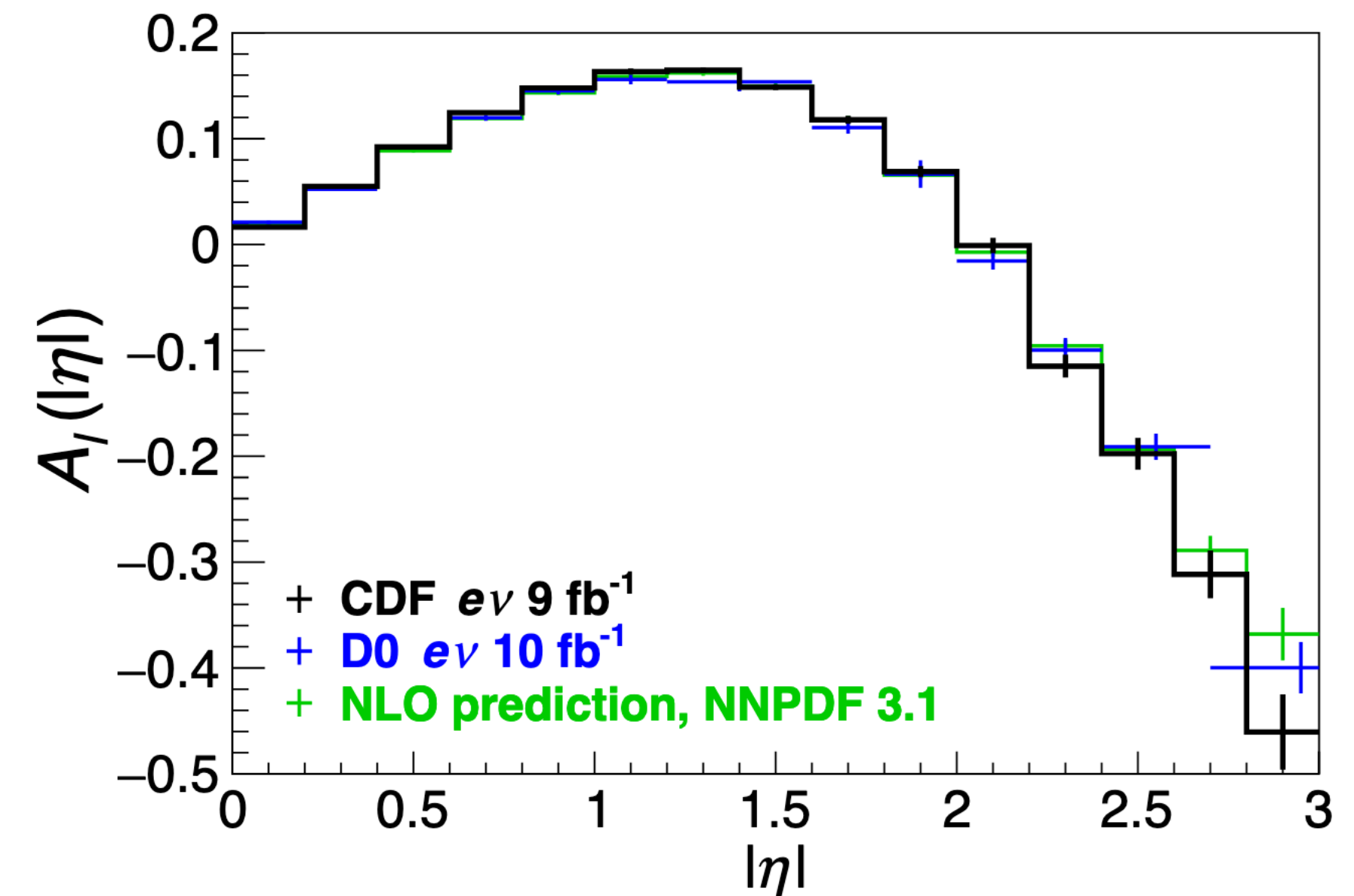
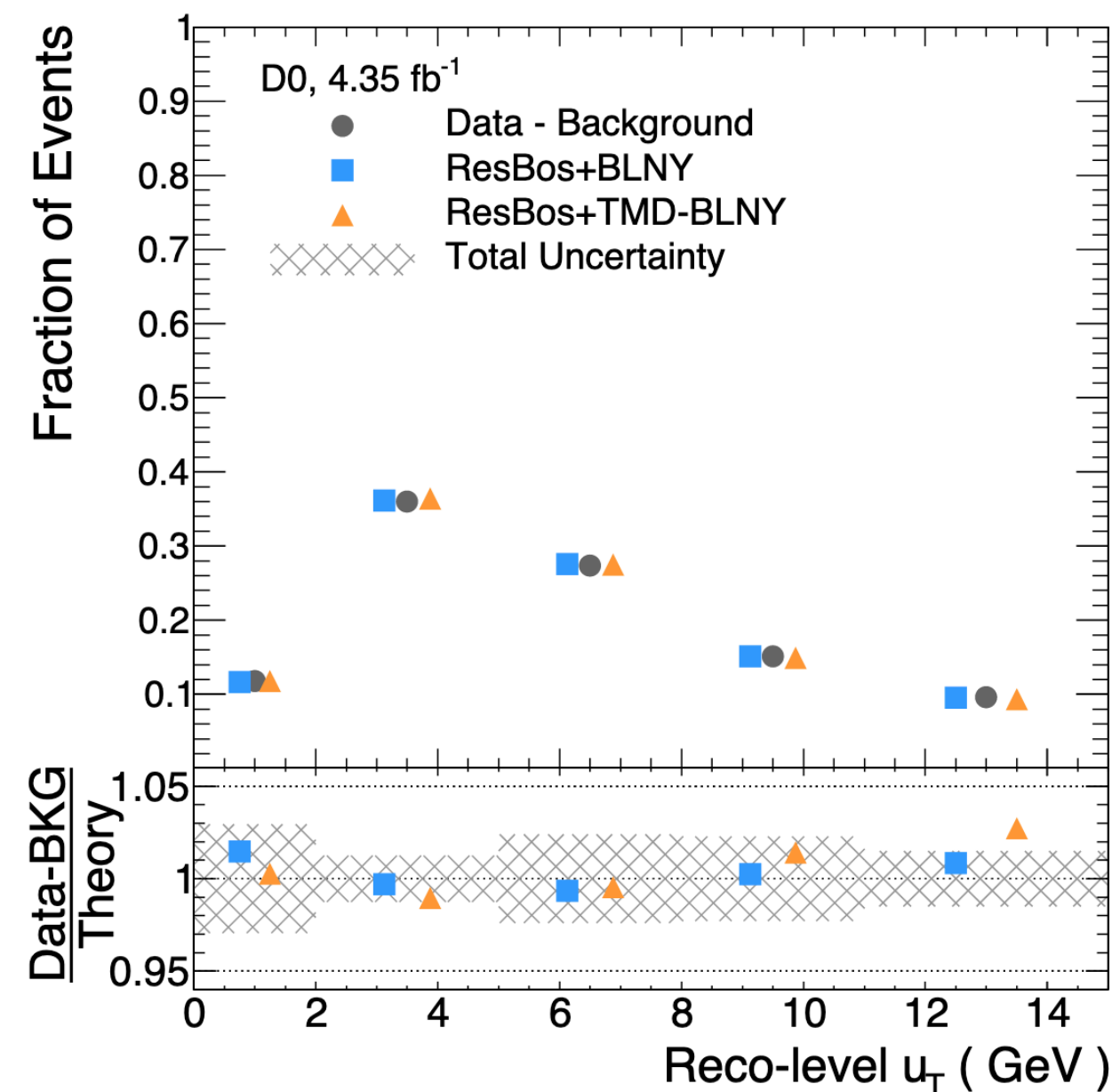
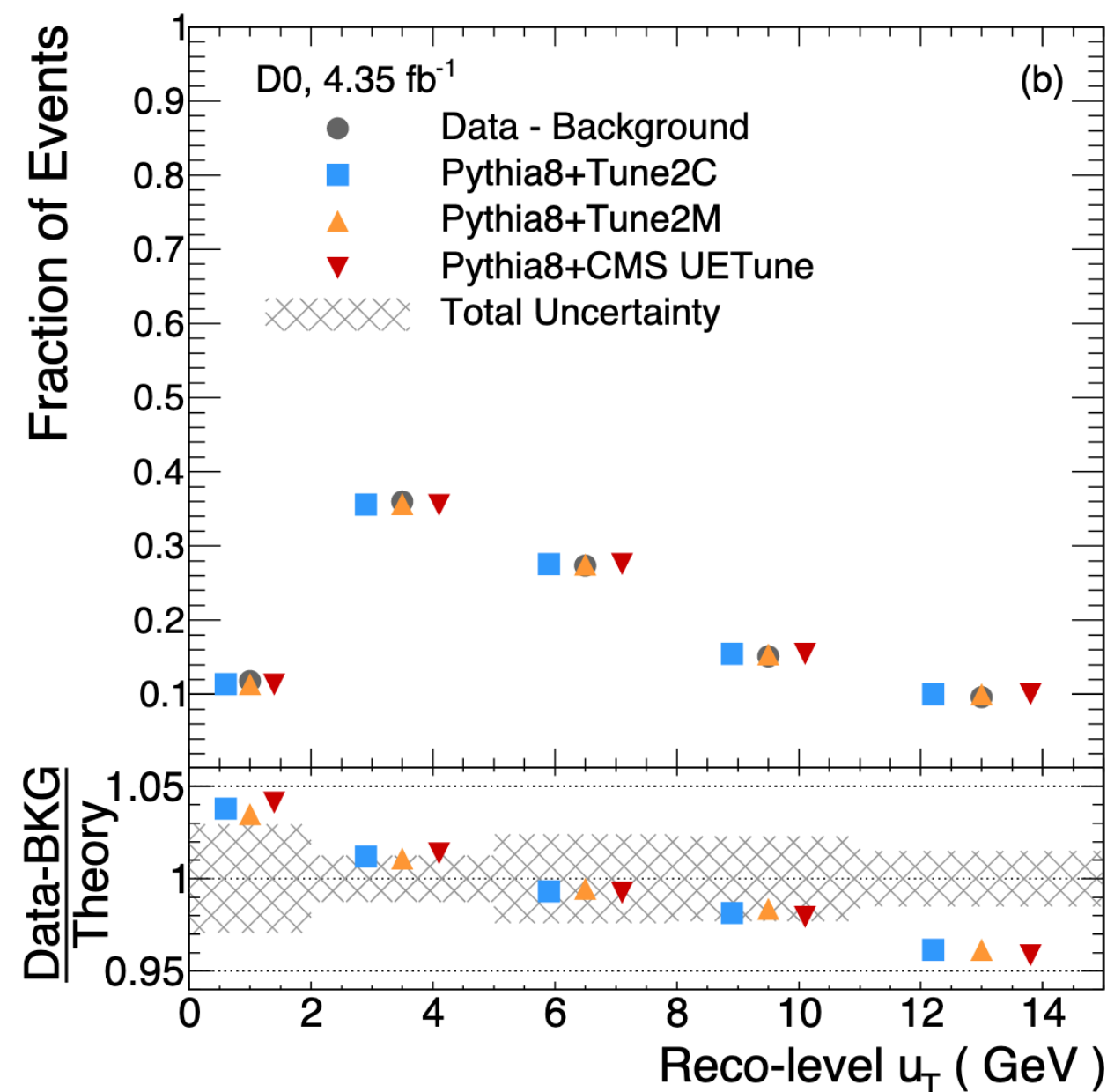
## D0: First inclusive $p_T(W)$ using Tevatron Run II data

- Using hadronic recoil
- Provides additional information for evaluating resummation calculation for W boson when production dominated by valence quarks
- LHC measurements involve also s-quark

## CDF: Charged-lepton asymmetry in W

- Full Tevatron dataset
- Sensitive to d/u valence quark ratio
- Improvements in global PDF expected

$$A_\ell = \frac{d\sigma_\ell^+ / d\eta - d\sigma_\ell^- / d\eta}{d\sigma_\ell^+ / d\eta + d\sigma_\ell^- / d\eta}$$



# Resolved tension in W decays: $R(\tau/\mu)$

Nature 17, 813 (2021)

All charged leptons ( $e, \mu, \tau$ ) couple to W boson with identical branching ratio

LEP reported  $2.6\sigma$  tension between  $\tau$  and  $e/\mu$

- BR of W to electrons and muons agree well

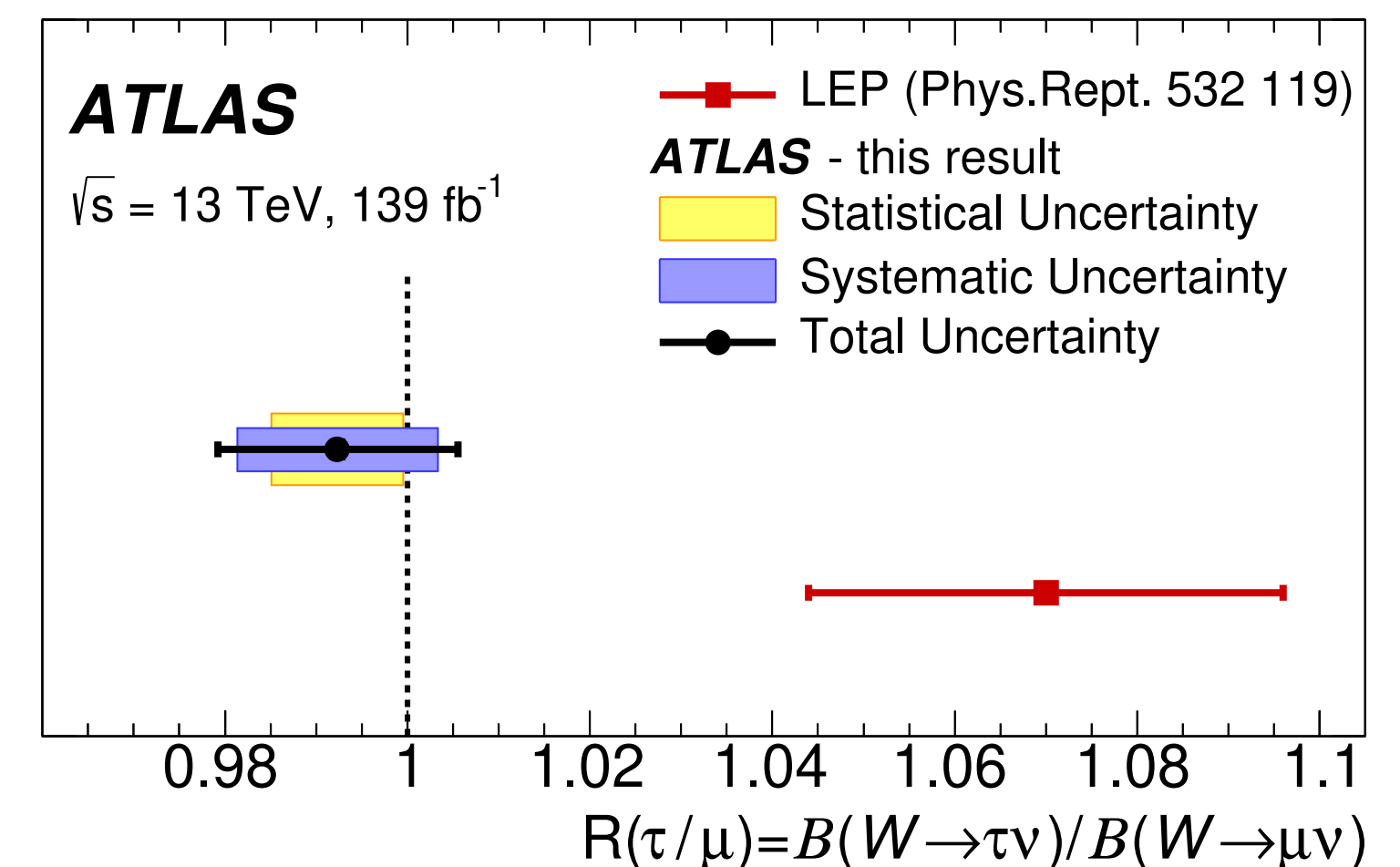
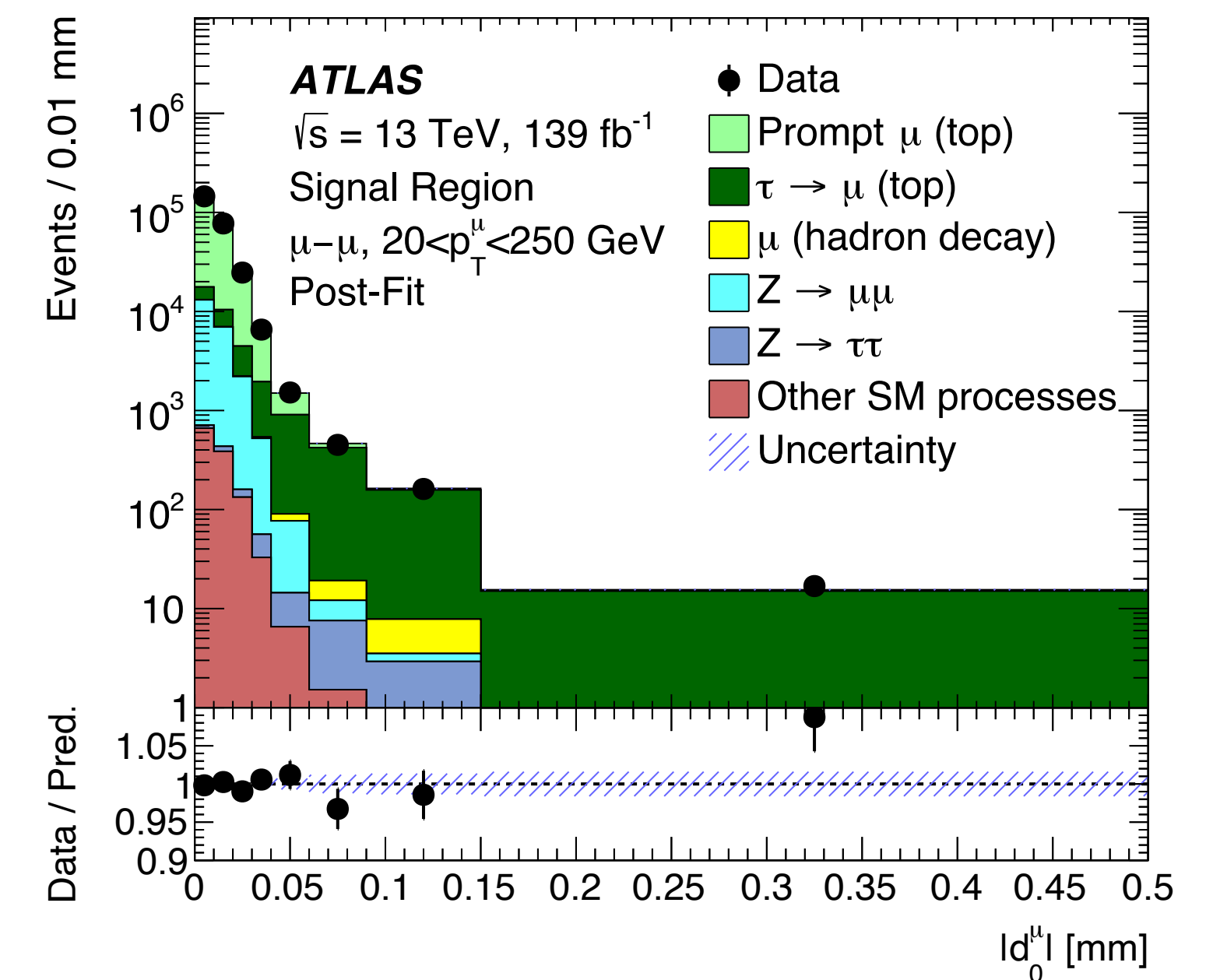
$$2\mathcal{B}(W \rightarrow \tau\bar{\nu}_\tau)/(\mathcal{B}(W \rightarrow e\bar{\nu}_e) + \mathcal{B}(W \rightarrow \mu\bar{\nu}_\mu)) = 1.066 \pm 0.025$$

ATLAS measures W decays in  $t\bar{t}$  events

- Muons classified to prompt W  $\rightarrow e/\mu$  decays,  $\tau$ -lepton decays and hadron decays fitted to muon transverse track impact parameter distribution  $d_0$  and muon  $p_T$

$$R(\tau/\mu) = 0.992 \pm 0.013 [\pm 0.007 \text{ (stat)} \pm 0.011 \text{ (syst)}]$$

- In agreement with SM, 2x more precise than LEP



# W decay branching fractions

CMS-PAS-SMP-18-011

## CMS uses $t\bar{t}$ , $tW$ , $WW$ , $W + \text{jets}$ events to study W decay BR

- Trailing lepton  $p_T$  used to discriminate between prompt  $W \rightarrow e/\mu$  decays and  $\tau$ -lepton in  $ee$ ,  $\mu\mu$ ,  $e\mu$

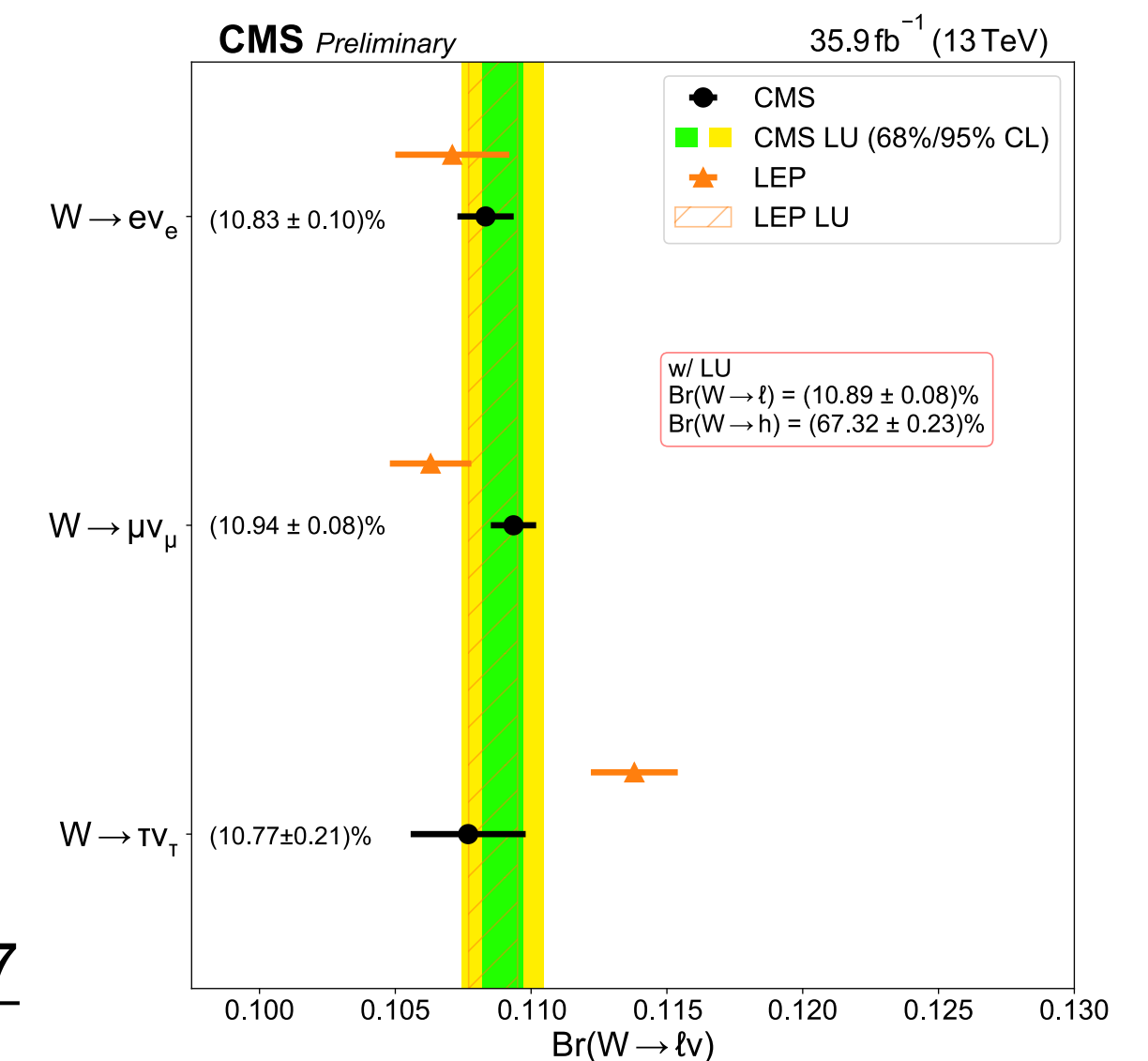
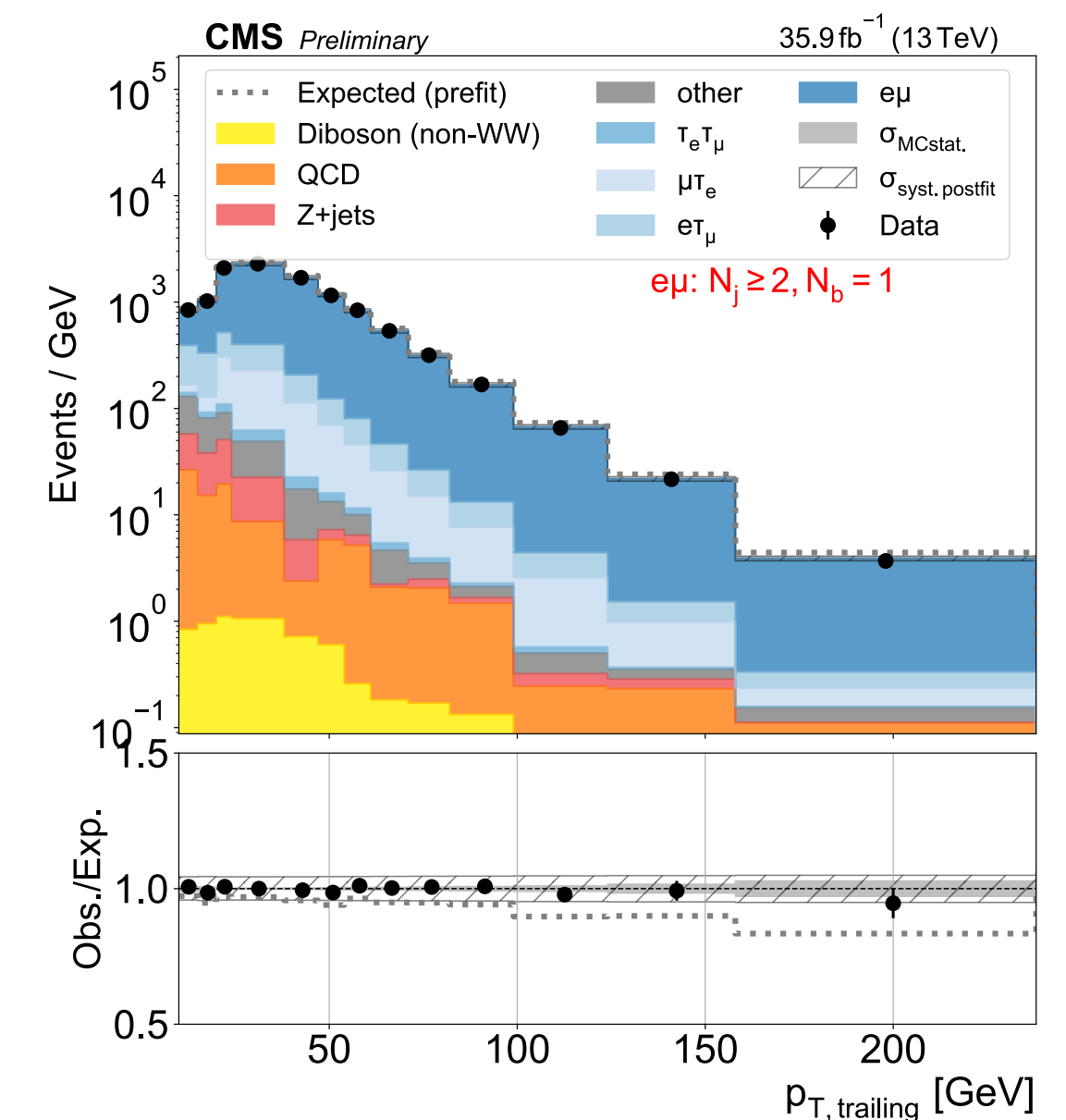
## Measured BR of individual lepton flavours

- While systematic uncertainties comparable to LEP, factor of 3-10 statistically more precise
- Assuming lepton universality  
 $\text{BR}(W \rightarrow \text{leptons}) = (10.89 \pm 0.01 \pm 0.08)\%$

	CMS	LEP
$\mathcal{B}(W \rightarrow e\bar{\nu}_e)$	$(10.83 \pm 0.01 \pm 0.10)\%$	$(10.71 \pm 0.14 \pm 0.07)\%$
$\mathcal{B}(W \rightarrow \mu\bar{\nu}_\mu)$	$(10.94 \pm 0.01 \pm 0.08)\%$	$(10.63 \pm 0.13 \pm 0.07)\%$
$\mathcal{B}(W \rightarrow \tau\bar{\nu}_\tau)$	$(10.77 \pm 0.05 \pm 0.21)\%$	$(11.38 \pm 0.17 \pm 0.11)\%$
$\mathcal{B}(W \rightarrow h)$	$(67.46 \pm 0.04 \pm 0.28)\%$	–

	CMS	LEP	ATLAS
$R_{\mu/e} = \mathcal{B}(W \rightarrow \mu\bar{\nu}_\mu) / \mathcal{B}(W \rightarrow e\bar{\nu}_e)$	$1.009 \pm 0.009$	$0.993 \pm 0.019$	<b><math>1.003 \pm 0.010</math></b>
$R_{\tau/e} = \mathcal{B}(W \rightarrow \tau\bar{\nu}_\tau) / \mathcal{B}(W \rightarrow e\bar{\nu}_e)$	$0.994 \pm 0.021$	$1.063 \pm 0.027$	–
$R_{\tau/\mu} = \mathcal{B}(W \rightarrow \tau\bar{\nu}_\tau) / \mathcal{B}(W \rightarrow \mu\bar{\nu}_\mu)$	$0.985 \pm 0.020$	$1.070 \pm 0.026$	$0.992 \pm 0.013$
$R_{\tau/\ell}$	$1.002 \pm 0.019$	$1.066 \pm 0.025$	–

ATLAS: [EPC 77 \(2017\) 367](#)



# Lepton universality at high-mass DY

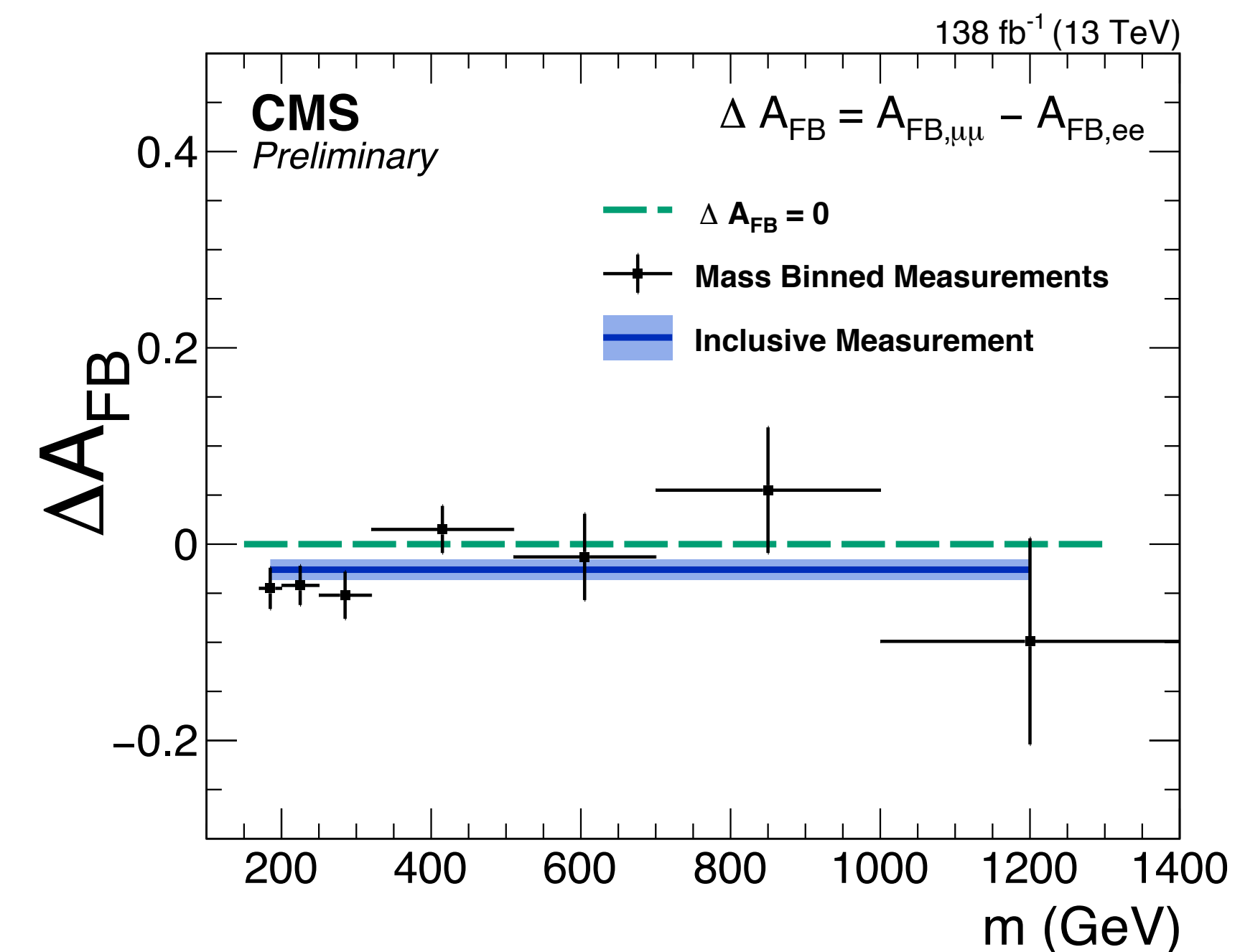
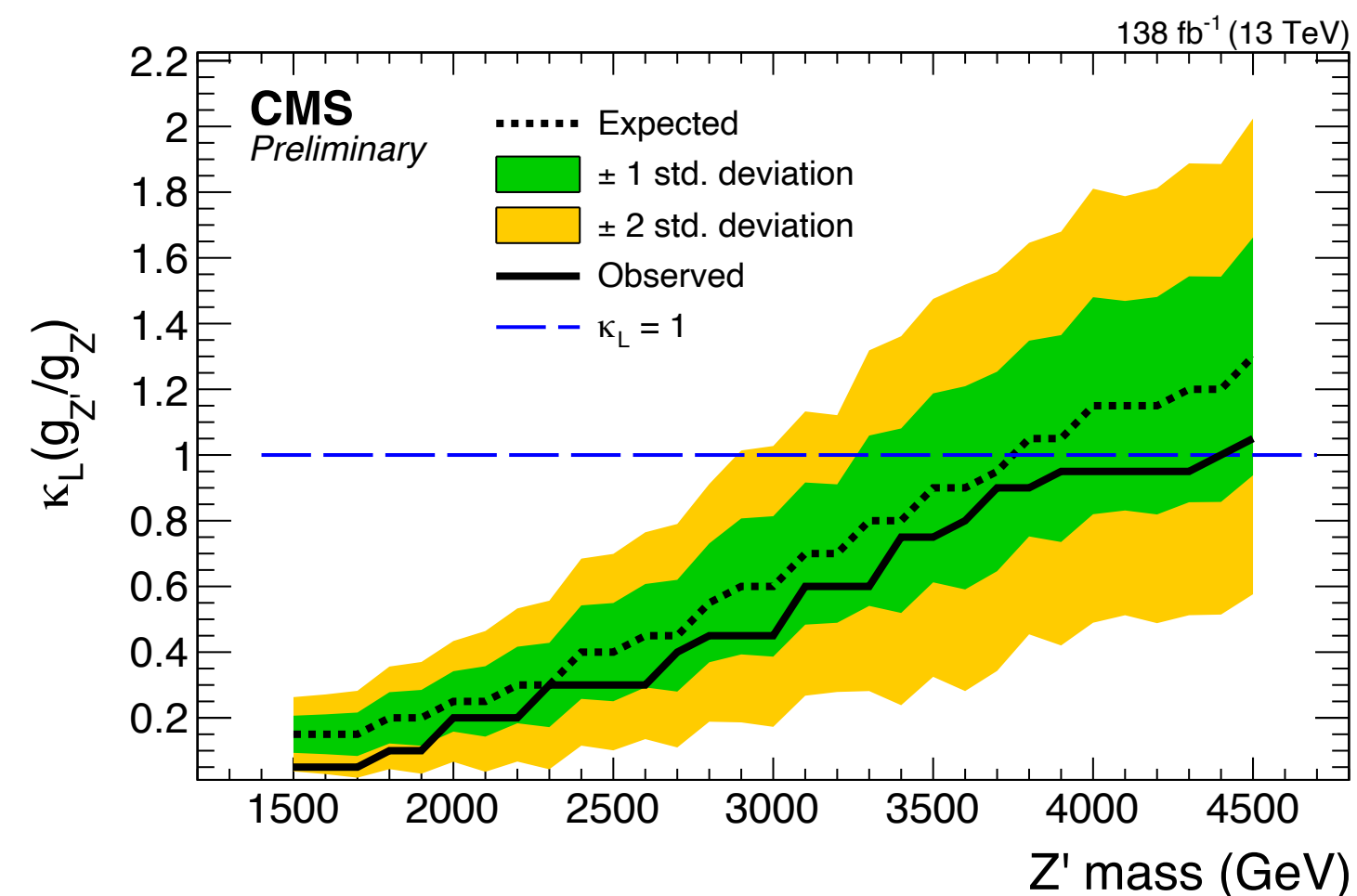
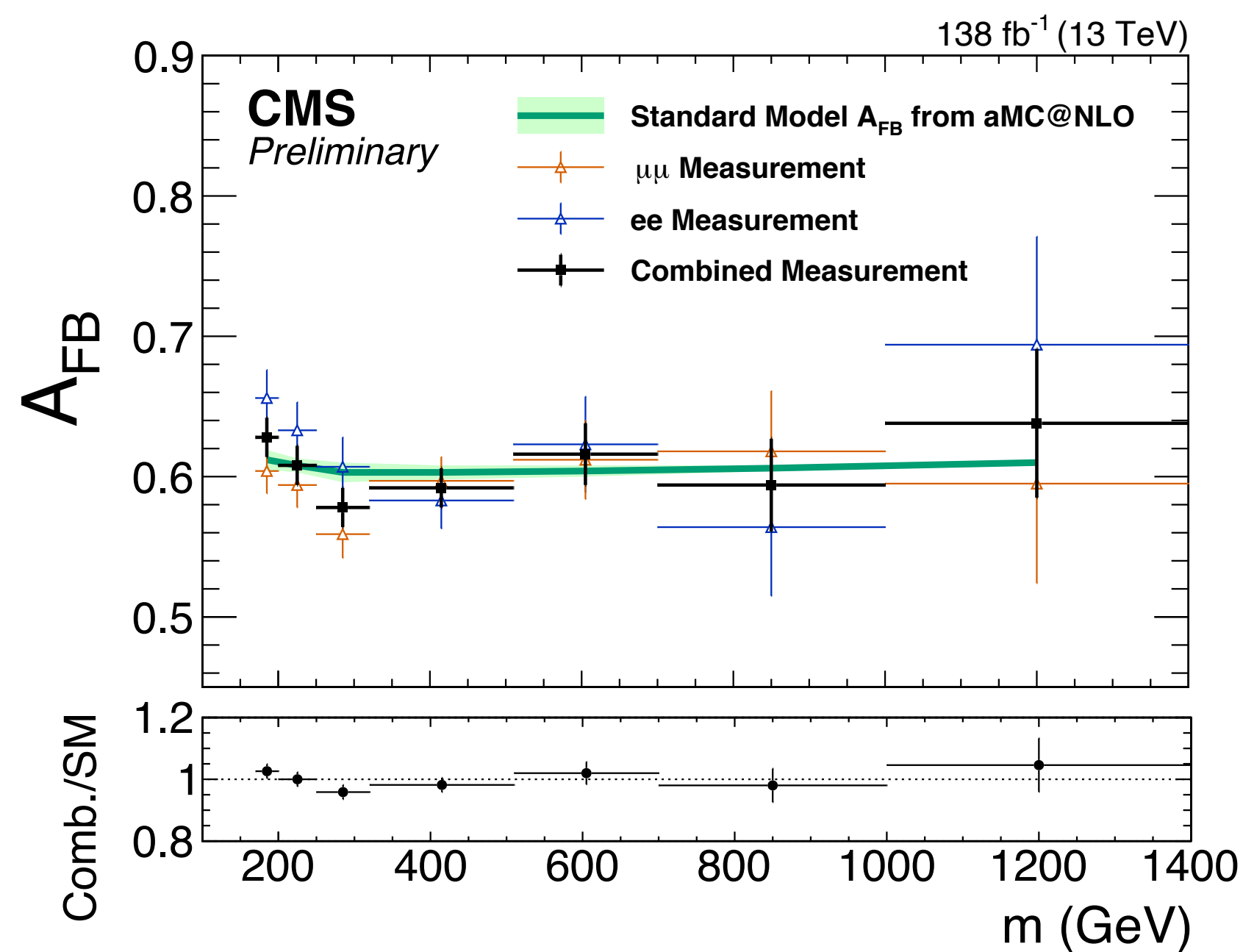
CMS-PAS-SMP-21-002

## CMS $A_{FB}$ measurement for $m_{ll} > 170$ GeV

- $A_{FB}$  non-zero and positive above Z-peak due to  $\gamma^*/Z$  interference
- Sensitive to BSM through interference with heavy vector boson

## $A_{FB}$ for ee and $\mu\mu$ allows to test lepton universality

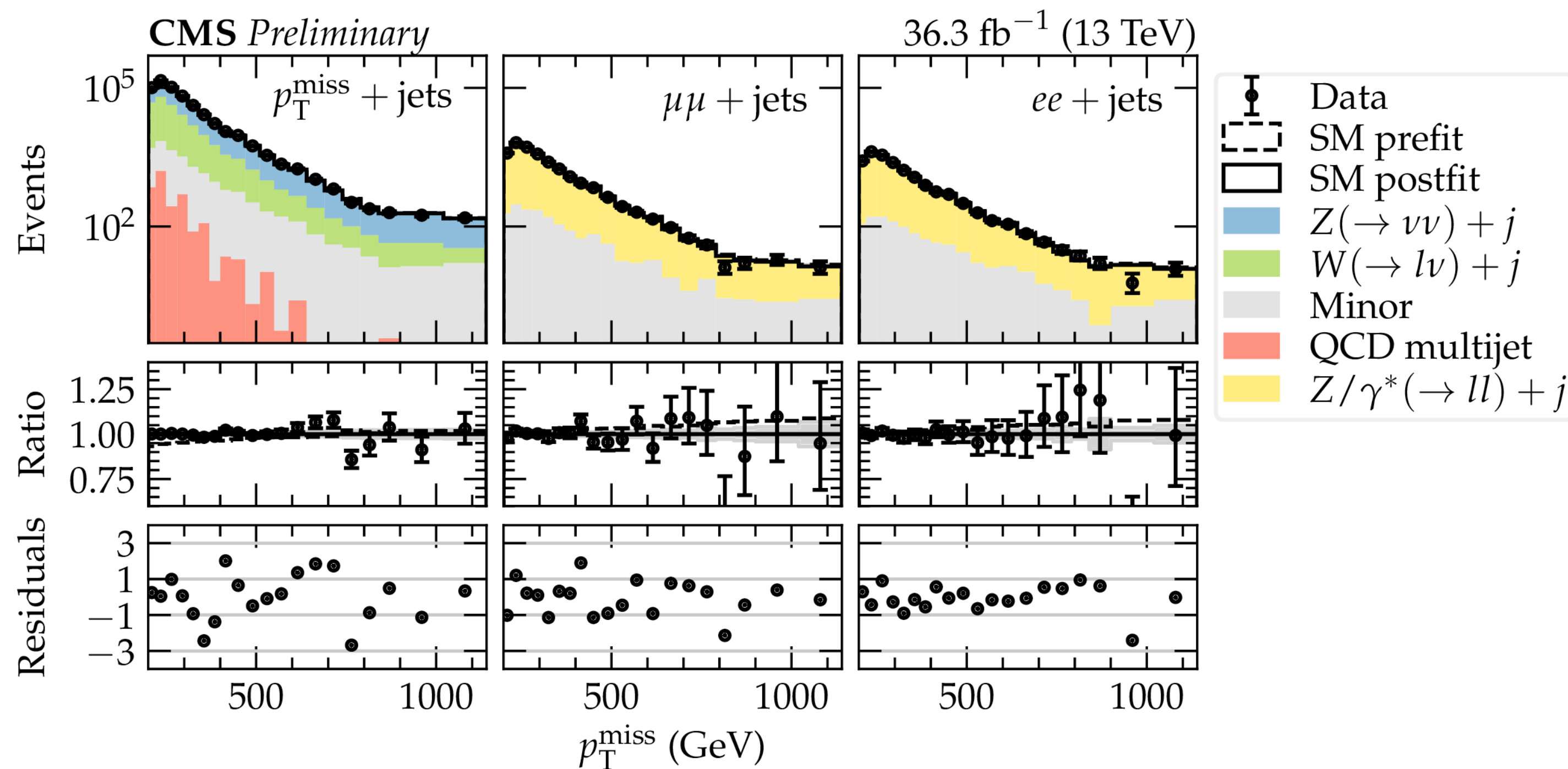
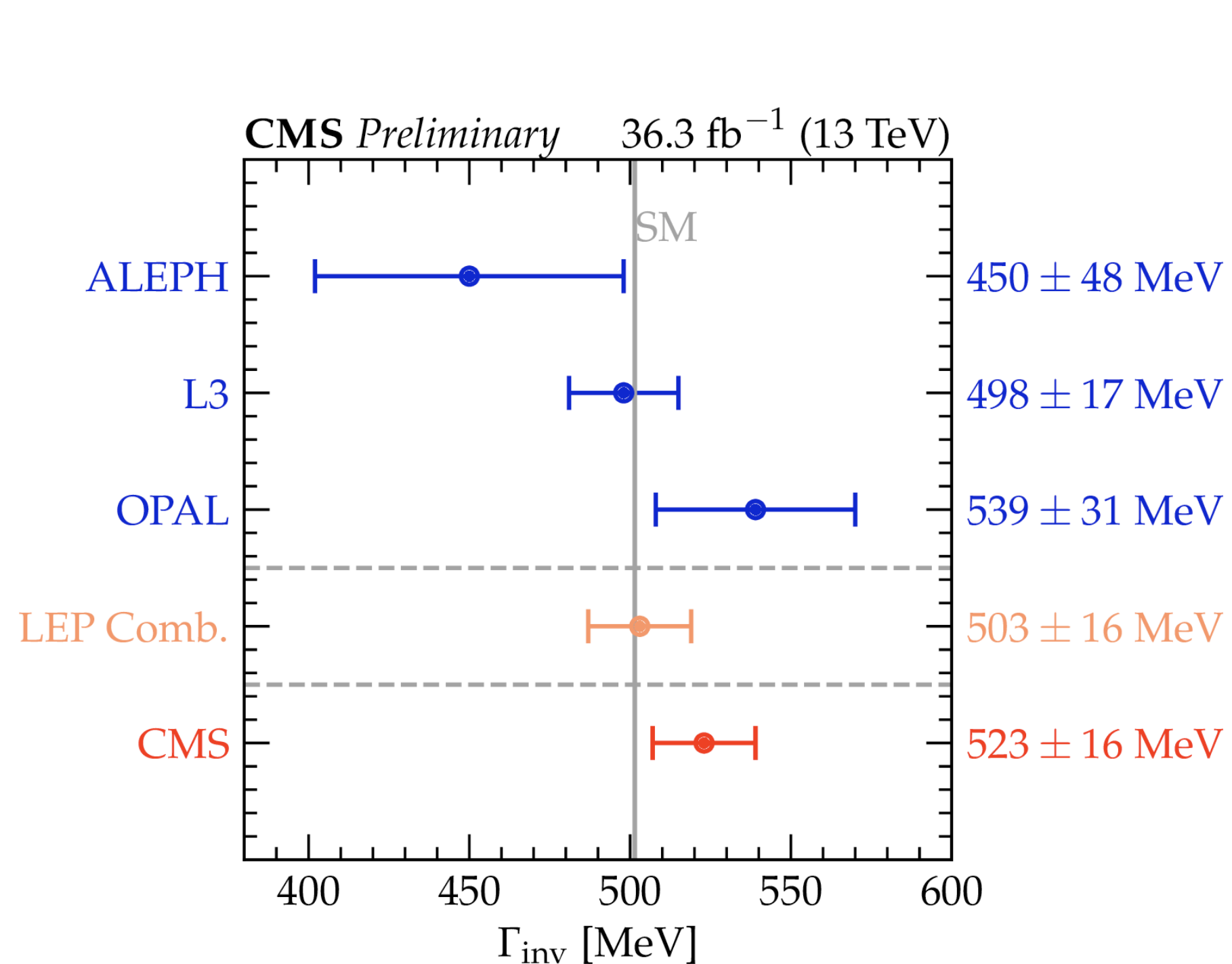
- In the inclusive  $\Delta A_{FB}$ , e/ $\mu$  are found to deviate at  $2.4\sigma$



## First measurement of Z invisible width at hadron collider

- Sensitive to number of neutrinos coupling to Z boson
- Ratio  $\sigma(Z \rightarrow \nu\nu) / \sigma(Z \rightarrow ll)$  interpreted as  $\Gamma_{inv}$
- Largest background  $W \rightarrow lv$  estimated using a control region

## Competitive precision with LEP direct measurements



# Diboson and triboson measurements

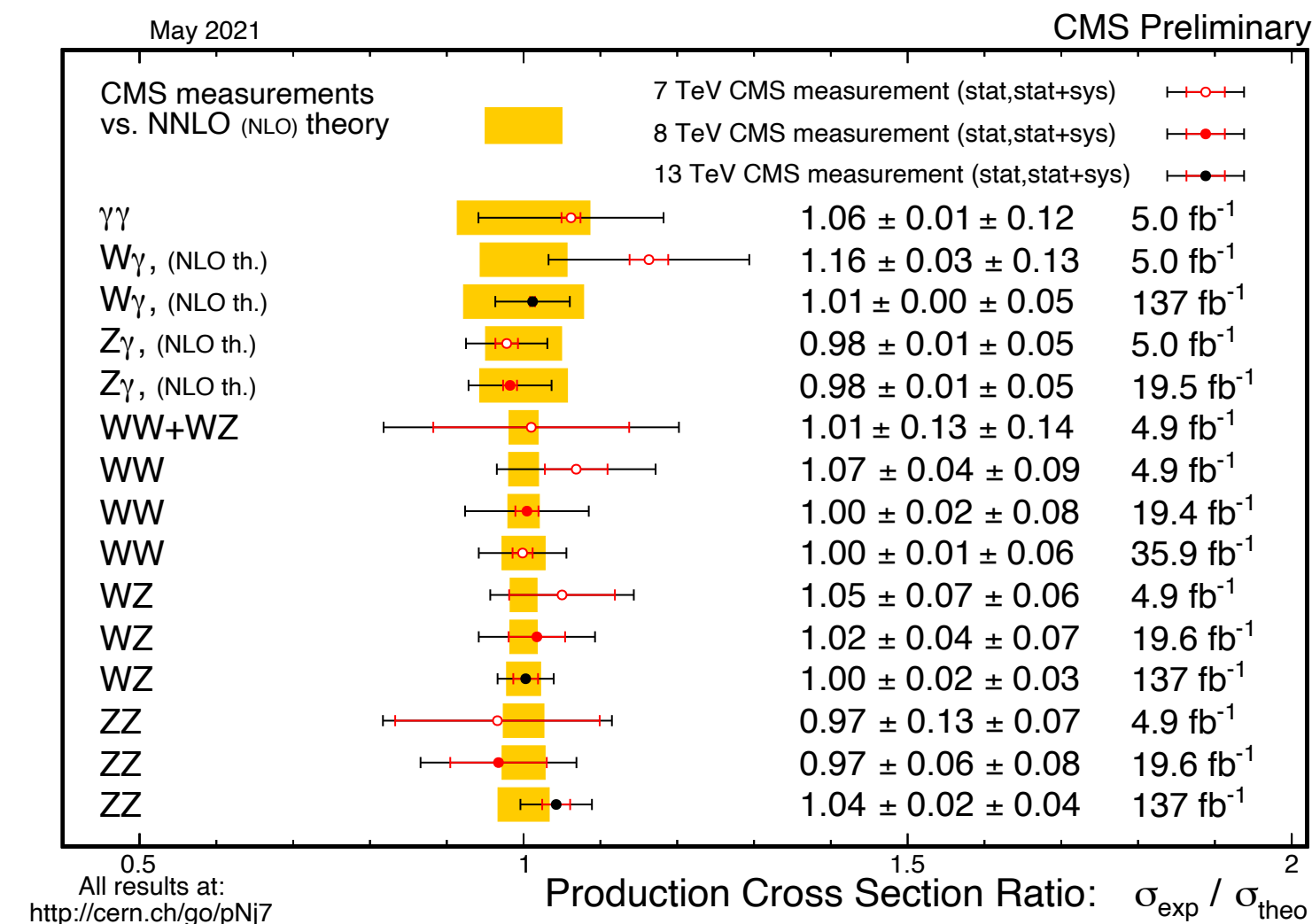
## All diboson production mechanisms have been measured by ATLAS and CMS

- Different decay modes of the vector bosons
- Mostly 13 TeV data, but some measurement still in progress with full Run 2 data
- Precision with Run 2 dataset is limited by systematic uncertainties
- In agreement with state-of-the-art theoretical calculations (see [ATLAS](#) and [CMS](#) summaries)

	Diboson	Triboson
not measured	$W\gamma$ <a href="#">ATLAS@7TeV</a> <a href="#">CMS</a>	$\gamma\gamma\gamma$ <a href="#">ATLAS</a>
8TeV	$Z\gamma$ <a href="#">ATLAS(lly, vv)</a> <a href="#">CMS@8TeV(lly, vv)</a>	$Z\gamma\gamma$ <a href="#">ATLAS@8TeV</a> <a href="#">CMS</a>
13 TeV	$WW$ <a href="#">ATLAS(incl, <math>\geq 1j</math>)</a> <a href="#">CMS</a>	$W\gamma\gamma$ <a href="#">ATLAS@8TeV</a> <a href="#">CMS</a>
	$WZ$ <a href="#">ATLAS</a> <a href="#">CMS</a>	$WW\gamma$ <a href="#">ATLAS</a> <a href="#">CMS</a>
	$ZZ$ <a href="#">ATLAS (4l, 2l2v,</a> <a href="#">CMS(4l, 4l+jets)</a>	$WZ\gamma$ <a href="#">ATLAS (WV)</a> <a href="#">CMS (WV)</a>

## Many triboson processes studied

- Low cross section require use of hadronic decays of massive gauge bosons
- First observation of the 3 massive gauge boson process at the LHC



$ZZ\gamma$	
$WWW$	<a href="#">ATLAS</a> <a href="#">CMS</a>
$WWZ$	<a href="#">ATLAS ( WV )</a>
$WZZ$	<a href="#">ATLAS ( WV )</a>
$ZZZ$	

results at lower  $\sqrt{s}$  not shown, adapted from [P. Sommer](#)

# BSM representation using EFT

## Characterise deviations from SM using SM Effective Field Theory (SMEFT)

$$\mathcal{L}_{\text{SMEFT}} \approx \mathcal{L}_{\text{SM}}^{(4)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)}$$

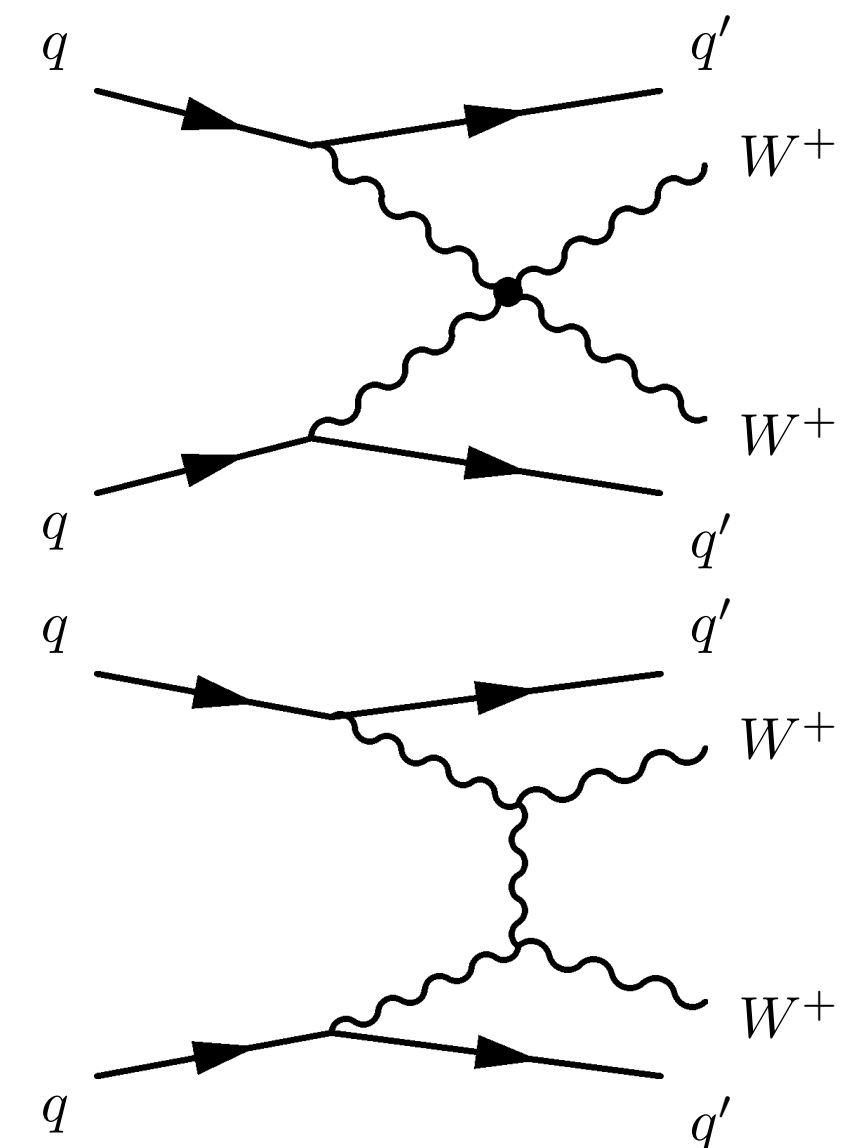
- Expansion in terms of scale of new physics  $1/\Lambda$
- Typically constrain one Wilson coefficient at a time in single measurement

## First attempts of global SMEFT interpretations using dim6 operators

- ATLAS: Combined interpretation of WW, WZ, 4l, EW Zjj measurements or  $H \rightarrow WW^*$  and WW processes, see [ATL-PHYS-PUB-2021-022](#), [ATL-PHYS-PUB-2021-010](#)

## VBS processes - laboratory to test SM EW symmetry breaking

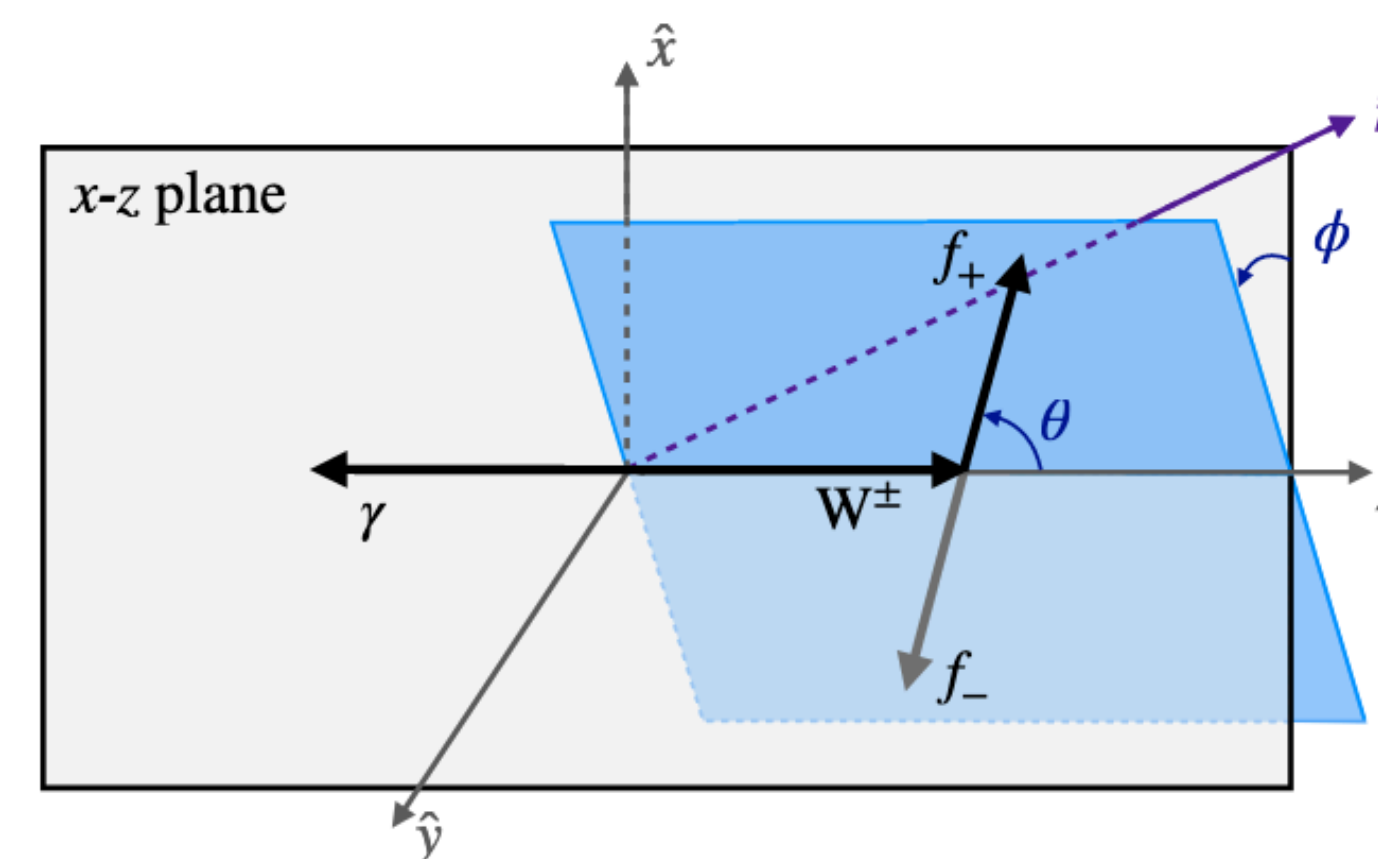
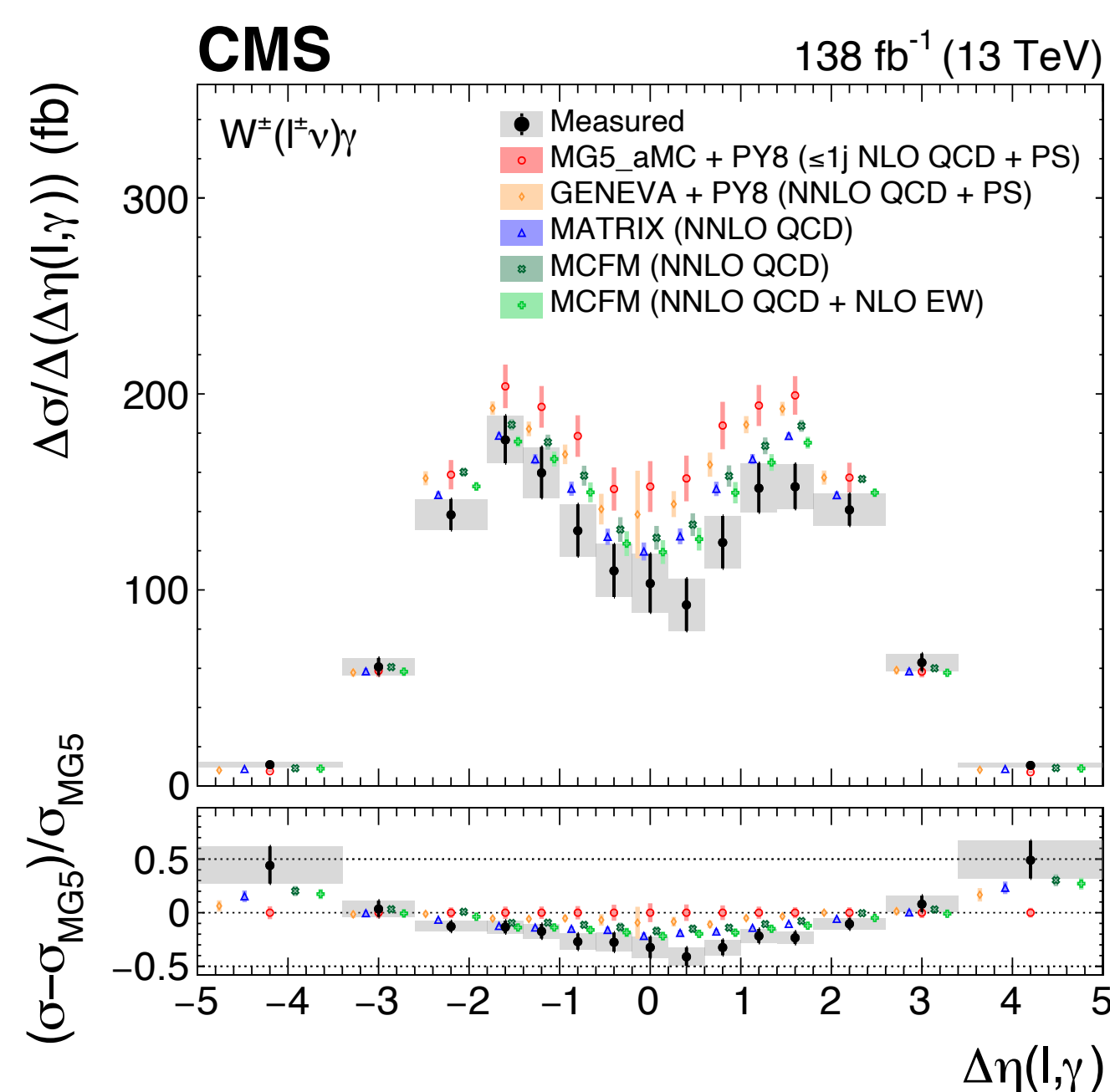
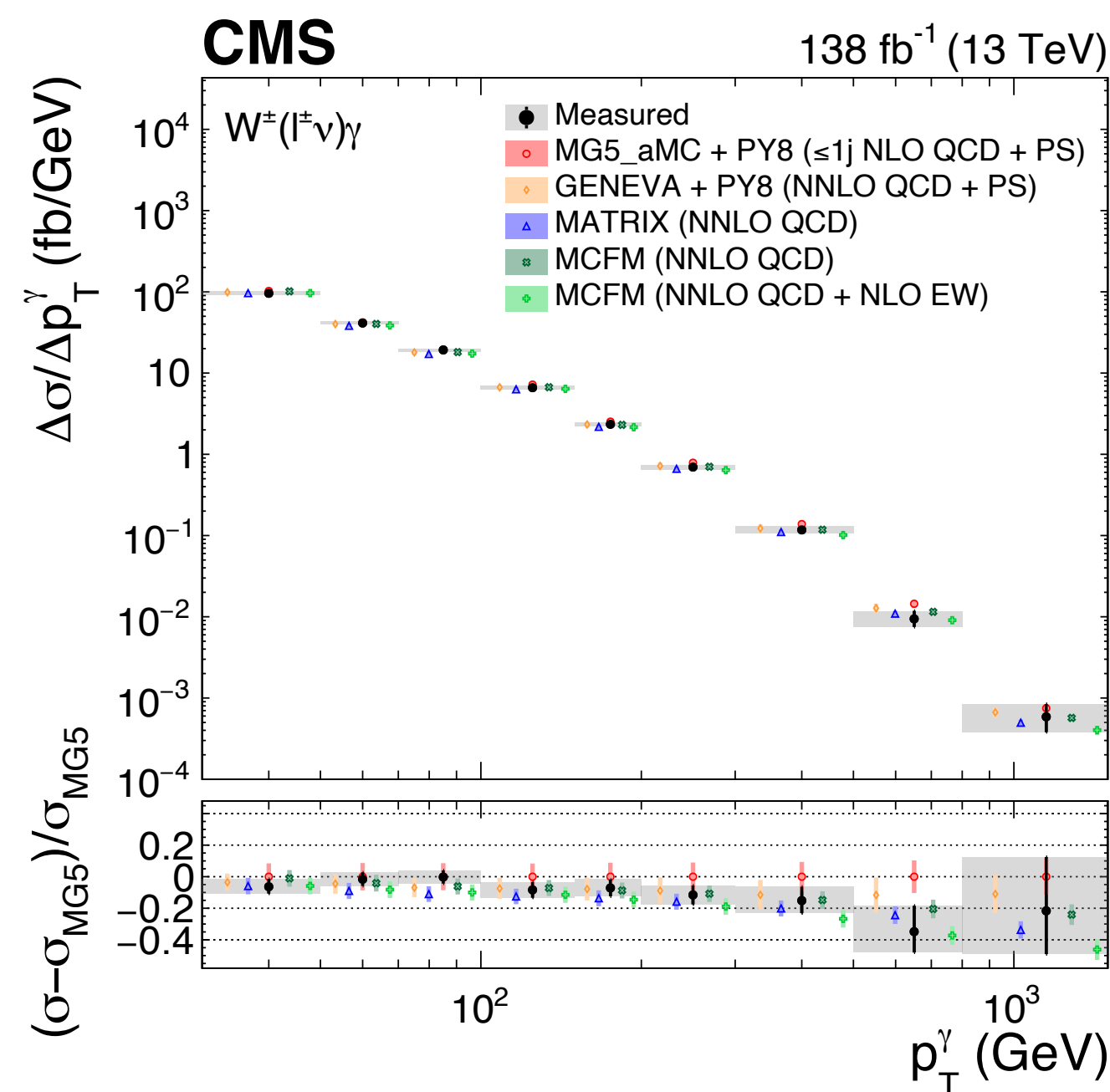
- Leading divergence (due to  $V_L$ ) cancellation between TGC and QGC contributions
- Higgs diagrams needed to avoid violation of unitarity
- Dim-8 operators can modify VVjj production via aQGC
- Experiments should use same unitarisation procedure allowing reinterpretation/combinaton (clipping, ...)





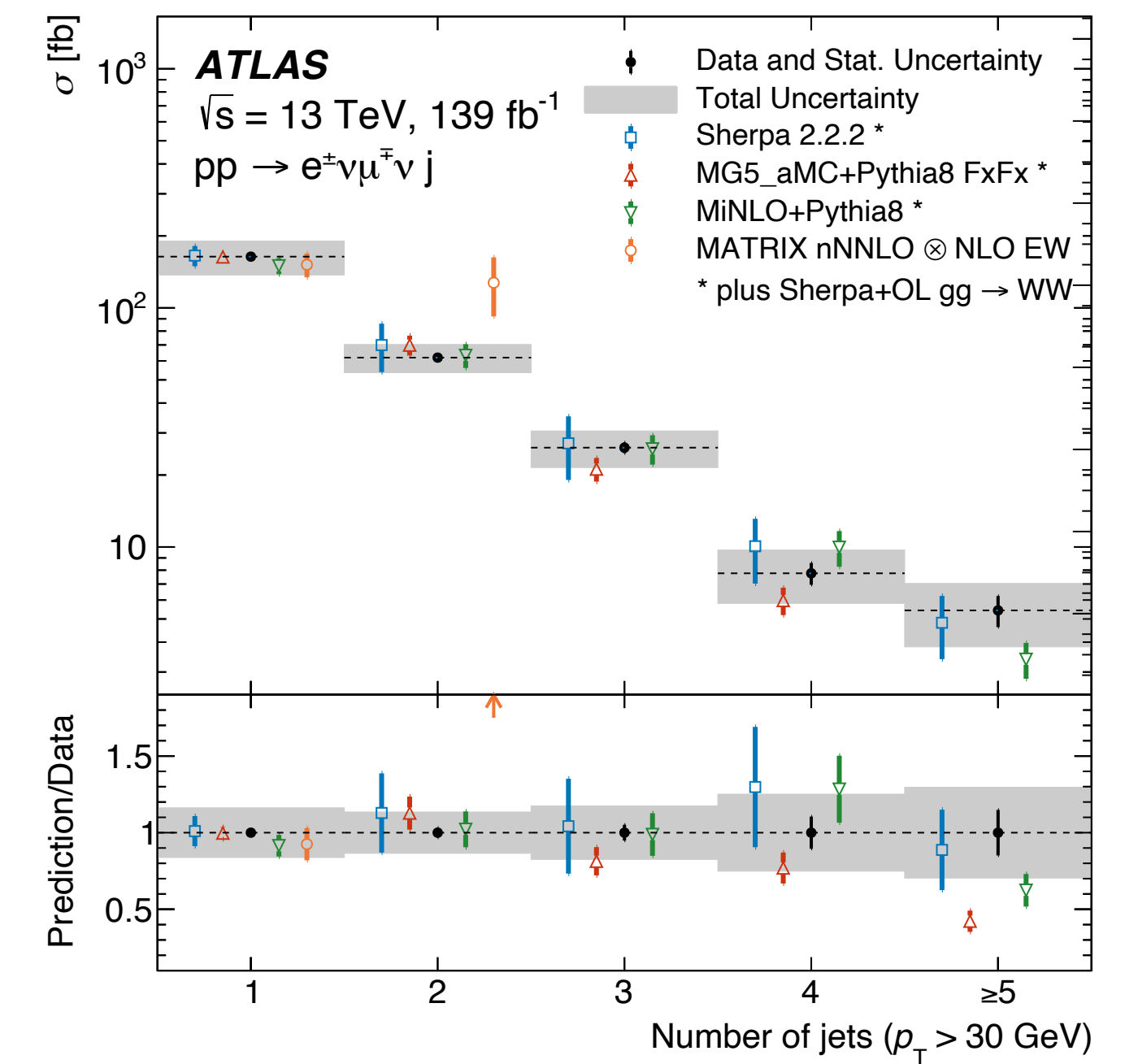
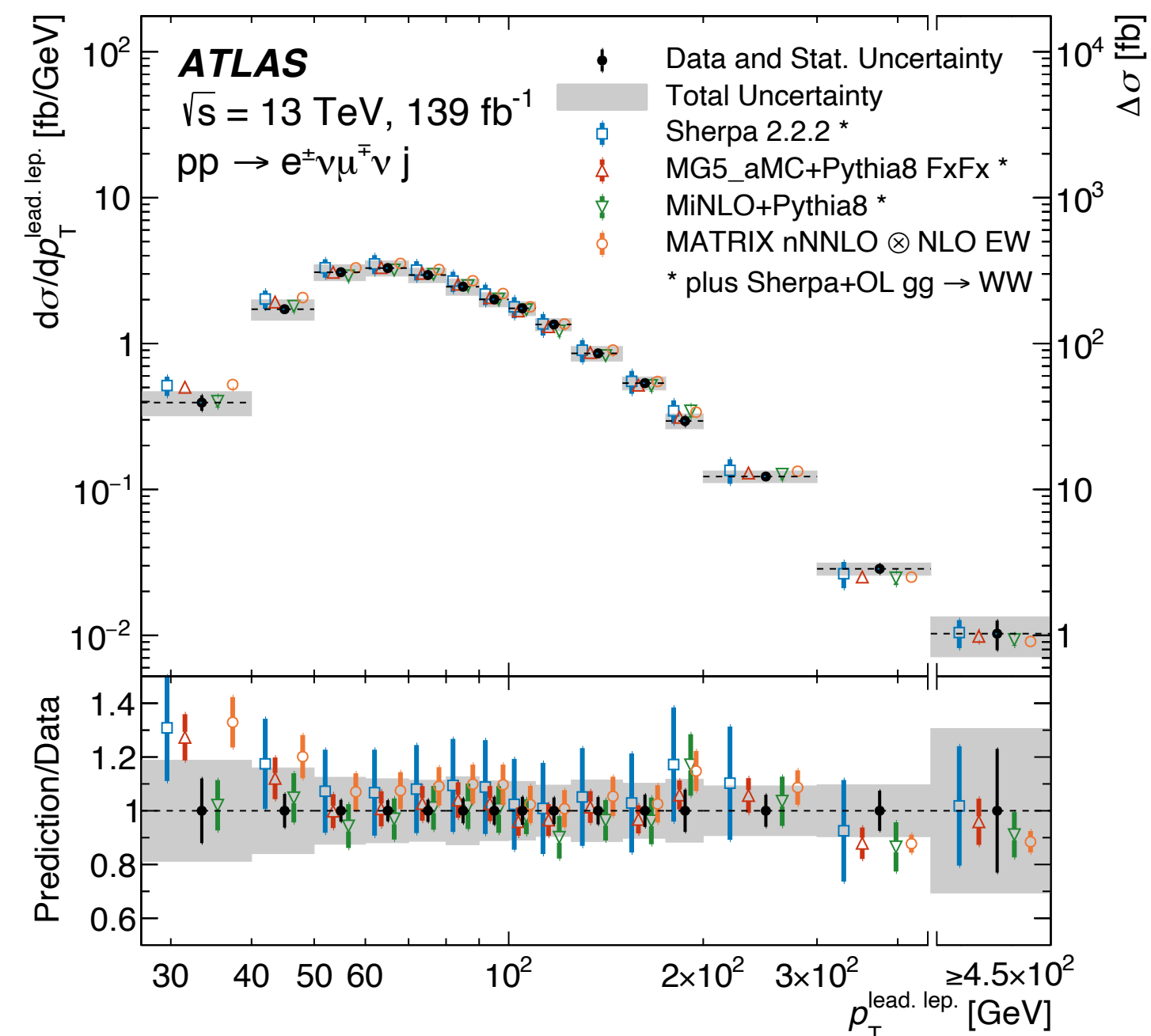
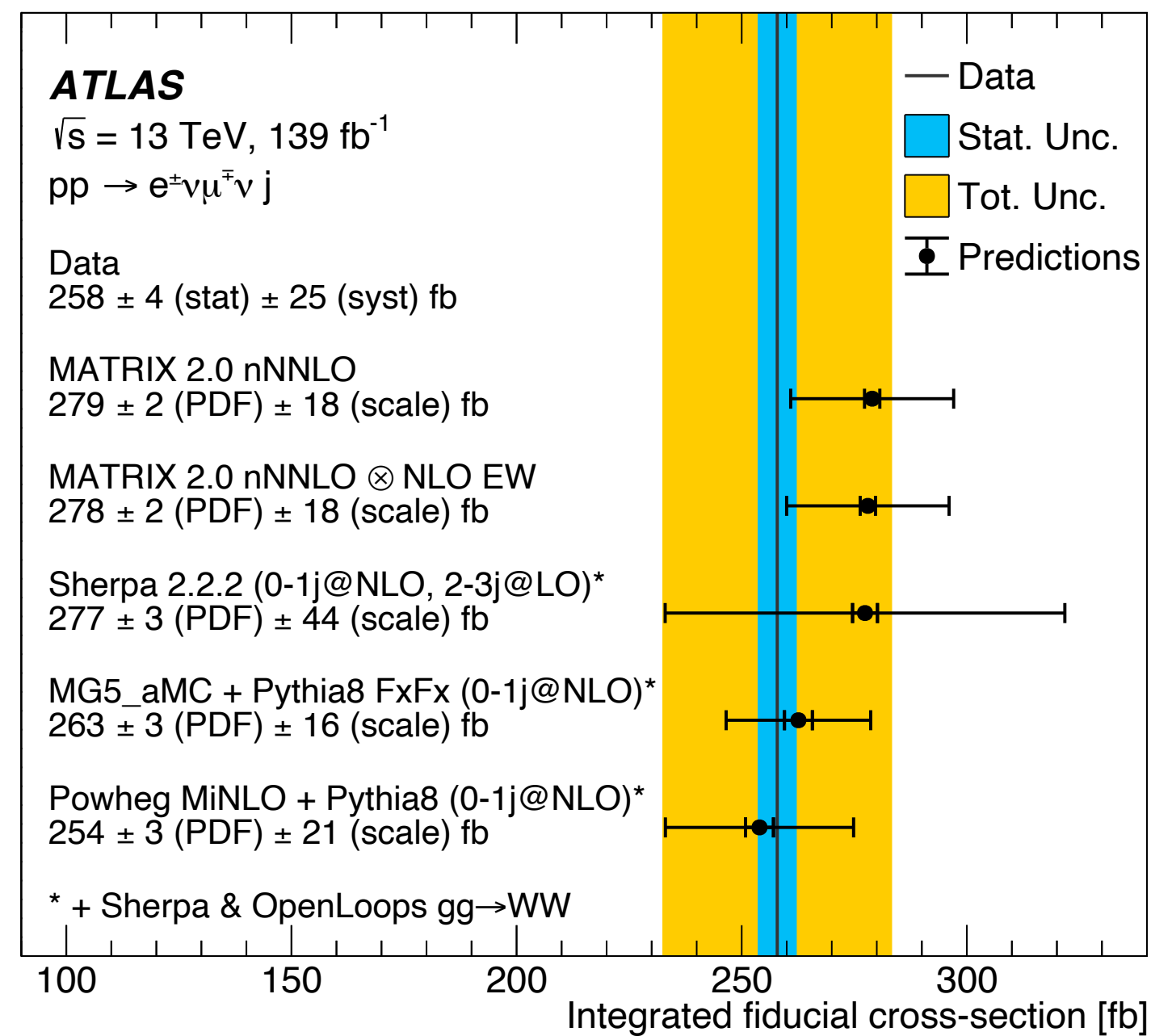
## Full Run 2 measurement of W $\gamma$ by CMS

- Previous result only at 7TeV, inclusive and differential cross section measurements, jet observables
- Radiation-zero due to destructive interference of LO diagrams
  - observed in rapidity difference between lepton/ $\gamma$ , sensitive to BSM contributions
- $\phi$  variable defined in the W $\gamma$  rest frame increases sensitivity for EFT constraints
  - enhancement of interference between SM and BSM differentially, which is zero for inclusive observable (linear term resurrection)



## Probing QCD higher order contributions in WW production: $pp \rightarrow e^\pm \nu \mu^\mp \bar{\nu} j$

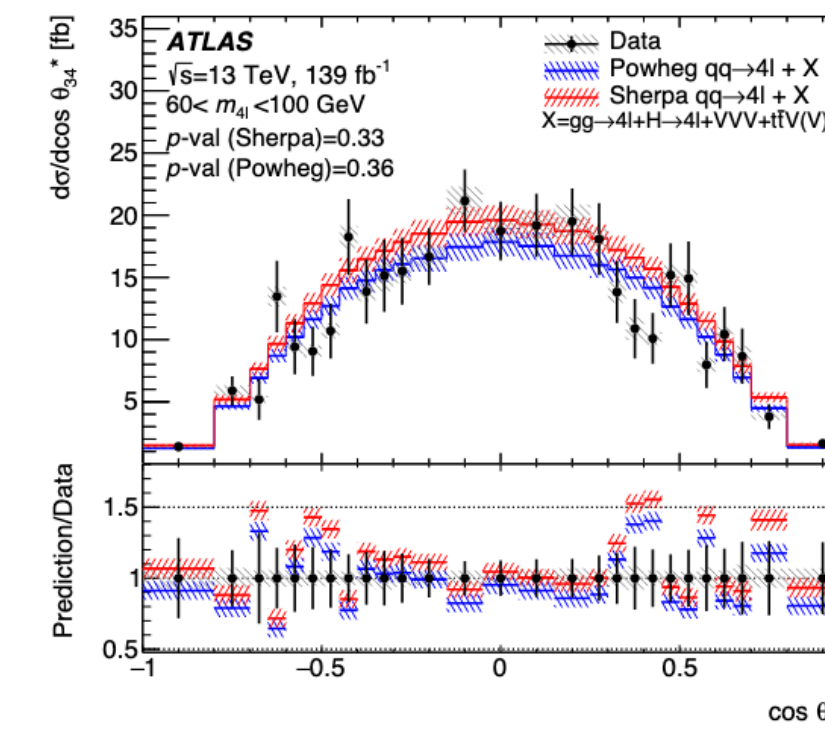
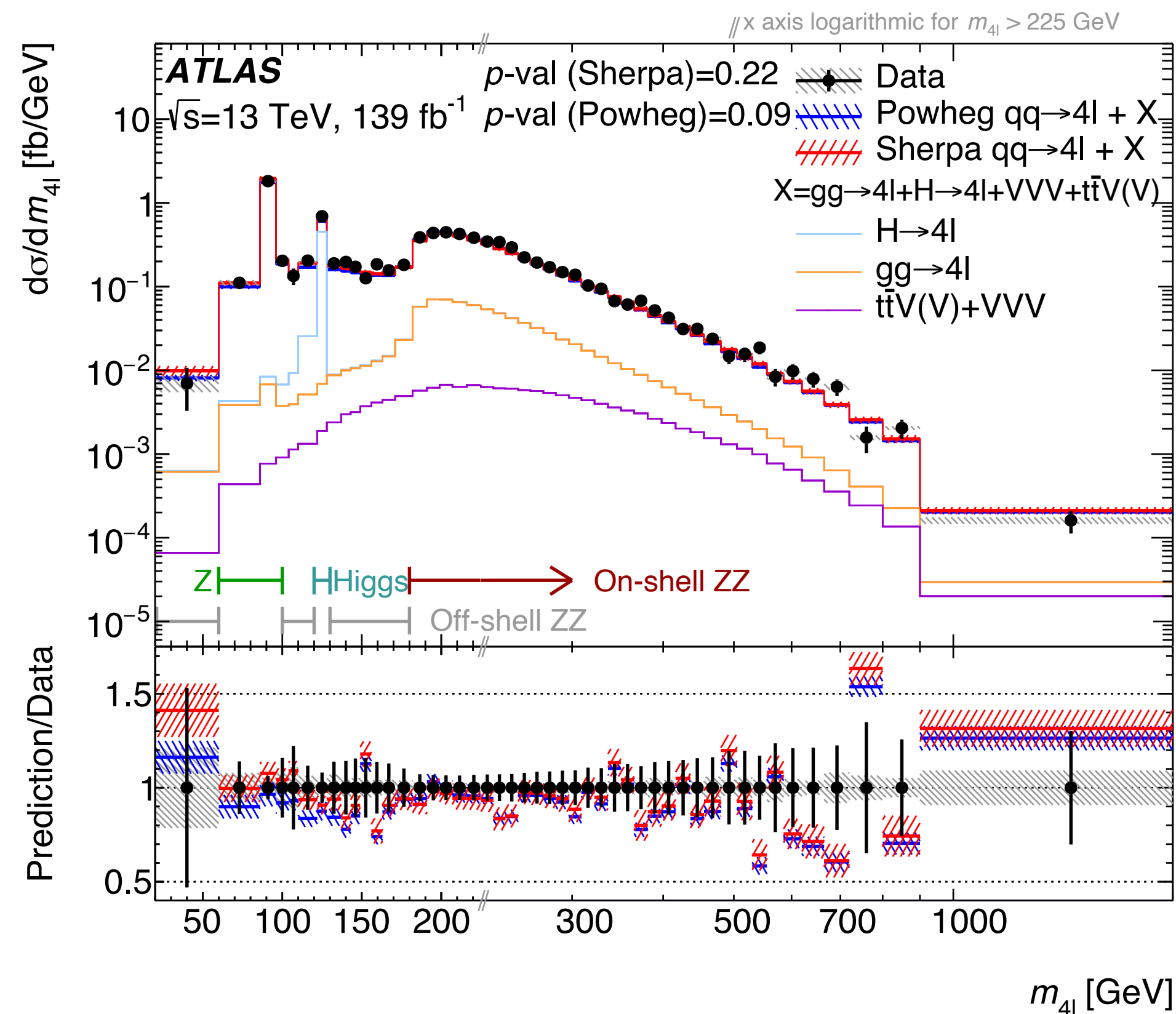
- Precision differential measurement of lepton and jet observables with full Run 2
- Data driven top background estimate, which is large after b-jet veto
- NLO theoretical prediction model well previously unexplored phase-space up to 5 jets



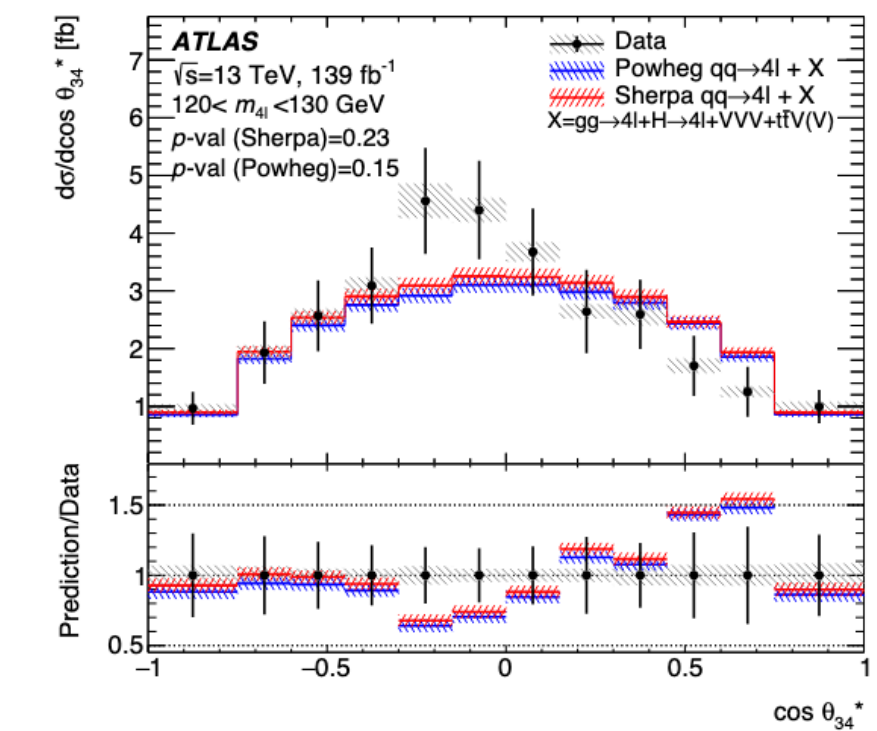
# 4-lepton production

## Differential measurement of 4l production in ATLAS

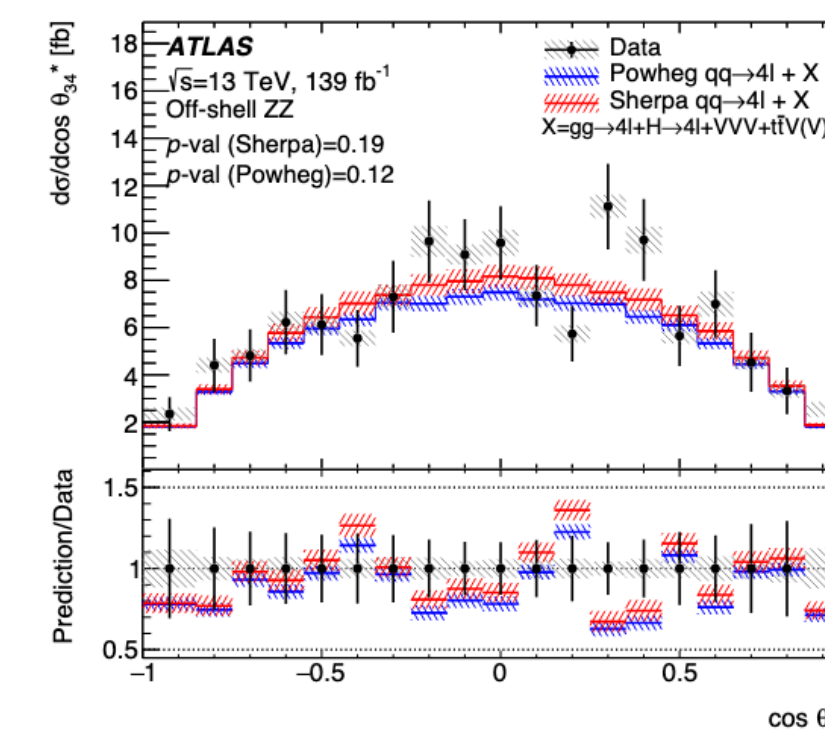
- $Z \rightarrow 4l$ ,  $H \rightarrow 4l$ , on-shell ZZ and off-shell regions
- Many variables measured including angular correlation
- Statistically limited even with full Run 2



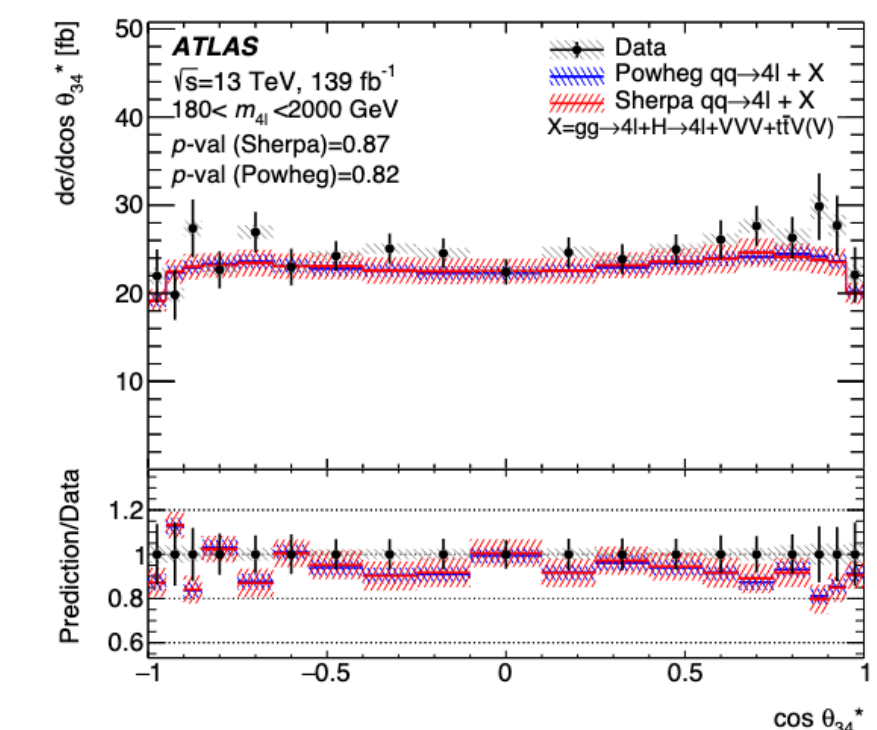
(a)  $Z \rightarrow 4\ell$  region



(b)  $H \rightarrow 4\ell$  region



(c) Off-shell ZZ region

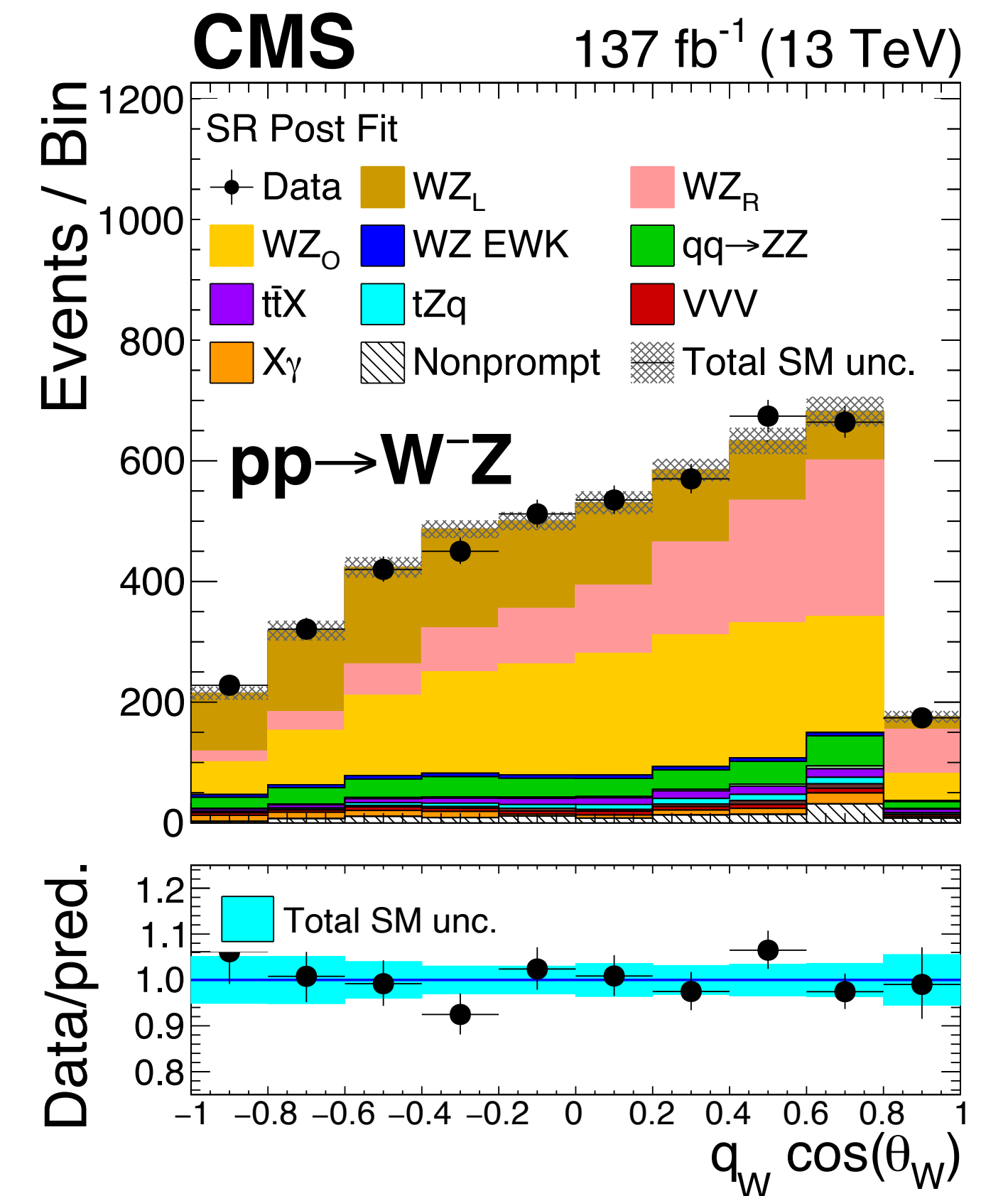
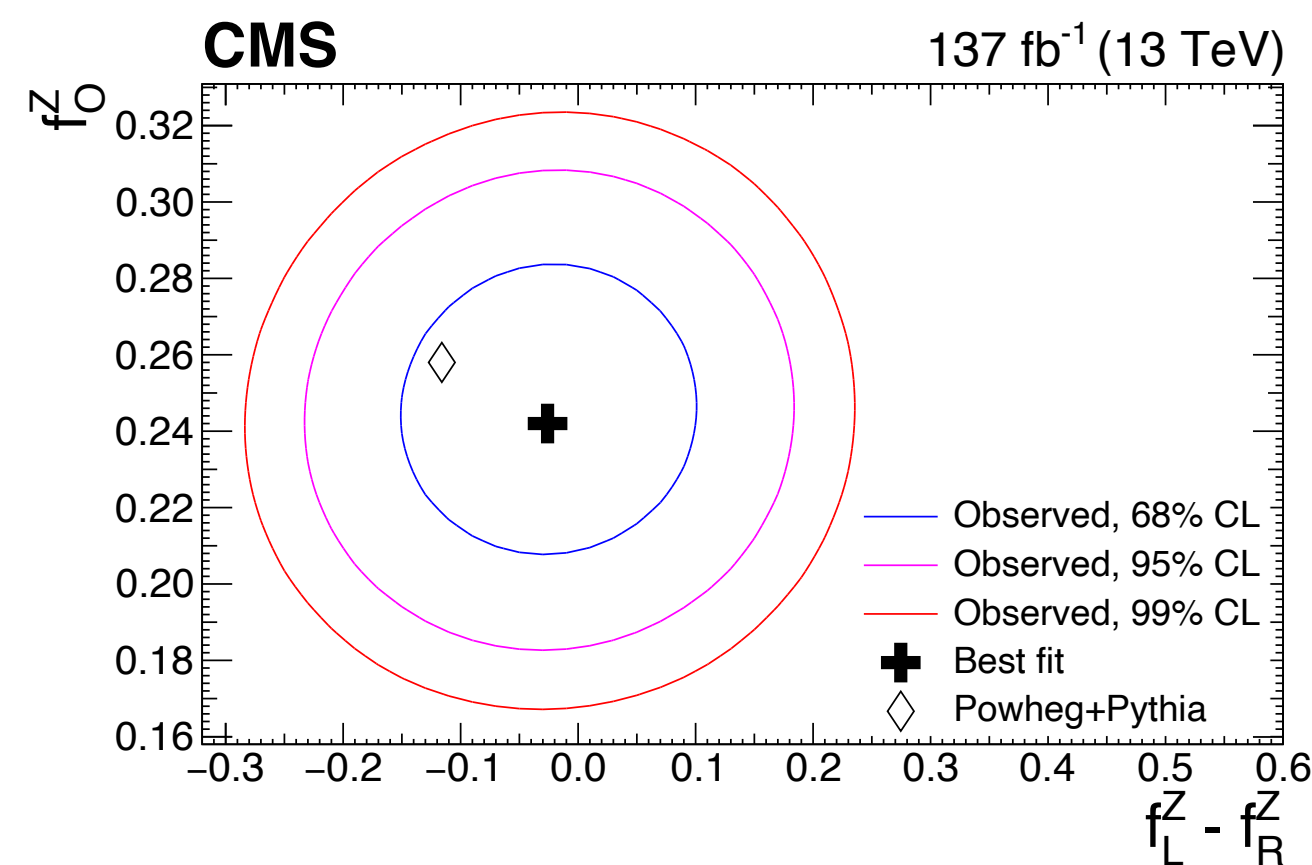
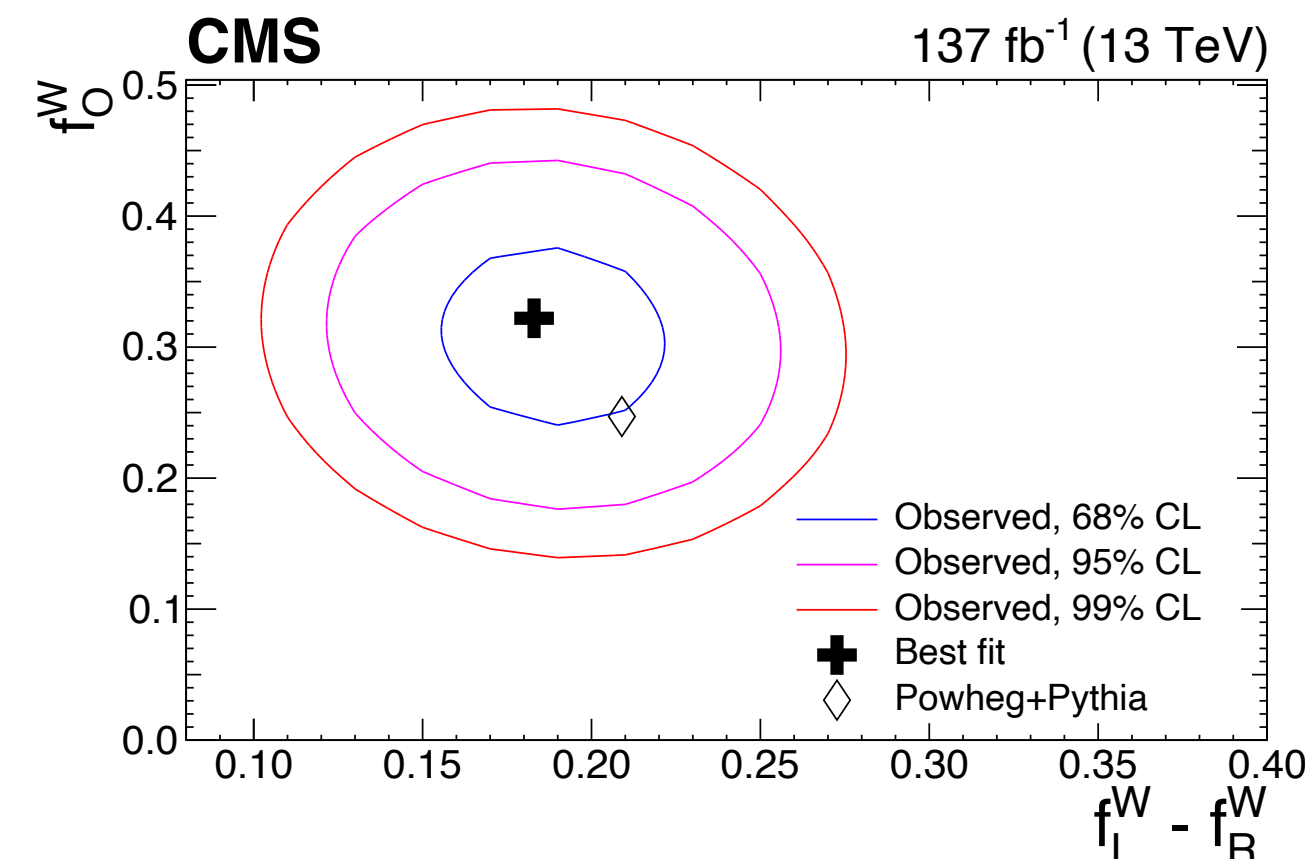
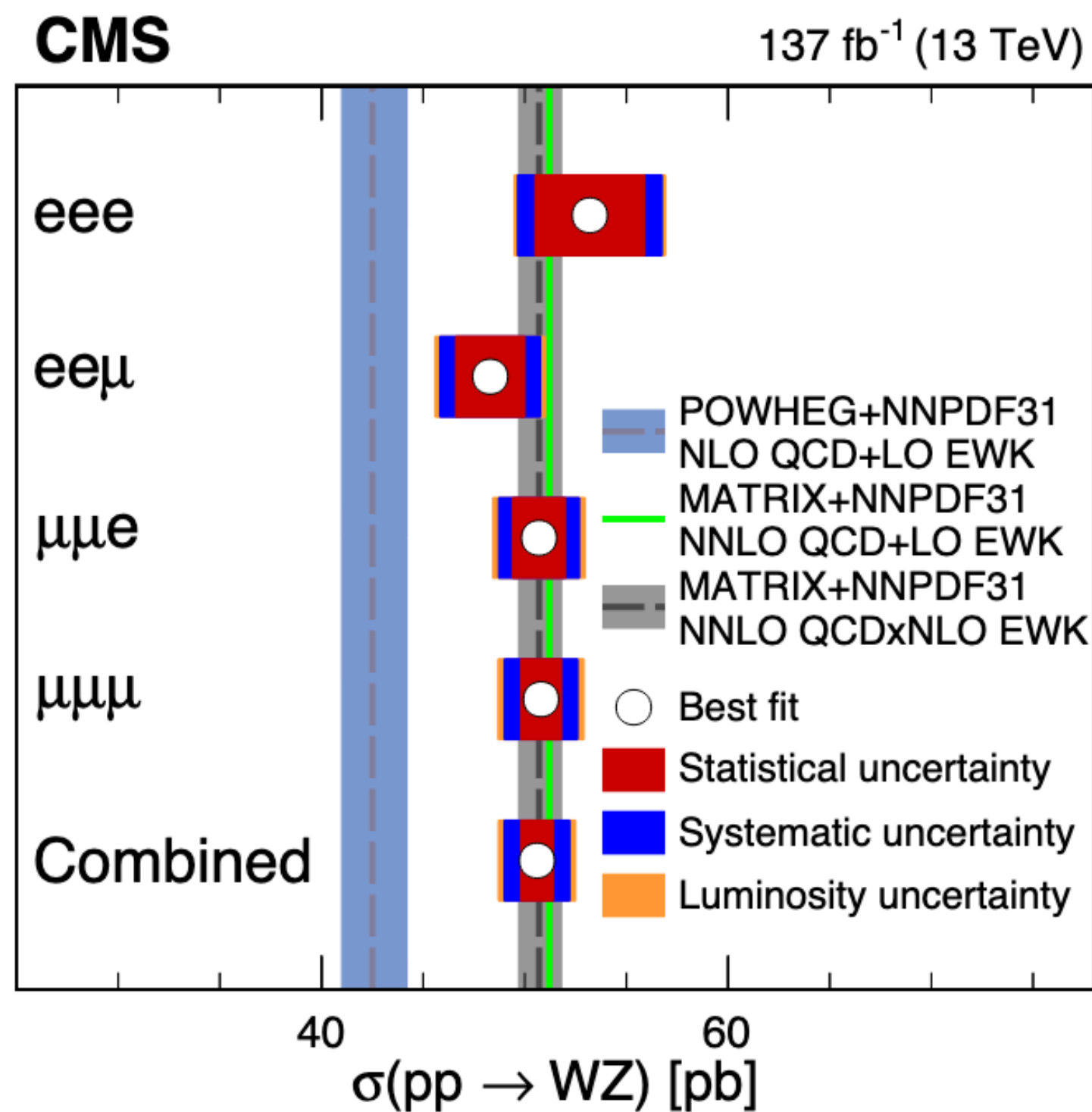


(d) On-shell ZZ region

# WZ cross section and polarisation CMS-PAS-SMP-20-014

## Full Run 2 measurement of WZ by CMS

- Improved precision compared to ATLAS/CMS measurement using smaller dataset
- Differential cross section measurement, charged asymmetry and (single-boson) polarisation
- Longitudinal polarisation from additional degree of freedom introduced by Higgs mechanism
- NNLO QCD gives better description

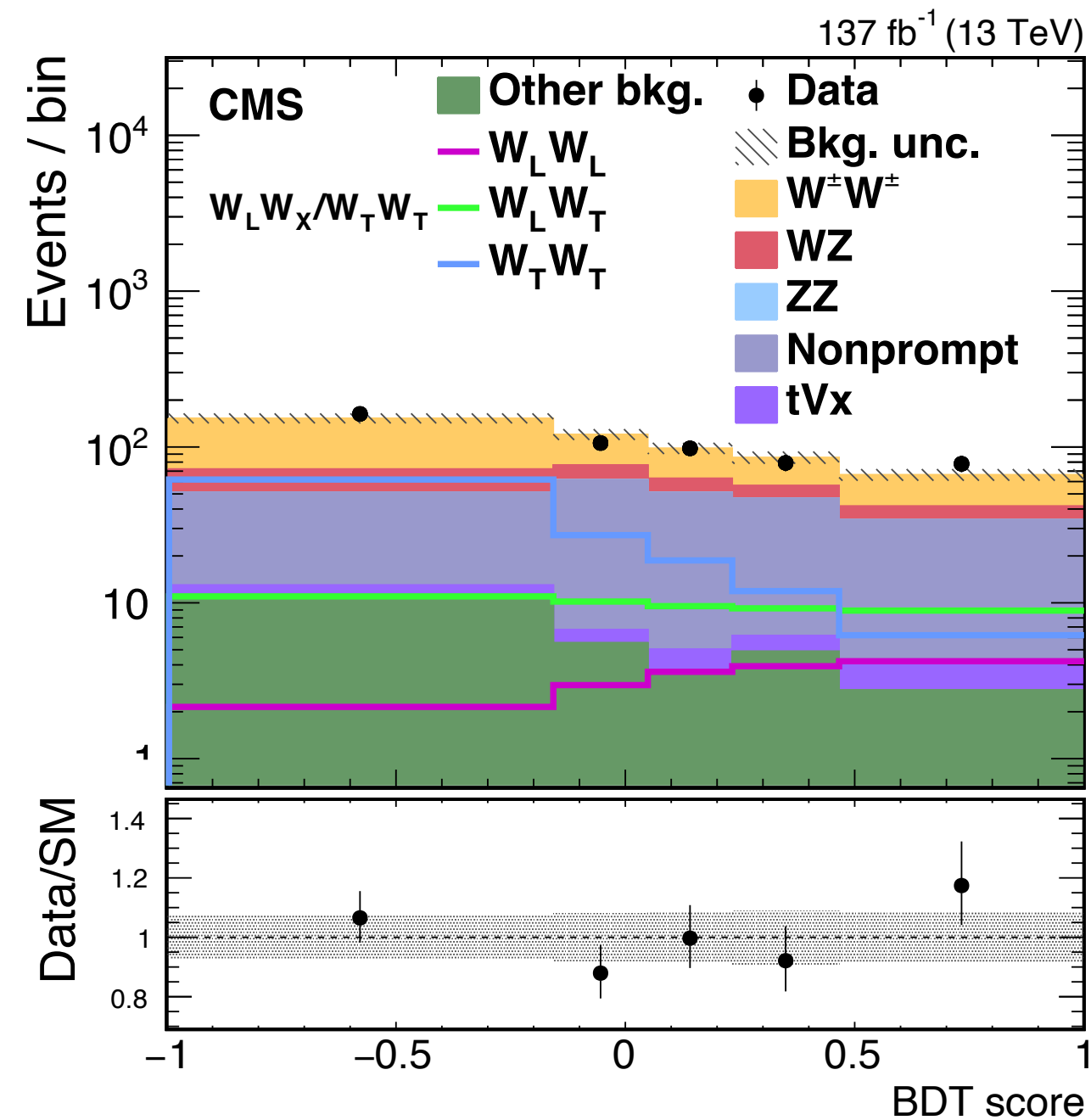
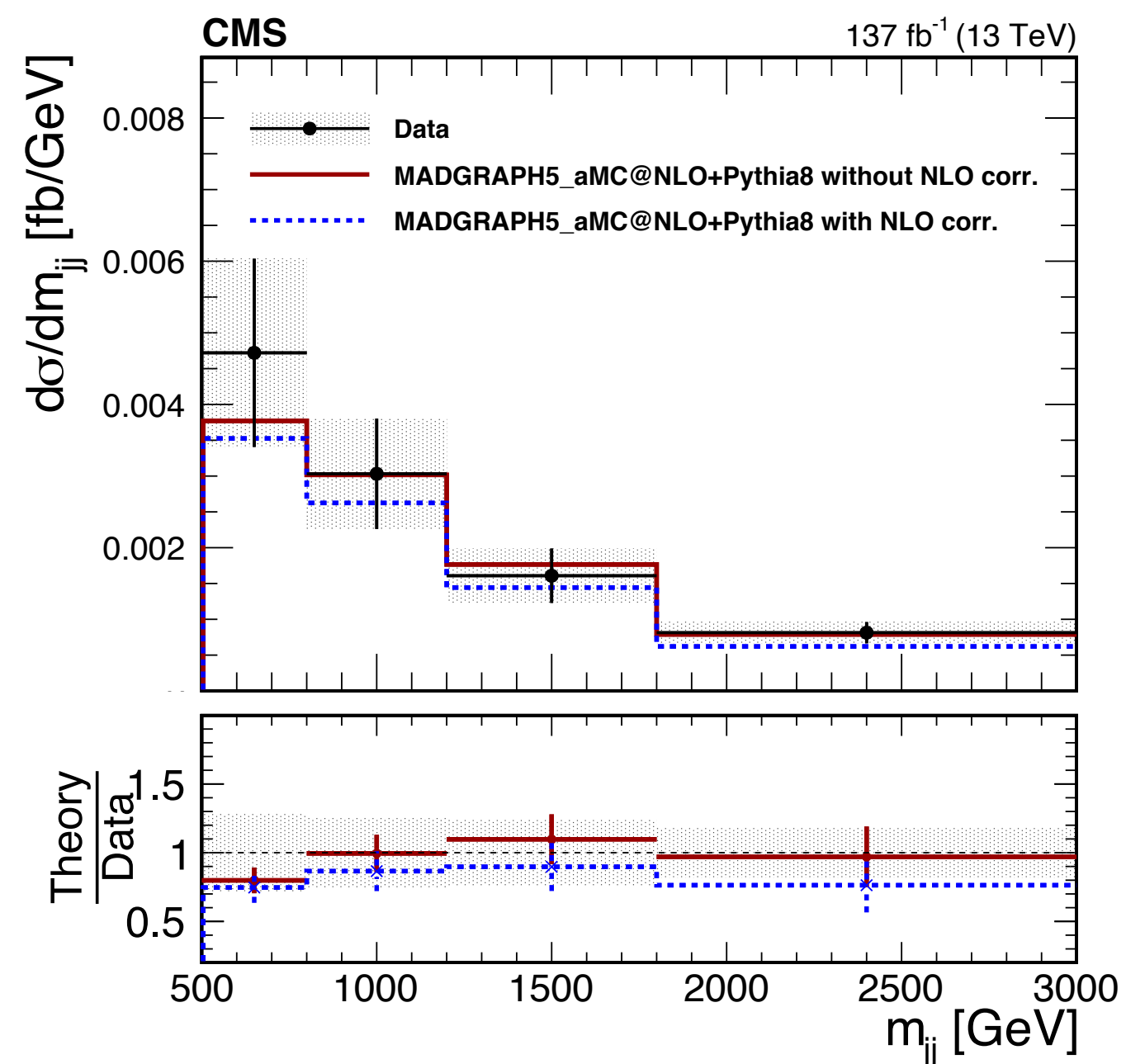


**Same-sign** electroweak  $W^\pm W^\pm jj$  and  $WZjj$  productions observed by ATLAS/CMS with early Run2

- processes well understood at NLO EW and NLO QCD [arXiv:2106.01393](https://arxiv.org/abs/2106.01393)

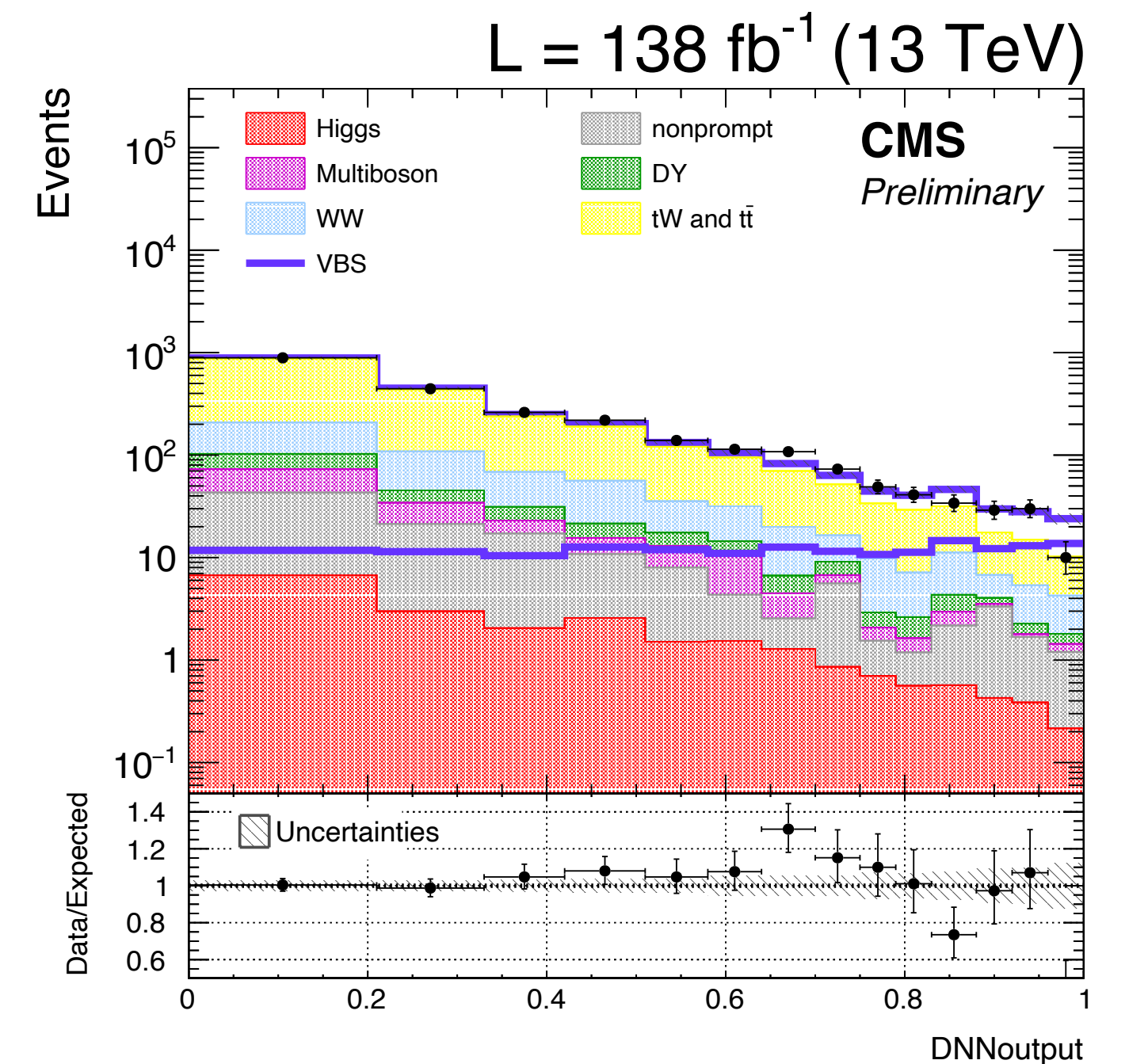
**Update with full Run 2 and first measurement of  $W^\pm W^\pm jj$  polarisation fractions by CMS**

- Significance for  $W_L W_X$  of  $2.3\sigma$  ( $3.1\sigma$  exp.)  
evidence for  $W_L W_L$  possible at HL-LHC, see [FTR-21-001](https://arxiv.org/abs/2106.01393)



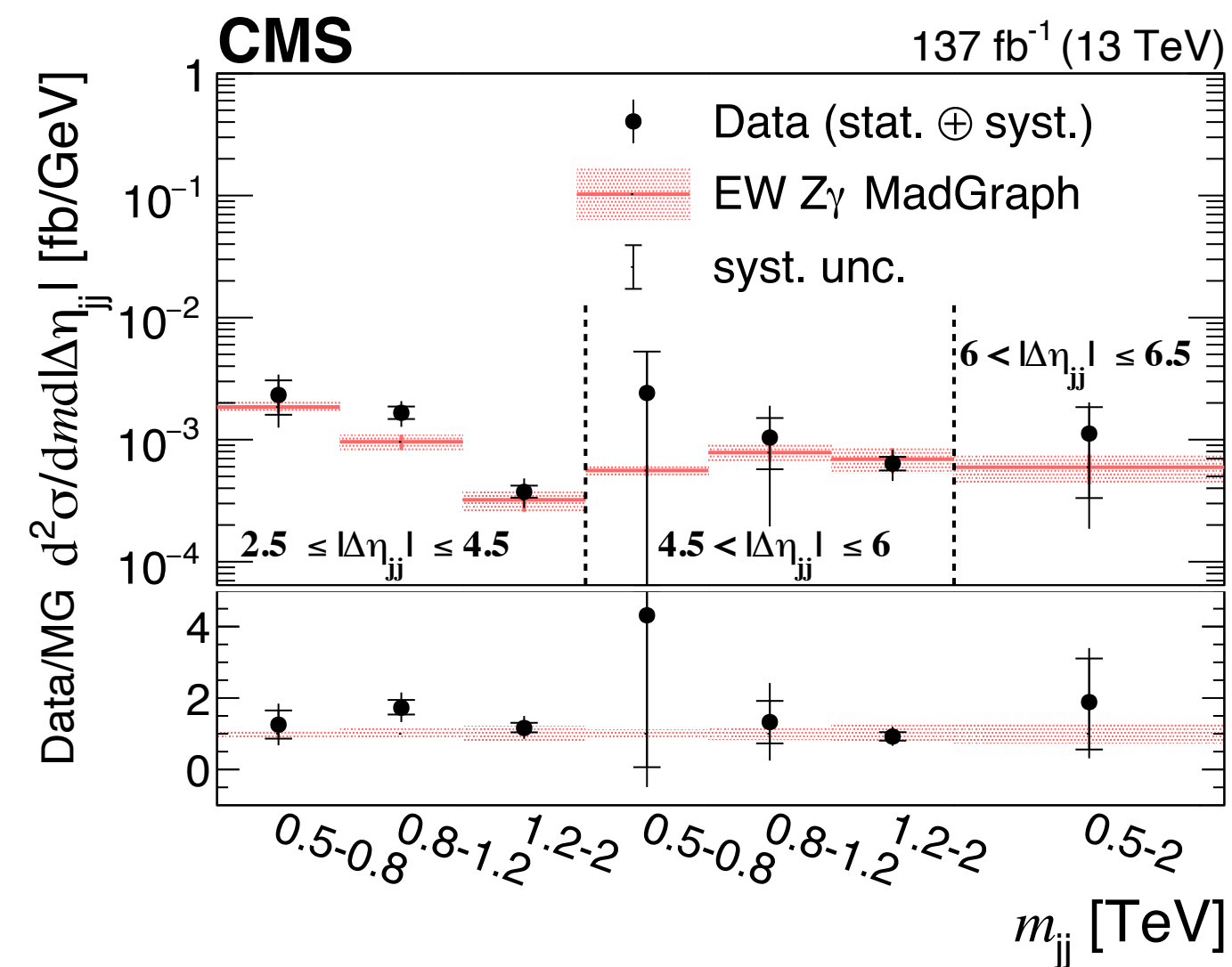
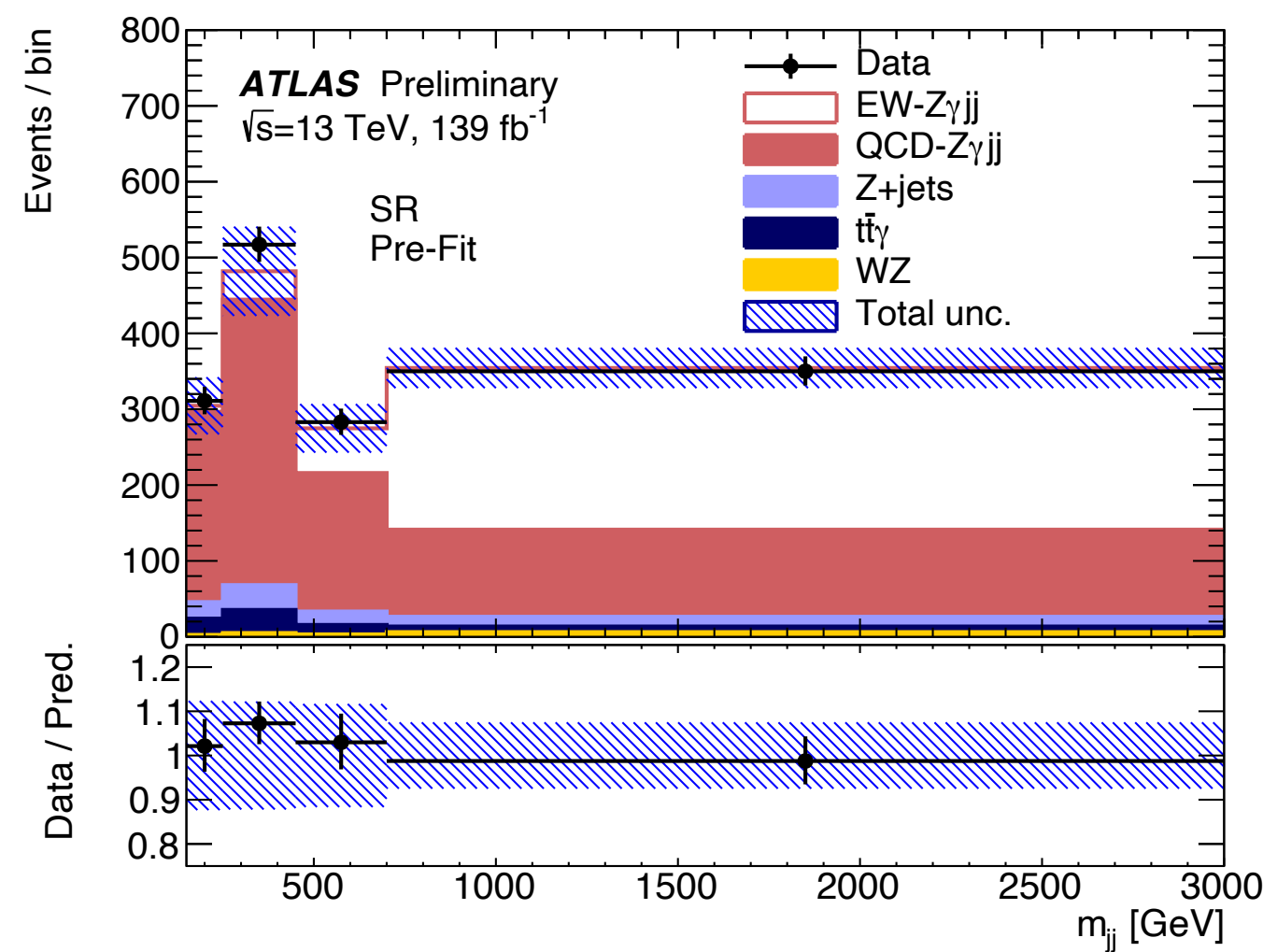
**Opposite-sign  $W^+W^-jj$  production observed by CMS with Run 2**

- Significance  $5.6\sigma$  ( $5.2$  exp.)



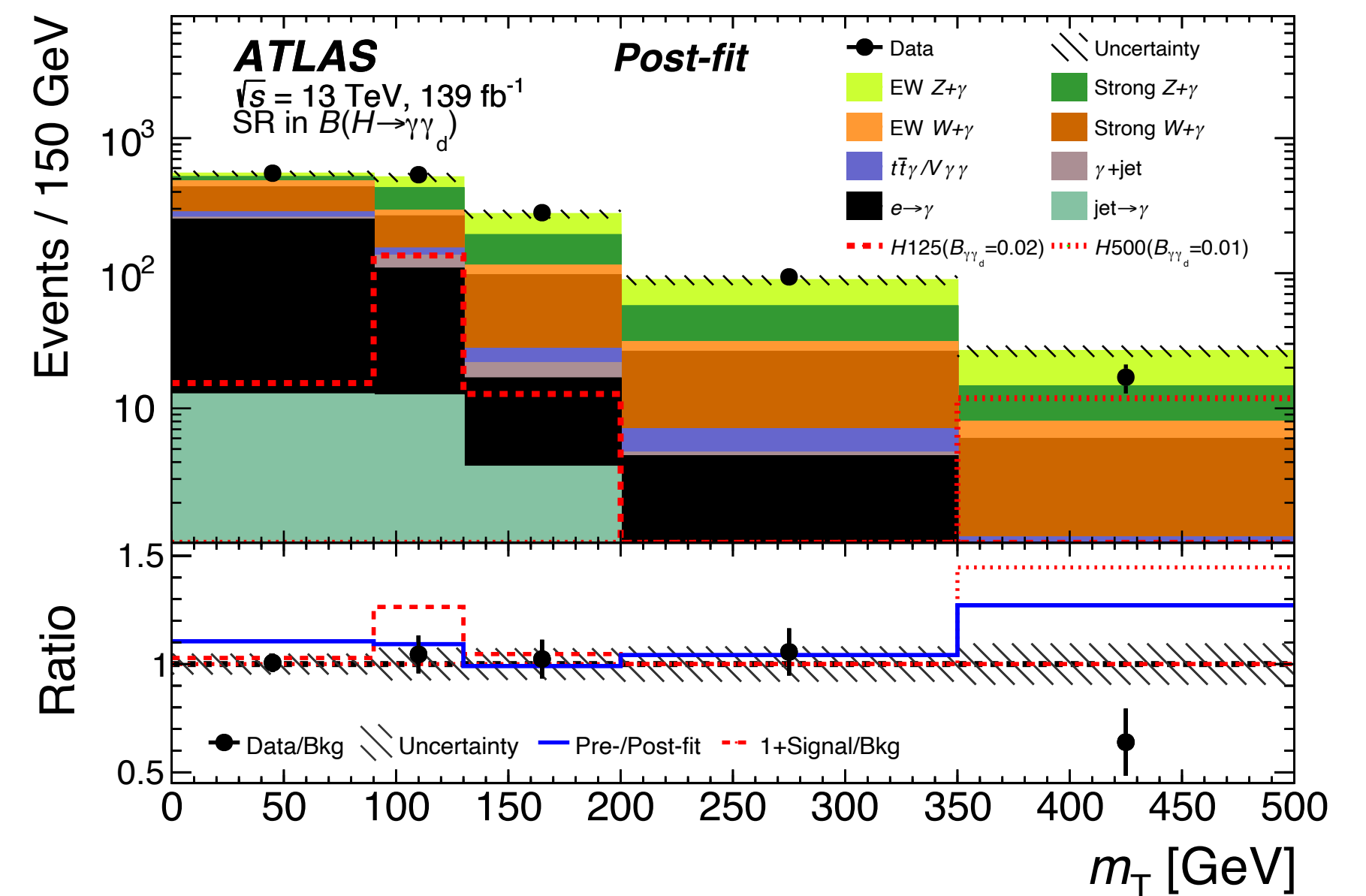
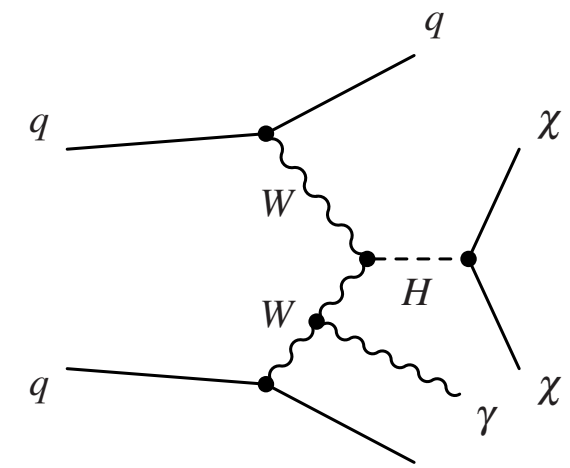
## ATLAS/CMS cross section measurement of EW production $Z(\rightarrow ll)\gamma$

- Single and double differential distribution measured ( $m_{jj}$ ,  $\Delta\eta_{jj}$ )
- Most stringent limit to date on the aQGC parameters  $F_{T9}/\Lambda^4$  by CMS



## First ATLAS observation of EW production of $Z(\rightarrow \nu\nu)\gamma$

- Limits on branching ratio of Higgs to invisible particles
- Best limits on Higgs branching ratio to  $\gamma\gamma_D$  (dark photon) at  $m_H=125$  GeV



## LHC as a photon-photon collider to test electroweak physics

## Observation of photon-induced $WW$ process in ATLAS

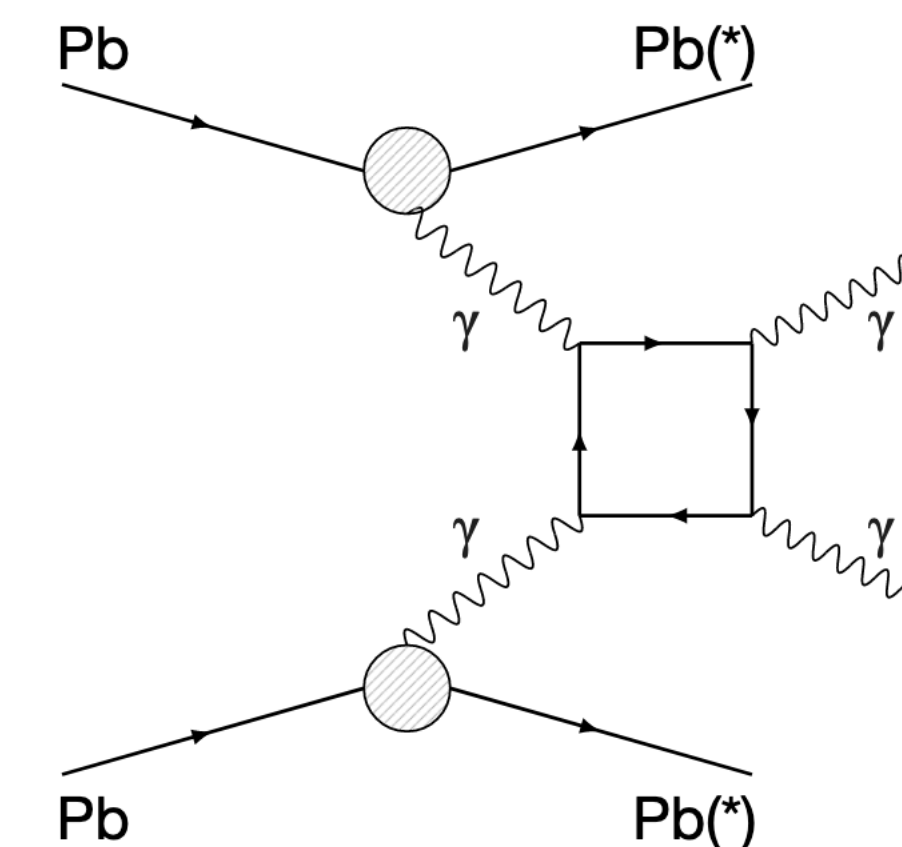
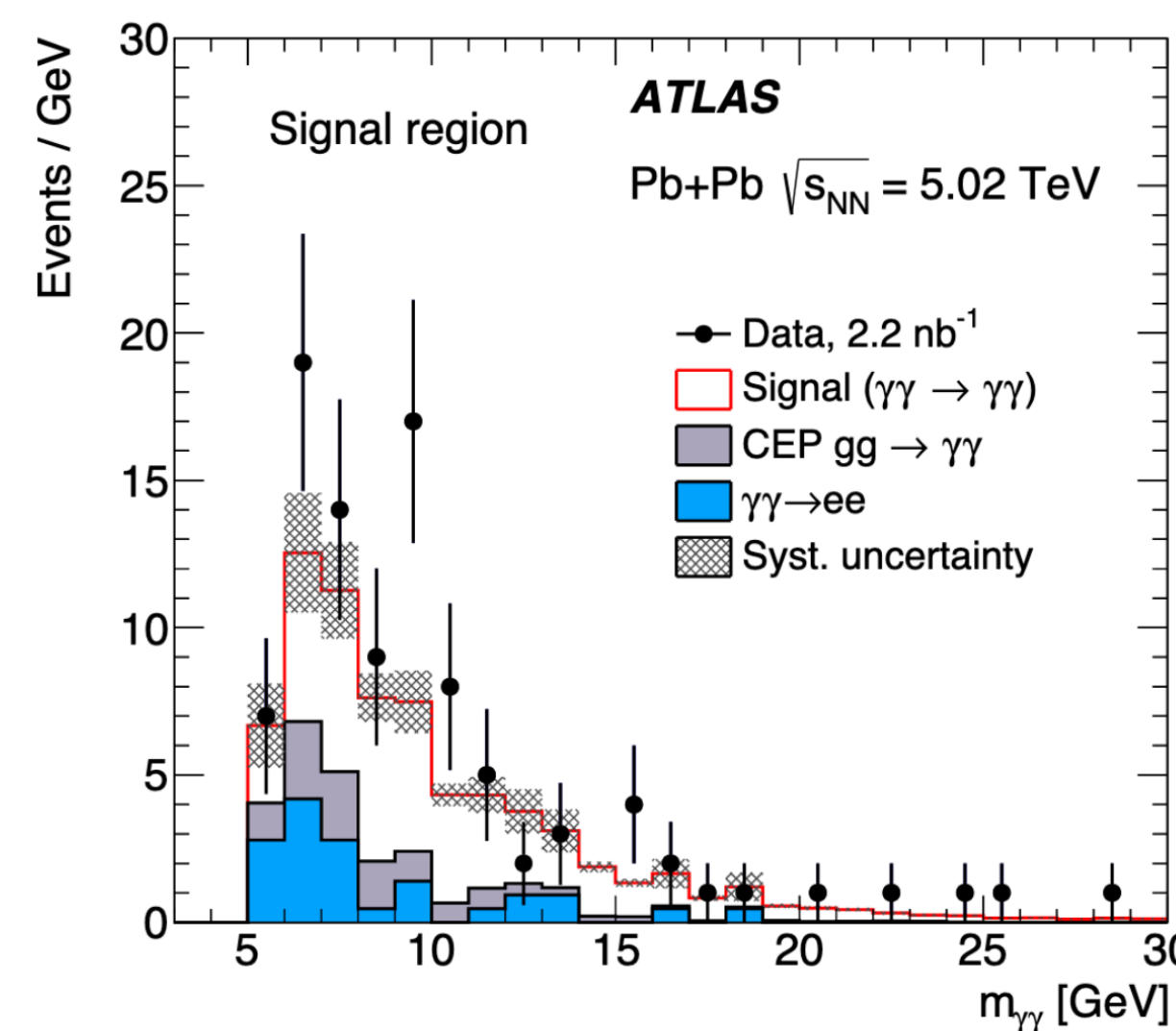
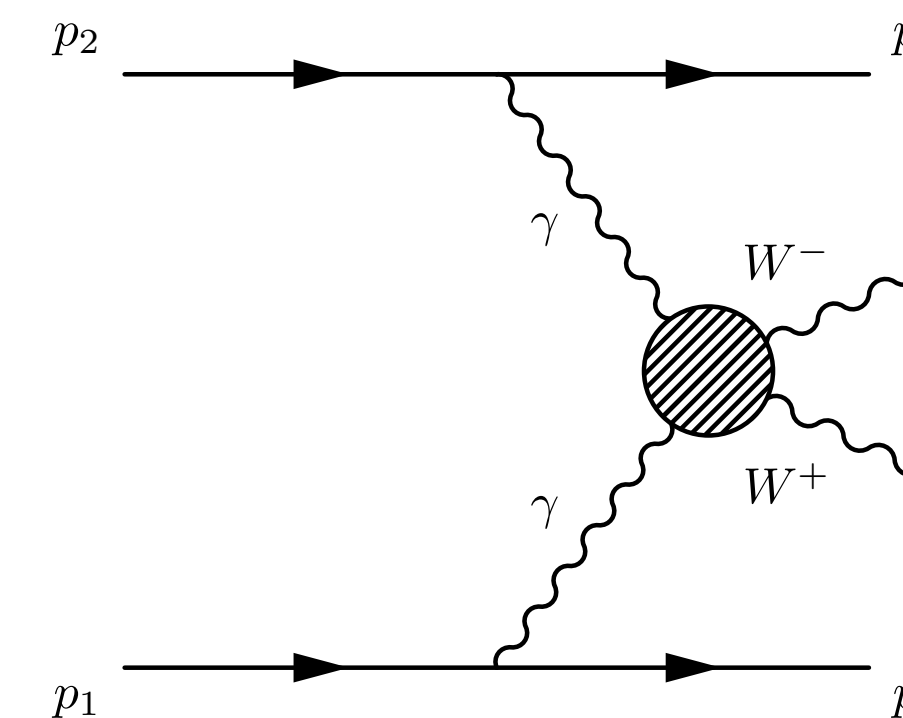
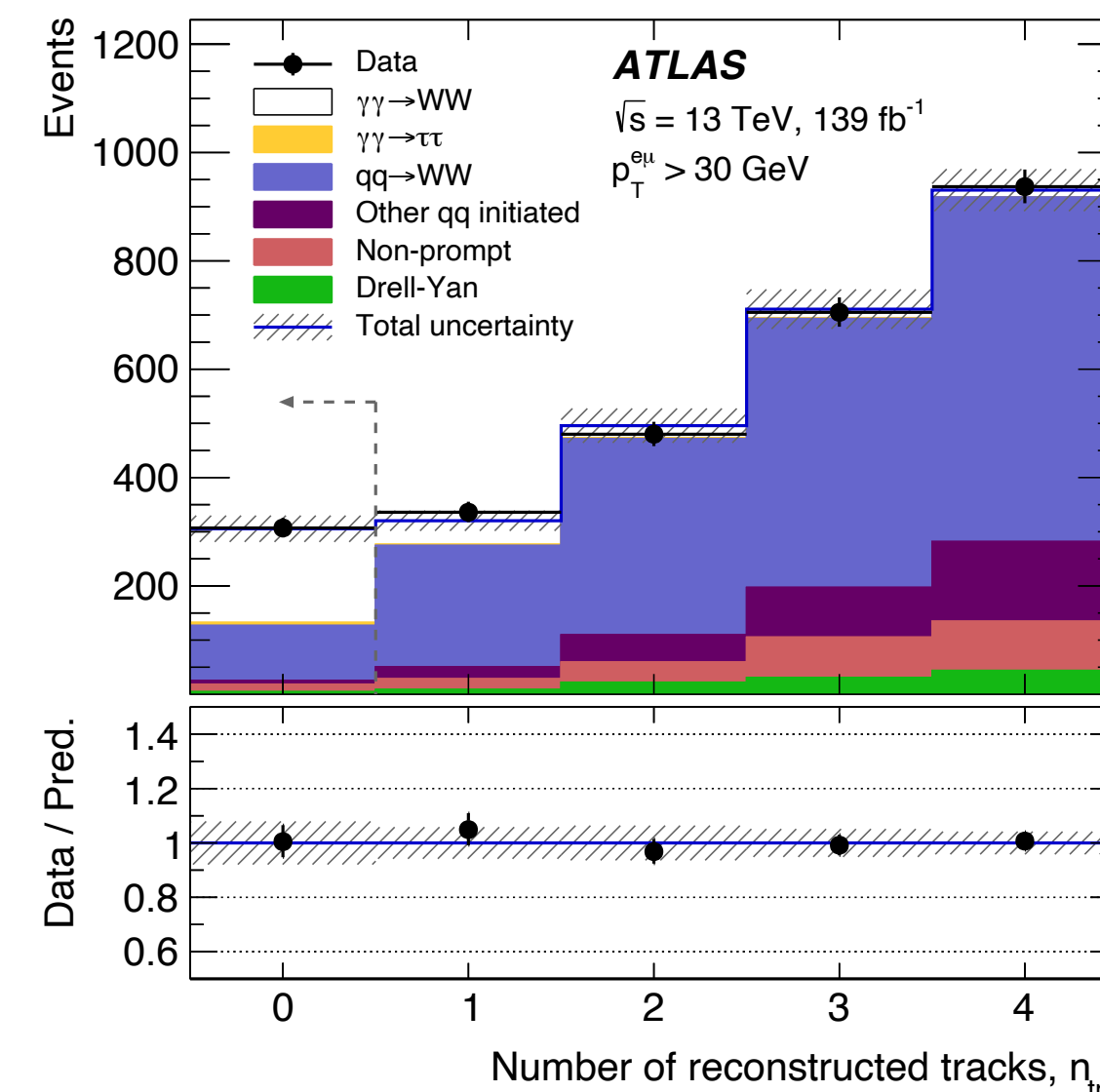
- Signal selected by requirement 0 tracks associated with reconstructed vertex (except selected  $e\mu$ )

## Measurement of light-by-light scattering in PbPb collisions

- Slight excess, consistent within  $2\sigma$  with SM

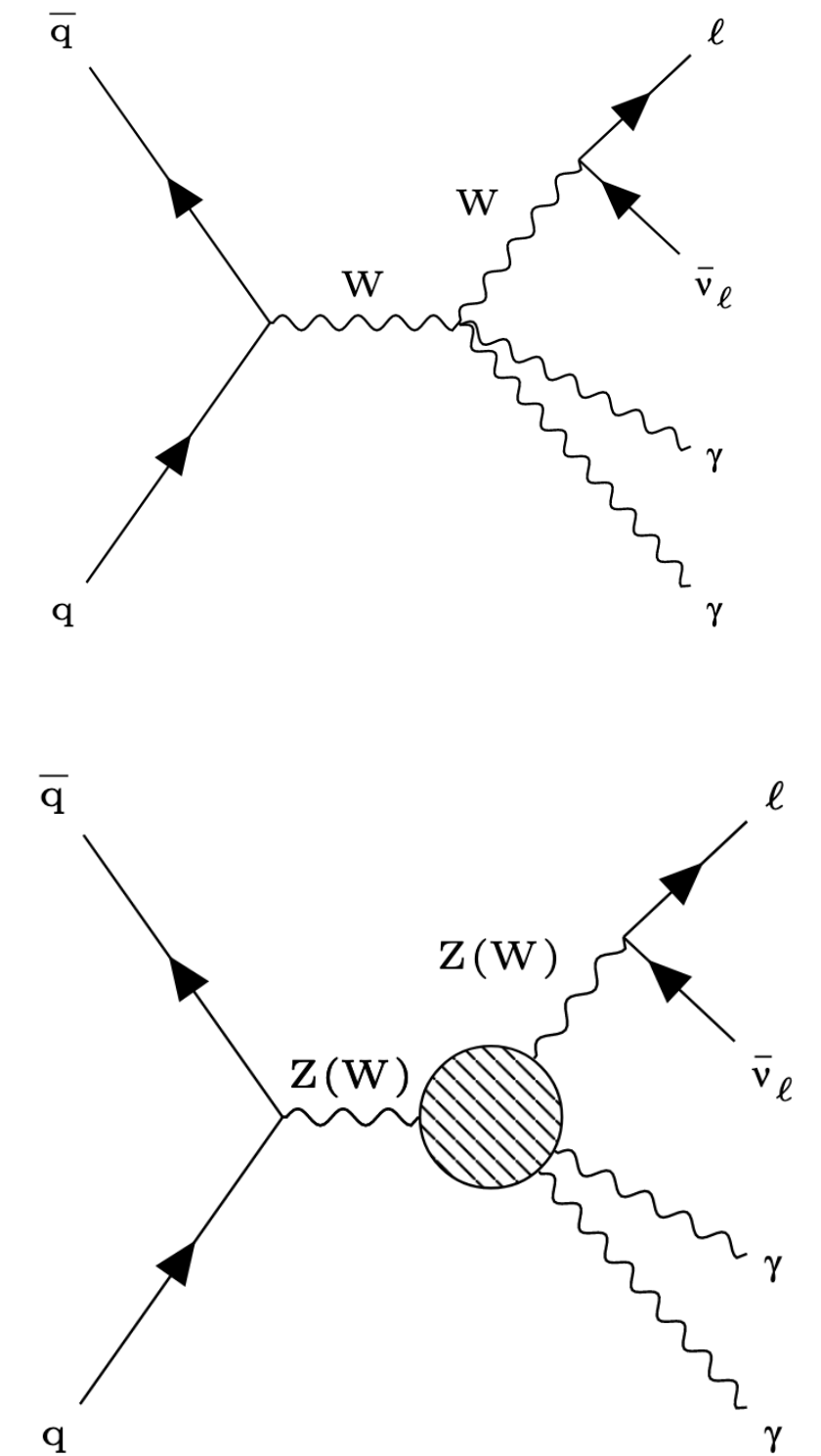
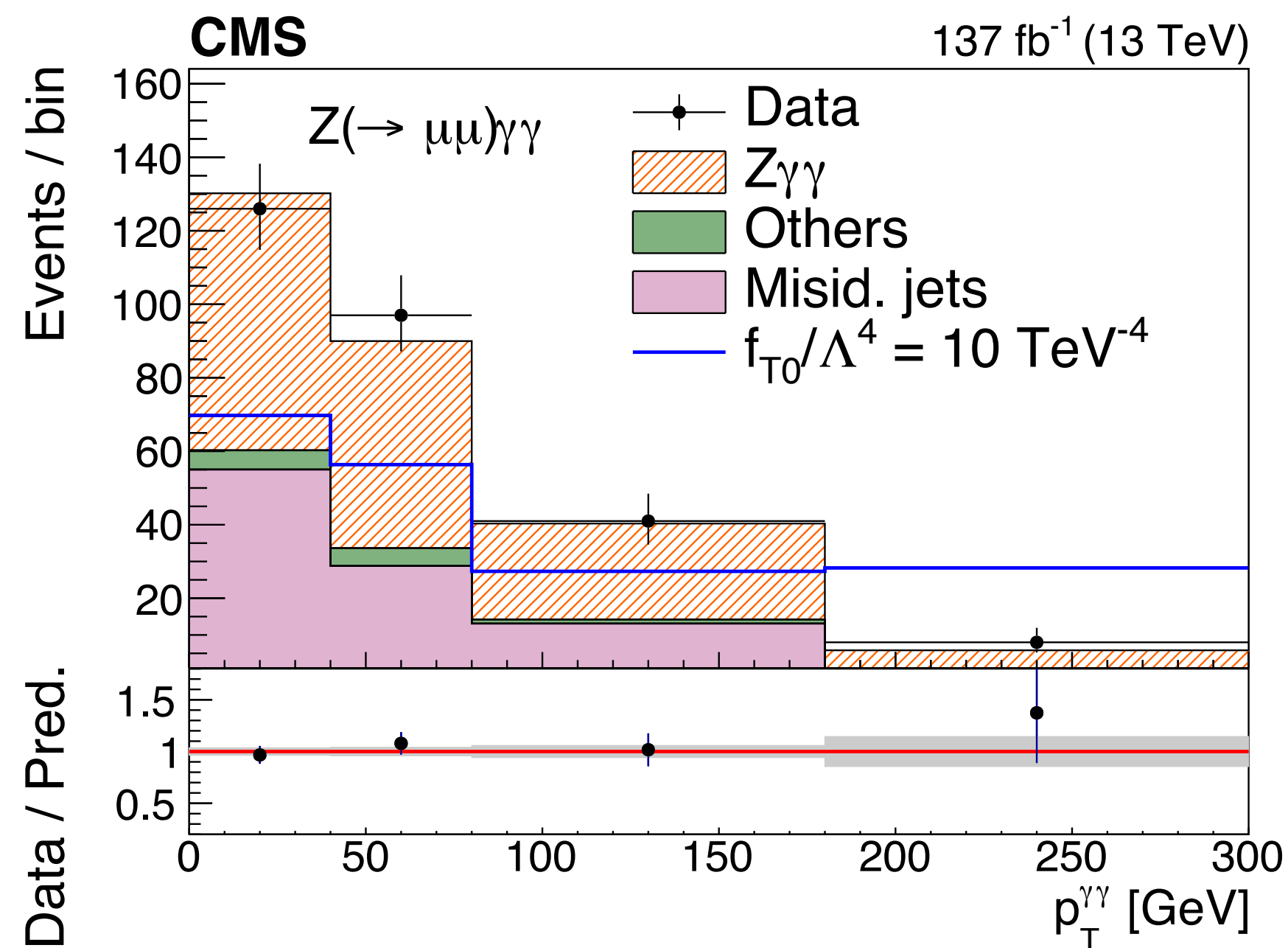
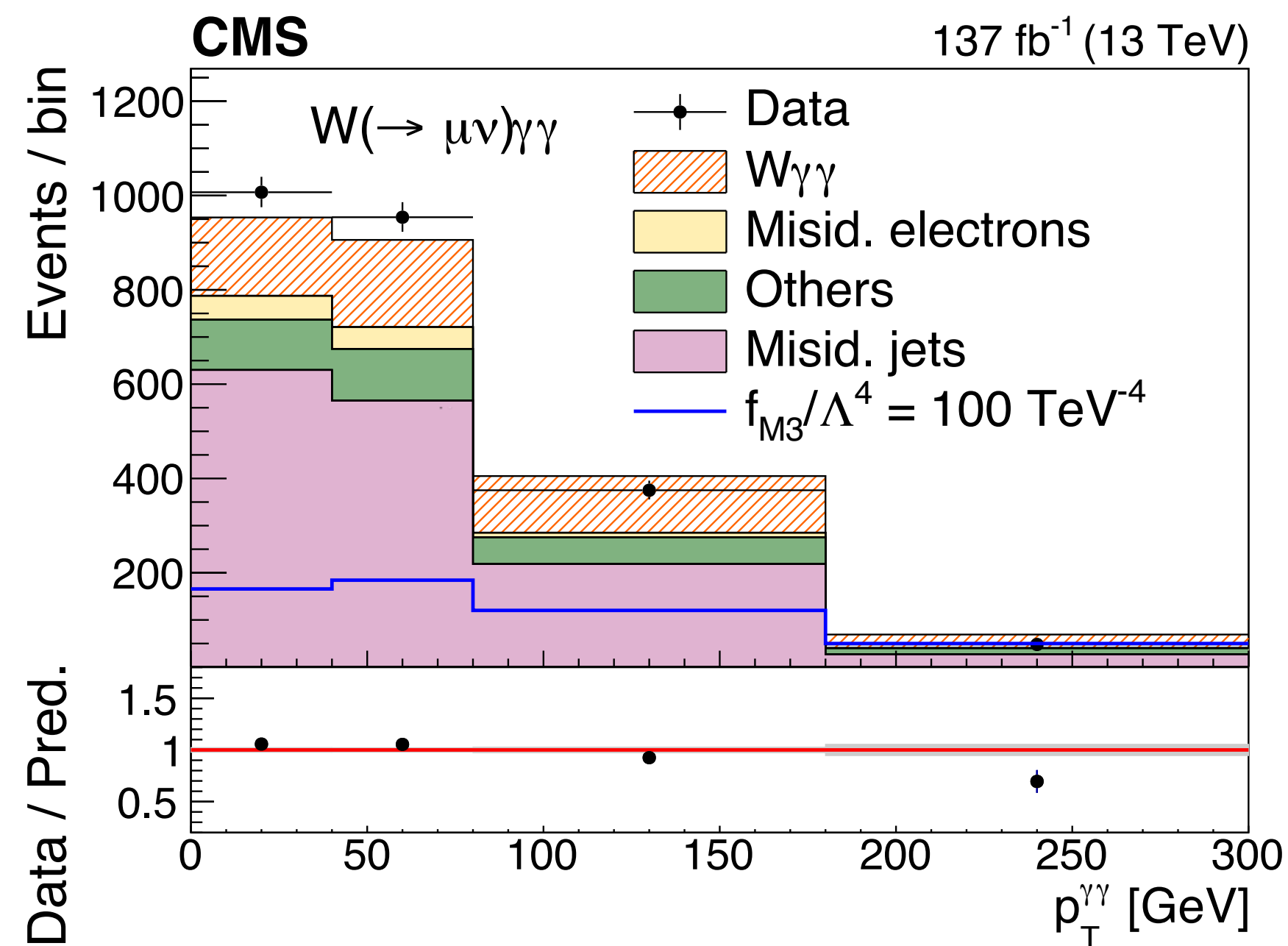
## Forward proton tagging in ATLAS/CMS

- Ongoing effort to use dedicated detectors installed  $\sim 220$  m from the interaction point to detect scattered protons



## New result on V $\gamma\gamma$ by CMS

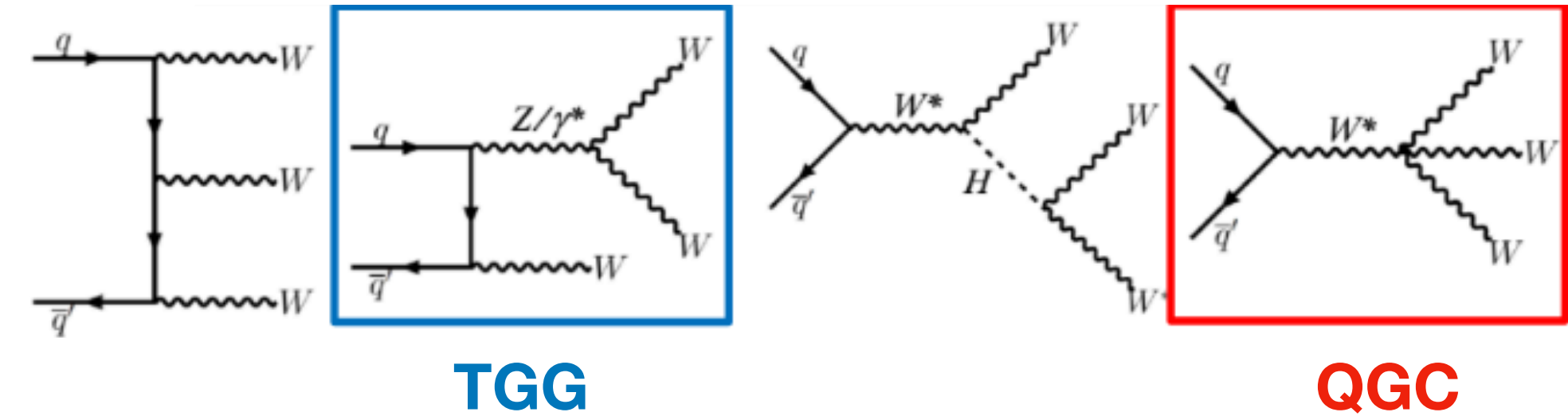
- Previous measurements only at 8 TeV
- Experimental limitations are the photon reconstruction (efficiency) and background (jet faking photons)
- Background only hypothesis reject with  $3.1\sigma$  and  $4.8\sigma$  significance
- Measurement compatible with SM, limits on aQGC





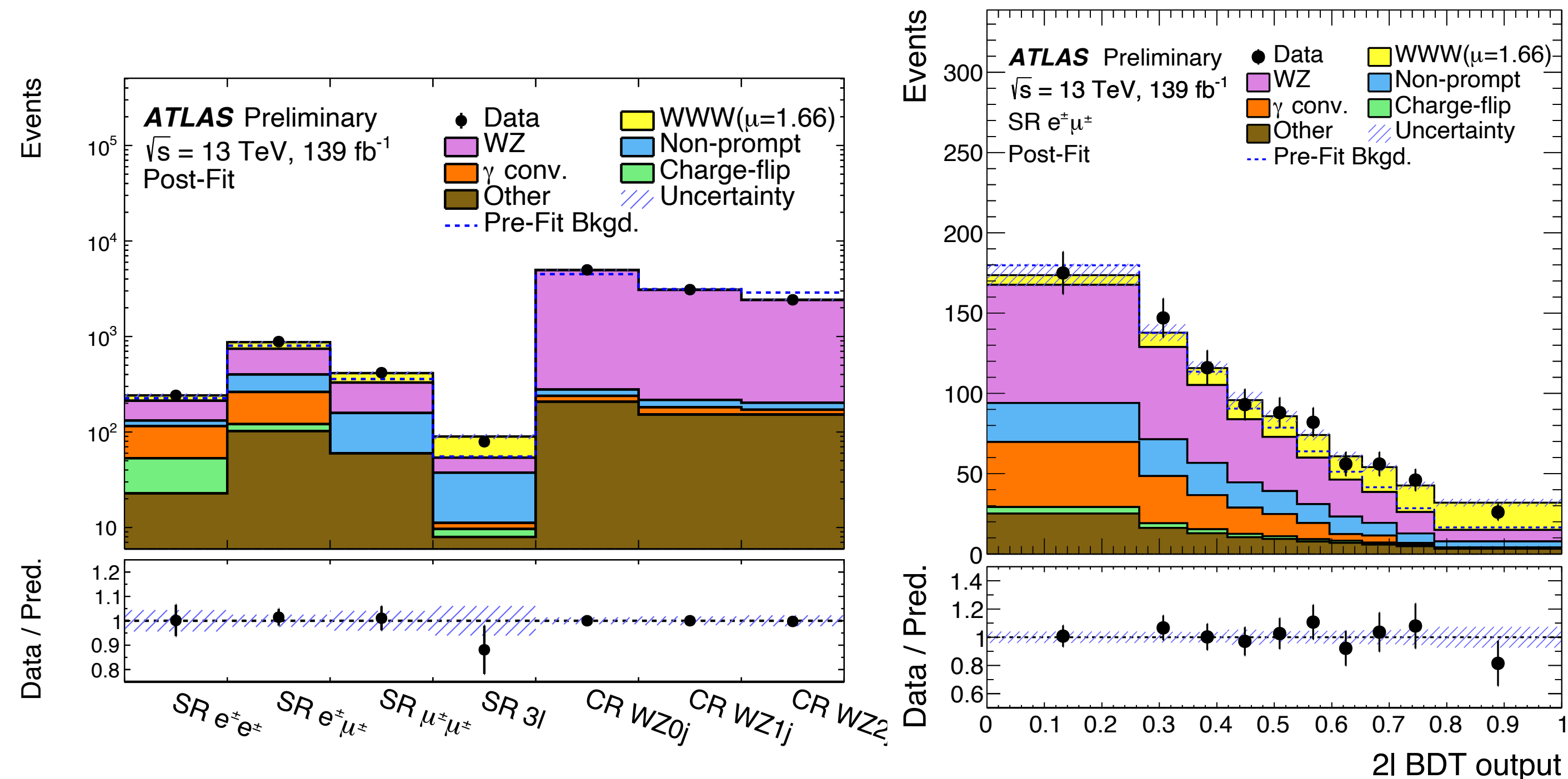
## Observation of three massive gauge bosons at the LHC

- Combination of WWW and  $WH \rightarrow WWW^*$
- Leptonic and semi-leptonic decays - 2 and 3 lepton signal categories, using BDT to enhance sensitivity
- Systematic uncertainties dominated by the non-prompt background and prompt background modelling



## Measured total WWW cross section

- Observed significance  $8.2\sigma$
- Signal strength =  $1.66 \pm 0.28$
- About  $2.4\sigma$  tensions between measured and predicted cross sections



# Summary

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**EW precision parameters with full Run 2 data still to come**

**Diboson cross section all measured and in good agreement with state-of-the art predictions**

- Precision measurements and improvements in the tails

**First observation of three heavy gauge bosons at the LHC**

**Many VBS/VBF processes observed**

- More precise measurement will benefit from Run 3 dataset and HL-LHC

**No significant deviation from Standard Model observed**

- Ongoing effort to constrain EFT operators, ensure publishing data in a format to allow combination and re-interpretation

# Backup

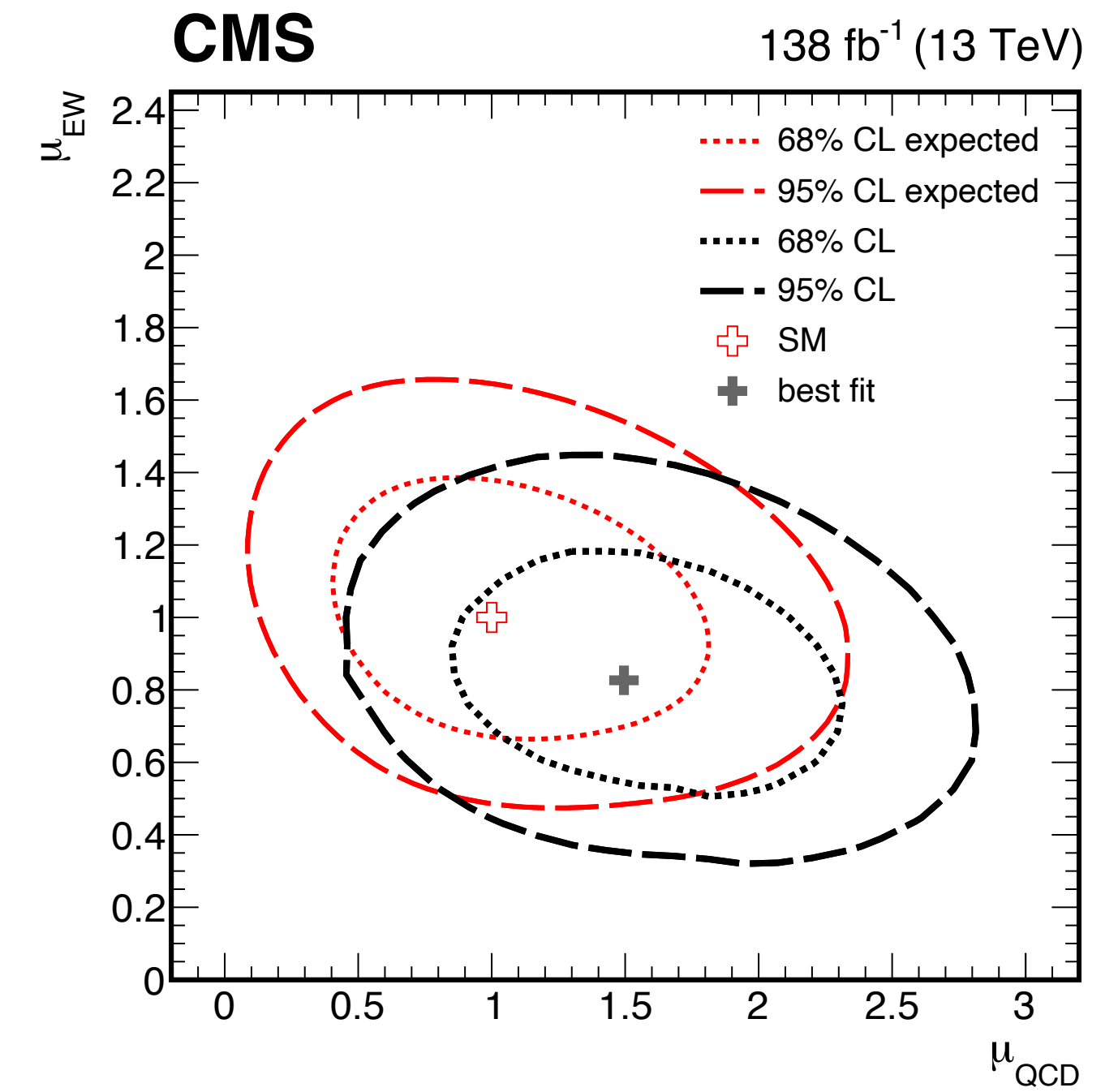
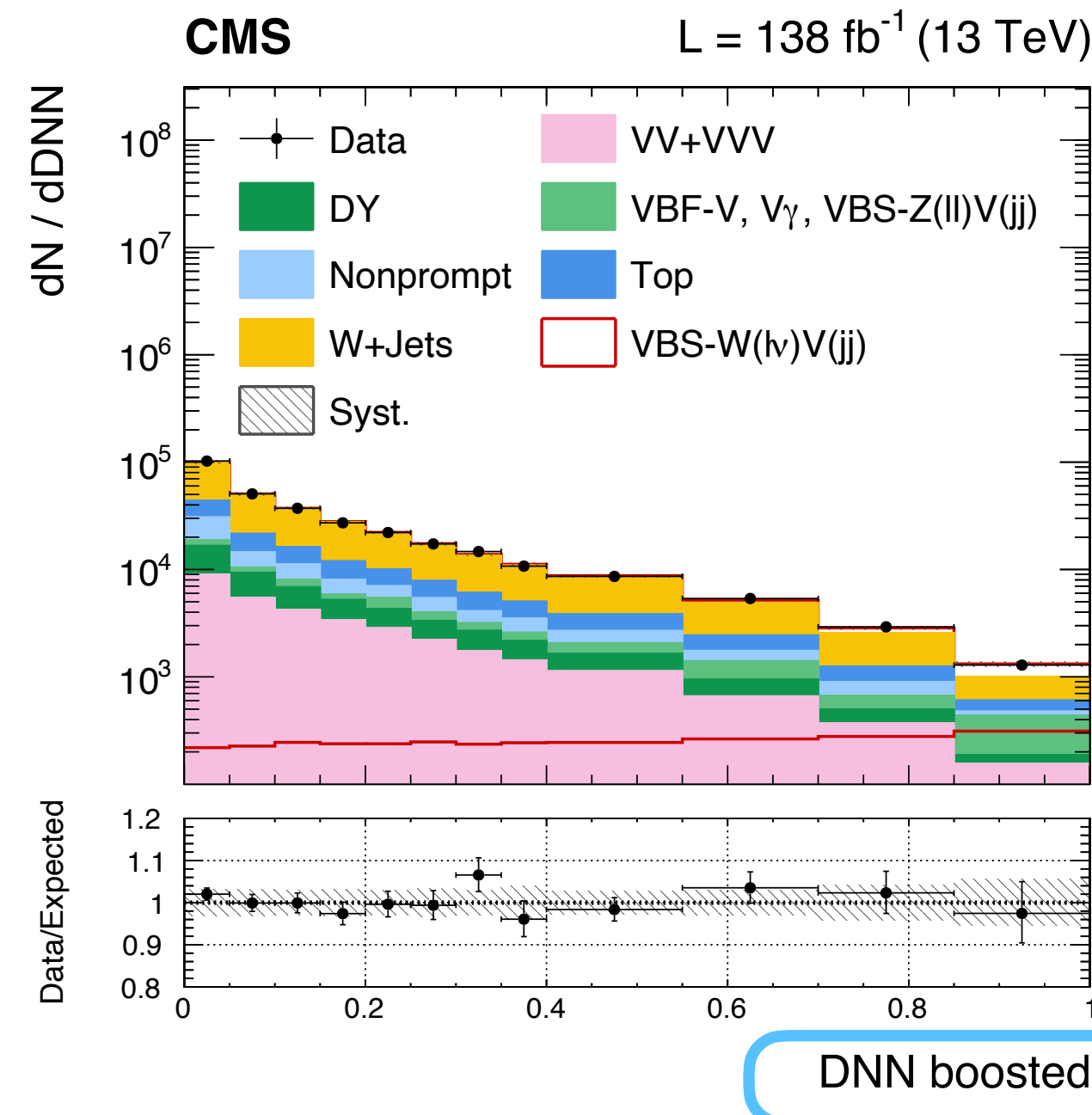
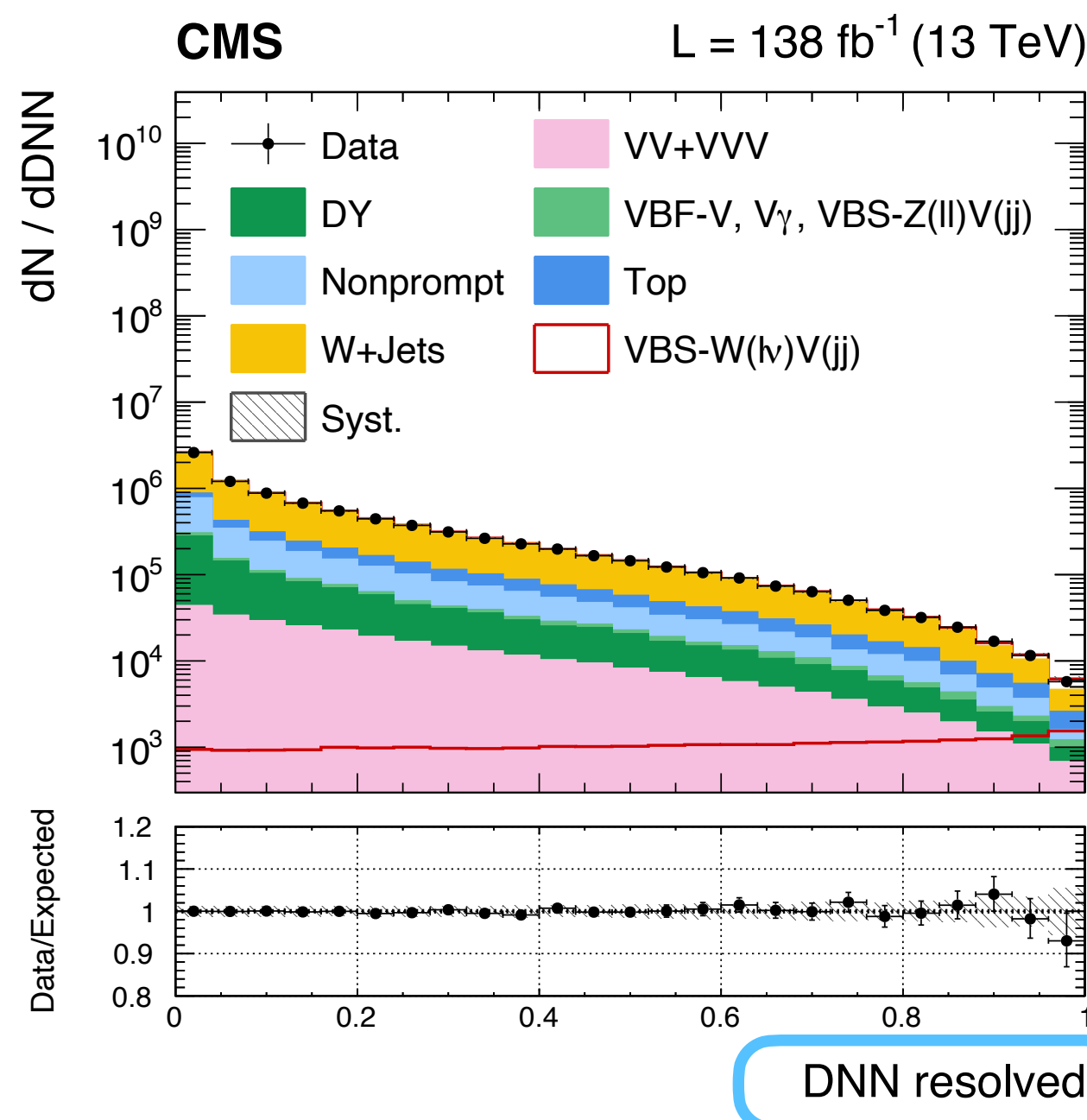
# Search for semi-leptonic VBS WW

arXiv:2112.0525

## Measurement in semi-leptonic WW decays, $WV(\rightarrow lvjj)jj$

- Large backgrounds from W+jet and top quark
- Separated using DNN classifier, resolved and boosted categories
- Background-only hypothesis rejected with  $4.4\sigma$  ( $5.1\sigma$  expected)

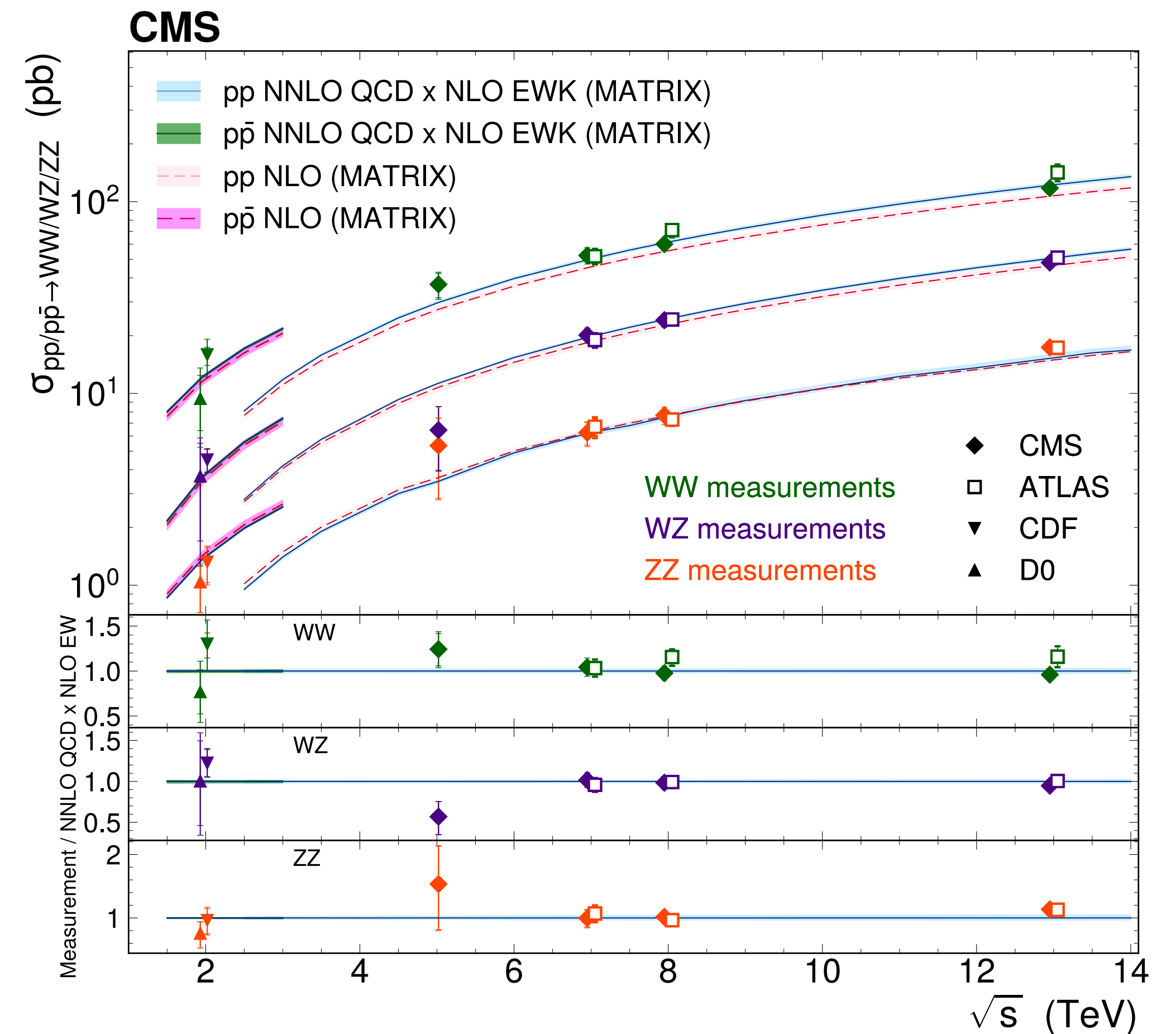
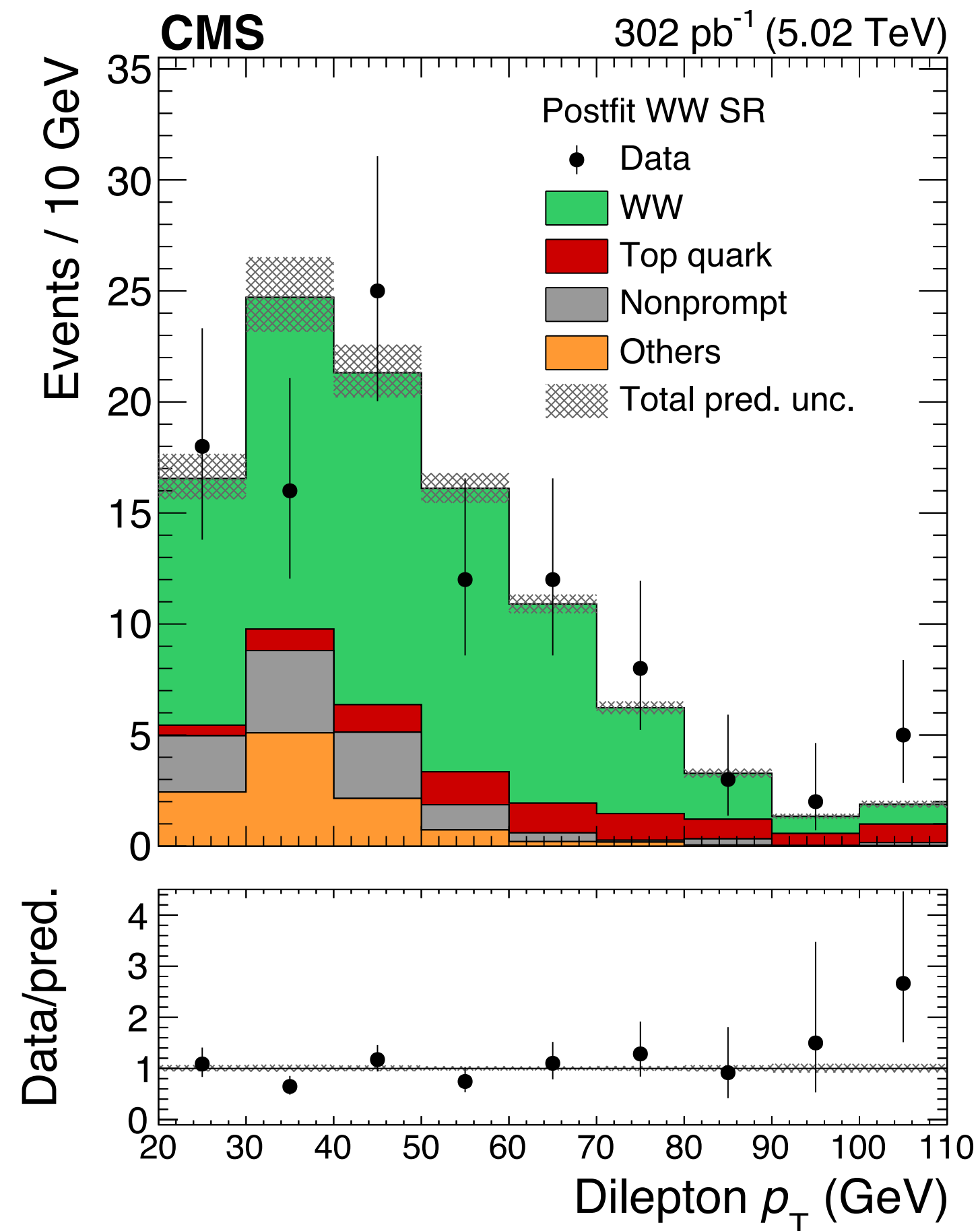
Previous results using  $35 \text{ fb}^{-1}$  data  
by ATLAS and CMS



# Dibosons at lower energy

## CMS measured WW, WZ, ZZ cross section at 5.02 TeV

- Good agreement seen for  $\sqrt{s}$  evolution, smaller statistical precision



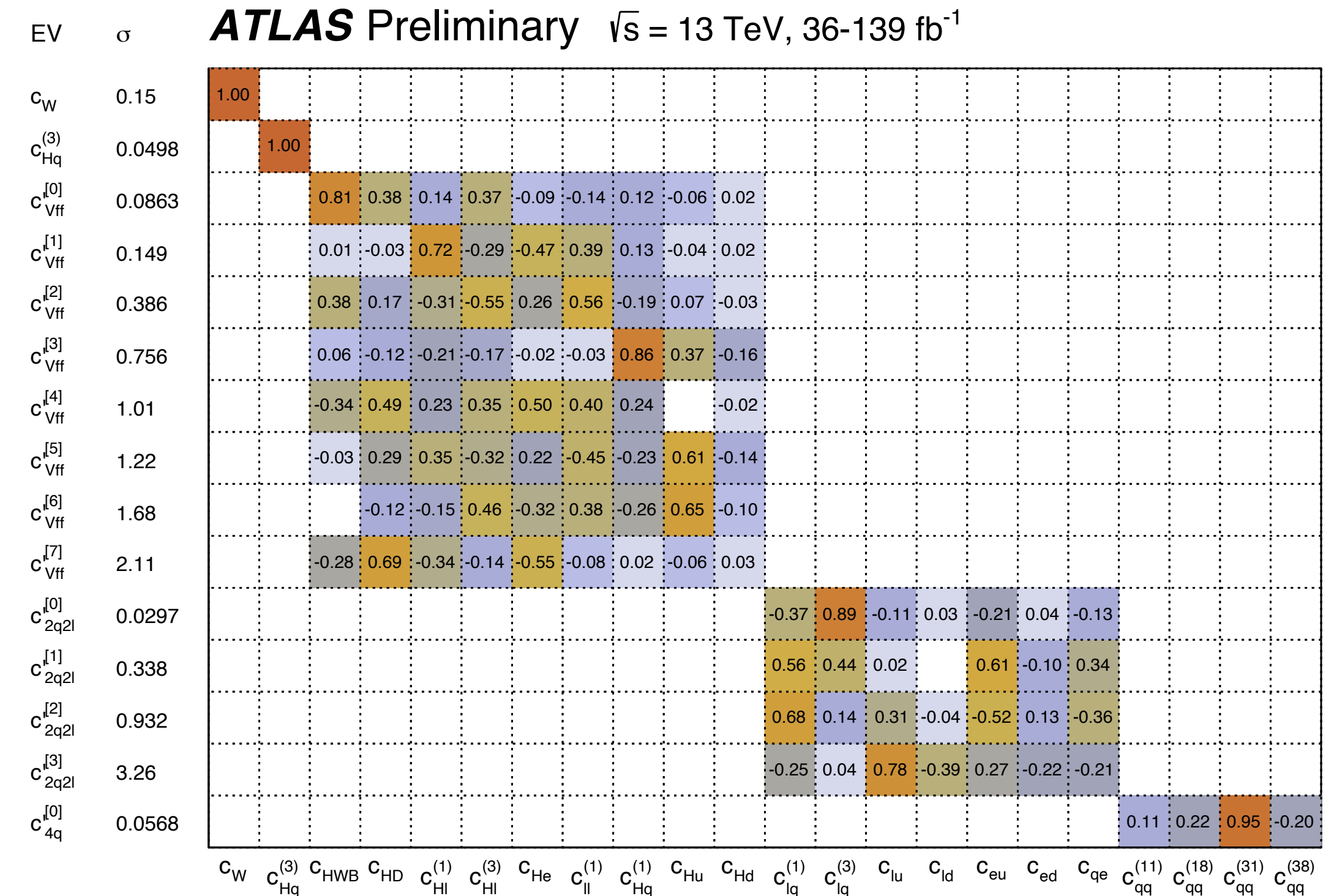
# Global SMEFT combination

ATL-PHYS-PUB-2021-022

- Simultaneous constrains of multiple Wilson coefficients achieved by complementarity of WW, WZ, 4l, EW Zjj measurements
  - Limited information prevents constraining all coefficients
  - Sensitivity directions determined using eigenvalue decomposition of the covariant matrix in space of Wilson coefficients

$$\sigma = \frac{1}{\sqrt{\lambda}} \quad \text{Expected uncertainty of eigenvector}$$

- Limits are set on linear combination of Wilson coefficients



# Global SMEFT combination

ATL-PHYS-PUB-2021-022

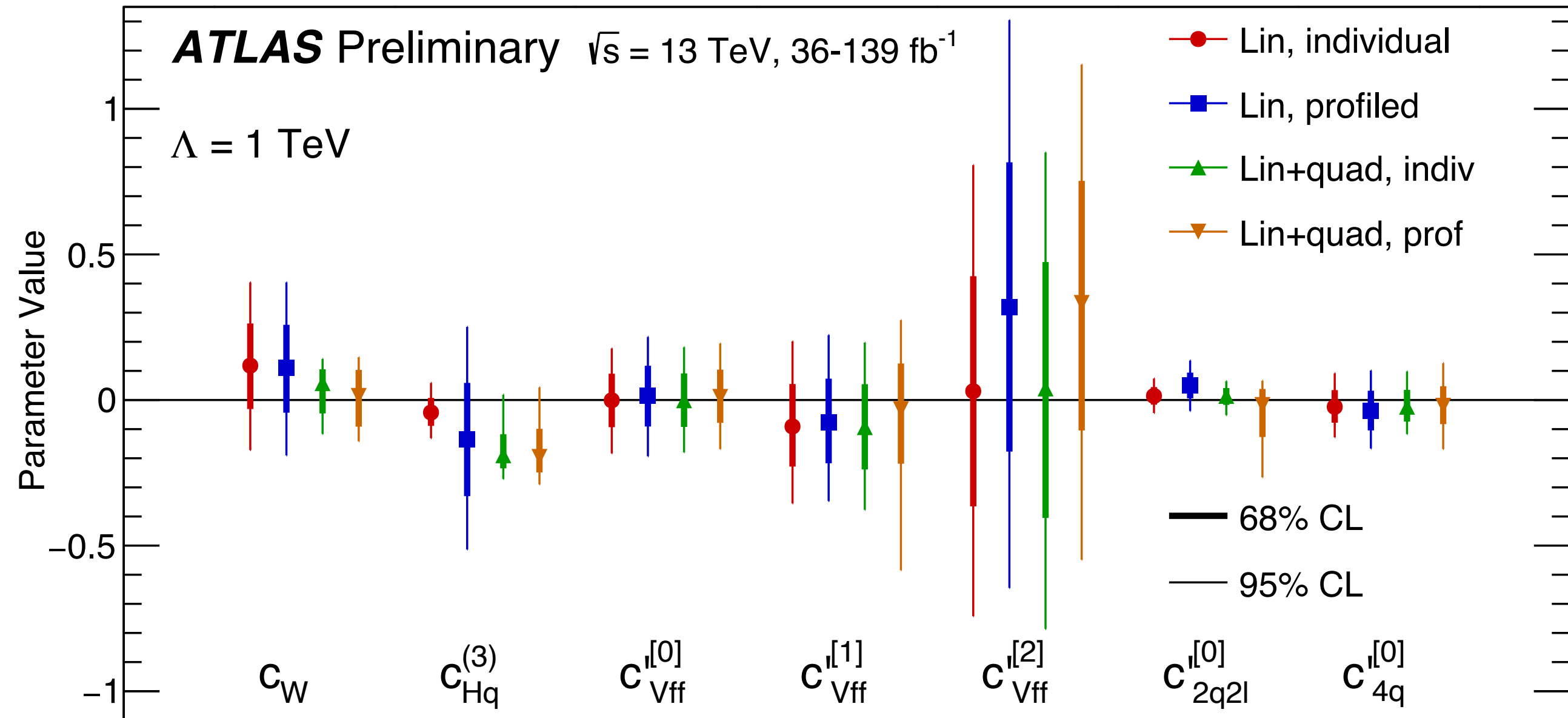
$$\sigma \propto |\mathcal{M}_{\text{SMEFT}}|^2$$

$$= |\mathcal{M}_{\text{SM}}|^2 + \sum_i \frac{c_i^{(6)}}{\Lambda^2} 2\text{Re}(\mathcal{M}_i^{(6)} \mathcal{M}_{\text{SM}}^*) + \sum_i \frac{(c_i^{(6)})^2}{\Lambda^4} |\mathcal{M}_i^{(6)}|^2 + \sum_{i < j} \frac{c_i^{(6)} c_j^{(6)}}{\Lambda^4} 2\text{Re}(\mathcal{M}_i^{(6)} \mathcal{M}_j^{(6)*})$$

linear

quadratic

cross



$$c_{Vff}^{ll[0]} \approx 0.81c_{HWB} + 0.38c_{HD} + 0.13c_{HI}^{(1)} + 0.37c_{HI}^{(3)} - 0.14c_{ll}^{(1)} + 0.12c_{Hq}^{(1)}$$

$$c_{2q2l}^{ll[0]} \approx -0.37c_{lq}^{(1)} + 0.89c_{lq}^{(3)} - 0.11c_{lu} - 0.21c_{eu} - 0.13c_{qe}$$

$$c_{Vff}^{ll[1]} \approx 0.73c_{HI}^{(1)} - 0.28c_{HI}^{(3)} - 0.48c_{He} + 0.38c_{ll}^{(1)} + 0.13c_{Hq}^{(1)}$$

$$c_{4q}^{ll[0]} \approx 0.11c_{qq}^{(11)} + 0.22c_{qq}^{(18)} + 0.95c_{qq}^{(31)} - 0.2c_{qq}^{(38)}$$

$$c_{Vff}^{ll[2]} \approx 0.37c_{HWB} + 0.17c_{HD} - 0.31c_{HI}^{(1)} - 0.53c_{HI}^{(3)} + 0.25c_{He} + 0.59c_{ll}^{(1)} - 0.21c_{Hq}^{(1)}$$

